

School Choice in Chile

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Centralized school admission mechanisms are an attractive way of improving social welfare and fairness in large educational systems. In this paper we report the design and implementation of the newly established school choice system in Chile, where over 274,000 students applied to more than 6,400 schools. The Chilean system presents unprecedented design challenges that make it unique. First, it is a simultaneous nationwide system, making it one of the largest school choice problems worldwide. Second, the system is used for all school grade levels, from pre-K to 12th grade. One of our primary goals is to favor the assignment of siblings to the same school. By adapting the standard notions of stability, we show that a stable assignment may not exist. Hence, we propose a heuristic approach that elicits preferences and breaks ties between students in the same priority group at the family level. In terms of implementation, we adapt the Deferred Acceptance algorithm as in other systems around the world.

Key words: school choice, matching, two-sided market

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1. Introduction

According to the Duncan Segregation Index, Chilean schools are extremely socially segregated (Bellei 2013, Valenzuela et al. 2014). Several authors have shown that the costs of school segregation are high, including low social cohesion and lack of equal opportunities and social mobility

(Villalobos and Valenzuela 2012, Wormald et al. 2012). While the drivers of school segregation include societal aspects well beyond school choice, social movements and politicians were probably right in blaming features of the admissions system.

The School Inclusion Law marks a breaking point in the organization and functioning of the school system. The law, promulgated in 2015, changed the old admissions process drastically by (i) eliminating co-payments in publicly subsidized schools; (ii) forbidding publicly subsidized schools from selecting their students based on social, religious, economic, or academic criteria; and (iii) defining priorities that must be used to assign students to schools.¹

In this paper we report the results of an ongoing collaboration with the Chilean Ministry of Education (MINEDUC) addressing the practical challenges of implementing the School Inclusion Law. To this end, we designed and implemented a centralized system that (i) provides information about schools to help parents and students in building their preferences; (ii) collects families' preferences through an online platform, reducing the time and cost that visiting each school involved in the past; and (iii) assigns students to schools using a transparent and fair procedure.

One of the distinctive features of this new school choice system is its universality, as it is used nationwide and for all school grade levels (from pre-K to 12th grade). We agreed with MINEDUC that one of the primary goals of the system should be to obtain an assignment that favors the joint allocation of siblings to the same school, although it is not required by law. The reason is that, since the new law does not include walk-zone priorities and there is no public provision of school transportation, the simplest way to reduce the travel time of families with multiple children is to increase the chances of them getting assigned to the same school. This objective imposes several challenges, as it introduces complementarities into the preferences of families, similar to those in the matching with couples literature. Indeed, we show that a stable assignment may not exist if families are allowed to report their preferences regarding the joint assignment of their children. For this reason, we adopt a heuristic approach where we allow families to submit one preference list per child, and we also allow them to report whether they want to prioritize the joint assignment of their children. If this is the case, our heuristic automatically updates the preferences of the younger siblings to account for the assignment of older siblings and to leverage the siblings' priority. We explore the stability and incentives of the mechanism when families' preferences are consistent with this heuristic, which we call higher-first preferences (see Sections 4.1 and 4.1.4, respectively). Further, we break ties between students in the same priority group at the family level within each school, as opposed to having one lottery number per student. Lotteries over families create correlations between the priorities of siblings applying to different grades in a given school. We show, theoretically and through simulations, that this correlation increases the probability that siblings are assigned to the same school.

Apart from obtaining a fair allocation that favors the assignment of siblings to the same school, our implementation—based on the Deferred Acceptance (DA) algorithm introduced in the seminal paper by Gale and Shapley (1962)—needs to accommodate several elements required by law and by MINEDUC. In particular, the system needs to consider a set of priority groups—students with siblings in the school, students with parents that work in the school, and students returning to the school—that are served in strict order of priority. The system also needs to fill quotas for students (i) with special educational needs and disabilities, (ii) with high academic performance, and (iii) from disadvantaged environments. The law requires ties between students be broken at the school level, and that students who are currently enrolled but are trying to transfer to a different school be guaranteed the option to enroll in their current school if they cannot improve their assignment.

The results reported in this paper consider the admissions process of 2018—for students who started the academic year in March 2019—which includes all regions except the Santiago metropolitan area, involving 274,990 students and 6,421 schools in the main round. In this admissions process students applied to 3.18 schools on average, and 59.2% of students were assigned to their top preference. Moreover, 82.5% of the students were assigned to one of the schools in their preference list, 8.6% were assigned to their current school, and only 8.9% were unassigned. In addition, there were 10,301 family applications involving 21,424 students and 65.3% of these were successful, i.e., siblings got assigned to the same school, while 3% were partially successful, i.e., only a subset of siblings got assigned together.² We also provide simulations evaluating different elements of our design, including: (1) the use of a family application (as opposed to no updating of preferences); (2) the use of a family lottery (as opposed to a student lottery); (3) modifying the order in which we process quotas; and (4) processing grade levels in decreasing order (as opposed to doing so in increasing order).

Contributions. Designing, implementing, and improving the Chilean school choice system has resulted in many contributions that could help other practitioners design large-scale clearinghouses. From a theoretical standpoint, we contribute to the existing literature by introducing the notion of family applications. We show that a stable matching may not exist, and we provide heuristics that are successful at increasing the fraction of siblings assigned to the same school. In addition, our results show that having lotteries over families significantly increases the fraction of siblings assigned to the same school.

From a practical standpoint, a key lesson is that maintaining continuous communication and collaboration with policy-makers is essential, as many practical issues arise and must be incorporated into the design. In addition, decomposing the implementation into a given number of steps allowed us to gain experience, solve unexpected problems, and continuously improve the system.

As centralized procedures to assign students to schools are becoming the norm in many countries, we expect that the lessons and solutions offered in this work will be deemed useful in other implementations.

The remainder of the paper is organized as follows. In Section 2 we describe the school choice problem in Chile. In Section 3 we discuss how this paper relates to several strands of the literature. In Section 4 we present our model and describe its implementation. In Section 5 we present the results. In addition, we evaluate the effects of (i) family applications and (ii) quotas for disadvantaged students via simulations. Finally, in Section 6 we conclude and provide directions for future work.

2. The School Choice Problem in Chile

Depending on their type of funding, schools can be classified into three types: (1) private, i.e., schools that are independent and privately funded; (2) voucher, i.e., schools where families make co-payments to complement state subsidies; and (3) public, i.e., schools that are fully funded and operated by local governments. Voucher and public schools, which are the focus of this paper, account for more than 90.3% of the total number of students in primary and secondary education (MINEDUC 2018). These schools can offer a subset of the fourteen grades that are part of the Chilean school system (from pre-kindergarten to 12th grade), but their lowest grade must be one of the following five entry-level ones: pre-kindergarten, kindergarten, 1st, 7th, and 9th grade.

Before the introduction of the School Inclusion Law, schools ran their admission processes independently, often selecting their students based on arbitrary rules, such as interviews with the students and their parents, results of unofficial admissions exams, past academic records, and more. Since the admissions processes were not coordinated, in many cases parents were forced to decide whether to accept an offer or to reject it and wait until other schools released their admissions offers, and declined seats were not efficiently reassigned. Moreover, many schools used “first-come first-served” rules to prioritize students, resulting in parents waiting in long overnight queues to secure a seat for their children. Overall, the freedom of schools to choose their students and the existence of voucher schools are considered among the main reasons for the polarization and segregation of the Chilean school system (Valenzuela et al. 2014).

To address these problems, the School Inclusion Law forbids any sort of discrimination in the admissions processes of schools that receive (partial or full) government funding, and mandates schools to use a centralized system that collects families’ and students’ preferences and returns a fair allocation. In this system, students and families can access a platform where they collect information—number of open seats, number of students per classroom and grade, educational project, rules and values, co-payments required, and more—to build their preferences. Later, they

can use this information to apply to as many schools as they want by submitting a strict order of preferences. The system collects all these applications and runs a mechanism that aims to assign each student to their top preference provided that there are enough seats available. More specifically, if the number of applicants is less than the number of open seats, the law requires that all students applying to that school be admitted, unless they can be allocated to a school they prefer to it. On the other hand, for schools that are over-demanded, the law defines a set of priority groups that are used to order students. In particular, there are three priority groups, which are processed in strict order of priority:

1. *Sibling*. This group consists of students that have a sibling already enrolled or admitted at the school.
2. *Working parent*. This group consists of students that have a parent working at the school.
3. *Returning student*. This group consists of students that were enrolled at the school in the past and were not expelled from it.

In addition to these priorities, the law specifies three different types of quotas:

1. *Special needs*. This quota prioritizes students with disabilities. It reserves at most two seats per classroom per school and must be processed before any other priority group or quota. The quota only applies to schools that have a validated special program.
2. *Academic excellence*. This quota prioritizes students with high academic performance. It must be processed right after the special needs quota and assigns between 30% and 85% of the total number of seats depending on the school. MINEDUC allows only a subset of pre-selected schools to implement this quota in the 7th and 9th grades, and schools can rank students based on an admissions exam only.³
3. *Disadvantaged*. This quota prioritizes the most vulnerable students (the bottom third in terms of income according to the Social Registry of Homes). At each grade in every school, 15% of the seats are reserved for disadvantaged students, and this group is processed right after students with siblings.

Finally, the School Inclusion Law sets three additional requirements: (1) ties between students in the same priority group must be broken at each school independently, i.e., a Single Tie-Breaking (STB) rule cannot be implemented;⁴ (2) if a student that is currently enrolled in a school participates in the system with the aim of transferring but remains unassigned, the system must guarantee that student the option to enroll in her current school; and (3) students that are left unassigned must be allocated to the school with remaining seats that is closest to their home. We refer to this as *assignment by distance*.⁵

To accommodate all these requirements, the first step was to decide which mechanism to use to perform the allocation. The law only requires that the resulting assignment be fair, and so

we considered two alternatives: the Deferred Acceptance algorithm, and the Top-Trading Cycles (TTC) algorithm. We decided to opt for the former because communicating the results of the assignment—especially to families that are unhappy with the allocation—is much simpler under DA. Moreover, this mechanism has been used in many other school districts worldwide. A second major choice was how to handle families with multiple children participating in the system. As opposed to many frameworks for inclusion in education around the world, the School Inclusion Law intentionally excludes walk-zone priorities due to the high urban segregation that characterizes most of the major cities in Chile. In addition, public provision of school transportation is almost nonexistent, making families responsible for getting their children to school. Hence, having siblings assigned to different schools can dramatically increase the transportation time and cost for families, and therefore the assignment of siblings to the same school is a priority in our design.

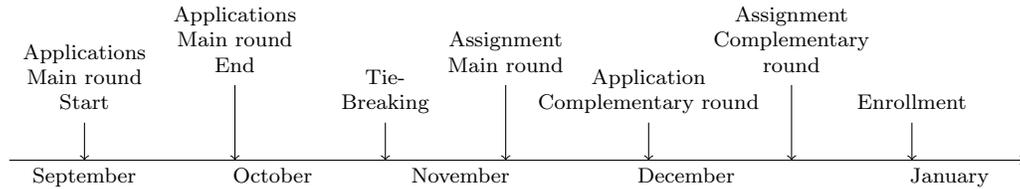
As discussed in the previous section, accommodating this goal is challenging because it introduces complementarities into the preferences of families with multiple children participating in the system, similar to those in the literature on matching with couples. However, the problem with families is even more complicated because families may have more than two children participating in the system, increasing the complexity of eliciting their preferences for their children’s joint allocation. For instance, a family with three children interested in applying to three schools would have to submit a preference list with 27 triplets to exhaust all possibilities. Moreover, due to the siblings’ priority, the ordering of students by schools becomes *dynamic*, as at any given iteration of the mechanism a student may be tentatively assigned to a school, thereby increasing the priority of her siblings being assigned to that school. This feature is in sharp contrast with the standard implementation of DA, where students’ preferences and schools’ priorities are fixed and known.

To address these challenges, we made three important decisions. First, instead of eliciting tuples of preferences to account for all siblings, we ask families to report one preference list per child and whether they want to prioritize the joint assignment of their children in the same school over each child’s individual preferences. We refer to this feature as a *family application*. Second, to avoid the problems associated with simultaneously finding an assignment (for all grade levels) that satisfies families’ preferences and schools’ priorities, we process grade levels sequentially and in decreasing order. More specifically, we start by solving the allocation of students in the 12th grade. Then, we use this allocation—and the enrollment of siblings not participating in the system—to update schools’ priorities in all lower grades to account for the siblings’ priority. We also use this allocation to update the preferences of siblings in a family application. In particular, if a family submits a family application and the older sibling is assigned to a school included in the preference lists of the younger siblings, the preference lists of the younger siblings are updated to place that school as their top preference.⁶ Based on the updated students’ preferences and schools’ priorities, we

then obtain the next grade’s allocation. We repeat this process until we obtain the allocation for the lowest grade (pre-K). Finally, the third choice we make to favor the joint allocation of siblings is to break ties between students in the same priority group (if any) at the family level in each school. As we show theoretically in Section 4 and through simulations in Section 5, this approach to breaking ties considerably helps to increase the fraction of siblings assigned to the same school.

2.1. Timeline of Process

Figure 1 Timeline of the Admissions Process



We summarize the timing of the admissions process in Figure 1. Families submit their preference lists between September and October. The centralized mechanism collects all these lists, generates the lotteries used to order students in over-demanded schools, and executes the main round of the process to obtain the allocation. Families have five days to make one of the following decisions: (1) to accept their initial assignment, (2) to wait in case of improvement from movements in the waiting lists, and accept the resulting assignment, (3) to reject the assignment, and (4) to reject it but also wait in case of improvement from movements in the waiting lists. Families that do not accept their assignment or are unassigned, together with those that do not participate in the main round, can participate in the complementary round by submitting a new preference list that only includes schools with available seats. Students that are unassigned in the complementary round are assigned to the closest school with available seats that does not charge a co-payment. The complementary round results are published in mid-December, and families can choose to either accept their assignment or reject it and reach out to MINEDUC to find a better assignment directly. Notice that students and families have incentives to accept their main round allocation, as there are fewer seats available in the complementary round, and rejecting the main round allocation entails giving up both their assignment and their right to enroll in their current school (if any). The last step in the timeline is enrollment, which takes place in late December. The system grants students the right to enroll in the school to which they were assigned. However, students can also enroll in schools outside the centralized system (private schools and some other exceptions), or they can directly contact schools that are part of the system to check whether they can accommodate them. However, direct contact can happen only after the official enrollment is over. In Appendix B.5 we discuss the results of the enrollment process.

3. Literature

This paper is related to five strands of literature: (1) school choice, (2) implementation of large-scale clearinghouses, (3) affirmative action, (4) matching with externalities, and (5) tie-breaking.

School Choice. In the past two decades, starting from the theoretical formalization of the school choice problem by Abdulkadiroğlu and Sönmez (2003), there have been reforms to the school choice system in many places worldwide. New York City introduced the first major reform, implementing a variation of the Deferred Acceptance (DA) algorithm with restricted lists (Abdulkadiroğlu et al. 2005a). In 2005, the Boston public school system decided to switch from the so-called Boston Mechanism (BM), also known as Immediate Acceptance (IA) mechanism, to DA to address the strategic incentives introduced by the former algorithm (Abdulkadiroğlu et al. 2005b). Since then, other school systems, e.g., Barcelona (Calsamiglia and Güell 2018), Amsterdam (Gautier et al. 2016), and New Orleans (Abdulkadiroğlu et al. 2017), have implemented centralized school choice systems using some variant of DA, BM, or Top-Trading Cycles (TTC). Abdulkadiroğlu and Sönmez (2003) also initiated a large literature that theoretically analyzes the school choice problem. Recent papers have extended it by including multiple priorities and quotas (see discussion below), allowing different admissions processes to run simultaneously (Manjunath and Turhan 2016) or sequentially (Andersson et al. 2018), optimizing other distributional goals (Bodoh-Creed 2020), and more. This paper contributes to this literature by adding a feature that has not been explored in previous literature: favoring siblings' joint allocation to the same school.

Priorities and Affirmative Action. Many school choice systems include affirmative action policies to promote diversity in the classroom. Ehlers (2010) explores DA under type-specific quotas, finding that the student-proposing DA is strategy-proof for students if schools' priorities satisfy responsiveness. Kojima (2012) studies the implementation of majority quotas and shows that these may hurt minority students. Consequently, Hafalir et al. (2013) propose using minority reserves to overcome this problem, and show that DA with minority reserves Pareto dominates DA with majority quotas. Ehlers et al. (2014) extend the previous model to account for multiple disjoint types and propose extensions of DA to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima (2015), who study problems with distributional constraints motivated by the Japanese Medical Residency program. Dur et al. (2016a) and Dur et al. (2016b) analyze the Boston and Chicago school systems, respectively.

Within the literature on affirmative action, the line of research closest to our paper is that on multiple reserves and overlapping types. Kurata et al. (2017) study this problem and show that a stable matching might not exist even in the soft-bound minority quota scenario. As a solution, they propose a model in which they recover stability by assuming that students have

preferences over contracts that specify the school and the type of seat to be used, while schools have preferences over contracts specifying the student and the type of seat. A similar setting is assumed by Aygün et al. (2020), who propose a mechanism to transfer seats from low-demand groups to high-demand ones in order to reduce the number of unassigned seats. Recently (following the implementation of the system in Chile), two papers—Sönmez and Yenmez (2020) and Delacrétaz (2020)—consider overlapping types. Both papers axiomatically characterize desirable properties and propose algorithms to find an allocation satisfying them. Sönmez and Yenmez (2020) aim to maximize the number of targeted students receiving a seat, while Delacrétaz (2020) focuses on respecting priorities and treating all target groups identically. Although both goals are valid, we cannot implement the algorithm of Sönmez and Yenmez (2020) in our setting. The reason is that their algorithm assumes that all seat types rank students according to the same *baseline priority order*, which is not the case in Chile. For instance, academic excellence seats are assigned based on test scores, special needs seats are assigned based on the fit of students with the infrastructure available to accommodate them, and general seats are assigned based on a random lottery. In this sense, the algorithm of Delacrétaz (2020) is closer to ours, as it allows students to have different priority orders for each type of seat. Nevertheless, the fact that most students have at most one type (93.8% in 2018)⁷ guarantees that our approach—based on Kurata et al. (2017)—incorporates quotas as minimum guarantees (Hafalir et al. 2013, Sönmez and Yenmez 2020), which is precisely our aim.

Matching with Externalities. The allocation of siblings is related to the work on matching with externalities, which extends the standard setting by allowing agents to have preferences over the allocation of other agents (Pycia and Yenmez 2015). An example of this is the labor market of medical residents, where couples prefer (in general) to be allocated to the same city. Roth (1984) shows that one cannot guarantee the existence of a matching without justified envy when couples have arbitrary preferences over pairs of hospitals. Kojima et al. (2013) show that a stable matching exists if the number of couples is relatively small and preference lists are sufficiently short relative to the size of the market. Another positive result is presented by Ashlagi et al. (2014), who introduce a new algorithm that finds a stable matching with high probability (in large matching markets) and where truth-telling becomes an approximate equilibrium for the induced game. Another example where complementarities are important is in the school choice context, where students may prefer to be assigned to the same school as their neighbors. Ashlagi et al. (2014b) show that using correlated lotteries, which maintain marginal assignment probabilities but increase the chance that students from the same neighborhood are assigned together, can significantly increase community cohesion. Dur and Wiseman (2019) study the case where neighbors share a subset of schools that they prefer to attend together and, beyond that subset, each of them has an individual ranking of schools.

The authors show that a stable matching may not exist, that the student-proposing DA algorithm is neither stable nor strategy-proof, and that there exists a variation of this algorithm to alleviate these problems. To our knowledge, the only paper that studies complementarities in the context of school choice with families is Dur et al. (2019b). The authors focus on the particular case where an assignment is feasible only if all family members submit the same preference list and all of them are assigned to the same school (or all of them are unassigned). These constraints may be too restrictive in a large system like the Chilean one, especially considering that most schools offer only a limited subset of school grade levels. Therefore, our paper expands their setting by introducing the family application and using lotteries at the family level to increase the probability that siblings are assigned to the same school. Moreover, we contribute to this literature by showing that a stable matching may not exist when there are family applications and by introducing a new heuristic that can solve this problem.

Tie-breaking. A common approach to breaking ties between students in the same priority group is to use random tie-breaking rules, such as Single Tie-Breaking (STB)—all schools use the same ordering for breaking ties—and Multiple Tie-Breaking (MTB)—each school uses a different random order. Abdulkadiroğlu et al. (2009) are the first to compare these tie-breaking rules empirically, and they find that there is no stochastic dominance between these tie-breaking rules in New York City’s school choice system. De Haan et al. (2015) obtain a similar result for Amsterdam. These findings are consistent with the theoretical results in Arnosti (2015) and Ashlagi et al. (2019). Arnosti (2015) shows that there is no first-order stochastic dominance among these two tie-breaking rules when preferences are short, as STB assigns more students to their top preferences, while MTB leads to more students being assigned. Similarly, Ashlagi et al. (2019) find no stochastic dominance when there is low competition. However, they also show that when there is a shortage of seats, STB almost dominates MTB and leads to a lower variance in students’ assignment preferences. We contribute to this literature by studying the effect of breaking ties at the family level to increase the probability that the mechanism assigns siblings to the same school.

Implementation of Large-Scale Clearinghouses. Our paper also contributes to the literature on designing large-scale clearinghouses. Laws, institutional details, and special requirements often forbid the use of tools directly taken from the theory, and other engineering aspects become relevant in the design and implementation. Special attention has been devoted to redesigning medical labor markets (Roth and Peranson 2002, Alon et al. 2018), college admissions systems (Biró 2008, Baswana et al. 2019, Rios et al. 2020), kidney exchange programs (Roth et al. 2004, Anderson et al. 2015), and the assignment to (pre)military branches and programs (Sönmez and Switzer 2013, Gonczarowski et al. 2019). We contribute to this literature by adding an example of successful implementation of a large-scale clearinghouse in the school choice context, and we also share some lessons that can be useful to other practitioners implementing large-scale clearinghouses.

4. Model

The Chilean school choice problem can be formalized as follows. Let S be a set of students and T be a set of traits. Each student s has a subset of traits $\tau(s) \subseteq T$, which captures special characteristics of the student, such as socioeconomic status, academic performance, and more. We say that $\tau(s)$ is the type of student⁸ s , and we denote by $S^t = \{s \in S : t \in \tau(s)\}$ the set of students with trait $t \in T$. In addition, let F be a partition of students into families, and let $f(s) \in F$ be the family of student s . Then, we say that students s and s' are siblings if and only if $f(s) = f(s')$, and we say that s is an only child if $f(s) \neq f(s')$ for all $s' \in S \setminus \{s\}$. Finally, let G be the set of all grade levels from pre-K to 12th grade. Each student $s \in S$ belongs to a grade in G that we denote by $g(s)$, and we denote by S_g the set of students in grade g .

On the other side of the market, let C be the set of schools. Without loss of generality we assume that each school $c \in C$ offers a number of seats q_{cg} in each grade $g \in G$, and we use $q_{cg} = 0$ to represent that grade g is not offered at school c . In addition, each school c in each grade g has a number of seats reserved—also referred to as *quotas*—for each student trait t , p_{cg}^t , and so $\sum_{t \in T} p_{cg}^t \leq q_{cg}$.

In a slight abuse of notation, we assume that each family $f = \{f_1, \dots, f_{|f|}\} \in F$ can be ordered and written as a tuple $(f_1, \dots, f_{|f|})$. Then, each (ordered) family f has a preference order \succ_f over possible assignments of its members, i.e., over tuples in the set $(C \cup \{\emptyset\})^f$, where the symbol \emptyset represents that a student is unassigned. For instance, if $f = (f_1, f_2)$, then for any $c, c', c'', c''' \in C \cup \{\emptyset\}$, $(c, c') \succ_f (c'', c''')$ implies that family f prefers that students f_1 and f_2 attend schools c and c' over schools c'' and c''' , respectively. On the other side of the market, each school has a preference order \succ_c over feasible subsets of assigned students, i.e., over sets in the power set of S , $\mathcal{P}(S)$. This preference order can be obtained as a result of considering priority groups, tie-breaking rules, and reserved seats.

An *assignment* is a function $\mu : S \cup C \rightarrow S \cup C \cup \{\emptyset\}$ such that (i) $\mu(s) \in C \cup \{\emptyset\}$ for every student s , (ii) $\mu(c) \subseteq S \cup \{\emptyset\}$ for every school c , and (iii) $\mu(s) = c$ if and only if $s \in \mu(c)$. In words, $\mu(s)$ represents the school where student s is assigned to, and $\mu(c)$ represents the set of students assigned to school c . In another abuse of notation, we denote by $\mu(f) = (\mu(f_i))_{i=1}^{|f|}$ the assignment of the members of family f , and by $\mu_{cg} = \{s \in S : \mu(s) = c, g(s) = g\}$ the subset of students assigned to grade g in school c .

An assignment μ is *feasible* if $|\mu_{cg}| \leq q_{cg}$; i.e., no school accepts more than the number of seats offered in each grade. There are two additional properties that are desirable in any assignment: *envy-freeness* and *non-wastefulness*. Given a feasible assignment μ , we say that a student s belonging to a family f has *justified envy* toward another student s' assigned to school c' if

1. $g(s) = g(s')$,
2. $(\mu(f_1), \dots, c', \dots, \mu(f_{|f|})) \succ_f \mu(f)$, and,
3. $(\mu(c') \setminus \{s'\}) \cup \{s\} \succ_{c'} \mu(c')$.

In words, student s has *justified envy* toward s' if both students are in the same grade, the family $f(s)$ prefers that student s be assigned to school c' to $\mu(s)$ given the assignment of her siblings $\mu(f(s) \setminus \{s\})$, and school c' prefers to exclude s' and accept s conditional on the other students admitted, $\mu(c') \setminus \{s'\}$. If there is no justified envy, we say that μ is envy-free. We say that μ is non-wasteful if no student claims an empty seat, i.e., there is no pair $s \in S$ and $c \in C$ such that $(\mu(f_1), \dots, c, \dots, \mu(f_{|f|})) \succ_{f(s)} \mu(f)$ and $|\mu_{cg(s)}| < q_{cg(s)}$. We say that a feasible assignment is *stable* if it is non-wasteful and there is no student that has justified envy.

As we will later show, these preferences are so general that a stable matching may not exist. Even if we further restrict our model to account for the special features of the Chilean case, the problem is still challenging due to the complementarities introduced by families' preferences and schools' priorities.⁹ For this reason, we make two simplifying assumptions. First, to deal with families' complementarities, we assume that each student submits a preference list and that some families prioritize the assignment of their children in higher grades over the assignment of their children in lower grades. As a result, we process grades sequentially in decreasing order, updating schools' priorities and students' preferences to account for the assignment in higher grades. Second, to deal with the complementarities generated by reserves, we assume that each reserve in each school is an independent sub-school with its own priorities and number of seats available, and that students have preferences for each sub-school. In Sections 4.1 and 4.2 we describe in detail the implementation of these assumptions.

4.1. Families

In Proposition 1 we show that a stable matching may not exist if the allocation is based on the joint preferences reported by families.¹⁰

PROPOSITION 1. *If families' preferences are arbitrary and schools' priorities are over students in each grade, then an envy-free and non-wasteful assignment may not exist, even with two schools and four students.*

Another issue of allowing families to report preferences over any tuple of schools is that it may be too complicated, as the number of combinations grows exponentially with the number of schools and with the number of siblings in a family. For instance, a family with two children where each of them prefers four schools to the outside option would require that the family apply to 16 pairs of schools to cover all combinations.

These issues suggest that we may need some more structure in families' preferences to guarantee the existence of a stable assignment and facilitate the reporting language to elicit families' preferences. Dur et al. (2019b) analyze a particular case of our model where families prefer having their siblings unassigned to having them assigned to different schools. By adapting the concept of justified envy to that setting, the authors show that an assignment satisfying their notion of stability always exists, and they propose an algorithm to find it. Although their assumption guarantees some notion of stability, the requirement that all siblings be assigned to the same school is too restrictive for our setting. For instance, many schools offer only a subset of grades, considerably reducing families' choice sets if we restrict ourselves to the setting in Dur et al. (2019b). In the next section, we discuss a different assumption on families' preferences that better fits the Chilean school choice problem and that guarantees the existence of a stable assignment.

4.1.1. Higher-First. To circumvent the aforementioned difficulties, we make further assumptions on families' preferences. In particular, we assume that families either (i) prioritize the assignment of their children in higher grades to the best possible schools and then prioritize the assignment of their siblings in lower grades to the same school, or (ii) prioritize the individual assignment of each child based on their individual preferences and not their joint preferences. If the former holds, we say that the family has *higher-first* preferences. To capture this, we make two important considerations. First, we simplify (and consequently restrict) the reporting language. In particular, we assume that each student s reports a strict preference order \succ_s over schools $c \in C \cup \{\emptyset\}$. In addition, we assume that each family f has the option to state whether their preferences satisfy *higher-first*, in which case we say that they submit a *family application*, or whether they prefer that the system treats each of their members independently.

DEFINITION 1. *Consider a family $f = (f_1, \dots, f_{|f|})$ with preference order \succ_f , and suppose that $g(f_1) \geq g(f_2) \dots \geq g(f_{|f|})$. Then, we say that \succ_f satisfies higher-first if there exist individual preferences $\{\succ_{f_i}\}_{i=1}^{|f|}$ such that for any $\vec{c}, \vec{c}' \in (C \cup \{\emptyset\})^f$, $\vec{c} \succ_f \vec{c}'$ if and only if, given $i = \arg \min \{j : c_j \neq c'_j, j \in \{1, \dots, |f|\}\}$, one of the following conditions holds:*

1. $c_i \succ_{f_i} c'_i$, and either (i) $c_i, c'_i \in \{c_1, \dots, c_{i-1}\}$ or (ii) $c_i, c'_i \notin \{c_1, \dots, c_{i-1}\}$; or
2. $c_i \in \{c_1, \dots, c_{i-1}\}$ and $c'_i \notin \{c_1, \dots, c_{i-1}\}$.

Second, we process grades sequentially and in decreasing order. More specifically, if grades $\{g_1, \dots, g_{|G|}\}$ are ordered in decreasing order (i.e., $g_1 = 12$ th grade and $g_{|G|} = \text{pre-K}$), we start by obtaining an assignment for g_1 using DA, while considering students' individual preferences and the siblings' priority for students with siblings enrolled in the corresponding school. Before processing g_2 , we update schools' priorities for g_2 to account for the siblings' new priorities that result from the allocation in grade g_1 . In addition, we update the individual preferences of students in g_2 who

have siblings assigned in g_1 and who are part of a family application, by moving the schools where their siblings were assigned to the top of their preference list while preserving their original order (see Example 1). Then, considering the updated students' preferences and schools' priorities, we obtain the allocation for g_2 , then move to the next grade, and then repeat the process until we obtain the assignment for grade $g_{|G|}$. Notice that, when updating students' preferences and schools' priorities in grade g_i , we consider the allocation of students in grades $\{g_1, \dots, g_{i-1}\}$, and preserve the original relative order of students and schools for priorities and preferences, respectively.

We jointly decided with MINEDUC to focus on a mechanism for *higher-first* preferences for three reasons. First, this mechanism increases the probability that siblings are assigned together. Second, it considerably simplifies the reporting language, making it easier to understand and reducing the complexity for families to build their preferences. Finally, it captures the fact that many families prioritize quality for their children's assignment in higher grades. By contrast, they prioritize convenience for their children's assignment in lower grades. The reason is that, to apply to most of the universities in Chile, students undergo a series of standardized national exams whose results are heavily correlated with the quality of the school that the students attended for their secondary education. Besides, students in lower grades are more dependent on their parents and older siblings for their transportation. Hence, families prioritize quality for their older children and convenience for their younger ones.¹¹

EXAMPLE 1. Consider a family $f = (f_1, f_2, f_3)$ with three members, and suppose that $g(f_1) > g(f_2) > g(f_3)$. In addition, suppose that the individual preferences are given by:

$$f_1 : c_1 \succ_{f_1} c_2 \succ_{f_1} c_3, \quad f_2 : c_2 \succ_{f_2} c_3 \succ_{f_2} c_1, \quad f_3 : c_3 \succ_{f_3} c_1 \succ_{f_3} c_2.$$

Suppose that f_1 is assigned to school c_1 . Then, if the family submits a family application, the preference list of f_2 is updated and becomes $c_1 \succ_{f_2} c_2 \succ_{f_2} c_3$. Notice that, in addition to this, f_2 receives the sibling priority in school c_1 , which further increases her chances of being assigned to the same school as f_1 . If f_2 is assigned to c_1 , then the preference list of f_3 becomes $c_1 \succ_{f_3} c_3 \succ_{f_3} c_2$, and she also receives the sibling priority in school c_1 . On the other hand, if f_2 is assigned to c_2 , then the preference list of f_3 becomes $c_1 \succ_{f_3} c_2 \succ_{f_3} c_3$, i.e., f_3 's preference list is updated to account for the assignment of f_1 and f_2 in schools c_1 and c_2 , respectively, but preserves the relative order between these two options (defined by f_3 's original preferences), and f_3 receives the sibling priority in both c_1 and¹² c_2 .

In Proposition 2 we show that a stable assignment always exists if preferences satisfy *higher-first* and grade levels are processed sequentially in decreasing order.

PROPOSITION 2. If grade levels are processed sequentially in decreasing order and the preferences of families satisfy *higher-first*, then the obtained assignment is stable.

4.1.2. Tie-Breaking. As discussed in Section 2, the priority groups included in the system define a partial order over students, and we must use a multiple random tie-breaking rule to obtain a strict order for each grade in each school. However, the School Inclusion Law does not further specify how to implement it. Since one of our primary goals is to favor the joint assignment of siblings, we propose to break ties at the family level in each school—using *family lotteries*—instead of breaking ties using student lotteries. Under this new approach, we first break ties between families and later use lotteries at the student level to break ties within each family. More precisely, for each school independently, we draw a uniformly random ordering over the families, and then we draw a uniformly random ordering over the members of each family. As a result, we obtain a strict ordering of students for each school and each grade.

EXAMPLE 2. Suppose that there is a school c and three families, $f = \{f_1, f_2\}$, $f' = \{f'_1, f'_2\}$, and $f(s) = \{s\}$ (an only-child student). Then, our procedure first draws an ordering of families uniformly at random, say $f' \succ_c f(s) \succ_c f$. Then, we draw an ordering of the members of each family also uniformly at random, say $f_1 \succ_c f_2$ for f and $f'_2 \succ_c f'_1$ for f' . The resulting ordering over all applicants is then $f'_2 \succ_c f'_1 \succ_c s \succ_c f_1 \succ_c f_2$. The same procedure is repeated independently for each school.

If there are no families with two or more applicants in the same grade, family lotteries induce the same distribution of assignments as the regular multiple tie-breaking rule within each grade. In Proposition 3 we show that using family lotteries increases the probability that families are assigned together if no family has two or more members in the same grade. This result implies that we can use family lotteries to increase the probability that siblings are assigned together without harming families with an only child participating in the system.

PROPOSITION 3. (Informal statement) Consider a family $f = \{f_1, f_2\}$ such that $g(f_1) > g(f_2)$. Given (and fixed) the students' preferences and schools' priorities, the probability that these siblings are assigned to the same school is larger under family lotteries than under student lotteries.

To ease exposition we defer the formal statement and proof of the latter proposition to Appendix A.3. Analyzing the case with siblings in the same grade is technically much more challenging, but arguably their effect in the system is small, as less than 2% of the applicants have a sibling applying to the same grade. Notice that the proposition works for a stylized case that simplifies many of the complexities of our problem. Nevertheless, this result sheds some light on why using family lotteries works, complementing the empirical analysis provided in Section 5.3.

4.1.3. Example: Family Applications and Lotteries. To illustrate the benefits of the family application and the lotteries by family we present the following example. Consider two schools, c and c' , that have a single seat in grades g_1 and g_2 , where the former is processed first. In addition, suppose that there is one family $f = \{f_1, f_2\}$ and two only-child students, s_1, s_2 , so that $g_1 = g(f_1) = g(s_1) > g(s_2) = g(f_2) = g_2$. Finally, suppose that $c \succ_s c'$ for all $s \in \{f_1, f_2, s_1, s_2\}$; i.e., all students prefer school c to c' . To illustrate the impact of the proposed policies, we compute the probability that the family is assigned together in each of the following four scenarios.

(i) *Lotteries by student, no family application.* The probability that the siblings are assigned together is equal to the probability that they are both assigned to either school c or c' . Since the probability that f_1 is assigned to c (or c') is $\frac{1}{2}$ and the probability that f_2 is assigned to c (or c') is also $\frac{1}{2}$, the overall probability that the siblings are assigned together is $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$.

(ii) *Lotteries by family, no family application.* There are three possible lottery outcomes for the family in school c , namely, being ranked first, second, or last. Each outcome has probability $\frac{1}{3}$. If the family is ranked first in c , f_1 and f_2 are assigned together in school c . If the family is ranked last in school c , both f_1 and f_2 are assigned to school c' . Finally, if the family is ranked second in school c , one child is assigned to school c and the other to c' . Then, the overall probability that the siblings are assigned together is $\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 = \frac{2}{3}$.

(iii) *Lotteries by student, with family application.* As in case (i), the probability that the siblings are assigned together in school c is $\frac{1}{4}$. However, the probability that the family is assigned to school c' is now $\frac{1}{2}$. The reason is that, once student f_1 is assigned to c' (which happens with probability $\frac{1}{2}$), we update f_2 's preferences so that she now prefers c' , and thus she gets assigned there for sure. Therefore, the overall probability that the siblings are assigned together is $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 1 = \frac{3}{4}$.

(iv) *Lotteries by family, with family application.* As in case (ii), the family may be ranked first, second, or last. Also, we know that the family is assigned together if it is ranked first or last. When the family is ranked second, either s_1 or s_2 is ranked first. In the former case, the family is assigned together because f_1 is assigned to school c' and so we update the preferences of f_2 (as in (iii)). In the latter case, the family is not assigned together because f_1 is assigned to school c and f_2 is assigned to c' . As a result, the overall probability that the siblings are assigned together is $\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{6}$.

4.1.4. Incentives In this section we analyze whether families have incentives to report their preferences truthfully. Certainly, this question is relevant only for families whose preferences are consistent with the reporting language, i.e., those with higher-first preferences. Recall that, under higher-first preferences, families report a preference list for each child (individual preferences) and whether they want to prioritize that their children are assigned together (by submitting a family

application). The mechanism is not strategy-proof in the general sense. However, as we point out in the following observations, the range of profitable deviations from reporting truthfully is relatively limited.

OBSERVATION 1. A family f with higher-first preferences cannot improve the assignment of one of its members $s \in f$ by misreporting the individual preference of s , given that f submits a family application. This comes from two facts: (1) if the individual preference of s is reported truthfully, then the true preference of f over assignments where only s goes to a different school is exactly the updated preference in the mechanism; and (2) using the updated preferences, the mechanism runs a student-proposing DA in each grade, which is incentive-compatible for the students.

OBSERVATION 2. If a family $f = (f_1, f_2)$ with higher-first preferences reports the individual preferences of f_1 and f_2 truthfully, then it is weakly optimal for them to submit a family application. This is because the only effect of submitting a family application is that the mechanism generates an updated preference for f_2 by moving to the top the school f_1 was assigned to, which is the true preference of f once the assignment of f_1 is fixed. Therefore, the same argument as in Observation 1 holds.

OBSERVATION 3. Consider a family $f = (f_1, f_2)$, with $g(f_1) > g(f_2)$, that has higher-first preferences. Under certain conditions f may improve the assignment of f_2 without changing the assignment of f_1 by misreporting the individual preference of f_1 .

OBSERVATION 4. A family f of three or more siblings with higher-first preferences may improve the assignment of one of its members without changing the assignment of the others by not submitting a family application, even if all individual preferences are reported truthfully.

OBSERVATION 5. In a large market with higher-first preferences our mechanism is essentially strategy-proof. In general, a stable matching can be characterized by market-clearing cutoffs per school such that a student is assigned to her most preferred school when she surpasses the cutoff (Biró 2008). When the market grows large the effect of a single student's preferences over the cutoffs vanishes (Abdulkadiroğlu et al. 2015, Azevedo and Leshno 2016). Therefore, the possible benefit a family gets from misreporting, which comes from manipulating the assignment in a higher grade to take advantage of the sibling priority in lower grades (as noted in Observations 3 and 4) also vanishes.

4.2. Quotas

As shown by Kurata et al. (2017), when student types overlap, the general concepts of stability with soft lower bounds proposed in the literature (Hafalir et al. 2013, Ehlers et al. 2014) are insufficient

to guarantee the existence of a stable matching. To overcome this difficulty, Kurata et al. (2017) propose a new model based on matching with contracts (Hatfield and Milgrom 2005). In this model, schools provide separate reserved seats for each student trait, and assignments are interpreted as contracts that explicitly state that a student is assigned to a particular reserved seat at a school, in contrast to previous models where a student is assigned to all the reserved seats for which she is eligible.

Due to its simplicity, we adapt their approach to our setting. First, we update students' preferences so that each student s has a strict preference order \succ_s over contracts of the form $(c, t) \in (C \times T) \cup \{\emptyset\}$. Second, we assume that each pair $(c, t) \in C \times T$ —which we refer to as a sub-school—has a weak priority profile \succeq_{ct} over students in¹³ $S \cup \{\emptyset\}$. Then, a *matching* is a function $\mu: S \cup (C \times T) \rightarrow S \cup (C \times T) \cup \{\emptyset\}$ such that:

1. $\mu(s) \in \{C \times T\} \cup \{\emptyset\}$ for all $s \in S$,
2. $\mu(c, t) \subseteq S$ for all $(c, t) \in C \times T$,
3. $\mu(s) = (c, t)$ if and only if $s \in \mu(c, t)$, for all $s \in S$ and for all $(c, t) \in C \times T$ and
4. $|\mu(c, t)| \leq p_c^t$ for all $(c, t) \in C \times T$.

In words, $\mu(s)$ represents the contract (or sub-school) to which student s is assigned; $\mu(c, t)$ represents the subset of students assigned to school c using the reserve for trait t . Note that this definition does not require matching students with trait t to seats reserved for this trait, providing extra flexibility if the reserves for some traits in some schools are not over-demanded. Another advantage of this formulation is that, based on the new preferences and priorities, the standard definition of stability directly applies, i.e., a matching is stable if and only if there is no pair $(s, (c, t)) \in S \cup (C \times T)$ such that, for some $s' \in S \setminus \{s\}$,

$$\mu(s') \succ_s \mu(s) \quad \text{and} \quad s \succ_{ct} s'.$$

In Section 4.2.1 we discuss in detail how we construct the preferences \succ_s and the priorities \succeq_{ct} .

4.2.1. Combining Quotas, Priorities, and Current Students. As discussed in Section 2, there are three priority groups (sibling, working parent, and returning student) and three quotas (special needs, academic excellence, and disadvantaged). In addition, the system must guarantee that students who aim to transfer to a different school have the option to enroll in their current school if they are not assigned to the other school they prefer. This feature of the problem has been previously studied in other settings, such as in house allocation (Guillen and Kesten (2012)) and teachers' assignment (Combe et al. 2016). Both cases use the same variant of DA to accommodate this requirement: they modify all houses/schools' priorities to rank their initial "owners" at the top of their priorities. In a recent paper, Combe (2018) shows that this variant of DA (called

Table 1 Weak priorities by type-specific seats. Lower numbers indicate higher priority.

Priority	Special Needs	Academic Excellence	Disadvantaged	No Trait
1	Current school	Current school	Current school	Current school
2	Special needs	Academic Excellence	Siblings	Siblings
3	Siblings	Siblings	Disadvantaged	Working parent
4	Working parent	Working parent	Working parent	Returning students
5	Returning students	Returning students	Returning students	No priority
6	No priority	No priority	No priority	

Table 2 Preferences of Students.

Currently Enrolled	Disadv.	Special Needs	Siblings	Preferences
Yes	Yes	Yes	Any	Special Needs \succ Disadvantaged \succ Regular \succ Academic Excellence
		No	Any	Disadvantaged \succ Regular \succ Academic Excellence \succ Special Needs
	No	Yes	Any	Special Needs \succ Regular \succ Disadvantaged \succ Academic Excellence
		No	Any	Regular \succ Disadvantaged \succ Academic Excellence \succ Special Needs
No	Yes	Any	Any	Special Needs \succ Academic Excellence \succ Disadvantaged \succ Regular
	No	Any	Yes	Special Needs \succ Academic Excellence \succ Regular \succ Disadvantaged
		Any	No	Special Needs \succ Academic Excellence \succ Disadvantaged \succ Regular

DA*) is a *justified-envy minimal* mechanism in the set of individually rational and strategy-proof mechanisms (Abdulkadiroglu et al. 2017),¹⁴ i.e., there is no other algorithm such that its set of blocking pairs (relative to the original preferences) is a subset of that of DA*. For this reason, we adopt a similar approach and make two important changes to adapt it: (1) we rank all students with current school at the top of their schools' priorities, and (2) we add their current school to the bottom of the preference list of each student seeking to transfer to another school that participates in the system.

Given the treatment of reserves described above, we model each trait as a separate sub-school with its number of seats (equal to the number of reserved seats for that trait) and its weak priority order. In Table 1, we describe the sub-schools' weak priorities over students depending on their traits. In each sub-school (c, t) , students currently enrolled at the school (who aim to transfer) have the highest priority in all reserves. Students with special needs and academic excellence have the second-highest priority in the corresponding reserves. The remaining students are ordered according to the priority groups defined by law (i.e., sibling, working parent, and returning student). Notice that, as required by law, students with siblings at the school have higher priority than disadvantaged students, even in seats reserved for that trait. Finally, in Table 2, we describe the preferences of students, which depend on their set of traits.

5. Results

In this section, we report the implementation results. We start by describing how the system evolved from 2016 to 2018. Then, we focus on the admissions process of 2018 and report the results of the

Table 3 Evolution of the System

	Main Round			Comp. Round		
	2016	2017	2018	2016	2017	2018
Regions	1	5	15	1	5	15
Schools	63	2,174	6,421	63	2,174	6,421
Students	3,436	76,821	274,990	439	9,507	46,698
1st preference [%]	57.0	56.2	59.2	81.3	81.8	46.7
Other preference [%]	27.4	26.8	23.4	12.5	14.3	19.0
Current school [%]	6.8	8.3	8.6	3.0	1.5	2.7
Not assigned/Distance [%]	8.8	8.7	8.9	3.2	2.4	31.6

main and complementary rounds in Sections 5.1 and 5.2, respectively. In Section 5.3, we study the impact of the family application and having lotteries at the family level. Finally, in Section 5.4, we analyze the effect of the quota for disadvantaged students.

In Table 3, we summarize the evolution of the admissions system. For 2016, we considered only the entry grades of the Magallanes region, located in the extreme south of the country. For 2017, the system was extended to all grades in Magallanes, and to entry grades in four more regions. For the 2018 admissions process, all the aforementioned regions’ grades were added, and all the remaining regions (except for the Metropolitan area) were included in their entry grades. For 2020, the system was implemented in the entire country and for all grades, i.e., from pre-K to 12th grade. As the table shows, most of the main round’s relevant performance metrics—fraction of students assigned to their top choice and unassigned—have remained stable over time.¹⁵

5.1. Main Round.

In 2018, 274,990 students and 6,421 schools—divided into 32,198 sections, i.e., school-grade pairs—participated in the system, with a total of 522,859 available seats (average of 16.2 seats per section). In Table 4 we classify students based on (1) their gender, (2) whether they have any priority in the schools they applied to, and (3) whether they are eligible for any quota in the schools of their choice. Notice that the percentage of disadvantaged students exceeds 50% of the total number of applicants. As the quota for this group is only 15%, an interesting design question is whether having a quota has any impact when the targeted population is relatively large. We analyze this in Section 5.4.

Analyzing the submitted preferences, we observe that students apply on average to 3.18 schools. Considering that there is no limit on the number of schools that students can include in their preference list, this number seems relatively low. One potential explanation is that students skip schools where they believe that their chances of being admitted are close to zero (Larroucau and Rios 2018). Another potential reason is that students make application mistakes due to lack of information, poor understanding of the mechanism, and other reasons. Recent literature has

explored similar application mistakes in other contexts, including college admissions (Artemov et al. 2017, Shorrer and S3vago 2017, Larroucau et al. 2021), the National Residency Match (Rees-Jones 2018, Rees-Jones and Skowronek 2018), the Israeli Market for Psychologists (Hassidim et al. 2021), among others. Understanding the drivers of this behavior in the school choice context is an interesting avenue for future research.

Overall, 73.1% of the applications are to public schools and 26.9% to voucher schools, although only 11% of the total seats available are of the latter type. Out of the 485,905 applications submitted by disadvantaged students, 22.0% are to voucher schools, which is significantly less than the general population. These differences are not surprising considering that disadvantaged students have fewer resources, and therefore their willingness to pay is probably lower.

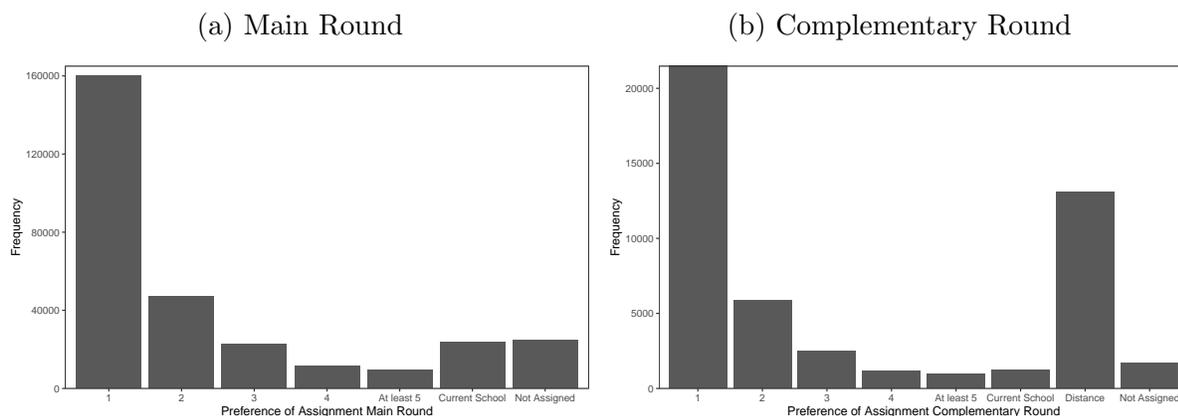
Table 4 Characterization of Applicants

		Main Round		Comp. Round	
		<i>N</i>	%	<i>N</i>	%
Gender	Female	134,973	49.1	23,063	49.4
	Male	140,016	50.9	23,635	50.6
Priority	Siblings	66,743	24.3	5,443	11.7
	Working parent	3,700	1.3	328	0.7
	Returning students	9,165	3.3	2,441	5.2
Quota	Special needs	1,631	0.6	-	-
	Academic excellence	6,534	2.4	-	-
	Disadvantaged	150,287	54.7	23,414	50.1

In Figure 2a we present the distribution of assignments by preference. We observe that 59.2% and 12.8% of the applicants are assigned to their first and second preference, respectively. In addition, 8.6% are assigned to their current school, and 8.9% are left unassigned (recall that these students—the unassigned—have the chance to participate in the complementary process, whose results are described in Section 5.2).

5.2. Complementary Round.

Overall, 46,698 students participated in the complementary round, including new applicants, unassigned students from the main round, and students who rejected their assignment from the main round. In Table 4 we characterize these students based on their gender, priority type, and eligibility for the disadvantaged quota, as the other quotas are not considered in the complementary round. In general, we observe that there are no significant differences relative to the main round. In Figure 2b we present the distribution of preferences of assignment in the complementary round. We observe that the results are not as good as in the main round, as 47% are assigned to their top choice, 28% are assigned by distance, and 3.6% are left unassigned.

Figure 2 Distribution of Preference of Assignment

Recall that the unassigned students from the complementary round are assigned to the nearest public school (within 17 km) with remaining open seats—referred to as a distance assignment. Indeed, 13,064 students were assigned by distance. The average distance for these students was 2.17 km, compared to 2.19 km for those assigned to one of their preferences in the complementary process and 3.35km for those assigned to their current school. Finally, only 1,691 students—0.6% of the total number of applicants in both rounds—were unassigned and were manually allocated by MINEDUC.

In Appendix B.5 we report the results of the enrollment process. Overall, 72.7% (214,209) of the students who applied to the system enrolled in the school they were assigned to (either in the main or in the complementary round). Among the remaining 60,139 students, 8.6% (5,192) did not enroll in any school, 20.1% (12,076) enrolled in a school that did not participate in the centralized system, and 71.3% (42,871) enrolled in a school that participated. The latter includes students who enrolled in the closest school with remaining seats after the complementary round, students who remained in their current school, and students who directly contacted a school to request a seat.

5.3. Assignment of Families.

Besides finding a fair allocation as required by law, one of our primary goals is to favor the joint allocation of siblings. In 2018, a total of 21,424 students were part of 10,301 family applications in the main round, with 2,869 (27.9%) having students belonging to the same grade and 7,432 (72.1%) having at least two students in different grades.¹⁶ Out of these family applications, 6,725 (65.3%) were *fully successful*—i.e., all siblings that were part of these were assigned to the same school—and 307 (3%) were *partially successful*—i.e., a subset of the siblings (among families with 3 or more applicants) were assigned to the same school.

As discussed in previous sections, we make three important decisions to favor the joint assignment of siblings:

1. Update preferences of younger siblings to accommodate the assignment of older siblings.
2. Use lotteries at the family level as opposed to the student level.
3. Process grades sequentially in decreasing order, i.e., starting from 12th grade and finishing with pre-K.

To assess the impact of these decisions, in Table 5 we compare the fraction of family applications that are *fully* and *partially* successful obtained from (i) updating/not updating the preferences of younger siblings, and (2) using lotteries at the family/student level. For simplicity, we focus on the main round, and for each combination we report the mean and standard deviation (in parentheses) obtained from 10,000 simulations.¹⁷

Table 5 Effect of Lotteries and Updating of Preferences in Family Application: Simulation

	Updating		No Updating	
	By Family	By Student	By Family	By Student
Fully Successful [%]	65.50 (0.21)	61.56 (0.23)	57.34 (0.25)	52.87 (0.27)
Partially Successful [%]	2.94 (0.07)	3.00 (0.07)	3.34 (0.08)	3.40 (0.08)

First, we observe that using lotteries at the family level increases the number of successful family applications by 3.9% when combined with updating preferences. The improvement is 4.4% when no updating of preferences occurs. On the other hand, the number of partially successful applications remains almost the same when a family lottery is combined with updating of preferences, while it increases by 0.06% when no updating is in place. These results suggest that using family lotteries can largely increase the number of successful family applications. Second, comparing the results of updating/not updating preferences (for a fixed type of lottery), we observe that our proposed mechanism significantly increases the fraction of fully successful family applications (by 8.2% and 8.7% for family and student lotteries, respectively). At the same time, it slightly decreases the number of partially successful family applications (by 0.44% and 0.4% for family and student lotteries, respectively).

To assess whether the order in which grades are processed matters, we also ran simulations where we sequentially process grades in increasing order, i.e., from pre-K to 12th grade. We do not find a significant effect, as this results in 65.48% and 2.82% fully and partially successful applications, respectively. Overall, these results suggest that updating preferences of younger siblings and using lotteries at the family level explain the improvement obtained from our mechanism, and that the former explains a higher fraction of the improvement.

Although the School Inclusion Law requires that each school has its own lottery to break ties (i.e., it enforces the use of a Multiple Tie-Breaking (MTB) rule), in Table 6 we compare this to

using a Single Tie-Breaking (STB) rule combined with updating preferences and holding lotteries at the family and student levels.

Table 6 Effect of Lotteries and Tie-Breaking Rule in Family Application: Simulation

	MTB		STB	
	By Family	By Student	By Family	By Student
1st preference [%]	59.21 (0.04)	59.12 (0.04)	62.05 (0.03)	62.05 (0.03)
Other preference [%]	23.31 (0.05)	23.40 (0.05)	20.14 (0.04)	20.14 (0.04)
Current school [%]	8.57 (0.02)	8.57 (0.02)	8.51 (0.02)	8.51 (0.02)
Not assigned [%]	8.91 (0.02)	8.90 (0.02)	9.31 (0.02)	9.30 (0.02)
Fully Successful [%]	65.50 (0.21)	61.56 (0.23)	65.47 (0.21)	61.94 (0.23)
Partially Successful [%]	2.94 (0.07)	3.00 (0.07)	2.93 (0.07)	3.00 (0.07)

Similar to previous findings in the literature, we observe that STB leads to more students assigned to their top choice, while MTB leads to fewer unassigned applicants. In addition, given a fixed tie-breaking rule (MTB or STB), we observe no significant effect of using lotteries by family on the outcomes of interest that are not related to family applications (i.e., on the distribution of preferences of assignment). This suggests that using lotteries at the family level significantly increases the number of siblings assigned to the same schools without having a major effect on other aggregate outcomes of interest. Finally, we observe that using lotteries at the family level increases the rate of success of family applications regardless of the tie-breaking rule considered (STB or MTB).

5.4. Quotas.

As discussed in previous sections, another requirement by law is the inclusion of a quota for disadvantaged students. If the goal is to benefit the group that is targeted by the quota, previous literature suggests that preferences should be updated to process it after the “regular” seats (see Dur et al. (2016b), Hassidim et al. (2018), Rios et al. (2020)). All these papers focus on settings where the quota serves a minority of students, and thus the goal is to increase their admission chances. However, in our setting the quota serves the majority of students—disadvantaged students represent 54.7% of the participants in the system—and the goal is to reduce school segregation and increase diversity within schools. For this reason, we decided to process the quota first.

To assess the effect of this decision and the overall impact of including the quota, we compare three policies:

1. Quota First: this policy corresponds to the actual implementation described in Section 4.2, i.e., assigning first the quota seats and then the regular seats in the applicants' preference lists.
2. No Quota: this policy assumes that there is no quota.
3. Quota Last: this policy simulates the opposite case, i.e., assigning first the regular seats and then the quota in the applicants' preference lists.

In Table 7, we report the results obtained from 10,000 simulations of each policy, where we consider only the main round of the process for simplicity.¹⁸ For each policy, we report the average and the standard deviation of the percentage of students (1) assigned to their top choice, (2) to a lower preference, (3) to their current school, and (4) not assigned. Also, for each policy, we compute a measure of school diversity given by the percentage of disadvantaged classmates that each student has in her school and grade (see Appendix B.3 for more details).

First, comparing the actual implementation with the case with no quotas (i.e., Quota First vs. No Quota), we observe that disadvantaged students perform better when there is a quota, but the differences are relatively small. One possible reason is that, by processing the quota first, disadvantaged students with high priority fill the quota. However, these students would also be admitted under the regular admissions process, and thus other disadvantaged students would benefit by processing the quota last. This becomes clear when comparing the results of our implementation with those in the last two columns in Table 7 (i.e., Quota First vs. Quota Last). We observe that disadvantaged students are significantly better off when the quota is processed last. For instance, the fraction of disadvantaged students assigned to their top choice increases by 2.5%, while the fraction that is unassigned decreases by 1.3%. Finally, we observe that processing the quota first helps improve the diversity of schools, which is the main objective of having the quota in our setting. Specifically, Quota First has a diversity of 0.566 (similar to No Quota) while Quota Last has a diversity of 0.574, where a number close to 0.54 would be optimal.¹⁹ But we also observe that processing the quota last reduces the variance in the fraction of disadvantaged classmates, which is also desirable. These results confirm that the order in which quotas are processed matters. Processing the quota last helps the group of students targeted by this policy, even when they represent most of the process participants, but can diminish the diversity of the schools.

6. Conclusions

Centralized procedures to assign students to schools are becoming the norm in many countries. This trend highlights the need to study these systems beyond the stylized models in the literature, as specific practical nuances can play a critical role. In this paper, we describe the design and implementation of the new school choice system in Chile, which expands previous applications by focusing on increasing the chances that siblings are assigned to the same school. In particular,

Table 7 Sensitivity to Different Variants of Socioeconomic Quota: Simulations

	Quota First		No Quota		Quota Last	
	Disadv.	Non-Disadv.	Disadv.	Non-Disadv.	Disadv.	Non-Disadv.
1st preference [%]	66.02 (0.06)	51.01 (0.07)	65.82 (0.06)	51.24 (0.07)	68.50 (0.06)	48.73 (0.07)
Other preference [%]	21.03 (0.06)	26.05 (0.08)	20.98 (0.06)	26.14 (0.08)	20.00 (0.06)	26.26 (0.07)
Current school [%]	7.30 (0.03)	10.10 (0.03)	7.38 (0.03)	10.01 (0.04)	7.06 (0.03)	10.41 (0.03)
Not assigned [%]	5.65 (0.03)	12.84 (0.05)	5.82 (0.04)	12.60 (0.05)	4.44 (0.03)	14.60 (0.05)
Diversity of schools						
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Fraction of disadv. classmates	0.566	0.217	0.565	0.220	0.574	0.211

we propose using two lotteries, one to order families and the other to break ties among siblings. Also, our mechanism updates students’ preferences to prioritize siblings getting assigned to the same school if they are part of a family application. Our results show that these features improve the fraction of siblings assigned to the same school by 13% compared to the standard approach of breaking ties at the student level. Apart from facilitating the joint allocation of siblings, our solution accounts for all the other requirements that are part of the system, including different priorities, quotas, and the assignment of students currently enrolled.

The experience of implementing a large-scale nationwide system stresses the importance of having a continuous collaboration with policy makers, and the need of implementing changes in small steps. Having a gradual implementation allows us to learn from the experience, continuously improve the system, and gives time to the general public—and final users of the system—to get information, learn, and understand the new system’s benefits. Overall, we will continue working to improve the system, increasing its efficiency and fairness to give all students equal opportunities, regardless of their background.

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Appendix A: Proofs

A.1. Proof of Proposition 1

Consider an instance with two schools and two grades, $C = \{c_1, c_2\}$, $G = \{g_1, g_2\}$, where g_1 is the higher grade; and two families $f = (f_1, f_2)$ and $f' = (f'_1, f'_2)$, where the sub-index denotes the grade the student belongs to. Suppose each school has exactly one seat in each grade. Suppose also that both schools' priorities are simply rankings over students in each grade, and that both prefer f_2 to f'_2 in grade g_2 , and both prefer f'_1 to f_1 in grade g_1 . Moreover, suppose the preferences of the families are

$$\begin{aligned} f &: (c_1, c_2) \succ_f (c_2, c_1) \succ_f (c_1, c_1) \succ_f (c_2, c_2) \\ f' &: (c_1, c_1) \succ_{f'} (c_2, c_2) \succ_{f'} (c_2, c_1) \succ_{f'} (c_1, c_2). \end{aligned}$$

Assume μ is a stable assignment. Since there are enough seats, every student should be assigned to one of the two schools in μ . We consider two possible cases:

1. If $\mu(f'_2) = c_1$, then f'_1 should be assigned also to school c_1 , because family f' prefers (c_1, c_1) to (c_1, c_2) , and f'_1 has priority over f_1 in both schools. Then, f_2 and f_1 are in school c_2 , but $(c_1, c_2) \succ_f (c_2, c_2)$ and f_2 has priority in grade g_2 , so μ cannot be envy-free.
2. If $\mu(f'_2) = c_2$, then f'_1 should be assigned to school c_2 , because $(c_2, c_2) \succ_{f'} (c_2, c_1)$ and f'_1 has priority over f_1 in both schools. But then both f_2 and f_1 are in school c_1 . Student f_2 has priority in grade g_2 and $(c_2, c_1) \succ_f (c_1, c_1)$, so μ cannot be envy-free. \square

A.2. Proof of Proposition 2

We show that the updated preference over schools that the algorithm uses for a student is exactly the one implied by their family's preference if it is higher-first, and therefore, the grade level stability implies overall stability.

Let μ be the resulting assignment from running the algorithm. Denote by $\succ_{s,ALG}$ the ranking over schools that the DA algorithm uses for student s in grade $g(s)$ and \succ_s the individual preference of student s in the definition of higher-first preferences. For a pair of schools $c, c' \in C$ we show that $c \succ_{s,ALG} c'$ if and only if $(c, \mu(f(s) \setminus \{s\})) \succ_{f(s)} (c', \mu(f(s) \setminus \{s\}))$. In fact, note that the latter holds if either

1. $\{c, c'\} \cap \mu(\{s' \in f(s) : g(s') > g(s)\}) = \{c\}$, or
2. $|\{c, c'\} \cap \mu(\{s' \in f(s) : g(s') > g(s)\})| \in \{0, 2\}$ and $c \succ_s c'$.

For $\succ_{s,ALG}$, in the first case the algorithm before processing grade $g(s)$ moves c to the beginning of the ranking of s because it assigned a sibling of s in a higher grade to c . In the second case, s has a sibling in a higher grade in both c and c' or in neither, in which case the algorithm respects

the originally reported preference of s . Thus, the ranking implied by the family preference and the one used by the algorithm are equal.

On the other hand, the priority in each school that the algorithm uses in grade g , $\succ_{c,ALG,g}$, is fixed once the assignment in grades $g' > g$ is fixed and is by definition the real priority in the school within grade g . Therefore, since DA produces a stable assignment within each grade g according to $\{\succ_{s,ALG}\}_{s \in S_g}$ and $\{\succ_{c,ALG,g}\}_{c \in C}$, and in general seats can be claimed and there can be justified envy only within grades, μ is stable in general. \square

A.3. Proof of Proposition 3

In this section we formally state and prove Proposition 3. Consider a family with two children, f_1 and f_2 applying to grades g_1 and g_2 , respectively. As usual, the set of schools is C ($|C| = m$). School $c \in C$ has q_{cg_1}, q_{cg_2} available seats in grades g_1 and g_2 , respectively. Without loss of generality, we assume there are no specific quotas or priorities.

Let S_g be the set of students applying to grade $g \in \{g_1, g_2\}$. Each student $s \in S_{g_1} \cup S_{g_2}$ has a preference profile over a subset $C_s \subseteq C$ denoted by \prec_s . For ease of presentation, the priorities of a student s in schools in C is given by a vector $u_s = (u_{sc})_{c \in C} \in [0, 1]^C$ so that the higher u_{sc} , the higher the priority of student s in school c (in the random priority model these numbers can be thought to be i.i.d. $U[0, 1]$ random variables, and in the following analysis we can obviate the null set where two students applying to the same grade get equal lottery numbers in the same school).

The following result concerns only grade g_1 and is related to Lemma 4 of Abdulkadiroğlu et al. (2015).

LEMMA 1. *Given \prec_s and u_s for all $s \in S_{g_1} \setminus \{f_1\}$ there exists a vector of cutoffs $(\tau_c)_{c \in C}$ such that for all preference profiles \prec_{f_1} and all vectors u_{f_1} , if $\mu(f_1)$ denotes the assigned school of f_1 in the DA mechanism, then $\mu(f_1) = c$ if and only if $u_{f_1 c'} < \tau_{c'}$ for all $c \prec_{f_1} c'$ and $u_{f_1 c} > \tau_c$.*

Proof. Recall that as proved by Dubins and Freedman (1981) the DA mechanism is truthful in the following sense: given the preferences of all students but f_1 and all priorities, for all pairs of preference profiles \prec_{f_1} and \prec'_{f_1} , if we denote as μ and μ' the assignments when student f_1 declares \prec_{f_1} and \prec'_{f_1} respectively, then $\mu'(f_1) \preceq_{f_1} \mu(f_1)$.

As is also known from the standard literature on Deferred Acceptance, the student-optimal assignment is unique and therefore independent of the order in which the student proposals are processed. Thus we may assume that an initial stable assignment has been reached without the participation of f_1 , who is then assigned her corresponding lottery number and inserted to allow the process to run to completion.

For each $c \in C$ we define τ_c as the minimum value that $u_{f_1 c}$ can take to get f_1 accepted to c if she were to apply to it as her first preference (note that these are well defined since the acceptance

or rejection to c as a first preference does not depend on the next ones). In this case it is clear that any value of u_{f_1c} higher than τ_c would also result in acceptance to c , and a priority number lower than τ_c would result in rejection by construction.

We will now show that this same vector $(\tau_c)_{c \in C}$ also works for an arbitrary preference profile \prec_{f_1} .

First we claim that if $u_{f_1c} < \tau_c$, then f_1 cannot be accepted to c . Indeed, suppose by contradiction that $u_{f_1c} < \tau_c$ and f_1 is accepted to c . Noting that the definitions of the τ_c 's do not depend on \prec_{f_1} we can assume that the altered profile \prec'_{f_1} , given by restricting \prec_{f_1} to start from school c , was the real preference profile and that \prec_{f_1} is a deviation from the truth. By definition of τ_c , f_1 will be rejected from c if she applies with profile \prec'_{f_1} , but accepted with profile \prec_{f_1} by hypothesis, which contradicts the truthfulness of the mechanism.

Returning to the proof of the lemma, to prove the right-hand implication suppose that f_1 is assigned to c . The inequality $u_{f_1,c} > \tau_c$ follows from the previous claim. If $u_{f_1c'} > \tau_{c'}$ for some $c \prec_{f_1} c'$, then once again f_1 could alter her preference profile to start from c' and by definition be accepted to her more preferred option c' , contradicting the truthfulness of the mechanism.

For the left-hand implication suppose by contradiction that the inequalities hold and there is a school $c' \neq c$ such that f_1 would be assigned to c' instead. From the claim and the inequalities $u_{f_1c'} < \tau_{c'}$ we get that it is not possible that $c \prec_{f_1} c'$. Also, if $c' \prec_{f_1} c$ we can once again consider the restricted preference profile starting from c and the inequality $u_{f_1c} > \tau_c$ to contradict the truthfulness of the mechanism. \square

With this lemma in hand we want to compare the probability that f_1 and f_2 get assigned to the same school if on the one hand we draw u_{f_1} and u_{f_2} as vectors of i.i.d. uniform random variables $U[0, 1]$, or on the other hand we draw u_{f_1} as a vector of i.i.d. random variables $U[0, 1]$ and set $u_{f_2c} = u_{f_1c}$. To this end we denote by \mathbb{P}_S the probability measure induced by the former situation (student lottery) and by \mathbb{P}_F the one for the latter situation (family lottery).

PROPOSITION 3. *(Formal statement) Given \prec_s and u_s for all $s \in S_{g_1} \cup S_{g_2} \setminus \{f_1, f_2\}$ then $\mathbb{P}_S(\mu(f_1) = \mu(f_2)) \leq \mathbb{P}_F(\mu(f_1) = \mu(f_2))$.*

Proof. We proceed by partitioning the event $\mu(f_1) = \mu(f_2)$ over the possible common school assignment $c \in C$. From Lemma 1 we know that the event $\mu(f_1) = c$ is equivalent to $u_{f_1c'} < \tau_{c'}$ for all $c \prec_{f_1} c'$ and $u_{f_1c} > \tau_c$. Therefore, since u_{f_1} is a vector of uniform i.i.d. random variables in $[0, 1]$, conditional on the event $\mu(f_1) = c$, we have that u_{f_1} is a vector of independent random variables but with $u_{f_1c} \sim U[\tau_c, 1]$, $u_{f_1c'} \sim U[0, \tau_{c'}]$ for $c' \succ_{f_1} c$, and $u_{f_1c'} \sim U[0, 1]$ for $c' \prec_{f_1} c$.

If we apply Lemma 1 to grade g_2 , we get certain cutoffs $(\bar{\tau}_c)_{c \in C}$ such that $\mu(f_2) = c$ if and only if $u_{f_2c'} < \bar{\tau}_{c'}$ for all $c' \prec_{f_2} c$ and $u_{f_2c} > \bar{\tau}_c$. Now, since under family lotteries $u_{f_1c'} = u_{f_2c'}$ for all $c' \in C$,

we have that $\mathbb{P}_F(u_{f_2c'} < \bar{\tau}_{c'} | \mu(f_1) = c) \geq \mathbb{P}_S(u_{f_2c'} < \bar{\tau}_{c'} | \mu(f_1) = c) = \mathbb{P}_S(u_{f_2c'} < \bar{\tau}_{c'})$ for all $c' \neq c$ and $\mathbb{P}_F(u_{f_2c} > \bar{\tau}_c | \mu(f_1) = c) \geq \mathbb{P}_S(u_{f_2c} > \bar{\tau}_c | \mu(f_1) = c) = \mathbb{P}_S(u_{f_2c} < \bar{\tau}_c)$. Then, because the variables in the vector u_{f_2} are independent, we can multiply the inequalities and to obtain that

$$\mathbb{P}_F(\mu(f_2) = c | \mu(f_1) = c) \geq \mathbb{P}_S(\mu(f_2) = c).$$

Note that for a given school $c \in C$, the marginal probabilities $\mathbb{P}_F(\mu(f_1) = c)$ and $\mathbb{P}_S(\mu(f_1) = c)$ are equal since they concern grade g_1 only. Hence, we can multiply by $\mathbb{P}_F(\mu(f_1) = c)$ on both sides of the previous inequality and sum over all $c \in C$ to obtain that

$$\sum_{c \in C} \mathbb{P}_F(\mu(f_2) = c, \mu(f_1) = c) \geq \sum_{c \in C} \mathbb{P}_S(\mu(f_2) = c, \mu(f_1) = c),$$

and therefore, $\mathbb{P}_F(\mu(f_1) = \mu(f_2)) \geq \mathbb{P}(\mu(f_1) = \mu(f_2))$. \square

A.4. Examples of Section 4.1.4

Example of Observation 3. Consider an instance with two grades $G = \{g_1, g_2\}$, where g_1 is the higher grade and g_2 the lower. We have three families $f = (f_1, f_2)$, $f' = (f'_1, f'_2)$, and s_1 (an only-child family applying to grade g_1) and three schools c_1, c_2 and c_3 . Schools c_1 and c_2 have one available seat in each grade and c_3 only has a seat in grade g_1 . The families have higher-first preferences given by the following individual preferences:

$$\begin{aligned} f_1 &: c_1 \succ c_2 \succ \emptyset, & f_2 &: c_2 \succ c_1, \\ f'_1 &: c_3 \succ c_1 \succ c_2, & f'_2 &: c_1 \succ c_2, \\ s_1 &: c_2 \succ c_1 \succ \emptyset. \end{aligned}$$

The tie-breaking rules in each school are given by:

$$\begin{aligned} c_1 &: s_1 \succ f' \succ f, \\ c_2 &: f \succ f' \succ s_1, \\ c_3 &: f \succ f' \succ s_1. \end{aligned}$$

If everyone reports truthfully, each student goes to her most preferred school in g_1 and there are no conflicts. If we denote the assignment by μ , we have that $\mu(f_1) = c_1$, $\mu(f'_1) = c_3$, and $\mu(s_1) = c_2$. However, in g_2 the mechanism updates the preference of f_2 to be $c_1 \succ c_2$ because her sibling was assigned to c_1 . Therefore, in the DA algorithm both f_2 and f'_2 propose to c_1 . Since f_1 was assigned to c_1 in g_1 , f_2 has now sibling priority, and so f'_2 is rejected and the resulting assignment is $\mu(f_2) = c_1$ and $\mu(f'_2) = c_2$.

We now consider the situation where all individual preferences are reported truthfully, except for f'_1 , whose reported preference is $c_1 \succ c_3 \succ c_2$. Denote the new assignment by μ' . In grade g_1 , in the DA algorithm, f_1 proposes to c_1 , f'_1 to c_1 , and s_1 to c_2 . The proposal of f_1 is rejected, so she proposes to c_2 . Then the proposal of s_1 is rejected, so she proposes to c_1 . Finally, the proposal of f'_1 is rejected, and so she proposes to c_3 . Thus, the resulting assignment is $\mu'(f_1) = c_2$, $\mu'(f'_1) = c_3$, and $\mu'(s_1) = c_1$. Note that the assignment of f'_1 is the same as in the case where her individual preference is reported truthfully. In g_2 the updated preference of f_2 is the same as her original individual preference, and so the assignment is $\mu'(f_2) = c_2$ and $\mu'(f'_2) = c_1$.

Example of Observation 4. Consider the same instance as in Observation 3, but add a higher level g_0 , a family s_0 (which is an only-child family applying to g_0), and a new member of f' , f'_0 , so that we get $f' = (f'_0, f'_1, f'_2)$. In g_0 schools c_1 and c_3 have one available seat and c_2 has no available seats. The individual preferences of the new students are given by

$$\begin{aligned} f'_0 &: c_3 \succ c_1 \\ s_0 &: c_3 \succ c_1, \end{aligned}$$

and s_0 has higher priority in all schools.

Assume all individual preferences are reported truthfully and that f submits a family application. In g_0 the assignment is $\mu(f'_0) = c_1$ and $\mu(s_0) = c_3$, regardless of whether f' submits a family application. However, if f' submits a family application, the mechanism updates the preference of f'_1 to be $c_1 \succ c_3 \succ c_2$, and we obtain the same situation as in the example of Observation 3.

Appendix B: Results

B.1. Main Round

B.1.1. Relation between Outcome and Number of Submitted Preferences. Figure 3 shows the fraction of students who (1) are assigned to one of their preferences, (2) are assigned to their current school, and (3) are left unassigned, conditional on the number of reported preferences. We observe that when the number of declared preferences increases so does the probability of being assigned, but the average preference of assignment also increases. Moreover, we find that students who are unassigned apply on average to fewer schools (3.36, with std. dev. 1.49) than those who are assigned (3.42, with std. dev. 1.83). Applicants assigned to their current school usually submit even fewer preferences (3.05, with std. dev. 1.49), which is expected as they have a secured option.

B.1.2. Relation between Number of Siblings and Success of Family Application. Figure 4 shows that larger families are less likely to be successful, which is intuitive as they require more students to be allocated to the same school. We refer to Appendix B.4 for results on family applications based on on the number of same schools a family declares.

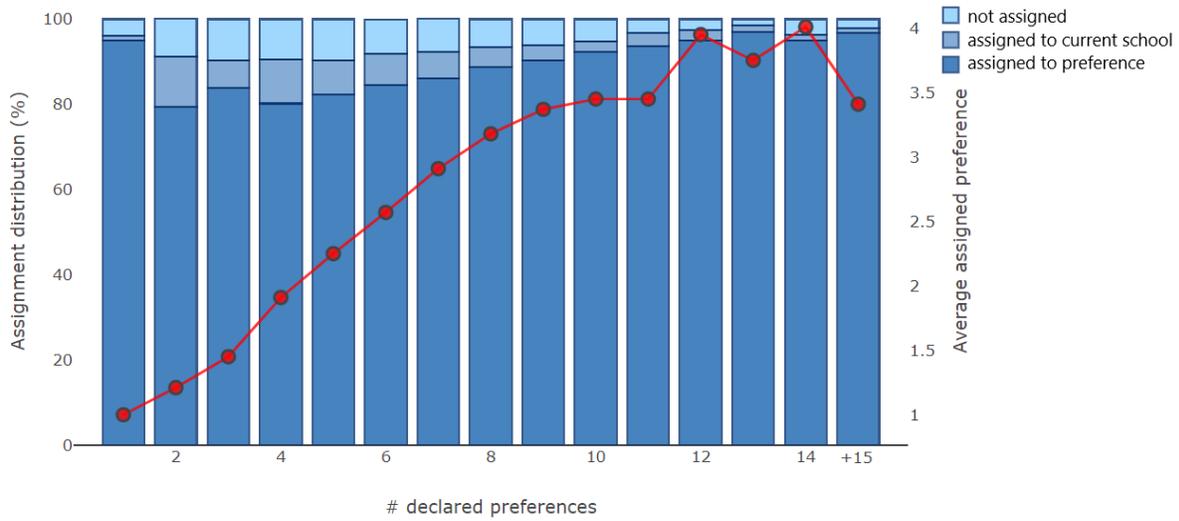


Figure 3 Assignment Distribution and Average Rank Distribution by Number of Declared Preferences: Main Round

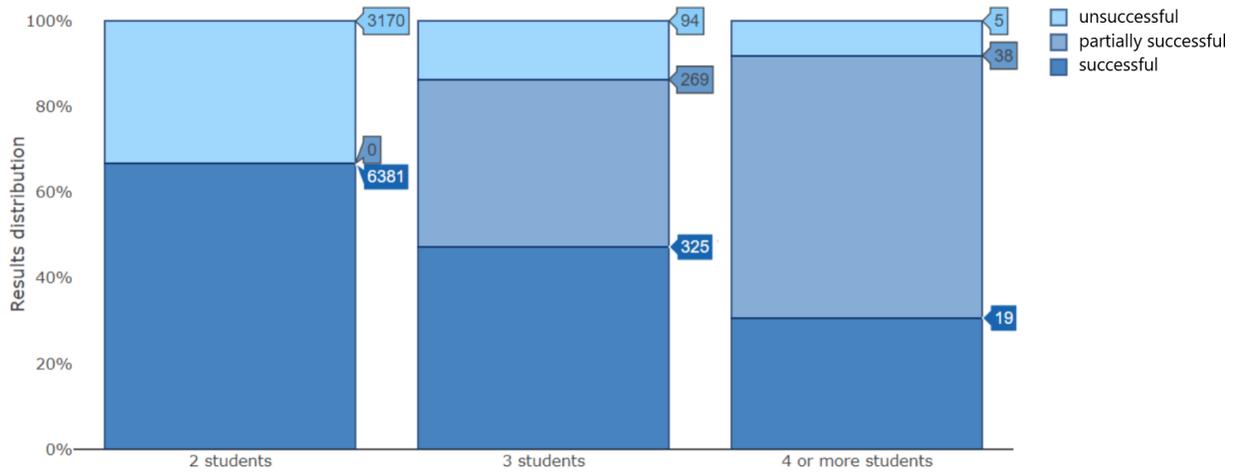


Figure 4 Number of Successful and Partially Successful Families by Size: Main Round

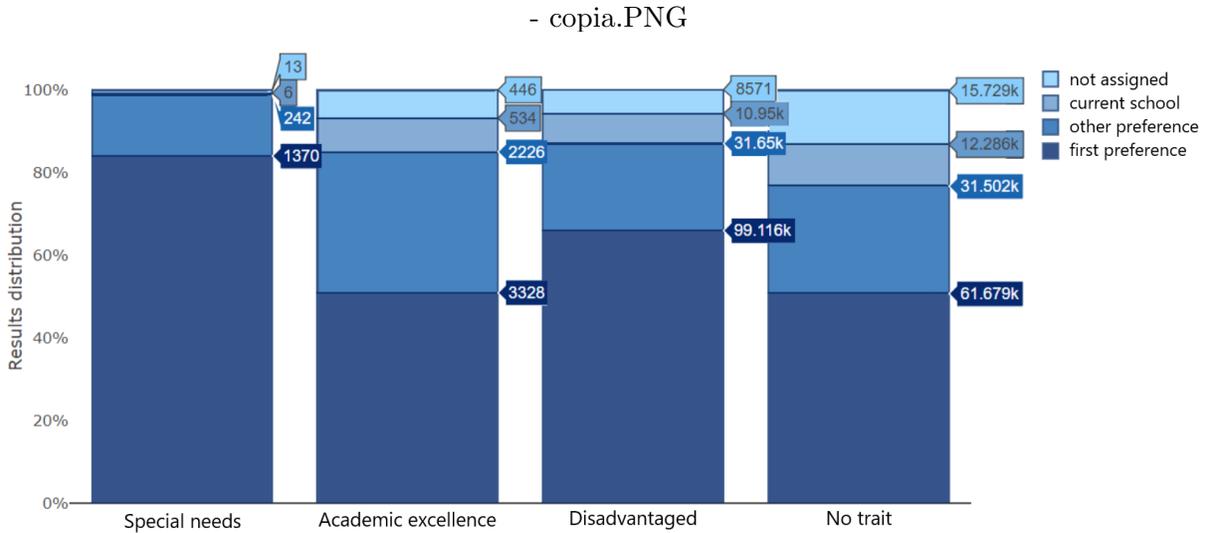
B.2. Other Quotas

Recall that students can belong to three quotas: (1) special needs, (2) academic excellence, and (3) disadvantaged. Students are indifferent between being assigned by any quota and by none of them and schools only declare their total available seats and the mechanism calculates seats for the different types of quotas that are allowed by the system. In Table 8 we show the distribution of the 524,178 declared seats for the 2018 process.

Figure 5 shows the results for students belonging to different quotas (recall that a student may belong to more than one quota). It is clear to see that students belonging to the special needs and disadvantaged quotas outperform students belonging to the academic excellence quota and

Table 8 Total Seats Declared by Schools: Main Round

Quota	# seats	% of total
Special needs	15,324	2.9%
Disadvantaged	43,336	8.3%
Academic Excellence	2,591	0.5%
No trait	462,927	88.3%

Figure 5 Results by quota - Main Round

students that do not belong to any quota in both percentage of students assigned to their first choice and percentage of unassigned students. The low performance of the academic excellence quota compared to the other two quotas could be explained by the fact that academic excellence students apply to a subset of very over-demanded schools.

B.3. Diversity Simulations

From the point of view of schools, MINEDUC seeks to have balanced and diverse schools with respect to the socioeconomic composition. In this sense, we analyze in both scenarios (with and without the quota) the balance of disadvantaged students in schools. To this end, we consider the following measure of diversity: among all the students in the first round that get an assignment, we pick a student uniformly at random and count the fraction of disadvantaged students that are assigned to her grade. This defines a random variable that depends both on the lottery used for the tie-breaking rule and on the selected student.

Let S_{dis} be the set of disadvantaged students that participate in the first round. Given an assignment μ , let $S(\mu) := \{s \in S : \mu(s) \neq \emptyset\}$ be the set of all students that get an assignment in μ ,

and, similarly, let $S_{\text{dis}}(\mu) := S(\mu) \cap S_{\text{dis}}$ be the set of all the disadvantaged students that get an assignment in μ .

For a fixed lottery, let μ^{lottery} be the assignment obtained from its induced tie-breaking and s be a student chosen at random from among all the students that get an assignment in μ^{lottery} . For a school $c \in C$ with $\mu(c) \neq \emptyset$, let $f^{\text{lottery}}(c) := \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|}{|\mu^{\text{lottery}}(c)|}$ be the fraction of disadvantaged students assigned to c in μ . Then, our random variable can be expressed as $f^{\text{lottery}}(\mu^{\text{lottery}}(s))$. Its conditional expectation given the lottery turns out to be the ratio of all the disadvantaged students assigned in μ^{lottery} to all the students assigned in μ^{lottery} , since

$$\begin{aligned} \mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \mid \text{lottery}] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} f^{\text{lottery}}(\mu^{\text{lottery}}(t)) \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} f^{\text{lottery}}(c) |\mu^{\text{lottery}}(c)| \\ &= \frac{|S_{\text{dis}}(\mu^{\text{lottery}})|}{|S(\mu^{\text{lottery}})|}. \end{aligned}$$

Its second moment, on the other hand, is given by

$$\begin{aligned} \mathbb{E} [(f^{\text{lottery}}(\mu^{\text{lottery}}(s)))^2 \mid \text{lottery}] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} ((f^{\text{lottery}}(\mu^{\text{lottery}}(t)))^2) \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} (f^{\text{lottery}}(c))^2 |\mu^{\text{lottery}}(c)| \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|^2}{|\mu^{\text{lottery}}(c)|}. \end{aligned}$$

We estimate $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$ by computing the average of $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \mid \text{lottery}]$ over the results of the 10,000 simulations. Similarly, we estimate $\text{Var}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))] = \mathbb{E}[(f^{\text{lottery}}(\mu^{\text{lottery}}(s)))^2] - \mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]^2$ by averaging $\mathbb{E}[(f^{\text{lottery}}(\mu^{\text{lottery}}(s)))^2 \mid \text{lottery}]$ over the 10,000 simulations and then subtracting the square of the estimator for $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$. Finally, we calculate the standard deviation as the square root of the variance.

B.4. Family Application

We measure the success of family applications of size two as a function of the number of schools their students declare in common in their preference lists. Table 9 shows, as expected, that the success rate increases with the number of common preferences, having its greatest increment when the number of common schools grows from one to two. Furthermore, in both rounds, blocks (families) of size two of the same grade were more successful in percentage than those of different grades. Indeed, the main round has 2,832 blocks of size two of the same grade and 6,719 of different grades, with success rates of 77.8% and 62.2%, respectively. For the complementary round, there are 362 blocks of size two of the same grade, with a success rate of 82.3%, and 1,059 of different grades, with a success rate of 70.7%.

Table 9 Results of Family Applications for Blocks of Size 2 by Number of Schools in Common

# schools in common	Main round		Complementary round	
	# blocks	% of success	# blocks	% of success
1	1,291	38.5%	497	67.7%
2	3,216	69.8%	2,245	76.6%
3	2,441	71.7%	1,750	77.8%
≥ 4	2,603	72.6%	1,889	80.5%
Total	9,551		1,421	

B.5. Enrollment

Notice that the assignment only grants the right to enroll in the school of assignment. However, families are free to look for better options of enrollment, e.g., by directly visiting schools with remaining seats after the complementary round or enroll in private schools. The School Inclusion Law did not change any aspect of the admissions process to private schools, so these are free to use any mechanism they were using in the past, i.e., interviews, entrance exams, etc. Also, many private schools carry out their admissions process all year around, while other (in general more selective) private schools run their admissions at the beginning of the academic year (i.e., between March and April) to decide admissions of students that start in the next academic year.

In Table 10 we provide summary statistics of the number of students that participate in each part of the process. *Applied* and *Assigned* represent the number of students that applied to at least one school and that were assigned to one of their reported preferences, respectively. Finally, *Enrolled* represents the number of students that enrolled in the school they were assigned to in the corresponding round.

Table 10 Summary Statistics: Process Funnel

	Main			Complementary		
	2016	2017	2018	2016	2017	2018
Applied	3,436	76,821	274,990	439	9,507	46,698
Assigned	3,133	70,145	250,469	424	9,285	31,962
Enrolled	2,071	47,248	194,003	268	5,720	21,212

From now on we will focus on the admissions process of 2018. First, we observe that 250,469 (91.1%) applicants in the main round are assigned to one of their preferences, and 194,003 (77.5%) of these students enrolled in the school they were assigned to. Out of the students that were assigned but did not enroll in their main-round assignment, only 10,605 participate in the complementary round, while the remaining 45,507 students look for other options outside the centralized system. Specifically, 4,061 students do not enroll in any school, 9,720 enroll in schools that are not part

of the centralized system (including private schools and other special cases), and 31,726 enroll in schools that are part of it. Most of the cases in the latter group are students assigned by distance (23,544 students are not assigned to any of their preferences and are automatically assigned to the closest school with remaining seats), but it is also possible that some families directly contacted schools to check whether there were seats available after the complementary round. Similarly, out of the students that apply and are assigned in the complementary round (31,962), 917 end up not enrolling in any school, 21,212 enroll in the school they were assigned to, and 9,833 enroll in a different school. Among the latter, 1,560 enroll in schools that do not participate in the centralized system, while 8,273 enroll in schools that are part of it.

Overall, 294,768 students apply in at least one of the rounds of the system, 274,348 get assigned, and 214,209 enroll in their assignment (either in the main or in the complementary round). Out of the remaining students, 5,192 do not enroll in any school, 12,076 enroll in schools that are not part of the centralized system, and 42,871 enroll in schools that are part of it. Among the latter, a total of 24,797 students are assigned to their current schools, and 13,064 are assigned to the closest school with remaining seats.

Endnotes

1. The Law also radically changed the way in which families apply and are assigned to schools, which made the transmission of information essential to the implementation.
2. This 3% corresponds to 307 partially successful family applications. However, only 750 family applications were of size 3 or more, and therefore this represents 41% of the possibly successful ones.
3. The use of entrance exams is only a temporary policy. Once the system reaches its full implementation, no entrance exams are allowed. Instead, all students that are in the top 20% of their grade will be eligible for the academic excellence quota.
4. The law explicitly states that each school must use a different random order to break ties, forbidding the use of STB. This is because authorities were concerned that a single tie-breaker would be unfair, as a low lottery number would harm students in all their applications. Nevertheless, the law allows the use of the same random tie-breaker for all the members of the same family within each school, and later breaks ties between siblings in the same grade randomly.
5. In case that there are no schools with available seats within 17 km, students remain unassigned and MINEDUC finds a solution for them.
6. If more than one older sibling is assigned or enrolled, then the preferences of the younger siblings are updated by moving those schools (if present) to the top of their preference list while

keeping their original order. By contrast, if a family with multiple children does not submit a family application, the preferences of younger siblings are not updated.

7. In 2018, there were 111,931 students with no trait; 133,198 disadvantaged students; 122,203 students of academic excellence; 530 students with special needs; 16,027 students of academic excellence and disadvantaged; 985 students with special needs and disadvantaged; 39 students of academic excellence and with special needs; and 77 students with all three traits.

8. Where $\tau(s) = \{\emptyset\}$ means s is a regular student with no trait.

9. For example, since the sibling priority applies to students with siblings currently enrolled and for students whose siblings are applying to the system and are tentatively assigned, the priority that a student gets depends on other students' allocations. Similar complementarities between students are introduced by the existence of multiple reserves with overlapping types.

10. All proofs are deferred to the Appendix.

11. In Section 5.3 we compare the results with processing grades sequentially in increasing order.

12. If f_3 had not applied to c_2 , then the updated preferences of f_3 would be $c_1 \succ_{f_3} c_3$ even if f_2 was assigned to c_2 .

13. A strict priority order is obtained by combining these weak priority orders with the random tie-breaking rule discussed in Section 4.1.2.

14. Kwon and Shorrer (2020) analyze the class of Pareto-efficient mechanisms, and show that Efficiency-Adjusted DA (EADA) is justified-envy minimal.

15. The only major difference is found in the complementary process of 2018. That year, there was a shortage of seats in pre-K in one region, which significantly worsened overall results. This is solvable by letting MINEDUC assign students to daycare institutions (not in the system) instead of schools, where there are available seats.

16. Family applications with siblings applying to the same grade are over-represented mostly because (i) families can decide whether or not to apply as a family, and (ii) only pre-K, K, 1st, 7th, and 9th grade are considered in the system, making it more likely to have siblings in the same grade.

17. We keep all the other elements of the algorithm fixed; i.e., we keep the same priorities and quotas, we solve the allocation sequentially in decreasing order starting from 12th grade, and we use different lotteries at each school.

18. For each policy and each simulation, we randomly drew the vector of family lotteries used to break ties in each school, and we solved for the main-round assignment.

19. The fraction of disadvantaged students in the entire population is 0.54. Deviations from this number imply that some group is over-represented in some schools.

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