

A Review of Operations Research in Mine Planning

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Applications of operations research to mine planning date back to the 1960s. Since that time, optimization and simulation, in particular, have been applied to both surface and underground mine planning problems, including mine design, long- and short-term production scheduling, equipment selection, and dispatching, inter alia. In this paper, we review several decades of such literature with a particular emphasis on more recent work, suggestions for emerging areas, and highlights of successful industry applications.

Key words: literature review; mine planning; mine design; production scheduling; equipment selection; dispatching; optimization; simulation; open-pit mining; underground mining.

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Mining is the process of extracting a naturally occurring material from the earth to derive a profit. In this paper, we confine our discussion of mining to the extraction of the following minerals: (1) metallic ores such as iron and copper, (2) nonmetallic minerals such as sand and gravel, and (3) fossil fuels such as coal. We omit literature regarding naturally occurring liquids such as petroleum and natural gas that have very different characteristics and require different extraction procedures. The oldest mine is located in southern Africa; about 43,000 years ago, prehistoric humans found iron (i.e., hematite) close to the earth's surface and used open-pit (i.e., surface) mining to remove ore and waste from the top downward in a methodical fashion. Ancient underground mines also exist. One such site in England was exploited from about 3,000 to 1,900 BC to recover flint from well beneath the earth's surface. Rather than completely exposing the mining face, miners dug shafts and used ladders to retrieve the hard stone before backfilling

the extracted areas to enforce stability in the excavated tunnels (Gregory 1980).

Mining has five stages: (1) prospecting, (2) exploration, (3) development, (4) exploitation, and (5) reclamation. In the first phase, geologists use visual inspection and physical measurements of the earth's properties to discover mineral deposits. In the exploration phase, geologists determine the value of the deposit by drilling holes to estimate the mineral concentration and its variability throughout the orebody. Interpolation techniques such as kriging (Krige 1951) and simulation techniques (Deutsch 2004) provide tonnage-grade curves representing the potential benefits of exploiting the orebody for a given set of economic parameters. The third stage, development, consists of obtaining rights to access the land and preliminarily preparing it to be mined by removing overlying waste by sinking shafts below the earth's surface. The development stage translates mine planning studies into mine design by

(1) determining the mining method, which consists of the geometrical arrangements of infrastructure; (2) estimating production capacity and infrastructure capital; and (3) performing detailed engineering design. In the exploitation stage, ore is removed from the ground via surface and (or) underground mining methods. It is transported to the surface in trucks via haulage ramps or in shafts. From there, it may be stockpiled (and eventually sent to a processing plant), sent directly to a processing plant, or taken to a dump. Finally, the fifth stage, reclamation, consists of restoring the area in which mining occurred to its natural state to the extent possible.

Operations research (OR) has been used in mining primarily for the development and exploitation stages. Mine planners must make decisions about when and how to perform both surface and underground extraction. Extraction decisions consist of determining (1) how to recover the material and (2) what to do with the extracted material. Because machines are used to extract the ore, decisions about which type of machines to use, how many machines to use, and how to allocate them also arise.

Topuz and Duan (1989) mention potential areas of mining applications, e.g., production planning, equipment selection and maintenance, mineral processing, and ventilation. Osanloo et al. (2008) review long-term optimization models for open-pit scheduling. Although the mining industry has been aware of OR methods for half a century, few places in the literature include a comprehensive, state-of-the-art review of the field and mention papers both from the OR and the mining-engineering literature. Our work provides an all-encompassing review of optimization and simulation models to improve mining operations. Although we limit ourselves primarily to papers appearing in the last three decades, we highlight directions for future research and note the progression of methodologies and theories throughout these decades. We hope to overcome existing shortcomings in the literature and to motivate researchers to continue to improve models and algorithms that have a direct application to the mining industry. We have organized the remainder of this paper as follows. In the *Surface Mining* section, we discuss surface-mining models; in the *Underground Mining* section, we give a corresponding treatment of underground models.

We separate both sections into subsections based on strategic, tactical, and operational models. Finally, we conclude with the *Emerging Areas and Conclusions* section, in which we point out emerging areas in mine planning research.

Surface Mining

Surface mining, which can be used when ore is close to the earth's surface, is an older and more productive method than underground mining. Open-pit mines differ depending on the nature of the material removed. Shallow mines from which gravel and sand are extracted are generally referred to as quarries; deeper, long mines from which coal is removed are known as strip mines. Figure 1 depicts a deep surface mine that is typical of mines with hard metal deposits such as copper. Overburden (i.e., waste) must be removed before extraction can begin. Haul roads wind up through the mine from the bottom of the pit to the surface. Extraction occurs from benches (i.e., open faces of material).

If prospectors deem the extraction of material in an open pit to be viable, they must determine both the pit design and the plan of operations. Pit design relies on preliminary analysis consisting of (1) an orebody model in which the deposit is discretized into a grid of *blocks*, each of which consists of a volume of material and the corresponding mineral properties; (2) the value of each block, which is determined by comparing market prices for ore with extraction and processing costs; and (3) a geometric model of the deposit. Blocks are used as spatial reference points. Geometric constraints, as Figure 2 depicts, ensure that the pit walls are stable and that the equipment can access the areas to be mined.

Solving the pit design problem yields the final pit boundary, i.e., the ultimate pit limit, while balancing the ore-to-waste (stripping) ratio with the cumulative value in the pit boundaries. This analysis requires that the *cutoff grade*, i.e., the grade that separates ore from waste, is fixed. Other aspects of open-pit mine design include the location and type of haulage ramps and additional infrastructure, as well as long-term decisions regarding the size and location of production and processing facilities.

Following the solution of the pit design problem, traditional open-pit production scheduling uses

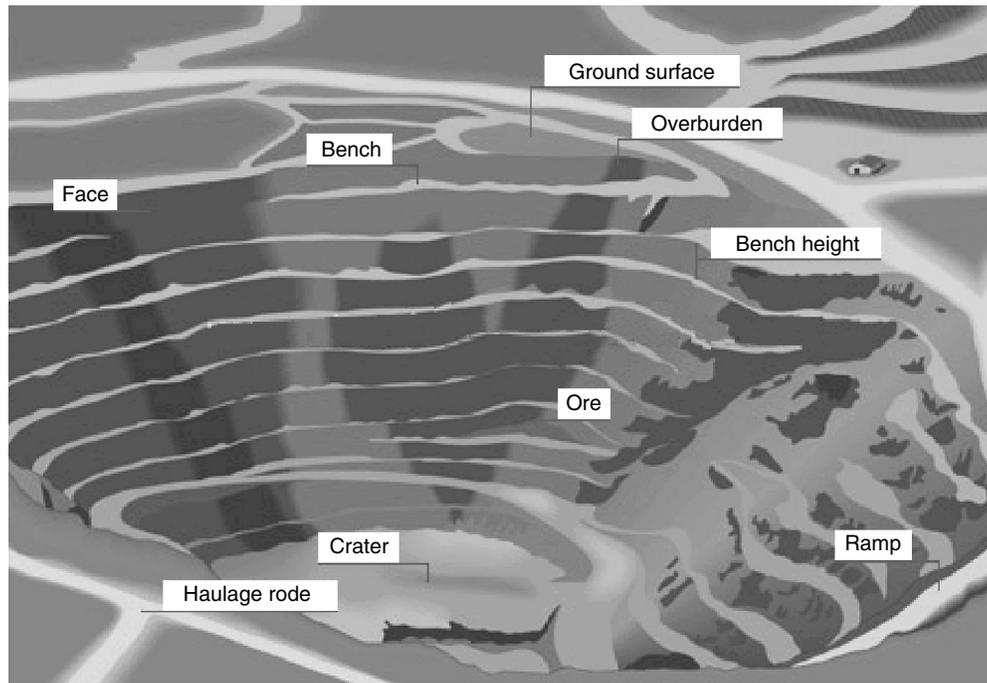


Figure 1: The schematic illustrates an open-pit mine.

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a discretized orebody model, i.e., the block model, assuming a fixed cutoff grade to determine a series of nested pits in which a given (notional) price is used to define one pit and increasing prices correspond to larger pits. From these pits, an ultimate pit is chosen. The nested pits within the ultimate pit are then grouped into pushbacks, where a single pushback is often associated with similar resource usage, e.g., extraction equipment. Within each pushback (which contains only a small subset of blocks within the block model), an extraction sequence is

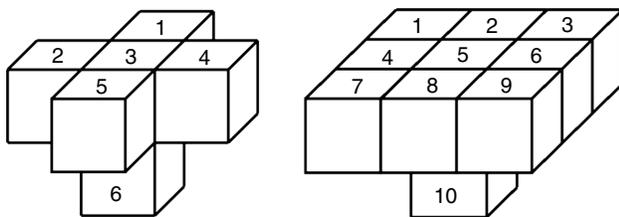


Figure 2: Sequencing rules can be based, for example, on the removal of five blocks above a given block (left) or on the removal of nine blocks above a given block (right).

then determined. The basic premise of this approach is that one can determine a cutoff grade policy to maximize net present value (NPV) subject to capacity constraints. Higher cutoff grades in the initial years of the project lead to higher overall NPVs; over the life of the mine, the tendency is to reduce the cutoff grade to a break-even level depending on the overall grade composition of the mine. Lane (1988), Fytas et al. (1987), and Kim and Zhao (1994) extensively discuss this process. The need to partition the problem in this way stems from the daunting and, until now, computationally intractable task of determining a start date (if any) for each production block in the entire geologic area of interest. Determining these dates for a subset of blocks within a predefined pushback is less challenging. However, three problematic aspects of this approach are (1) the assumption of a fixed cutoff grade, which depends on an arbitrary delineation between ore and waste; (2) the use of notional (and monotonically increasing) prices in determining the nested pits; and (3) the piecemeal approach to the entire optimization problem, which disregards the temporal interaction of resource requirements.

We now mention the progress from the traditional technique to advanced techniques that attempt to solve the entire mine scheduling problem as a mixed-integer program. Although the latter approach is newer and less tractable, there is promise that it will soon solve large enough models to become widely accepted. Bixby and Rothberg (2007) remind us of the continued improvements in hardware and software capabilities.

Strategic Ultimate Pit Limit Design and Mine Layout Models

Two principal classical methods determine the shape of a surface mine. The floating cone method (Laurich 1990) assumes a block as a reference point for expanding the pit upward according to pit slope rules. This upward expansion, which contains all blocks whose removal is necessary for the removal of the reference block's removal, forms a cone whose economic value we can compute. One can then take a second reference block and add to the value of the cone the incremental value associated with the removal of the additional blocks necessary to remove the second reference block; the process then continues. Problems with this method include the following: (1) the final pit design relies on the sequence in which reference blocks are chosen, and (2) many reference blocks might need to be chosen (and the associated value of the cone computed) to achieve a reasonable, although not even necessarily optimal, pit design.

Although the floating cone method is used widely in practice, the seminal work of Lerchs and Grossmann (1965), who provide an exact and computationally tractable method for open-pit design, and associated extensions appear more often in the literature. This problem can be cast as an integer program (Hochbaum and Chen 2000), as we describe below.

- $(b, b') \in B$: set of precedences between blocks (predetermined set).
- v_b : value obtained from extracting block b (parameter).
- y_b : 1 if block b is extracted, i.e., if the block is part of the ultimate pit, 0 otherwise (variable).

$$\begin{aligned} & \max \sum_b v_b y_b \\ & \text{subject to } y_b \leq y_{b'} \quad \forall (b, b') \in B, \\ & \quad 0 \leq y_b \leq 1. \end{aligned}$$

Note that this problem has a totally unimodular structure; therefore, solving the linear program relaxation of the model, as we show above, is sufficient. Specifically, Lerchs and Grossmann (1965) use a maximum-weight closure algorithm that exploits network structure to produce an optimal solution. Because of their algorithm's fast solve times and solution accuracy, many current commercial software packages that incorporate open-pit mine design (e.g., Whittle 2009, Maptek 2009, Datamine 2009) use it. Other authors seek to improve this algorithm. Underwood and Tolwinski (1998) solve the problem by developing a network-flow algorithm based on the dual of its linear programming formulation. The authors provide an interpretation of the graph theoretic methodology. Similarly, Hochbaum and Chen (2000) suggest a maximum-flow, push-relabel algorithm with improved theoretical complexity and faster run times on problem instances with varying mine characteristics such as ore-grade distribution. Wright (1989) argues that dynamic programming is an effective way to determine the ultimate pit limits, particularly because it allows identification of incremental pit boundaries. The boundaries define production requirements and equipment capacities, and these boundaries can then be used to determine those incremental pits that satisfy the corresponding constraints. Wright presents a case study.

Two authors propose extensions to the basic ultimate pit limit problem by incorporating stochasticity. Frimpong et al. (2002) argue that most models developed to address pit design lack the ability to incorporate structural, hydrological, and geotechnical elements. The authors discuss a case study on a gold mine in Zimbabwe using neural networks and artificial intelligence approaches. Jalali et al. (2006) propose the use of Markov chains to determine ultimate pit limits via spatial constraint aggregation. The authors assume an initial pit depth and then assign probabilities to the existence of lower pit depths based on the economic values of underlying blocks. They then present an application of the algorithm for two-dimensional problems; however, they concede that the method would be more difficult to apply to three-dimensional cases.

In addition to the design of the mine itself with respect to the pit limits, authors have also considered mine layouts. Specifically, Bradley et al. (1985)

determine the number of train tracks and silos to include at a mine site by using formulae to compute storage capacity requirements and minimum production rates as a function of demand. They then use simulation to analyze trade-offs between the number of loading trucks, storage capacity, production rates, and train filling rates. Using data from Wyoming's Powder River Basin mine, they measure the average train waiting time.

Tactical Block-Sequencing Models

Unlike the ultimate pit limit problem, the block-sequencing problem considers not only which blocks to remove, i.e., which blocks form part of the economic envelope, but also when to remove these blocks. The introduction of time into these sequencing models allows for the inclusion of resource constraints, e.g., production (extraction) and processing (milling). In addition, discounting can be used to more accurately reflect the value of a block as a function of its extraction date.

A typical formulation of such a model is as follows:

- $b \in B$: set of all blocks b .
- $t \in T$: set of periods within the horizon.
- B_b : set of blocks that must be excavated immediately before block b .
- v_{bt} : value associated with the extraction of block b in period t .
- c_b : consumption of resource associated with the extraction of block b (tons).
- \underline{C}, \bar{C} : minimum (maximum) resource bound in any period (tons).
- y_{bt} : 1 if block b is extracted in period t , 0 otherwise.

$$\begin{aligned} & \max \sum_{b \in B} \sum_{t \in T} v_{bt} y_{bt} \\ \text{subject to} & \sum_{t \in T} y_{bt} \leq 1 \quad \forall b, \\ & \underline{C} \leq \sum_{b \in B} c_b y_{bt} \leq \bar{C} \quad \forall t, \\ & y_{bt} \leq \sum_{\tau=1}^t y_{b'\tau} \quad \forall b, b' \in B_b, t, \\ & y_{bt} \in \{0, 1\} \quad \forall b, t. \end{aligned}$$

Life-of-mine instances of the above model contain many blocks and periods. Therefore, researchers often

assume a fixed cutoff grade and tend to aggregate entities (strata in early work and aggregated blocks later). Although authors also use linear programming, this method cannot accurately capture block precedence. Finally, authors also present decomposition techniques and heuristics.

Early work aggregated blocks into strata, or horizontal layers, subject to a simple set of constraints. Busnach et al. (1985) solve the problem of production scheduling in a phosphate mine in Israel. They determine which sublayers to extract at what time and to which extent (referred to as shallow or deep mining). The corresponding model maximizes the NPV (influenced by factors such as phosphate prices, transportation distance, and ratio of good material to waste) while ensuring that each sublayer is removed either via deep or shallow mining, and that only one sublayer is mined within a given period. The objective function is nonconcave; therefore, the authors use a tailored local search heuristic and demonstrate its use with a numerical example. Klingman and Phillips (1988) solve a similar problem (although they do not differentiate by shallow and deep mining, and they use a linear objective) also for phosphate; they note that their model has been used to make decisions worth millions of dollars. Gershon and Murphy (1989) determine which strata of material to mine, either as ore or waste, to maximize NPV, and they present a dynamic program that aggregates strata into layers, which are mined entirely as ore or waste. A spreadsheet model demonstrates the technique on an oil shale deposit. Samanta et al. (2005) extract layers of material to minimize deviation between target levels of two quality characteristics, silicon oxide and aluminum oxide, subject to sequencing constraints, for a bauxite mine. The authors employ a genetic algorithm on a data set containing 98 layers and 24 periods to arrive at "good" schedules.

Other early work treats blocks but ignores the binary nature of the decisions. Tan and Ramani (1992) consider both a linear program and a dynamic program to schedule extraction over multiple periods subject to equipment capacity constraints. The linear program assesses the differences in production schedules with varied interest rates and equipment availability. In the absence of discrete variables, block-sequencing decisions are not made. Fytas et al. (1993)

use simulation to determine long-term extracted material in each period subject to sequencing constraints and to minimum and maximum production bounds, processing bounds, waste stripping, and quality. They then use linear programming (under the assumption of partial block removal) to schedule blocks in the short term subject to constraints similar to those considered in the long term. The authors propose an iterative technique to obtain practical mining sequences.

In the 1980s, researchers were aware that sequencing decisions had to be made at the block level for reasonable schedule fidelity and also that such models were difficult to solve because of their problem structure and size. Some authors exploit the structure. An example of seminal work in this area is Dagdelen and Johnson (1986), who maximize NPV subject to constraints on production and block sequencing. Rather than relying on heuristics, the authors propose an exact approach, Lagrangian relaxation, that exploits the network structure of the problem if side constraints, e.g., a constraint on the maximum amount of material that can be removed per period, are dualized, i.e., placed in the objective with associated multipliers to induce the relaxed constraint(s) to be satisfied. They apply a subgradient multiplier updating scheme to small examples. Akaike and Dagdelen (1999) extend this work by iteratively altering the values of the Lagrangian multipliers until the solution to the relaxed problem meets the original side constraints, if possible. Kawahata (2006) expands on the Lagrangian relaxation procedure (Dagdelen and Johnson 1986) by including a variable cutoff grade (with a variable such as y_{blt} , where the l index would denote the location to which the extracted ore is sent), stockpiling (with the inclusion of an inventory variable and associated balance constraints), and waste-dump restrictions. To bound the solution space, he uses two Lagrangian relaxation subproblems, one for the most aggressive case of mine sequencing and the other for the most conservative case. Because he has difficulty obtaining feasible solutions for the relaxation, he adjusts bounds on capacities to ensure that the Lagrangian solution is feasible for this adjusted model. Cai (2001) also uses Lagrangian relaxation. His problem differs from conventional scheduling in

that he designs open-pit mine phases and incorporates constraints such as production capacity and sulphur content. The author concedes that there may be gaps between nested pits, precluding his solution technique from providing optimal multiperiod schedules; he presents a gold mine case study with 11 million blocks.

Although the above are exact techniques for addressing the monolithic problem, many authors attempt to sequentially determine the ultimate pit limits, and then the production schedule, by either successively or iteratively solving the problem. Dynamic programming is a popular approach because of its ability to allow the creation of solutions sequentially. For example, Dowd and Onur (1993) use dynamic programming for sequence and pit design via a variable cone algorithm. Onur and Dowd (1993) take the ultimate pit limits as given and schedule blocks to be extracted while including haul roads. The production schedule can be smoothed to include such roads after the pit design has been created. The authors' software generates a number of feasible roads from which the user can select based on various monetary and geological criteria. Eleveli (1995) presents a model that maximizes the NPV of extracted blocks subject to hard sequencing constraints and soft constraints on production and processing capacity. The author poses the problem as a dynamic program and uses local search techniques to solve it. He illustrates his methodology with a small example of approximately 1,000 blocks.

Two papers use a sequential approach; the first is Sundar and Acharya (1995), who consider one model to determine blocks to be blasted and a second model to subsequently find the benches and blocks to be excavated from those available from the blast, accounting also for mill and transport capacity. A chance-constrained program addresses the random nature of ore quality and quantity characteristics and variations in operation-time requirements. Blending of ore blocks meets demand. The authors test the program on a mine in India, demonstrating results with reduced equipment requirements for a given production level. The second paper, Sevim and Lei (1998), describes how the ultimate pit limits, the cutoff grade, the mining sequence, and the production rate interact in a circular fashion. The authors propose a methodology based on a combination of heuristics

and dynamic programming to obtain simultaneously the optimum mining sequence, ore and waste production, ultimate pit limits, and mine life. Their model has three phases. In the first phase, a block model is formed based on the deposit's boundaries. A bounding algorithm is then applied to this block model to determine the largest feasible pit. The second phase considers a spectrum of cutoff grades; for each cutoff grade, a series of nested pits is generated inside the largest feasible pit. These pits are generated in such a way that each pit contains the highest amount of metal among all possible pits of the same size. In the third phase, all possible sequences of pushbacks are formed with the generated pits and are evaluated with respect to their NPV.

Erarslan and Çelebi (2001) determine a production schedule to maximize NPV subject to such factors as grade, blending, and production constraints. They use dynamic programming to enumerate various volumes and determine the optimal pit size. This method ostensibly solves the ultimate pit limit problem and the block-sequencing problem simultaneously. Wang and Sun (2001) propose an approach using a dynamic pit-sequencing scheme to integrate cutoff-grade determination, production rates, the ultimate pit limits, and sequencing. In each period, different options for the next pit phase are modeled as a network.

Some of the above authors concede that dynamic programming is not tractable for larger problems. However, the Lagrangian approach described above remains a prominent method for addressing large problems while also (theoretically) ensuring an optimal solution. Other authors simply pose a monolithic integer program; e.g., Hoerger et al. (1999) develop a multiperiod mixed-integer programming model for Newmont's mining operations. The authors use LINGO (Lindo) to solve small instances.

The following two works use genetic algorithms to attempt to solve large integer programs more efficiently. Denby and Schofield (1994) concede that determining the final open pit and extraction schedule must be integrated. The authors define schedules as a combination of a final open pit and one extraction schedule. To generate the best "schedules" from these combinations, the authors use a genetic algorithm, employing the typical tools of crossover and mutations. They obtain good results, e.g., 6 percent

higher NPV, on small problems, but solution times increase rapidly with problem size. Zhang (2006) also uses a genetic algorithm and aggregates blocks a priori to reduce the problem size. The author tests the algorithm against the ability of CPLEX (IBM 2009) to solve the same instances from BHP Billiton and finds that CPLEX requires approximately two to four times longer to achieve solutions of the same quality. However, the author does not mention the practical consequences of aggregation or how to subsequently disaggregate.

Caccetta and Hill (2003) provide an exact approach by defining variables representing whether a block is mined by period t . (Using the definition of y_{bt} (above) and defining $w_{bt} = 1$ if block b is mined by period t and 0 otherwise, then $y_{b1} = w_{b1}$ and for $t > 1$, $y_{bt} = w_{bt} - w_{b,t-1}$.) The model contains constraints on the mining extraction sequence; mining, milling, and refining capacities; grades of mill feed and concentrates; stockpiles; logistics; and various operational requirements such as minimum pit-bottom width and maximum vertical depth. To solve this problem, the authors use a branch-and-cut strategy, which consists of a combination of breadth-first search and depth-first search to achieve a variety of possible pit schedules. A judicious choice of variable definitions and fixings and the implementation of a linear programming-based heuristic help obtain good bounds on the solutions. Bley et al. (2010) present an open-pit formulation defined by the same variables (see Caccetta and Hill 2003) that maximizes net present value subject to precedence and multiple upper bound resource constraints; cutoff grade is fixed. The authors develop variable reduction techniques and cuts based on the precedence-constrained knapsack structure of the problem and demonstrate how their developments significantly reduce solution time for problems containing hundreds of blocks and 5–10 time periods.

Halatchev (2005) maximizes revenues of gold, less fixed and variable operating (e.g., processing) costs, costs associated with handling waste, and fixed-capital costs. To adhere to sequencing and capacity constraints, the author enumerates all viable production sequences. However, he takes benches, which are generated via the ultimate pit limit, as given and uses

variants of bench-sequencing rules to provide flexibility to his production schedules, all of which must adhere to mill supply and demand constraints; cutoff grade is fixed. The author uses Monte Carlo simulation to generate a variety of data sets that reflect ore-grade variability; he provides a case study with a sample of schedules for various ore-grade data.

Aggregation combined with optimal solution strategies, as opposed to heuristics such as genetic algorithms, has also started to play a role in block sequencing. In short, authors seek to reduce model size by combining blocks with similar properties; they then exploit the problem structure with a view to using fast solution methods on the reduced problem. For example, Ramazan (2007) proposes an aggregation scheme in which he uses linear programming to construct “fundamental trees” to reduce the number of blocks to sequence. Each fundamental tree contains blocks within the smallest aggregate entity possible that can be mined without violating sequencing constraints and that have an overall positive value. The author applies his techniques to a copper mine consisting of about 40,000 blocks to be scheduled over eight years. Boland et al. (2009) use binary variables (similar to those in Caccetta and Hill 2003) to enforce precedence between *aggregates* of blocks while continuous-valued variables control the amount of material extracted both from each aggregate and from each block within an aggregate. The authors exploit the structure of the linear programming relaxation of their problem to develop partitions of the aggregates into sets of blocks such that these sets are optimal for the linear programming (LP) relaxation of their monolith, if possible, or show how to obtain good sets of such blocks. They then use these sets to approximate a solution for their original, mixed-integer program. The authors demonstrate their procedure using instances containing as many as 125 aggregates and nearly 100,000 blocks. Times to obtain a solution within 1 percent of optimality range from thousands to tens of thousands of seconds. Gleixner (2008) extends these results with a different type of aggregation and also presents ideas for using Lagrangian relaxation in this context. Bienstock and Zuckerberg (2010) extend the work of Boland et al. (2009) by developing a customized linear programming algorithm to solve the linear programming relaxation of the problem addressed in Boland

et al. The authors solve model instances containing between 8 and 100 time periods and between about 3,000 and 200,000 production blocks in seconds, whereas CPLEX consumes up to 4 orders of magnitude more time, if it can solve the instances at all.

Amaya et al. (2009) also use “by” variables but not aggregation. They maximize NPV based on a fixed cutoff grade subject to upper bounds on resource and block-sequencing constraints; the authors develop a random, local search heuristic that seeks to improve on an incumbent solution by iteratively fixing and relaxing part of the solution, with the relaxation determined through geometric strategies. This simple, effective idea produces solutions for instances containing up to four million blocks and 15 periods in four hours of computing time. These solutions are approximately 25 percent better than the solutions that standard heuristics provide, e.g., Gershon (1987); in some cases, a standard application of CPLEX requires multiple days to find any feasible solution. Chicoisne et al. (2009) extend the aforementioned work to include a customized algorithm to solve linear programming relaxations of large instances of the same problem; the authors also demonstrate that using heuristics similar to those in Amaya et al. (2009) on the solutions obtained from the LP relaxations produces results in about an hour that are within 5 percent of optimality for problem instances containing millions of blocks and 20 time periods.

Open-pit block sequencing is a heavily studied area, largely because of the many open-pit mines in existence today and the somewhat generic nature of the mines. Researchers are able to solve increasingly large models, which has enabled progress from solving an ultimate pit model and subsequently sequencing blocks within small nested pits to solving a monolith sequencing problem containing up to hundreds of thousands of blocks. However, work remains in terms of incorporating fidelity such as variable cutoff grades and inventory constraints into large models that produce optimal solutions in a reasonable amount of time.

Tactical and Operational Equipment-Allocation Models

In addition to determining a production schedule, a related question is determining the resources that

enable such a schedule. These are generally separate models in the literature; however, in theory, the decisions are connected. A tactical problem consists of the size and nature of the fleet, which can be determined by artificial and geological mine characteristics and by equipment capabilities. The operational problem of equipment allocation entails scheduling and dispatching strategies; decisions are based on haul route requirements and equipment limitations. To this end, both stochastic and deterministic models exist.

Both queuing theory and simulation are used as stochastic methodologies. For example, Oraee and Asi (2004) use simulation for truck scheduling, considering fuzzy parameters on outputs per shovel. The authors wish to attain desired production levels while ensuring that the ore sent to the mill is of sufficient quality. They present a case study for Songun Copper Mine in Iran.

Kappas and Yegulalp (1991) use queuing theory to analyze the steady-state performance of a typical open-pit truck and shovel system in which trucks, or customers, transport excavated material to an end location on a path consisting of a network of haul roads. Trucks also undergo repair and maintenance; although these facilities are capacitated, the authors assume that the haul roads have infinite capacity. Because no trucks enter or leave the system, it is closed; a matrix gives the probabilities of a truck transitioning from one (service) area to another. Because of factors such as the service-time distributions at various areas, the corresponding stochastic process is not Markovian. However, the authors derive results based on extensions of Markovian principles to estimate performance parameters such as the expected number of trucks in an area. The authors verify the accuracy of their derivations with simulation, benchmark tests against a purely Markovian approach, and present a small numerical example.

Najor and Hagan (2006) use queuing theory to model stochastic behavior of truck and shovel systems. A spreadsheet records truck productivity given truck payloads, crusher feed rate, and cycle time. The model analyzes equipment idle time and predicts lower material throughput when both truck and plant capacity are considered rather than simply the former. Numerical results on the Pilbara mine in Australia

show that ignoring queuing leads to overestimating production by about 8 percent.

Many equipment routing and selection models use optimization, i.e., integer programming, to determine fleet size and allocation. A few exploit the problem structure to arrive at network-like models or incorporate stochasticity. Weintraub et al. (1987) exploit network structure in their development of linear programming-based heuristics to route multiple trucks with varying capacities to minimize waiting time at servers, i.e., loaders that fill trucks with ore and waste for transport from the mine. The authors account for loading, unloading, and transportation times and use the underlying transportation network structure of the problem as a basis for their solution algorithm. This model results in an estimated 8 percent increase in productivity at Chuquicamata, a large open-pit mine in northern Chile. Similarly, Goodman and Sarin (1988) develop an integer program combined with a transportation model to determine an optimal equipment schedule and waste distribution. They successively solve the integer portion of the model, fix those values, and evaluate the fixed solution in the resulting transportation model. The authors' results provide insight as to which combinations of transport equipment result in the highest productivity. Soumis et al. (1989) also solve a model in phases. In short, they seek to maximize truck and shovel productivity while meeting demand requirements. The authors first determine the location of shovels; they then use a network model to establish an optimal production plan (considering waiting times) that includes access routes to the ore. Finally, they solve a real-time assignment model to dispatch trucks within the mine.

White and Olson (1992) suggest an operational truck-dispatching system based on network models, linear programming, and dynamic programming. The objectives maximize production, minimize material rehandling, and ensure that the plant is supplied with material while meeting blending constraints. The authors' three-stage process first finds shortest paths between all locations in the mine; the linear program then determines material flows along these paths. Finally, the dynamic program assigns trucks to operate between shovels and dumps. The authors' system,

which is in real-time use at more than 10 mines internationally, yields significant (10–20 percent, in most cases) productivity improvements in these mines.

Naoum and Haidar (2000) develop an integer program to choose a cost-minimizing set of equipment that satisfies maximum production requirements, minimum and maximum bounds on the number of pieces of each type of equipment and the operational hours for each piece of equipment, and mine-life duration. The different characteristics of each type of equipment, e.g., capital and maintenance costs and capabilities, make solving the problem difficult. The authors tailor a genetic algorithm to provide solutions that save 15 percent in equipment-selection costs to the mine for specific case studies. Burt et al. (2005) develop an integer program to determine the number of trucks assigned to a loader to minimize the cost of operating a truck and loader fleet subject to capacity, cycle time, fleet efficiency, and tonnage requirements. The fleet is heterogeneous, and some machines cannot work together. Operational costs increase and productivity decreases, both nonlinearly with equipment age. The authors linearize their objective and demonstrate their model on small instances containing fewer than 10 trucks and loaders.

Ta et al. (2005) extend optimization modeling to incorporate stochasticity by formulating a truck allocation model using chance-constrained stochastic optimization that accommodates uncertain parameters such as truckload and cycle time. The number and type of trucks that are allocated to shovels are the decision variables. The objective minimizes the operating and capital cost of ore delivery subject to production rates, shovel capacity, waste material removal, and truck availability. The crucial step in solving the chance-constrained problem is to convert the constraints into a deterministic form by identifying the confidence level that must be satisfied. Apparently, the technique is simple to implement and possesses reasonable computation times, thus making it applicable to real-time problems.

Rubio (2006) addresses the use of processing facilities rather than the routing or scheduling of transportation equipment. He takes the production schedule as given; for each block, he then determines the processing facility to which it is to be sent, if any,

to maximize revenue subject to processing time available at the mills. Rubio demonstrates the use of his model at the open-pit Grasberg Mine in Indonesia, compares the results against a heuristic, and concludes that using an optimization model can have significant benefits; however, he concedes that such a model should also incorporate block sequencing. McKenzie et al. (2008) address the question of how to locate equipment, specifically a feeder into which extracted aggregate (e.g., sand and gravel) is funneled. This feeder is attached to a conveyor belt that transports the material to a processing plant and can be moved by adding conveyor belt extensions. The trade-off is the proximity of the feeder to the operations, vice the (nonoperational) time required to relocate the feeder as the mining frontier progresses. The authors model the problem as a dynamic program and solve it as a shortest-path model in which an arc in the model represents moving the feeder along a linear path from location i to location j . Implementing the model results in savings of approximately 14 percent of total operational costs at the mine.

Munirathinam and Yingling (1994) discuss survey articles on open-pit truck dispatching; they classify truck-dispatching strategies, examine their underlying mathematical formulations in detail, and identify the strengths and weaknesses of alternate approaches. Alarie and Gamache (2002) analyze dispatching systems and the advantages and disadvantages of solution strategies, and they present the current challenges of dispatching systems such as real-time monitoring capabilities.

Underground Mining

Despite the relatively lower fixed infrastructure cost of an open-pit mine, surface mines necessitate significant extraction of waste. A mine can be or become cost prohibitive to operate when the ratio of extracted waste to ore becomes too high, when waste storage space is insufficient, when pit walls fail, or when environmental considerations outweigh extraction benefits. In these cases, underground mining begins.

The economic viability of an underground mine relies on much of the same economic analysis as an open-pit mine. Given viability, there are both pit design and operating decisions. However, in underground mine planning, there is no real equivalent to

determining ultimate pit limits. Underground mining methods are commonly categorized as unsupported methods, supported methods, and caving methods; the choice of which method to employ in a mine depends on the size and shape of the orebody, the characteristics of the surrounding rock, and the characteristics of the ore. Although there are many underground methods (and variants), we restrict ourselves to brief descriptions of the methods used in papers contained in our literature review.

Room and pillar (Figure 3, left panel) is an unsupported method designed for flat, relatively thin, homogeneous deposits such as coal. Regularly positioned pillars consisting of unrecoverable ore support “rooms” from which ore is blasted and then removed via trackless loaders. Bolts or pillars are used as ceiling supports. In a second phase of this method, retreat mining, the pillars are mined until the tunnel collapses.

Sublevel stoping (Figure 3, right panel) is another unsupported method used for steeply dipping orebodies with regular boundaries. The orebody is divided into separate stopes (i.e., large, vertical pipes of rock) into which ore is blasted from various drilled access levels. Ore is recovered from drawpoints at the bottom of each stope. Stopes are usually backfilled.

Caving methods rely on the rock breaking into pieces that are small enough to be retrieved from the

deposit and to flow into a recovery location without having to blast all the ore. Longwall mining (Figure 4) is a caving method used on long, thin deposits such as coal seams. If the rock (e.g., coal) is soft, it is simply mechanically cut from the mining face (without blasting). The continuous extraction of the material makes it amenable to transport via a conveyor belt. Mined-out areas collapse behind the currently active ones.

Sublevel caving (Figure 5, left panel) is a caving method that is used for long, relatively pure vein-like deposits. Ore is blasted and extracted from systematically laid-out tunnels (sublevels) parallel to each other at the same depth. Other series of parallel tunnels lie deeper in the mine. Because of the highly structured nature of the mine design, the surrounding rock must cave in a controlled fashion. Blasted ore is hauled via loaders to an ore pass; the ore falls down a chute to a haulage level from where it can be broken into rock small enough to be hoisted to the surface via a series of vertical shafts.

Finally, block caving (Figure 5, right panel) is a caving method with much less structure than sublevel caving. It is applied to orebodies of much lower quality in which large masses of rock are blasted; the ore is recovered through drawpoints at the bottom of the location at which the ore is undercut. Managing the size of the rock and the rate at which rock

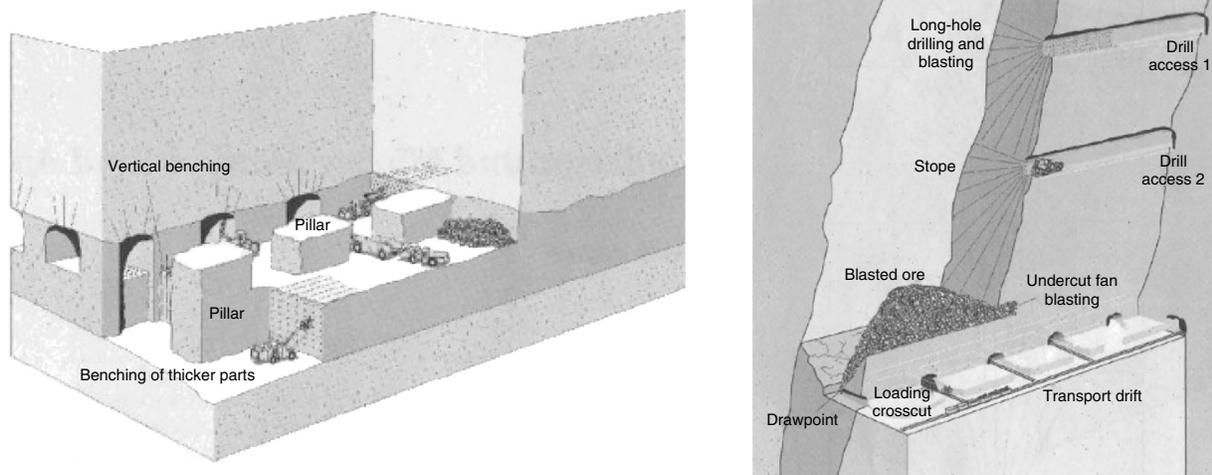


Figure 3: The diagram illustrates room-and-pillar mining (left) and sublevel stoping mining (right).

Source. Hamrin (2001).

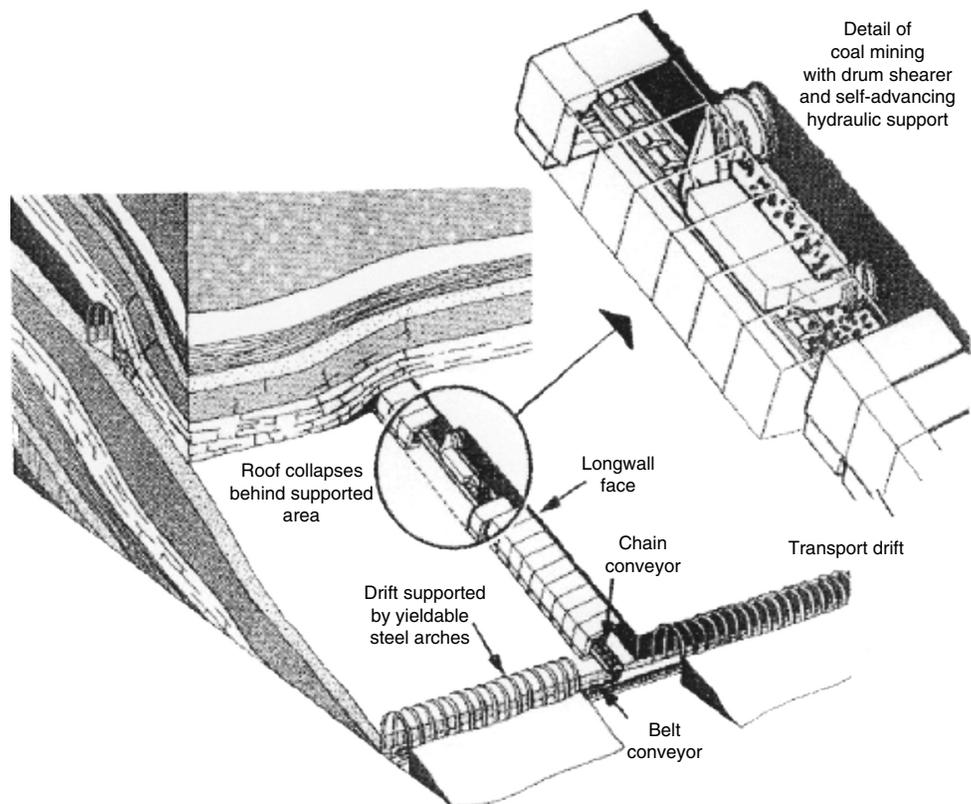


Figure 4: The diagram illustrates longwall mining.
Source. Hamrin (2001).

filters through the drawpoints, as well as continually monitoring the stability of the mine, are critical. Early applications of block caving relied entirely on the gravity flow of the rock. Now, large rock is redrilled and reblasted or broken with hydraulic hammers. As in sublevel caving, the rock is sent down a chute to a main haulage level before it is hoisted to the surface.

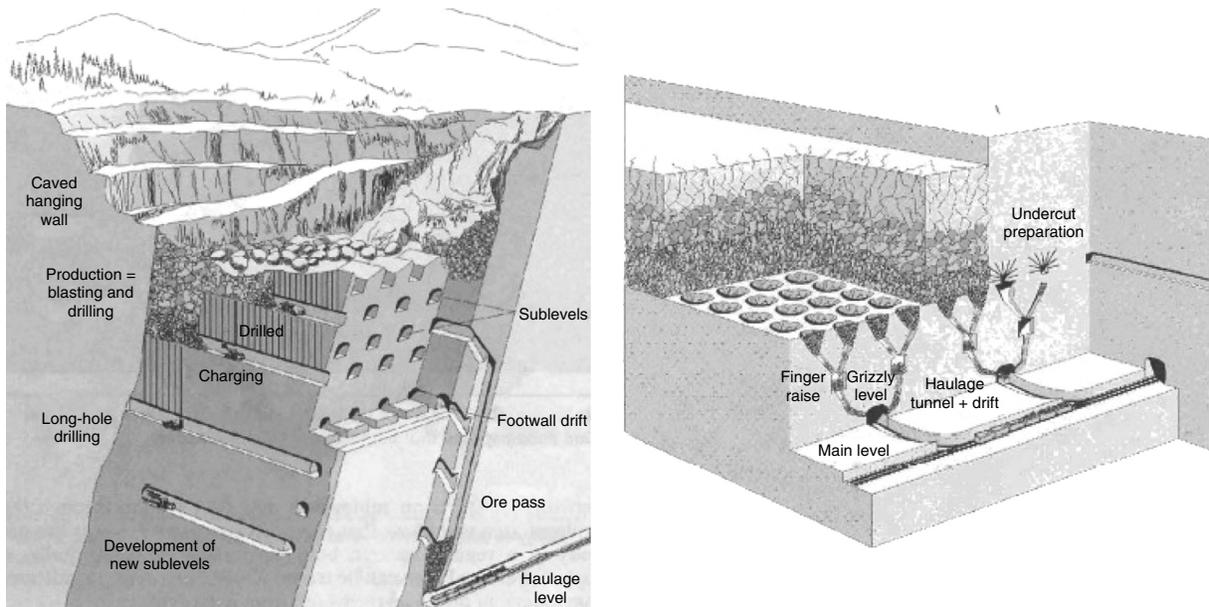
The literature on underground mining is more recent, partially because of the complicated nature of underground operations. A parallel to the Lerchs–Grossmann algorithm (1965) does not exist for underground mines; therefore, early optimization work does not necessarily rely upon network-based methods. Strategic decisions in underground mining may consider how to geographically position various facilities, e.g., mills, on the mine site. One may also be concerned with which mining method to choose and with mine layout, such as the design of haulage ramps and other infrastructure. Alternative mining methods

or haulage systems can be evaluated using simulation. Haulage ramps can be designed using geometric approaches. Long-term decisions, e.g., whether to operate a mine or to position a mill at a given location, are often made using integer programming optimization models.

Tactical production models generally consist of a mixed-integer program in which binary variables address longer-term block-extraction decisions and continuous variables address the related shorter-term decisions of how much ore should be extracted from a block. Transportation devices, e.g., conveyor belts in coal mines or front-end loaders, are usually assessed using tactical or operational optimization and simulation models.

Strategic Mine Layout and Design Models

Generally speaking, the mining method is determined via geotechnics rather than using OR techniques;



**Figure 5: The diagram illustrates sublevel-caving mining (left) and block-caving mining (right).
Source. Hamrin (2001).**

however, Qinglin et al. (1996) use neural networks based on a deposit's geotechnical and economic factors to optimally select an underground mining method. The case study the authors address is that of a gold mine in which they consider longwall, room-and-pillar, sublevel caving, and sublevel stoping methods.

Other prominent work on underground mine design concerns mine shape and layout given the method. Yun et al. (1990) use a genetic algorithm to determine the number and spacing of openings given restrictions on their relative placement. To value the quality of a given set of openings, the authors consider costs of development, drilling and blasting, underground pressure, transportation and dilution, and the revenue from the excavated ore. They apply their algorithm to a sublevel caving mine and show how they tune algorithmic parameters, e.g., crossover and mutation rates.

The following three articles pertain to underground sublevel stoping. Concerned with the shape of an underground mine, much like the shape of an ultimate pit for surface mining, Alford (1995) describes the floating stope method (analogous to the floating cone method for surface mines as the *Strategic*

Ultimate Pit Limit Design and Mine Layout Models subsection describes) as a tool for analyzing mineral reserves and stope geometry. Model inputs are the orebody model, the stope geometry, and a cutoff grade. The model minimizes waste, maximizes grade, or maximizes metal based on ore-quality restrictions such as minimum grade and maximum dilution from a stope. Brazil and Thomas (2007) provide theory and background for Brazil et al. (2003), who consider the strategic decision of determining the three-dimensional geometry of haulage ramps through a sublevel stoping mine to serve each stope in a given order while subscribing to maximum-gradient, minimum-curvature, and area-avoidance constraints. They use existing mathematical theory that projects their problem onto a two-dimensional space to find a minimum-cost path satisfying the above conditions with the exception of the gradient and area-avoidance constraints. These constraints are later imposed on the ramp geometry. The optimization model has been embedded into software and implemented at various mines in Australia. The authors specifically mention an 11 percent decrease in costs at one representative mine in Queensland.

Two early papers address underground facilities, e.g., production shafts, and those above ground, e.g., mills. Lizotte and Elbrond (1985) discuss the problem of siting a single facility to minimize demand-weighted transportation costs for a set of existing facilities. Using Euclidean distances, the authors formulate their problem as an unconstrained, nonlinear optimization model. Therefore, they propose using a hyperboloid approximation procedure based on methods of calculus and show how the procedure can be adapted to the problem of siting multiple facilities. They then describe two case studies in which they use their procedure to lay out underground levels and to locate a production shaft. Finally, they point out some shortcomings in their optimization model, e.g., assumptions that haulage and excavation costs are not necessarily directly proportional to Euclidean distance. Barbaro and Ramani (1986) formulate a mixed-integer programming model that can be used to determine if a mine produces in a given period, if market demand is satisfied in a given period, where to locate a processing facility, and the amount of ore to ship from a mine to a processing facility and then to a market. The objective function maximizes revenue less production, processing, waste disposal, and fixed costs. Constraints include minimum and maximum production requirements, market demand, quality, and limits on the number of mines and facilities open simultaneously. The authors use a coal system to demonstrate their model's capabilities.

Tactical Block-Sequencing Models

The literature on sequencing models for underground operations is relatively new. Early models use simulation to assess production schedules and linear programming to make decisions regarding ore extracted. Integer programming models are needed to determine whether to mine a given segment of ore in a particular period, e.g., to maximize NPV subject to, *inter alia*, complex sequencing constraints and minimum and maximum production (or draw) rates. These models, which have appeared relatively recently, were solved first via enumeration and later via more sophisticated techniques.

As is the case for underground mine design, in which it is not easy to identify a general mine-design formulation, it is also not easy to identify a generic

underground block-sequencing formulation. Such formulations possess the general flavor of those of open-pit block sequencing in that objective functions represent NPV and constraints generally encompass both precedence and resource restrictions. However, both sets of constraints are more complex and mining method (or even mine) specific. Typical decision variables in this context could include the following: (1) $y_{att'} = 1$ if we (start to) mine area a in period t and subsequently backfill it in period t' , and 0 otherwise (for some type of supported method); (2) $y_{akt} = 1$ if we (start to) mine area a with equipment of type k in period t , and 0 otherwise (Sarin and West-Hansen 2005); or (3) variables representing whether area a is developed, drilled, prepared, or extracted from in period t (Carlyle and Eaves 2001).

Linear programming enables the user to optimize yet does not account for discrete aspects such as block sequencing. Jawed (1993) uses linear programming to determine the amount of material to be extracted, e.g., via room and pillar. He minimizes deviation from prescribed production and cost targets subject to operational constraints, manpower requirements, extraction capacity, ventilation requirements, plant capacity, and lower bounds on extraction quantity. The author performs sensitivity analysis on the results for a typical mine; of special interest are changes in cost parameters and resource (mining equipment) availability. Similarly, Magda (1994) circumvents binary variables in a model that evaluates the economics of decision making in mine production processes. He gives formulas for the costs and benefits of investments as a function of time. Using geometry, the rate at which the project is constructed, construction costs, and commodity prices, he can maximize values, e.g., NPV or internal rate of return. The author presents an example of longwall coal mining in which, for a particular extraction sequence of panels, the length and width of exploitation panels and the number of longwall panels within an exploitation panel can be determined. An enumerative procedure yields the NPV for a given point in time and interactions between NPV and panel dimensions can be examined.

Authors combine simulation with optimization—the first article we describe here uses linear programming, and the second uses integer programming.

Winkler (1998) describes a production-scheduling model to determine the amount of ore to mine in each period from each production block. He minimizes weighted deviations from production goal averages in each of 14 to 28 daily periods subject to constraints on ore quality, a lower bound on demand and on the amount extracted from a block, and capacity restrictions on available ore. Linear programming solves a corresponding single-period model; simulation is then used to fix the current period's decisions and optimize over the successive period. The results for a case study involving an iron ore mine using sublevel caving are displayed in a three-dimensional environment. Chanda (1990) combines simulation with optimization in a model that uses an integer program to determine when ore should be extracted from drawpoints in an underground block-caving mine to minimize the differences in average grade between successive periods. Constraints include limits on production and grade, and on the operational aspects of block caving. The production schedule given by the integer program is used as input to a simulation model that considers constraints such as production capacity. Depletion of the ore as it is extracted is based on gravity flow principles. The author applies his model to a copper mine in Zambia and shows a decreased intertemporal ore-quality fluctuation and a decreased number of open drawpoints.

The presumed intractability of a mixed-integer program to model the complexities of tactical production scheduling is sometimes mitigated by the fact that much of the added detail is modeled with continuous, as opposed to binary, variables. As software (e.g., IBM's CPLEX) and hardware improved, researchers began to use mixed-integer programs. Trout (1995) might represent the first attempt to optimize underground mine production schedules, either at the strategic or at the tactical level, using integer programming. By maximizing NPV, the model schedules an underground stoping mine for base metals (e.g., copper sulphide). Binary variables control the timing of extraction from or backfilling of a stope. Continuous-valued variables track the material extracted from or backfilled into a stope in a given period. The constraint set incorporates (1) stope sequencing in terms of extraction, backfilling, and the corresponding

relationship in time; (2) stope extraction and backfill quantities; (3) equipment capacity; and (4) grade requirements on the metal recovered. The model contains a 17-period horizon in which the last 4 periods are aggregated into durations that are three times longer than the previous 13. Although the model runs out of memory, it yields a 25 percent improvement over the NPV generated by then-current operational policies.

Since the early 2000s, authors have routinely used increasingly sophisticated optimization models to derive tactical production schedules, in some cases for complex underground mines, that use a variety of methods. Carlyle and Eaves (2001) present a model that maximizes revenue from Stillwater's sublevel stoping platinum and palladium mine. Integer variables schedule the timing of various expansion-planning activities, such as development and drilling, and the number of stopes to prepare and produce from in a given period. Constraints include (1) sequencing of operations and stope preparation; (2) production limits of stopes, lower bounds on production targets, and upper bounds on processing per period; and (3) bounds on the change in crew size between periods. The authors obtain near-optimal solutions for a variety of scenarios over a 10-quarter time horizon. Their scenarios examine variations consisting of (1) constraint relaxation, (2) time-horizon expansion, and (3) mine-design (drift-spacing) changes, and the scenarios provide planners with insights into different tactical operating procedures. Smith et al. (2003) construct a production-scheduling model for a copper and zinc underground mine at Mount Isa, Australia. Decision variables represent the time at which to mine each production block to maximize NPV subject to operational constraints, e.g., ore availability, concentrator (mill) capacity, mine-infrastructure production capacity, grade (mineral quality) limits, continuous production rules, and precedence relationships between production blocks. However, the authors are unable to solve all instances of their problem in a reasonable amount of time.

Epstein et al. (2003) present a mathematical programming model to determine the levels of extracted ore from several different underground copper mines to maximize profit over a 25-year horizon subject

to demand, technical exploitation constraints, and environmental limitations. The mixed-integer program specifically defines the geological area of interest through profitable columns (or extraction points), which are the vertical aggregation of blocks; a mine or sector corresponds to a set of neighboring columns. Extracted material is sent through a network of alternative technologies and (or) infrastructure (e.g., mill) investments. This capacitated network contains hundreds of thousands of variables and constraints. The authors develop a rounding heuristic that provides implemented solutions that represent an improvement of more than 5 percent on current operations at El Teniente, the largest underground copper mine in the world.

Interestingly enough, several seminal works in this subfield do not consider monetary goals. The examples that follow use different methods—block caving and sublevel caving—and although the models are conceptually similar, it is interesting to note the difference in decisions because of the difference in mining methods. Rahal et al. (2003) describe a mixed-integer programming model to plan operations of a block-caving mine. Their mathematical program schedules drawpoint production to minimize deviations from preset demand levels while also minimizing deviations from a given draw profile. Constraints measure these deviations, restrict draw rates between minimum and maximum levels, limit waste content within a drawpoint, and establish precedence relationships between points from which ore is removed. Detailed implementation of these constraints can incorporate ore grade and quality. The authors develop life-of-mine draw profiles for notional scenarios and show that by using the results from their integer program, they greatly reduce deviations from ideal drawpoint depletion rates while adhering to a production target. Rubio and Diering (2004) expand the analysis to include cost considerations. Newman and Kuchta (2007), motivated by an underground mining operation at Kiruna, Sweden, formulate a multiperiod mixed-integer program for iron ore production. The optimization model determines an operationally feasible ore extraction sequence that minimizes deviations from planned production quantities. The authors design a heuristic based on solving a smaller, more tractable model in which they aggregate periods;

they then solve the original model using information gained from the aggregated model. They compute a bound on the worst-case performance of this heuristic and demonstrate empirically that this procedure produces good-quality solutions while substantially reducing computation time for problem instances; this model was implemented at the Kiruna mine (Kuchta et al. 2004).

Sarin and West-Hansen (2005) use decomposition to successfully solve the original problem. The authors maximize NPV for an underground coal mine that consists of sections mined with longwall, room-and-pillar, and retreat mining. Binary variables track whether a section is scheduled to start being mined by a given set of equipment at a given time, whereas continuous variables track the quality and production volume of the material extracted. The constraint set consists primarily of enforcing precedence, smoothing quality and production levels, and limiting the quantity of sections simultaneously mined. The authors tailor a Benders' decomposition technique, exploiting the structure of the master problem. A case study containing over 100 weekly periods suggests that their model can improve profits. Weintraub et al. (2008) also successfully exploit the problem structure to arrive at good solutions by developing an aggregation scheme based on cluster analysis for El Teniente, a large Chilean block-caving mine. The scheme reduces the size of the five-year production scheduling model, allowing it to be solved about one order of magnitude faster with an error of approximately 3 percent (on original, disaggregated instances) because of aggregation. Note that both this work and that of Epstein et al. (2003) use aggregation for underground block-sequencing operations and embed it in an optimization-based heuristic; we also mention this technique in open-pit block-sequencing models.

Underground OR problems, both strategic mine-layout and design models and tactical block-sequencing models, are not as heavily studied as their open-pit counterparts because fewer real-life instances of such models exist; each underground mine is unique in its design and operations. Mine planners work with existing software but try to meet conflicting objectives—to have software specifically tailored to the orebody at hand yet to also have the

same software that can handle a general underground orebody. Continuing to develop faster-solving and customized models is necessary. It remains to be seen whether such models could be developed into a generally applicable underground design and scheduling model.

Tactical and Operational Equipment-Allocation Models

The majority of equipment allocation concerns transportation systems in underground coal mines that have numerous alternatives, complicated systems, and different types of machines, e.g., conveyor belts and transport cars that are interdependent and require analysis to determine bottleneck locations. To allow managers to address uncertainty, these mines commonly use discrete event simulation as their analysis tool.

In the early 1980s, Topuz et al. (1982) simulated the performance of two different haulage systems for an underground coal mine. Their work compares a conventional shuttle car with a conveyor belt and a diesel shuttle car (without a conveyor belt). They vary performance parameters such as haulage distance and speed, the number and discharge rates of haulage units available for coal transport, and the capacity of a feeder located where the shuttle car dumps its output. They also perform a case study on a room-and-pillar mine to select feeder discharge rates, the number and type of haulage units, and a reasonable haulage distance to improve the overall production potential of the mine. Sevim (1987) uses simulation to examine a hydrotransport system that mixes coal with water upon extraction of the coal from the mine face; the coal is then pumped directly to a processing plant. Model events include (1) preparing coal to be extracted from a face, (2) pumping water into a pipeline to the requisite pressure level, (3) mining the coal and subsequently pumping it into the pipeline with the water, and (4) repositioning the equipment for new extraction. Between the third and fourth stages, a delay can occur because of factors such as equipment malfunction. The authors consider two particular operational characteristics: (1) the merging of pipelines from different areas of the mine and (2) the inclusion of a surge tank to store slurry. The authors present an instance using both room-and-pillar and longwall mining. Their simulation study

shows statistics on water and slurry pumping times, surge-tank overflows, the concentration of slurry fed to the preparation plant, etc., as well as associated operating costs for particular mine configurations. Mutagwaba and Hudson (1993) present a simulation model to assess underground transportation systems. Given a mine layout, a hoisting system, and desired production rates, their model evaluates the performance of various transport systems, such as trucks and conveyor belts. The authors apply their analysis to a mine in North Wales, UK to choose the equipment that best balances performance and costs. McNearney and Nie (2000) simulate a conveyor belt system used with longwall and continuous miner methods to transport coal from a mine face to the surface. The authors balance the cost of the conveyor belt system with overall performance. The study reveals bottlenecks and examines the effect of adding surge bins to remove or mitigate these bottlenecks. The authors experiment with various conveyor belt speeds and sizes and show that at a mine in southern Utah, productivity could increase by more than 13 percent. Simsir and Ozfirat (2008) construct a simulation model as a case study for a Turkish coal mine that uses longwall top-coal caving. The model assesses the efficiency of loaders, crushers, and conveyor belts; however, it omits the geomechanical effects on the mine of different types of extraction equipment.

In contrast, in another type of stochastic model that is applied to an iron ore mine, Huang and Kumar (1994) use queuing theory to determine the optimal number of load-haul-dump machines by considering their performance and price, required maintenance, operators' salaries, etc. An example from a Swedish mine illustrates that their queuing model can accurately calculate the probability that a given number of machines is sufficient for production. The authors argue that the number of machines estimated by using the queuing model is more accurate than a standard method using statistical distributions.

Optimization models are also used to allocate equipment in underground mines. These models are deterministic and less common; in general, they address a more limited system than simulation models do. Dornetto (1988) studies equipment that is designed to extract, store, and transport material in

an underground coal mine. The objective is to minimize the number of extraction cycles necessary to fill a shuttle car that transports coal from a surge bin to a belt feeder by determining optimal values for the position and rate of mining equipment. Constraints include lower and upper bounds on the mining position and rate, allowable amounts of coal mined in a cycle, and an upper bound on the distance that must be mined before the equipment can be moved. This nonlinear program is solved via analytical methods; the author shows that lost production decreases by using his (optimized) operational policy. Vagenas (1991) examines real-time dispatching for load-haul-dump units in underground mines. He considers bidirectional loader movement and a few alternate paths between an origin and a destination; he preliminarily uses Dijkstra's (1959) algorithm to determine a shortest path between an origin and a destination, and he subsequently develops algorithms that resolve vehicle conflicts either by slowing a loader, by stopping it completely, or by routing it to a different destination. The goal is to minimize loader delays. The author uses a simulation model to assess the performance of his algorithms via case studies. Gamache et al. (2005) extend Vagenas (1991) by considering routing on a network with load-haul-dump unit orientation and by accounting for time windows; the authors use Dijkstra's algorithm to solve these shortest-path problems. The approach has two drawbacks; it assumes deterministic travel times and the algorithm considers only a single vehicle at a time, thus ignoring the effect of the current vehicle on subsequent vehicle movements.

Kumral (2005) poses the problem of choosing the extent to which to improve the reliability of an underground mine consisting of five independently functioning subsystems: (1) drilling, (2) blasting, (3) loading, (4) hauling and hoisting, and (5) ventilation. The author assumes that each subsystem is either operational or not and defines a failure as the result of events such as uncontrolled caving, improper air circulation, or equipment breakdowns. He minimizes the cost of operating all subsystems at an acceptable level of reliability subject to bounds on reliability variances and reliabilities of the subsystems themselves. He presents a case study in which he uses a genetic algorithm to solve the nonlinear programming

problem and arrives at reasonable levels of minimum required reliability levels for each subsystem.

Operational decisions other than in situ transportation exist—specifically, issues outside of the mine such as stockpiling, transportation, and facility operating procedures. For example, Baker and Daellenbach (1984) use dynamic programming to determine optimal operating settings for a coal-fired power station and subsequently use simulation to determine corresponding coal mining and stockpiling policies. Everett (1996) uses a simulation model to reduce fluctuations in iron ore composition by intelligently stacking the ore in stockpiles and subsequently recovering it prior to placing it on a ship for overseas transport. Pendharkar (1997) presents a short-term model to determine the quantity of coal to ship from a mine to a processing facility and from the processing facility to a market in each period of the planning horizon. Pendharkar and Rodger (2000) extend the aforementioned model to include a nonlinear objective. Binkowski and McCarragher (1999) use queuing theory to determine the optimal number and size of stockpiles in a yard to maximize throughput. Ore arrives at the yard by train, is deposited into a single stockpile, can be blended, and awaits transportation by ship. The authors demonstrate how system parameters such as stockyard capacity, ship capacity, and arrival and service rates hypothetically influence the performance of the stockyard and its configuration.

Emerging Areas and Conclusions

As hardware, software, and solution techniques improve, we can expect to see models that are more realistic and include more detail. For example, stochastic mine planning is relevant given the long time horizons involved, the large initial investments and operational budget required, and the historical fluctuations of metal prices. In addition, the percentage of ore contained in each block of a deposit is uncertain, and significant costs are involved in determining ore content with accuracy. Brennan and Schwartz (1985) use real options theory to evaluate natural resource investment. To date, the heavy computational burden has precluded using this methodology in mathematical programming models with a large search space; however, the concepts are highly

relevant to stochastic planning scenarios. Cortazar et al. (1998) develop a model that determines the optimum output price level for a firm to invest in environmental technologies and the main parameters that affect this decision.

With respect to mine planning (as we discuss specifically in this paper), Lemelin et al. (2007) consider the collection of geologic block data, the completion of mine design and production planning, and the performance of financial analysis based on the outcome from the mine design and planned production. The authors criticize approaches that accept subjective, if uncertain, parameter values and that ignore the relationship between production schedules and information availability. The authors propose modeling prices according to well-regarded stochastic models in which operational plans are updated as prices become available. They present a case study in which a mine's value is substantially different using their approach when compared with a conventional approach, primarily because the authors consider shutdown options for unprofitable portions of the mine. Carvallo et al. (2009) consider production planning for an underground mine in Chile. They maximize NPV subject to sequencing and mining-rate constraints and reduce the problem size by aggregating mineable areas; however, because the authors introduce price uncertainty, which they characterize in various scenarios, the problem is large. They use Lagrangian relaxation to dualize the nonanticipativity constraints so that each scenario can be solved independently as a deterministic problem. Although the algorithm produces more robust results when it considers price uncertainty than when it does not, it does so at the expense of computer time.

Ramazan and Dimitrakopoulos (2004) and Gholamnejad and Osanloo (2007) consider ore-grade uncertainty in open-pit mines. Ramazan and Dimitrakopoulos (2004) mention the traditional block-sequencing problem and illustrate a schedule obtained from such a formulation for a case study involving a nickel laterite deposit. They then introduce a model with a modified objective in which, in addition to the NPV obtained from the extraction of a block, a penalty is incurred for production capacity-constraint violations. The authors use various simulated data sets to show that the modified

model produces a schedule with a more implementable mine sequence, i.e., one in which equipment movement is reduced substantially. The modified model also accounts for geological uncertainty, which (the authors claim) helps to produce a schedule that can more readily meet production constraints. Gholamnejad and Osanloo (2007) present a problem in which they determine which blocks to extract in an open-pit mine; they consider block-grade uncertainty in which each block has a probability distribution function obtained using geostatistical simulation. They transform the probabilistic problem (i.e., the problem that has probabilities of satisfying the constraints) into a nonlinear integer program. Because of the resulting model's complexity, the authors propose using a genetic algorithm; however, they do not give details or examples. Askari-Nasab et al. (2007) use a stochastic simulation model to develop an open-pit schedule based on a geometric elliptical frustum model. They use a simulator to evaluate a variety of open-pit expansions associated with a schedule or volume of ore removed (i.e., processed, sent to waste, or stockpiled) in each period. A case study from an iron ore mine with 114,000 blocks and over 20 periods demonstrates an improvement in NPV using their elliptical frustum model with stochasticity compared with the NPV obtained from a parametric analysis using commercial software (Whittle 2009). Finally, Boland et al. (2010) extend Boland et al. (2009) to incorporate ore-grade uncertainty (note that the online version of Boland et al. 2010 was available as of December 2008); the authors update ore-grade data in their model instances as information becomes available.

The following article addresses geological uncertainty in sublevel stoping mines. Grieco and Dimitrakopoulos (2007) present a strategic model for determining the design of a sublevel stoping mine under ore-grade uncertainty. The design parameters of interest are the location, size, and number of stopes. Their model seeks to determine if a given ring should be included in a panel for extraction to maximize a probability-weighted metal content across all rings and panels subject to constraints on the minimum and maximum number of rings allowed in a panel, a minimum acceptable risk level (as defined by the probability of a ring in a panel meeting a specified

cutoff grade), and stope-size constraints in terms of an acceptable number of rings mined in adjacent panels. The authors apply their model to a case study of a polymetallic mine in Ontario, Canada.

Other emerging areas include a more holistic view of the mine planning process, e.g., the entire supply chain, including the decisions of investing, mining, and processing (Caro et al. 2007). The authors argue that although these decisions are often made independently, they should be made simultaneously. They describe in detail the various resources necessary to extract ore from a deposit and the processes involved in obtaining saleable metal from the extracted ore; they mention the transportation issues associated with processing facilities (e.g., leaching plants versus refineries) and the nature of the markets in which the metal is sold. Other recent models address decisions involving distinct stages in mine life. In other words, these models address multiple different decisions that must be made over the life of the mine rather than decisions that are made throughout the geographical space of the production supply chain. To date, the most prominent of these temporal decisions is the transition from open-pit to underground mining (Chen et al. 2003, Newman et al. 2009). These models take a long-term view of the mine planning process, anticipating that a deposit will be mined both via surface and subsequently via underground methods; the authors seek to optimize the timing of the transition.

Bley et al. (2009) address an open-pit block-sequencing model in which ore grade is variable and an option to stockpile ore before sending it to a processing plant exists. To preserve ore grade in a stockpile, the authors use (nonconvex) blending constraints. They examine the performance of a variety of off-the-shelf commercial solvers (i.e., BARON (University of Illinois Urbana-Champaign), Couenne (Lehigh University), SBB (ARKI Consulting and Development), and SCIP (TU Braunschweig, TU Darmstadt, and Siemens AG)) to solve this problem and show that such solvers can obtain very good solutions, thus precluding the need for specialized solution algorithms. This work exhibits the important point that commercial solvers are becoming more powerful; thus, they can be used more easily and effectively on complex problems.

We have presented a review of OR mining literature. Researchers propose different techniques to

handle various strategic-, tactical-, and operational-level decisions. Many authors use case studies to demonstrate the advantages of OR, which has played a particularly important industry role in long-range planning. Industry has begun to incorporate the ideas mentioned here in software. For example, MineMax (2006) and Maptek (2009) use CPLEX to provide underlying optimization algorithms to determine block-sequencing schedules to maximize NPV subject to sequencing and operational constraints. Such software, which can account for blending and cutoff grade, also contains underground schedulers. Whittle (2009) and Gemcom (2009) address many of these elements; the former includes an optimization model that considers uncertainty. Datamine (2009) focuses on pushback design and production-scheduling optimization in open-pit mines. Unfortunately, because the details of these models are proprietary, stating the types of decision variables, objectives, and constraints the underlying optimization models have or how they perform optimization is impossible. However, academics are increasingly seeking ties with companies in the mining industry; to some extent, they are also openly incorporating the state of the art into real mining operations at the level of detail that individual companies seek.

The direction of research in the mining industry is toward solving larger and more complicated (i.e., more detailed and realistic) models faster. Specifically, we see a trend in the open-pit block-sequencing literature that abandons the traditional approach in which models are solved in stages and adopts one in which researchers successfully solve large-scale life-of-mine models in their monolithic form. With advances in hardware and software, researchers are exploring methods to exploit problem structures to tackle more detailed (e.g., stochastic), more complex (e.g., nonlinear), and larger problems. These methods include limiting the size of very large problems and subsequently demonstrating that these limits do not significantly compromise the solution quality. Some researchers are using heuristics based on aggregation, including optimization-based heuristics; others are using exact decomposition techniques such as Lagrangian relaxation. This trend is also being followed in the underground arena; however, the more complex sequencing constraints currently preclude

fast, exact solutions for large models. We anticipate a trend in both open-pit and underground mine planning toward models that are more realistic and faster to solve and that the industry will implement to an ever-increasing extent.

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