Lagrange Duality in Online Scheduling with Resource Augmentation and Speed Scaling

Nguyen Kim Thang
(university Evry, France)

Resource Augmentation

- Worst-case paradigm: strength and weakness.

Competitive ratio = $\max_I ALG(I)/OPT(I)$

Non-clairvoyant scheduling $1|r_j, pmtn|\sum_j (C_j - r_j)$

Jobs: arrive over time

Goal: minimize the total flow-time without knowledge of jobs' processing times.

- Resource augmentation model (Kalyanasundaram & Pruhs)
  - Algorithms with additional resource vs optimal algorithms.
  - s-speed r-competitive algorithms.

Design & Analysis

- Potential function method: most successful but not much insight.
- Principled methods

Primal program ⟷ Dual program

  - Dual fitting (Anand et al. '12)
    - Construction of feasible dual variables.
    - Keep reasonable ratio ALG/OPT
  - Non-linear primal-dual (Gupta et al. '12)
    - Increase rate of the dual proportional to the primal

useful: convex objectives, water-filling algorithms

Lagrangian duality-based scheme

Primal: $\min f_0(x)$ constraints $f_i(x) \leq 0 \; \forall 1 \leq i \leq m.$

Lagrangian Dual: $\max_{\lambda} \min_{x} L(x, \lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

- Construction of dual variables.

weak duality

$$\min_{x} L(x, \lambda) \leq \min f_0(x) \leq r \cdot \min_{x} L(x, \lambda)$$

critical

- Difficulty: dual is not in closed form
- Game between ALG vs ADV
- Advantages:
  - non-convex programs; non water-filling algorithms
  - well capture the resource augmentation model

Non-clairvoyant scheduling

Non-clairvoyant scheduling $1|r_j, pmtn|\sum_j (C_j - r_j)$

Jobs: arrive over time

Goal: minimize the total flow-time without knowledge of jobs' processing times.

- EQUI: the machine shares its resource equally to the pending jobs.

- Theorem (Edmonds '00): EQUI is $\dfrac{1}{1/2-\epsilon}$ -speed, $\dfrac{1}{\epsilon}$ -competitive

Analysis

- Variables: C_j : completion time of job j

  $x_j(t)$: processing rate on job j at time t

- Primal: $$\min \quad \sum_j \frac{C_j - r_j}{p_j} \cancel{(C_j - r_j)} \int_{r_j}^{C_j} x_j(t)dt$$

$$\int_{r_j}^{C_j} x_j(t)dt = p_j \quad \forall j$$

λ_j = (marginal total flow-time due to job j)/$2p_j$

$$\sum_{j=1}^{n} x_j(t) \le 1 \quad \forall t$$

$\gamma(t)$ = 0

$$x_j(t) \ge 0 \quad \forall j, t$$

$\mu_j(t)$ = 0

- Dual: $$\max_{\lambda,\gamma,\mu} \min_{x,C} L(x, C, \lambda, \gamma, \mu)$$

$$\ge \min_{x,C} \sum_j \int_{r_j}^{C_j} \frac{C_j - r_j}{p_j} x_j(t)dt + \sum_j \lambda_j \left(p_j - \int_{r_j}^{C_j} x_j(t)dt \right)$$

EQUI

- λ_j could be explicitly computed.
- Lemma: $$\frac{1}{p_j}\Big(\lambda_j p_j - (t - r_j)\Big) \leq N(t)$$
- Theorem (Edmonds '00): EQUI is $\dfrac{1}{1/2-\epsilon}$ -speed, $\dfrac{1}{\epsilon}$ -competitive

- Proof:

$$\min_{x,C} \sum_j \int_{r_j}^{C_j} \frac{C_j - r_j}{p_j} x_j(t)dt + \sum_j \lambda_j \left(p_j - \int_{r_j}^{C_j} x_j(t)dt\right)$$

$$\geq \min_{x,C} \sum_j \lambda_j p_j - \sum_j \int_{r_j}^{C_j} x_j(t) \cdot \left(\lambda_j - \frac{C_j - r_j}{p_j}\right) dt$$

$$\geq \mathcal{F}/2 - \min_x \sum_j x_j(t) \int_0^{\infty} N(t)dt \geq \epsilon\mathcal{F}$$

resource augmentation: $\sum x_j(t) \leq 1/2 - \epsilon$

Results & Conclusion

- ☑ Approach
    - Energy-aware scheduling for unrelated machines.
    - Scheduling to minimize general cost functions of flow-time on unrelated machines.
- ☐ Applying the approach for other online problems.

Thank you!