Ann Oper Res
DOI 10.1007/s10479-009-0676-3

Design of insurance contracts using stochastic programming in forestry planning

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Abstract This work addresses a tactical planning problem faced by a forestry firm, deciding which timber units to harvest and what roads to build to obtain the greatest possible benefits. We include uncertainty in prices by means of utility theory. This enables solutions to be found that the firm finds preferable to those obtained when risk aversion is ignored and makes it possible to design insurance contracts that benefit the firm while also being attractive to an insurer. Two types of contract are designed; one dependent on the firm's operating result and the other independent of it. Metrics are then developed to quantify the benefits conferred by a contract, demonstrating that the latter contract type dominates the former. These results are then illustrated by applying them to a simplified planning problem of a forest owned by the Chilean forestry operator Millalemu.

Keywords Utility theory · Insurance · Stochastic programming · Forestry planning

1 Introduction

Since the 1960s, many studies have used deterministic models in forestry planning (Andalaft et al. 2003) to address the major sources of uncertainty that exist in relevant factors such as prices, timber sales, the real productivity of harvest units, future plagues and fires and real extraction costs (Weintraub and Romero 2006). Yet despite its significance and the theoretical advances in dealing with uncertainty, few applications have been made to the

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Published online: 29 December 2009

forestry industry, perhaps due to the lack of reliable data and the difficulties of implementation (Martell et al. 1998).

Forest management can be viewed as a controlled Markov process in which prices and growth vary as discrete events due to economies of scale (Lohmander 2007). This paper presents various mathematical tools employing a stochastic approach based on adaptive optimization.

One of the few theoretical treatments of uncertainty in the forestry industry uses stochastic programming under price uncertainty, defined through future price scenarios with defined probabilities (Quinteros et al. 2009). Solutions are sought which maximize the expected value of net revenues subject to satisfying constraints under all scenarios. A coordinated branching algorithm which decomposes the problem is developed to solve the harvest and road construction planning problem.

The present study also addresses the problem of harvest and road planning in a forestry operation, but differs from previous works in that it incorporates the attitude of the planner toward risk.

The major contribution of this paper is the introduction in the field of forestry planning of the issue of the firm's risk aversion and how it is willing to pay to cover itself for the risk of scenarios with lower prices. While Quinteros et al. (2009) dealt with forest harvest scheduling under uncertain market scenarios, supporting decisions by a forest manager, in our case, the decisions we support belong to the financial area. The approach proposed here supports the general or financial manager, who is concerned with protecting the firm from risks due to losses or low profits. As far as we know, this is the first work in forestry to introduce these concepts.

The remainder of this paper is organized as follows. In Sect. 2, we posit the problem the firm faces, incorporating risk aversion into the formulation. Section 3 introduces two types of contracts, calculating and comparing their economic gains. Section 4 suggests a method to find efficient contracts. Section 5 applies the results of these methods to a real-world case, integrating operational and financial decisions. Finally, Sect. 6 concludes the study, commenting on the results and possibilities for future extensions of this work.

2 Description of the forestry problem

In forestry planning, each forest unit is associated with a given timber availability and harvest cost. The timber harvested in each unit must be produced at a single origin. To transport the timber from the origins to the demand destinations, existing roads may be used or new ones built. All timber arriving at a destination is sold at a price that is precisely known only for the first period. The harvest and road construction planning problem is about finding the best plan for harvesting and road construction.

Clearly the plan must depend on future unknown timber prices. In this paper we use scenario forecasting of prices, which follow various sequences of values. These scenarios are the only uncertain element in our model. Figure 1b, Sect. 5.1, gives an example with six possible scenarios of timber prices. This example, which is a simplification of the Quinteros et al. (2009) model, enables us to analyze and highlight the impact of the risks.

The common attribute for comparing alternatives is the expected total net revenues. However it does not take into account the planner's risk attitude (which is usually one of risk aversion). Ignoring risk aversion eliminates possible actions by the planner, such as insurance contracts, which protect the forest firm against unfavorable scenarios.

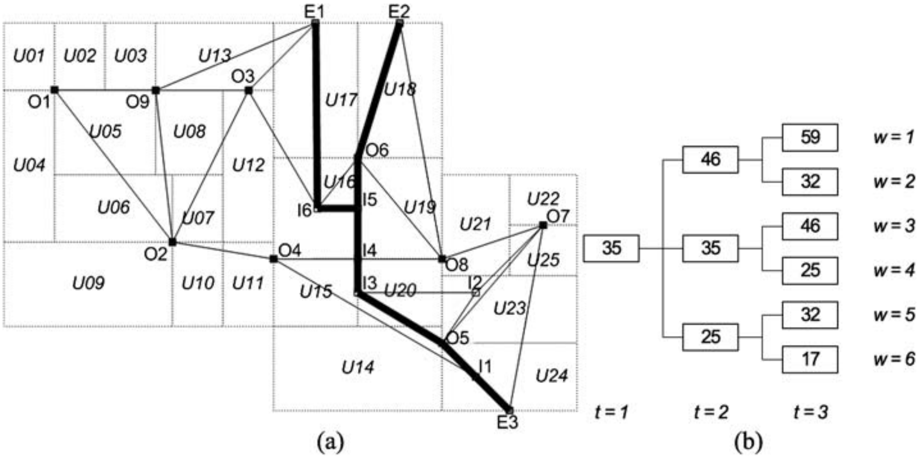

Fig. 1 (**a**) Diagram of the Los Copihues forest. The blocks represent harvesting units, the arcs represent roads (arcs in *bold lines* represent roads already built before the planning horizon starts). (**b**) The price-scenarios, in dollars per cubic meter of timber

2.1 The mathematical model

A forest firm, referred to as the firm, decides on its activities for a series of periods $\tau = \{1, \dots, n\}$. Here a compact model is described which highlights the main ingredients related to price uncertainty. A full linear programming model which captures the many activities of the firm relating to harvest and road planning is presented in Appendix.

For each period $t \in \tau$, the vector of net prices (prices minus costs) for the various activities is denoted by a_t. Over the time horizon τ the sequence of vectors of net prices is $(a_1, \dots, a_n)$. This sequence is a random variable whose sample points, called (forecast) scenarios, are the elements of a set Ω. If $w \in \Omega$ occurs, the scenario is $(a_1^w, \dots, a_n^w)$. The probability, p^w, of each scenario is known.

The alternatives for the firm are presented by the harvesting, transporting and road building decision variables x_t^w for period $t \in \tau$ and scenario $w \in \Omega$. Given decisions x_t^w and the occurrence of scenario w, the firm's immediate net revenue in t is the inner product $a_t^w \cdot x_t^w$. Its profit is the sum of the net revenues, $\sum_{t\in\tau} a_t^w \cdot x_t^w$. The set of feasible decisions is denoted X and includes *non-anticipativity* constraints (Rockafellar and Wets 1991). These constraints, in the case that two scenarios represent the same realization up to period t, obligate the decisions of both scenarios to be the same up to that period.

When the firm is risk neutral, the firm maximizes its expected profit solving:

$$(ER) \qquad \max_{x_t^w} \sum_{w\in\Omega} p^w \sum_{t\in\tau} a_t^w \cdot x_t^w$$

$$\left(x_t^w\right)_{t\in\tau}^{w\in\Omega} \in X.$$

Present values of net revenues are obtained by multiplying net prices by discount factors.

For an example, consider the timber prices in Fig. 1b, Sect. 5.1. The sets τ and Ω are respectively $\{1, 2, 3\}$ and $\{1, 2, \dots, 6\}$. The six possible forecast scenarios are[1]:

[1] It is assumed (w.l.g.) that the only aleatory component of vector a_t^w is the first one, $(a_t^w)_1$.

$(a^1)_1 = (35, 46, 59)$, $(a^2)_1 = (35, 46, 32)$, $(a^3)_1 = (35, 35, 46)$, $(a^4)_1 = (35, 35, 25)$, $(a^5)_1 = (35, 25, 32)$ and $(a^6)_1 = (35, 25, 17)$ each with $p^w = 1/6$.

For Fig. 1b, the non-anticipativity constraints are: $x_1^1 = x_1^2 = x_1^3 = x_1^4 = x_1^5 = x_1^6$, $x_2^1 = x_2^2$, $x_2^3 = x_2^4$ and $x_2^5 = x_2^6$.

2.2 Incorporating risk aversion

We assume that the planner's attitude toward risk is captured by a utility function over monetary values (von Neumann and Morgenstern 1994). This function will be denoted as $u(\cdot)$. Previous models (e.g. Quinteros et al. 2009) do not include any risk considerations, assuming a linear utility function as captured by the previous optimization problem.

The model solved by the firm is to maximize the expected utility of profit

$$(EU) \qquad \max_{x_t^w} \sum_{w\in\Omega} p^w u\left(\sum_{t\in\tau} a_t^w \cdot x_t^w\right)$$
$$\left(x_t^w\right)_{t\in\tau}^{w\in\Omega} \in X.$$

The optimal value of (EU) is denoted $\hat{u}$.

Since u is non-linear, solving (EU), as well as the models that follow, usually amounts to some search algorithm. Solving it, using linear programming, when u is a piecewise linear function, is applied later.

3 The insurance contract

To bring insurance contracts into the model we assume the existence of a second entity, an insurer. As insurers are large enough to be practically risk neutral with respect to the particular status of a single firm, this entity is, conventionally, risk neutral. The inclusion of an external insurer in order to define, evaluate and choose insurance contracts is the main contribution of this paper.

We assume that both parties have full information on the scenarios, their probabilities and other parameters of the model. We consider two types of contracts: a dependent contract and an independent contract. The difference, as will be detailed next, is the guaranteed coverage: the amount of money the insurer pays the firm. In both types the coverage depends on the time $t \in \tau$ and scenario $w \in \Omega$, and the firm pays a premium, D, to the insurer.[2]

3.1 The dependent insurance contract

The feature of this contract is the following guaranteed coverage: For each $t \in \tau$ and $w \in \Omega$ a value C_t^w is given. When the immediate net revenue, $a_t^w \cdot x_t^w$, of the firm is less than C_t^w the insurer pays the firm the difference, $C_t^w - a_t^w \cdot x_t^w$. The set of all guaranteed coverage, $C = (C_t^w)_{t\in\tau}^{w\in\Omega}$, is called the coverage set.

A dependent contract is denoted (C, D). Over the planning horizon τ the firm maximizes the expected utility of profit subject to the conditions of the contract (C, D) and the feasible set X. The model the firm solves is:

$$(IEU)_{C,D} \qquad \max_{x\in X} U(x, C, D)$$

[2] D is the total discounted sum of payments by the firm to the insurer.

where

$$U(x, C, D) = \sum_{w \in \Omega} p^w u\left(\sum_{t \in \tau}(a_t^w \cdot x_t^w + [C_t^w - a_t^w \cdot x_t^w]^+) - D\right)$$

is the expected utility of the firm.[3] The optimal value of $(IEU)_{C,D}$ is denoted $\hat{U}(C, D)$

3.2 Gains from the dependent contract

In monetary terms,[4] the firm's gain from entering into a dependent contract (C, D) is the increment in utility it receives by so doing. This is defined as

$$G^F(C, D) = u^{-1}(\hat{U}(C, D)) - u^{-1}(\hat{u}),$$

where $\hat{u}$ is the optimal solution of (EU).

We define expected co-payment as the expected monetary value the insurer pays the firm over the horizon. This value is:

$$V(x, C) = \sum_{w \in \Omega} p^w \sum_{t \in \tau} \left[C_t^w - a_t^w \cdot x_t^w\right]^+ .$$

Notice that the value of $V(x, C)$ depends on the actual values of the decision variables that the firm applies, which we denote by $\hat{x}$. So, given a dependent contract (C, D) the gain to the insurer is

$$G^I(C, D) = D - V(\hat{x}, C).$$

One does not expect a contract to be signed when the firm is risk neutral. Indeed, when $u(v) = v$

$$\begin{aligned}\hat{U}(C, D) &= \sum_{w \in \Omega} p^w \left(\sum_{t \in \tau} \left(a_t^w \cdot \hat{x}_t^w + \left[C_t^w - a_t^w \cdot \hat{x}_t^w\right]^+\right)\right) - D \\ &= \sum_{w \in \Omega} p^w \left(\sum_{t \in \tau} a_t^w \cdot \hat{x}_t^w\right) + V\left(\hat{x}, C\right) - D \le \hat{u} + V\left(\hat{x}, C\right) - D = \hat{u} - G^I(C, D).\end{aligned}$$

The inequality $\hat{U}(C, D) \le \hat{u} - G^I(C, D)$ shows that the insurer gains, $G^I(C, D) > 0$, if and only if $\hat{U}(C, D) < \hat{u}$, when the firm loses.

From a financial point of view, a dependent contract seems very attractive as it guarantees the firm a lower bound to its profits. However, by making the payments depend on the net revenues, the firm may manipulate its decisions (decreasing the net revenues to below the coverage in certain periods) in order to maximize its profit. This may affect the willingness of the insurer to take this contract upon itself, unless the premium is relatively high. The next type of contract avoids this situation.

[3] $[x]^+ \equiv \max(x, 0)$.

[4] The monetary equivalent of a utility value is the inverse utility function.

3.3 The independent insurance contract

In an independent contract the guaranteed coverage is independent of the firm's revenues. Given a scenario $w \in \Omega$ and a period $t \in \tau$, the insurer pays the firm R_t^w. Contrary to the dependent contract the coverage depends only on the realizations of the uncertain prices but not on the decisions taken by the firm. An independent contract is denoted (R, D).

The model the firm solves is:

$$(IEW)_{R,D} \qquad \max_{x \in X} W(x, R, D)$$

where

$$W(x, R, D) = \sum_{w \in \Omega} p^w u\left(\sum_{t \in \tau} (a_t^w \cdot x_t^w + R_t^w) - D\right).$$

The optimal value of $(IEW)_{R,D}$ is denoted $\hat{W}(R, D)$.

3.4 Gains from the independent contract

Analogously to the dependent contract type, the gain to the firm from entering into an independent contract (R, D) is

$$H^F(R, D) = u^{-1}\left(\hat{W}(R, D)\right) - u^{-1}(\hat{u})$$

where $\hat{u}$ is the optimal solution of (EU).

The gain to the insurer from entering into an independent contract (R, D) is:

$$H^I(R, D) = D - \sum_{w \in \Omega} p^w \sum_{t \in \tau} R_t^w.$$

As with the dependent type, independent contracts are also of no interest if the firm is risk neutral.

$$\begin{aligned}
\hat{W}(R, D) &= \sum_{w \in \Omega} p^w \sum_{t \in \tau} (a_t^w \cdot \hat{x}_t^w + R_t^w) - D \\
&= \sum_{w \in \Omega} p^w \sum_{t \in \tau} a_t^w \cdot \hat{x}_t^w - H^I(R, D) \le \hat{u} - H^I(R, D).
\end{aligned}$$

The inequality $\hat{W}(R, D) \le \hat{u} - H^I(R, D)$ shows that the insurer gains, $H^I(R, D) > 0$, if and only if $\hat{W}(R, D) \le \hat{u}$, and the firm loses.

3.5 Comparison of dependent and independent contracts

An important advantage of the independent contract over the dependent one is that for the latter the insurer must check the net revenue made by the firm in each period, whereas for the independent contract this value is irrelevant. More important, the following theorem states that for any dependent contract we can find an independent one that is better for the firm while the insurer does not lose out. This advantage causes the dependent contracts to be less interesting to study as the independent contracts dominate them.

Theorem *Let* (C, D) *be a dependent contract and let* $\hat{x}$ *solve* $(IEU)_{C,D}$. *For each pair* $(w, t) \in \Omega \times \tau$ *define* $R_t^w = [C_t^w - a_t^w \cdot \hat{x}_t^w]^+$. *The independent contract* (R, D) *satisfies*

- $\hat{W}(R, D) \geq \hat{U}(C, D)$
- $H^I(R, D) = G^I(C, D)$

Proof

- $\hat{W}(R, D) \geq W(\hat{x}, R, D) = U(\hat{x}, C, D) = \hat{U}(C, D)$
- $H^I(R, D) = D - \sum_{w\in\Omega} p^w \sum_{t\in\tau} R_t^w = D - \sum_{w\in\Omega} p^w \sum_{t\in\tau} [C_t^w - a_t^w \cdot \hat{x}_t^w]^+ = D - V(\hat{x}, R) = G^I(C, D)$ □

This theorem shows that for any dependent contract there is an independent one that dominates it by fixing the coverage $R_t^w = [C_t^w - a_t^w \cdot \hat{x}_t^w]^+$.

4 Design of efficient contracts

We are interested in contracts that benefit both parties in the forestry planning, and that both are willing to sign. Furthermore, we wish to find efficient contracts, namely those where no party can increase its gain without decreasing the other party's gain. From the previous theorem we can concentrate on designing independent contracts only. This section provides an algorithm to build the best possible independent contracts for a given instance.

Mathematically speaking, given the utility function u, we wish to find a pair (R, D) yielding $\hat{W}(R, D)$ so that both $H^F(R, D)$ and $H^I(R, D)$ are positive. Given two contracts, (R_A, D_A) and (R_B, D_B), the first dominates the second, denoted $(R_A, D_A) \succ (R_B, D_B)$, when $H^I(R_A, D_A) \geq H^I(R_B, D_B)$ and $H^F(R_A, D_A) \geq H^F(R_B, D_B)$ with at least one strict inequality. A contract which is not dominated is called efficient and the efficient frontier represents, in terms of monetary gains, the set of efficient contracts. Usually, there are many (perhaps an infinite number of) efficient contracts, each of which has the potential to be signed, but the negotiation mechanism which determines which one (if any) is signed, is beyond the scope of this paper.

To get the efficient frontier we first find the highest possible gain to the insurer provided the firm does not decrease from its initial utility. The model which solves this problem is

$$(BCU)_1 \qquad \max_{x,R,D} H^I(R, D)$$
$$W(x, R, D) \geq \bar{u}, \quad 0 \leq R_t^w, x \in X.$$

The optimal value is denoted $\bar{H}$. The solution yields an efficient contract, though the firm gains nothing.

Now by varying the level of the insurer's gain we can find the efficient frontier. This is done by locating the best contracts for the firm among those that guarantee the insurer a fraction $\lambda (0 \leq \lambda \leq 1)$ of $\bar{H}$:

$$(BCU)_\lambda \qquad \max_{x,R,D} W(x, R, D)$$
$$H^I(R, D) \geq \lambda \bar{H}, \quad 0 \leq R_t^w.$$

Or equivalently,

$$(BCU)_\lambda \qquad \max_{x,R,D} \sum_{w\in\Omega} p^w u\left(\sum_{t\in\tau}\left(a_t^w \cdot x_t^w + R_t^w\right) - D\right)$$

$$D - \sum_{w\in\Omega} p^w \sum_{t\in\tau} R_t^w \geq \lambda \bar{H}, \quad 0 \leq R_t^w.$$

This approach, which serves as a basis for finding a subset of the efficient frontier, is a common approach in bi-criteria mathematical modeling by which one objective serves as a constraint and the other as an objective function (e.g., Yu 1985). To approximate the frontier, we make as fine a grid as is desired on the interval [0, 1], then solve $(BCU)_\lambda$ taking the value of λ equal to each of the grid points and calculate a collection of corresponding contracts (R_λ, D_λ) and their gains for both parties.

5 Application to a forestry planning problem

The concepts developed above were applied to the real-world case of a forest area known as Los Copihues, owned by the Chilean firm Millalemu. The results obtained for the contracts just discussed based on actual company data are presented below. The details of the mixed integer program model for solving the forestry planning problem under uncertainty but without taking attitude to risk into account is described in Appendix. The following case captures only a portion of the price sequences and the forest size values of a real forestry planning problem, as our intention is to highlight the impact of including risk considerations on optimal decisions and not to solve a large instance, as in Quinteros et al. (2009). Notice, however, that introducing a non-linear utility function may require extra calculation time, but using a piece-wise linear function still allows the use of linear programming optimization methods.

5.1 Description of the case

The forest is divided into 25 units and covers a total area of approximately 300 hectares with a total timber volume of 150,000 m^3 (Fig. 1a). Based on timber prices for 2004 we constructed six possible scenarios for a time horizon of three years (Fig. 1b).

The scenarios: In 2005 the price may rise 30%, remain the same or fall 30% with equal probability. In 2006, given the figure for 2005, the price may rise or fall 30% with equal probability. Monetary values in the rest of this section are given in thousands of dollars.

The activities of the firm concerning harvesting, timber selling, road construction and timber transporting were optimized by the mixed integer linear programming model described in Appendix. All the models were solved by the mixed integer linear programming models codifying in GAMS with the solver COIN.

First the risk-neutral case, model (*ER*), was solved, yielding an expected profit of 4.714 million $ (see Table 1).

Table 1 Summarized comparison between (*ER*) and (*EU*)

	(*ER*)	(*EU*)
Expected profit	4706	4661
Standard deviation	1192	721
Expected utility	80282	82391
Certainty equivalence	4070	4598

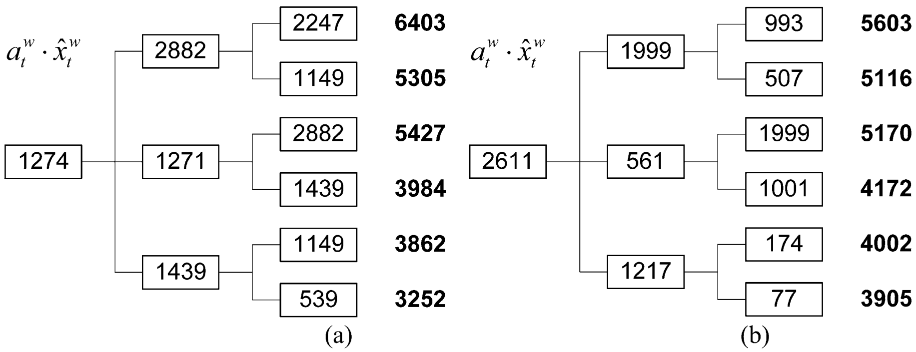

Fig. 2 Immediate optimal net revenues for each t and w and total profit (in 1000's of $) (**a**) for a solution to (*ER*); (**b**) for a solution to (*EU*)

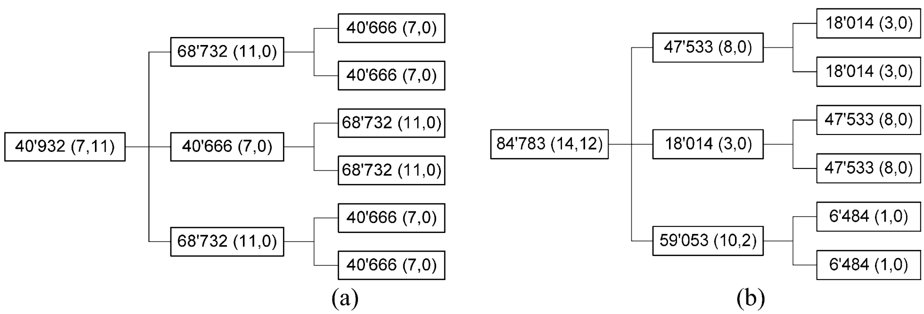

Fig. 3 Timber harvested (in cubic meters) and units harvested and roads built (in parentheses) for each t and w (**a**) for a solution to (*ER*); (**b**) for a solution to (*EU*)

5.2 Incorporation of risk aversion

To analyze optimal planning under risk aversion, we assumed the following concave (reflecting risk aversion) piece-wise linear utility function:

$$u(v) = \begin{cases} 20 \cdot v & \text{if } v \leq 4{,}000 \\ 4 \cdot (v - 4{,}000) + 80{,}000 & \text{if } 4{,}000 \leq v \leq 6{,}000 \\ 2 \cdot (v - 6{,}000) + 88{,}000 & \text{if } 6{,}000 \leq v \end{cases}$$

Figures 2 and 3, and Table 1 show the effects of accommodating this utility function on the model (*ER*) and (*EU*).

Each box in Fig. 2 indicates net revenues according to the time period t in $\{1, 2, 3\}$ and each of the six scenarios. The last column of numbers is the total profits according to the scenarios. Notice that in the (*ER*) solution they range from 3252 to 6403 whereas in the (*EU*) solution, which takes risks into account, they have a smaller range, only from 3905 to 5603. These differences are summarized in Table 1.

Notice in Fig. 3 the huge difference between the (*ER*) and the (*EU*) models regarding the first year activities: 40,932 (7 stands, 11 roads) versus 84,783 (14 stands, 12 roads). Actually, the decisions in this period are the only relevant decision variables as other activities are contingent on the realizations of the scenarios in the future. Clearly, they result from risk considerations, harvesting and selling more logs at the price of 35 in $t = 1$, avoiding the risk of much lower prices in the future. Another difference in (*EU*) is that two additional roads are built on the second year in case the price is low, which allows harvesting more stands at the price of 25 avoiding the risk of a price even lower in the third year. Model (*ER*) instead, does not build more roads during the time horizon. Both solutions harvest all stands.

Table 1 presents the following results:

- Expected profit: $EP \equiv \sum_{w\in\Omega} p^w \sum_{t\in\tau} a_t^w \cdot \hat{x}_t^w$, where $\hat{x}$ indicates the related optimal values. Obviously, it is greater for the risk-neutral model as expected profit is maximized.
- Standard deviation: $ST \equiv \sqrt{\sum_{w\in\Omega} p^w (\sum_{t\in\tau} a_t^w \cdot \hat{x}_t^w - EP)^2}$. Risk aversion usually leads to elimination of extreme values of profits and losses.
- Expected utility: $\hat{u} \equiv \sum_{w\in\Omega} p^w u(\sum_{t\in\tau} a_t^w \cdot \hat{x}_t^w)$, where $\hat{x}$ indicates the related optimal values. Obviously, it is greater for the risk-averse model as expected utility is maximized.
- Certainty equivalence: $CE \equiv u^{-1}(\hat{u})$.

As expected by definition, the expected profit declines when risk is relevant to the decisions but the certainty equivalence increases by 15%. Notice the decrease in the standard deviation, indicating reduction of extreme profits and losses.

5.3 Design of independent contracts

Given $\lambda \in \lfloor 0, 1 \rfloor$, the model $(BCU)_\lambda$ indicates a point on the efficient frontier of independent contracts. In order to visualize the efficient frontier, the interval $\lfloor 0, 1 \rfloor$ was discretized into eleven equidistant points.

Table 2 shows the gains for both parties for the computed contracts and Fig. 4 shows the details of one of these contracts.

Given $\lambda \in \lfloor 0, 1 \rfloor$ or equivalently a point of the efficient frontier, there may be several contracts (R_λ, D_λ) which are solution of $(BCU)_\lambda$. Among all of them, the premium D_λ can be minimized. Table 2 also shows the minimum possible value of D_λ, denoted as $\hat{D}_\lambda$.

As an example, Fig. 4 shows two contracts which solve $(BCU)_\lambda$ for $\lambda = 0.5$. In general, efficient contracts showed higher coverage at low-price scenarios.

Both contracts on Fig. 4 are in the efficient frontier. The difference between them is that contract (a) was computed directly by solving $(BCU)_{0.5}$ and contract (b) was selected minimizing the premium $D_{0.5}$ using a simple methodology not shown in this paper.

6 Conclusions

The approach presented in the paper could very well be adopted by forest firms, in particular medium size firms which are more risk averse, and create a market for insurance against price volatility and other uncertain parameters. In the present world situation of very high

Table 2 Gains (in 1000's of $) for the insurer and the firm for efficient independent contracts

λ	$H^I(R_\lambda, D_\lambda)$	$H^F(R_\lambda, D_\lambda)$	$\hat{D}_\lambda$
0.00	0	91	371
0.10	9	82	389
0.20	18	73	407
0.30	27	64	425
0.40	36	55	443
0.50	45	45	468
0.60	55	36	506
0.70	64	27	561
0.80	73	18	615
0.90	82	9	670
1.00	91	0	725

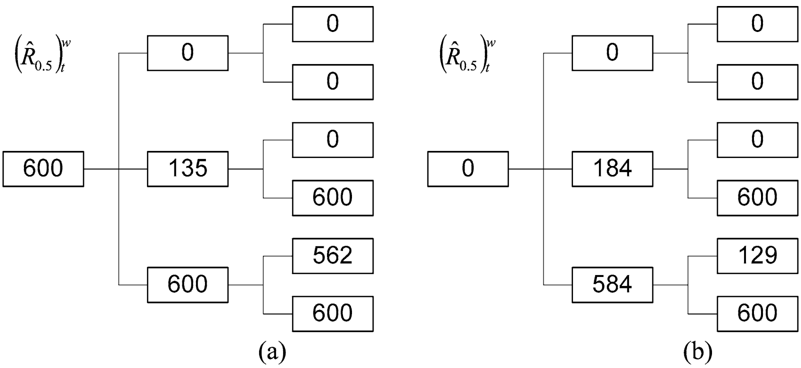

Fig. 4 Two possible Coverage sets (in 1000's of $) which solve $(BCU)_\lambda$ for $\lambda = 0.5$. The premium of these contracts are respectively $D_{0.5} = 1184$ and $D_{0.5} = 468$

uncertainty and variability in the commodities markets, implementing such tools could be a positive step for both the forest firms and large insurers. Specifically, the paper has demonstrated the following ideas:

- Including risk aversion in the optimization model reflects, more faithfully, the planner's preferences than just maximizing expected profit. As demonstrated above, and in accord with common commercial practice, when the utility function of the firm is linear, there is no need for insurance.
- The inclusion of risk attitude in the models allows us to find contracts where both the firm and the insurer gain.
- Two types of insurance contracts were presented: The dependent and the independent contract. The difference between them is the dependence of the coverage on the operating result of the firm. For each of them the gain to both parties was quantified. Of these two types, the independent contract is superior with regard to the gains of both parties.[5]

[5]Calculations which are not presented in this paper showed that the independent contracts in the efficient frontier were about 60% better than the best dependent contracts found. These dependent contracts were not necessarily efficient.

- Varying the optimization model parametrically, the efficient frontier for the independent contracts was computed.
- The above models were applied to a real-world problem of forestry planning, obtaining a solution that delivered a 15% improvement in the certainty equivalence due to the incorporation of risk aversion (Table 1). Contracts were found that bettered both the forestry firm's and the insurer's situations if they negotiated, achieving an additional gain of up to 2%.[6]

More important perhaps is that the models can serve as a basis for other creative ideas to reduce risks in the timber market which, like other commodity markets, is volatile. The paper also emphasizes that optimization in the forest firms should not be restricted to physical activities like harvesting, transportation and sales but to financial activities as well. So further research may be required to seek and create financial tools which are tailor made for the forest industry and to establish econometric methods to assess the firms' attitude toward risk.

Appendix: original forestry planning model

The mixed integer programming model for planning the activity with price scenarios is an adaptation from a previous model (Quinteros et al. 2009). This simplified version considers only one type of timber and one type of road.

Notation (including terms defined earlier)

Sets

- τ: Planning horizon time periods
- O: Origins
- J: Intersections
- S: Destinations
- K: Forest nodes, $K = O \cup J \cup S$
- U: Harvest units
- Ω: Possible scenarios
- XR: Arcs $(k, k') \in K \times K$ for which a road exists
- PR: Arcs $(k, k') \in K \times K$ for which a road can be constructed
- $U(o)$: Units $u \in U$ that can be accessed from origin $o \in O$
- Γ: Triples $(w, w', t) \in \Omega \times \Omega \times \tau$ such that scenarios w and w' are equal in t

Parameters

- b_u^t: Productivity of unit $u \in U$ in $t \in T$
- A_u: Area of unit $u \in U$
- $U_{k,k'}$: Road capacity $(k, k') \in XR \cup PR$
- q_o^t: Unit production cost at origin $o \in O$ in $t \in \tau$
- e_u^t: Unit harvest cost for unit $u \in U$ in $t \in \tau$
- $d_{k,k'}$: Unit transport cost for road $(k, k') \in XR \cup PR$
- $h_{k,k'}$: Road construction cost $(k, k') \in PR$
- $r^{w,t}$: Unit timber price in $t \in \tau$ if $w \in \Omega$ occurs

[6] From Table 2, the contract obtained for $\lambda = 0$ reports an additional gain of 91, which is a 2.3% to the initial certainty equivalent 4008 from Table 1.

Variables

$Z_u^{w,t}$: For any $w \in \Omega$, if unit $u \in U$ is harvested in $t \in T$ then $Z_u^{w,t} = 1$, otherwise $Z_u^{w,t} = 0$
$z_s^{w,t}$: Timber sold at destination $s \in S$ in $t \in \tau$ if $w \in \Omega$ occurs
$Y_{k,k'}^{w,t}$: for any $w \in \Omega$, if road $(k, k') \in PR$ is constructed then $Y_{k,k'}^{w,t} = 1$, otherwise $Y_{k,k'}^{w,t} = 0$
$y_{k,k'}^{w,t}$: Timber transported by road $(k, k') \in PR \cup XR$ in $t \in \tau$ if $w \in \Omega$ occurs

A.1 Constraints

1. Conservation of flow: For each period $t \in \tau$ and scenario $w \in \Omega$
   (a) At the origins: $o \in O$

$$\sum_{k:(o,k)\in XR\cup PR} y_{o,k}^{w,t} - \sum_{k:(k,o)\in XR\cup PR} y_{k,o}^{w,t} = \sum_{u\in U(o)} b_u^t A_u Z_u^{w,t}$$

   (b) At the intersections: $\forall j \in J$

$$\sum_{k:(k,j)\in XR\cup PR} y_{k,j}^{w,t} - \sum_{k:(j,k)\in XR\cup PR} y_{j,k}^{w,t} = 0$$

   (c) At the destinations: $\forall s \in S$

$$\sum_{k:(k,s)\in XR\cup PR} y_{k,s}^{w,t} - \sum_{k:(s,k)\in XR\cup PR} y_{s,k}^{w,t} = z_s^{w,t}$$

2. Road capacity: For each period $t \in \tau$ and scenario $w \in \Omega$
   (a) Existing roads: $\forall (k, k') \in XR$, $y_{k,k'}^{w,t} \le U_{k,k'}$
   (b) Potential roads: $\forall (k, k') \in PR$, $y_{k,k'}^{w,t} \le U_{k,k'} \sum_{t'\le t} Y_{k,k'}^{t',w}$
3. One-time decisions: For each scenario $w \in \Omega$
   (a) Road construction: $\forall (k, k') \in PR$, $\sum_{t\in\tau} Y_{k,k'}^{w,t} \le 1$
   (b) Harvest: $\forall u \in U$, $\sum_{t\in\tau} Z_u^{w,t} \le 1$
4. Non-anticipation: For each period $t \in \tau$ and scenario pair $w, w' \in \Omega$ such that $(w, w', t) \in \Gamma$
   (a) Units: $\forall u \in U$, $Z_u^{w,t} = Z_u^{w',t}$
   (b) Roads: $\forall (k, k') \in PR$, $Y_{k,k'}^{w,t} = Y_{k,k'}^{w',t}$
   (c) Transport: $\forall (k, k') \in PR \cup XR$, $y_{k,k'}^{w,t} = y_{k,k'}^{w',t}$
   (d) Sales: $\forall s \in S$, $z_s^{w,t} = z_s^{w',t}$
5. Nature of the variables: $\forall w \in \Omega, t \in \tau$
   (a) Units: $\forall u \in U$, $Z_s^{w,t} \in \{0, 1\}$
   (b) Roads: $\forall (k, k') \in PR$, $Y_{k,k'}^{w,t} \in \{0, 1\}$
   (c) Transport: $\forall (k, k') \in PR \cup XR$, $y_{k,k'}^{w,t} \ge 0$
   (d) Sales: $\forall s \in S$, $z_s^{w,t} \ge 0$

A.2 Objective function

$$\max \sum_{w\in\Omega} p^w \left[\sum_{t\in\tau} WSI^{w,t} - PHC^{w,t} - WTC^{w,t} - RBC^{w,t} \right],$$

where

- Timber sales revenue: For each period $t \in \tau$ and scenario $w \in \Omega$

$$WSI^{w,t} = \sum_{s \in S} r^{w,t} z_s^{w,t}$$

- Production and harvest cost: For each period $t \in \tau$ and scenario $w \in \Omega$

$$PHC^{w,t} = \sum_{o \in O} \sum_{u \in U(o)} (q_o^t b_u^t + e_u^t) A_u Z_u^{w,t}$$

- Timber transport cost: For each period $t \in \tau$ and scenario $w \in \Omega$

$$WTC^{w,t} = \sum_{(k,k') \in XR \cup PR} d_{k,k'} y_{k,k'}^{w,t}$$

- Road construction cost: For each period $t \in \tau$ and scenario $w \in \Omega$

$$RBC^{w,t} = \sum_{(k,k') \in PR} h_{k,k'} Y_{k,k'}^{w,t}$$

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