Robust Optimization Models for Energy-limited Wireless Sensor Networks under Distance Uncertainty

Wei Ye and Fernando Ordóñez

University of Southern California, ISE,

3715 McClintock Ave., GER-240, Los Angeles, CA 90089.

yewei@usc.edu, fordon@usc.edu

September, 2006

Abstract

The distance between nodes in a wireless sensor network (WSN) is an important factor in the performance that can be extracted from the network for many tasks. However, the distance values are typically subject to uncertainty as they might have been indirectly estimated through signal strength or have changed because of node movement. In this paper we propose robust optimization models that take this uncertainty into account for three operational problems in energy limited WSNs: maximizing the data extracted, minimizing the energy consumed, and maximizing the network lifetime. In a robust optimization model the uncertainty is represented by considering that the uncertain parameters belong to a bounded, convex uncertainty set $\mathcal{U}$. A robust solution is the one with best worst case objective over this set $\mathcal{U}$. We show that solving for the robust solution in these problems is just as difficult as solving for the problem without uncertainty. Our computational experiments show that as the uncertainty increases a robust solution for these problems provides a significant improvement in worst case performance at the expense of a small loss in optimality when compared to the optimal solution of a fixed scenario.

1 Introduction

In recent years, the rapid pace of improvements in micro-processor and wireless technology have enabled the rapid development of WSNs. Sensors network can remotely monitor and track many objects in unfriendly physical environments such as remote geographic regions or toxic urban locations [1-3]. These sensor networks are poised to revolutionize information gathering and processing in many applications. Since sensors have limited energy, it becomes important to identify efficient operating polices. This has lead to research on a number of different problems, such as identifying the maximum data that can be extracted for a given amount of energy, minimizing the energy consumed to extract certain amount of data and maximizing network lifetime for a given amount of data transfer.

For these problems, distance is an important factor that influences the consumption of energy, and hence the efficiency with which the network operates. In many applications, the distance measurements are subject to uncertainty as they might have been indirectly estimated through signal strength or due to unfriendly conditions during the WSN's deployment or operation [6,18-19]. The effect of ignoring distance uncertainty in planning the operations of a WSN can be varied, in particular optimized operating practices can turn out to be inefficient if the problem parameters change. A more successful strategy in a problem with uncertainty can be a solution that is less optimal for a particular distance vector but obtains efficient solutions for all likely distance measurements.

In this paper, we discuss models of these problems that are robust with respect to the distance uncertainty. The paper is organized as follows: we review related literature in Section 2. In Section 3 we provide some background on the WSN problems we investigate in this work: the max data extraction problem, min energy consumption problem, and max network lifetime problem. In Section 4 we introduce the robust optimization methodology and build the robust counterpart of these problems under distance uncertainty. We discuss performance criteria and present computational results that assess what is the efficiency of a robust solution when compared to a deterministic solution in Section 5. We finish the paper with concluding remarks in Section 6.

2 Related Work

Our work involves the use of robust optimization methods in wireless sensor networks problems, therefore in this section we first review the relevant papers on robust optimization methods and then discuss relevant literature on the wireless sensor network problems considered in this paper. The robust optimization methodology that we use in this work is presented in [4] for different convex optimization problems and various uncertainty models. Since that work, a number of applications of robust optimization have been studied, such as portfolio optimization [5], supply chain control [7], finite Markov Decision Processes [8], and general network flow problems [9]. An important conclusion of these works is that in many cases the robust solution is able to significantly reduce the worst case performance while only suffering from a limited loss in optimality. This is a consequence of the reduced variability of the robust solution under the uncertainty considered. To our knowledge, there is no research on robust or worst case models for problems in WSNs. In particular we note that the network flow model in [9] only considers uncertainty in the objective function coefficients.

In energy limited WSN the decisions of how to route information influence the amount of energy consumed and thus the overall efficiency of the operation. In this paper we focus on three optimization problems that involve the decision of routing information in a energy constrained system. A number of papers consider the problem of maximizing the amount of data that can be outputted of an energy limited WSNs, see [2,3,10,21,22]. A closely related problem to max data extraction is the problem of determining the minimum amount of energy to output a given amount of data. This problem is important in designing WSNs to perform efficiently, see [10-11]. The problem of maximizing the network lifetime studied in [12-17,23] maximizes the time until the first node depletes its energy. Although these three problems have mathematical formulations that depend on the distances between nodes, there is no wort to data that significantly addresses optimal operating practices conditions when there is uncertainty in this distance.

3 PROBLEM DEFINITION

We consider a WSN with n fixed sensor nodes that gather data and send it to a sink node, denoted as node $n{+}1$. Let $D^i_{\max}$ be the total amount of the data (bytes) collected by node i, and $E^i_{\max}$ be the total energy of node i. Let d_{ij} be the Euclidean distance between nodes i and j. We denote by N the set of sensor nodes and A the set of directed arcs (i, j) in the complete graph $i \in N, j \in N \cup \{n{+}1\}$. The energy consumed in transmitting data from one sensor node to another depends on the distance between them according to the following radio model also used in [2,3,20]: We consider that a radio dissipates 400nJ/byte to run the transmitter or receiver circuitry and 800pJ/byte/m^2 for the transmitter amplifier. This means that the energy consumed in transmitting k units of data from i to j is given by $E_{Tx}(k, d_{ij}) = \varepsilon_{\text{elec}} k + \varepsilon_{\text{amp}} k d^2_{ij}$ where $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. The energy consumed by sensor i in receiving a unit of data is given by: $E_{Rx}(k) = \varepsilon_{\text{elec}} k$.

To simplify notation, we have normalized the energy in terms of receptions, that is to say, each reception consumes a unit of energy, while each transmission from the node i to the node j consumes $1 + \beta d^2_{ij}$, where $\beta = \frac{\varepsilon_{amp}}{\varepsilon_{elec}}$. The amount of data that can be received and transmitted by a node is limited by the energy of the node, which leads to the normalized energy value $E^i = \frac{E^i_{\max}}{\varepsilon_{elec}}$.

3.1 Maximum Data Extraction Problem

The problem of maximizing data output, where nodes have a limited energy and use the above energy expenditure in transmissions and receptions, can be written as a linear programming problem, see [2,3]. Let f_{ij} be the amount of the data transmitted from node i to node j, we formulate the maximal data extraction problem as follows:

$$
\begin{aligned}
\max \quad & \sum_{\{i,j=n+1 \mid (i,j)\in A\}} f_{ij} \\
\text{s.t.} \quad & \\
& \sum_{\{j \mid (i,j)\in A\}} f_{ij}(1+\beta d_{ij}^2) + \sum_{\{j \mid (j,i)\in A\}} f_{ji} \le E^i && \forall i \in N \\
& \sum_{\{j \mid (i,j)\in A\}} f_{ij} - \sum_{\{j \mid (j,i)\in A\}} f_{ji} \le D^i_{\max} && \forall i \in N \\
& \sum_{\{j \mid (i,j)\in A\}} f_{ij} - \sum_{\{j \mid (j,i)\in A\}} f_{ji} \ge 0 && \forall i \in N \\
& f_{ij} \ge 0 && \forall (i,j) \in A
\end{aligned}
$$

In this problem, the first constraint is the energy constraint. The amount of data transmitted and received by a sensor is limited by the energy of the sensor. The second and third constraints represent conservation of flow constraints. The difference of the amount of data transmitted and received at each node must be less than or equal to the amount of the data collected at that node, and also must be greater than or equal to 0. The last constraint states the non-negativity of flow. In the remainder of the paper we represent the set of flows that satisfy the last three constraints as a polyhedron P, therefore we replace these constraints by $f \in P$.

3.2 Minimum Energy Consumption Problem

Minimizing energy consumption problem is closely related to the problem of maximizing data extraction in energy limited WSNs, and can also be written as a linear programming problem, see [10]. The objective of the problem is to minimize the amount of energy consumed to extract $f_{\min}$ information to the sink node. Our minimum energy consumption model is given by the linear program below:

$$\begin{aligned}
&\min \quad \sum_{i \in N} \Big(\sum_{j|(i,j)\in A} f_{ij}(1+\beta d_{ij}^2) + \sum_{j|(j,i)\in A} f_{ji} \Big) \\
&\text{s.t} \\
&\quad \sum_{(i,n+1)\in A} f_{i(n+1)} \geq f_{\min} \qquad\qquad \forall i \in N \\
&\quad f \in P \ .
\end{aligned}$$

The information requirement can enforce, for example, that a given percentage p of the available information must reach the sink. In this case we set $f_{\min} = p \sum_i D_{\max}^i$.

In this problem, the first constraint references the minimum data extraction constraint: we should guarantee that at least f_{min} information is extracted to the sink node. The polyhedral constraint represents the conservation of flow and non-negativity of flow constraints, similar to the constraints in the maximum data extraction problem.

3.3 Maximum Lifetime Problem

The lifetime T_i of node i is defined as the expected time for the energy E_i to be exhausted, where each node i has the limited energy E_i of node i to be exhausted. We define the lifetime T of the system as the time when the first sensor i is drained of its energy, that is to say, the system lifetime T of a sensor network is the minimum lifetime of all nodes of the network, $T = \min\{T_1, T_2, ..., T_n\}$. The problem of maximum network lifetime of WSN is discussed in papers [12-17] and is described as the following optimization problem:

$$
\begin{aligned}
&\max \quad T \\
&\text{s.t.} \\
&\sum_{(i,n+1)\in A} f_{i(n+1)} \geq f_{\min} && \forall i \in N \\
&T\Big(\sum_{j|(i,j)\in A} f_{ij}(1+\beta d_{ij}^2) + \sum_{(j,i)\in A} f_{ji}\Big) \leq E^i && \forall i \in N \\
&f \in P
\end{aligned}
$$

The constraints include the minimum data extraction constraint, energy constraints, and the same polyhedral constraints representing conservation of flow and non-negativity. Clearly, the problem above is not linear because of the products Tf_{ij}. It is natural only to consider $E^i > 0$ for all $i \in N$, otherwise simply remove the node from the network. This implies that $T > 0$ and we can obtain an equivalent linear program using a new variable $q = \frac{1}{T}$. The objective function becomes to minimize q, and the non-linear constraint can be equivalently written in linear form, yielding the linear program:

$$
\begin{aligned}
&\min \quad q \\
&\text{s.t} \\
&\sum_{(i,n+1)\in A} f_{i(n+1)} \geq f_{\min} && \forall i \in N \\
&\sum_{j|(i,j)\in A} f_{ij}(1+\beta d_{ij}^2) + \sum_{(j,i)\in A} f_{ji} \leq qE^i && \forall i \in N \\
&f \in P
\end{aligned}
$$

4 Methodology and Uncertainty Set

4.1 Robust Optimization Methodology

We address the uncertainty in problems through the robust optimization methodology introduced for convex optimization by Ben-Tal and Nemirovski [4]. The robust solution for an optimization problem under uncertainty is defined as the solution that has the best objective value in its worst case uncertainty scenario. Attractive features of a robust solution are that while it is only close to optimal for any specific scenario, it behaves well over all likely uncertainty outcomes. To introduce the robust optimization methodology, we consider the following optimization problem under uncertainty: $\min f(x,u)$ s.t. $g(x,u) \leq 0$ where u is an uncertainty parameter that belongs to a closed bounded and convex uncertainty set $u \in \mathcal{U}$. A robust solution is feasible for all $u \in \mathcal{U}$ and optimizes a worst case objective function. In other words, the robust solution is obtained by solving the following Robust Counterpart problem (RC):

$$
\begin{array}{rl}
\min_x \max_u & f(x,u) \\
\text{s.t.} & g(x,u) \leq 0 \quad \forall u \in \mathcal{U}
\end{array}
\tag{1}
$$

or equivalently

$$
\begin{array}{rl}
\min_{x,\gamma} & \gamma \\
\text{s.t.} & g(x,u) \leq 0 \quad \forall u \in \mathcal{U} \\
 & f(x,u) \leq \gamma \quad \forall u \in \mathcal{U}\ .
\end{array}
\tag{2}
$$

In many settings finding the robust solution is no harder than solving the deterministic problem. The complexity of solving problem (RC) has been shown to be the same as the complexity of solving the deterministic problem (fixed $u \in \mathcal{U}$) for various problems and uncertainty sets. For example, the robust counterpart of an LP is equivalent to an LP when $\mathcal{U}$ is a polyhedron and to a quadratically constrained convex program when $\mathcal{U}$ is a bounded ellipsoidal set; In addition, the size of the resulting RC problem is bounded by a polynomial of the deterministic problem's dimensions [4]. Thus given a certain optimization problem the definition of the uncertainty set is key to be able to formulate a robust counterpart that can be solved efficiently.

4.2 Uncertainty Set

Distance measurements among nodes in sensor networks can suffer from uncertainty due to a variety of reasons: distance measurement methods, unfriendly conditions, or existence of arbitrary noise and rugged land surface. Here, we consider the uncertainty in the vector $d = (d_{ij})_{(i,j)\in A}$ of distances between nodes by allowing it to belong to an uncertainty set $d \in \mathcal{U}$. The set $\mathcal{U}$ defines distance vectors that are a given deviation from a given estimate of the distance vector between nodes d^0. We consider the following two types of uncertainty sets in this paper, each used for different problems to obtain an efficient solution procedure:

$$\mathcal{U}^1 \equiv \left\{ d \mid d_{ij}^2 = (d_{ij}^0)^2 + \xi_{ij} R,\ \sum_{i,j} \xi_{ij} \leq \tau M,\ \xi \geq 0 \right\} \tag{3}$$

or

$$\mathcal{U}^2 \equiv \left\{ d \mid d_{ij}^2 = (d_{ij}^0)^2 + \xi_{ij} R,\ \sum_{j} \xi_{ij} \leq \tau M_i\ \forall i,\ \xi \geq 0 \right\}\ . \tag{4}$$

These sets consider that the square of every distance measurement can vary up-wards by at most R. To exclude overly conservative distance measurements, the sets limit either the total variation by τM or the variation out of every node by τM_i, where $\tau \in [0, 1]$ is a parameter that controls the size of the uncertainty set.

5 Robust Counterpart Problem

In this section we formulate robust counterpart problems, following the robust optimization methodology, for the different problems of interest in WSNs data-centric networks: maximum data extraction for a given amount of energy, minimum energy consumption for a given amount of data transfer, and maximum network lifetime for a given data transfer. We present three propositions in these WSNs problems and explain how to express the robust counterpart problem of each problem. We note that the robust counterpart problem in each case is of the same form as the original problem, therefore the complexity of solving for the robust solution is the same as the complexity of solving a deterministic problem.

Proposition 1 *Consider the maximum data extraction problem for a given amount of energy with uncertainty distance, where $d \in \mathcal{U}$ given by equation (4). The robust counterpart of this problem is equivalent to*

$$\begin{aligned}
&\max \sum_{\{i,j=n+1|(i,j)\in A\}} f_{ij} \\
&s.t. \\
&\sum_{\{j|(i,j)\in A\}} f_{ij}(1+\beta(d^0_{ij})^2) + \theta_i\tau M_i + \sum_{\{j|(j,i)\in A\}} f_{ji} \le E^i && \forall i\in N \\
&\theta_i \ge f_{ij}\beta R && \forall (i,j)\in A, \forall i \in N \\
&\theta_i \ge 0 && \forall i \in N \\
&f \in P
\end{aligned}$$

Proof: The robust counterpart of the maximum data extraction is the problem of maximizing $\sum_{\{i,j=n+1|(i,j)\in A\}} f_{ij}$ under the robust energy constraints due to the distance uncertainty

$$\sum_{\{j|(i,j)\in A\}} f_{ij}(1+\beta((d^0_{ij})^2+\xi_{ij}R)) + \sum_{\{j|(j,i)\in A\}} f_{ji} \le E^i\,, \quad \forall \sum_{\{j|(i,j)\in A\}} \xi_{ij} \le \tau M_i,\ i\in N,\ \xi\ge 0\,,$$

and $f \in P$. Then, from strong LP duality we have that for every $i \in N$

$$\max_{\substack{\sum_j \xi_{ij} \le \tau M_i \\ \xi_{ij}\ge 0}} \sum_{\{j|(i,j)\in A\}} f_{ij}\beta R\xi_{ij}$$

has the same optimal value as $\min_{\substack{\theta_i \ge f_{ij}\beta R, \\ \theta_i \ge 0}} \theta_i\tau M_i$. Therefore we can express the inequality constraint as the following system of inequalities

$$\sum_{\{j|(i,j)\in A\}} f_{ij}(1+\beta(d^0_{ij})^2) + \theta_i \tau M_i + \sum_{\{j|(j,i)\in A\}} f_{ji} \le E^i, \forall i \in N$$
$$\theta_i \ge f_{ij}\beta R,$$
$$\theta_i \ge 0 \ .$$

Considering $f \in P$, we prove Proposition 1. ■

Proposition 1 shows that the (RC) problem of a maximum data extraction problem for a given amount of energy with $\mathcal{U}$ given by set (4) is similar to the deterministic problem without uncertainty. Different from the deterministic problem, the (RC) has n additional non-negative variables θ_i, $|A| + n$ additional constraints $\theta_i \ge f_{ij}\beta R$, and the energy is reduced to $E^i - \theta_i \tau M_i$ for $i \in N$. But this (RC) problem is still a linear programming problem, which also implies that solving the (RC) is just as difficult as solving the deterministic problem.

Proposition 2 *Consider the Minimum Energy Consumption Problem for a given amount of data transfer with uncertainty distance, where $d \in \mathcal{U}$ given by equation (3). The robust counterpart of this problem is equivalent to*

$$\begin{aligned}
\min \quad & (\sum_{i\in N}(\sum_{j|(i,j)\in A} f_{ij}(1+\beta(d^0_{ij})^2) + \sum_{j|(j,i)\in A} f_{ji}) + \theta\tau M) & \\
s.t & & \\
& \sum_{(i,n+1)\in A} f_{i(n+1)} \ge f_{\min} & \forall i \in N \\
& \theta \ge f_{ij}\beta R & \forall (i,j) \in A \\
& \theta \ge 0 & \\
& f \in P &
\end{aligned}$$

Proof: The (RC) problem of the minimum energy consumption model is:

$$\min \left(\sum_{i\in N} \left(\sum_{j|(i,j)\in A} f_{ij}(1+\beta(d^0_{ij})^2) + \sum_{j|(j,i)\in A} f_{ji} \right) + \max_{\substack{\sum_i \sum_j \xi_{ij} \leq \tau M \\ \xi_{ij} \geq 0}} \sum_{i\in N} \sum_{j|(i,j)\in A} f_{ij}\beta\xi_{ij}R \right)$$

with the deterministic problem constraints. Then, from LP duality, we have that the dual of

$$\max_{\substack{\sum\limits_i \sum\limits_j \xi_{ij} \leq \tau M \\ \xi_{ij} \geq 0}} \sum_i \sum_{j|(i,j)\in A} f_{ij}\beta\xi_{ij}R$$

is equivalent to $\min_{\theta \geq f_{ij}\beta R, \theta \geq 0} \tau M\theta$. So the objective function becomes

$$\min \left(\sum_i \left(\sum_{j|(i,j)\in A} f_{ij}(1+\beta(d^0_{ij})^2) + \sum_{j|(j,i)\in A} f_{ij} \right) + \min_{\theta\geq f_{ij}\beta R, \theta\geq 0} \tau M\theta \right)$$

Considering $f \in P$, we prove Proposition 2. ■

Proposition 2 also shows that the (RC) problem of a minimum energy consumption problem for a given amount of data transfer with uncertainty set $\mathcal{U}$ given by (3) is similar to the deterministic problem without uncertainty. Different from deterministic problem without uncertainty, the (RC) has an additional non-negative variable θ, $|A|$ additional constraints $\theta \geq f_{ij}\beta R$, and the objective function has an extra term $\theta\tau M$. But this (RC) problem is still a linear problem and this implies that solving the (RC) is just as difficult as solving the deterministic problem.

Proposition 3 *Consider the Maximum Lifetime Problem for a given amount of data transfer with uncertainty distance, where $d \in \mathcal{U}$ given by equation (4). The robust counterpart of this problem is equivalent to*

$$
\begin{aligned}
&\min \quad q \\
&s.t \\
&\sum_{(i,n+1)\in A} f_{i(n+1)} \geq f_{\min} && \forall i \in N \\
&\sum_{j|(i,j)\in A} f_{ij}(1+\beta(d_{ij}^{0})^{2}) + \theta_i \tau M_i + \sum_{j|(j,i)\in A} f_{ji} \leq qE^{i} && \forall i \in N \\
&\theta_i \geq f_{ij} R\beta && \forall (i,j) \in A \\
&\theta_i \geq 0 && \forall i \in N \\
&f \in P
\end{aligned}
$$

Proof: Analogous to the proof of Proposition 1. ■

Proposition 3 shows that the (RC) problem of a maximum lifetime problem for a given amount of data transfer with $\mathcal{U}$ given by set (4) is similar to the deterministic problem without uncertainty. Different from deterministic problem, the (RC) has n additional non-negative variables θ_i, $|A| + n$ additional constraints $\theta_i \geq f_{ij}\beta R$, and the energy is reduced to $qE^i - \theta_i \tau M_i$ for $i \in N$. But this (RC) problem is still a linear problem and this implies that solving the (RC) is just as difficult as solving the deterministic problem.

The three propositions show robust counterpart problems in maximum data extraction for a given amount of energy, minimum energy consumption for a given amount of data transfer, and maximum network lifetime for a given data transfer are of the same form as the original problem and imply the complexity of solving for the robust solution is the same as the complexity of solving a deterministic problem without uncertainty.

6 Computational Experiments

We conducted computational experiments that investigate the relative merit of the robust solution when compared to the deterministic solution of the problem without uncertainty for all three type of problems considered. We present three types of experiments: (1) we investigate the maximum protection that the robust solution can provide in the worst case and at what cost, (2) we conduct a simulation to observe the practical performance of a robust and deterministic solutions, and (3) we study the effect of varying problem parameters on the robust and deterministic solutions.

6.1 Experimental Set Up

These experiments were coded in AMPL and are solved with CPLEX 8.1. In our simulation, there are 50 nodes randomly deployed in 0.5km $\times$ 0.5km area, the sink node located at (0.25km, 0.5km). Each node has $E^i = 250,000$ or $E^i = 100,000$ units of energy and $D^i_{\max} = 10,000$ units of data. We use the $R = 0.1$ or 0.005, $M_i = 2$ or $1 (\forall i \in N)$, $M = 50$ $p = 90\%$ or 50% in the uncertainty set $\mathcal{U}$.

We compare the performance of the robust solution to the deterministic solution obtained for d^0, some nominal (average estimate) of the uncertain distances. In case of a minimization problem, we consider a pair of ratios that compare the robust and deterministic solutions on their respective worst case and on the nominal data:

$$R^{wc} = \frac{Dsol(d^{wc}) - Rsol(d)}{Dsol(d^{wc})}, \quad R^{ac} = \frac{Rsol(d^0) - Dsol(d^0)}{Dsol(d^0)} \ ,$$

where

- $Dsol(d^0)$: optimal value of deterministic solution
- $Dsol(d^{wc})$: objective value of deterministic solution in its worst case scenario
- $Rsol(d)$: optimal value of robust solution
- $Rsol(d^0)$: objective value of robust solution in the deterministic scenario

The first ratio R^{wc} measures the relative increase of the deterministic solution in the worst case, while the second ratio R^{ac} quantifies the relative loss of optimality of the robust solution on the nominal data. Therefore the ratio R^{wc} measures the maximum protection that a robust solution can provide, while R^{ac} is the percent increase in cost for this protection. Similar ratios are defined for a maximization problem simply by flipping the signs.

For each problem, we solve the deterministic problems for the nominal distance scenario d^0 and robust problems with appropriate distance uncertainty sets to get the objective value $Dsol(d^0)$ and $Rsol(d)$. Obtaining the other values requires to solve related optimization problems in which: we fix the deterministic solution to determine its worst distance case for $Dsol(d^{wc})$ and compute the value of the robust solution in the deterministic scenario in $Rsol(d^0)$. We compute the ratios above for 30 randomly generated networks and report the mean ratio values.

We also present numerical results that illustrate the performance in practice of the robust and deterministic solutions on a fixed network. Given a robust and deterministic routing solutions, f^R and f^D respectively, we conduct the following simulation to compute their practical performance. We randomly generate the uncertainty parameters ξ_{ij} in the uncertainty set (4) to obtain an actual sampled distance vector d_{ij} (where we fix d^0_{ij}). The robust and deterministic solutions are scaled, αf^R and αf^D respectively, to guarantee feasibility and efficiency for the problem defined with the sampled distance values d. We conduct 100 random experiments and report mean and standard deviation of these simulations for the robust and deterministic solutions.

In each of the next three subsections we present the results for each of the three problems considered in this work: the maximum data extraction, the minimum energy consumption, and the maximum lifetime problems. For each problem we present the three types of experiments: a study of the trade-offs of the robust solution (ratios R^{ac} and R^{wc}), the simulated performance, and the sensitivity of the robust solution to changes in the problem parameters.

6.2 Results for Maximum Data Extraction Problem

The trade-off between robust solutions and deterministic solutions through the comparison of the ratios R^{ac} and R^{wc}. In Fig.1 we present R^{ac} and R^{wc} for different energy $E1^i = 250,000$ and $E2^i = 100,000$. We observe that the robust solutions are able to improve the worst case scenario with relatively little loss in optimality. With increasing τ, we observe a faster increase in R^{wc} than R^{ac}, which shows that the robust solutions become more attractive as the distance uncertainty increases and can be able to compensate for uncertainty while suffering small performance losses.

A comparison of the practical performance for robust solutions and deterministic solutions for a given network under different uncertainty levels. Fig.2 presents the mean and standard deviations for the robust solutions and deterministic solutions over the 100 samples of all uncertain distance values. Given a sampled vector of distances we scale the robust and deterministic routing solutions so that they are feasible and maximize the objective value. This plot shows that the mean value of the adjusted robust solution is always better than the mean value of the adjusted deterministic solution. In addition we note that the standard deviation, represented by the vertical segments is much smaller for the robust solution. This shows that on a simulation the robust solution is observed to perform better on average and with a smaller standard deviation than a deterministic solution for any uncertainty level.

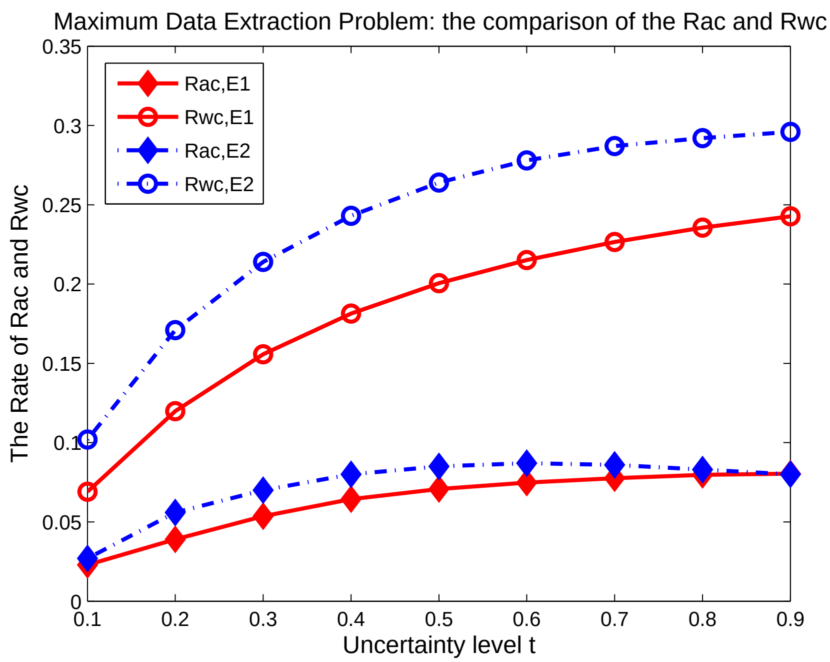

Figure 1: The comparison of R^{ac} and R^{wc} in maximum data extraction problems. $E1^i = 250000$, $E2^i = 100000$, $D^i_{\max} = 10000$, $R = 0.005$

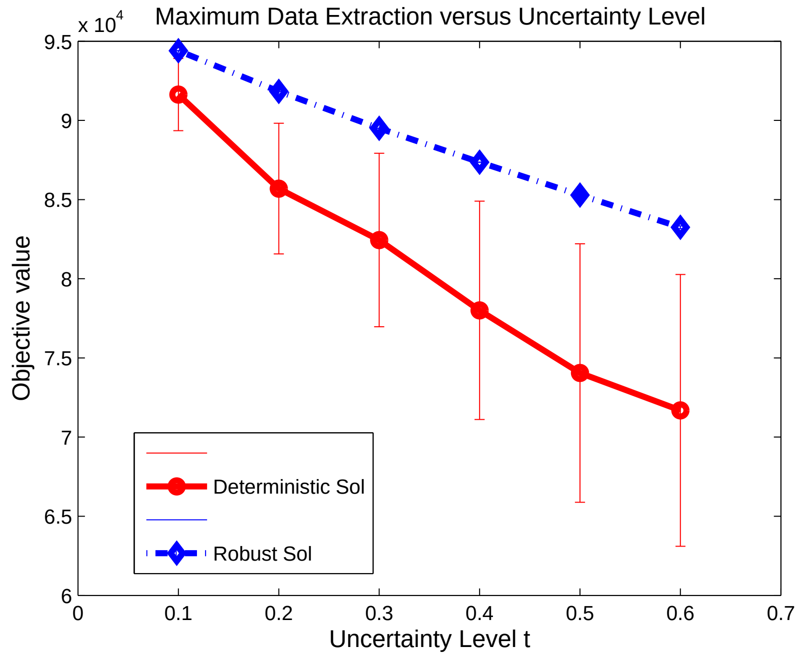

Figure 2: The mean and standard deviations of objective value for the deterministic and robust solutions in maximum data extraction problem for different uncertainty level. E^i=250000, $D^i_{\max}$=10000, R=0.005

For the maximum data extraction problem we conduct a sensitivity analysis on the available energy E^i and observe its effect on the trade-off ratios. Fig.3 shows the rate values R^{wc} and R^{ac} for $\tau 1 = 0.1$ and $\tau 2 = 0.9$ and different available energy E on each node. Notice that R^{wc} is higher than R^{ac} for any energy level, with most ratios remaining below 5%, except for a middle range of energy. In this middle range the robust solution appears more attractive as there is a bigger gap between R^{wc} and R^{ac}. Note also that the ratios R^{wc} and R^{ac} are almost 0 when the available energy E is small or large. In these two cases the levels of energy in the nodes encourage every node to route directly to sink for both the deterministic and robust solutions. This occurs because, either the node does not have enough energy to transmit someone else's information or has enough energy to transmit everything directly to the sink.

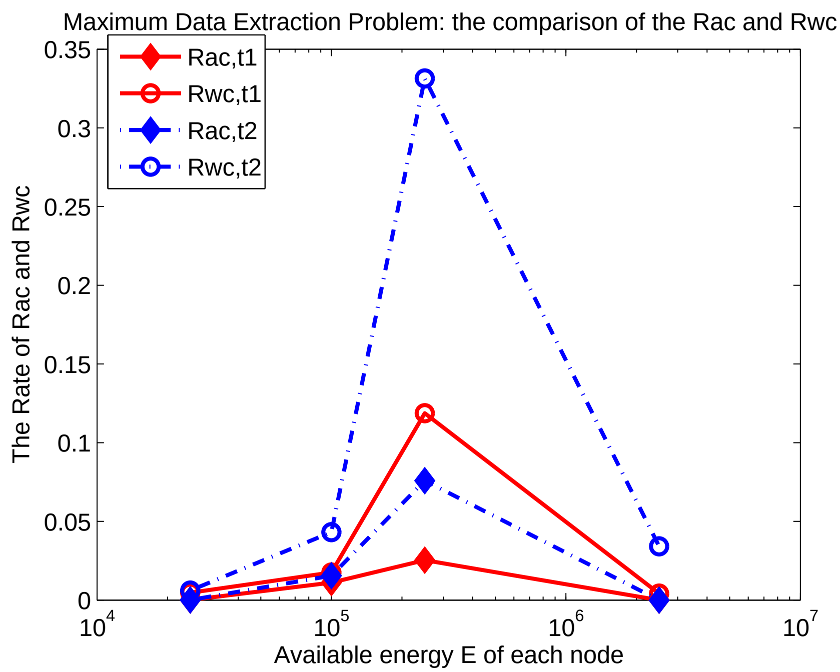

Figure 3: The comparison of R^{ac} and R^{wc} in maximum data extraction problems as a function of the available energy E of each node. $D^i_{\max}$=10000, $\tau 1$=0.1, and $\tau 2$=0.9

6.3 Results for Minimum Energy Consumption Problem

The trade-off between robust solutions and deterministic solutions through the comparison of the ratios R^{ac} and R^{wc}. In Fig.4 we present the ratios R^{ac} and R^{wc} for different information extraction requirements, $f_{\min} = p \sum_{i \in N} D^i_{\max}$. We present the ratios R^{ac} and R^{wc} for $p1 = 90\%$ and $p2 = 50\%$. The graph presents the ratios R^{ac} and R^{wc} as a function of τ. We observe that ratio R^{wc} is larger than R^{ac} with this difference being accentuated as τ increases. This shows that the robust solutions become more attractive as the distance uncertainty increases and can be able to compensate for uncertainty while suffering small performance losses.

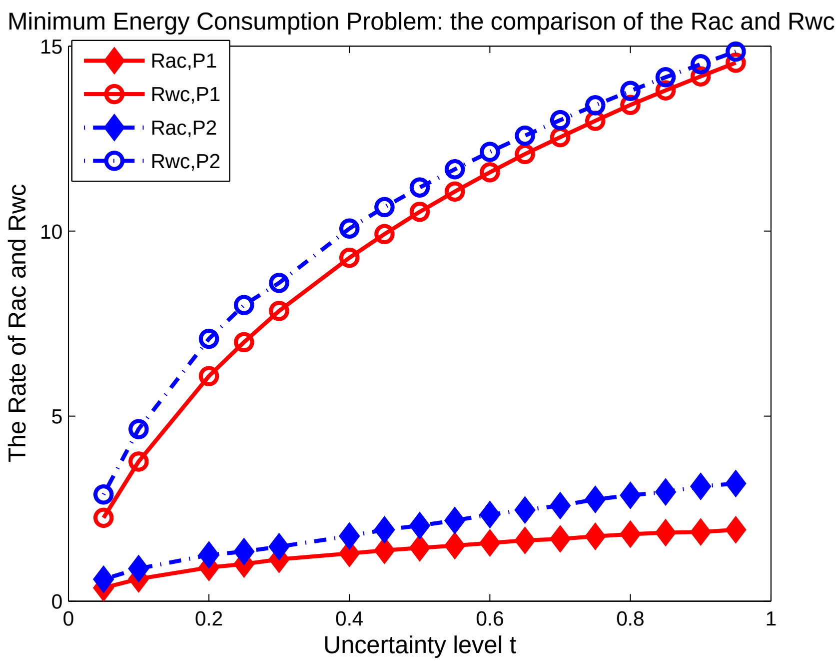

Figure 4: The comparison of R^{ac} and R^{wc} for objective value in minimum energy consumption problems. $p1 = 90\%$, $p2 = 50\%$, $E^i = 250000$, $D^i_{\max} = 10000$, $R = 0.1$

The comparison of the practical performance for robust solutions and deterministic solutions under different uncertainty levels. We present the mean and standard deviation for these simulation results in Fig.5. We observe that, similarly to the maximal data extraction problem results showed in Fig.2, the mean value of the adjusted robust solution is always better than the mean value of the adjusted deterministic solution, also the standard deviation of the simulation is smaller for the robust solution. This shows that on a simulation the robust solution is observed to perform better on average and with a smaller standard deviation than a deterministic solution for any uncertainty level.

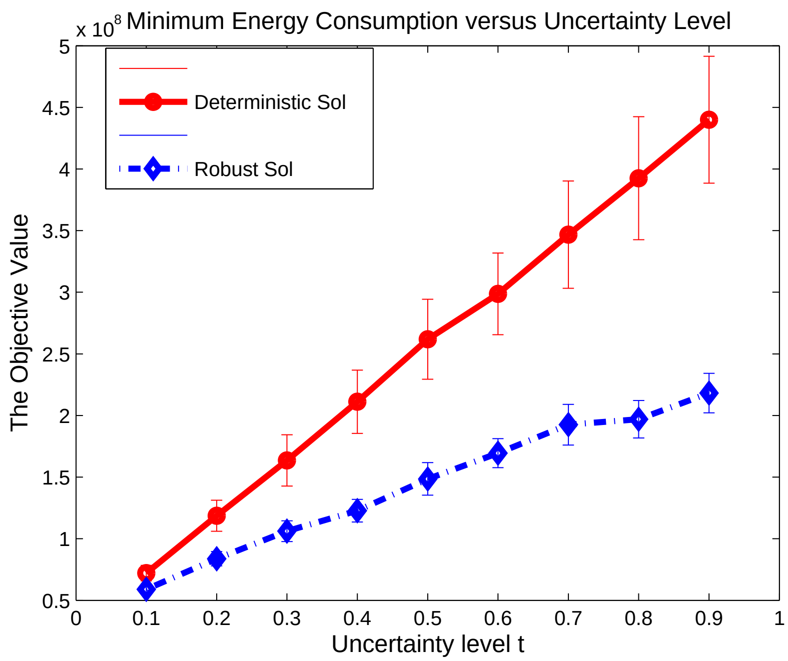

Figure 5: The mean and standard deviations of objective value for the deterministic and robust solutions in minimum energy consumption problem for different uncertainty level. E^i=250000, $D^i_{\max}$=10000, p=90%, M_i=2, R=0.1

We conduct a sensitivity analysis on the trade-off ratios, R^{ac} and R^{wc}, as we vary the minimal amount of information that must be sent. In Fig.6, we present the ratios R^{ac} and R^{wc} for different percentages p of the total information that must be sent to the sink, $f_{\min} = p \sum_{i \in N} D^i_{\max}$. We observe that R^{wc} is always higher than R^{ac} for every level of minimal amount of information, which means that the robust solutions are attractive as they provide a higher protection in the worst case than additional cost on the nominal data. Note also that the benefit of the robust solution increases for a larger uncertainty τ.

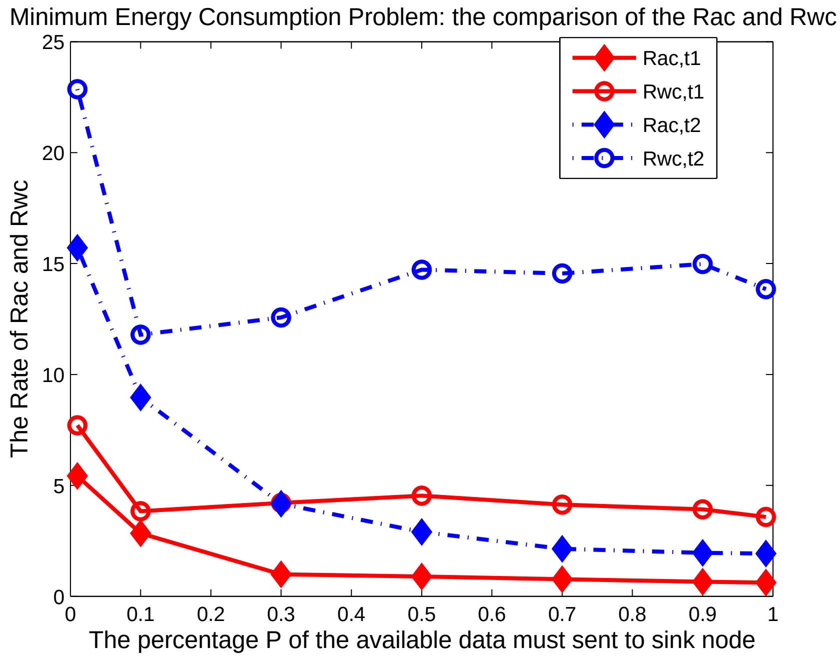

Figure 6: The comparison of R^{ac} and R^{wc} of the objective value in minimum energy consumption problem on different percentage P of total data that must be sent to the sink. E^i=250000, $D^i_{\max}$=10000, $\tau 1$=0.1, and $\tau 2$=0.9

6.4 Results for Maximum Lifetime Problem

The trade-off between robust solutions and deterministic solutions through the comparison of the ratios R^{ac} and R^{wc}. In Fig.7 we present the ratios R^{ac} and R^{wc} for different information extraction requirements. We plot the ratios R^{ac} and R^{wc} for $p1 = 90\%$ and $p2 = 50\%$. The graph presents the ratios R^{ac} and R^{wc} as a function of τ. The results here mimic the results found in the other two examples, that is the robust solution can

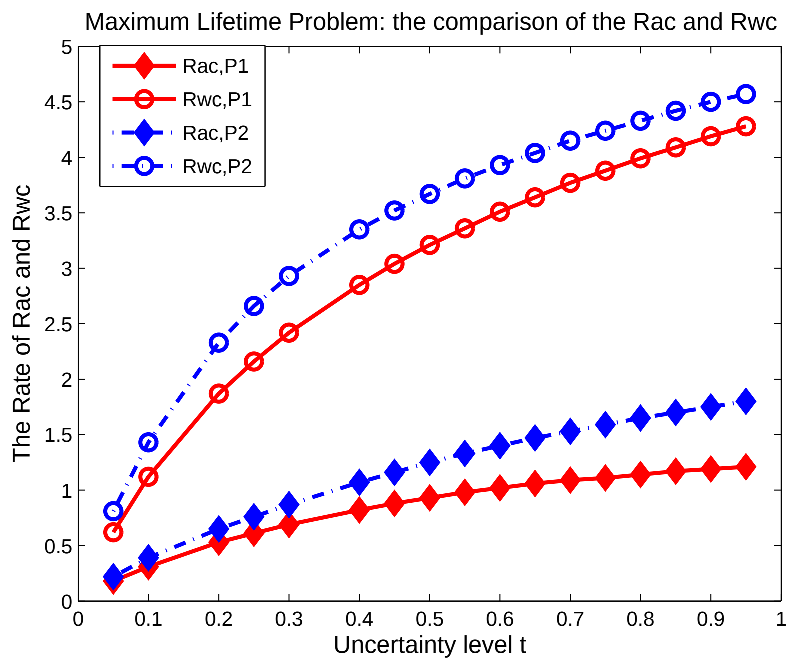

Figure 7: The comparison of R^{ac} and R^{wc} for objective value q (q=1/T) in maximum lifetime problems. $p1 = 90\%,\ p2 = 50\%,\ E^i = 250000,\ D^i_{\max} = 10000,\ R = 0.1$

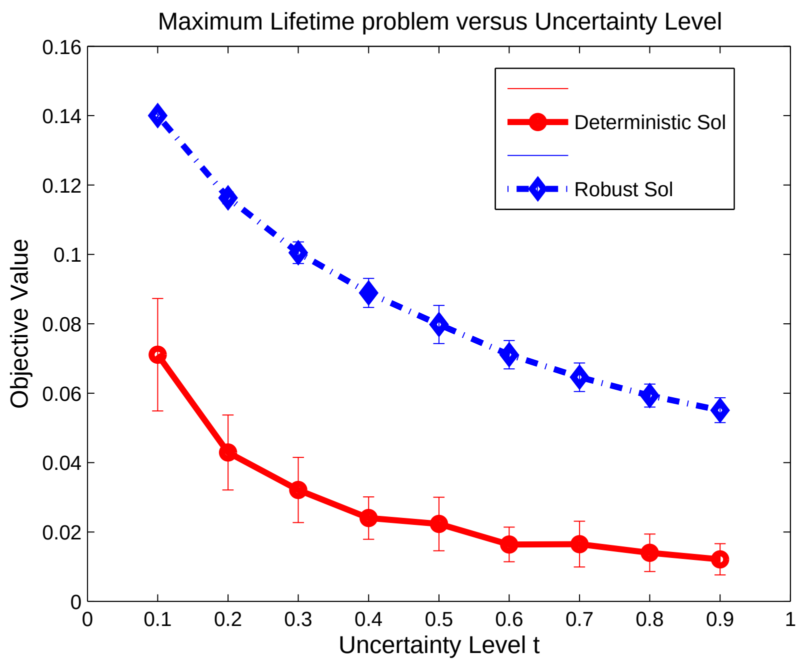

Figure 8: The mean and standard deviations for the deterministic and robust solutions of objective value T in maximum lifetime problem for different uncertainty level. T=1/q, E^i=250000, $D^i_{\max}$=10000, p=90%, M_i=2, R=0.1

significantly reduce the worst case cost, as the uncertainty increases, while increasing at a slower rate the loss of optimality of the robust solution in the nominal case, which shows the robust solutions can be able to compensate for uncertainty while suffering small performance losses.

The comparison of the practical performance for robust solutions and deterministic solutions under different uncertainty levels. Fig.8 presents the mean and standard deviation for these simulation results. We observe that, as it was with the previous two types of problems, the robust solution outperforms the deterministic solution in mean value for any uncertainty level, also the standard deviation of the objective function is smaller for the robust solution. This shows that under a simulation the robust solution of the maximum lifetime problem is observed to perform better on average and with smaller standard deviation than a deterministic solution for any uncertainty level.

For the maximum lifetime problem, we study the sensitivity of the ratios R^{ac} and R^{wc} to the percentage p of total data that must be sent to the sink, $f_{\min} = p \sum_{i \in N} D^i_{\max}$. Fig.9 shows that R^{wc} is much higher than R^{ac} for problems with little uncertainty $\tau 1 = 0.1$. However, the two ratios are comparable for large uncertainty sets, $\tau 2 = 0.9$, with R^{ac} beating R^{wc} for small percentages of information $p \leq 0.3$. Therefore, for this problem the benefits of a robust solution depend on the amount of uncertainty and amount of information that is being sent to the sink. There are conditions where the robust solution becomes overly conservative, costing in the nominal case more than the protection it can provide in the worst case.

7 CONCLUSIONS

Many planning and operational problems on energy limited wireless sensor networks must operate in conditions with significant uncertainty in distances between nodes. Op-

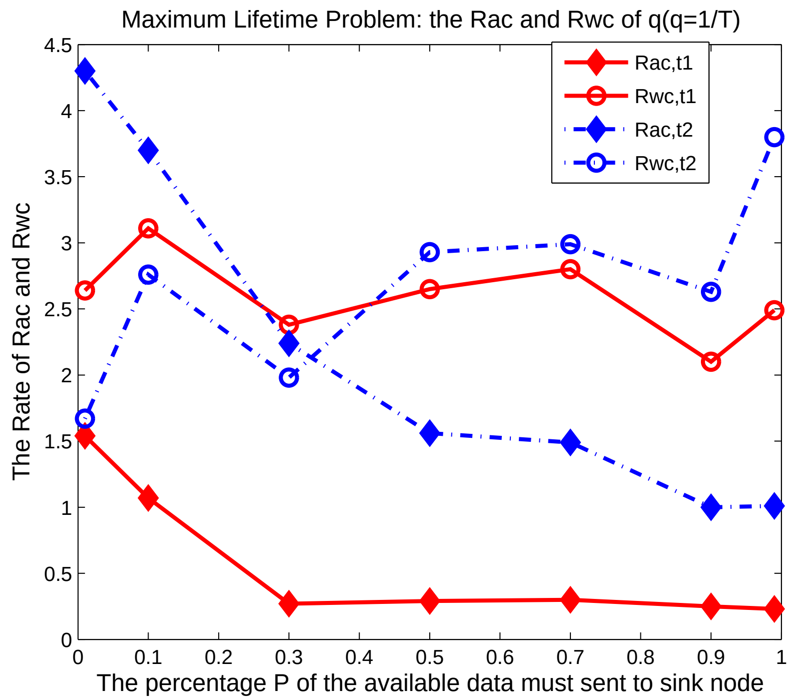

Figure 9: The comparison of R^{ac} and R^{wc} of the objective value q (q=1/T) for maximum lifetime problem for different percentage p of total data to sink. E^i=250000, $D^i_{\max}$=10000, $\tau1$=0.1, $\tau2$=0.9

timal solutions that do not take into consideration this uncertainty may be inefficient solutions in practice. In this paper, we present robust optimization models to address distance uncertainty for three optimization problems related to the operation of energy limited wireless sensor networks: the maximum data extraction, minimum energy consumption, and maximum lifetime problems.

For these three problems we proved that computing the robust solution, i.e. the solution with best worst case objective over the uncertainty set, is no harder than solving the deterministic version of the problem. The specific form of the uncertainty sets considered is fundamental to be able to compute the robust solution efficiently. Our computational experiments investigate whether a robust solution can be an attractive solution in practice. We find that the robust solution can provide significant worst case protection while often incurring in a small additional expense over the optimal solution for a nominal data instance. In addition we showed through simulations that a robust solution can exhibit better mean objective value and smaller standard deviations, and that these results hold for a wide setting of problem parameters.

This work showed that for a specific uncertainty set, the robust solution can be computed efficiently and it can be an attractive solution in practice. Future work will study what are representative models of the uncertainty faced by sensor networks in different applications and develop the problem formulations and algorithms to compute the robust solution efficiently. Our computational experiments also showed that for some problem instances the robust solution could be overly conservative, incurring in a large cost over a nominal optimal solution. An important future research direction is method to identify when a robust solution will be competitive from the problem instance and uncertainty set considered.

8 REFERENCES

[1] D. Estrin, R. Govindan, J. Heidemann, and S. Kumar,"Next Century Challenges: Scalable Coordination in Sensor Networks", MOBICOM, 1999.

[2] W. Ye and F. Ordóñez, "A Sub-Gradient Algorithm For Maximal Data Extraction In Energy-limited Wireless Sensor Networks", Proc. of IEEE Wirelesscom 2005

[3] N. Sadagopan, and B. Krishnamachari,"Maximizing Data Extraction in Energy-Limited Sensor Networks", INFOCOM, 2004.

[4] A. Ben-Tal and A. Nemirovski, "Robust Convex Optimization. Mathematics of Operations Research", 23(4):769-805, 1998.

[5] D. Goldfarb and G. Iyengar, "Robust Portfolio Selection Problems", Mathematics of Operations Research, 28(1):1-38, 2003.

[6] Lance Doherty, Kristofer S. J. Pister, Laurent El Ghaoui, "Convex Position Estimation in Wireless Sensor Networks", In Proceedings of IEEE Infocom, 2001.

[7] D. Bertsimas and A. Thiele, "A robust optimization approach to supply chain management", Technical report, MIT, LIDS, 2003.

[8] L. El Ghaoui and A. Nilim, "Robust solutions to markov decision problems with uncertain transition matrices", Operations Research, 2004.

[9] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows", Mathematical Programming, 98(1-3):49-71, 2003.

[10] F. Ordóñez and B. Krishnamachari, "Optimal information extraction in energy-limited wireless sensor networks", IEEE JSAC, 22(6):1121-1129, 2004.

[11] S. Doshi and T. X. Brown, "Minimum Energy Routing Schemes for a Wireless Ad

hoc Network", INFOCOM, 2002.

[12] C. Hua and T. Yum, "Optimal Routing for Maximizing lifetime of Wireless Sensor Networks", INFOCOM, 2005.

[13] A. Sankar and Z. Liu, "Maximum Lifetime Routing in Wireless Ad-hoc Networks", INFOCOM 2004.

[14] R. Madan and S. Lall, "Distributed algorithms for maximum lifetime routing in WSNs", IEEE GLOBECOM, 2004.

[15] H. Zhang and J. C. Hou, "Maximizing a-Lifetime for Wireless Sensor Networks", IPSN05, 2005.

[16] A. Giridhar and P.R. Kumar,"Maximizing the Functional Lifetime of Sensor Networks", IPSN05, 2005.

[17] R. Madan and S. Lall, "Distributed Algorithms for Maximum Lifetime Routing in Wireless Sensor Networks", IEEE GLOBECOM, 2004.

[18] K. Whitehouse, C. Karlof, A. Woo, F. Jiang, and D. Culler, "The Effects of Ranging Noise on Multihop Localization: An Empirical Study", IPSN05, 2005.

[19] N. Patwari, A. Hero, M. Perkings, N. Correal, and R. O'Dey, "Relative location estimation in wireless sensor networks", IEEE Transactions on Signal Processing, 51(8):2137-2148, 2003.

[20] W. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for WSN", Intl' Conference on System Sciences, Hawaii, 2000.

[21] A. Krause, C. Guestrin, A. Gupta, J. Kleinberg, "Near-optimal Sensor Placements: Maximizing Information while Minimizing Communication Cost", IPSN06

[22] B. Hong and V.K. Prasanna, "Maximum Data Gathering in Networked Sensor

Systems", Int. Journal of Distributed Sensor Networks, vol.1, pp:57-80,2006

[23] R. Subramanian, F. Fekri, "Sleep Scheduling and Lifetime Maximization in Sensor Networks: Fundamental Limits and Optimal Solutions", IPSN06