



Lagrange Duality in Online Scheduling with Resource Augmentation and Speed Scaling

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Resource Augmentation

- Worst-case paradigm: strength and weakness.

$$\text{Competitive ratio} = \max_I \text{ALG}(I) / \text{OPT}(I)$$

Non-clairvoyant scheduling $\sum_j (C_j - r_j)$

Jobs: arrive over time

Goal: minimize the total flow-time without knowledge of jobs' processing times.

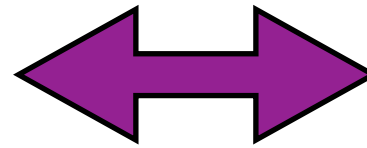
- **Resource augmentation model** (Kalyanasundaram & Pruhs)
 - Algorithms with additional resource vs optimal algorithms.
 - s-speed r-competitive algorithms.

Design & Analysis

- Potential function method: most successful but not much insight.

- **Principled methods**

Primal
program



Dual
program

- Dual fitting (Anand et al. '12)

- ▶ Construction of feasible dual variables.
- ▶ Keep reasonable ratio ALG/OPT

- Non-linear primal-dual (Gupta et al. '12)

- ▶ Increase rate of the dual proportional to the primal

useful: convex objectives, water-filling algorithms

Lagrangian duality-based scheme

Primal: $\min f_0(x)$ constraints $f_i(x) \leq 0 \forall 1 \leq i \leq m.$

Lagrangian Dual: $\max_{\lambda} \min_x L(x, \lambda) = f_0(x) + \sum_{i=1}^m \lambda_i f_i(x)$

□ Construction of dual variables.

weak duality

$$\min_x L(x, \lambda) \leq \min_x f_0(x) \leq r \cdot \min_x L(x, \lambda)$$

□ Difficulty: dual is not in closed form

critical

□ Game between ALG vs ADV

□ Advantages:

- non-convex programs; non water-filling algorithms
- well capture the resource augmentation model

Non-clairvoyant scheduling

Non-clairvoyant scheduling $\sum_j (C_j - r_j)$

Jobs: arrive over time

Goal: minimize the total flow-time without knowledge of jobs' processing times.

□ **EQUI:** the machine shares its resource equally to the pending jobs.

✓ **Theorem** (Edmonds '00): EQUI is $\frac{1}{1/2 - \epsilon}$ -speed, $\frac{1}{\epsilon}$ -competitive

Analysis

- Variables: C_j : completion time of job j
 $x_j(t)$: processing rate on job j at time t

- Primal:
$$\min \sum_j \left(\frac{C_j - r_j}{p_j} \right) \int_{r_j}^{C_j} x_j(t) dt$$

$$\int_{r_j}^{C_j} x_j(t) dt = p_j \quad \forall j$$

$\lambda_j =$ (marginal total flow-time due to job j)/ $2p_j$

$$\sum_{j=1}^n x_j(t) \leq 1 \quad \forall t$$

$\gamma(t) = 0$

$$x_j(t) \geq 0 \quad \forall j, t$$

$\mu_j(t) = 0$

- Dual:
$$\max_{\lambda, \gamma, \mu} \min_{x, C} L(x, C, \lambda, \gamma, \mu)$$

$$\geq \min_{x, C} \sum_j \int_{r_j}^{C_j} \frac{C_j - r_j}{p_j} x_j(t) dt + \sum_j \lambda_j \left(p_j - \int_{r_j}^{C_j} x_j(t) dt \right)$$

EQUI

□ λ_j could be explicitly computed.

✓ **Lemma:** $\frac{1}{p_j} \left(\lambda_j p_j - (t - r_j) \right) \leq N(t)$

✓ **Theorem** (Edmonds '00): EQUI is $\frac{1}{1/2 - \epsilon}$ -speed, $\frac{1}{\epsilon}$ -competitive

○ **Proof:**

$$\begin{aligned}
 & \min_{x, C} \sum_j \int_{r_j}^{C_j} \frac{C_j - r_j}{p_j} x_j(t) dt + \sum_j \lambda_j \left(p_j - \int_{r_j}^{C_j} x_j(t) dt \right) \\
 & \geq \min_{x, C} \sum_j \lambda_j p_j - \sum_j \int_{r_j}^{C_j} x_j(t) \cdot \left(\lambda_j - \frac{C_j - r_j}{p_j} \right) dt \\
 & \geq \mathcal{F}/2 - \min_x \sum_j x_j(t) \int_0^\infty N(t) dt \geq \epsilon \mathcal{F}
 \end{aligned}$$

resource augmentation: $\sum x_j(t) \leq 1/2 - \epsilon$

Results & Conclusion

☑ Approach

- Energy-aware scheduling for unrelated machines.
 - Scheduling to minimize general cost functions of flow-time on unrelated machines.
- Applying the approach for other online problems.

Thank you!