School choice and segregation*

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Abstract

Segregation in schools is prevalent in many cities around the world. In this paper, we analyze the impact on segregation and efficiency of affirmative action policies in school choice programs. In a large market model, we show that minority reserves–that guarantee a number of seats to minority students–are an effective tool to reduce segregation in schools. More subtle, minority reserves increase the number of students assigned to their first preferences and improve efficiency. The main cost of increasing minority reserves is leaving more students unassigned. Each of these predictions from the stylized model is confirmed by field evidence from school choice programs in the largest urban centers in Chile. In our data, minority reserves can reduce the Duncan segregation index in more than 20% and improve the efficiency of the system.

1 Introduction

Recent school choice programs around the world use centralized procedures to assign students to schools (Boston, NYC, Amsterdam, Chile, Paris). Based on the celebrated Gale-Shapley deferred acceptance algorithm (Gale and Shapley 1962), these programs result in assignment processes that are considered successful by both scholars and policy makers. Yet, our understanding of the impact of alternative design decisions on segregation and other market outcomes is rather limited. The main goal of this paper is to uncover some of the tradeoffs that market designers and policy makers face when trying to reduce segregation in schools by using affirmative action policies.

We provide theoretical results and field evidence from several Chilean cities on the impact of affirmative action policies on segregation and efficiency in school choice programs. Our theoretical results are

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derived in a large market model (Abdulkadiroglu, Che, and Yasuda 2015, Azevedo and Leshno 2016). In the model, a continuum of students apply to a finite number of schools. A student is either a regular or a minority student. Schools are in two tiers, 1 and 2. Each school ranks students randomly. In any given school, a minority student is weakly more likely than a regular student to have a low ranking in a given school. We thus allow schools to rank all students uniformly. Tier 1 schools are popular and over-demanded, while tier 2 schools are unpopular and under-demanded. Importantly, we assume that regular students apply more intensely to tier 1 schools than minority students. In the absence of reserves, regular students are over-represented in tier 1 schools while minority students get mostly assigned to the low popularity tier 2 schools.

A key assumption in our model is that minority students are less likely to apply to high demand schools than regular students. This assumption is motivated by evidence from several cities. Laverde (2020) shows that Black and Hispanic families are less likely than other groups to rank popular, high-achievement schools in the Boston Public Schools program. We also provide evidence that socially disadvantaged minority students are much less likely to list popular schools than regular students applying in the main Chilean cities. Our assumption implies that even when school choice programs may treat all students equally by using uniform random lotteries to rank applicants, that may not be enough to integrate schools due to differences in application patterns.

We explore the impact of minority reserves on several outcomes. Minority reserves guarantee a given number of seats to minority students whenever minority students demand a school, but otherwise respect each school ranking (Hafalir, Yenmez, and Yildirim 2013, Ehlers, Hafalir, Yenmez, and Yildirim 2014, Echenique and Yenmez 2015). Our theoretical results describe the impact of minority reserves on segregation, the rank distributions of the assignment, and efficiency.

Increasing minority reserves reduces segregation, unless too many seats are reserved for minority students. Minority reserves also impact the rank distribution of assignments for both groups. Minority students are favored by the introduction of reserves as they face less competition for some seats. Thus, minority reserves favor minority students and decrease segregation in schools.

Increasing minority reserves may both leave more students assigned to their top schools and reduce the inefficiencies of the assignment. Two observations are useful to understand these results. First, increasing minority reserves replaces regular students by minorities in popular tier 1 schools. Second, a minority student applying to a tier 1 school is more likely than a regular student to list the school as her top choice. Increasing minority reserves also reduces the number of seats available to regular students and triggers system-wide effects that need to be taken care of. We thus show that, unless the number of seats in popular schools is relatively large, the assignment under a higher minority reserve has more students that obtain their top choices.[1] We measure the inefficiency of the assignment by the Pareto improving pairs, which is the number of applicants that are better off by exchanging seats. Our results show that increasing minority reserves improves the efficiency of the system by reducing the number of

[1]The system-wide effects of an increase in minority reserves explain why we need an upper bound on the number of seats to derive this comparative statics results. We discuss all these effects in detail right after Proposition 2.

Pareto improving pairs.

The analysis also exposes the costs of reducing segregation in schools. We show that increasing minority reserves leaves more students assigned to less attractive schools. Formally, we prove that as minority reserves increase, the cumulative rank distributions cross once and, as a result, cannot be compared in the first order stochastic dominance sense. In particular, increasing the total number of students assigned to unattractive schools is an important consequence of a rise in minority reserves.

Our results uncover the impact that changes to minority reserves have on several market outcomes. While our matching model is stylized and abstracts away from several features, each of our main theoretical findings is confirmed by field data from the assignment processes in Chilean cities.

We use applications to pre-Kinder from the three largest urban centers in Chile. The Ministry of Education in Chile uses a version of the Gale-Shapley deferred acceptance algorithm with multiple lotteries (Abdulkadiroğlu, Pathak, and Roth 2009). By law, 15% of the seats in each school are reserved for socially disadvantaged minority students. We show that minority students in Chilean cities are much less likely to apply to high demand schools and simulate the assignment for different values of the minority reserves.

The simulations show that minority reserves impact several market outcomes. Segregation is U-shaped and an increase in minority reserves reduces the Duncan segregation index in more than 20% in each of the three cities (Duncan and Duncan 1955). Raising minority reserves reduces the number of students in Pareto improving pairs, and also increases the number of students assigned to their top schools and the students assigned to unattractive schools (or unassigned). The simulations thus show that, as a policy decision, minority reserves are quantitatively important. We also explore the role of alternative design decisions, including double and set aside reserves (Echenique and Yenmez 2015, Dur, Kominers, Pathak, and Sönmez 2018). These exercises illustrate that minority reserves are a key design decision in our application.

Segregation in schools is pervasive in cities around the world. How segregated schools are impacts both learning outcomes and social attitudes (Karsten 2010, Rao 2019). Centralized school choice programs are often seen as providing equal access to schools.[2] Yet, recent research shows that systematic differences in the application patterns of different groups may limit the efficacy of school choice programs at reducing social, ethnic, or racial segregation in schools. Laverde (2020) shows that in some dimensions the outcome of the school choice program in Boston is similar to the outcome generated by an assignment based on proximity between residences and schools. Kutscher, Nath, and Urzua (2020) show that the introduction of school choice in Chile has had an extremely limited impact on segregation in schools. We build from the main premise of these papers –that school choice alone may not be enough to integrate schools– and explore how affirmative action policies in school choice programs impact several market outcomes, including segregation in schools.

[2]For example, the law that introduced the school choice program in Chile in 2016 is named the *Inclusion Law* (or Ley de Inclusión Escolar). As Laverde (2020) notes, the Boston Public Schools' superintendent wrote "My overall goal is to create a student assignment plan that provides all Boston students with high-quality desegregated education".

Our results are relevant for policy discussion. The number of students assigned to their top schools is an important quantity usually considered by policy makers. Abdulkadiroğlu, Pathak, and Roth (2009) discuss how the greater number of students assigned to their top choices was key to favor single over multiple-tie breaking in the New York city school match.[3] When discussing the Boston school choice experience, Abdulkadiroglu, Pathak, Roth, and Sönmez (2006) argue that "the ability to tell the public that a high proportion of students receive their top choices may be a reason for the widespread popularity of the Boston mechanism." The number of Pareto improving pairs in the assignment is an (in-)efficiency measure authorities look at. In Amsterdam, in order to reduce the number of Pareto improving pairs, the deferred acceptance algorithm with single-tie breaking replaced the deferred acceptance algorithm with multiple-tie breaking in 2016 (Ashlagi and Nikzad 2016, De Haan, Gautier, Oosterbeek, and Van der Klaauw 2015).[4] Thus our results are of interest to policy makers, who at the same time may combat segregation in schools, increase the number families obtaining their top choices, reduce the number of applicants in Pareto improving pairs, but incur the costs of leaving more students unassigned.

Abdulkadiroğlu and Sönmez (2003) apply matching theory to school choice problems. Recent work has explored several design issues, such as how different tie-breaking rules impact efficiency (Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Ashlagi and Nikzad 2016). Our work explores how another policy decision–minority reserves–impacts both segregation and efficiency. More broadly, the school choice literature has shown that the design of matching mechanisms involves complex tradeoffs. For example, the deferred acceptance algorithm results in a stable but inefficient matching while the top trading cycle algorithm yields an efficient but unstable matching (Gale and Shapley 1962, Roth and Sotomayor 1990, Che and Tercieux 2019). Reducing segregation in schools is another important desideratum and we believe that our results expose new forces that are important to practical implementations of school choice programs.

An important and extensive literature studies matching problems with affirmative action considerations.[5] Throughout the paper, we adapt several definitions and concepts from these works, particularly from Hafalir, Yenmez, and Yildirim (2013), Echenique and Yenmez (2015), and Dur, Kominers, Pathak, and Sönmez (2018). We contribute to this literature by deriving new theoretical results on the impact of affirmative action policies on important but unexplored market outcomes, such as segregation, the rank distributions of assignment, and the number of applicants in Pareto improving pairs. We also provide field evidence that confirms our theoretical predictions.

Finally, the present paper is related to the growing literature using large market models to shed light

[3]As Abdulkadiroğlu, Pathak, and Roth (2009) observe: "The greater number of students obtaining one of their top choices in a similar simulation and in the first year of submitted preference data convinced New York City to employ a single tiebreaker in their assignment system."

[4]See also the discussion in Alvin Roth's blog at https://marketdesigner.blogspot.com/2015/12/amsterdam-school-choice-next-year-will.html

[5]The list of papers exploring affirmative action in school choice includes Abdulkadiroğlu (2005), Kojima (2012), Hafalir, Yenmez, and Yildirim (2013), Ehlers, Hafalir, Yenmez, and Yildirim (2014), Echenique and Yenmez (2015), Kominers and Sönmez (2016), Fragiadakis and Troyan (2017), Dur, Kominers, Pathak, and Sönmez (2018), Nguyen and Vohra (2019), Aygun and Bó (Forthcoming).

on market design issues (Abdulkadiroglu, Che, and Yasuda 2015, Azevedo and Leshno 2016, Ashlagi and Nikzad 2016, Che and Tercieux 2019). The large market assumption in our model allows us to derive clean comparative statics results on the impact of reserves on various market outcomes. To the best of our knowledge, this paper is the first one exploring the impact of affirmative action policies in a large market model.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 introduces minority reserves and provides our main comparative statics results. Section 3 also discusses variations of our model. Section 4 confirms and quantifies our theoretical results using application data from Chilean cities. Section 5 concludes.

2 Model

2.1 Environment

We consider a school choice problem with a continuum of students and a finite number of schools (Abdulkadiroglu, Che, and Yasuda 2015, Azevedo and Leshno 2016). There is a measure 1 of regular students (r), and a measure $\beta \in]0,1]$ of minority students (m). So, a student is characterized by $s = (t, x) \in (\{m\} \times [0, \beta]) \cup (\{r\} \times [0, 1])$. The set of all students is denoted S.

The set of schools is $C = \{1, \dots, n, n+1, \dots, n+N\}$. Schools $c \in C_1 = \{1, \dots, n\}$ are tier 1 , while schools $c \in C_2 = \{n+1, \dots, n+N\}$ are tier 2. Tier i schools have capacity k_i.

For $l \leq n$, we define the set $Z(l)$ of complete and transitive preferences over schools such that the l-most prefered schools are all tier 1 schools, but the school ranked $l+1$ is tier 2.[6] Thus, $Z(n)$ is the set of all preferences $\succ$ such that for all $c_1 \in C_1$ and all $c_2 \in C_2$, $c_1 \succ c_2$. We also denote by $\bar{Z}$ the set of all preferences $\succ$ such that for some $c_2 \in C_2$, $c_2 \succ c$ for all $c \in C \setminus \{c_2\}$.

For $t \in \{r, m\}$, a fraction $\alpha_t \in [0, 1]$ of group t students have preferences uniformly distributed over $Z(l_t)$, with $1 \leq l_t \leq n$, while the remaining $1 - \alpha_t$ have preferences uniformly distributed over $\bar{Z}$. For both types, the preference profile of a student (t, x) is entirely determined by (t, x).[7]

We assume that $\alpha_r > \alpha_m$ and $l_r > l_m$. The restriction $\alpha_r > \alpha_m$ captures the fact that minority students are less likely to rank first a popular school. The restriction $l_r > l_m$ models the idea that even restricting attention to students that rank first a popular school, minority students are less likely to apply to other popular schools than regular students. These assumptions are motivated by evidence from cities in Chile and the US. For example, Laverde (2020) shows that white families are more likely than Black and Hispanic families to rank high-achievement schools in Boston Public Schools choice system.[8] In Section 4.2, we also provide evidence from the main Chilean cities (Valparaíso, Concepción

[6] In particular, for a preference that belongs to $Z(l)$, with $l \leq n-1$, the school ranked $l+2$ could be tier 1 or tier 2.

[7] For example, let $\eta = |Z(l_r)|$ and divide the interval $[0, \alpha_r]$ of regular students into η subsets such that each subset is assigned a unique preference in $Z(l_r)$. Analogously, let $\bar{\eta} = \bar{Z}$ and divide $[\alpha_r, 1]$ into $\bar{\eta}$ intervals such that each interval maps to a unique preference in $\bar{Z}$.

[8] Laverde (2020) additionally shows that travel costs are a key driver of these differences in Boston. Consistent with these results, in Appendix C.2 we show that minority students tend to live farther away from popular schools than regular students

and Santiago) that minority students are less likely to apply to high-demand schools.

Tier 1 schools are overdemanded, but the total capacity of the market exceeds total demand. We thus assume that $nk_1 \leq \alpha_r + \beta\alpha_m$ and $nk_1 + mk_2 > 1 + \beta$.

In many school choice systems, schools rank students independently and uniformly. We allow some more generality and assume that schools ranks are not necessarily uniform. A student $s = (t, x)$ at each school c draws $\omega_c^s \in [0, 1]$ independently from the cumulative distribution G_t on $[0, 1]$. The number ω_c^s represents the priority that student s has in school c. A higher number ω_c^s implies that the student has higher priority in school c. We will refer to ω_c^s as the *score* that student s has in school c. We assume that regular students tend to have higher scores than minority students so that G_r dominates G_m in the first order stochastic sense: $G_r(\omega) \leq G_m(\omega)$ for all $\omega \in [0, 1]$. This assumption is natural in college admission in which socially disadvantaged students are likely to perform worse in entrance exams. The assumption that $G_r(\omega) \leq G_m(\omega)$ is also appropriate in school choice programs in which schools rank students according to academic performance, or in school systems in which siblings or children whose parents work in the school have higher priority. Under all these criteria, a minority student is less likely to be highly ranked in a school than a non-minority student. When $G_r = G_m$ equals the uniform distribution on $[0, 1]$, schools rank all students uniformly as in many school choice programs.

Our two-tier model is natural in school choice applications in which parents tend to value similar attributes of schools. In our model, a tier 1 school tends to be more attractive than a tier 2 school for all students. However, a given minority student is less likely than a regular student to apply to a tier 1 school. When $\beta = 0$, $\alpha_r = 1$, and $l_r = n$, our model has only one group and all students in the group prefer a tier 1 school over a tier 2 school. In this case, our model is analogous to the limit model in Che and Tercieux (2019).

2.2 Matchings and cutoffs

A matching is a function $\mu \colon S \cup C \to C \cup 2^S$ such

i. For all $s \in S$, $\mu(s) \in C$;

ii. For all $c \in C_i$, $\mu(c) \subseteq S$ with $|\{s \mid \mu(s) = c\}| \leq k_i$;

iii. For all $c \in C$ and all $s \in S$, $\mu(s) = c$ iff $s \in \mu(c)$.

The first condition says that each student is assigned to a school, the second condition says that each school is assigned to a measure of students that does not exceed its capacity, the third condition says that a student is assigned to a school iff the school is assigned to that student.

A matching μ is *stable* if for all $c \in C_i$ and all $s = (t, x) \in S$ with $c \succ_s \mu(s)$, the following two conditions hold: (i) $|\{s \mid \mu(s) = c\}| = k_i$; and (ii) $\omega_c^s < \omega_c^{s'}$ for all $s' = (t', x')$ with $\mu(s') = c$. Intuitively, a matching is stable if there is no pair (s, c) that can block the matching (Gale and Shapley 1962).

Following Abdulkadiroglu, Che, and Yasuda (2015) and Azevedo and Leshno (2016), we can characterize a stable matching by means of cutoffs $p_c \in [0, 1]$, for all $c \in C$. A cutoff p_c determines the

in Chilean cities. Yet, for our analysis all what matters is that minority students tend to apply less to popular schools.

lowest lottery number ω_c that a student can have to be admitted to school c. The highest the cutoff p_c, the harder it is to get to school c. Two observations simplify the characterization of cutoffs. First, schools within the same tier are symmetric and therefore $p_c = p_{c'}$ for all $c, c' \in C_i$. Second, in any stable matching a tier two school will have excess capacity and therefore its cutoff will equal 0. We can therefore characterize a stable matching by means of a single cutoff p that clears the market for tier 1 schools:

$$\alpha_r \sum_{q=1}^{l_r} \frac{1}{n} G_r(p)^{q-1}(1 - G_r(p)) + \alpha_m \beta \sum_{q=1}^{l_m} \frac{1}{n} G_m(p)^{q-1}(1 - G_m(p)) = k_1. \tag{2.1}$$

The left hand side in equation (2.1) is the demand for a school $c \in C_1$ when the admission cutoff in all schools is p. The first term on the left hand side of (2.1) is the demand for school c of regular students. For each school $c \in C_1$, α_r/n regular students will rank the school in the q-th position. A student that ranks school c in the q-th position will demand school c if her scores in schools ranked above c are below the cutoffs (which happens with probability $G_r(p)^{q-1}$) but her score in school c is above the cutoff (which happens with probability $1 - G_r(p)$). The second term on the left of (2.1) is the demand for school c of minority students. The measure of minority students that demand some tier 1 school is $\alpha_r \beta$. Those students have preferences uniformly distributed over $Z(l_r)$. Therefore for each tier 1 school c and each $q \in \{1, \ldots, l_r\}$, a fraction $1/n$ of minority students that demand some tier 1 schools will rank school c in the q-th position.

The unique solution $\bar{p} \in [0, 1]$ to equation (2.1) is characterized by

$$\frac{\alpha_r}{n}\Big(1 - (G_r(\bar{p}))^{l_r}\Big) + \frac{\alpha_m \beta}{n}\Big(1 - (G_m(\bar{p}))^{l_m}\Big) = k_1. \tag{2.2}$$

Naturally, $\bar{p}$ increases when the supply of tier 1 schools, nk_1, decreases or when the demand for tier 1 schools, α_t, β, and l_t, increases, for $t = r, m$.

In the unique stable matching, minority students are underrepresented in tier 1 schools. Indeed, the ratio of minority to regular students in the whole population equals β, while the ratio of minority to regular students assigned to a tier 1 school is

$$\frac{\alpha_m \beta\big(1 - G_m(\bar{p})^{l_m}\big)}{\alpha_r\big(1 - G_r(\bar{p})^{l_r}\big)} < \beta.$$

Minority students are less likely than regular students to list any tier 1 school (as $\alpha_m < \alpha_r$). Compared to a regular student, when a minority student does include a tier 1 school in her application, she is less likely to apply to other tier 1 schools (as $l_m < l_r$), her scores are likely to be lower ($G_m(\bar{p}) \geq G_r(\bar{p})$), and therefore $\frac{1-G_m(\bar{p})^{l_m}}{1-G_r(\bar{p})^{l_r}} < 1$. These forces combine to result in school segregation.

3 Minority reserves, segregation and efficiency

We now introduce minority reserves and explore their impact on several market outcomes. We also discuss variations of our main model.

3.1 Stable matching under minority reserves

A minority reserve ensures that whenever the number of minority students in a school c is below the reserve, all other minority students must be assigned to schools that they strictly prefer to c. We adapt Hafalir, Yenmez, and Yildirim (2013) to model minority reserves as follows. Let $\rho = (\rho_1, \rho_2)$ be a vector of minority reserves in tier 1 and tier 2 schools. A matching μ is *stable under reserves* if for all $c \in C_i$ and all $s = (t, x) \in S$ with $c \succ_s \mu(s)$, the following three conditions hold:

i. $|\{s \mid \mu(s) = c\}| = k_i$;

ii. if $|\{s' = (t', x') \mid \mu(s') = c, t' = m\}| \geq \rho_i$, then $\omega_c^s < \omega_c^{s'}$ for all $s' = (t', x')$ with $\mu(s') = c$; and

iii. if $|\{s' = (t', x') \mid \mu(s') = c, t' = m\}| < \rho_i$, then $t = r$ and $\omega_c^s < \omega_c^{s''}$ for all $s'' = (r, x'') \in \mu(c)$.

A matching is stable under reserves ρ if whenever a student s would like to move to another school c, that school is filling its seats, it is admitting students having higher priority and exceeding the minority reserves, and if it is not exceeding the minority reserves then s is a regular student having a score below the lowest score of regular students assigned to c. Note that when $\rho \equiv 0$, a matching is stable under reserves ρ iff it is stable.

A matching μ that is stable under reserves always exists. It can be computed by the deferred acceptance algorithm by either properly defining a choice function or by making a copy of each school that targets minority students (Hafalir, Yenmez, and Yildirim 2013). Note that since our model has a continumm of students, the deferred acceptance algorithm need not converge in finite time (Abdulkadiroglu, Che, and Yasuda 2015).

We now characterize the unique stable matching under reserves ρ. First note that if $\rho_1 < \frac{\alpha_m \beta}{n}(1 - (G_m(\bar{p}))^{l_m})$, then the stable matching characterized by cutoffs $\bar{p}$ is stable under reserves ρ. This simply follows from the observation that the minority reserve ρ_1 is already filled in tier 1 schools and therefore Conditions ii. and iii. in the definition of stability under reserves are equivalent to Condition ii in the definition of stability. Second, note that when $\rho_1 > \min\{\alpha_m \beta / n, k_1\}$, the reserve either is above the number of minority students that demand the school, or exceeds the capacity of the school. We thus define $R = [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p})^{l_m}), \min\{\alpha_m \beta / n, k_1\}]$.

Take a reserve $\rho_1 \in R$. We can characterize stability under reserves by means of cutoffs p_c^t that depend on the school c and the types $t \in \{r, m\}$ of the applying students. Similar to the analysis in Subsection 2.2, we can restrict attention to cutoffs such that $p_c^t = p_{c'}^t$ for all $c, c' \in C_1$ and $p_c^t \equiv 0$ for all $c \in C_2$ and all t. It is therefore enough to characterize the cutoffs p_m and p_r, with $p_m \leq p_r$, that minority and regular students face in tier 1 schools. First, the market clearing condition can be written as:

$$\frac{\alpha_r}{n}\Big(1-(G_r(p_r))^{l_r}\Big)+\frac{\alpha_m\beta}{n}\Big(1-(G_m(p_m))^{l_m}\Big)=k_1. \tag{3.1}$$

This is similar to equation (2.1), but in this market clearing condition different groups face different cutoffs. Second, the minority reserve condition must hold. Since $\rho_1 \geq \frac{\alpha_m\beta}{n}(1-G_m(p)^{l_m})$, the reserve must bind and therefore the number of minority students in a tier 1 school equals the reserve:

$$\alpha_m\beta\sum_{q=1}^{l_m}\frac{1}{n}G_m(p_m)^{q-1}\Big(1-G_m(p_m)\Big)=\rho_1. \tag{3.2}$$

Equivalently, the minority reserve condition can be written as

$$\alpha_m\beta\Big(1-G_m(p_m)^{l_m}\Big)=\rho_1. \tag{3.3}$$

We can solve for p_m and p_r to deduce

$$G_r(p_r)=\Big(1-\frac{n(k_1-\rho_1)}{\alpha_r}\Big)^{1/l_r} \quad G_m(p_m)=\Big(1-\frac{n\rho_1}{\alpha_m\beta}\Big)^{1/l_m}$$

Figure 1 illustrates how cutoffs are determined. Note that increasing ρ_1 moves the minority reserve condition (3.3) to the left in Figure 1. So, after an increase in minority reserves, p_m decreases and p_r increases. Increasing ρ_1 makes the access to tier 1 schools easier for minority students and harder for regular students. We denote by μ_ρ the stable matching under reserves ρ.

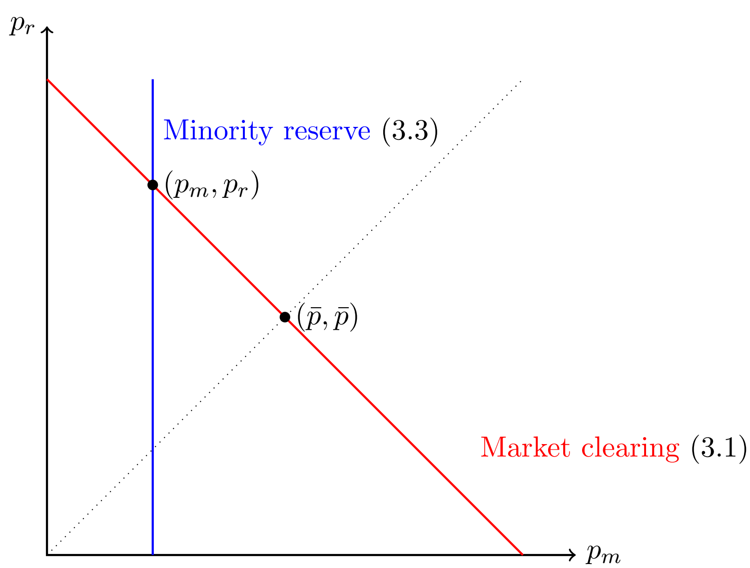


Figure 1: The market clearing condition and the minority reserve condition determine cutoffs p_r and p_m. The cutoff $\bar{p}$ is in the intersection of the market clearing condition and the 45 degree line.

The main focus of the paper is the impact of reserves ρ on several market outcomes. In many systems, minority reserves are defined as a constant fraction $f \in [0,1]$ of seats in each school. Thus, naturally, $\rho_i = fk_i$ (our field data exercise will be parameterized in this manner). Note that since tier 2 schools have excess capacity, ρ_2 is irrelevant for the allocation. We explore the role of reserves by stating several comparative statics results with respect to ρ_1.

3.2 Segregation

There are several ways to measure segregation in schools, but one of the the most common ones is the Duncan index (Duncan and Duncan 1955). Given a matching μ, the *Duncan index* D_μ is defined by

$$D_\mu = \frac{1}{2}\sum_{c=1}^{n+N}\left|\eta^r_\mu(c) - \frac{\eta^m_\mu(c)}{\beta}\right| \in [0,1]$$

where $\eta^t_\mu(c)$ is the mass of students of type t assigned to school c in the matching μ. The index equals 0 under perfect integration, where each school is filled by exactly the same number of students of each type. More generally, the Duncan index can be interpreted as the mass of regular students that would need to be moved to different schools so that every school had the same proportions of students of each group.

Given a reserve ρ_1, we denote $D(\rho_1) = D_{\mu_\rho}$.

Proposition 1. *$D(\rho_1)$ is nonincreasing over $\rho_1 < \frac{\beta}{1+\beta}k_1$ and is non-decreasing over $\rho_1 > \frac{\beta}{1+\beta}k_1$.*

This result shows that reserves have an impact on segregation in schools. The Duncan segregation index is minimized when the fraction of seats reserved to minority students, ρ_1/k_1, equals the share of minority students in the population, $\beta/(1+\beta)$. Actually, in the proof we show a slightly stronger result: Segregation in each school c, $\left|\eta^\rho_{\mu_r}(c) - \frac{\eta^m_{\mu_\rho}(c)}{\beta}\right|$, is non-increasing over $\rho_1 < k_1\frac{\beta}{1+\beta}$ and non-decreasing over $\rho_1 > k_1\frac{\beta}{1+\beta}$. Intuitively, when $\rho_1 < k_1\frac{\beta}{1+\beta}$, minority students are underrepresented in tier 1 schools and overrepresented in tier 2 schools, and increasing ρ_1 moves minority students from tier 2 to tier 1 schools. This stronger property also implies that the index we actually use to measure segregation in our model is rather irrelevant for the Proposition.[9]

3.3 Rank distribution and efficiency

We now explore how ρ_1 impacts the efficiency of the assignment. Changing ρ_1 does not Pareto improve the assignment for students. We thus evaluate the assignment using two measures. The first measure is the rank distribution of students, which is a function that for each $q \in \{1,\ldots,l_r+1\}$ yields the fraction of students assigned to one of their q most preferred schools. Our second measure is the number of students that belong to a Pareto improvement pair. The main results in this Subsection show how ρ_1 can change both measures.

[9]Proposition 1 and our field evidence also apply to alternative segregation indexes, such as the ones discussed by Hutchens (2004) or Frankel and Volij (2011). See Appendix B.

Since a type t student ranks at most l_t tier 1 shools and tier 2 schools always have free slots, type t students are assigned to one of their $(l_t + 1)$-most preferred schools. The share of type t students assigned to their q-th preference is

$$f_t(q) = \begin{cases} \alpha_t(1 - G_t(p_t)) + (1 - \alpha_t) & \text{if } q = 1 \\ \alpha_t G_t(p_t)^{q-1}(1 - G_t(p_t)) & \text{if } 2 \leq q \leq l_t \\ \alpha_t G_t(p_t)^{l_t} & \text{if } q = l_t + 1. \end{cases}$$

The cumulative rank distribution for type t students is thus

$$F_t(q) = \sum_{q' \leq q} f_t(q') = \begin{cases} \alpha_t\big(1 - G_t(p_t)^q\big) + (1 - \alpha_t) & \text{if } q \leq l_t, \\ 1 & \text{if } q = l_t + 1. \end{cases}.$$

We will sometimes emphasize the dependance of these distributions on ρ_1 by writing $F_t(q, \rho_1)$.

Lemma 1. *Take $\rho_1 \in R$. Then, $\frac{\partial}{\partial \rho_1} F_m(q, \rho_1) > 0$ for all $q \leq l_m$ and $\frac{\partial}{\partial \rho_1} F_r(q, \rho_1) < 0$ for all $q \leq l_r$.*

This lemma says that increasing ρ_1 reduces (in the first order stochastic dominance sense) the cumulative rank distribution for minority students and increases the rank distribution of regular students. In other words, reserves favor the assignment for minority students but hurt regular students.[10] Figure 3 illustrates Lemma 1.

Our main focus is the impact of reserves on the overall efficiency of the assignment. We thus define the total cumulative rank distribution as

$$F(q) = \frac{1}{1 + \beta}\Big(\beta F_m(q) + F_r(q)\Big)$$

which measures the fraction of students assigned to a school ranked q or below. The following is the first main result in this Subsection.

Proposition 2. *Take $\rho_1 \in R$ and suppose that $\alpha_r\Big(1 - \big(\frac{l_m}{l_r}\big)^{\frac{l_r}{l_r - 1}}\Big) > nk_1$. Then, there exists $\bar{q} \in \{1, \ldots, l_m\}$ such that*

$$\frac{\partial F}{\partial \rho_1}(q, \rho_1) > 0 \text{ for } q \leq \bar{q} \quad \text{and} \quad \frac{\partial F}{\partial \rho_1}(q, \rho_1) \leq 0 \text{ for } q > \bar{q}.$$

Proposition 2 shows conditions under which raising ρ_1 increases the mass of students assigned to their first preferences. Moreover, increasing the reserve ρ_1 leaves more students assigned to one of their $\bar{q}$-most preferred schools. But there is no free lunch: Increasing ρ_1 also increases the mass of students assigned to schools that are not highly ranked. The condition under which this result applies says that

[10] This is related to Hafalir, Yenmez, and Yildirim (2013). Their Theorem 2 shows, in a general matching model, that the introduction of minority reserves favor at least one minority student. They also provide restrictions on preferences such that all minority students are better off when reserves are introduced. Lemma 1 thus complements these results.

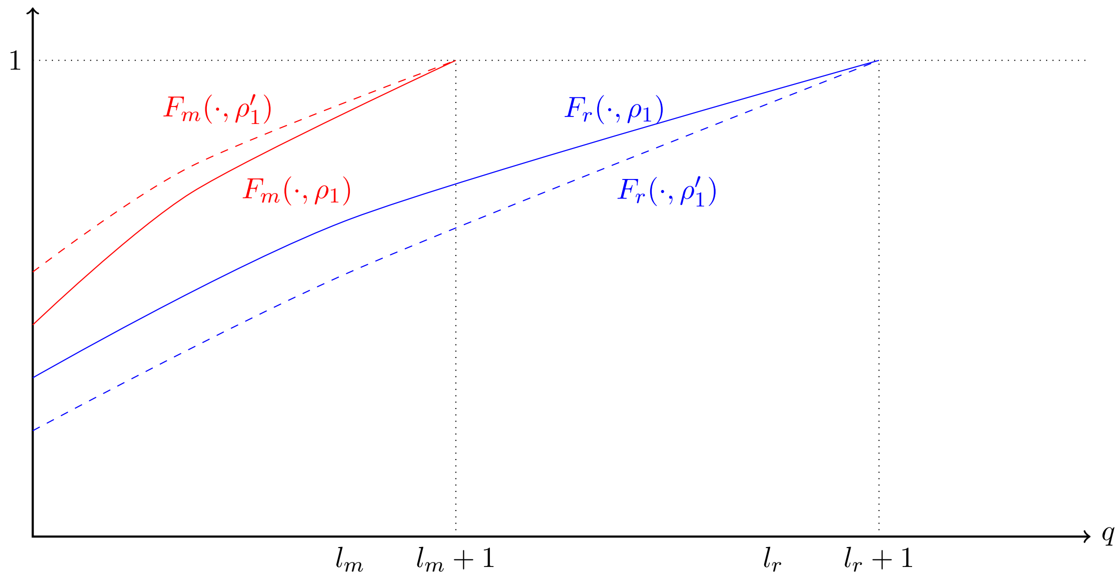


Figure 2: Cumulative rank distributions for minority and regular students as the reserve changes. Solid lines represent the distributions for $\rho_1 \in R$. Dashed lines represent the distributions for $\rho_1' > \rho_1$.

the market should be relatively tight in the sense that the total capacity of the popular tier 1 schools, nk_1, is below a threshold. This bound is more likely to hold when minority students apply to fewer tier 1 schools (l_m decreases), or when regular students apply more intensely to tier 1 schools (α_r and l_r increase). Figure 3 illustrates Proposition 2. Proposition 5 and Example 2 show that the tightness of the market is important for these results.

There are two main forces behind the result that more students are assigned to their most preferred schools when ρ_1 increases. First, by increasing ρ_1, some regular students are replaced by minority students in tier 1 schools. But minority students apply to tier 1 schools with lower intensity: they are more likely to rank first a tier 2 schools and, when they do rank first a tier 1 school, they are likely to include more tier 2 schools in the rest of the application. For students that rank first a tier 2 school, the reserve ρ_1 does not make any difference. For minority students that actually apply to tier 1 schools, the reserve does make a difference. An increase in the reserve will bring some of those students into a tier 1 school, and those students will replace regular students. Thus, the increase in the reserve ρ_1 will replace regular students by minority students for whom the tier 1 school is likely to be very attractive.

The second important force is more subtle and explains why the market should be tight for reserves to increase the total number of students assigned to top choices. Reserves create competition among regular students applying to popular schools. There are two reasons for this competition. The first reason is that when more seats are reserved for minority students in a tier 1 school c, some regular students are displaced and compete for seats in other schools. This stronger competition for seats in other schools displaces regular students, who in turn may demand seats in school c. The second reason is

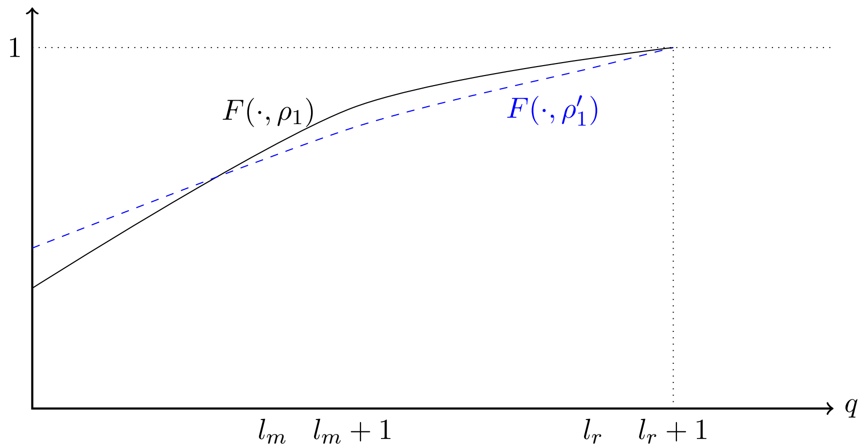


Figure 3: Total cumulative rank distributions as the reserve changes. The solid lines represents the distribution for $\rho_1 \in R$. The dashed line represents the distributions for $\rho_1' > \rho_1$.

that when more seats are reserved for minority students in tier 1 schools other than c, displaced regular students also compete for seats in school c. As a result, p_r will be increasing in ρ_1 and decreasing in k_1.

When k_1 is relatively large, there is little competition in tier 1 schools and most regular students are assigned to their top school. But increasing the reserves ρ_1 activates the two competitive forces mentioned above and, to balance supply and demand in each tier 1 school, p_r increases substantially. This increase in p_r translates into a substantive reduction in the mass of regular students that are assigned to their top school, $\alpha_r(1 - G_r(p_r)) + (1 - \alpha_r)$. Thus, increasing the reserve ρ_1 reduces the total number of students assigned to their top choice even when most (even all) minority students that are now accepted in a tier 1 school c rank the school as their top choice. On the other hand, when slots are scarce and k_1 is relatively small, competition is already intense among regular students and increasing the reserves ρ_1 increases competition moderately. This means that the overall number of students assigned to their most preferred schools increases with the reserve.

To see why increasing ρ_1 decreases $F(q)$ for q large enough, note that all minority students are assigned to one of their top $l_m + 1$ schools. As Lemma 1 shows, the cumulative rank distribution for regular students is increasing in ρ_1. As a result, for $q > l_m$, as ρ_1 raises, fewer students are assigned to schools they rank below q.[11]

Given a matching μ, students $s = (t, x)$ and $s' = (t', x')$ are a *Pareto improving pair* if $c' = \mu(s') \succ_s c = \mu(s)$ and $c \succ_{s'} c'$. In this case, we say that s is in a Pareto improving pair. Let $P(\rho_1)$ be the total measure of students s that are in a Pareto improving pair. Arguably, $P(\rho_1)$ measures the inefficiency of the matching. The following proposition shows that minority reserves have an unambiguous effect on $P(\rho_1)$.

Proposition 3 (Pareto improvements)**.** *Under the conditions of Proposition 2, $P(\rho_1)$ is decreasing in*

[11] In the proof of Proposition 1, we characterize $\bar{q}$. We show that $\bar{q} \leq l_m$, but the equality may or may not hold.

ρ_1

When ρ_1 increases, fewer students are in a Pareto improving pair. Thus, a higher reserve increases the efficiency of the matching. In the proof, we show that a student s can Pareto improve by switching school iff s is assigned to a tier 1 school that is not her top choice. Thus, Proposition 3 follows immediately from Proposition 2.

3.4 Discussion

We now discuss variations of our model and results. We explore the role our assumption on preferences on our main results. We also show how our results apply when the affirmative action policy is a set aside reserve (Dur, Kominers, Pathak, and Sönmez 2018). We finally refine Proposition 2 and provide a result for slack markets.

3.4.1 Preferences

In our main model, regular students concentrate their applications on high demand schools, while minority students apply with lower intensity to overdemanded schools. In theory (but not in our field data), segregation could arise because minority and regular students concentrate their applications on different sets of schools. The following example shows that under this type of preferences, the number of students assigned to their top school need not increase with reserves.

Example 1. *We restrict our main model to $n = 2$, $\beta = 1$, $k_1 \leq 1$, $k_2 = 2$, but now we assume preferences are given by*

$$r : c_1 \succ c_2 \succ c_3 \quad m : c_2 \succ c_1 \succ c_3.$$

Schools rank students uniformly and independently. In this setup, while all students prefer tier 1 schools over the tier 2 school (school c_3), minority students prefer c_2 to c_1 while regular students prefer c_1 to c_2.

When no reserve is imposed, it is relatively simple to find the cutoff $\bar{p} = \sqrt{1 - k_1}$ for each tier 1 school. As a result, in the stable matching without reserves, minority students are underrepresented in c_1, and a fraction $F(1) = 1 - \sqrt{1 - k_1}$ of all students are assigned to their top school.

Now, we impose a reserve $\rho_1 \in [\sqrt{1 - k_1}(1 - \sqrt{1 - k_1}), k_1]$. We can characterize the stable matching by solving the market clearing conditions:

$$k_1 - \rho_1 = 1 - p_1^r \quad \rho_1 = p_2(1 - p_1^m) \quad k_1 = 1 - p_2 + p_1^r(1 - p_2)$$

where p_1^r (resp. p_1^m) is the cutoff faced by non-disadvantaged (resp. disadvantaged) students in school c_1. Solving the system of equations, we deduce that the fraction of students assigned to their top school is

$$F(1, \rho_1) = \frac{k_1 - \rho_1}{2} + \frac{1}{2} \frac{k_1}{2 - k_1 + \rho_1}.$$

The function $F(1, \rho_1)$ is decreasing in ρ_1.

The example shows that when both groups of students concentrate their applications in different schools, imposing a reserve reduces the number of students assigned to their top schools.[12] There are two forces behind this result. First, after the reserve is imposed in c_1, regular students are replaced by minority students for whom c_1 is not their most preferred school. Second, displaced regular students demand school c_2 and thus $1 - p_2$ decreases. As a result, fewer minority students are assigned to school c_2.

3.4.2 Set aside reserves

We have interpreted the affirmative action policy as a minimum guarantee for minority students. As noted by Dur, Kominers, Pathak, and Sönmez (2018), an alternative interpretation of an affirmative action policy is to set aside seats for minority students. Under a set aside policy, a school first assigns the $k_1 - \rho_1$ open seats and reserves the remaining ρ_1 seats for minority students. In this Subsection, we extend our results to this alternative interpretation.

To characterize a stable matching under set aside reserves, we again consider cutoffs p_r^{SA} and p_m^{SA} that apply to regular and minority students in tier 1 schools under the set aside policy. The market clearing and reserve conditions for a set aside policy are

$$\frac{\alpha_r}{n}\left(1 - G_r(p_r^{SA})^{l_r}\right) + \frac{\beta\alpha_m}{n}\left(1 - G_m(p_m^{SA})^{l_m}\right) = k_1$$

and

$$\frac{\beta\alpha_m}{n}\left(G_m(p_r^{SA})^{l_m} - G_m(p_m^{SA})^{l_m}\right) = \rho_1. \tag{3.4}$$

Equation (3.4) is the set aside condition. Motivated by Dur, Kominers, Pathak, and Sönmez (2018), the set aside condition says that the number of minority students with scores below the regular cutoff p_r and that get admitted to a school should equal the reserve ρ_1. In contrast to minority reserves, under this interpretation of the affirmative action policy, the number of minority students effectively admitted to a tier 1 school exceeds the reserve ρ_1.

We define by $F^{SA}(q, \rho_1)$ as the fraction of students assigned to a school ranked q or below under a set aside affirmative action policy ρ_1. Analogously, we define $P^{AS}(\rho_1)$ as the total measure of students than can Pareto improve in the matching with set aside reserves ρ_1. The following result shows that the main insights from Propositions 2 and 3 extend to the set aside policy.

Lemma 2. *There exists $\bar{k} = \bar{k}(l_r, l_m, G_r, G_m, \alpha_r, \alpha_m, \beta, n)$ such that for all $k_1 < \bar{k}$ and all $\rho_1 < \bar{k}$,*

$$\frac{\partial F^{SA}(1, \rho_1)}{\partial \rho_1} > 0 \text{ and } \frac{\partial P^{SA}(\rho_1)}{\partial \rho_1} < 0.$$

The main intuition behind this result is similar to the ones in Subsection 3.3 and therefore omitted.

[12] Note that this holds for all $k_1 \leq 1$. In particular, it holds even if the market is slack.

As Dur, Kominers, Pathak, and Sönmez (2018) show, the precedence order with which reserves are processed has an impact similar to adjusting reserve sizes. We derive a similar result in our framework.

Proposition 4. *Take $\rho_1 \in R$ and suppose that $\alpha_r\Big(1-\big(\frac{l_m}{l_r}\big)^{\frac{l_r}{l_r-1}}\Big) > nk_1$. Then,*

$$F^{SA}(1,\rho_1) > F(1,\rho_1) \text{ and } P^{SA}(\rho_1) < P(\rho_1).$$

Fewer minority students are assigned to tier 1 schools under minority reserves than under set aside.

Changing the interpretation of the affirmative action policy from minority reserves to set asides increases the number of students assigned to their top schools and reduces the number of students who can Pareto improve by switching schools. Obviously, compared to the minority reserve policy, the set aside policy may or may not reduce segregation by placing more minority students in tier 1 schools.

Proposition 4 can be understood graphically. As shown in Figure 4, the cutoffs p_m^{SA} and p_r^{SA} are entirely determined by the intersection of the market clearing (3.1) and set aside (3.4) conditions. The set aside condition is to the left of the minority reserve condition (see also Figure 1) and therefore $p_r^{SA} > p_r$ and $p_m^{SA} < p_m$. By increasing ρ_1, the minority reserve condition (3.3) moves to the left. As a result, we can find $\rho_1' > \rho_1$ such that the cutoffs p_m' and p_r' under minority reserves ρ_1' satisfy $p_m^{SA} = p_m'$ and $p_r^{SA} = p_r'$. Since $\alpha_r\Big(1-\big(\frac{l_m}{l_r}\big)^{\frac{l_r}{l_r-1}}\Big) > nk_1$, Proposition 2 implies that for a fixed ρ_1 more students are assigned to their top school under set asides than under minority reserves.

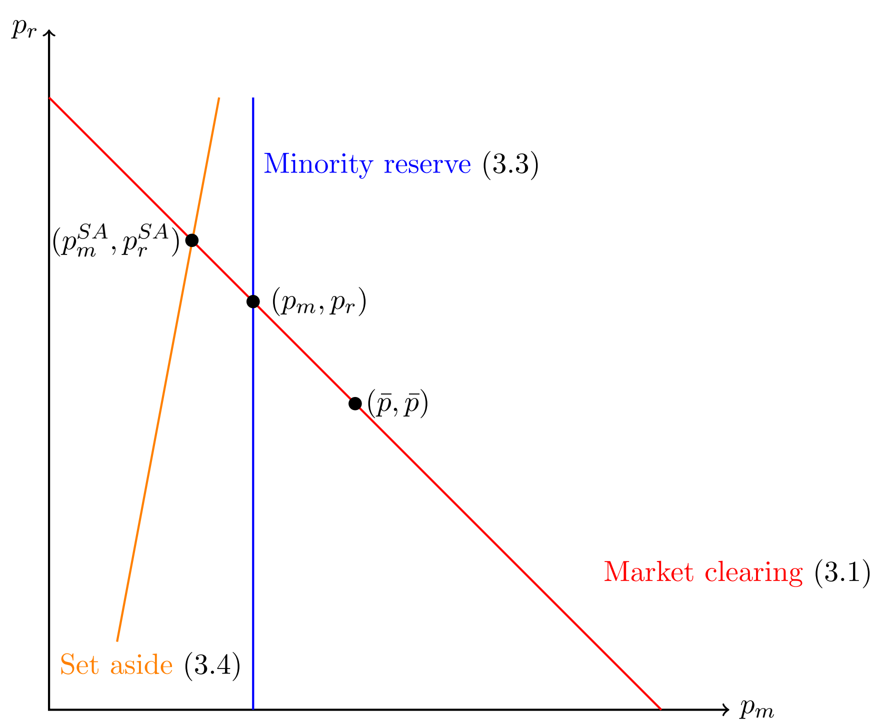


Figure 4: The market clearing condition and the set aside condition determine cutoffs p_r^{SA} and p_m^{SA}. For a given ρ_1, the set aside condition is to the left of the minority reserve condition.

3.4.3 Slack markets

Propositions 2 and 3 apply under the assumption that the market is tight. We now show that the tightness of the market is important for these results.

The following result fully characterizes the environments in which minority reserves reduce the number of students assigned to their first preference and increase the number of students that can Pareto improve by switching schools.

Proposition 5. *Let $\rho_1 \in R$. Then,*

$$\frac{\partial F(1,\rho_1)}{\partial \rho_1} < 0 \text{ iff } \frac{\partial P(\rho_1)}{\partial \rho_1} > 0 \text{ iff } \ln(\frac{l_r}{l_m}) + \frac{l_r - 1}{l_r} \ln\Big(1 - \frac{n(k_1 - \rho_1)}{\alpha_r}\Big) < \frac{l_m - 1}{l_m} \ln\Big(1 - \frac{n\rho_1}{\alpha_m \beta}\Big).$$

One implication of this result is that when the market is slack and k_1 is relatively big, then increasing the minority reserve both reduces the number assigned to their first preferences and increases the number of students that can Pareto improve by switching schools. The next example shows that the set of parameters under which this happens is nonempty.

Example 2. *We take our model with $\beta = 1$, $n = 2$, $N = 1$, $k_2 = 2$, $k_1 = 8/9$ $\alpha_r = 1$, $l_r = 2$, $\alpha_m = 1$, $l_m = 1$. In this model, all regular students apply to tier 1 schools, while minority students apply first to a tier 1 school and second to the tier 2 school. In the model without reserves, $\bar{p} = (-1/2) + \sqrt{1/4 + 2(1 - 8/9)} \approx 0.18$. Now, $R \approx [0.4, 0.5]$ and it is relatively simple to show that for any reserve $\rho_1 \in R$, the number of students assigned to their top choice decreases iff $8/9 - 3/8 > \rho_1$. Thus, for any $\rho_1 < 8/9 - 3/8 \approx 0.51$ with $\rho_1 \in R$, increasing the reserve reduces the number of students assigned to their top choice and increases the number of students that can Pareto improve by switching schools.*

4 Field evidence

4.1 School choice in Chilean cities

Chile initiated its school choice system gradually in 2016. The current system runs nationwide and througout all school levels. All students in the country that enter the system or want to switch school access to a platform and fill a rank order list. A centralized algorithm ran by the Ministry of Education assigns students to schools, using as inputs the students' preferences and the schools seats. Schools rank students using a variety of criteria, but many of them are relevant for a small fraction of the applicants. Many students cannot be ranked by schools simply using any of the priority criteria.[13] For those cases, each school runs a lottery over its whole set of applicants.

The Law regulating the admission process to schools also reserves 15% of seats in each school to minority students. A student is considered a minority student if her social background impairs her

[13] For details on the Chilean system, see Correa, Epstein, Escobar, Rios, Bahamondes, Bonet, Epstein, Aramayo, Castillo, and Cristi (2019).

education process and outcomes. The Ministry of Education carries out an objective evaluation of the socioeconomic environment of each student using health, housing, and income information to determine whether a student is *socially disadvantaged*.[14] This reserve policy is an explicit attempt to promote social inclusion in schools, but as our simulations show it has a modest effect on outcomes.

Students are assigned to schools by running a Gale-Shapley deferred acceptance algorithm (Gale and Shapley 1962). The assignment process is a multiple lottery deferred acceptance algorithm with minority reserves (Abdulkadiroğlu, Pathak, and Roth 2009, Hafalir, Yenmez, and Yildirim 2013) that runs as follows:

Step 1: Each student proposes to her first choice. Each school tentatively assigns seats to its proposers, following the priority and lottery orders and respecting the minority reserves.

Any remaining proposers are rejected.

Step k: Each student rejected in the previous step proposes to her next best choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students following the priority and lottery orders and the minority reserves.

Any remaining proposers are rejected. Go to Step $k + 1$.

The algorithm terminates either when there are no new proposals or when all rejected students have exhausted their preference lists. Note that while some students may end up unassigned, the Chilean system allows families to submit rank order lists of arbitrary length.

We focus on the admission process for Pre-Kinder in 2019. The data we use is publicly available from the Ministry of Education.[15] Pre-Kinder is the first level for which the system applies. As a result, to get admission to this level, all students need to apply through the centralized platform.[16] Pre-Kinder is therefore a natural grade to test our theoretical results.

While the system runs nationwide and a student could apply to any school in the country, virtually all students apply exclusively within their province or district. The system is thus composed of several isolated markets. We concentrate on the markets from the three main urban centers in Chile: Santiago, Valparaiso, and Concepcion. Each of these markets is indeed isolated and virtually independent from the rest of the markets in the country. [17] The following table presents a brief summary for each market:

[14] See https://sep.mineduc.cl/alumnos-prioritarios-preferente/ for details.

[15] See http://datos.mineduc.cl/dashboards/20514/descarga-bases-de-datos-sistema-de-admision-escolar/.

[16] While the system runs throughout all levels, the majority of students in levels other than Pre-Kinder do not attempt to switch school and thus do not participate in the platform.

[17] See Appendix C.1 for details. We have also ran simulations for smaller Chilean cities and obtained results that are similar to the ones reported in this paper.

Table 1: Valparaiso, Concepcion and Santiago markets

	Valparaíso	Concepción	Santiago
Number of schools	275	250	1,214
Total capacity (seats)	8,754	9,199	56,331
Number of students	6,819	7,523	49,108
Minority students	2,994 (43.91%)	3,233 (42.97%)	18,399 (37.47%)
Mean number of submitted preferences	2.99	3.15	3.36

Note that in each of the cities, the percentage of minority students far exceeds the current minority reserve of 15%. While families are allowed to submit lists of arbitrary length, applications are relatively short.[18]

4.2 Popular schools and application patterns

Schools face different demand levels. Following Ashlagi and Nikzad (2016), we can measure the popularity of a school c as the the ratio between the number of students for whom school c is their top choice and the capacity of school c. More formally, let $p_1(c)$ be the number of students that list school c as their top choice and let q_c be the number of seats that school c has. The *popularity* of school c is given by

$$pop(c) = \frac{p_1(c)}{q_c}.$$

A school c such that $pop(c) \geq 1$ will fill its seats under different variations of the deferred acceptance algorithm.[19]

Table 2 shows the popularity of schools across markets. For example, close to a quarter of schools in Valparaíso and Santiago have popularity above 1.

	Valparaíso	Concepción	Santiago
First quartile	0.32	0.23	0.33
Median	0.53	0.44	0.60
Mean	0.74	0.67	0.80
Third quartile	0.96	0.86	1.03

Table 2: Popularity of schools

[18]Each of our markets include some rural areas in which the supply of schools is limited and therefore naturally families apply to few schools.

[19]The characteristics or attributes of popular schools is not central for our analysis. In Appendix C.1, we show that students attending popular schools tend to have higher scores in standardized tests.

Minority students tend to apply less to popular schools. For each group of students, we compute the cumulative distribution function for the popularity of the schools ranked first.[20] The cumulative distribution function for the popularity of the school ranked first of regular students dominates (in the first order stochastic sense) the distribution for minority students in all three markets.[21] Figure 5 shows the distributions.

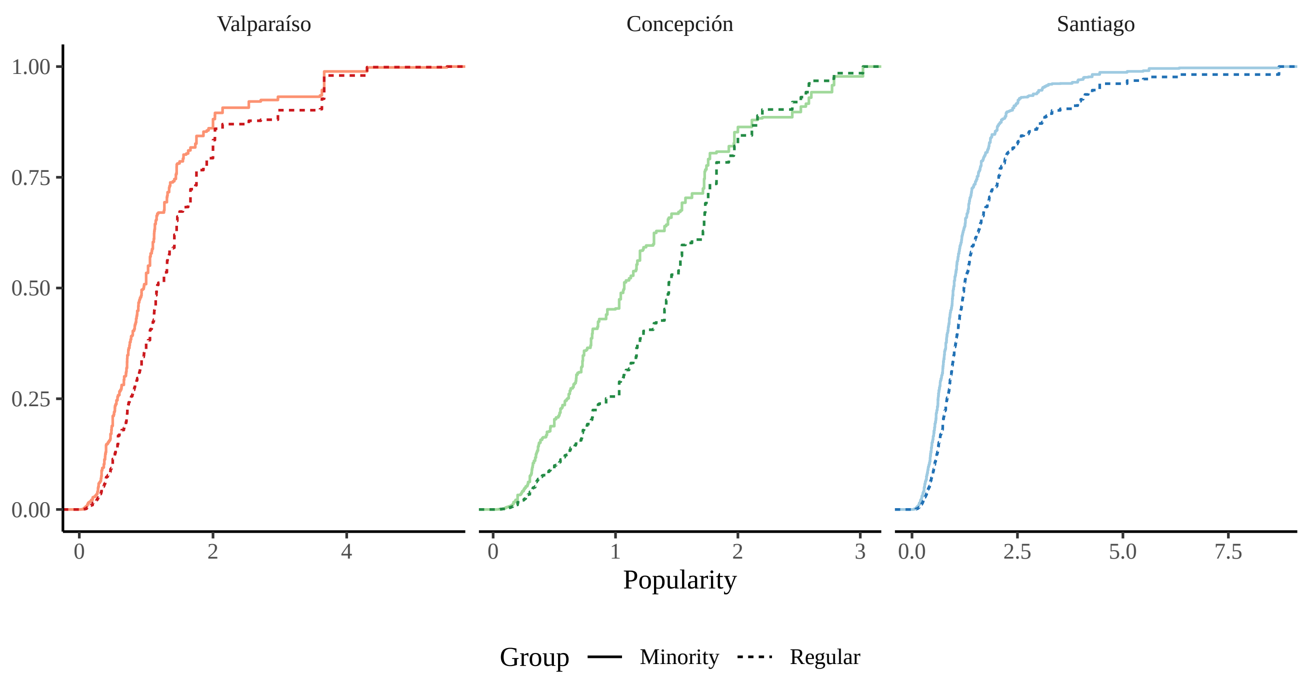


Figure 5: Cumulative distributions of the popularity of schools listed first.

Minority students that apply first to a popular school are also less likely to list other popular schools. Restricting attention to students whose first school has popularity above 1, we compute the cumulative distributions for the popularity of the school ranked second. Under this restriction, the popularity of the school listed second by minority students is below than that for regular students. Figure 6 shows the distributions.

[20]We also considered the distributions of the sum of the popularities in the rank order list, and the sum of the popularities of the first three schools in the list. We obtained similar results.

[21]In Concepción, the distributions cross at popularity close to 2. In the Appendix C.2, we show the results of a Kolmogorov-Smirnov showing the stochastic dominance of the distributions.

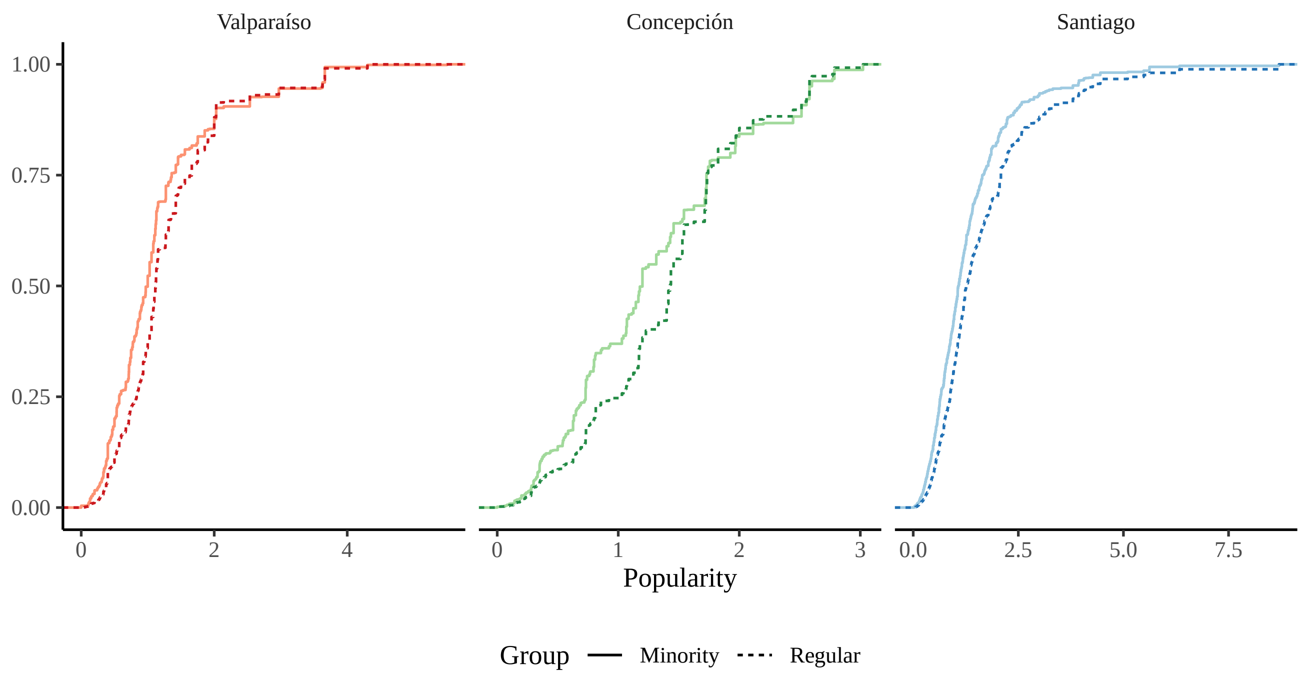


Figure 6: Cumulative distributions of the popularity of schools listed second, conditional on listing first a school with popularity above 1.

In sum, some schools face a lot of demand while others are barely demanded. Regular students are more likely to apply to schools that are popular than minority students.

4.3 Main simulation results

We now present the simulation results for Valparaíso, Concepción and Santiago. For each market, we run the algorithm used by the Ministry of Education for different minority reserves.[22] Concretely, for each market and for each $f \in \{0, \ldots, 100\}$, we run 50 simulations of the algorithm where minority reserves in each school equals $f\%$ of its seats.[23] We assume that variations in the algorithm do not change students applications. This assumption is justified since the deferred acceptance algorithm with minority reserves is strategy-proof (Hafalir, Yenmez, and Yildirim 2013).[24] For each simulation, we compute the Duncan index, the empirical rank distributions of the assignments, and the number of students in Pareto improving pairs.[25]

For each market, Table 3 reports outcomes with no reserves, reserves equal to $15\%, 75\%, 100\%$ and

[22]In particular, our simulation considers all the criteria used by the Ministry of Education to rank students in each school, including sibling priority (Correa, Epstein, Escobar, Rios, Bahamondes, Bonet, Epstein, Aramayo, Castillo, and Cristi 2019).

[23]For each simulation, we draw a new independent realization of the lotteries ranking students.

[24]See Correa, Epstein, Escobar, Rios, Bahamondes, Bonet, Epstein, Aramayo, Castillo, and Cristi (2019) for further discussion in the Chilean context.

[25]In our field data, some students are not assigned. To compute the empirical distributions of the assignments, if a student is unassigned and her rank order list has length l, we consider that student was assigned to her $(l + 1)$-th school.

equal to the fraction of minority students in the market. The simulations confirm each of our theoretical results.

Valparaíso	$f = 0\%$	$f = 15\%$	$f = 44\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.316 (0.005)	0.312 (0.005)	0.247 (0.003)	0.311 (0.003)	0.317 (0.002)
Minority students assigned to their top choice (Lemma 1)	71.73 (0.52)	72.05 (0.4)	78.89 (0.38)	88.69 (0.29)	89.54 (0.25)
Regular students assigned to their top choice (Lemma 1)	63.67 (0.37)	63.45 (0.48)	59.88 (0.35)	56.25 (0.35)	55.96 (0.27)
Students assigned to their top choice (Proposition 2)	67.21 (0.23)	67.23 (0.33)	68.23 (0.25)	70.49 (0.22)	70.7 (0.21)
Students assigned to their fourth choice or worst (Proposition 2)	7.32 (0.21)	7.29 (0.17)	7.6 (0.17)	8.24 (0.16)	8.31 (0.16)
Students unassigned (Proposition 2)	9.46 (0.16)	9.48 (0.19)	9.81 (0.14)	10.56 (0.13)	10.63 (0.13)
Students in Pareto improving pairs (3)	7.81 (0.53)	7.9 (0.51)	6.68 (0.57)	3.66 (0.26)	3.3 (0.33)
Concepción	$f = 0\%$	$f = 15\%$	$f = 43\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.353 (0.004)	0.342 (0.005)	0.264 (0.003)	0.355 (0.003)	0.358 (0.003)
Minority students assigned to their top choice (Lemma 1)	67.77 (0.43)	68.57 (0.44)	76.74 (0.37)	88.01 (0.2)	88.46 (0.22)
Regular students assigned to their top choice (Lemma 1)	58.06 (0.47)	57.4 (0.46)	54.11 (0.39)	50.68 (0.33)	50.5 (0.27)
Students assigned to their top choice (Proposition 2)	62.23 (0.29)	62.2 (0.33)	63.84 (0.26)	66.73 (0.24)	66.81 (0.17)
Students assigned to their fourth choice or worst (Proposition 2)	11.42 (0.21)	11.41 (0.25)	11.46 (0.21)	11.95 (0.15)	11.97 (0.16)
Students unassigned (Proposition 2)	12.65 (0.16)	12.75 (0.13)	13.08 (0.15)	13.61 (0.1)	13.63 (0.1)
Students in Pareto improving pairs (Proposition 3)	12.31 (0.41)	12.34 (0.54)	10.25 (0.44)	5.64 (0.37)	5.35 (0.3)
Santiago	$f = 0\%$	$f = 15\%$	$f = 37\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.312 (0.002)	0.303 (0.002)	0.246 (0.001)	0.328 (0.001)	0.331 (0.001)
Minority students assigned to their top choice (Lemma 1)	70.86 (0.19)	71.58 (0.17)	77.81 (0.16)	90.79 (0.08)	91.35 (0.07)
Regular students assigned to their top choice (Lemma 1)	56.48 (0.17)	56.15 (0.19)	54.05 (0.12)	50.29 (0.11)	50.1 (0.1)
Students assigned to their top choice (Proposition 2)	61.87 (0.12)	61.93 (0.13)	62.95 (0.09)	65.46 (0.08)	65.55 (0.07)
Students assigned to their fourth choice or worst (Proposition 2)	12.3 (0.09)	12.32 (0.1)	12.49 (0.09)	13.14 (0.07)	13.19 (0.06)
Students unassigned (Proposition 2)	12.49 (0.06)	12.49 (0.05)	12.67 (0.06)	13.1 (0.05)	13.13 (0.04)
Students in Pareto improving pairs (Proposition 3)	9.5 (0.2)	9.43 (0.19)	8.06 (0.16)	4.52 (0.13)	4.39 (0.12)

Table 3: Impact of minority reserves on market outcomes

Note: Excluding the Duncan index, all values are percentages. Standard deviations inside parenthesis.

As Proposition 1 shows, the Duncan index is U-shaped. In each of the markets the Duncan is above 0.3 with no reserves and with a reserve equals to 15% (as currently written in the Inclusion Law). The Duncan index can be reduced in close to 20% when the reserve is close to the fraction of minority students in the market. A reserve above the fraction of minority students in the market worsens segregation. All of this is consistent with Proposition 1. Figure 7 shows the Duncan index for each minority reserve.[26]

Table 3 also confirms the prediction from Lemma 1 that after the reserve increases, a higher (resp. lower) fraction of minority students (resp. regular students) get their top schools. Moreover, the rank distributions of the assignments for each group at each market move precisely as shown in Lemma 1. Indeed, Figure 8 shows how the empirical rank distributions for minority and regular students change

[26] In Appendix C.3, we show how the popularity of each school determines how the minority reserve changes the composition of students.

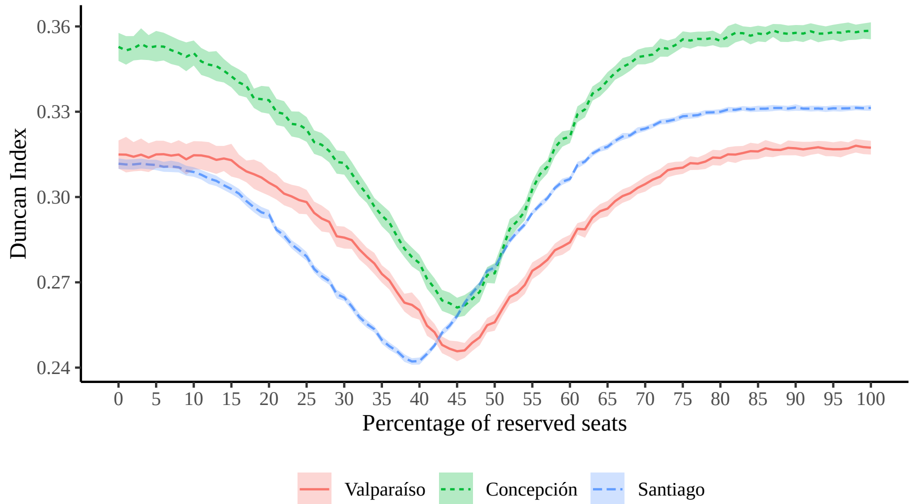
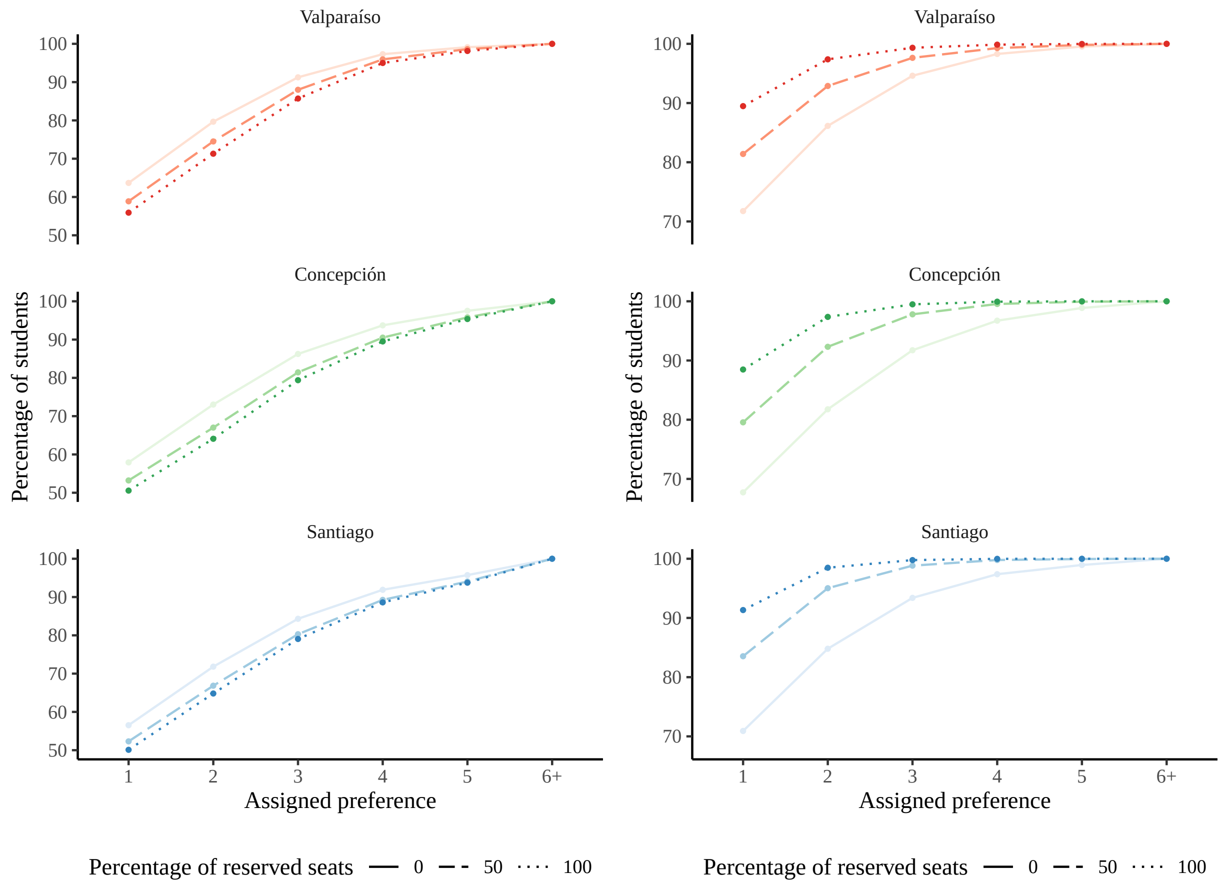


Figure 7: Duncan index.

with the reserves.

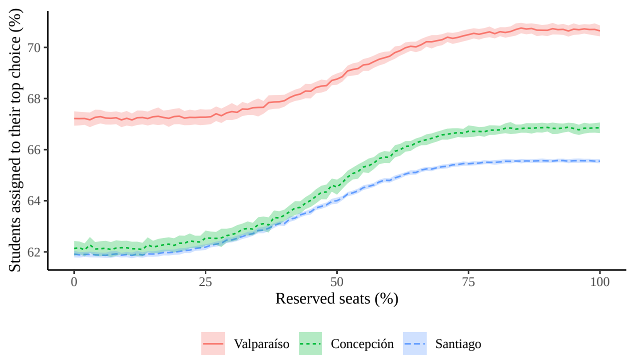


(a) Regular (b) Minority

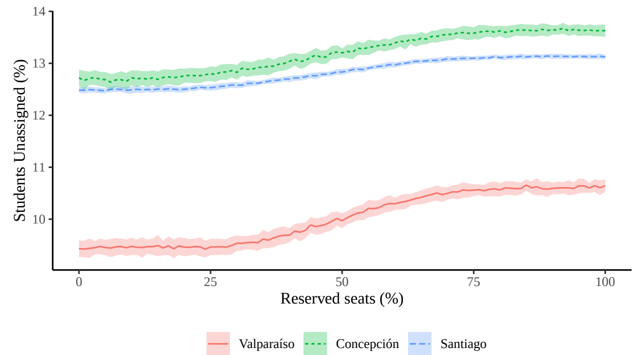
Figure 8: Cummulative rank distributions of assignments for each group.

Proposition 2 is perhaps the most subtle prediction from our model. This result is also consistent with our simulations. Table 3 shows that the total number of student assigned to their top school and the total number of students assigned to schools that are not very attractive (ranked fourth or below) move precisely as predicted by Proposition 2. Figure 9 plots the percentage of students assigned to their top choices and students not assigned, as a function of the reserve.[27]

[27]Even though in our theoretical framework all the students are assigned, in the simulations we have computed the fraction of unassigned students as a measure of students whose assignment is unattractive. The fact that more students are unassigned as we increase the reserve is related to the result in Proposition 2 that the fraction of students assigned to schools that are not highly ranked increases with the reserve.



(a) Percentage of students assigned to their top choices.



(b) Percentage of students unassigned.

Figure 9: Students assigned to their top choices and unassigned students.

Proposition 2 also shows that the rank distributions cross as minority once reserves increase. Figure 10 illustrates this result in our simulations. Consistent with Proposition 3, Table 3 also shows that in our simulations the percentage of students in Pareto improving pairs falls as minority reserves increases.

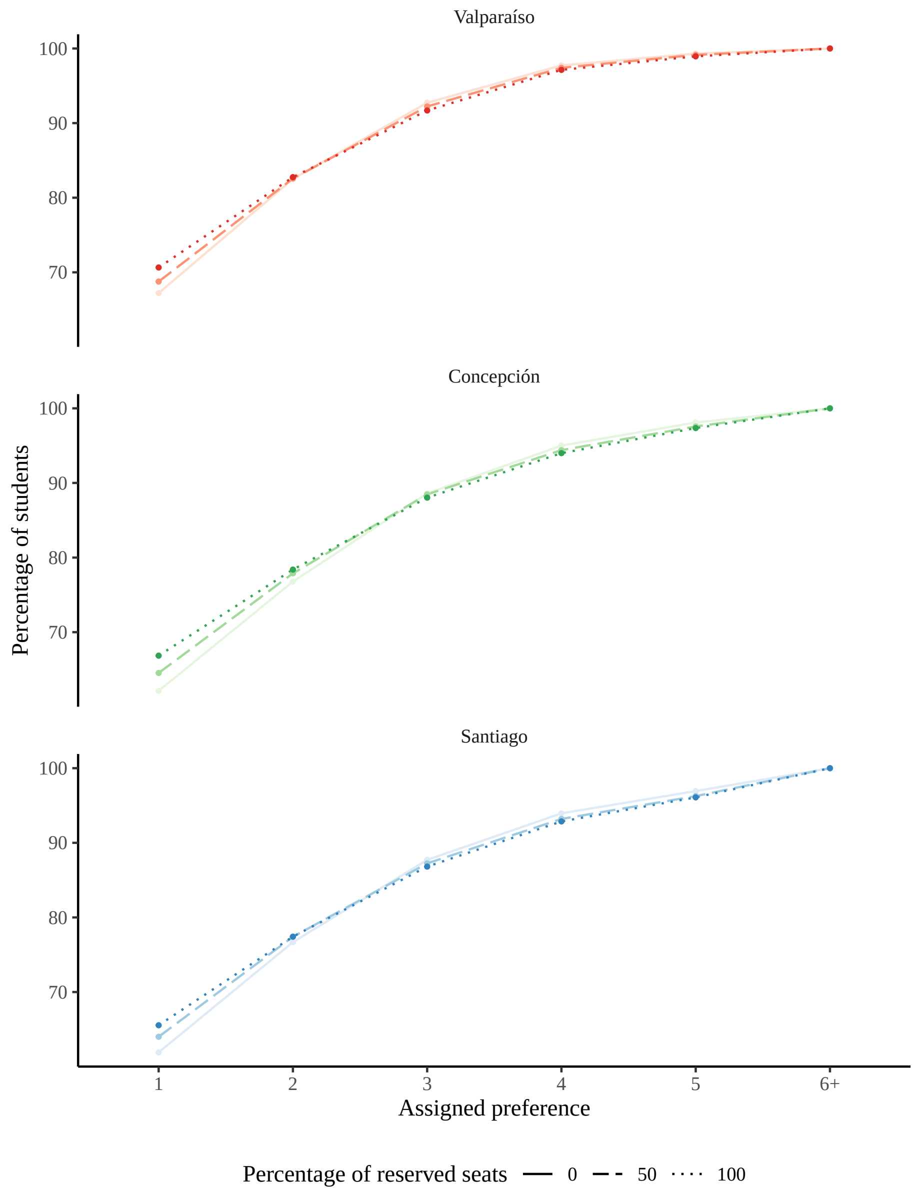


Figure 10: Cummulative rank distribution of assignment. As minority reserves increase, the rank distributions cross.

4.4 Minority reserves and other policy decisions

To put the design of the minority reserve policy in perspective, we now discuss the impact of other policies on market outcomes.

4.4.1 Set asides

We also simulated each of the markets using the set aside affirmative action policy. Consistent with Lemma 2, increasing the magnitude of the affirmative action policy has similar impacts under minority reserves and set asides. Tables 3 and 4 also confirm the prediction of Proposition 4 that fixing r, changing the interpretation of the affirmative action policy from minority reserves to set asides increases the number of students assigned to top schools and reduces the number of Pareto improving pairs. Under set asides, segregation is minimized for a reserve below the proportion of minority students in the population.

Valparaíso	$f = 0\%$	$f = 15\%$	$f = 44\%$	$f = 75\%$	$f = 100\%$
Duncan index	0.315 (0.004)	0.285 (0.004)	0.303 (0.003)	0.317 (0.003)	0.317 (0.002)
Minority students assigned to their top choice	71.88 (0.51)	80.01 (0.49)	87.71 (0.26)	89.52 (0.23)	89.41 (0.26)
Regular students assigned to their top choice	63.64 (0.56)	59.28 (0.48)	56.67 (0.26)	55.94 (0.31)	55.95 (0.31)
Students assigned to their top choice	67.26 (0.32)	68.38 (0.29)	70.3 (0.16)	70.69 (0.2)	70.64 (0.21)
Students assigned to their fourth choice or worst	7.28 (0.19)	7.7 (0.19)	8.24 (0.18)	8.29 (0.17)	8.28 (0.18)
Students unassigned	9.44 (0.16)	9.95 (0.13)	10.49 (0.15)	10.6 (0.12)	10.59 (0.14)
Students in Pareto improving pairs	7.78 (0.54)	6.6 (0.48)	3.99 (0.32)	3.36 (0.3)	3.4 (0.31)
Concepción	$f = 0\%$	$f = 15\%$	$f = 43\%$	$f = 75\%$	$f = 100\%$
Duncan index	0.351 (0.005)	0.309 (0.004)	0.347 (0.003)	0.359 (0.003)	0.358 (0.003)
Minority students assigned to their top choice	67.78 (0.48)	75.91 (0.42)	87.52 (0.27)	88.49 (0.2)	88.48 (0.22)
Regular students assigned to their top choice	58.03 (0.46)	53.96 (0.38)	50.85 (0.32)	50.47 (0.32)	50.45 (0.29)
Students assigned to their top choice	62.22 (0.29)	63.39 (0.27)	66.61 (0.22)	66.81 (0.2)	66.79 (0.21)
Students assigned to their fourth choice or worst	11.39 (0.2)	11.53 (0.2)	11.91 (0.15)	11.96 (0.14)	11.96 (0.18)
Students unassigned	12.71 (0.14)	12.98 (0.14)	13.58 (0.12)	13.65 (0.1)	13.61 (0.11)
Students in Pareto improving pairs	12.29 (0.52)	10.68 (0.43)	5.77 (0.35)	5.31 (0.36)	5.32 (0.31)
Santiago	$f = 0\%$	$f = 15\%$	$f = 37\%$	$f = 75\%$	$f = 100\%$
Duncan index	0.312 (0.002)	0.279 (0.001)	0.308 (0.001)	0.331 (0.001)	0.331 (0.001)
Minority students assigned to their top choice	70.91 (0.19)	80.22 (0.18)	88.35 (0.12)	91.33 (0.08)	91.34 (0.06)
Regular students assigned to their top choice	56.5 (0.16)	52.75 (0.14)	50.78 (0.12)	50.12 (0.12)	50.13 (0.1)
Students assigned to their top choice	61.9 (0.11)	63.05 (0.1)	64.85 (0.09)	65.56 (0.08)	65.57 (0.07)
Students assigned to their fourth choice or worst	12.3 (0.08)	12.7 (0.08)	13.04 (0.07)	13.2 (0.08)	13.19 (0.06)
Students unassigned	12.49 (0.05)	12.71 (0.05)	13.01 (0.05)	13.13 (0.04)	13.13 (0.05)
Students in Pareto improving pairs	9.46 (0.2)	8.09 (0.18)	5.53 (0.15)	4.38 (0.13)	4.35 (0.14)

Table 4: Impact of set asides on market outcomes

4.4.2 Double reserves

Another measure that can be used to promote integration in schools is to reserve seats for *both* types of students. This policy results in an ideal point choice rule, for which Echenique and Yenmez (2015) provide an axiomatic justification. We compare the single reserve policy to a double reserve policy, where reserved seats are guaranteed to both groups. Under double reserve, for each group we reserve a fraction of seat equals to the proportion of the group in the market.

	Single minority reserve	Double reserve
Valparaíso		
Duncan index	0.247 (0.003)	0.231 (0.002)
Minority students assigned to their top choice	78.89 (0.38)	76.75 (0.37)
Regular students assigned to their top choice	59.88 (0.35)	61.35 (0.35)
Students assigned to their top choice	68.23 (0.25)	68.11 (0.23)
Students assigned to their fourth choice or worst	7.6 (0.17)	7.41 (0.2)
Students unassigned	9.81 (0.14)	9.71 (0.16)
Students in Pareto improving pairs	6.68 (0.57)	6.64 (0.48)
Concepción		
Duncan index	0.264 (0.003)	0.245 (0.003)
Minority students assigned to their top choice	76.74 (0.37)	74.57 (0.42)
Regular students assigned to their top choice	54.11 (0.39)	55.71 (0.34)
Students assigned to their top choice	63.84 (0.26)	63.81 (0.25)
Students assigned to their fourth choice or worst	11.46 (0.21)	11.38 (0.2)
Students unassigned	13.08 (0.15)	13.08 (0.16)
Students in Pareto improving pairs	10.25 (0.44)	10.23 (0.47)
Santiago		
Duncan index	0.246 (0.001)	0.232 (0.001)
Minority students assigned to their top choice	77.81 (0.16)	74.79 (0.14)
Regular students assigned to their top choice	54.05 (0.12)	55.77 (0.16)
Students assigned to their top choice	62.95 (0.09)	62.89 (0.12)
Students assigned to their fourth choice or worst	12.49 (0.09)	12.37 (0.08)
Students unassigned	12.67 (0.06)	12.66 (0.05)
Students in Pareto improving pairs	8.06 (0.16)	8.06 (0.17)

Table 5: Single and double reserve

As Table 5, moving from single to double reserves has smaller impact than moving from no reserve to minority reserve. Intuitively, reserving seats to regular students does not change the outcomes significantly as the schools where they are under-represented (tier 2 schools in our model) are unpopular and reserves make no difference in those schools. In the data (and in contrast to our model), we observe few popular schools where minority students are over-represented and, thus, introducing double reserve reduces segregation in less than 10% and marginally changes the rank distribution and efficiency.[28]

5 Conclusions

This paper provides theoretical results and field evidence about the impact of minority reserves on segregation and efficiency in school choice programs. We show that minority reserves are an important tool to reduce segregation in schools. Minority reserves increase the number of students assigned to their first preferences and improve efficiency, but more students are unassigned or assigned to unattractive schools. This paper contributes to the market design literature by making explicit the impact that minority reserves have on several market outcomes.

The fact that low income groups apply less to high demand institutions is key for our results. These patterns have been documented in other contexts, such as the school match in Boston (Laverde 2020) and college admission in the US (Hoxby and Avery 2013). We thus hope that our findings are deemed relevant when discussing tools to reduce segregation in different markets.

Information and busing policies may also impact segregation in schools by changing the application patterns of minority students. These policies may determine the effectiveness of minority reserves. Reserves will remain a relevant policy instrument inasmuch as minority students apply less intensely to high demand schools. Our focus has been on the short run impacts of changes to minority reserves. Other impacts, including migration of regular students to private schools or changes in the application patterns as a result of differences in school compositions, could be relevant in the long run but are absent in our analysis. These are important questions that are left for future research.

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[28]Note that the introduction of double reserve tends to reduce the number of students assigned to their top schools, similar to the model in Subsection 3.4.1.

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Appendix

This Appendix has three parts. Appendix A contains proofs. Appendix B provides a version of Proposition 1 for alternative segregation indexes. Appendix C provides details about our field data.

A Proofs

Proof of Proposition 1. Note that for $\rho_1 \in [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))^{l_m}), \min\{\frac{\alpha_m \beta}{n}, k_1\}]$, each tier 1 school has $k_1 - \rho_1$ regular students and ρ_1 minority students, while a tier 2 school has $(1 - n(k_1 - \rho_1))/N$ regular students and $(\beta - n\rho_1)/N$ minority students.[29] Therefore,

$$D(\rho) = \frac{1}{2}\Big\{n|\frac{k_1 - \rho_1}{1} - \frac{\rho_1}{\beta}| + N|\frac{(1-(k_1-\rho_1)n)/N}{1} - \frac{(\beta - n\rho_1)/N}{\beta}|\Big\}.$$

The first (resp. second) term inside the bracket captures the summation defining $D(\rho)$ over tier 1 schools (resp. tier 2 schools). Thus, for $\rho_1 \in [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))^{l_m}), \min\{\frac{\alpha_m \beta}{n}, k_1\}]$,

$$D(\rho) = \frac{1}{2}\Big\{n|k_1 - \rho_1(1 + \frac{1}{\beta})| + n|k_1 - \rho_1(1 + \frac{1}{\beta})|\Big\}.$$

Note that when $\rho_1 \leq \frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))^{l_m})$, or when $\rho_1 \geq \min\{\frac{\alpha_m \beta}{n}, k_1\}$, $D(\rho)$ is flat. The result follows. $\square$

Proof of Proposition 2. We first note that for $q \leq l_t$,

$$F_t(q) = \alpha_t\Big(1 - G_t(p_t)^q\Big) + (1 - \alpha_t).$$

Thus,

$$\begin{aligned}(1+\beta)\frac{\partial F}{\partial \rho_1}(q) &= \frac{\partial}{\partial \rho_1}\Big\{-\alpha_r\Big(1 - n\frac{k_1 - \rho_1}{\alpha_r}\Big)^{q/l_r} + (-\alpha_m\beta)\Big(1 - \frac{n\rho_1}{\alpha_m\beta}\Big)^{q/l_m}\Big\} \\ &= \frac{-nq}{l_r}\Big(G_r(p_r)^{q-l_r}\Big) + \frac{qn}{l_m}\Big(G_m(p_m)^{q-l_m}\Big).\end{aligned}$$

We deduce that for $q \leq l_m$

$$\frac{\partial F}{\partial \rho_1}(q) < 0 \text{ (resp. } > 0) \quad \text{iff} \quad \ln\Big(\frac{l_r}{l_m}\Big) + (l_r - q)\ln(G_r(p_r)) < (l_m - q)\ln(G_m(p_m)) \text{ (resp. } >). \quad \text{(A.1)}$$

Now, note that

$$\frac{\partial F}{\partial \rho_1}(1) > 0 \text{ iff } \ln\Big(\frac{l_r}{l_m}\Big) + (l_r - 1)\ln(G_r(p_r)) > (l_m - 1)\ln(G_m(p_m)).$$

[29] To see the distribution of students in tier 2 schools, note that $1 - n(k_1 - \rho_1)$ regular students are not assigned to tier 1 schools. Regular students that are not assigned to tier 1 schools demand tier 2 schools uniformly.

Since $G_r(p_r) = \left(1 - \frac{n(k_1-\rho_1)}{\alpha_r}\right)^{1/l_r}$ is increasing in ρ_1 and $G_m(p_m) = \left(1 - \frac{n\rho_1}{\alpha_m\beta}\right)^{1/l_m}$ is decreasing in ρ_1, it follows that

$$\ln\left(\frac{l_r}{l_m}\right) + \frac{(l_r-1)}{l_r}\ln\left(1 - \frac{nk_1}{\alpha_r}\right) > \frac{l_m-1}{l_m}\ln\left(1\right) \Rightarrow \frac{\partial F}{\partial\rho_1}(1) > 0.$$

Rewritting the condition on the right hand side, we deduce that $\frac{\partial F}{\partial\rho_1}(1) > 0$ when $\alpha_r\Big(1 - \left(\frac{l_m}{l_r}\right)^{\frac{l_r}{l_r-1}}\Big) > nk_1$.

Now, when $G_r(p_r) \geq G_m(p_m)$, (A.1) implies that for all $q \leq q' \leq l_m$,

$$\frac{\partial F}{\partial\rho_1}(q) < 0 \Rightarrow \frac{\partial F}{\partial\rho_1}(q') < 0$$

As a result, if $G_r(p_r) \geq G_m(p_m)$, there exists $\bar{q} \leq l_m$ such that for all $q \in \{1, \ldots, l_m\}$

$$\frac{\partial F}{\partial\rho_1}(q) > 0 \text{ iff } q \leq \bar{q}.$$

From Lemma 1, for $q \in \{l_m+1, \ldots, l_r\}$,

$$\frac{\partial F}{\partial\rho_1}(q) = \frac{\partial}{\partial\rho_1}\left(\frac{\beta + F_r(q)}{1+\beta}\right) < 0.$$

It thus follows that when $G_r(p_r) \geq G_m(p_m)$, we can find $\bar{q} \leq l_m$ such that Proposition 2 holds.

To see the case $G_m(p_m) > G_r(p_r)$, note that (A.1) and $\frac{\partial F}{\partial\rho_1}(1) > 0$ imply that $\frac{\partial F}{\partial\rho_1}(q) > 0$ for all $q \leq l_m$.[30] Again, using Lemma 1, we can set $\bar{q} = l_m$ to deduce Proposition 2. □

Proof of Proposition 3. Consider any student s who is assigned to a tier 1 school $c = \mu_\rho(s)$ that is not her top choice . Let $\bar{c}$ be the top choice of student s. Consider the (positive measure) set $\bar{S} \subset S$ of all students such that they rank school c first, and school $\bar{c}$ second. Define $\hat{S} \subseteq \bar{S}$ by $\hat{S} = \{s' \in \bar{S}, \omega_c^{s'} < p_\rho < \omega_{\bar{c}}^{s'}\}$. By construction, $\hat{S}$ has positive measure. For any $s' \in \hat{S}$, $c \succ_{s'} \mu_\rho(s') = \bar{c}$. As a result, s can Pareto improve by switching school with $s' \in \hat{S}$.

If s is assigned to a tier 1 school that is her top choice, then it is clear that s cannot Pareto improve by switching school.

If s is assigned to a tier 2 school, then s is either assigned to her top choice or s would prefer a tier 1 school. If s is assigned to her top choice, then s cannot Pareo improve by switching school. If s would like to move to some tier 1 school, then all students assigned to that tier 1 school prefer their current school to the tier 2 school s is assigned to. So, s cannot Pareto improve by switching school.

It thus follows that

$$P(\rho_1) = 1 - F(1,\rho_1) - \frac{\alpha_r G_r(p_r)^{l_r} + \alpha_m\beta G_m(p_m)^{l_m}}{1+\beta} = 1 - F(1,\rho_1) - \frac{\alpha_r + \alpha_m\beta - nk_1}{1+\beta} \qquad \text{(A.2)}$$

[30] Note that $p_m < p_r$, but it is possible that $G_m(p_m) > G_r(p_r)$. For example, when $G_m(x) > G_r(x)$ for all $x < 1$ and ρ_1 is close but above $\frac{\alpha_m\beta}{n}(1 - G_m(\bar{p})^{l_m})$, p_m and p_r are both close to $\bar{p}$ and, thus, $G_m(p_m) > G_r(p_r)$. Now, when $G_m \equiv G_r$, it follows that $p_m = G_m(p_m) < G_r(p_r) = p_r$.

which is decreasing in ρ_1 under the conditions of Proposition 2. $\square$

Proof of Proposition 5. The proof of this result follows from the proofs of Propositions 2 and 3. To see this, note that the first equivalence follows from equation (A.2), while the second equivalence follows from (A.1). $\square$

Proof of Lemma 2. Write the conditions defining the cutoffs as

$$G(g_r)^{l_m} - g_m^{l_m} = \frac{\rho_1 n}{\beta \alpha_m} \tag{A.3}$$

and

$$\frac{\alpha_r}{n}(1 - g_r^{l_r}) + \frac{\beta \alpha_m}{n}(1 - g_r^{l_m}) = k_1 - \rho_1 \tag{A.4}$$

with $g_r = G_r(p_r^{AS})$, $g_m = G_m(p_m^{AS})$ and $G = G_m \circ G_r^{-1}$. The number of students assigned to their top school is

$$\frac{1}{1+\beta}\Big(\alpha_r(1-g_r) + (1-\alpha_r) + \beta\alpha_m(1-g_m) + \beta(1-\alpha_m)\Big).$$

Thus, $\frac{\partial F(1,\rho_1)}{\partial \rho_1} > 0$ iff

$$-\alpha_r \frac{\partial g_r}{\partial \rho_1} - \beta \alpha_m \frac{\partial g_m}{\partial \rho_1} > 0. \tag{A.5}$$

Taking derivatives with respect to ρ_1 in (A.3), we deduce that

$$l_m G(g_r)^{l_m - 1} G'(g_r) \frac{\partial g_r}{\partial \rho_1} - l_m g_m^{l_m - 1} \frac{\partial g_m}{\partial \rho_1} = \frac{n}{\beta \alpha_m}.$$

We can thus solve for $\frac{\partial g_m}{\partial \rho_1}$ and plug it into (A.5) to deduce that $\frac{\partial F(1,\rho_1)}{\partial \rho_1} > 0$ iff

$$-\alpha_r \frac{\partial g_r}{\partial \rho_1} - \beta \alpha_m \frac{l_m G(g_r)^{l_m-1} G'(g_r) \frac{\partial g_r}{\partial \rho_1} - \frac{n}{\beta \alpha_m}}{l_m g_m^{l_m - 1}} > 0.$$

Taking derivative with respect to ρ_1 in (A.4)

$$(-\frac{\alpha_r}{n}) l_r g_r^{l_r - 1} \frac{\partial g_r}{\partial \rho_1} + (-\beta \frac{\alpha_m}{n}) l_m G(g_r)^{l_m - 1} G'(g_r) \frac{\partial g_r}{\partial \rho_1} = -1.$$

We can solve for $\frac{\partial g_r}{\partial \rho_1}$ to deduce that $\frac{\partial F(1,\rho_1)}{\partial \rho_1} > 0$ iff

$$l_m g_m^{l_m - 1} < l_r g_r^{l_r - 1}. \tag{A.6}$$

Now, note that g_m can be solved from equation (A.3) so

$$g_m^{l_m} = G(g_r)^{l_m} - \frac{\rho_1 n}{\beta \alpha_m}$$

and therefore to deduce (A.6) it is enough to show

$$l_m G(g_r)^{l_m-1} < l_r g_r^{l_r-1}.$$

Take $\bar{g} = \bar{g}(l_m, l_r, G_m, G_r) < 1$ such that the condition above holds provided $g_r > \bar{g}$. From (A.4), it follows that g_r is decreasing in $k_1 - \rho_1$ and $g_r \to 1$ as $k_1 - \rho_1 \to 0$. In particular, there exists $\bar{k}$ such that for all k_1 and all $\rho_1 < k_1$, $g_r > \bar{g}$. The result follows. □

B Other segregation indexes

We adapt Proposition 1 for the Hutchens index (Hutchens 2004):

$$H_\mu = 1 - \sum_{c \in C} \sqrt{\eta_\mu^r(c) \cdot \frac{\eta_\mu^m(c)}{\beta}}$$

Note first that H_μ does not depend on ρ_1 when $\rho_1 \notin [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))_m^l, \min\{\frac{\alpha_m \beta}{n}, k_1\}]$.

Recall that for $\rho_1 \in [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))_m^l, \min\{\frac{\alpha_m \beta}{n}, k_1\}]$, each tier 1 school has $k_1 - \rho_1$ regular students and ρ_1 minority ones. Each tier 2 school has $\frac{1 - n(k_1 - r_1)}{m}$ regular students and $\frac{\beta - n r_1}{m}$ minority ones. Thus, for ρ_1 in this range, the H-index is computed as:

$$H_\mu = 1 - \underbrace{n\sqrt{\frac{\rho_1(k_1 - \rho_1)}{\beta}}}_{H_1} - \underbrace{m\sqrt{\frac{(1 - n(k_1 - \rho_1))(\beta - n\rho_1)}{m^2 \beta}}}_{H_2}$$

where the terms H_1 and H_2 correspond to the sum across tier 1 and tier 2 schools respectively.

Taking derivatives we get that:

$$\frac{\partial H_\mu}{\partial \rho_1} = -\frac{n}{2\sqrt{\beta}} \left(\frac{k_1 - 2\rho_1}{\sqrt{\rho_1(k_1 - \rho_1)}} + \frac{\beta - 1 + n(k_1 - 2\rho_1)}{\sqrt{(1 - n(k_1 - \rho_1))(\beta - n\rho_1)}} \right)$$

And also that:

$$\frac{\partial^2 H_\mu}{\partial \rho_1^2} = \frac{n}{4\sqrt{\beta}} \left(\frac{k_1^2}{[\rho_1(k_1 - \rho_1)]^{3/2}} + \frac{n(\beta + 1 - nk_1)^2}{[(1 - n(k_1 - \rho_1))(\beta - n\rho_1)]^{3/2}} \right) > 0$$

So we deduce that H_μ is a strictly convex function. Since $\frac{\partial H_\mu}{\partial \rho_1} = 0$ when $\rho_1 = \frac{\beta}{1+\beta} k_1$, the result follows.

The Atkinson index (Frankel and Volij 2011) can be defined in our setup as:

$$A_\mu = 1 - \left[\sum_{c \in C} \eta_\mu^r(c)^\delta \cdot \left(\frac{\eta_\mu^m(c)}{\beta} \right)^{1-\delta} \right]^{\frac{1}{1-\delta}}$$

Where $\delta \in (0, 1)$ is a fixed weight. In the symmetric case in which both types are treated equally in the segregation index, $\delta = \frac{1}{2}$ and thus the Atkinson index is obtained by an increasing transformation of the Hutchens index. The result follows.

C Field data

C.1 Markets, popularity and standardized tests

The Valparaiso market includes each school located in the provincial department of Valparaiso. The Concepcion market includes each school located in the provincial department of Concepcion. The Santiago market includes each school located in the Metropolitan Region of Santiago. As Santiago is the capital city of the country, the provincial department of Santiago excludes several towns close to Santiago whose students apply to schools in the city. So, the boundary of each of our markets follows administrative definitions.

For each market, we consider all students that apply exclusively within the market. Thus a student with a rank order list including some schools in Valparaiso and others outside Valparaiso is excluded from our exercise. This set of students is small as big urban centers heavily concentrate applications. In our database, 99.76% of all nationwide applications listing some school in the Santiago market list exclusively schools in Santiago. The numbers for Valparaiso and Concepcion are 98.85% and 99.66%, respectively. The following table shows the characterization for each market:

Table 6: Valparaiso, Concepcion and Santiago markets

	Valparaíso	Concepción	Santiago
Number of provincial departments	1	1	7
Number of counties	10	12	52
Number of schools	275	250	1,214
Applicants to the market applying exclusively inside the market	98.85%	99.66%	99.76%

Thus, in practical terms, each of our markets is isolated and independent from all other markets in the country.

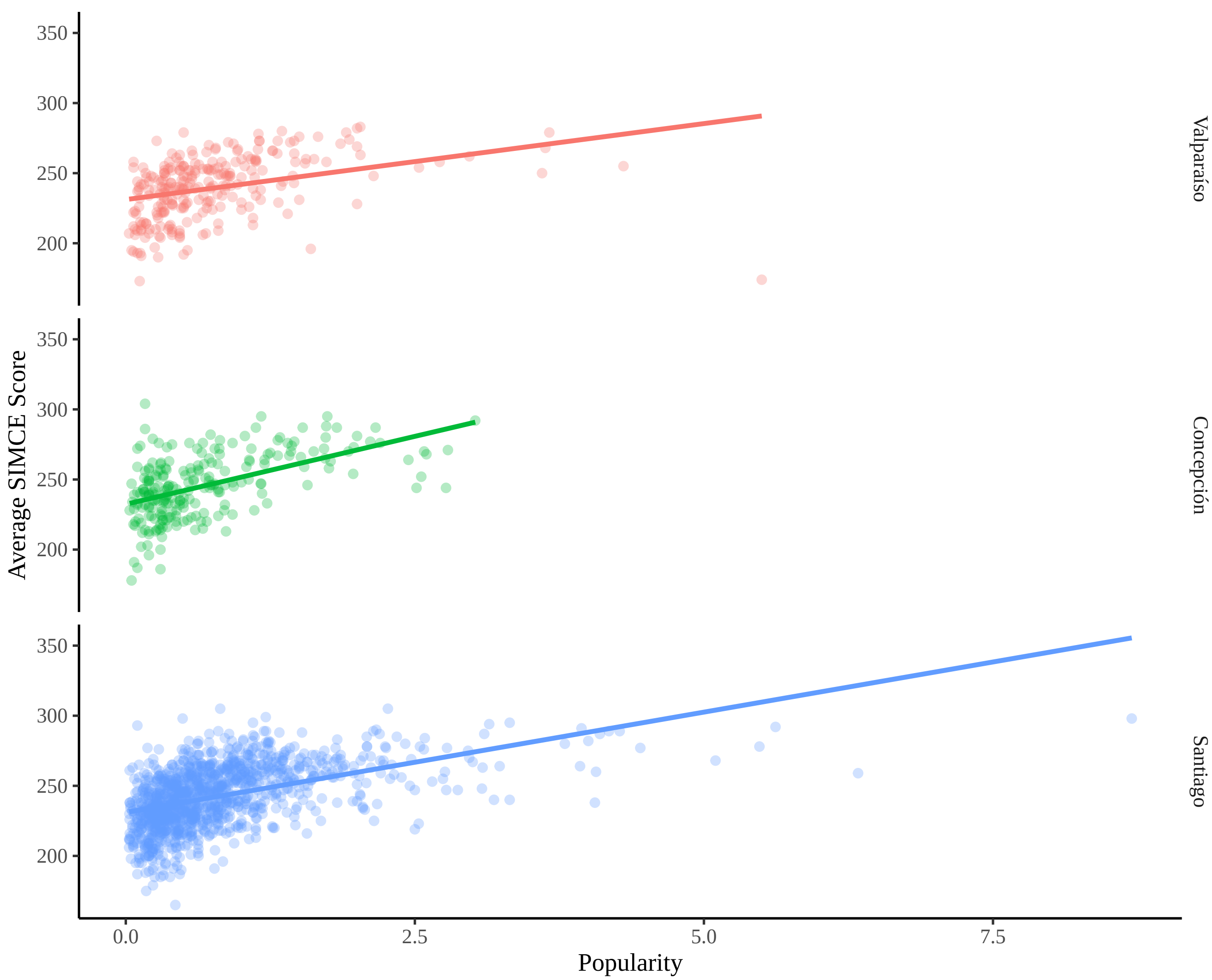
As discussed in the text, popular schools tend to perform better in standardized tests. For each market, we restrict our set of schools to those such that: (1) took part in SIMCE 2015 test[31] (2) reported valid SIMCE scores . We only use data from the Language test of second degree students in 2015. Popular schools are those such that $pop(c) > 1$.

[31]SIMCE is a standardized test taken to all students in the country

Table 7: SIMCE scores

	Valparaíso		Concepción		Santiago	
	Not popular	Popular	Not popular	Popular	Not popular	Popular
Sample (number of schools)	200	61	185	53	884	302
First quartile	222	244	224	259	223.75	248.00
Median	239	259	239	269	236	260
Mean	234.84	254.21	239.21	267.47	236.87	259.10
Third quartile	250	269	252	277	251	271

Figure 11: Popular schools tend to have higher SIMCE scores



These results show that popular schools have better performance in standardized tests. Obviously, this is just illustrative and we are not claiming any causality.

C.2 Application patterns and distance

Denote by F the distribution of the popularity of schools ranked first by minority students, and G the distribution of the popularity of the schools ranked first by regular students. Let $F_n(w), G_n(w)$ be the corresponding empirical distributions. Following McFadden (1989), we use a Kolmogorov-Smirnov-type statistic for testing if F first order stochastically dominates G. Thus, for each market, we conduct a one-sided two-sample Kolmogorov-Smirnov test for the null hypothesis $H_0 : F(w) \leq G(w)$ for some $w \in [0,1]$ against the alternative $H_1 : F(w) > G(w)$ for each $w \in [0,1]$ using the statistic:

$$D^+ = \max_w \{F(w) - G(w)\}$$

Using the KS.TEST function of the STAT package from the R Statistical Software (Core Team 2020) we obtain the following results:

	Valparaíso		Concepción		Santiago	
Obs. (M - R):	2,994	3,825	3,233	4,290	18,378	30,545
Statistic D^+:	0.18501		0.20398		0.16423	
P-Value:	$< 2 \cdot 10^{-16}$		$< 2 \cdot 10^{-16}$		$< 2 \cdot 10^{-16}$	

Table 8: Two Sample KS Tests - Popularity of first choices

Similarly, we run the Kolmogorov-Smirnov test using the popularity of the schools listed second, restricting attention to students applying first to a school having popularity at least 1. We obtain the following results:

	Valparaíso		Concepción		Santiago	
Obs. (M - R)	1,463	2,439	1,759	3,187	9,030	19,934
Statistic D^+:	0.1621		0.1591		0.1255	
P-Value:	$< 2 \cdot 10^{-16}$		$< 2 \cdot 10^{-16}$		$< 2 \cdot 10^{-16}$	

Table 9: KS Test - Popularity of second choices conditional on applying first to a popular school

Understanding why minority students apply less to popular schools is beyond the scope of this paper. We observe that distance may be playing a role because minority students tend to live farther away from popular schools. To see this, in each market, we restrict our set of students to those that are market as *properly georeferenced* by the Chilean Ministry of Education[32]. For these set of students, we compute

[32]Students that shared their location when applying on the platform or those whose location held a unique response and

the distance to the closest popular school ($pop(c) > 1$) using the Vincenty (ellipsoid) method provided by the GEOSPHERE package from the R Statistical Software. The resulting distributions are presented below:

	Valparaíso		Concepción		Santiago	
	Regular	Minority	Regular	Minority	Regular	Minority
Sample (number of students)	2494	1833	2907	2029	22508	13089
First quartile	0.47	0.52	0.42	0.48	0.38	0.42
Median	0.81	0.92	0.70	0.82	0.63	0.69
Mean	2.86	1.37	1.13	1.27	0.97	1.19
Third quartile	1.36	1.59	1.16	1.38	1.02	1.07

Table 10: Distance (Km.) to the closest popular school

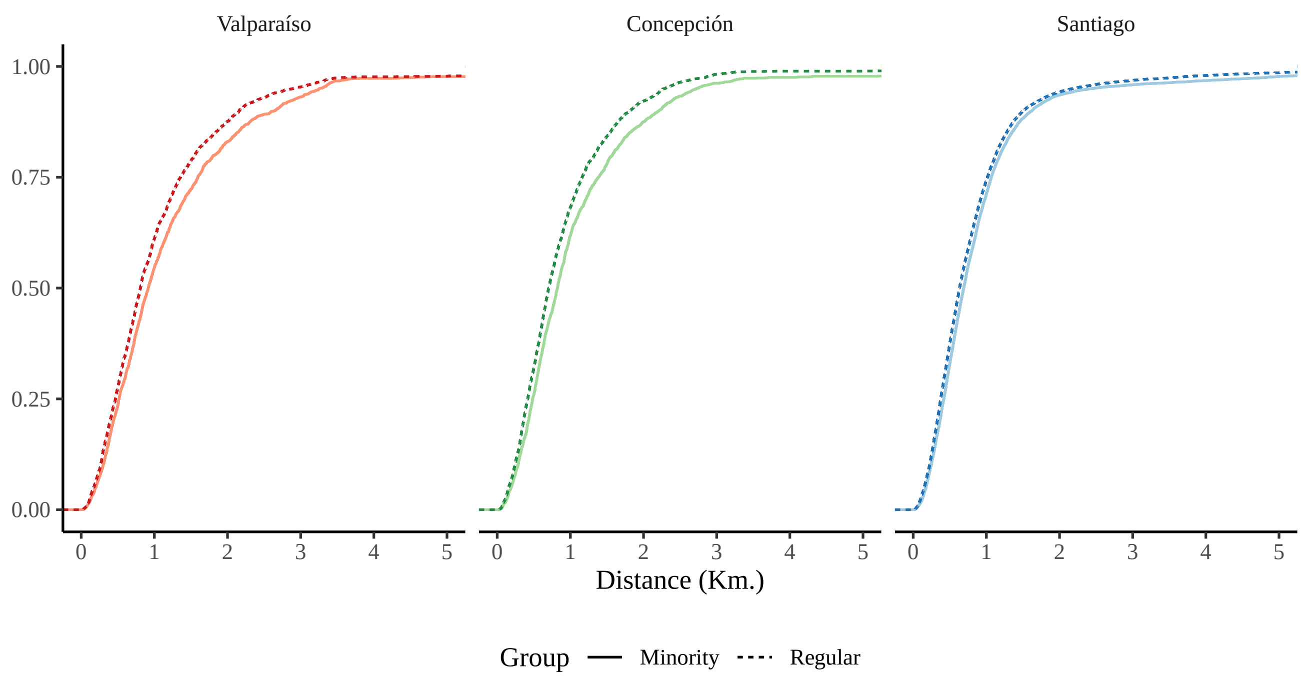


Figure 12: Distance to closest popular school. Minority students live farther away from popular schools than regular students.

was marked as “rooftop” or “range_interpolated” in the “location_type” variable of Google’s Geocoding API.

C.3 Segregation in schools

In the main body of the paper, we have explored how an aggregate segregation index (the Duncan index) changes as minority reserves increase. Figure 13 shows how segregation in each school is determined by its popularity and by the minority reserve. Each school is an observation. As can be seen popular schools tend to have a lower fraction of minority students. The upper graphs are derived without any minority reserve. The lower graphs are derived with minority reserves equal to the fraction of minority students in the population.

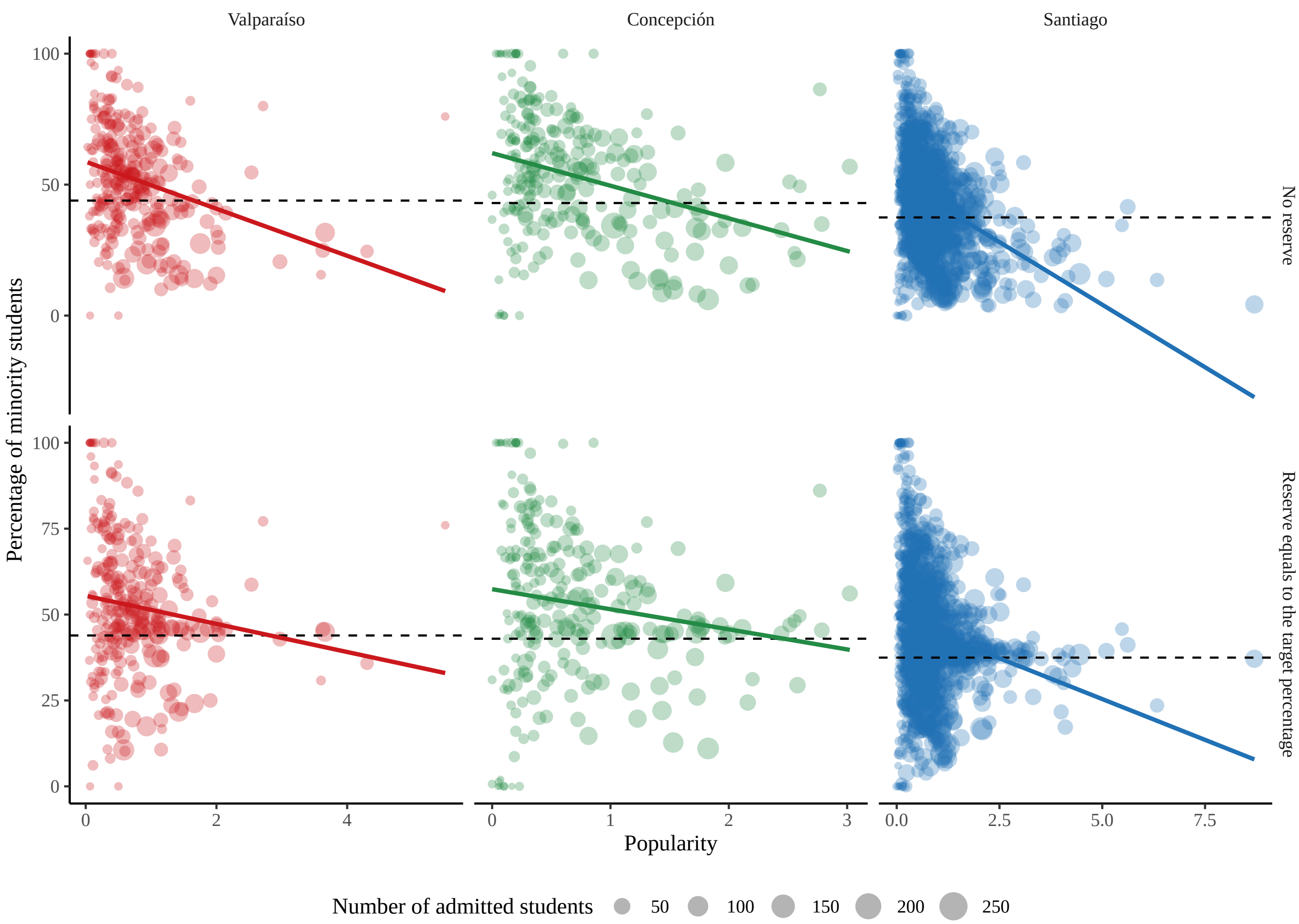


Figure 13: Schools composition. Minority students are under-represented in popular schools.

The following table also shows the number of schools that become less segregated after reserves are introduced. We say that a school becomes less segregated if, after introducing the reserve, the distance between the share of minority students in the school get closer to the proportion of minority schools in the population.

Table 11: School that reduce segregation after introducing reserves

Market	Total number of schools	Improving in segregation
Valparaíso	272	163 (59.93)
Concepción	245	153 (62.45)
Santiago	1206	795 (65.92)