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Integer programming in forestry

Andres Weintraub P.

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Introduction

After finishing my Ph.D. in Berkeley in 1971, I needed a job in the San Francisco Bay Area, while my wife was finishing her Ph.D. in statistics. It was a tough year jobwise and the only available position was a part-time job with the US Forest Service Station in Berkeley, which funded a position as a Research Engineer at the OR Center at UC Berkeley.

My boss and friend up to today, Daniel Navon had just developed TIMBERAM (Navon, 1971), the first widely used LP model for forest planning. There was even a US Congress directive compelling the use of FORPLAN in the next 10 year planning cycle. One of our main projects was extending TIMBERAM to include decisions on road building. Road building is not a minor issue, as it accounts for about 40% of operational costs. The planning process in those days was done separately for road building and harvesting. Road engineers took as input the harvesting plans from foresters and built a road network to access areas as they needed to be harvested. Intuitively this seemed suboptimal, and was proved so later by Jones, Hyde III, and Meacham (1986). So, it seemed a good idea to try a model that integrated harvesting and road building, which was my first project.

My relation with the US Forest Service lasted almost two decades, most of them part time as I went back to Chile in 1974 to teach at the University of Chile. It was a complex bureaucratic, but legal set up. I would leave time cards signed with the secretary for about 25% time, and she would hand them in every month, while I was in Chile. I would do the work there, and come to Berkeley about twice a year. Stuart Dreyfus was the academic formally responsible, and he was always very helpful. I had been his TA in Dynamic Programming and we met several times for dinner while I was a student, so it was a comfortable

A. Weintraub P. (✉)
University of Chile, Industrial Engineering Republica 701. Santiago, Chile
e-mail: aweintra@dii.uchile.cl

relation. We would discuss briefly the projects when we met and then talk about general topics.

I have continued working in forest modeling up to now. That 1970's harvesting-roading problem was one of the first ones of spatial nature, leading to MIP models where 0-1 decisions reflected specific decisions on the ground.

In the years to come, spatial problems would become very prevalent in forest planning, driven in part by the need to locate spatial activities like road building, harvesting machinery and mostly due to the growing importance of environmental concerns, such as protection of wildlife, which leads to spatial constraints of how areas can be harvested.

I would like to discuss in this note how OR methods have been used to solve these problems, and the algorithmic challenges involved in them, and the people I have been fortunate to meet.

Many authors have worked on these problems. While this note will concentrate the work we have carried out, I will try to point out at least some of the relevant global contributions.

1 The road building problem

Deciding on specific road building to access harvest areas as needed was started in the mid-70's. The work we did with the US Forest Service was one of the first attemps at solving this MIP problem (Weintraub and Navon, 1976).

A more efficient approach was proposed by Kirby, Hager, and Wong (1986) and models since then have followed essentially that pattern, with constraints allowing flow on arcs up to capacity only if the corresponding road was built, while flow conservation insures that roads need to be built to access harvest areas. This mixed-integer problem can be difficult to solve. While I went back to Chile in 1974, my association part time with the US Forest Service continued. I spent a sabbatical in Berkeley, 1977 on these projects, plus teaching a class at UC Berkeley in the IEOR Department. By that time I started working also with Mel Kirby and later, Greg Jones from Montana. The roading was one of the main problems we worked on. In the 80's we solved it combining LP with heuristics to round of fractional 0-1 variables and obtain approximate solutions. This model was used by the US Forest Service to support decisions in a planning cycle in several regions (Weintraub et al., 1994).

In the 1990's we worked on a similar harvesting and road building problem for a Chilean forest firm, Millalemu. The system was implemented and used successfully for tactical planning. Also, Millalemu at times purchased the right to harvest timber lands owned by third parties. The value of such a right depends significantly on how it integrates into the overall operation. For example, distance to mills is relevant due to high transportation costs. Running the model with and without that specific timber land gave a value to the firms of those rights (Epstein et al., 1999). The system implemented was again based on LP and heuristics. Difficult instances appeared when there were many potential roads, forming multiple cycles. We could solve this problem quite well using strengthening of the formulation through capacity reduction on roads, adding logical constraints, logical lifting, and Lagrangean relaxation.

Using all these tools, we could improve for difficult instances from a gap of around 40% in 10 hours to a gap of 1% in 1 hour (Andalaft et al., 2003).

The work in Chile involved several students; Nicolas and Pablo Andalaft, and a colleague from Wharton, Monique Guignard, with whom I had started collaborating in a relation which has been fruitful till today, and has developed into a close friendship, which includes her husband, Kurt Spielberg. They are frequent visitors to Chile, where Monique often brings her PhD students and I have also been a glad guest at their home in Philadelphia. With Kurt, among other things we share a passion for football (soccer if you wish). Monique and I have gone together to many conferences, many in Latin America where she is by now an honorary citizen, often including our students.

Other work in solving this problem includes among others (Clark, Meller, and McDonald, 2000). We feel that with present computer power and the above described algorithmic enhancements this problem is solvable for most difficult, large instances.

2 The machine location problem

A typical harvesting problem is that of locating the harvesting machinery and building access roads. In steep areas towers or cable logging bring logs up to the road while in flat area tractors are used. A road network to access the machines needs to be built.

This is a practical problem that is traditionally solved by a forest engineer using topographical maps. In the early 90's we built a system based on GIS information on topography, timber existences and geographical accidents. The system was based on a friendly visual interactive input and heuristics to obtain solutions, and was considered the most advanced for this problem.

This development was led by Rafael Epstein and John Sessions of Oregon State had also an important participation. The system is used by Chilean forest firms and one in Colombia Epstein and Weintraub (2005). These projects, with others used by Chilean forest firms on short term harvesting and transportation won us the INFORMS Edelman Prize Competition of 1998 (Epstein et al., 1999).

Rafael Epstein was a student of mine in Chile, and after his PhD at MIT joined our academic staff. We have been partners in joint projects of all kinds for two decades now. Most of our projects, many in forestry, involve use of Integer Programming. I believe he is one of the top researchers in the world in his capacity to take a real, complex problem, in the frontier of knowledge and develop it successfully. In an area where a large percentage of such projects fail, his batting average is about 1.0, that is practically all the projects are successfully implemented and running after years. We have become close friends and share many hours on projects, research, academic issues of our department, and discussions on life in general, including soccer.

The machine location problem can be viewed as a combination of a plant location problem (where the locations of machines act as plants and timber cells are the customers) and a fixed costs network flow problem for the roading part. This is a difficult problems and we have not been able to solve it in an exact formulation. One approach, using Lagrangean relaxation and strengthening of the formulation could only solve moderate size problems (Vera et al., 2003). We had very good results using Tabu Search, where we could get solutions close to upper bounds in small CPU times (Diaz et al., 2004).

3 The spatial harvesting problem

Basic forest planning models are LP models which allow to decide which areas to harvest to satisfy demand in each period.

In the 70's a growing concern appeared in particular in native forests in relation to environmental concerns. These considerations include wildlife protection, avoiding erosion, water quality, preserving scenic beauty.

It is not clear how to express these multiple concerns in harvesting patterns. However, proxy management options have been established. The most common is the called adjacency or maximum opening harvesting constraints. These establish the maximum contiguous area that may be harvested in a given time period. Neighboring areas can then be harvested only when the trees in the first area have grown to a minimum size. Reasons for this measure include prevention of erosion, scenic beauty and wildlife protection. For example, elks will not feed on openings created by harvesting, unless they are relatively near the protection provided by grown trees.

Mandated maximum contiguous areas that can be harvested are between 30 and 50 hectares for different countries and regions. Forests are typically composed of basic indivisible cells of 3 to 20 hectares. Traditionally, a forest engineer supported by a GIS would construct harvesting blocks within the maximum area allowed. Then, harvesting patterns are feasible only if rules are imposed which do not allow harvesting adjacent blocks in the same period (or a couple of periods if trees require longer green-ups). When adjacency constraints are imposed, we are dealing with 0-1 LP models, as a block is completely harvested or not in a given period. In order to solve these problems as mandated, solution approaches were developed based on heuristics. In the 80's Montecarlo Simulation was used, and replaced by Metaheuristics, mostly Tabu search from the 90's on.

Proposals to solve exact formulations of the problems were also formulated. Note that a straightforward formulation is to add one constraint for each pair of adjacent blocks, to allow at most one of them to be harvested. This formulation has a very large number of adjacency constraints, and is weak, as it leads to very fractional solutions.

3.1 Model

Let $X_i^t = 1$ if block i is harvested in t.

A simple expression of the models is:

$$\text{Max} \sum_i \sum_t C_i^t X_i^t$$

$$\text{s.t.} \sum_i a_{it} \cdot x_i^t = b^t \tag{1}$$

$$X_i^t + X_j^t \leq 1 \quad (i, j) \text{ adjacent, all } t. \tag{2}$$

$$X_i^t = 0, 1 \tag{3}$$

where C_i^t is the profit of harvesting block i in t, a_{it} is the timber produced by block i when harvested in t and b^t is the demand in t. To improve on this formulation, Murray and Church (1996) proposed strengthening the formulation by replacing the pairwise constraints by clique constraints. We developed an approach based on a master problem composed of harvesting plans which were feasible with respect to adjacency. These columns were generated solving a stable set problem. Barahona, Weintaub, and Epstein (1992). One comment we received at the time was that this seemed to be the first time a stable set problem was solved in relation to a real problem. Hogason and Borges (1998) developed a dynamic programming approach to solve moderate size problems.

This theme was the thesis of Rafael Epstein, our first joint work. Francisco Barahona was a student of mine in Chile, and is currently a well known researcher in combinatorics, working at the IBM Watson Rearch Center. We have collaborated over the years, as he comes often to Chile. We also share tennis games, but I have lots of trouble getting a set from him once in a while.

In the mid 90’s a more sophisticated approach was developed, where decisions on how to form the blocks from basic cells was incorporated into the model (Murray, 1999). It could be shown that this approach led to significantly better solutions than forming the harvesting blocks a priori (Murray and Weintraub, 2002).

This however, is naturally a far more difficult problem to solve. We start with basic cells and must form blocks or clusters of contiguous cells no larger than the allowed dimensions, and the selected block must then satisfy adjacency.

These problems have been solved in practice through Metaheuristics, in particular Tabu Search (Boston and Bettinger, 2002; Barret and Guilles, 2000). Exact approaches are being proposed now.

We can view a straight forward formulation as follows. Let C be the set of all feasible clusters (adjacent cells not exceeding the maximum opening size). Two clusters are incompatible if they either share one or more cells or are adjacent. Then, if $X_s^t = 1$ when cluster s is harvested in period t, the problem can be stated as.

$$\text{Max } \sum_{s,t} C_s^t \cdot X_s^t$$

$$\text{s.a. } \sum_{\text{all } s/i \in \text{cluster } s} X_s^t \leq 1, \quad \text{for all cells } i, \text{ periods } t. \tag{4}$$

$$X_{s^1}^t + X_{s^2}^t \leq 1 \quad \text{for all cluster } s^1,\ s^2 \text{ incompatible, all periods } t. \tag{5}$$

$$x_s^t = 0,\ 1 \tag{6}$$

where C_s^t is the net present benefit of harvesting cluster S in period t. Constraints (4) insure that each cell is harvested at most once and constraints (5) allow at most one of two incompatible clusters to be chosen. This formulation again has too many constraints and is weak.

This is a new area of research and in the last few years several papers have attempted to solve this problem. We could solve reasonably well problems of moderate size defining a formulation of clusters and using clique constraints of the original graph projected onto the clusters graph (Murray, Goycoolea, and Weintraub, 2004).

An extension of this work, Vielma et al. (2005), allows to solve problems with many periods by elasticizing (relaxing and penalizing in a dynamic form) constraints which link timber production between periods.

This is an area of current high research interest, so we can expect significant contributions on it in the near future.

I have worked on this with several colleagues. I have known Alan Murray currently at Ohio State since he was a Ph.D. student at UC Santa Barbara with Rick Church. We have been collaborating for years now. More recent is the contact with David Ryan, from the University of Auckland, New Zealand. One of our first contacts was in 1999, when I was President of IFORS, the International Federation of Operations Research Societies, and I invited him to present the opening plenary at our Triennial Conference in Beijing. His talk was excellent, on the relevance of applications in OR. Our groups share a similar philosophy of giving high value to frontier applied work, and developing algorithms to solve these problems, which in most cases involve IP.

Several students have worked their thesis in this area. Marcos Goycoolea and Juan Pablo Vielma are currently doing their PhDs at Georgia Tech.

4 Extension of spatial problems

As mentioned, adjacency reflects only one form of environmental concerns. Other considerations include:

- Considering also the perimeter and area of mature tree patches, as some animal species have requirements that need either of these characteristics.
- Some planning strategies require to have corridors of grown trees linking mature
- tree patches, to allow animal movement.

These problems have only been solved via heuristics or Metaheuristics (Caro et al., 2003; Sessions and Sessions, 1991).

Felipe Caro worked on this problem as part of his thesis with me. He has just finished his Ph.D. at MIT, and is starting as an academic at UCLA.

Another obvious problem is integrating into harvest planning both adjacency constraints and road building. These problems have been approached only via heuristics and metaheuristics, and for small instances exact formulations (Richards and Gunn, 2000).

We developed an LP based rounding off heuristic for this problem with reasonable results (Weintraub et al., 1995).

5 Integer programming and South America

While this note concentrates on forestry problems, there have been many other areas where problems in IP have been relevant. Both in our case and with colleagues in Chile and South America. Other problems our group has looked at related to IP include combinatorial auctions, led by Rafael Epstein to support how contracts for meals are awarded to private firms to supply over one million meals a day for students in lower income schools. Use of IP to decide on best offers led to savings of 40 million US per year (Epstein et al., 2002).

We have scheduled also the 2005 professional soccer season. Several friends and colleagues do intensive work in this area. Nelson Maculan, a close friend for over 25 years, with whom we founded ALIO, the Latin American Association of OR Societies, along with Hugo Scolnik and Roberto Galvao in 1982, is well known for his work in Steiner trees and telecommunications: He is now Secretary of the Brazilian government for higher education. Roberto Galvao has done excellent work in location problems, Celso Ribeiro is working now in metaheuristics and in applications in sports, while Abilio Lucena has developed models for Brazilian airlines. More theoretical work on graphs is carried out by Jayme Szwarcfiter and groups in Brasil and Argentina which include Guillermo Duran, now working in our Department. Given the importance of research in IP and combinatorics, several excellent workshops have been organized in the last few years on these topics, in Brazil and Chile. In particular we had in 2003 the EURO/ALIO workshop in combinatories and applications and a 2004 workshop in Santiago on combinatories, IP and applications. A presentation at this workshop by George Neumhauser on scheduling the US baseball season led to our scheduling the soccer season in Chile, a project led by Guillermo Duran.

Overall, I think that while other areas such as non linear programming have excellent researchers, probably IP is the strongest area in our subcontinent, maybe driven by the many applications which require use of IP.

6 Conclusion

Spatial considerations have led to several forms of MIP problems. Planners commonly use commercial packages for smaller size problems or heuristics and Metaheuristics for larger problems. I hope I have been able to show that these problems, which are of applied interest are often of high algorithmic challenge, some not solved yet.

For example, most of the extensions shown can be considered an area of interesting research. At least, there is where I am going to and I hope with many friends.

References

Murray, A.T., M. Goycoolea, and A. Weintraub. (2004). "Incorporating Average and Maximum area Restrictions in Harvest Scheduling Models." *Canadian Journal of Forest Research*, 34, 456–464.

Andalaft, N., P. Andalaft, M. Guignard, M. Adrian, W. Alexis, and W. Andrés. (2003). "A Problem of Forest Harvesting and Road Building Solved Through Model Strengthening and Lagrangean Relaxation." *Operations Research*, 51(4), 613–628.

Barahona, F., A. Weintaub, and R. Epstein. (1992). "Habitat Dispersion in Forest Planning and the Stable Set Problem." *Operations Research*, 40(1).

Barrett, T.M. and J.K. Gilless. (2000). "Even-Aged Restrictions with Sub-Graph Adjacency." *Annals of Operations Research*, 95, 159–175.

Boston, K. and P. Bettinger. (2002). "Combining Tabu Search and Genetic Algorithm Heuristic Techniques to Solve Spatial Harvest Scheduling Problems." *Forest Science*, 48, 35–58.

Clark, M.M., Meller, and T.P. McDonald. (2000). "A Three-Stage Heuristic for Harvest Scheduling with Access Road Network Development." *Forest Science*, 46, 204–218.

Caro, F., M. Constantino, I. Martins, and A. Weintraub. (2003). "A 2 Opt. Tabu Search Procedure for the Multi-Period Forest Harvesting Problem with Adjacency, Green-up, Old Growth and Even Flow Constraints." *Forest Science*, 49(5), 738–751.

Diaz, A., J. Ferland, C. Ribeiro, J. Vera, and A. Weintraub. (2005). "A Tabu Search Approach for Solfing a Difficult Forest Harvesting Machine Location Problem." In Review process in *European Journal of Operational Research*.

Epstein, R., R. Morales, J. Serón, and A. Weintraub. (1999). "Use of OR Systems in the Chilean Forest Industries." *Interfaces*, 29, 7–29.

Epstein, R. and A. Weintraub. (2005). A Mixed Integer Programming Approach for Solving Machinery Location and Road Design in Forestry Planning", submitted to *Operations Research*.

Epstein, R., L. Henríquez, J. Catalán, G. Weintraub, and C. Martínez. (2002). "A Combinatorial Auction Improves School Meals in Chile." *Interfaces*, 32(6), 1–14.

Goycoolea, M., A. Murray, F. Barahona, R. Epstein, and A. Weintraub. (2005). "Harvest Scheduling Subject to Maximum Area Restrictions Exploring Exact Approaches." To appear in *Operations Research*.

Hogason, H.M. and J.G. Borges. (1998). "Using Dynamic Programming and Overlapping Sub Problems to Address Adjacency in Large Harvest Scheduling Problems." *Forest Science*, 44, 526–538.

Jones, J.G., J.F.C. Hyde III, and M.L. Meacham. (1986). "Four Analytical Approaches for Integrating Land Management and Transportation Planning on Forest Lands." Research Paper INT—361, U.S. Department of Agriculture, Forest Service, Intermountain Research Station, p. 33.

Kirby, M.W., W. Hager, and P. Wong. (1986). "Simultaneous Planning of Wildland Transportation Alternatives." In *TIMS Studies in the Management Sciences*, Vol. 21, New York, Elsevier Science Publishers, pp. 371–387.

McDill, M.E., S.A. Rabin, and J. Braze. (2002). "Harvest Scheduling with Area-Based Adjacency Constraints." *Forest Science*, 48, 631–642.

Murray, A.T. (1999). "Spatial Restrictions in Harvest Scheduling." *Forest Science*, 45, 1–8.

Murray, A.T. and R.L. Church. (1996). "Analyzing Cliques for Imposing Adjacency Restrictions in Forest Models." *Forest Science*, 42, 166–175.

Murray, A.T. and A. Weintraub. (2002). "Scale and Unit Specificaton Influences in Harvest Scheduling with Maximum Area Restrictions." *Forest Science*, 48, 779–789.

Murray, A.T., M. Goycoolea, and A. Weintraub. (2004). "Incorporating Average and Maximum Area Restrictions in Harvest Scheduling Models." *Canadian Journal of Forest Research*, 34, 456–464.

Navon, D.I. (1971). "Timber RAM... A Long-Range Planning Method for Commercial Timber Lands Under Multiple-Use Management." USDA Forest Service, Research Paper PSW—70, Berkeley, California, 22.

Richards, E.W. and E.A. Gunn. (2000). "A Model and Tabu Search Method to Optimize Sand Harvest and Road Construction Schedules." *Forest Science*, 46, 188–203.

Vera, J., A. Weintraub, M. Koenig, G. Bravo, M., Guignard, and F. Barahona. (2003). "A Lagrangean Relaxation Approach for a Machinery Location Problem in Forest Harvesting." *Journal of the Brazilian OR Society*, 23, 111–128.

Vielma, J., A. Murray, D. Ryan, and A. Weintraub. (2004). "Improving Computational Capabilities for Addressing Volume Constraints in Forest Harvest Scheduling Problems". *In Review European Journal of Operations Research*.

Weintraub, A. and D. Navon. (1976). "A Forest Management Planning Model Integrating Silvicultural and Transportation Activities." *Management Sciences*, 22(12), 1299–1309.

Weintraub, A., G. Jones, A. Magendzo, M. Meacham, and M. Kirby. (1994). "Heuristic System to Solve Mixed Integer Forest Planning Models." *Operations Research*, 42(6), 1010–1024.

Weintraub, A., G. Jones, M. Meacham, M. Magendzo, A. Magendzo, and D. Malchuk. (1995). "Procedures for Solving Mixed-Integer Harvest Scheduling Transportation Planning Models." *Canadian Journal of Forest Research*, 25, 1618–1626.