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CROSS CUTTING AREAS

Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines

Rafael Epstein, Marcel Goic, Andrés Weintraub
Industrial Engineering Department, University of Chile, 8370439 Santiago, Chile
{repstein@dii.uchile.cl, mgoic@dii.uchile.cl, aweintra@dii.uchile.cl}

Jaime Catalán, Pablo Santibáñez, Rodolfo Urrutia
Fundación para la Transferencia Tecnológica, 8370481 Santiago, Chile
{jcatalan@dii.uchile.cl, psantiba@dii.uchile.cl, rurrutia@dii.uchile.cl}

Raúl Cancino, Sergio Gaete, Augusto Aguayo
Codelco Chile, División Norte, El Teniente, División Andina, 8370424 Santiago, Chile
{rcancino@codelco.cl, sgaete@codelco.cl, aaguayo@codelco.cl}

Felipe Caro
UCLA Anderson School of Management, Los Angeles, California 90095, fcaro@anderson.ucla.edu

We present a methodology for long-term mine planning based on a general capacitated multicommodity network flow formulation. It considers underground and open-pit ore deposits sharing multiple downstream processing plants over a long horizon. The purpose of the model is to optimize several mines in an integrated fashion, but real size instances are hard to solve due to the combinatorial nature of the problem. We tackle this by solving the relaxation of a tight linear formulation, and we round the resulting near-integer solution with a customized procedure. The model has been implemented at Codelco, the largest copper producer in the world. Since 2001, the system has been used on a regular basis and has increased the net present value of the production plan for a single mine by 5%. Moreover, integrating multiple mines provided an additional increase of 3%. The system has allowed planners to evaluate more scenarios. In particular, the model was used to study the option of delaying by four years the conversion of Chiquicamata, Codelco's largest open-pit mine, to underground operations.

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1. Introduction

Basically, there are two major forms of mining: underground and open pit. For both, the use of analytical models to support extraction planning has a long practice. In particular, the notion of a cutoff grade (Lane 1988) has been a fundamental concept in many of these models, where the objective is to find a threshold that determines whether a block, the basic geological planning unit, is profitable to mine.

Given the advances in software and hardware, the use of operations research (OR)—in particular, mixed-integer programming (MIP)—has gained strength (Alford et al. 2007, Cacceta 2007). This approach—in contrast with cutoff grade models—allows for a detailed representation of the mine, where the block of rock is the minimum element that can be defined. Moreover, MIP-based models can incorporate in a natural form constraints related to geomechanical, economic or extraction sequencing conditions. The down side is that the size of such models makes the formulations difficult to solve, and the literature so far reports few applications to large-scale mines. In that respect, this paper presents and solves a MIP model that has been successfully used in Chilean copper mines by Codelco, a state-owned company, for both underground and open-pit extraction.

Codelco is the largest copper enterprise in the world, and it has been a fundamental pillar of Chile's economy for decades. As such, it has placed strong emphasis on achieving excellence in every aspect of its operations, ranging from extraction planning to safety and the working conditions of its miners. A telling fact is that in August 2010, when 33 miners from a small company got trapped 700 meters below the surface—a story that received worldwide attention—the technical leader of the successful rescue effort was the General Manager of Mining from El Teniente, Codelco's largest underground mine (he also happened to be a main counterpart for this project).

Our collaboration with Codelco started in 1999. First, we developed a model for long-term planning at El Teniente. Improvements attributable to the model led to an increase in value of the mine of over \$100 M. Then, the model was extended to incorporate open-pit mines. It was implemented at Andina and Codelco North Division, which are two clusters of mines located in northern Chile. In these clusters, several mines share common plant installations, which include flotation, leaching, and bioleaching processes. Moreover, at Andina—and soon at Codelco North Division—open-pit and underground mines coexist. Our model is specifically suited for these cases, and since 2001 it has been used on a regular basis.

Overall, our paper makes the following contributions.

- It integrates the extraction phase with downstream processes, showing the advantages of integrating planning along the whole production chain until the final product—refined copper—is obtained to be shipped. Traditionally, mining and plant processes have been solved separately, iterating between the two problems to match supply and demand.
- It considers, as is the case in Codelco, the interaction of multiple mines, multiple products, and multiple downstream processes. In particular, it incorporates the notion of material from different mines competing for limited downstream processing capacity. Traditionally, planning has been carried out not considering this multiplicity.
- It develops a comprehensive model that is an alternative to the traditional approach based on cutoff grades, and it generalizes the concept of underground and open-pit mining as part of a capacitated network. This allows representing any form of mining as a process in the production chain, including the transition from open-pit to underground operations.
- It establishes the complexity of the mining problem for both underground and open pit. For underground mines, we solve the problem by taking advantage of their nature, where ore grades decrease vertically at each drawpoint. For open pit, we present an extended formulation with a tight linear relaxation. In both cases, we use a rounding heuristic to find integer solutions. This approach was favored over other methods, such as decomposition, because in our experience it required less on-site OR expertise, which facilitated the adoption and widespread use of the model at Codelco.
- It reports two independent case studies at Codelco North Division and El Teniente, where the model was used with gains between 5% and 8% in net present value (NPV). A third study at Andina had a similar impact.

The reminder of the paper has the following structure. In the next section we present a brief literature review. In §3 we describe the problem, and we provide some background on mining operations. In §4 we present our framework based on a general capacitated multicommodity network flow problem. In §5 we introduce the customized rounding heuristics used to generate approximate solutions. In §6 we provide an overview of the model's implementation and impact at Codelco. In the final section we conclude. The theoretical proofs are given in an online companion to this paper, an electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

2. Literature Review

Most mining optimization models available in the literature have been developed for open-pit mines, and they solve only a partial version of the long-term planning problem (Newman et al. 2010b). Historically, the problem has been divided in two: the ultimate pit problem, which determines the best final pit; and the production scheduling problem that considers which blocks to remove and when.

There are two main approaches to solve the ultimate pit problem: one based on cutoff grades and the other based on OR techniques. The optimal cutoff grade methodology was made popular through the work of Lane (1988). See Kim and Zhao (1994) and Poniewierski et al. (2003) for extensive discussions. The basic premise of this approach is that one can use cutoff grades to maximize NPV subject to capacity constraints, with higher cutoffs in the initial years leading to higher overall profits. The approach has important operational advantages, and it is embedded in the background of most mining practitioners. However, the assumption of a fixed cutoff grade—which depends on an aggregated delineation between ore and waste—generates suboptimal solutions because it ignores that the value of a block is not inherent to the block but rather depends on the interaction with the rest of the mine and the capacity of the downstream processes.

The use of OR techniques to solve the ultimate pit problem started with the classic "moving cone" heuristic; see, for instance, Kim (1978) or Laurich (1990). This approach takes a block as a reference point and expands the pit upward according to pit slope rules. This solution can be suboptimal, but it is intuitively appealing. Among the algorithms that are guaranteed to reach the optimum, historically the most important are Lerchs and Grossmann (1965) and Picard (1976). The first one is based on graph theory, but its structure is very similar to the dual simplex method; see Underwood and Tolwinski (1998). The algorithm by Picard reduces the ultimate pit problem to finding a maximum closure in a graph, so it can be solved as a maximum flow. A comparison of both procedures with an extensive literature review can be found in Hochbaum and Chen (2000).

OR techniques have also been used to solve the production scheduling problem for a single mine. Among the papers based on optimization methods, Gershon (1983) presents a MIP formulation, Dagdelen and Johnson (1986) suggest a Lagrangian relaxation, Caccetta and Hill (2003) use branch and cut, and Tolwinski and Underwood (1996) propose a dynamic programming approach. Given the complexity of the problem, several papers have resorted to heuristic methods such as simulation (Fytas et al. 1993, Erarslan and Celebi 2001), neural networks (Denby and Schofield 1995), and simulated annealing (Kumral and Dowd 2005). For other approaches that combine optimization and heuristic methods, see Sevim and Lei (1988), Sarin and West-Hansen (2005), Busnach et al. (1985), Dowd and Onur (1993), Elevli (1995), Wang and Sun (2001), Ramazan (2007), and Chicoisne et al. (2009).

The literature on underground mining is more recent, partially due to the complicated nature of its operations. Indeed, there is no equivalent to the Lerchs-Grossman algorithm or the work by Picard for open pit. Yun et al. (1990) use a genetic algorithm to determine the number and spacing of openings given restrictions on their relative placement. Alford (1995) describes the floating stope method as a tool for analysis of mineral reserves. Barbaro and Ramani (1986) formulate a MIP model that can be used to determine whether to produce in a given time period. Kuchta et al. (2004, 2003) describe the implementation of a production scheduling MIP model at a Swedish underground mine. Their objective is to minimize the deviation from a pre-specified demand target per period, which contrasts with our model that maximizes NPV. The sizes of the instances are not comparable either, but there are some similarities with our project in terms of the methods used to reduce the size of the model.

As noted in Hochbaum and Chen (2000), a comprehensive model of mining operations has seldom been addressed in the literature. Even fewer implementations of such models have been reported in practice. The work by Carlyle and Eaves (2001) is a noteworthy attempt. They developed a MIP model for an underground platinum and palladium mine and considered several planning decisions, but it was applied to only one sector of the mine, and near optimal integer solutions were obtained with a commercial software package. In our case, we consider all the sectors simultaneously, and we even allow for multiple mines. In fact, we are unaware of other implementations that solve an integrated long-term planning problem with the level of detail considered here.

3. Problem Description

Most mines have a long life-cycle. Therefore, the planning horizon is somewhere between 10 and 30 years, with shorter periods at the beginning where more detail is required. In general, planning decisions are final and cannot be reversed, meaning that they can drastically affect the future of the mine. The long-term planner must solve three main problems: (i) Investment: determine the selection and timing of investments; (ii) Extraction: determine the production in the mine; and (iii) Processing: determine the operation of the plants. Solving each one of these problems, even separately, is already a complex task. However, the need for an integrated planning approach cannot be dismissed. For instance, the real value of an investment project is appreciated only once its coherence is verified with respect to the extraction and processing decisions. Our methodology explicitly solves problems (ii) and (iii), i.e., extraction and processing. The investment decisions are usually restricted to a small set of options and can be evaluated with our model by running different scenarios.

3.1. Underground Mining

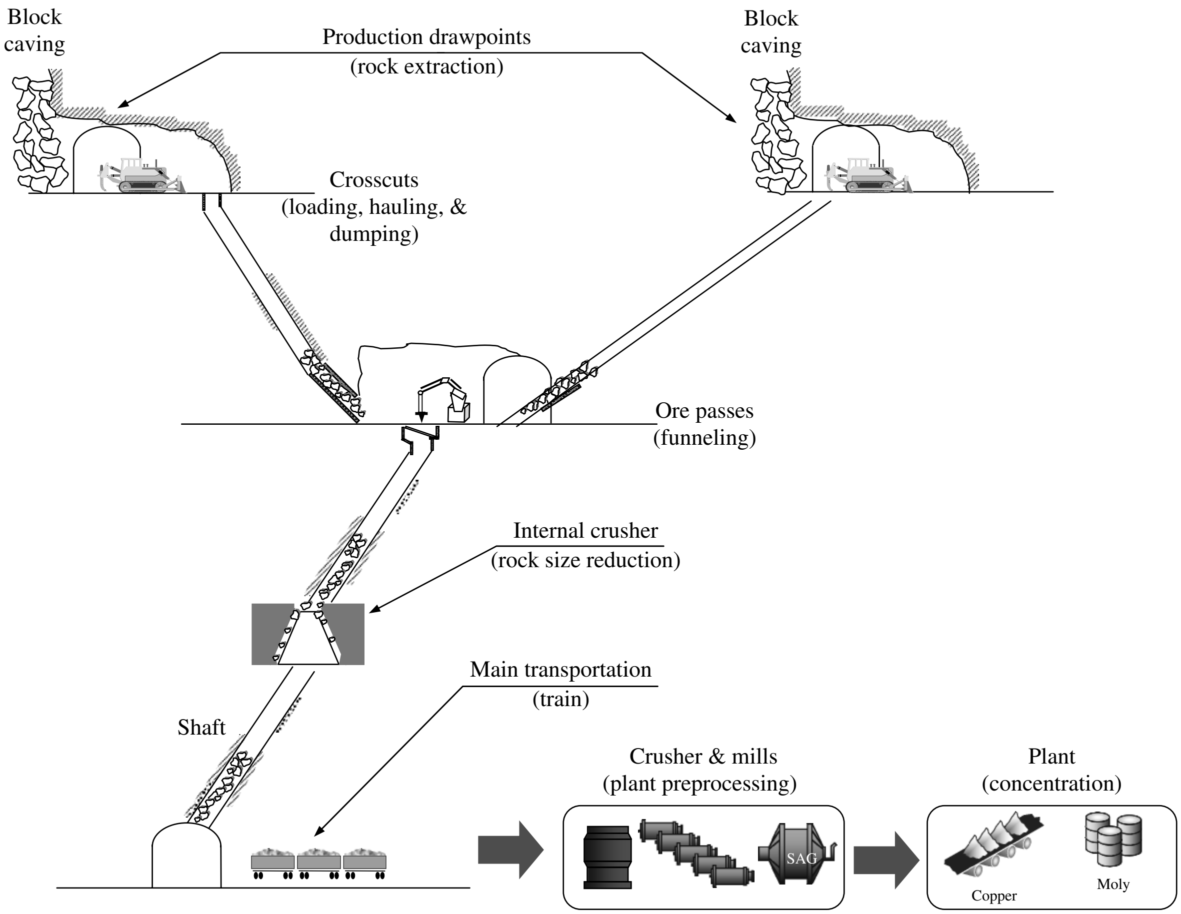
Here we present a brief description of the operations at an underground mine. The first step is the selection of reserves. This consists in defining the boundaries of the orebody, delimiting the material to be removed, and it is done according to economic criteria that determine what is profitable based on the grade distribution. Large underground mines typically are subdivided in sectors by design, and the selection of reserves is made for each sector.

For planning and operational purposes, the geological configuration of the whole deposit is expressed through a block model. Each block is uniquely identified together with its geological characteristic, in particular the ore grades. These values are estimated using geostatistic procedures such as kriging. A column or drawpoint is the vertical aggregation of blocks, and a mine or sector corresponds to a set of neighboring columns.

Once the drawpoints are specified, the following step is to program the ore extraction on a time scale taking into account geological restrictions and the downstream capacities. There are several mining methods for underground mines (Newman et al. 2010b). In our model, we consider the block caving method, which is the prevalent underground mining method at Codelco. In a nutshell, the block caving method consists in creating a void at each drawpoint so that the rock breaks and falls due to its own weight. For this to happen, the rock extraction pattern must follow specific rules: (i) the columns have to enter production in a particular sequence to generate a "wave" that breaks the rock; (ii) the wave has to advance smoothly, which requires regularity in heights among neighboring columns; and (iii) at each drawpoint there is maximum extraction rate to prevent the roof from collapsing, and there is a minimum number of blocks that must be removed in order to avoid solid pillars.

Figure 1 shows the typical flow in an underground mine. The broken rock is removed from the drawpoints, which are arranged along parallel crosscuts. Each crosscut has enough space for an LHD machine that hauls the material to a dumping point.[1] The material is then funneled through ore passes and reaches the internal crusher, where the rock is crushed to a size that can be transported by train to the processing stages outside the mine. For our purposes, the flow finishes at the concentration plants where the profitable minerals are obtained by eliminating waste. Each one of these intermediate processes can be characterized through technical coefficients, such as its capacity and variable cost.

Figure 1. Description of underground mining operations.

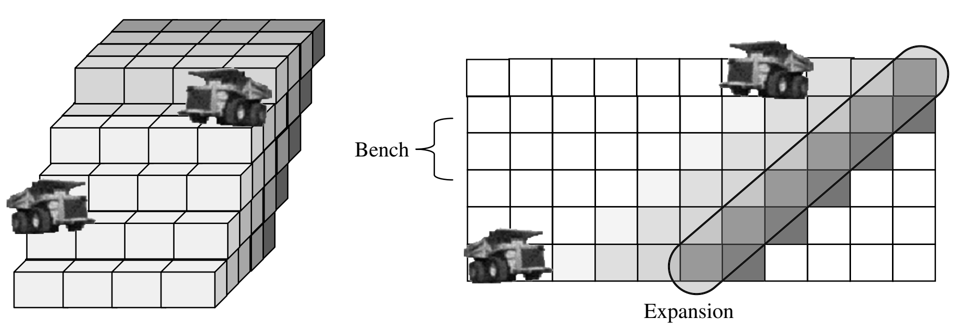


3.2. Open-Pit Mining

Open-pit mining is conceptually similar to underground mining, except that the orebody is reached from above, which requires removing plenty of overburden from the soil. A preliminary definition of the regions to be mined is done using geological models that take into account the geometry of the pit and technical requirements (e.g., the maximum slope to prevent the walls of the pit from collapsing). The regions are called expansions, which are also known as pushbacks or phases. In figurative terms, an expansion can be viewed as a "slice" of the pit. Each expansion is subdivided in benches of a predetermined height that must be extracted in order, from top to bottom. Figure 2 illustrates the geometry of expansions and benches, and Figure 3 shows a view from above.

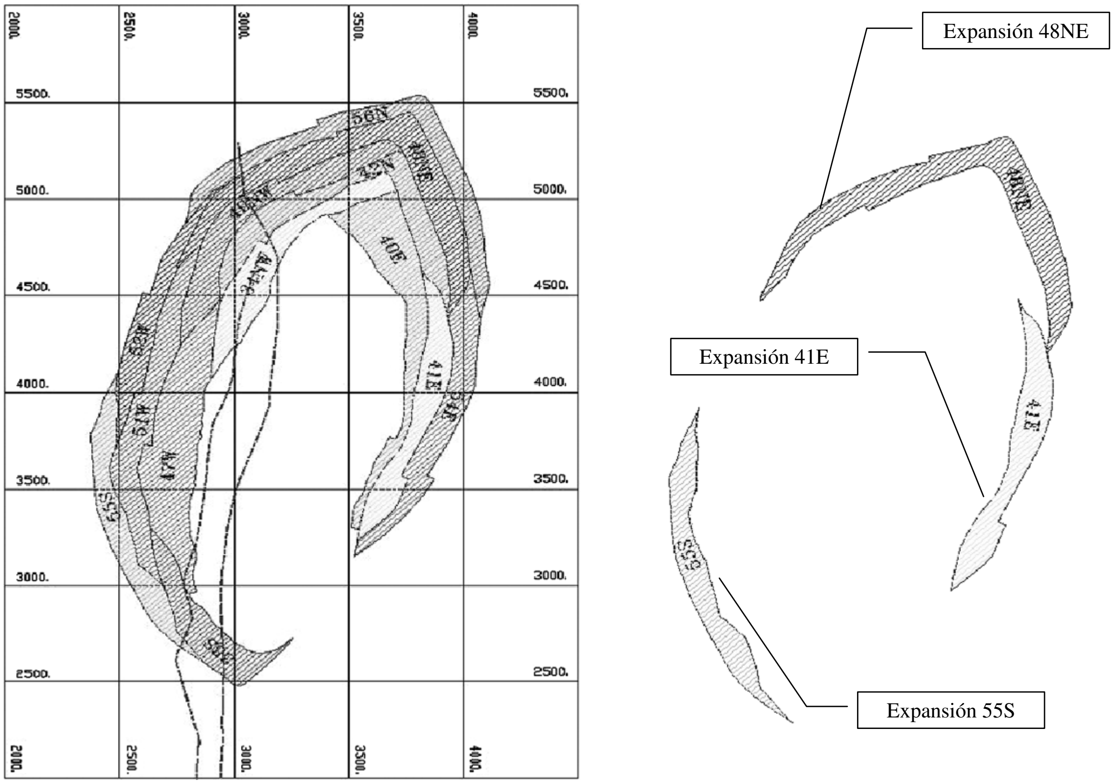
The bench is the minimal extraction decision unit. When a bench is mined, all its material needs to be removed from the pit—including waste, which has no economic value. There are three processes involved in removing a bench: perforation and blasting, where the bench is separated from

Figure 2. Description of open-pit mining operations.



Note. Within an expansion, benches are extracted from top to bottom.

Figure 3. Expansions of an open-pit mine, viewed from above.



the soil using dynamite; loading, where all the material is loaded into large size trucks; and transportation, where the material is taken to either a metallurgic process or a waste dump. It is worth mentioning that perforation and blasting impose severe constraints on the sequence in which expansions are extracted. For instance, there is a minimum distance required between two expansions in operation on the same wall of the pit to avoid rock spillage.

As in underground mining, the sequence of downstream processes involves rock size reduction and ore recovery. However, there are some important differences. First, the existence of stockpiles plays an important role in balancing the plant inflow, and they are held for several years before entering the metallurgic processes. Second, a significant fraction of the material extracted from the mine is taken to the waste dumps, which might be located several miles away from the pit. This introduces an important trade-off between sending low-grade ore to the downstream processes or to the costly dumps.

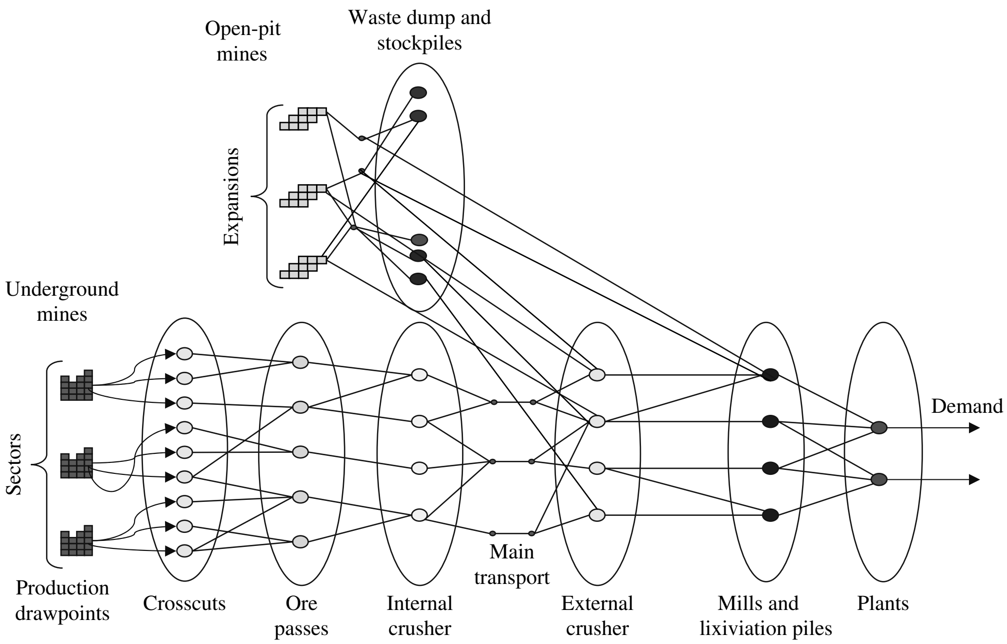
In the case of copper, the existence of two ore types, sulfides and oxides, is another key element of an integrated plan. Although the geological reserves typically have both types, in the traditional planning scheme, mining plans are defined considering only one main metallurgic process for a particular type. Thus, an integrated plan considers different ore types in alternative concentration lines that would be otherwise discarded or sent to processes with low yields.

4. Model Formulation and Discussion

In this section we present a mathematical programming approach to optimize long-term production plans in open-pit and underground mines that share downstream processes (we refer the reader to the online companion for a description of the legacy planning approach). A graphical representation of the different stages in the model is given in Figure 4, where material flows from left to right. The network of mining operations begins with the production or rock extraction phase that takes place at sectors for underground mines and at expansions for open pit. This phase corresponds to the first stage on the left-hand side of Figure 4. Then the extracted rock is fed into the network of downstream processes that involves size reduction and chemical reactions. The last stage is the concentration phase, which yields the three final products: copper, molybdenum, and arsenic. The first two have commercial value while the third one is a contaminant on which environmental restrictions are imposed.

In what follows, we present the model with its essential components. When necessary, we use the subindices O and U to denote open-pit and underground mines, respectively. In §4.1, we first introduce a unified model for the network of downstream processes. This part is shared by all the mines regardless of their type. Then, in §§4.2 and 4.3 we present a specific model of the production phase that captures the unique characteristics of underground and open-pit mining, respectively.

Figure 4. Network flow representation for mining operations that share downstream processes.



4.1. Unified Model Formulation for Downstream Processes

The downstream processes are modeled as a capacitated multicommodity network flow problem. The starting nodes $s \in S = S_U \cup S_O$ represent the different sectors (S_U) for underground mines and expansions (S_O) for open-pit mines. The intermediate nodes $u, v \in V$ represent stocking points and processing stages that precede the concentration plants, which are the final nodes (F) in the network. Multiple products flow through the network. A given product $k \in K$ represents material (i.e., rock) with an average rock size and a grade of copper, molybdenum, and arsenic within a certain range. Depending on the characteristics of the ore deposits, the product definition might also include other relevant attributes such as the chemical nature (sulfide or oxide) or geological properties (e.g., rock hardness). The set of commercial products is denoted COM, whereas the set of contaminants is denoted CONT.

The flow of product $k \in K$ going from node u to node v in period $t \in T = \{1, \ldots, T\}$ is denoted f_{uvkt} and is measured in tons. As the material flows through the downstream processes, the average rock size is reduced, and the valuable minerals are separated from waste. Let $\mathrm{TC}_{vkk'}$ denote the transformation coefficient at node $v \in V$, which represents the amount of output k' obtained for each ton of input k, with $\sum_{k' \in K} \mathrm{TC}_{vkk'} = 1$, $\forall v \in V, k \in K$. Let c_{vkt} denote the cost of processing a ton of product k at node v in period $t \in T$, while the maximum capacity across all products at node v in period t is given by CP_{vt}. For nodes $v \in V$ that can hold inventory, let y_{vkt} represent the amount of product k available at the beginning of period t, and let CS_{vt} be the maximum stock level per period in tons (if the node cannot hold inventory, then $\mathrm{CS}_{vt} = 0$). If k is a contaminant, then the maximum amount that can be released per period is E_{kt} (otherwise, $E_{kt} = \infty$). We assume that the firm is a price-taker and that all the production of product k in period t can be sold at the market price p_{kt} (clearly, $p_{kt} = 0$ for noncommercial products).

The link of the downstream processes with the upstream production phase is established through the auxiliary variables $\mathrm{ProdCost}_{st}$ and x_{skt}. The former denotes the total production cost, while the latter represents the tons of product k produced in sector/expansion s in period t. The mathematical formulation of the downstream processes is the following:

$$\max \sum_{t=1}^{T} \sum_{k \in K} \sum_{u \in V \cup S} \beta^{t-1} \left(\sum_{v \in F} \sum_{k' \in \mathrm{COM}} p_{k't} \mathrm{TC}_{vkk'} f_{uvkt} - \sum_{v \in V \cup F} c_{vkt} f_{uvkt} \right) - \sum_{t=1}^{T} \sum_{s \in S} \beta^{t-1} \mathrm{ProdCost}_{st}, \tag{1}$$

subject to

$$x_{skt} = \sum_{v \in V \cup F} f_{svkt} \quad \forall\, s \in S,\ k \in K,\ t \in T, \tag{2}$$

$$\sum_{u \in V \cup S} \sum_{k \in K} \mathrm{TC}_{vkk'} f_{uvkt} + y_{vk't-1} = \sum_{u \in V \cup F} f_{vuk't} + y_{vk't} \quad \forall\, v \in V,\ k' \in K,\ t \in T, \tag{3}$$

$$\sum_{u \in V \cup S} \sum_{k \in K} f_{uvkt} \leqslant \mathrm{CP}_{vt} \quad \forall\, v \in V \cup F,\ t \in T, \tag{4}$$

$$\sum_{k \in K} y_{vkt} \leqslant \mathrm{CS}_{vt} \quad \forall\, v \in V,\ t \in T, \tag{5}$$

$$\sum_{u \in V \cup S} \sum_{v \in F} \sum_{k \in K} \mathrm{TC}_{vkk'} f_{uvkt} \leqslant E_{k't} \quad \forall\, k' \in \mathrm{CONT},\ t \in T, \tag{6}$$

$$f_{uvkt}, y_{vkt} \geqslant 0 \quad \forall\, u, v \in V,\ k \in K,\ t \in T. \tag{7}$$

The objective of the model is to maximize the NPV of the deposits over the next T periods. The first term in the objective function (1) corresponds to the discounted benefits obtained from selling the final products. The second and third terms represent the discounted processing and production costs, respectively (here the parameter $\beta < 1$ is the discount factor). Constraints (2) feed the production from each sector/expansion into the network of downstream processes. Constraints (2) ensure flow conservation and inventory balance at each node. Constraints (2) and (4) impose maximum processing capacities and stock levels respectively. Finally, Constraints (5) limit the release of pollutants at the final stage, and Constraints (6) are the usual nonnegativity conditions on flows and inventory levels.

4.2. Underground Extraction

To model the production phase for underground mines, we assume that the mining method is block caving, as explained in §3.1. Let the index $i \in I(s)$ denote a drawpoint or column in sector $s \in S_U$ and let the index $n \in N(i)$ denote the blocks in that column that are numbered 0 to $|N(i)| - 1$ from bottom to top. The main decision in the production phase is when to remove each block. For that, we introduce the binary variable z_{int}—which equals one if the extraction of block n in column i is initiated in period t—and the continuous variable e_{int}, which represents the fraction of the block that is removed in each period.

As part of the model's input, we need the following sets and parameters. Let TON_{ink} be the amount (tons) of product k contained in block (i, n). Let $E\,\text{MIN}(i)$ denote the minimum set of blocks of column i that must be removed once production at that column is initiated. Let $I_U = \bigcup_{s\in S_U} I(s)$ be the set of drawpoints/columns in all the underground mines considered. Let Δ denote the maximum number of periods a drawpoint can remain open. Let α_{in} be the height of block (i, n), and let the variable h_{it} denote the height of column i in period t that is given by the blocks that have been removed so far. Let δ_s denote the maximum height differential allowed between two neighboring columns in sector s, and let IJ be the set of column pairs that are close enough to be considered neighbors.

The extraction precedence relationship among columns is given by the set SEC_U, where $(i, j) \in \text{SEC}_U$ if the extraction of column i must begin before it takes place at column j. Initiating production at column i incurs a fixed cost a_i and let b_{st} be the variable cost per ton of rock extracted from sector s in period t. Removing block (i, n) takes a minimum of γ_{in} days, and let DP_t denote the duration of period t. Let MAX_AREA_{st} (MIN_AREA_{st}) and MAX_EXT_{st} (MIN_EXT_{st}) be the maximum (minimum) incorporated area and extracted tons allowed in sector s in period t, respectively. Finally, let $\text{WIN}(s)$ denote the production time window for sector s. The following constraints complete the model formulation for underground mines.

$$x_{skt} = \sum_{i\in I(s)} \sum_{n\in N(i)} \text{TON}_{ink}\, e_{int} \quad \forall\, s \in S_U,\ k \in K,\ t \in T, \tag{8}$$

$$\text{ProdCost}_{st} = \sum_{i\in I(s)} a_i z_{i0t} + b_{st} \sum_{k\in K} x_{skt} \quad \forall\, s \in S_U,\ t \in T, \tag{9}$$

$$\sum_{t\in T} z_{int} \leqslant 1 \quad \forall\, i \in I_U,\ n \in N(i), \tag{10}$$

$$\sum_{g=1}^{t} e_{ing} \leqslant \sum_{g=1}^{t} z_{ing} \quad \forall\, i \in I_U,\ n \in N(i),\ t \in T, \tag{11}$$

$$\sum_{t=1}^{T} \sum_{n\in E\,\text{MIN}(i)} e_{int} \geqslant \sum_{t=1}^{T} |E\,\text{MIN}(i)| z_{i0t} \quad \forall\, i \in I_U, \tag{12}$$

$$\sum_{g=1}^{t} Z_{in+1g} \leqslant \sum_{g=1}^{t} e_{ing} \quad \forall i \in I_U,\ n \in N(i),\ t \in T, \tag{13}$$

$$\sum_{g=1}^{t} z_{j0g} \leqslant \sum_{g=1}^{t} e_{i0g} \quad \forall\, (i, j) \in \text{SEC}_U,\ t \in T, \tag{14}$$

$$z_{i0t} + \sum_{g=t+\Delta}^{T} e_{ing} \leqslant 1 \quad \forall\, i \in I_U,\ n \in N(i),\ t \in T, \tag{15}$$

$$\sum_{n\in N(i)} \gamma_{in} e_{int} \leqslant \text{DP}_t \quad \forall\, i \in I_U,\ t \in T, \tag{16}$$

$$h_{it} = h_{it-1} + \sum_{n\in N(i)} \alpha_{in} e_{int} \quad \forall\, i \in I_U,\ t \in T, \tag{17}$$

$$h_{it} - h_{jt} \leqslant \delta_s \quad \forall\, i, j \in IJ,\ s \in S_U,\ t \in T, \tag{18}$$

$$\text{MIN\_AREA}_{st} \leqslant \sum_{i\in I(s)} \text{AREA}_i\, z_{i0t} \leqslant \text{MAX\_AREA}_{st} \quad \forall\, s \in S_U,\ t \in T, \tag{19}$$

$$\text{MIN\_EXT}_{st} \leqslant \sum_{k\in K} x_{skt} \leqslant \text{MAX\_EXT}_{st} \quad \forall s \in S_U, t \in T, \tag{20}$$

$$\sum_{i\in I(s)} z_{i0t} = 0 \quad \forall\, s \in S_U,\ t \in T \setminus \text{WIN}(s), \tag{21}$$

$$z_{int} \in \{0, 1\},\ e_{int},\ h_{it} \geqslant 0 \quad \forall\, i \in I_U,\ n \in N(i),\ t \in T. \tag{22}$$

Constraints (8) define the production per sector, product, and period. Constraints (9) define the production cost, which has a fixed cost component for each column that starts production and a variable cost per ton of rock removed. Constraints (10) ensure that each block is removed at most once. Constraints (11) establish the logical link between variables e_{int} and z_{int} so that the extraction of a block cannot occur in a period prior to when it is initiated. Constraints (12) ensure that the minimum set of blocks is removed if a column enters production. Constraints (13) require the extraction of block n to finish before the extraction of block $n+1$ can start at each drawpoint. Constraints (14) restrict the order in which the drawpoints can enter production.[2] Constraints (15) impose the maximum duration of a drawpoint. Constraints (16) limit the extraction rate of each column. Constraints (17) define the height of the column in each period, and Constraints (18) guarantee that neighboring columns have similar heights so that the rock breaks smoothly. Constraints (19) and (20) impose bounds on the area incorporated and the rock extracted per period, respectively. Constraints (21) ensure that a sector is extracted within its time window, and Constraints (22) restrict the domain of the decision variables.

The formulation above captures the specific elements of underground operations, but the amount of detail comes at a cost. Indeed, the following proposition formally states the complexity of the underground mining problem (the proof is in the online companion):

PROPOSITION 1. *The long-term production planning problem for underground mines given by Equations* (1)–(22) *is strongly NP-hard.*

The previous result justifies the use of approximate solution methods. Fortunately, the nature of underground mining gives us a hand. Notice that the binary requirement on the variables z_{int} ensures that the blocks of a column are extracted one by one from bottom to top. However, if the blocks at the bottom have the highest copper grades—which by design is usually the case—then the binary requirement can be relaxed because the sole fact that this is a maximization problem will force the extraction to occur in the desired order. Hence, in that case the LP relaxation provides a near-integer solution. This is an important observation that is pivotal to the solution method described later. Therefore, we state it formally in the following lemma, proved in the online companion.

LEMMA 2. *If the blocks $n \in N(i)$ in column i are identical except for the grade of each commercial product $k \in$ COM, which is decreasing from the bottom to the top of a column, then the optimal solution to the LP relaxation satisfies $z_{int} \in \{0, 1\}$ for all blocks except the first and last one extracted in period t.*

Clearly, the lemma also holds if the commercial products have decreasing (nonincreasing) grades and the noncommercial products—in particular, the contaminants—have increasing (nondecreasing) grades. In practice, because the mine is designed primarily to extract copper, the columns exhibit vertically decreasing copper grades, while for the other secondary products the grades are relatively constant.

4.3. Open-Pit Extraction

In contrast with underground mines, the mining method for open pit excavations allows for a more aggregated model because there is no need to get into the details at the block level. Here the index $i \in I(s)$ represents a drawpoint or bench in expansion $s \in S_O$, which are numbered 1 to $|I(s)|$ from top to bottom. The decision variables are $z_{it} \in \{0, 1\}$, which equals one if the extraction of bench i is initiated in period t; and $e_{it} \geqslant 0$, which is a continuous variable that represents the tonnage extracted per period. Let $I_O = \bigcup_{s\in S_O} I(s)$ be the set of drawpoints/benches across all open-pit mines. Let $S(m)$ be the set of all the expansions in the open-pit mine $m \in M_O$. Let SEC_O denote the set of all pairs (i, j), such that the extraction at bench i must be initiated before it takes place at bench j. The pair $(i, j) \in \text{SEC}_O$ could be benches from different expansions due to geomechanical requirements. All the other parameters are the same as in the underground case. The following constraints complete the production plan optimization model for open-pit mines.

$$x_{skt} = \sum_{i\in I(s)} \text{TON}_{ik}\, e_{it} \quad \forall\, s \in S_O,\, k \in K,\, t \in T, \tag{23}$$

$$\text{ProdCost}_{st} = \sum_{i\in I(s)} a_i z_{it} + b_{st} \sum_{k\in K} x_{skt} \quad \forall\, s \in S_O,\, t \in T, \tag{24}$$

$$\sum_{t\in T} z_{it} \leqslant 1 \quad \forall\, i \in I_O, \tag{25}$$

$$\sum_{g=1}^{t} e_{ig} \leqslant \sum_{g=1}^{t} z_{ig} \quad \forall\, i \in I_O,\, t \in T, \tag{26}$$

$$\sum_{t=1}^{T} e_{it} \geqslant \sum_{t=1}^{T} z_{it} \quad \forall\, i \in I_S, \tag{27}$$

$$\sum_{g=1}^{t} Z_{i+1g} \leqslant \sum_{g=1}^{t} e_{ig} \quad \forall i \in I_O,\, t \in T, \tag{28}$$

$$\sum_{i\in I(s)} \gamma_i e_{it} \leqslant \text{DP}_t \quad \forall\, s \in S_O,\, t \in T, \tag{29}$$

$$\sum_{i\in I(s)} z_{it} = 0 \quad \forall\, s \in S_O,\, t \in T \setminus T(s), \tag{30}$$

$$\sum_{g=1}^{t} z_{jg} \leqslant \sum_{g=1}^{t} e_{ig} \quad \forall\, (i, j) \in \text{SEC}_O,\, t \in T, \tag{31}$$

$$\text{MIN\_EXT}_{mt} \leqslant \sum_{s\in S(m)} \sum_{k\in K} x_{skt} \leqslant \text{MAX\_EXT}_{mt} \quad \forall\, m \in M_O,\, t \in T, \tag{32}$$

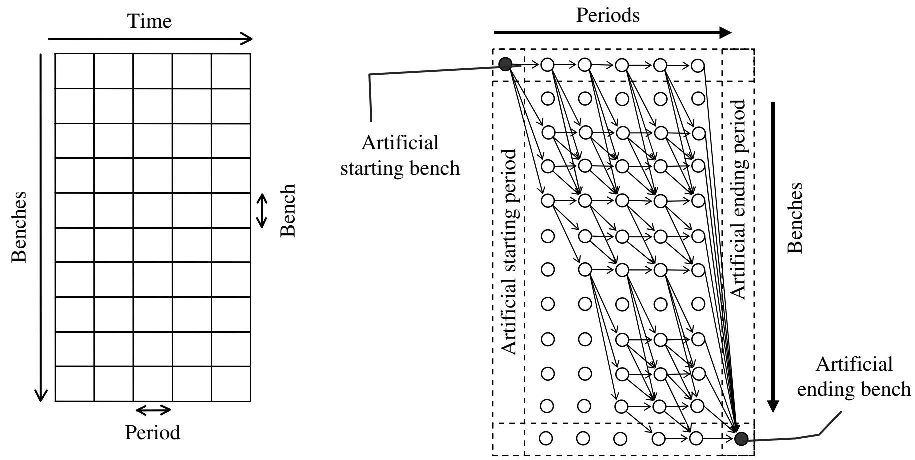
$$z_{it} \in \{0, 1\}, \quad e_{it} \geqslant 0 \quad \forall\, i \in I_O,\ t \in T. \tag{33}$$

Constraints (23) and (24) define the production and costs variables, respectively, similar to Constraints (8) and (9) in the underground case. Constraints (25) ensure that each bench is removed at most once. Constraints (26) establish the logical link between variables e_{it} and z_{it} so that the extraction of a bench cannot occur in a period prior to when it is initiated. Constraints (27) ensure that bench is not partially extracted. Constraints (28) require the extraction to take place top down within an expansion. Constraints (29) limit the extraction rate per expansion, and Constraints (30) enforce the extraction time window. Constraints (31) impose the bench extraction sequence, and Constraints (32) limit the total extraction per period. Finally, Constraints (33) restrict the domain of the decision variables.

The following proposition is equivalent to Proposition 1 for open-pit mines.

PROPOSITION 3. *The long-term production planning problem for open-pit mines given by Equations* (1)–(7) *and Equations* (23)–(33) *is strongly NP-hard.*

Figure 5. Extraction graph of the production phase for open-pit mines.



For open pit, there is no equivalent to Lemma 2. In fact, several benches of overburden must be removed before the ore deposit is reached, and even then, the best ore grades are usually deeper in the ground. In terms of our model, this means that the solution of the LP relaxation is highly fractional because it is optimal to reach the lower benches as soon as possible. Indeed, most of the z_{it} variables have small fractional values, very close to zero. This makes it hard to construct feasible integer solutions based on the LP relaxation. In particular, branch-and-bound or rounding heuristics become very inefficient. To overcome this obstacle, we strengthen the formulation for open-pit mines by adding a network structure to the production phase. This is explained next.

For each expansion, we generate a directed graph where the nodes represent a bench-period grid, as shown in the left-hand side of Figure 5. At a given node, there is an outgoing arc to all the benches that can be reached in the next period, according to the conditions imposed by Constraints (25)–(30). An artificial starting bench and period is added to the upper level of the grid, and similarly an artificial ending bench and period is added to the lower part (see the right-hand side of Figure 5). Hence, timing the extraction of the expansion is equivalent to moving one unit of flow from the upper left corner to the lower right of the grid.

We introduce some additional notation to formulate the extraction graph for an expansion $s \in S_O$. Let $t = 0$ and $t = T + 1$ denote the artificial starting and ending periods respectively. Similarly, let $\mathrm{Or}(s)$ and $\mathrm{De}(s)$ denote the artificial starting and ending benches. Let $\mathrm{AN}(i, t)$ and $\mathrm{SU}(i, t)$ denote the antecessors and successors of bench i in period t in the extraction graph, so $\mathrm{AN}(i, t)$ is some interval terminating in i, and $\mathrm{SU}(i, t)$ is an interval starting in i. Let w_{ijt} be a binary decision variable that equals one if the unit of flow (i.e., the extraction) goes from bench i to bench j in period t. In other words, if $w_{ijt} = 1$, then it implies that the extraction of benches i and j must begin in periods $t-1$ and t, respectively; and given the precedence, Constraints (28), all the benches in between must be fully extracted in period t. Finally, we add an arc from node $(\mathrm{Or}(s), T)$ to $(\mathrm{De}(s), T+1)$ so the model can still decide not to extract an expansion at all (see Figure 5). The formulation of the extraction graph is the following:

$$\sum_{j \in \mathrm{SU}(\mathrm{Or}(s),\,0)} w_{\mathrm{Or}(s)j1} = 1 \quad \forall\, s \in S_O, \tag{34}$$

$$\sum_{j \in \mathrm{AN}(\mathrm{De}(s),\,T+1)} w_{j\,\mathrm{De}(s)T+1} = 1 \quad \forall\, s \in S_O, \tag{35}$$

$$\sum_{j \in \mathrm{AN}(i,\,t)} w_{jit} = \sum_{j \in \mathrm{SU}(i,\,t)} w_{ij(t+1)} \quad \forall\, i \in I_O,\ t \in T, \tag{36}$$

$$z_{it} = \sum_{h=0}^{i-1} \sum_{\substack{j \in \mathrm{SU}(h,\,t-1)\\ j \geqslant i}} w_{hjt} \quad \forall\, i \in I_O,\ t \in T, \tag{37}$$

$$w_{ijt} \in \{0, 1\} \quad \forall\, i, j \in I_O,\ t \in T\,. \tag{38}$$

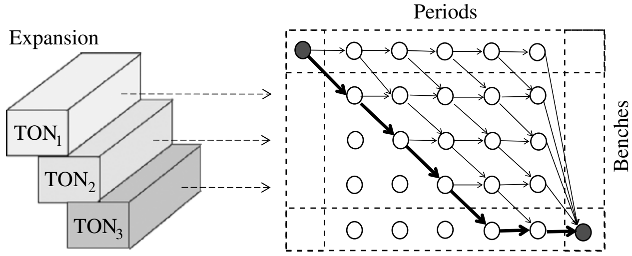
Constraints (34) and (35) impose that a unit of flow starting from the upper left node of the extraction graph in period $t = 0$ must finish at the lower right in period $t = T + 1$. Constraints (36) establish the flow conservation at each node of the graph. Constraints (37) link the flow variables w_{ijt} with the z_{it} variables, and Constraints (38) impose the binary condition.

This strengthening procedure resembles the lift and project technique described in Balas (2001). On one hand, the introduction of the w_{ijt} variables and Equations (34)–(36) represents a higher dimensional representation of the extraction phase, which can be viewed as an extended formulation—or lifting—of the original formulation given by Equations (25)–(30). On the other hand, Equation (37) projects the w_{ijt} variables to the space of the z_{it} variables. The advantage of doing this is summarized in the following observations: (i) the extended formulation (34)–(38) is valid because the sets $\mathrm{AN}(i, t)$ and $\mathrm{SU}(i, t)$ are constructed from Equations (25)–(30), so only feasible extraction sequences are allowed; (ii) Equations (34)–(36) are an integral polyhedron because it is a model of a flow of one unit through a network; and (iii) its projection is an integral polyhedron because the extreme points of the projection all arise from extreme points of the extended polyhedron (see Balas 2001, Proposition 1). Therefore, in the absence of constraints linking different expansions, the extended formulation is stronger than the original formulation because it yields an integral polyhedron in the z_{it} variables, which was not the case in the original formulation. The following example illustrates the overall idea. Consider a single expansion with three benches that have the same characteristics, except that the total copper content is increasing: $\text{TON}_1 < \text{TON}_2 < \text{TON}_3$ (see Figure 6). For simplicity, assume that there are no other products, and ignore any production costs. Moreover, assume that the extraction rate allows at most one bench to be extracted in each period. The LP relaxation of the original formulation (25)–(30) has the fractional solution $z_{it} = 1/3$, $\forall\, i, t = 1, 2, 3$, while it can be easily verified that the extended formulation imposes $z_{21} = z_{31} = z_{32} = 0$, so its LP relaxation yields $z_{11} = z_{22} = z_{33} = 1$. If we now consider n benches with increasing grades, then the solution of the original formulation is $z_{it} = 1/n$, $\forall\, i, t = 1, \ldots, n$, which shows that the fractional values can be arbitrarily small. Note that when different expansions are linked—e.g., by the downstream processes—the integrality of the extended formulation is lost. However, it still provides a good starting point for the rounding heuristic described in the next section.

5. Solution Approach

Our approach to solve the unified model for open-pit and underground mines consists of solving the LP relaxation and then using a specific rounding heuristic for each type of mine to find a feasible integer solution. We chose this approach because it required less technical intervention on behalf of the users. Other methods, such as Lagrangian relaxation, were more likely to require specific knowledge and expertise on site for tuning and calibrating the parameters. We believe our choice facilitated the adoption of the model within Codelco. Moreover, due to the strong model

Figure 6. A three-bench expansion (left) and its extraction graph (right) when one bench can be extracted per period.



Note. The bold arrows represent the solution in the extended formulation.

formulation, the rounding heuristics have worked very well, and full-size instances can be solved within a few hours—which is remarkable given that it took several weeks with the legacy planning approach.

Here we describe the solution approach for open-pit mines (see the online companion for underground mining). Note that in the relaxed problem we can assume with no loss of optimality that Equation (11) holds as an equality so the z_{int} and e_{int} variables take the same values. Therefore, we omit the latter. Similarly, we can assume that Equation (26) holds as an equality, so we omit the e_{it} variables because they are equal to z_{it}, $\forall\, i, t$. Note that the solution approach applies to any mineral, not only copper.

The heuristic to solve the open pit problem is performed in an iterative fashion. The premise of the heuristic, as in most rounding procedures, is that the LP solutions are good starting points to build near-optimal integer solutions. This was definitely the case with the extended formulation described in §4.3. Therefore, the starting point is the solution of the LP relaxation. Then a subset of extraction variables is fixed, and the LP relaxation is solved again. This procedure continues until there are no fractional variables remaining. The overall structure of the heuristic is the following.

Step 0. *Initialization*. Solve the LP relaxation of the open-pit problem, and let $z_{it}^{\text{LP}(q)}$ denote the solution for the extraction variables where q represents a counter. Initialize $q = 1$ and initialize $\Lambda(m)$ to contain all the expansions in the open-pit mine $m \in M_O$.

Step 1. *Extraction simulation*. For each expansion $s \in \bigcup_{m \in M_O} \Lambda(m)$, apply Algorithm 1—described in the online companion—to simulate its extraction. Start with the inner expansions of the open pit and continue outward. Let $z_{it}^{\text{SIM}(q)}$ denote the output of the simulation algorithm and let $Buf(s)$ contain the last bench that is reached in each period of the simulated extraction of expansion s.

Step 2. *Expansion selection*. For each mine $m \in M_O$, select the expansion that minimizes the squared difference between the extraction simulation (from the previous step) and the solution of the qth LP relaxation. In formal terms, let

$$s_q^*(m) = \arg\min_{s \in \Lambda(m)} \sum_{i \in I(s), t \in T} (z_{it}^{\text{SIM}(q)} - z_{it}^{\text{LP}(q)})^2.$$

Step 3. *Tighten LP formulation*. Add the following two constraints to the open-pit problem:

$$z_{it} = z_{it}^{\text{SIM}(q)} \quad \forall\, i \in I(s_q^*(m)),\ m \in M_O,\ t \in T,\ (i, t) \notin Buf(s_q^*(m)), \tag{39}$$

$$z_{it} + z_{it+1} = 1 \quad \forall\, (i, t) \in Buf(s_q^*(m)),\ m \in M_O. \tag{40}$$

Solve the $q + 1$th LP relaxation and let $z_{it}^{\text{LP}(q+1)}$ denote the solution for the extraction variables. For each $m \in M_O$, redefine $\Lambda(m) = \Lambda(m) \backslash \{s_q^*(m)\}$. If $\bigcup_{m \in M_O} \Lambda(m)$ is empty, then STOP. Otherwise, increment the counter $q = q + 1$ and go to Step 1.

After initializing the variables (Step 0), the outcome of the qth LP relaxation is used in Step 1 to build an integer solution for each expansion, which is denoted $z_{it}^{\mathrm{SIM}(q)}$. The superscript SIM stands for "simulation" of the extraction phase. In Step 2, the expansions to be fixed are selected based on how much the extraction simulation $z_{it}^{\mathrm{SIM}(q)}$ differs from the LP solution $z_{it}^{\mathrm{LP}(q)}$. Finally, in Step 3, additional constraints are added to the open-pit problem to fix the extraction variables for the expansions selected in Step 2. Constraint (39) sets the variable z_{it} equal to 0-1 according to $Z_{it}^{\mathrm{SIM}(q)}$. For a bench that is extracted in two consecutive periods, Constraint (40) is imposed, which allows the LP relaxation to decide what fraction of the bench to extract in each period. These benches act as buffers between periods and give the LP relaxation more flexibility to satisfy all the constraints. Note that when there are few expansions left to be fixed, usually it is possible to solve the MIP formulation to optimality in a reasonable time.

6. Implementation and Impact at Codelco

All the data needed by the model were extracted directly from Codelco's databases. The investments in equipment, machinery, and facilities that had lifespans shorter than the planning horizon were prorated and included as variable costs, but in what follows, they are still referred to as *investments*. Some standard preprocessing routines similar to Kuchta et al. (2003) were performed in order to reduce the number of binary variables. Representative instance sizes after preprocessing are shown in Table 1.

The model was implemented using the modeling language GAMS and was solved using the parallel barrier interior point algorithm of CPLEX on a computer running Microsoft Windows. Given the large timeframe spanned by the project, it has benefited from several upgrades in commercial software and hardware performance. In §6.1 we describe a large-scale case study performed at Codelco North Division, and in §6.2 we summarize the current uses of the model and its impact on Codelco's operations. A case study at El Teniente is reported in the online companion.

6.1. Case Study at the North Division

Codelco North Division is located 1,650 kilometers north of Santiago and at approximately 3,000 meters above sea level. It comprehends several open-pit mines at different stages of their life-cycles. Currently, the two most important mines are Chuquicamata (better known as "Chuqui") and Radomiro Tomic (RT). Chuquicamata has been in operation for nearly a century, and its pit today covers more than 1,300 hectares and is almost 1 kilometer deep. In contrast, the exploitation of RT started only in 1995, but it ramped up production very quickly. While Chuquicamata is mainly composed of sulfides, RT's reserves are mostly oxides.

The case study analyzed at Codelco North Division considered both mines, Chuquicamata and RT, and we used the geological, technical, and economic information available in 2004. We considered a 10-year planning horizon divided in yearly periods, with a fixed price for copper of 85 ¢/lb. As before, sunk costs were excluded from the evaluation and investments were mostly the acquisition of major extraction machinery including trucks, drills, and loaders.

Prior to this project, extraction planning for Chuquicamata and RT had been done separately. To isolate the impact of the model from the mere benefits due to integrating both mines, we compared our proposed solution against three benchmarks:

(i) Legacy-based independent plans ($l - ind$): This is the optimized baseline where we impose the flow levels determined by the legacy planning approach, and then we solve the model to find the best solution under such conditions. Here, different mines are planned independently, so the downstream processes are not shared.

(ii) Model-based independent plans ($m - ind$): In this instance the two mines are not allowed to interact (as in $l - ind$), but we no longer impose the flow levels from the legacy approach.

(iii) Legacy-based integrated plan ($l - int$): In this instance the flow levels are determined by the legacy planning process (as in $l - ind$), but the two mines are communicated, so it provides an estimation of the value of integrated planning under the legacy approach.

We denoted our proposed solution $m - int$, which stands for model-based integrated plan. In Table 2 we compare $m - int$ against the three benchmarks. The total economic benefit from using the model-based methodology corresponds to a gain of 8.2% with respect to the optimized baseline $l - ind$. The total impact has the following breakdown: 3.2 percentage points (pp) are due to integrated planning ($l - int$ vs. $l - ind$), 4.7 pp are due to the model-based approach ($m - ind$ vs. $l - ind$), 0.2 pp can be attributed to synergies between integrated planning and the model-based approach—to see this, note that the increase from $m - ind$ to

Table 1. Example of instance sizes after pre-processing.

Model	No. of constraints	No. of variables	0-1 variables
Underground mine	446,521	535,639	196,386
Open-pit mine	245,391	898,742	160,386

Table 2. Economic evaluation of the model-based plan ($m - int$) vs. benchmarks at Codelco North Division.

	$m - int$ (%)	$l - ind$ (%)	$m - ind$ (%)	$l - int$ (%)
Revenues	1.4	0.0	1.7	1.1
Production costs	−1.6	0.0	0.8	0.0
Investments	−27.3	0.0	−31.8	0.0
NPV	8.2	0.0	4.7	3.2

Note. Figures are shown as percentage changes with respect to the baseline ($l - ind$).

$m - int$ is 3.4% versus only 3.2% from $l - ind$ to $l - int$—and finally, 0.1 pp is the multiplicative effect. In other words, the specific impact of the model-based approach is in the order of 5%, which is consistent with the result obtained in the case study done at El Teniente (see the online companion). The fact that there are synergies is another interesting observation, meaning that integrated planning and the model-based approach reinforce each other.

In Table 3 we compare our solution $m - int$ with the three benchmark plans. We observe that the total mined material is almost identical in all instances, so the total costs are comparable (see Table 2). This is intuitive because the ore with higher grades is located at the bottom of the pit, and to get there all the material above it needs to be extracted. The total extraction might be different if the model decides not to extract a whole expansion, but that did not happen in this exercise. The economic improvement of the model-based over the legacy approach comes mainly from two sources. First, the model-based approach requires less investment in machinery because it can achieve better utilization by smoothing production whereas the legacy approach needs to make additional investments to accommodate higher production peaks.[3] Second, and more importantly, the model-based approach produces more copper than the legacy approach. In particular, with respect to the baseline $l - ind$, the solution $m - int$ sends 5.5% more ore to the downstream plants reducing the average grade by 2.6%. Unlike traditional methodologies that use a cutoff grade per mine and period, the model-based approach simultaneously considers multiple elements to decide when and where to process each bench of the pit, resulting in a better usage of the available capacity. For instance, we verified that in our solutions when the benches are close to the plant, the model decides to send ore of relatively lower grade than that sent from benches that are further away.

An interesting observation comes from comparing the independent and integrated model-based solutions ($m - ind$ and $m - int$, respectively). The former has a better recovery rate and produces more copper but has a lower NPV than the latter. The reason is that the integrated solution redirects mineral from Chuquicamata to a plant next to RT that had some slack capacity (<10%) in the nonintegrated solution $m - ind$. However, this move is counterintuitive because the mineral from Chuquicamata is mostly sulfides, and sulfides have a lower recovery rate at the RT plant that is mostly for oxides. The up side is that the processing cost at the RT plant is lower, and although there is an additional transportation cost, it does not offset the savings because the material sent from Chuquicamata comes from the bench that is the closest to RT. Hence, the nonintegrated solution ($m - ind$) recovers more copper but at a higher cost, which can be seen in Table 2. This kind of subtlety would be harder to identify using the legacy approach.

Table 3. Mining plan summary of the model-based vs. benchmarks at North Division.

	$m - int$	$l - ind$	$m - ind$	$l - int$
Total extraction (Kton)	2,882,457	2,889,825	2,874,050	2,889,825
Ore to process (Kton)	1,468,104	1,391,384	1,456,711	1,418,244
Average ore grade (%)	0.660	0.678	0.663	0.674
Average recovery rate (%)	80.4	81.0	82.1	80.5
Copper produced (Kton)	7,792	7,643	7,926	7,693

Note. Material flows are shown in kiloton, grades are shown in percentage.

6.2. Current Uses of the Model

This project began in 1999, when the model was first implemented and validated in the case study described in §0. To date, the system, known as MUCH ("model of the University of Chile") has been implemented in all of Codelco's major divisions. In what follows we present some examples of how the model is being used to support current planning decisions.

(1) *Routinely planning support*: Several MUCH runs are carried out every week. The planners need to continuously evaluate production decisions as the input data change over time, such as prices, costs, grade of mineral, and work interruptions caused by machine failures or strikes. Based on the multiple runs made with MUCH, the planners work out the details of a definite plan at the different decision levels. MUCH also provides insight for initiatives Codelco engineers want to test, such as determining which expansions become attractive if future copper prices are higher or the impact of adding a \$4 M truck to the current fleet of about 80 trucks. In addition, Codelco carries out two yearly formal plans for the following year. One is an exploratory, tentative plan done at midyear, and a final proposal is developed at the end of the year. This is important because it suggests major investments, like the purchase of a crusher, with a cost of about \$100 M. In this case, MUCH is used to evaluate the impact of different capacity levels, e.g., 50, 100, or 150 thousand tons/day. These investments translate into alterations of the nodes and arcs in the network, and the model allows visualizing how the overall production chain would perform.

(2) *Simultaneous planning of underground and open pit mines*: At Andina, located 80 km north of Santiago, the model has been used since 2006 to plan a mixed operation mine with open-pit and underground extraction. To the best of our knowledge it is the first system to optimize simultaneously both types of mine operations that compete for plant processing capacity. Compared to the 2008 legacy plan, using the model separately for each mine led to an improvement of 5% in net income, equivalent to \$180 M. Integrating both mines led to an additional increase of 3%, which is consistent with the results obtained at Codelco North Division (see §6.1). Also, several exercises have been

carried out to evaluate different scenarios. For instance, the model was used to analyze the effect of delaying the extraction of an expansion from 2009 to 2011. The results showed that the delay would have led to a loss of $114 M, so it was avoided. In another scenario, a 10-year delay for a sector in the underground mine was studied and the result was a loss of $654 M. However, this was assuming a conventional fixed extraction rate of 165 K ton/day for the open-pit mine and a very restrictive time window for a particular expansion. In a follow-up study, these assumptions were removed, and the model showed that it was optimal to extract material from the open-pit mine at a rate of 233 K ton/day, which contributed with higher grade ore to the plant, and delaying the underground sector led to a profit of $184 M.

(3) *Analysis of strategic decisions*: In addition to the regular use of MUCH for planning purposes, the model has been used to support basic strategic decisions. For instance, a main sulfides concentration plant is located at Chuquicamata in the North Division. The mine RT is located 7 km away. There is a lixiviation plant located at RT, but with a lower recovery factor for sulfides than the plant at Chuquicamata. The question was: should the sulfides material be sent from RT to the Chuquicamata plant, at a higher transportation cost, instead of being processed at the RT plant? The decision was not trivial because it had to consider the costs of processing, transportation, and copper recovery under multiple scenarios for copper prices, fuel costs, and mineral grades. After many MUCH runs it could be established that using the plant at Chuquicamata was the better option, and this decision was implemented in the annual plans.

Another important issue dealt with the mine Chuquicamata. As the open-pit mine started one hundred years ago, it became too deep, and Codelco developed a project to convert the mine to underground operation. Codelco has used the MUCH optimization system to evaluate the timing of this huge transformation. Certainly, a structural project like this involved many other studies and elements that impact such important and strategic decisions. The conversion is now set to take place in 2018, and it involves an investment of $2,000 M. It represents one of the major strategic projects that Codelco has put forward to strengthen its competitive position during the next decades (Economía y Negocios 2010).[4]

7. Conclusions

In this paper we have presented an approach to optimize long-term production plans for copper mines considering multiple deposits, multiple products and multiple processing plants. The model is a multicommodity capacitated network flow, where nodes correspond to extraction or processing points. The flow components are copper and molybdenum in different stages of production, as well as the main pollutant, arsenic. Arc or node capacities reflect limits on extraction, transportation, and processing, while investments are represented as costs. Constraints include bounds on extraction rates, restrictions induced by safety considerations related to rock spillage and instability of walls, as well as limits on pollutants. One major decision is how to allocate mineral from the different mines to the downstream processes.

An important challenge here was the algorithmic solution. Given that there are hundreds of thousands of integer variables, an approximation heuristic was used. In the case of underground mines, an iterative rounding approach worked well, aided by the vertically decreasing rate of copper at each drawpoint. For the open-pit mines, an extended network flow formulation proved to be quite strong, leading to less fractional values.

As shown by the different mines of Codelco where the system was implemented, there was an important impact on the decision process due to the introduction of the model-based approach. In particular, at Codelco North Division the use of the model led to an increase of 8% in NPV, which can be decomposed in 3% due to mine integration and 5% due to the optimized plan. Similar results were obtained at El Teniente and Andina. For several years now, the system has been used on a regular basis for routine long-term planning. Through the analysis of scenarios, the model has also been used to support strategic decisions such as the conversion of Chuquicamata to an underground mine.

Overall, the system has become an essential planning tool to support Codelco's mission of increasing its long-term value and thus its contribution to the country. Given the success of this project, further collaboration between the University of Chile and Codelco has been established. Current work focuses on short-term planning models and how to incorporate uncertainty, mostly due to price volatility in the copper market.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Endnotes

1. LHD stands for loading, hauling, and dumping.
2. Note that the left-hand side of Constraints (11), (13), and (14) is formulated as a cumulative sum over time, which helps tighten the LP relaxation.
3. The reduction in investments can be seen in Table 2. Percentage-wise, these are important reductions. However, in absolute terms they are small compared to total revenue and costs.
4. We recently became aware of the paper by Newman et al. (2010a), who specifically study the conversion from open pit to underground for a South African mine, although this work has not been implemented.

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