A spatial theory of urban segregation*

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Abstract

We provide a competitive equilibrium theory of urban segregation in a linear city. Households demand consumption and housing along the city and are exposed to neighborhood externalities. We show that equilibria that are robust to small coalitional deviations are completely segregated. Our results explain urban segregation in a standard neoclassical framework and emphasize the difficulties faced by authorities to integrate cities.

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1 Introduction

Urban segregation by race or income is a pervasive feature of cities around the world (Boustan 2011, Musterd, Marcińczak, Van Ham, and Tammaru 2017). In the US, for example, urban segregation in most major cities has persisted despite the fact that the 1968 Fair Housing Act banned discrimination in the housing market and the black-white income gap has shrunk (Sethi and Somanathan 2004). The distribution of households across a city partly determines the socioeconomic outcomes of different groups (Ananat 2011, Card and Rothstein 2007, Chetty, Hendren, Kline, and Saez 2014). As urban segregation is deemed detrimental for minorities and low income households, several policies–ranging from housing subsidies to public infrastructure improvements in minority ghettos–have been proposed and implemented to promote urban integration. Yet, it is difficult to design and evaluate measures intended to combat segregation without understanding its causes and consequences.

We propose a competitive theory of urban segregation based on the idea of neighborhood externalities in a linear city. Neighborhood externalities refer to a variety of interactions among residents that are determined by location (Durlauf 2004). For example, a given house can be more attractive for a family when neighbors are richer, more educated, or have the same race. Neighborhood effects have been documented in a number of empirical studies[1] and, not surprisingly, have been incorporated into most models of sorting and location decisions. Yet, the existing literature either fails to explain urban segregation by rather showing *stratification* in settings of isolated jurisdications (de Bartolome 1990, Benabou 1996, Epple and Platt 1998, Becker and Murphy 2000, Sethi and Somanathan 2004), or has entirely neglected the role of prices in the housing market by assuming houses are bartered (Schelling 1969, Pancs and Vriend 2007).[2] This paper fills this gap by showing that urban segregation naturally arises in a spatial neoclassical model of a city under neighborhood externalities. We additionally obtain a pricing equation for each location (which can be used to infer preferences) and show that common policy interventions are unlikely to reduce segregation.

We model a city as an interval. Households decide where to live and how much to consume. Consumption and housing are provided competitively. Households are of two types, types 1 and 2, which can be interpreted broadly, including race (black and white) or income (rich and poor). How attractive a given location is for a household depends on the household type and on how the entire population of households is distributed across the city. Consumption is supplied inelastically and

[1] See, for example, Ioannides (2002) and Rossi-Hansberg, Sarte, and Owens III (2010).
[2] We discuss the literature in detail later.

houses can be frictionlessly traded at equilibrium rental prices. An equilibrium is a configuration in which, given rental prices, each household chooses a bundle (location and consumption) optimally, and markets clear.

The main driver of urban segregation in our model comes from the idea that different groups of households have different willingness to trade consumption and locations in the city. Formally, we introduce a *single crossing condition*, which captures differences in marginal rates of substitution and requires that the indifference curves of type 1 and type 2 agents cross at most once. The single crossing condition is easy to check in practice. Notably, single crossing is a rather general condition that accommodates a number of attitudes towards neighborhood composition, including setups in which one type of household, say type 1, imposes a positive externality on every household (Guerrieri, Hartley, and Hurst 2013), and models in which each household' ideal configuration involves some level of urban integration as in Schelling's (1969) groundbreaking analysis.

We first observe that a perfectly integrated equilibrium–in which households are evenly distributed along the city and rental prices are constant–always exists. Our first main result, Theorem 1, is an equilibrium segregation theorem showing that, under the single-crossing condition, in any equilibrium other that the perfectly integrated one, households are segregated. This means that the city can be divided into disjoint intervals, each of them being a ghetto where only one type of household lives. This result rules out, for example, allocations in which households are unevenly distributed over the whole city or in which some neighborhoods are perfectly integrated. Behind the equilibrium segregation theorem is the intuition that the price system cannot bring together to an integrated neighborhood households having different willingnesses to pay for location, unless the housing market is degenerate and offers exactly one option of neighborhood composition. In other words, in equilibrium the price scheme can integrate households only if integration is perfect.

Our second main result, Theorem 2, complements Theorem 1 by exploring equilibrium robustness to coalitional deviations. We say that an equilibrium is *stable* if no small coalition of households can redistribute their equilibrium allocations to attain higher utility. Theorem 2 shows that, under relatively weak restrictions, the only stable equilibrium outcome is complete segregation. The main idea behind our result is that complete segregation is stable when type 1 households are more willing than type 2 households to give up consumption in order to live in neighborhoods with higher concentrations of type 1 agents. Theorems 1 and 2 together point to the market forces that move equilibrium outcomes away from integration and towards segregated allocations.

Our competitive theory of urban segregation brings important lessons. First, the empirical literature has emphasized a number of mechanisms through which segregation can arise in a city, including black-self segregation, white collective action, and white individual action (Boustan 2011). We show that segregation is a plausible outcome under a variety of attitudes towards the composition of neighborhoods in a full fledged spatial and competitive equilibrium model. Notably, our results apply for a rich set of specifications about how households rank different neighborhoods.

Second, our model points to the difficulties faced by authorities attempting to reduce urban segregation. We modify our setup to show that commonly used policies, such as subsidies and place-based investments, are unlikely to integrate a city. On the contrary, authorities that want to fight urban segregation need to resort to much more aggressive measures, including minority quotas and social housing spread throughout different neighborhoods in the city.[3]

Third, applied work using equilibrium models in linear cities ignores nonsegregated allocations and restricts attention to segregated ones (Guerrieri, Hartley, and Hurst 2013). Our results provide a foundation for this restriction by deriving conditions under which, without loss, researchers can focus on segregated equilibria in neoclassical models over linear cities. More generally, we hope our model can be used as a workhorse for applied research.

As in Schelling (1969), in our model small preferences for segregation may result in a completely segregated city. In contrast to Schelling (1969) and the ensuing literature (Pancs and Vriend 2007, Fagiolo, Valente, and Vriend 2009), in our model houses are traded through the market, instead of bartered. This brings our work closer to both reality and the urban economics literature (Fujita and Ogawa 1982, Lucas and Rossi-Hansberg 2002, Guerrieri, Hartley, and Hurst 2013, Card, Mas, and Rothstein 2008), and allows us to evaluate the impact of common policy interventions under a standard microeconomics framework.

Our paper is also related to the literature on neighborhood sorting, including de Bartolome (1990), Benabou (1996), Epple and Platt (1998), Becker and Murphy (2000), and Sethi and Somanathan (2004).[4] Similar to Tiebout (1956), households can choose among different *jurisdictions* and how attractive a given jurisdiction is depends entirely on its own composition but not on how other jurisdictions are composed.[5] In these models, equilibria are typically stratified: within each jurisdiction there is only one type of household.The spatial nature of cities–that has long been recognized by the urban economics literature–is absent in these papers and, as a result, their

[3]Quotas for different ethnic groups at the block and neighborhood levels are in place in Singapore (Wong 2013).
[4]The literature is vast. See Durlauf (2004) and Kuminoff, Smith, and Timmins (2013) for surveys.
[5]Another way to put it is that in these models, neighborhoods are non-overlapping.

stratification results cannot be interpreted as urban segregation.[6]

Earlier papers attempted to derive urban segregation patterns as equilibrium outcomes in a linear city. Courant and Yinger (1977) restricted attention to completely segregated equilibria and urban segregation was an assumption (rather than a result). Yinger (1976) and Kern (1981) focused on both completely segregated and perfectly integrated outcomes under the *assumption* that externalities are entirely determined by the next-door neighbors.[7] Importantly, this earlier literature recognized the importance of exploring models in which externalities are determined by averaging compositions over different locations but, as Yinger (1976) points out, in such models progress was unfeasible “because the mathematics is not tractable.” We contribute to this literature by providing a framework under which the analysis is tractable and results in novel economic mechanisms.

Finally, our formulation follows the urban economics literature by considering both the housing market and externalities in a model of a spatial city (Fujita and Ogawa 1982, Lucas and Rossi-Hansberg 2002). We share with this literature the focus on understanding how equilibrium forces determine the use of different locations. However, our goal is different as we explore how distinct types of households compete for land and how this competition determines urban segregation.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 states and discusses our main results. Section 4 presents policy implications and extensions. Supporting material is relegated to the Appendix.

2 Model

A city is modeled as the interval $[0, 1]$. The total mass of households is 1. At each $t \in [0, 1]$, there is capacity for dt households, which means that the supply of housing has a constant unitary density. There are two types of households, $i \in \{1, 2\}$. We say that a household type i initially assigned at $h \in [0, 1]$ is *endowed* with a unit of housing at h. At each h, a fraction β of households are type 1, whereas a fraction $(1 - \beta)$ are type 2. Thus, households are initially uniformly assigned across the city and β is the fraction of type 1 households in the whole population.[8] We explore the resulting

[6] To see this, take $N \geq 2$ (even) jurisdictions and assume that half of them are type 1 and half of them are type 2. Since each jurisdiction is isolated from others, arranging jurisdictions back-to-back in a linear city does not alter the equilibrium but results in a rather integrated city.

[7] These are models of jurisdictions and, as we explained above, result in stratification but not necessarily in urban segregation.

[8] The initial allocation of households is irrelevant for our main results.

outcomes when households can trade.

2.1 Allocations, neighborhood composition, and equilibrium

A *housing allocation* is $x = (x_1, x_2)$, with $x_i \colon [0,1] \to [0,1]$. We interpret an allocation x by saying that households of type i endowed with housing at $h \in [0,1]$ are assigned to $x_i(h)$. Any such allocation determines a distribution of households of type i across the city by

$$\begin{aligned} D_i(t) &= dX\left(x_i^{-1}([0,t])\right) \\ &= dX\left(\{\text{Household type } i \text{ endowed with housing at some } h \in [0,1] \mid x_i(h) \le t\}\right) \end{aligned}$$

for any $t \in [0,1]$. $D_i(t)$ is the measure of type i households living within distance t of the origin.[9] In general, the distribution D_i need not have a density. We will restrict attention to housing allocations such that the induced distribution D_i is absolutely continuous and therefore has a (unique) density $d_i(t)$, for $i = 1, 2$.[10]

Given a housing allocation x, we define $\gamma_i \colon [0,1] \to [0,1]$ as the fraction of type i agents that live at each $t \in [0,1]$:

$$\gamma_i(t) = \begin{cases} \frac{d_i(t)}{d_1(t)+d_2(t)} & \text{if } d_1(t) + d_2(t) > 0 \\ 1 & \text{if not.} \end{cases}$$

For our main results, whether we normalize by $d_1(t)+d_2(t)$ or not is immaterial since $d_1(t)+d_2(t) = 1$ for any feasible allocation. We denote $L^\infty = \{\gamma \colon [0,1] \to \mathbb{R}_+ \mid \sup \mathrm{ess}_{y\in[0,1]} \gamma(y) < \infty\}$.[11]

In order to capture rich neighborhood externalities, we formulate our model so that the utility of a household of type i living at t depends not only on $\gamma_i(t)$ but on the entire function $\gamma_i \colon [0,1] \to 1$. Formally, a type i agent located at t has a *perception* $\Gamma_i(t)$ about her neighborhood defined by

$$\Gamma_i(t) = \int_0^1 f_t(y)\gamma_i(y)dy \tag{2.1}$$

where $f_t \colon [0,1] \to \mathbb{R}_+$ is a *weight function* and it is assumed continuous in (t, y) and $\int_0^1 f_t(y)dy = 1$. $\Gamma_i(t)$ measures the perception that a household living at t has about the distribution of types in the

[9]Note that $D_1(t) \le \beta$ and $D_2(t) \le 1 - \beta$ for all t.

[10]This restriction is without losee since we assume that at each t, there is capacity for dt households. Thus, in the model, it is not feasible to assign a positive measure of agents at any t.

[11]Here, $\sup \mathrm{ess}_{x\in X}\ f(x)$ denotes the *essential supremum* of a function f, defined as the smallest number such that $\{x \in X \mid f(x) > \alpha\}$ has measure 0.

city. The larger $\Gamma_i(t)$, the bigger the perception that a household at t has about the prevalence of type i agents when living at t. We denote $\Gamma(t) = (\Gamma_1(t), \Gamma_2(t))$. Denoting $\Delta = \{p \in \mathbb{R}^2_+ \mid p_1 + p_2 = 1\}$ the 1-simplex, $\Gamma(t) \in \Delta$ by construction.

Each household h consumes housing at some $t \in [0, 1]$ in addition to a composite consumption good $c \in \mathbb{R}$. Given x, let us denote $\Gamma^x(t)$ the neighborhood composition function induced by x. Then the utility enjoyed by a type i household that consumes $c \in \mathbb{R}$ and lives in $t \in [0, 1]$ is

$$U_i(c, \Gamma^x(t)) = c + v_i(\Gamma_i^x(t)),$$

with $v_i \colon [0, 1] \to \mathbb{R}$ a continuously differentiable function. Assuming that v_i is a function of Γ_i simplifies exposition.

There are several sources of neighborhood externalities (Durlauf 2004). The composition of a neighborhood may determine the quality of public goods (parks, schools) or the social interactions (peer effects, role models) that a household living in t may have. All these externalities are captured by $\Gamma_i(t)$.

Consumption is supplied elastically at price $p_c = 1$. The price of housing services at location t is denoted $R(t)$. Thus, the initial wealth of a household assigned at h is $R(h)$.

We now define an equilibrium for our model.

Definition 1. *An* equilibrium *is a price function $R \colon [0, 1] \to \mathbb{R}$, housing and consumption allocations $x_i \colon [0, 1] \to \mathbb{R}$ and $c_i \colon [0, 1] \to [0, 1]$, $i = 1, 2$, that induce distributions D_i and densities d_i such that*

i. For all $t \in [0, 1]$

$$d_1(t) + d_2(t) = 1$$

ii. For all i and all $h \in [0, 1]$, $(x_i(h), c_i(h))$ solves

$$\max_{t \in [0,1], \tilde{c} \in \mathbb{R}} \left\{ U_i(\tilde{c}, \Gamma_i^x(t)) \mid \tilde{c} + R(t) \leq R(h) \right\}$$

This is the standard definition of competitive equilibrium for an economy with a continuum of goods and externalities. The first condition ensures the housing market at each t clears. The second condition imposes each household chooses consumption and location optimally.

Observe that if $R \colon [0, 1] \to \mathbb{R}$ is an equilibrium price function, then so is $\tilde{R}(t) := R(t) + r$ for any constant r. So, an equilibrium price function is defined up to translation.

The model is constructed abstracting away from many features. Later, we extend our results to allow for nonquasilinear utility functions, more than two groups, and production.

We observe that an equilibrium always exists.

Lemma 1. *An equilibrium in which households are uniformly distributed across the city always exists.*

To see why this lemma holds, define $R(t) = 1$ and allocations x such that $d_1(t) = \beta$ and $d_2(t) = 1 - \beta$. This means that each group of households is uniformly distributed across the city and thus all locations are equally attractive. It is thus immediate to see that the initial allocation of households is an equilibrium of the model. This equilibrium is integrated. Our main results characterize other equilibria.

2.2 Segregated and integrated allocations

It will be useful to distinguish different types of locations and housing allocations.

Definition 2. *Let $x = (x_1, x_2)$ be a housing allocation.*

a. A location $t \in [0,1]$ is mixed *if there exists $\epsilon > 0$ such that for almost every $t' \in (t-\epsilon, t+\epsilon)$, $\gamma_1(t'), \gamma_2(t') \in (0,1)$;*

b. A location $t \in [0,1]$ is segregated *if there exists $\epsilon > 0$ and $i \in \{1,2\}$ such that for almost every $t' \in (t-\epsilon, t+\epsilon)$, $\gamma_i(t') = 1$;*

c. A location $t \in [0,1]$ is frontier *if it is neither of the above.*

By construction, every location is either mixed, segregated, or frontier. These definitions are illustrated in Figure 1.

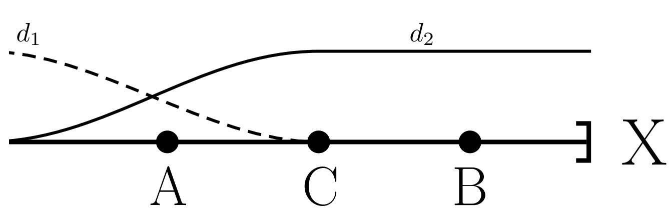
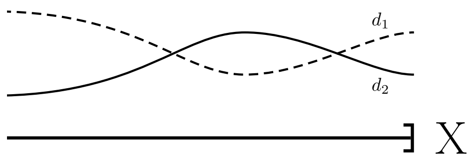


Figure 1: Location A is mixed, location B is segregated, location C is frontier.

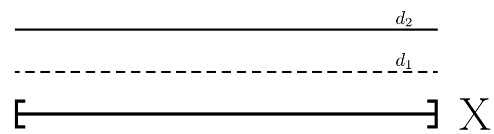
We now distinguish different allocations.

Definition 3. *a. A housing allocation x is* integrated *if every location is mixed. It is* perfectly integrated *if, in addition, there exists $\bar{\gamma} \in \mathbb{R}$ such that $\gamma_1^x(t) = \bar{\gamma}$ for almost every $t \in [0, 1]$.*

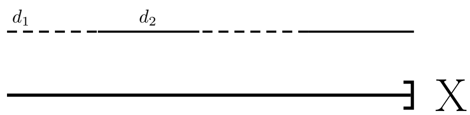
b. A housing allocation x is segregated *if there are no mixed locations. It is* completely segregated *if in addition there is only one frontier location.*



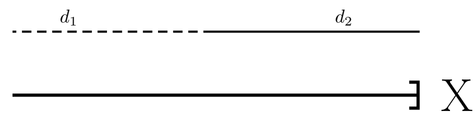
(a) Integrated allocation



(b) Perfectly integrated allocation



(c) Segregated allocation



(d) Completely segregated allocation

Figure 2: Representation of different allocations in Definition 3.

As seen in Figure 2a, there are some integrated allocations for which the neighborhood composition might change, for example if d_1 increases at the same time that d_2 decreases, but they are both positive. Those allocations are integrated, but not perfectly. As shown in Figure 2d, if a segregated allocation has exactly one frontier location, then the city is divided into two connected areas each of which is occupied by a different type of household. If there were more frontier locations, then there would be two non connected areas occupied by the same type of household, as in Figure 2c.

3 Analysis

This section states and discusses our main theorems. Subsection 3.1 introduces and illustrates a single-crossing condition. Subsection 3.2 imposes restrictions on the weight function f_t that are motivated by our our location model.

The main results are presented in Subsections 3.3, 3.4, and 3.5. In Subsection 3.3, we show that other than trivial perfectly integrated equilibrium, all equilibria are segregated. In Subsection 3.4, we show that any equilibrium that is robust to small coalitional deviations must be completely segregated. Finally, in Subsection 3.5 we derive comparative statics results for the completely segregated equilibrium.

3.1 The single crossing condition

In our model, preferences for neighborhood composition are captured by the function v_i. We do not assume any functional form on v_i but rather impose an assumption that allows us to capture several attitudes towards neighbors, ranging from homophily to one group generating positive externalities on every household.

The following definition is key to derive our results.

Definition 4. *.*

a. *The* single-crossing condition (SCC) *holds if* $v_1(P) - v_2(1-P)$ *is one-to-one over* $P \in [0,1]$.

b. *The* strong single crossing condition (S-SCC) *holds if* v_1 *and* v_2 *are differentiable and* $v_1'(P) + v_2'(1-P) > 0$ *for all* $P \in [0,1]$.

Under the SCC, the indifference curves of group i and group $-i$ cross once. To see this, take $(c', P'), (c, P) \in \mathbb{R} \times \Delta$ that belong to the same indifference curve for group i, with $P_i \neq P_i'$. Then, $v_i(P_i) - v_i(P_i') = c_i' - c_i$. When the SCC holds, $v_{-i}(P_{-i}) - v_{-i}(P_{-i}') \neq v_i(P_i) - v_i(P_i') = c_i' - c_i$ and therefore (c, P) and (c', P') are not in the same indifference curve for group $-i$. In other words, the SCC models situations in which the consumption compensation that a household should receive to be willing to move to a neighborhood with a different composition depends on the household group.

Since v_i is continuous and, under the SCC, also one-to-one, $P \mapsto v_1(P) - v_2(1-P)$ is either strictly increasing or strictly decreasing. Under the S-SCC, $P \mapsto v_1(P) - v_2(1-P)$ is strictly incresing.[12] Under the S-SCC, $v_1'(P) > -v_2'(1-P)$ and therefore type 1 households are willing to give up more consumption than type 2 agents to increase the share of type 1 households in their neighborhood.

The following example discusses different specification of the model that satisfy the SSC and that have been frequently employed in the literature (Schelling 1969, Sethi and Somanathan 2004, Guerrieri, Hartley, and Hurst 2013).

Example 1. *i. Own group's preferences: For all* i, $v_i(\Gamma_i)$ *increases in* $\Gamma_i \in [0,1]$. *Then,* $v_1(\Gamma_i) - v_2(1-\Gamma_i)$ *is strictly increasing in* Γ_i. *See Figure 3a.*

[12]The S-SCC is strictly more demanding than the condition $P \mapsto v_1(P) - v_2(1-P)$ being strictly increasing. Indeed, a strictly increasing function could have a derivative that equals 0 in some points.

ii. Schelling's preferences (Schelling 1969, Sethi and Somanathan 2004): $v_1 = v_2 = v$ *is concave,* v *is maximized when* $\Gamma = \rho$*, with* $\rho \in [1/2, 1]$*, and* $v'(\rho - P) > -v'(\rho + P)$ *for all* $P \in]0, 1-\rho]$*. Schelling's preferences capture the idea that there is an ideal segregation level* ρ *but that a household would prefer to be slightly over-represented rather than under-represented and therefore the utility function over* $P > \rho$ *is flatter than over* $P < \rho$*.*[13]

To see that the SCC holds, note that if $1 - \rho < P < \rho$, $v'(P) + v'(1-P) > 0$*. If* $P < 1 - \rho$*, then* $-v'(1-P) = -v'(\rho + (1 - P - \rho)) < v'(2\rho - 1 + P) < v'(P)$ *and therefore* $v'(P) + v'(1-P) > 0$*. Finally, if* $P > \rho$, $-v'(P) = -v'(\rho + (P - \rho)) < v'(2\rho - P) < v'(1-P)$ *and thus* $v'(P) + v'(1-P) > 0$*. Thus, the function* $P \mapsto v(P) - v(1-P)$ *is strictly increasing as its derivative is strictly positive on* $[0,1] \setminus \{\rho, 1-\rho\}$*. The S-SCC holds when we additionally impose that* $v_1'(\rho) + v_2'(1-\rho) > 0$ *and* $v_1'(1-\rho) + v_2'(1-\rho) > 0$*.*[14] *See Figure 3b.*

iii. Positive externalities I: $v_1(\Gamma_1)$ *increases in* Γ_1 *and* $v_2(\Gamma_2)$ *decreases in* Γ_2*. The S-SCC holds provided* $v_1'(P) + v_2'(1-P) > 0$*. Similar to Guerrieri, Hartley, and Hurst (2013), these preferences capture the idea that while both types of agents prefer to live close to type 1 households (so* $v_1' > 0$ *but* $v_2' < 0$*). When there are positive externalities it is reasonable to suppose that the willingness to pay for living closer to type 1 households is larger for type 1 than for type 2 agents. See Figure 3c.*

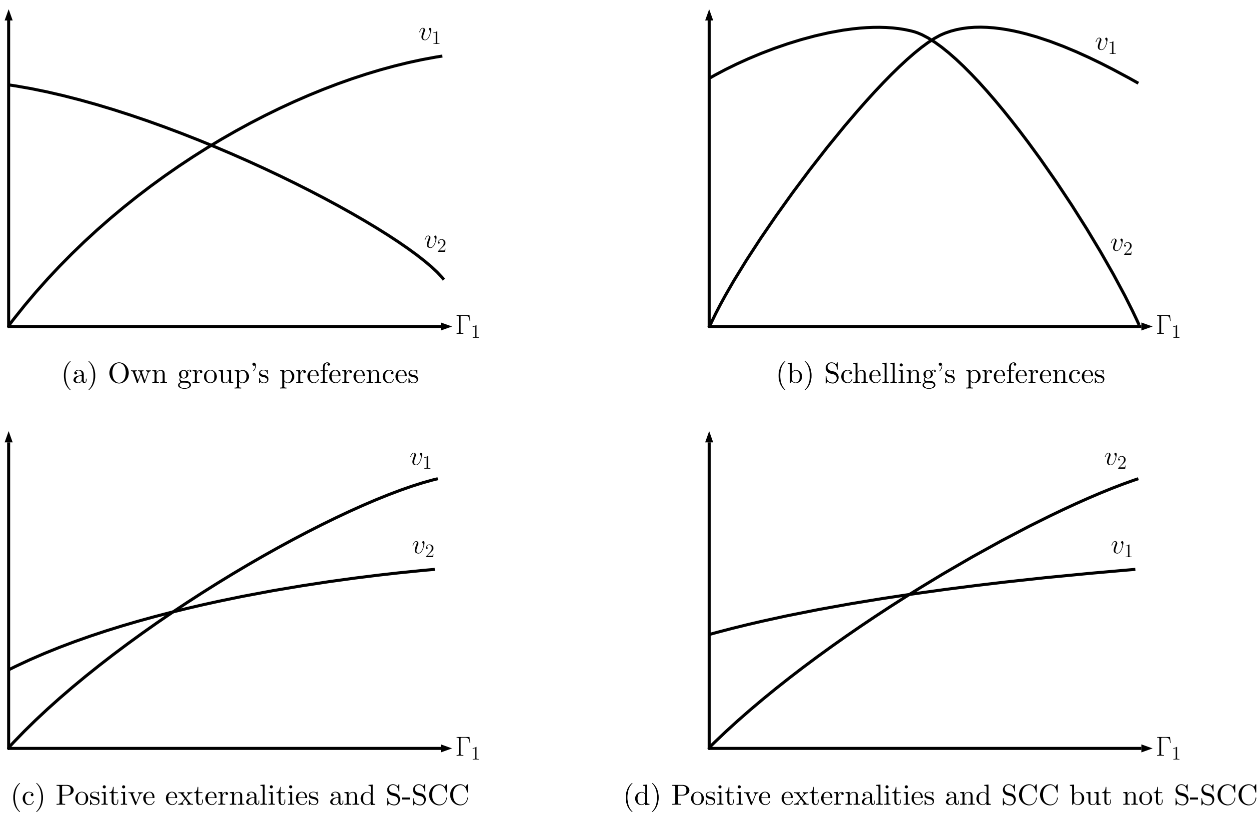
iv. Positive externalities II: $v_1(\Gamma_1)$ *increases in* Γ_1 *and* $v_2(\Gamma_2)$ *decreases in* Γ_2*. It is theoretically possible that type 2 agents would trade more consumption for living closer to group 1. In this case,* $v_1'(P) + v_2'(1-P) < 0$*, the SCC holds but S-SCC does not. See Figure 3d.*

3.2 Perceptions for neighborhood composition

We now impose some restrictions on the weight function f_t that are motivated by the fact that households use it to form perceptions about neighborhoods. Recall that the perception $\Gamma_i(t)$, defined on equation (2.1), is determined both by γ_i (which is an endogenous variable) and the weight function f_t.

[13] As Schelling (1969) put it: "Whites and blacks may not mind each other's presence, even prefer some integration, but, if there is a limit to how small a minority either color is willing to be, initial mixtures more extreme than that will lose their minority members..." Wong (2013) provides support to Schelling's preferences by presenting evidence that households prefer some degree of racial integration.

[14] To see an example of this type of preferences where the SCC holds but S-SCC does not, suppose that $\rho = 1/2$ and therefore $v_1'(\rho) = v_2'(1-\rho) = 0$.



(a) Own group's preferences

(b) Schelling's preferences

(c) Positive externalities and S-SCC

(d) Positive externalities and SCC but not S-SCC

Figure 3: Indifference curves

Definition 5. *We say that households* perceive differences in city composition *if for any open interval $T \subseteq [0,1]$ such that $\Gamma_i(t) = \Gamma_i(t')$ for almost every $t, t' \in T$, there exists $\bar{\gamma} \in [0,1]$ such that $\gamma_i(t) = \bar{\gamma}$ for almost every $t \in [0,1]$.*

This restriction captures the idea that differences in the composition of agents along the city γ_i imply differences in how households perceive the group composition across different locations Γ_i. Household perceive differences in city composition if whenever the fraction of agents of type i, γ_i, is not constant, then in any open interval T we can find locations $t, t' \in T$ where the city is perceived differently: $\Gamma_i(t) \neq \Gamma_i(t')$.

The idea that households perceive differences in city composition is natural in our model of neighborhood effects. Indeed, under the extremely opposite assumption that perceptions $\Gamma^x(t)$ do not depend on x (or γ), the neighborhood composition would be irrelevant for households and would be undetermined in equilibrium. Our goal is to explore the impact of neighborhood effects on equilibrium choices and, accordingly, Definition 5 says that the distribution of households over the city matters for households decisions.

The following example shows a density for which households do not perceive differences in city composition.

Example 2. *Take* $0 < a < b < 1$ *and define* $r_1 = \ln\left(\frac{1+e^a}{2}\right)$ $(< a)$ *and* $r_2 = -\ln\left(\frac{e^{-b}+e^{-1}}{2}\right)$ $(> b)$. *Consider the housing allocation* x *where intervals* $(0, r_1)$ *and* (b, r_2) *consists of type 1 households, intervals* (r_1, a) *and* $(r_2, 1)$ *consists only of type 2 households, and* (a, b) *is a mixed interval where households type 1 and 2 live in equal proportions and are uniformly distributed along* (a, b)*. For this allocation,* $\gamma_1(t) = \mathbb{1}_{(0,r_1)\cup(b,r_2)}(t) + \frac{1}{2}\mathbb{1}_{(a,b)}(t)$*, where* $\mathbb{1}_A$ *represents the characteristic function of the set* A*.*

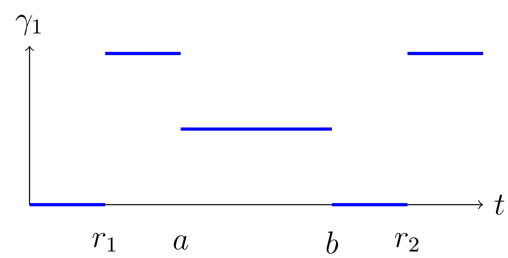


Figure 4: This figure shows the fraction of type 1 households, $\gamma_1(t)$, at each location $t \in [0, 1]$.

Consider the weight function $f_t(y) = \exp(-|t-y|)\kappa(t)$ *where* $\kappa(t) = \left(\int_0^1 \exp(-|t-y|)\, dy\right)^{-1}$. *For this weight function, households do not perceive differences in city composition. Indeed,* $\forall t \in (a, b)$, $\Gamma_1^x(t) = \frac{1}{2}$.[15] *Thus when* f_t *is used to form perceptions* $\Gamma(t)$*, agents perceive exactly the same neighborhood composition at each location* $t \in (a, b)$ *even though locations across* $t \in (a, b)$ *seem qualitatively very different.*

The restriction to households perceiving differences in city composition is related to the idea of bounded completeness of a family of measures used in statistics (Casella and Berger 2002). The following result is a corollary to well-known characterizations in the statistics literature.

Lemma 2. *Suppose that* $f_t(y) = \exp\big(\lambda(t)q(y) + V(y) + W(t)\big)$*, where* λ*,* q*,* V*, and* W *are continuous,* $\{\lambda(t) \mid t \in T\}$ *is open for any open set* $T \subseteq [0, 1]$*, and* q *is one-to-one. Then, households perceive differences in city composition.*

[15] To see this, note that $\kappa(t) = \frac{1}{\int_0^1 \exp(-|t-y|)dy} = \frac{1}{2-\exp(-t)-\exp(t-1)}$, whereas $\int \gamma_1(y)\exp(-|t-y|)dy = \frac{1}{2}\Big(2-\exp(-t)-\exp(t-1)\Big)$ for $t \in (a, b)$.

We can use this lemma to specify several densities f_t for which households perceive differences in city composition.

Example 3. *For each of the following densities, households perceive differences in city composition:*

i. $f_t(y) = \exp\left(\frac{-(t-y)^2}{2\sigma^2}\right)\kappa(t)$ *where* $\kappa(t) = \left(\int_0^1 \exp\left(\frac{-(t-y)^2}{2\sigma^2}\right)dy\right)^{-1}$. *This density is a location family with truncated Gaussian kernel.*

ii. $f_t(y) = \exp(-V'(t)y + V(y) + W(t))$, *where* $V(y)$ *is twice continuously differentiable and strictly concave. For fixed* t, *this density attains its maximum at* $y = t$, *is increasing over* $y \in [0,t]$ *and decreasing over* $y \in [t,1]$. *This density extends the location family with Gaussian kernel.*

iii. $f_t(y) = \frac{(2-y)^{\frac{2-t}{2+t}}}{2+y}\kappa(t)$, *where* $\kappa(t) = \left(\int_0^1 \frac{(2-y)^{\frac{2-t}{2+t}}}{2+y}\right)^{-1}$. *This weight function is obtained from Lemma 2 by setting* $\lambda(t) = \frac{2-t}{2+t}$, $q(y) = \ln(2-y)$, *and* $V(y) = \ln(2+y)$. *For fixed* t, *this density attains its maximum at* $y = t$.

We also consider the following restrictions on the weight function f_t.

Definition 6. *i. Households* care most about next-door neighbors *if* $\{t\} = \arg\max_{y\in[0,1]} f_t(y)$ *for all* t.

ii. Households have monotone perceptions *if for all* $y \in [0,1]$, $t \mapsto F_t(y) = \int_0^y f_t(s)ds$ *is strictly decreasing.*

iii. Households have regular perceptions *when they care most about next-door neighbors, have monotone perceptions, and perceive differences in city composition.*

On the one hand, households care most about next-door neighbors when they put most weight on the location at which they are evaluating the distribution of households. The densities in Example 3 model perceptions in which households located at t put the highest weight on the next-door neighbors $y = t$. These densities model situations in which agents care most about neighbors closer to them. On the other hand, noting that $F_t(y)$ is the perception that a household living at t has about a ghetto with all its members living in $[0, y]$, households have monotone perceptions when their perceptions about the ghetto reduce as they move to the right.[16] For the family of distributions illustrated in Lemma 2, we can derive a simple sufficient condition for monotone perceptions.

[16]Obviously, monotone perceptions for all x is equivalent to F_t being increasing in t in the first order stochastic dominance sense.

Lemma 3. *For any weight function $f_t(y)$ in Lemma 2, households have monotone perceptions if λ and q are both non-decreasing or both non-increasing.*

This lemma immediately implies that for all the weight functions in Example 3, households have monotone perceptions. Thus, for all distributions in Example 3, households have regular perceptions.

3.3 Equilibrium segregation

The following result says that segregated allocations are the only non-trivial equilibrium outcomes.

Theorem 1. *Suppose that households perceive differences in city composition and that SCC holds. Then, any equilibrium allocation is either segregated or perfectly integrated.*

This result shows that, other than the trivial perfectly integrated equilibrium, equilibrium allocations are segregated and the city can be partitioned in intervals, each of them containing only one type of household. This result rules out configurations in which some only some neighborhoods are integrated, or in which all neighborhoods are partially integrated.

The main force behind this theorem is the following. Take an equilibrium that is not perfectly integrated such that there is some open interval $T \subseteq [0, 1]$ in which both groups live. Take any two locations t' and t'' in the mixed interval T, and note that prices $R(t')$ and $R(t'')$ must ensure that households of type $i = 1, 2$ find t' and t'' equally attractive. Since the allocation is not perfectly integrated, both types of households perceive locations t' and t'' differently. Prices $R(t')$ and $R(t'')$ must therefore attract both types of households $i = 1, 2$ to locations t' and t'' that are different. This is impossible under the SCC, since households have different willingness to pay for locations t' and t''. The equilibrium must therefore be segregated.

We now explore the existence of a completely segregated equilibrium. Let us take a completely segregated allocation in which type 1 households live in $[0, \beta]$, while type 2 households live in $[\beta, 1]$. Then let us define the rent price function:

$$R(t) = \begin{cases} v_1(F_t(\beta)) & \text{if } t \leq \beta, \\ v_2(1 - F_t(\beta)) + \eta & \text{if } t > \beta. \end{cases}$$

where $\eta \in \mathbb{R}$ is to be determined. Given R, all type 1 households find any location $t \in [0, \beta]$ equally attractive. In the Appendix, we prove that when perceptions are monotone and S-SCC holds, type

1 households strictly prefer to live in $[0,\beta]$ over $]\beta,1]$ and households type 2 strictly prefer to live in $]\beta,1]$ over $[0,\beta]$ iff

$$\eta = v_1(F_\beta(\beta)) - v_2(1 - F_\beta(\beta)).$$

We observe that in a completely segregated equilibrium the price function R is continuous even at $t=\beta$.

We state our main existence result.

Proposition 1. *Suppose that households have monotone perceptions at any y and SCC holds. Then, a completely segregated equilibrium exists iff S-SCC holds.*

The proof of this result is constructive. In Appendix A.4, we provide necessary and sufficient conditions for the existence of a segregated equilibrium that is not completely segregated.

3.4 Coalitions, stability and segregation

In this section we discuss a refinement of equilibria that borrows some ideas from the core allocation of the economy (Aumann 1964). Formally, take a subset of households $B_i \subseteq [0,1]$ for each i. We say the assignment (x^B, c^B) is *built* from (x,c) by coalition B if

(i) All households outside of the coalition maintain their consumption and housing, that is, $x_i^B(h) = x_i(h)$ and $c_i^B(h) = c_i(h)$ for all $h \notin B_i$; and

(ii) Consumption and housing are redistributed among households inside B, that is,

$$\beta \int_{B_1} c_1^B(h)dh + (1-\beta)\int_{B_2} c_2^B(h)dh = \beta \int_{B_1} c_1(h)dh + (1-\beta)\int_{B_2} c_2(h)dh$$

and

$$d_1^B(t) + d_2^B(t) = 1 \quad \forall t \in [0,1]$$

where d_i^B is the density induced by the distribution $D_i^B(t) = |\{h \in [0,1] \mid x_i^B(h) \le t\}|$.

We say that coalition B *can block* (x,c) if (x^B, c^B) can be built by B and members in B get higher utility under (x^B, c^B) than under (x,c), that is, for all $h \in B_i$,

$$c_i^B(h) + v_i(\Gamma_i^{x^B}(x_i^B(h))) > c_i(h) + v_i(\Gamma_i^x(x_i(h))).$$

In words, a coalition B can block (x, c) if agents in B can redistribute their housing and consumption assignments so that all members in B get higher utility.

Definition 7. *We say that an equilibrium allocation* (x, c) *is* stable *if there exists* $\epsilon > 0$ *such that for any coalition* B *such that* $|B_i| < \epsilon$, $i = 1, 2$, (x, c) *cannot be blocked by* B. *An equilibrium allocation* (x, c) *that is not stable is said to be* unstable.

A stable equilibrium allocation is robust to small coalitional deviations. In our specific context of housing decisions, the idea that an allocation (x, c) that can be blocked by some small coalition B should be deemed fragile seems particularly appropriate, as private developers or real state agents could coordinate some households to exclude others from some neighborhoods. This is consistent with the "white collective action" view under which white homeowners or businesses exclude black households from white areas (Boustan 2011). We therefore use the idea of small blocking coalitions to define our stability notion.

Our definition of stable equilibrium allocations is reminiscent of (and indeed inspired by) the core concept (Aumann 1964, Telser 1994). One distinction is that our definition of stability only allows for small blocking coalitions. We note that while our model has a continuum of agents, *core equivalence* does not hold due to neighborhood externalities (Aumann 1964). A full characterization of core allocations is beyond the scope of this paper.

We are ready to state our main result about stable equilibrium allocations.

Theorem 2. *Suppose that S-SCC holds and that households have regular perceptions. Then, a completely segregated equilibrium exists and is stable. Moreover, any stable equilibrium must be completely segregated.*

This theorem provides conditions under which completely segregated allocations are the only stable equilibria. Theorem 2 complements Theorem 1 by ruling out any equilibrium that is not completely segregated.

Theorem 2 assumes the S-SCC. Under the S-SCC, for $P > P'$, $v_1(P) + v_2(1 - P') > v_1(P') + v_2(1-P)$. This means that if a type 1 household is living under perception P and a type 2 household is living under perception $1 - P'$, with $P' < P$, they would get lower total utility by switching housing. In particular, in a completely segregated equilibrium, households type i and type $-i$ would not benefit from switching housing as they would both end up under lower perceptions and would get lower total utility.

To see that no other equilibrium can be stable, take the perfectly integrated allocation. Under a perfectly integrated equilibrium, type 1 households have perception β while type 2 households have perception $(1-\beta)$. Since S-SCC holds, $v_1'(\beta)+v_2'(1-\beta) > 0$. This means that if a small mass of type 1 and type 2 households switched housing, both types of households would end up having higher perceptions and would therefore be willing to trade housing. As a result, the perfectly integrated equilibrium cannot be stable.

Theorem 2 fails when we only impose SCC instead of S-SCC. Consider Schelling's preferences in Example 1 with $\rho = 1/2$ and $\beta = 1/2$. Under perfect integration, $\Gamma_i^x(t) = 1/2$ and therefore perfect integration maximizes the total sum of households' utilities. Thus, no coalition can block perfect integration.

3.5 Comparative statics

When perceptions are monotone, $F_t(\beta)$ increases in t. This means that for the segregated allocation in which type 1 households live in ghetto $[0,\beta]$, a household living in t perceives more type 1 agents than a household in $\bar{t}$, for $t < \bar{t} < \beta$. Whether this translates in lower or higher rental prices depends on v_1. Indeed, for $t < \beta$

$$R'(t) = v_1'(F_t(\beta))\frac{\partial F_t(\beta)}{\partial t} \tag{3.1}$$

and therefore the sign of the derivative of R equals minus the sign of $v_1'(F_t(\beta))$.[17] The following example shows the behavior of the rental price function for different specifications under the assumption that perceptions are monotone.

Example 4. *i. Own group's preferences. Since $v_i(\Gamma_i)$ increases, $R(t)$ decreases over $[0,\beta]$ and increases over $[\beta,1]$.*

ii. Schelling's preferences. The utility function v attains a maximum at ρ. Suppose $F_0(\beta) > \rho$ and $F_\beta(\beta) < \rho$. Then, R will be increasing over $[0,\hat{t}]$ and decreasing over $[\hat{t},\beta]$, where $F_{\hat{t}}(\beta) = \rho$.

iii. Positive externalities. Since $v_1' > 0$ and $v_2' < 0$, $R(t)$ is decreasing over $[0,1]$.

One important implication from these comparative statics results is that housing prices in different locations can be used to infer preferences. For example, consider a city consisting of two

[17]Note that since perceptions are monotone, $\frac{\partial F_t(\beta)}{\partial t} \leq 0$.

ghettos, one is black and the other white. If rental prices increase as we move from the core of the black ghetto towards its frontier, this suggests that black households prefer whiter neighborhoods. This would be consistent with preferences in which whites impose positive externalities on blacks.

4 Discussion

4.1 Policy interventions

There are several reasons authorities may want to fight segregation. First, there is no reason to believe that the competitive equilibrium in a model with externalities will be Pareto efficient.[18] Indeed, each household ignores the impact of its location decision on the well being of others and thus the welfare theorems need not apply. Second, authorities may want to improve the material conditions of households living in isolated low income segregated neighborhoods. Families in segregated metropolitan areas have lower earnings as they have fewer employment opportunities and harmful social interactions (Boustan 2011). Finally, segregation may damage societal values such as inter group tolerance and social attitudes (Rao 2019).

Two measures are typically used to fight segregation. On the one hand, housing or loan subsidies help individual home buyers or renters gain access to consolidated well off neighborhoods. On the other hand, place-based investments improve the housing stock or amenities in black or low income neighborhoods to encourage white or affluent households to move in. We use our model to shed light on the difficulties these policies face to modify a segregated outcome.

For concreteness, suppose that type 2 households are the minority group living in a low income ghetto $[\beta, 1]$ in a completely segregated equilibrium as in Section 3.5. To see the impact of subsidies, suppose now that type 2 households receive a subsidy $s > 0$ whenever they buy a house in the type 1 ghetto $[0, \beta]$. Right after the subsidy s is introduced, type 2 households find attractive locations in $[0, \beta]$ that are close to β. This creates excess demand and therefore prices should rise. It is hard to examine the adjustment equilibrium dynamics. In the new equilibrium, segregation will prevail. Moreover, we can now strengthen Theorem 1 to show that *all equilibria must be segregated.* Indeed, the perfectly integrated allocation cannot be part of an equilibrium. To see this, note that if type 1 households demand all locations in [0,1], $R(t) = R(0)$ for all $t \in [0, 1]$. If type 2 households demand all locations in $[0, 1]$, $R(t) - s = R(t')$ for all $t < \beta$ and all $t' > \beta$. These two conditions

[18] In our environment, it is relatively easy to formulate the problem of maximizing welfare over all housing allocations x. Analytically solving the problem of optimal welfare seems unfeasible.

are incompatible when $s \neq 0$. Adapting Theorem 1 to our model with subsidy $s > 0$, it follows that any equilibrium must be segregated.

When the subsidy can condition on the location where the household chooses to live, the logic above no longer applies. Indeed, we show in Appendix A.6 that given any housing allocation x, there exists a location-dependent subsidy policy such that allocation x is an equilibrium. We think this result is of limited practical interest as it requires a rather complex subsidy policy that depends on parameters that are unknown to authorities.

Consider now a place-based investment policy that improves the type 2 ghetto $[\beta, 1]$. Concretely, supposes that the government develops an infrastructure project such that a household living in $t \in [\beta, 1]$ gets an additional consumption utility $I(t) > 0$. Similar to the introduction of a subsidy, right after the public infrastructure is built there will be excess demand for housing in $[\beta, 1]$. In the new equilibrium, if households type 1 and 2 live in a mixed neighborhood, it must be the case that the infrastructure discounted rental price makes both types indifferent between all locations in the mixed neighborhood and therefore the allocation is perfectly integrated. Again, once public infrastructure is introduced, the equilibrium must be segregated or perfectly integrated.[19]

4.2 Extensions

We present two extensions of our model. The first extension allows for production of houses. The second extension allows for several groups and more general utility functions. Other extensions, such as to two-dimensional cities, are promising and left for future research.

Production. In our model, we have assumed that the supply of houses if fixed. Our model can be extended to allow for an endogenous supply of houses. Suppose that the cost of supplying $d(t) \geq 0$ units of housing is $C(d(t))$, with C strictly increasing, convex, and $C(0) = 0$. At each t, housing is supplied by a competitive firm. In this extension, households are not initially endowed with housing. In equilibrium, at each $t \in [0, 1]$, the competitive production of houses $d(t)$ will satisfy $C'(d(t)) = R(t)$. Each household $h \in [0, 1]$ of type i will choose $(x_i(h), c_i(h))$ to solve

$$\max_{t\in[0,1],\tilde{c}\in\mathbb{R}} \left\{ U_i(\tilde{c}, \Gamma^x(t)) \mid \tilde{c} + R(t) \leq \pi_i(h) \right\}$$

where $\pi_i(h)$ is the sum of profits received by the household. The market clearing is written as

[19]Different from the subsidy policy, perfect integration is an equilibrium of the model with public infrastructure. Indeed, to sustain the perfectly integrated assignment it is enough to set $R(t) = I(t)$, where we extend $I(t) = 0$ for $t < \beta$. Yet, similar to Theorem 2, a perfectly integrated equilibrium will be unstable.

$d_1(t) + d_2(t) = d(t)$ for all $t \in [0, 1]$, where d_i is built from x_i as in our main model. Extending Theorems 1 and 2 to this general environment is immediate.

General utility functions and several groups. Suppose now that there are $n \geq 2$ groups, with $\beta_i \geq 0$ the fraction of households of type i ($\sum_{i=1}^n \beta_i = 1$). For each allocation $x = (x_i)_{i=1}^n$, we define $d_i(t)$, $\gamma_i(t) = d_i(t)/(\sum_{j=1}^n d_j(t))$, and $\Gamma_i(t)$ analogously to the main model. We now assume that the utility of a household of type i that consumes $c \in \mathbb{R}$ and lives in t is given by $U_i(c, \Gamma(t))$, where $\Gamma(t) = (\Gamma_j(t))_{j=1}^n$. Denoting $\Delta = \{p \in \mathbb{R}_+^n \mid p_1 + \cdots + p_n = 1\}$, it follows that $\Gamma(t) \in \Delta$. In this extension, U_i need not be quasilinear in c. Our baseline model obtains when $n = 2$, $\beta_1 = \beta$, $\beta_2 = (1 - \beta)$, and $U_i(c, \Gamma(t)) = c + v_i(\Gamma_i(t))$.

Assume that the wealth of each household of type i is given by $\pi(i)$. We say that the *single-crossing condition* holds if for all r, r', all $P, P' \in \Delta$, with $(r, P) \neq (r', P')$, and all $i \neq j$

$$U_i(\pi(i) - r, P) = U_i(\pi(i) - r', P') \text{ implies } U_j(\pi(j) - r, P) \neq U_j(\pi(j) - r', P').$$

This condition says that if bundles $(\pi(i) - r, P)$ and $(\pi(i) - r', P')$ are in the same indifference curve for a household of type i, then a type j household would not find $(\pi(j) - r, P)$ and $(\pi(j) - r', P')$ equally attractive. To see how Theorem 1 can be extended, consider a mixed neighborhood T in which households type i and j live and note that for $t, t' \in T$, $U_k(\pi(k) - R(t), \Gamma^x(t)) = U_k(\pi(k) - R(t'), \Gamma^x(t'))$ for $k = i, j$. It must be the case that $R(t) = R(t')$ and $\Gamma^x(t) = \Gamma^x(t')$ and, since households perceive differences in city composition, $\gamma^x(t) = \bar{\gamma}$ for $t \in [0, 1]$. This means that the allocation must be perfectly integrated.

A Appendix

A.1 Proofs for Lemmas

Proof of Lemma 2. The definition of households perceiving differences in city composition boils down to the claim that for any open interval $T \subseteq [0,1]$, the family $(f_t)_{t\in T}$ is boundedly complete, as in Definition 6.2.21 in Casella and Berger (2002) (setting $T(x) = x$). The lemma follows from Theorem 6.2.25 in Casella and Berger (2002). Just note that since q is one-to-one, to check completeness we can assume without loss that $q(y) = y$.

$\square$

Proof of Lemma 3. Note that $F_t(x)$ is decreasing in t provided

$$\lambda'(t)\int_0^x q(y)\exp(\lambda(t)q(y)+V(y)+W(t))dy \leq -W'(t)\int_0^x \exp(\lambda(t)q(y)+V(y)+W(t))dy.$$

Since $\int_0^1 f_t(y)dy = 1$, $W'(t) = -\mathbb{E}_t[q(y)]\lambda'(t)$ where the expectation is taken assuming that y distributes according to the density $f_t(y)$. Thus, $\frac{\partial}{\partial t}F_t(x) \leq 0$ provided

$$\lambda'(t)\mathbb{E}_t[q(y) \mid y \leq x] \leq \lambda'(t)\mathbb{E}_t[q(y)]. \tag{A.1}$$

When λ and q are both increasing, $\lambda'(t) \geq 0$ and $E_t[q(y) \mid y \leq x] \leq E_t[q(y)]$. When p and q are both decreasing, $\mathbb{E}_t[q(y) \mid y \leq x] \geq E_t[q(y)]$ but $\lambda'(t) \leq 0$. In both cases, condition (A.1) holds. $\square$

A.2 Proof of Theorem 1

Proof of Theorem 1. Consider an equilibrium allocation x that is not segregated and let $\bar{t}$ be a mixed location. Thus, there exists $\epsilon > 0$ such that for almost every $t' \in (\bar{t}-\epsilon, \bar{t}+\epsilon)$, $\gamma_i(t') \in (0,1)$. For any such t' and t'' and for each i, take households h_i' and h_i'' living in t' and t''. Then

$$U_i(R(h_i') - R(t'), \Gamma^x(t')) \geq U_i(R(h_i') - R(t''), \Gamma^x(t''))$$

and

$$U_i(R(h_i'') - R(t''), \Gamma^x(t'')) \geq U_i(R(h_i'') - R(t'), \Gamma^x(t')).$$

Equivalently, $R(t'') - R(t') \geq v_i(\Gamma_i^x(t'')) - v_i(\Gamma_i^x(t'))$ and $R(t') - R(t'') \geq v_i(\Gamma_i^x(t')) - v_i(\Gamma_i^x(t''))$. In other words, for all $i = 1, 2$,

$$R(t') - R(t'') = v_i(\Gamma_i^x(t')) - v_i(\Gamma_i^x(t'')).$$

In particular,

$$v_1(\Gamma_1^x(t')) - v_1(\Gamma_1^x(t'')) = v_2(\Gamma_2^x(t')) - v_2(\Gamma_2^x(t''))$$

The SCC implies that $\Gamma_i^x(t') = \Gamma_i^x(t'')$ for $i = 1, 2$. Defining $T = (\bar{t} - \epsilon, \bar{t} + \epsilon)$, it follows that $\Gamma_i^x(t') = \Gamma_i^x(t'')$ for almost all $t', t'' \in T$. Since households perceive differences in city composition, $\gamma_i(t)$ is constant in $[0, 1]$. It follows that the allocation is perfectly integrated. □

A.3 Proof of Theorem 2

The proof of Theorem 2 follows from two lemmas.

Lemma 4. *Suppose that S-SCC holds and that households have monotone perceptions. Then, a completely segregated equilibrium is stable.*

Proof. Take a completely segregated allocation x and suppose all type 1 agents are in the interval $[0, \beta]$. For any type 1 agent living in some $t \leq \beta$, $\Gamma_1^x(t) = F_t(\beta)$. For any type 2 agent living in $t' \geq \beta$, $\Gamma_2^x(t) = 1 - F_t(\beta)$. Since perceptions are monotone, for $t \neq t'$ with $t \leq \beta \leq t'$, $\Gamma_1^x(t) > \Gamma_1^x(t')$. Since $v_1(x) - v_2(1 - x)$ is increasing, $v_1(\Gamma_1^x(t)) - v_2(\Gamma_2^x(t)) > v_1(\Gamma_1^x(t')) - v_2(\Gamma_2^x(t'))$ and therefore $v_1(\Gamma_1^x(t)) + v_2(\Gamma_2^x(t')) > v_1(\Gamma_1^x(t')) + v_2(\Gamma_2^x(t))$. As a result, there exists $\epsilon > 0$ such that for any any coalition B, with $B_1 \cap B_2 = \emptyset$, $|B_i| < \epsilon$, and $t \in B_1, t' \in B_2$, $v_1(\Gamma_1^x(t_1)) + v_2(\Gamma_2^x(t_2)) > v_1(\Gamma_1^B(t_1)) + v_2(\Gamma_2^B(t_2))$ for all $t_i \in B_i$. Integrating over the set of agents switching housing, we get

$$\beta \int_{B_1} v_1(\Gamma_1^x(x_1(h)))dh + (1-\beta) \int_{B_2} v_2(\Gamma_2^x(x_2(h)))dh > \beta \int_{B_1} v_1(\Gamma_1^B(x_1^B(h)))dh + (1-\beta) \int_{B_2} v_2(\Gamma_2^B(x_2^B(h)))dh$$

It thus follows that for any feasible consumption allocation c^B

$$\begin{aligned} &\beta \int_{B_1} (c_1(h) + v_1(\Gamma_1^x(x_1(h))))dh + (1 - \beta) \int_{B_2} (c_2(h) + v_2(\Gamma_2^x(x_2(h))))dh \\ &\quad > \beta \int_{B_1} (c_1^B(h) + v_1(\Gamma_1^B(x_1^B(h))))dh + (1 - \beta) \int_{B_2} (c_2^B(h) + v_2(\Gamma_2^B(x_2^B(h))))dh \end{aligned}$$

As a result, there exists a positive measure set of households such that $c_i(h) + v_i(\Gamma_i^x(x_i(h))) >$

$c_i^B(h) + v_i(\Gamma_i^B(x_i^B(h)))$. It thus follows that coalition B cannot block (x, c). $\square$

Lemma 5. *Suppose that S-SCC holds and that households care most about next-door neighbors. Then, any stable equilibrium must be completely segregated.*

Proof. We first show that a perfectly integrated allocation x is not stable. For simplicity, assume that $x_i(h) = h$. By definition, at each t, there is βdt of type 1 households, and $(1-\beta)dt$ of type 2 households. Take intervals $I_1 \subset [0, 1/2]$ and $I_2 \subset [1/2, 1]$ with $(1-\beta)|I_1| = \beta|I_2|$. Assume these intervals are small enough so that for all $t \in I_1$

$$\min_{y \in I_1} f_t(y) > \max_{y \in I_2} f_t(y). \tag{A.2}$$

and for all $t \in I_2$, $\min_{y \in I_2} f_t(y) > \max_{y \in I_1} f_t(y)$. Take $B_1 = I_2$, $B_2 = I_1$, and x^B such that the resulting densities are given by

$$d_1^B(t) = \begin{cases} \beta & t \notin I_1 \cup I_2 \\ \beta + \epsilon_1 & t \in I_1 \\ \beta - \epsilon_2 & t \in I_2 \end{cases}$$

and

$$d_2^B(t) = \begin{cases} 1-\beta & t \notin I_1 \cup I_2 \\ 1-\beta-\epsilon_1 & t \in I_1 \\ (1-\beta) + \epsilon_2 & t \in I_2 \end{cases}$$

where $\epsilon_1, \epsilon_2 > 0$ are small enough and $\epsilon_1|I_1| = \epsilon_2|I_2|$. It then follows that for $t \in I_1$

$$\int \gamma_1^B(y) f_t(y) dy - \int \gamma_1(y) f_t(y) dy > 0 \text{ iff } \epsilon_1 \int_{I_1} f_t(y) dy > \epsilon_2 \int_{I_2} f_t(y) dy.$$

From (A.2), for all $t \in I_1$,

$$\epsilon_1 \int_{I_1} f_t(y) dy \geq \epsilon_1 |I_1| \min_{y \in I_1} f_t(y) > \epsilon_2 |I_2| \max_{y \in I_2} f_t(y) \geq \epsilon_2 \int_{I_2} f_t(y) dy$$

and therefore

$$\int \gamma_1^B(y) f_t(y) dy > \int \gamma_1(y) f_t(y) dy = \beta. \tag{A.3}$$

Analogously, for all $t' \in I_2$,

$$\int \gamma_2^B(y) f_{t'}(y) dy > (1-\beta). \tag{A.4}$$

Note that as I_1 and I_2 get small and close enough, for all $t \in I_1$ and all $t' \in I_2$,

$$|(\int \gamma_1^B(y) f_t(y) dy - \beta) - (\int \gamma_2^B(y) f_{t'}(y) dy - (1-\beta))| \to 0.$$

Since $(A.3) - (A.4)$ and $y \mapsto v_1(\beta + y) + v_2(1 - \beta + y)$ is strictly increasing close to $y = 0$, we can therefore take I_1 and I_2 so that for all $t \in I_1$ and all $t' \in I_2$,

$$v_1(\int \gamma_1^B(y) f_t(y) dy) + v_2(\int \gamma_2^B(y) f_{t'}(y) dy) > v_1(\beta) + v_2(1-\beta).$$

We now construct the consumption assignment c^B. For all $h \in B_i$, let $\tilde{c}_i(h)$ be defined by

$$\tilde{c}_i(h) + v_i\big(\Gamma_i^{x^\epsilon}(x_i^\epsilon(h))\big) = c_i(h) + v_i\big(\Gamma_i^x(x_i(h))\big)$$

Integrating,

$$\begin{aligned}\beta \int_{h \in B_1} \tilde{c}_1(h) + v_1\big(\Gamma_1^{x^\epsilon}(x_1^\epsilon(h))\big) dh + &(1-\beta) \int_{h \in B_2} \tilde{c}_2(h) + v_2\big(\Gamma_2^{x^\epsilon}(x_2^\epsilon(h))\big) dh \\ &= \beta \int_{h \in B_1} c_1(h) + v_1\big(\beta\big) dh + (1-\beta) \int_{h \in B_2} c_2(h) + v_2\big(\beta\big) dh\end{aligned}$$

and therefore

$$\beta \int_{h \in B_1} \tilde{c}_1(h) dh + (1-\beta) \int_{h \in B_2} \tilde{c}_2(h) dh < \beta \int_{h \in B_1} c_1(h) dh + (1-\beta) \int_{h \in B_2} c_2(h) dh$$

We thus can build c^B such that $c_i^B(h) = c_i(h)$ for $h \notin B_i$, $c_i^B(h) > \tilde{c}_i^B(h)$ for $h \in B_i$ with

$$\beta \int_{h \in B_1} c_1^B(h) dh + (1-\beta) \int_{h \in B_2} c_2^B(h) dh = \beta \int_{h \in B_1} c_1(h) dh + (1-\beta) \int_{h \in B_2} c_2(h) dh.$$

Since $c_i^B(h) > \tilde{c}_i(h)$ for $h \in B_i$, $c_i^B(h) + v_i(\Gamma_i^{x^B}(x_i^B(h))) > c_i(h) + v_i(\Gamma_i^x(x_i(h))))$. Thus, coalition B blocks (x, c).

Finally, take now a segregated equilibrium that is not completely segregated. Take two frontier locations $0 < r < s < 1$, and assume that in $[0, r[$ all households are type 1, whereas in $]r, s[$ all households are type 2. Note that in equilibrium, all type 1 households get utility u_1 which equals the utility level of type 1 households living in r. Analogously, all type 2 households get utility u_2 which equals the utility level of type 2 households living in s. Without loss, assume that $\Gamma_1^x(r) \geq \Gamma_1^x(s)$ and $\Gamma_2^x(r) \leq \Gamma_2^x(s)$. Since households care most about next-door neighbors, we can take $\epsilon > 0$ small enough such that

$$\min_{y\in[r,r+\epsilon]} f_r(y) - \max_{y\in[s,s+t]} f_r(y) > 0.$$

Move all type 1 households in $[s, s+\epsilon]$ to $[r, r+\epsilon]$, and move all type 2 households in $[r, r+\epsilon]$ to $[s, s+\epsilon]$. Call B the coalition of agents that were reassigned x^ϵ the new allocation. Now, for all $t \in [r, r+\epsilon]$, $\Gamma_1^{x^\epsilon}(t) > \Gamma_1^x(r)$. By continuity, for all $t \in [r, r+\epsilon]$, $\Gamma_1^{x^\epsilon}(t) > \Gamma_1^x(r) \geq \Gamma_1^x(s)$. Analogously, $\Gamma_2^{x^\epsilon}(t) > \Gamma_2^x(r)$ for all $t \in [s, s+\epsilon]$. Since the S-SCC holds, the coalition can block x by means of a consumption assignment identical to the one built in the paragraph above (c^B). $\square$

A.4 Constructing a segregated equilibrium

We construct an equilibrium that is segregated but not completely. Take $x_0 = 0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 1$ such that for any l odd (resp. even) $I_l :=]x_{l-1}, x_l[$ only has type 1 (resp. type 2) households. In equilibrium, the rental price function is pinned down by the condition

$$R(t) = \begin{cases} v_1\big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\big) & \text{if } t \in I_l, l \text{ even} \\ v_2\big(\sum_{m \text{ odd}} F_t(x_m) - F_t(x_{m-1})\big) + \eta & \text{if } t \in I_l, l \text{ odd} \end{cases}$$

where η is to be determined.[20] In equilibrium, the rental price $R(t)$ must be continuous and thus

$$v_1\big(\sum_{m \text{ even}} F_{x_l}(x_m) - F_{x_l}(x_{m-1})\big) = v_2\big(\sum_{m \text{ odd}} F_{x_l}(x_m) - F_{x_l}(x_{m-1})\big) + \eta$$

for all $l = 1, \ldots, n-1$. The SCC implies that for all $l = 2, \ldots, n-1$,

$$\sum_{m \text{ even}} F_{x_l}(x_m) - F_{x_l}(x_{m-1}) = \sum_{m \text{ even}} F_{x_1}(x_m) - F_{x_1}(x_{m-1}). \tag{A.5}$$

Together with the condition $\sum_{l \text{ odd}} x_l - x_{l-1} = \beta$, these $n-1$ conditions result in candidates for the $n-1$ parameters, $x_1, \ldots, x_{n-1}$, characterizing a segregated equilibrium with n ghettos.

Since the equilibrium is not completely segregated, there exists $\bar{l} \in \{2, \ldots, n-1\}$ such that

[20] Adding a constant $\bar{\eta}$ to the rental price function when l is even is immaterial.

$x_{\bar{l}-1} \neq 0$ and $x_{\bar{l}} \neq 1$ and suppose first that only type 2 households live in $]x_{\bar{l}-1}, x_{\bar{l}}[$. The incentive condition for type 1 to prefer $x_{\bar{l}-1}$ and $x_{\bar{l}}$ over any $t \in]x_{\bar{l}-1}, x_{\bar{l}}[$ is

$$\begin{aligned}
R(h) - R(x_{\bar{l}-1}) + v_1\Big(\sum_{m \text{ even}} F_{x_{\bar{l}-1}}(x_m) - F_{x_{\bar{l}-1}}(x_{m-1})\Big) &= R(h) - R(x_{\bar{l}}) + v_1\Big(\sum_{m \text{ even}} F_{x_{\bar{l}}}(x_m) - F_{x_{\bar{l}}}(x_{m-1})\Big) \\
&\geq \sup_{t \in]x_{\bar{l}-1}, x_{\bar{l}}[} \Big\{ R(h) - R(t) + v_1\Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big)\Big\} \\
&= \max_{t \in [x_{\bar{l}-1}, x_{\bar{l}}]} \Big\{ R(h) - R(t) + v_1\Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big)\Big\}
\end{aligned}$$

where the last equality follows by continuity of $R(t)$ and $t \mapsto F_t(x)$. The incentive condition above is equivalent to

$$\begin{aligned}
v_1\Big(\sum_{m \text{ even}} F_{x_{\bar{l}-1}}(x_m) - F_{x_{\bar{l}-1}}(x_{m-1})\Big) - v_2\big(\sum_{m \text{ odd}} F_{x_{\bar{l}-1}}(x_m) - F_{x_{\bar{l}-1}}(x_{m-1})\big)& \\
= v_1\Big(\sum_{m \text{ even}} F_{x_{\bar{l}}}(x_m) - F_{x_{\bar{l}}}(x_{m-1})\Big) - v_2\big(\sum_{m \text{ odd}} F_{x_{\bar{l}}}(x_m) - F_{x_{\bar{l}}}(x_{m-1})\big)& \\
= \max_{t \in [x_{\bar{l}-1}, x_{\bar{l}}]} v_1\Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big) - v_2\big(\sum_{m \text{ odd}} F_t(x_m) - F_t(x_{m-1})\big)&
\end{aligned}$$

This means that the function

$$t \in [x_{\bar{l}-1}, x_{\bar{l}}] \mapsto v_1\Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big) - v_2\big(\sum_{m \text{ odd}} F_t(x_m) - F_t(x_{m-1})\big) \tag{A.6}$$

attains its maximum at $t \in \{x_{\bar{l}-1}, x_{\bar{l}}\}$. Analogously, assuming that only type 1 households live in neighborhood $[x_{\bar{l}-1}, x_{\bar{l}}]$, the function

$$t \in [x_{\bar{l}-1}, x_{\bar{l}}] \mapsto v_1\Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big) - v_2\big(\sum_{m \text{ odd}} F_t(x_m) - F_t(x_{m-1})\big) \tag{A.7}$$

attains it minimum at $t \in \{x_{\bar{l}-1}, x_{\bar{l}}\}$. For neighborhood $[0, x_1]$, it must be that the function above attains its minimum at $t = x_1$, while for neighborhood $[x_{n-1}, 1]$ the minimum (resp. maximum) is attained at $t = x_{n-1}$ when only type 1 (resp. type 2) households live on it. All these conditions are necessary and sufficient for a segregated equilibrium. When the S-SCC holds, these conditions require that the function

$$t \in [0, 1] \mapsto \Big(\sum_{m \text{ even}} F_t(x_m) - F_t(x_{m-1})\Big)$$

has a periodic behavior, crossing through the points x_l for $l = 1, \ldots, n-1$ downwards (resp. upwards) if l is odd (resp. even).

A.5 Constructing a completely segregated equilibrium

A perfectly segregated allocation is characterized by a cutoff $\bar{x} \in]0,1[$ such that type 1 (resp. type 2) households live in $[0, \bar{x}]$ (resp. $[\bar{x}, 1]$). Define the price function:

$$R(t) = \begin{cases} v_1(\int_0^{\bar{x}} f_t(y)dy) & \text{if } t \leq \bar{x}, \\ v_2(\int_{\bar{x}}^1 f_t(y)dy) + \eta & \text{if } t > x. \end{cases}$$

where $\eta \in \mathbb{R}$ is to be determined. We first argue that given R, households type 1 (resp. type 2) optimally demand housing in $[0, \bar{x}]$ (resp. in $[\bar{x}, 1]$). Let $u_i(h)$ be the optimal utility level that household h of type i gets. Then, for all h, $-R(t) + v_1(\int_0^{\bar{x}} f_t(y)dy)$ equals 0 for $t \in [0, \bar{x}]$ and thus $u_1(h) = R(h)$. Analogously, $-R(t) + v_2(\int_{\bar{x}}^1 f_t(y)dy) = -\eta$ for $t > \bar{x}$ and $u_2(h) = R(h) - \eta$. By demanding housing at any $t > \bar{x}$, household type 1 gets at most

$$\begin{aligned} \sup_{t>\bar{x}}\{R(h) - R(t) + v_1(\int_0^{\bar{x}} f_t(y)dy)\} =& \sup_{t>\bar{x}}\{R(h) - R(t) + v_2(\int_{\bar{x}}^1 f_t(y))dy + v_1(\int_0^{\bar{x}} f_t(y)dy) - v_2(\int_{\bar{x}}^1 f_t(y))dy\} \\ =& u_2(h) + \sup_{t>\bar{x}}\{v_1(\int_0^{\bar{x}} f_t(y)dy) - v_2(\int_{\bar{x}}^1 f_t(y))dy\} \\ =& R(h) - \eta + \sup_{t>\bar{x}}\{v_1(\int_0^{\bar{x}} f_t(y)dy) - v_2(\int_{\bar{x}}^1 f_t(y))dy\}. \end{aligned}$$

Thus, it is optimal for household type 1 to demand housing in $[0, \bar{x}]$ provided

$$-\eta + \sup_{t>\bar{x}}\{v_1(\int_0^{\bar{x}} f_t(y)dy) - v_2(\int_{\bar{x}}^1 f_t(y)dy)\} \leq 0.$$

Analogously, it is optimal to demand housing in $t > \bar{x}$ for a household type 2 provided

$$\sup_{t\leq\bar{x}}\{R(h) - R(t) + v_2(\int_{\bar{x}}^1 f_t(y)dy)\} \leq R(h) - \eta$$

which is equivalent to

$$\eta + \sup_{t\leq\bar{x}} v_2(\int_{\bar{x}}^1 f_t(y)dy) - v_1(\int_0^{\bar{x}} f_t(y)dy) \leq 0.$$

Thus, it is enough to take η such that

$$\sup_{t>\bar{x}}\{v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x}))\} \leq \eta \leq \inf_{t\leq\bar{x}}\{v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x}))\}. \tag{A.8}$$

To complete the equilibrium construction, assign all households type 1 (resp. type 2) uniformly in $[0, \bar{x}]$ (resp. in $[\bar{x}, 1]$). The market clearing condition holds when $\bar{x} = \beta$ and we have therefore constructed a perfectly segregated equilibrium.

We are now in position to prove Proposition 1.

Proof of Proposition 1. We first claim that for $\bar{x} = \beta$,

$$\sup_{t>\bar{x}}\{v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x}))\} = \inf_{t\leq\bar{x}}\{v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x}))\} = v_1(F_{\bar{x}}(\bar{x})) - v_2(1 - F_{\bar{x}}(\bar{x})).$$

To see this, note that $t \mapsto F_t(\bar{x})$ is decreasing and therefore its maximum value over $[\bar{x}, 1]$ is attained at $t = \bar{x}$. Analogously, $t \mapsto F_t(\bar{x})$ is minimized over $t \leq \bar{x}$ at $t = \bar{x}$. Since $v_1(x) - v_2(1 - x)$ is increasing, for all $t > \bar{x}$,

$$v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x})) \leq v_1(F_{\bar{x}}(\bar{x})) - v_2(1 - F_{\bar{x}}(\bar{x}))$$

while for all $t \leq \bar{x}$,

$$v_1(F_t(\bar{x})) - v_2(1 - F_t(\bar{x})) \geq v_1(F_{\bar{x}}(\bar{x})) - v_2(1 - F_{\bar{x}}(\bar{x})).$$

Now, it immediately follows that $\bar{x} = \beta$ and $\eta = v_1(F_\beta(\beta)) - v_2(1 - F_\beta(\beta))$ satisfy (A.8). We have therefore found a completely segregated equilibrium.

Now, suppose that a completely segregated equilibrium does not exist. Then, it must be the case that (A.8) does not hold at $\bar{x} = \beta$. Therefore, there exist $\bar{t} > \beta$ and $\underline{t} < \beta$ such that

$$v_1(F_{\bar{t}}(\beta)) - v_2(1 - F_{\bar{t}}(\beta)) > v_1(F_{\underline{t}}(\beta)) - v_2(1 - F_{\underline{t}}(\beta))$$

Since $\underline{t} < \bar{t}$, $F_{\bar{t}}(\beta) < F_{\underline{t}}(\beta)$. But the SCC holds and therefore $x \mapsto v_1(x) - v_2(1 - x)$ is strictly decreasing. $\square$

A.6 Implementing a housing allocation using a general tax policy

Suppose now that a type 2 household that chooses location t receives $\phi(t)$. The definition of equilibrium is similar to Definition 1, but now each household chooses consumption and location optimally given the tax policy ϕ: For all i and all $h \in [0,1]$, $(x_i(h), c_i(h))$ solves

$$\max_{t\in[0,1],\tilde{c}\in\mathbb{R}} \left\{ U_i(\tilde{c}, \Gamma^x(t)) \mid \tilde{c} + R(t) \leq R(h) + \phi(t) \right\}$$

Proposition 2. *Let x be any housing allocation. There exists a tax policy for group 2 ϕ: $[0,1] \rightarrow [0,1]$ such that x is part of an equilibrium*

Proof. Take

$$R(t) = v_1(\Gamma^x_1(t))$$

and

$$R(t) - \phi(t) = v_2(\Gamma^x_2(t)).$$

It is clear that households are indifferent over any $t \in [0,1]$ and therefore x is part of an equilibrium. □

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