INGENIERIA INDUSTRIAL
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Financial liberalization, market structure and credit penetration

F. Balmaceda

R. Fischer

F.J. Ramírez

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F. Balmaceda — U. de Chile

R. Fischer — U. de Chile

F. J. Ramírez — U. de Chile *

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Abstract

This paper shows that the effects of financial liberalization on the credit market of a small and capital constrained economy depend on the market structure of domestic banks prior to liberalization. Specifically, under perfect competition in the domestic credit market prior to liberalization, liberalization leads to lower domestic interest rates, in turn leading to increased credit penetration. However, when the initial market structure is one of imperfect competition, liberalization can lead to the exclusion of less wealthy entrepreneurs from the credit market. This provides a rationale for the mixed empirical evidence concerning the effects of liberalization on access to credit in developing markets. Moreover, the analysis provides new insights into the consequences of foreign lenders' entry into developing economies.

*Center for Applied Economics (CEA) and Finance Center (FC), Department of Industrial Engineering, University of Chile, Av. República 701, Santiago, Chile. Tel. +56-2-978-4072. Fax: +56-2-689-7895.

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Emails: fbalmace@dii.uchile.cl; rfischer@dii.uchile.cl and felipram@gmail.com.

1 Introduction

The last three decades have witnessed a wave of financial liberalization that has increased less developed countries' access to international capital markets and to foreign financial banks. Financial liberalization was regarded as a means to foster competition in the financial sector, increasing access to credit for firms. As a result, an increased growth rate was expected. In some countries this was observed (Claessens, Demirguc-Kunt, and Huizinga, 2001; Micco, Panizza, and Yañez, 2007). In others, while the access and conditions faced by larger firms improved, the effects on smaller firms have been mixed or negative. For example, focusing on a specific type of liberalization, i.e., the entry of foreign banks, the study of Gormley (2010) for India showed a reduction in lending to small and medium enterprises after financial liberalization. What could explain this wide divergence in the results of financial liberalization? We suggest that one possible explanation for this contradictory evidence is that the impact of financial liberalization depends on the market structure of the domestic banking system prior to liberalization.

Assume that financial liberalization is defined as the entry of foreign banks coupled with an improved access to international capital markets. We show that if the domestic banking market is competitive, financial liberalization leads to the expected positive results. On the other hand, when markets are initially non-competitive, liberalization can lead to the exclusion of smaller (or weaker) businesses from the credit market, while the stronger companies are served by international banks. Thus, the *ex ante* competitive situation in the financial market can explain the diverging observations on the effects of financial liberalization.

The proponents of financial liberalization argue that the entry of foreign banks reduces costs by the use of improved financial technology because: (i) they would demand improved financial regulation; and (ii) they would be less corrupt and would not lend to related firms in banking conglomerates. Opponents of liberalization point out that foreign entrants are at a disadvantage because: (i) they have less information about clients and the legal system; and (ii) their lobbying capacity is lower than that of domestic banks. One implication of informational disadvantages is that for foreign banks, the *entrepreneurial rent* received by entrepreneurs should be higher, *ceteris paribus*, than for domestic banks. Since increased competition by foreign banks will weaken domestic banks and reduce their lending, these two effects could combine to reduce total lending in the economy post-liberalization.

This combination of advantages and weaknesses suggests that foreign banks tend to cherry pick firms with good internal accounting systems, solid financial positions and therefore a low probability of default. Given their better technology and lower costs of funds, they can outcompete the domestic banks for these clients, leaving the riskier and more information-demanding clients for domestic banks, which can apply their relatively better monitoring ability. In these conditions, each type of bank specializes in the type of firm in which it has a comparative advantage, and this should lead to an increase in efficient lending. However, this conclusion rests on the assumption that the entry of foreign banks does not increase the cost of funds for domestic banks. If the entry of foreign banks were to raise the cost of funds for domestic banks, the effects of entry would be different. In particular, if the domestic banking system is imperfectly competitive, and the cost of funds is initially kept artificially low due to the lack of alternatives for savers, post-liberalization, the increased access to the international financial markets and the increased competition for domestic funds by foreign banks leads to an increase in the cost of funds for local banks. This may lead to a reduction in lending to the riskiest clients.

There is empirical evidence for these effects. Rashid (2011), using data from 81 developing and emerging countries for the period 1995-2009, shows that foreign banks compete for deposits with the domestic banking system, and that this leads to an increase in the reliance on costlier (and more volatile) non-deposit funds by domestic banks. Moreover, the author presents evidence that increases in the share of deposits by foreign leads to a decline in credit to the private sector. Another paper, by Detriagache, Gupta, and Tressel (2008), uses data from the banking sector of 62 low income countries. They show that increased foreign bank penetration is associated with lower access to credit by the private sector and that foreign banks have a safer (less risky) loan portfolio.[1]

Why would foreign banks lend to a less risky loan portfolio than domestic banks in emerging economies? One possibility is that there are costs associated to operating in countries in environments that are different, with legal systems that are often inefficient or corrupt. In this regard, Mian (2006) has shown, using a sample of 80.000 loans in Pakistan, that the greater the physical and cultural distance between the headquarters of a foreign bank and a local subsidiary, the less willing they are to grant loans that are informationally

[1]In this regard, it is interesting to note that similar effects were observed by Berger, Kashyap, Scalise, Gertler, and Friedman (1995, pp73-80, 91-93), who review the effects of deregulation in the banking industry in the US. The authors show that following the liberalization of bank deposit rates, funding costs for banks rose and profitability fell, and that domestic bank lending fell. According to the authors, foreign banks more than offset the decline in US bank lending, but this lending was restricted to highly rated corporations, which received lower rates than the average customer of domestic banks. Small businesses were more likely to borrow from local banks and faced a steep decrease in lending. Lending fell by 34.8% and 41.9% for small and very small borrowers, respectively, in the period 1989-1994.

complex, even though they are willing to lend to sound firms requiring relational contracting. Greater distance also makes banks less likely to bilaterally renegotiate loan contracts, and less successful at recovering from defaults.

Several models have used information problems as the basis of their explanations for the observed phenomena. In their paper, Detriagache et al. (2008) propose a model in which, prior to the entry of foreign banks, domestic banks may find it convenient to reduce monitoring costs by pooling creditors, therefore implicitly providing a subsidy to weaker borrowers. When foreign banks arrive, with improved ability to screen hard numerical information, they compete in advantageous conditions for the financial needs of larger firms. This produces a segmentation of the credit market, reducing the average quality of firms being pooled by domestic banks and, under certain conditions on the monitoring cost, a pooling equilibrium is no longer feasible. In a separating equilibrium, loans become too expensive for weak borrowers, who drop out of the market. Foreign bank entry only benefits more transparent firms, while other firms are either indifferent or worse off. Rueda Maurer (2008), using the arguments of Detriagache et al. (2008), notes that different levels of credit protection may lead to credit constraints for opaque small and medium sized enterprises (SMEs), and tests this conclusion in a cross-section of 22 transition countries. Similarly, Gormley (2011) uses a model where information acquisition is relatively more expensive for foreign banks but these banks have a lower cost of funds. Initially, if screening costs are high, there is a pooling equilibrium. This equilibrium is broken by the entry of foreign firms, which cream-skim the firms that can use large amounts of capital (so that the lower financing cost of foreign banks offsets their higher screening costs), lowering the average quality of firms that remain for domestic banks. Now pooling may not be feasible and if screening costs are high, some banks exit the market, reducing lending to weaker firms.

In this paper we present a complementary explanation for the observation that foreign entry may either increase or reduce firm's access to loans. In our model it is the initial intensity of competition in the domestic financial market that drives the results. Consider a continuum of potential entrepreneurs with different wealth levels, who can invest in risky projects but whose individual assets are insufficient to carry out their projects without a loan from the credit market. Moreover, the ownership of assets by agents is private information and there is ex-post moral hazard, modeled as an incomplete verifiability of returns. The fact that a share of the returns is non-verifiable ex-post gives rise to *entrepreneurial rent*, as defined in Holstrom and Tirole (2011). The extent of entrepreneurial rent depends on factors such as the quality of the legal institutions in a country.

The moral hazard problem reduces the surplus that can be contracted with a lender to an amount smaller than the surplus of the project. This means that entrepreneurs with small amounts of wealth will not receive loans and will be unable to set up their projects, even though it would be efficient to do so, if the capital were available. Moreover, we assume that domestic banks have the advantage of facing a lower rate of entrepreneurial rents, relative to foreign lenders. This is explained by better knowledge of the idiosyncracies of the legal system, or better monitoring ability due to immediacy. However, foreign banks have the advantage of a lower cost of funds, either because they have better access to less expensive international funds, or because of better financial and administrative technology.

Consider a small, closed and capital constrained economy, so that under a competitive domestic financial market, the internal deposit rate is higher than the world rate. When the economy liberalizes, the entry of foreign banks does not change the competitive situation of the internal financial system. However, the supply of loanable funds ceases to be restricted, as the domestic financial system has access to lower cost foreign capital. Since foreign banks have a lower cost of funds, but face higher entrepreneurial rents, they lend to select wealthier entrepreneurs, whose contractible surplus relative to the loan size is higher.[2] Therefore, these agents face lower lending costs after entry through two channels: the lower financing costs after liberalization and by the fact that they operate with more efficient foreign banks. The remaining entrepreneurs who gain access to credit operate with domestic banks, which face relatively lower entrepreneurial rents than foreign banks. These entrepreneurs benefit from the lower costs of funds after liberalization because competition transfers the lower rates from banks to borrowers. Moreover, the lower costs of funds leads to an increase in the number of potential entrepreneurs with access to loans for their projects. With a supply of capital that is no longer constrained by domestic supply, there is an increase in the number of entrepreneurs that receive funding.

However, when the initial market structure is one of imperfect financial markets, the results are different. The reason is that both the credit and the deposit market are non-competitive. This means that banks pay a lower rate to savers than the rate paid in the case of perfect competition. When the economy liberalizes, the deposit rate for domestic banks rises because the foreign banks compete for deposits with the domestic monopoly bank. Again, foreign banks lend to the wealthier entrepreneurs, leaving the weaker

[2]The fact that wealthier individuals do not ask for larger loans is an artifact of the fixed investment size of the projects. For our purposes, the additional complications of a variable investment size serve no purpose.

entrepreneurs for the domestic banks. However, the increase in the cost of funds for the domestic bank may lead it to stop lending to marginal entrepreneurs, who are no longer profitable. Hence, the model is capable of explaining the conflicting evidence on the effects of liberalization on access to credit, via a mechanism that is complementary to those based on the observation that foreign entry leads to cream-skimming, making pooling equilibria inviable.

While in our formal model we have imposed interest rate equalization in order to simplify the presentation, this is not needed for our results, which depend only on the fact that under imperfect competition, liberalization could result in the costs of funds for domestic banks going up. If, on the other hand, international interest rates are lower than those paid on deposits under an oligopsonistic banking system, the negative effects of liberalization disappear, i.e., the results are analogous to those obtained under a competitive financial system. Similarly, if there are competitive alternatives for savers in the closed economy, so there is no financial repression even under imperfect competition in the banking system, we recover the qualitative results obtained under a competitive financial system.

We conduct a comparative statics analysis to investigate how changes in the various parameters of the economy alter the effects of financial liberalization. An important parameter is the entrepreneurial rent rate, which reflects, among other factors, the efficiency of the domestic legal system. When there is competition in the closed economy, a fall in the rate of entrepreneurial rents facing domestic and foreign banks increases welfare through two channels. Lower levels of entrepreneurial rents means that foreign banks can cut into the the segment served by domestic banks and with their lower cost of funds, they increase the welfare of those entrepreneurs. Similarly, when domestic banks that face a lower rate of entrepreneurial rents, firms that were excluded from the loan market now have access to it, which improves the welfare of their owners.

When domestic banks are not competitive, lowering the rate of entrepreneurial rents facing the domestic bank increases access to credit, raising welfare as before. However, when it is the foreign banks that face lower rates of entrepreneurial rents, there is an additional effect. There is a direct effect because foreign banks can lend to more firms, increasing the welfare of the owners due to their lower costs. But there is also an indirect effect, because when agents switch to foreign banks, the domestic bank begins to lend to entrepreneurs that were not being served before. Our model explains the observation of Rueda Maurer (2008), who found that foreign entry interacts favorably with increases in creditor protection (i.e., reductions in the rate of entrepreneurial rents), leading to increases in lending to the smallest SMEs.

Regarding the wealth distribution, the most interesting results are those that relate to ex ante competition. In that case, we show that more inequality in the distribution of wealth leads to higher benefits from liberalization in economies that are wealth constrained i.e., relatively capital poor. The opposite holds when the economies are relatively capital rich. The intuition is that in capital poor countries, concentrating wealth leads to a reduction in the number of agents that are excluded from the loan market because of the level of entrepreneurial rents, and in the open economy, these additional agents are not constrained in obtaining credit by the limited amounts of domestic capital.

1.1 Literature Review

There is a large empirical and theoretical literature on the effects of financial liberalization. Levine (2001) asseses theory and evidence to evaluate the effects of financial liberalization and concludes that the effects are positive. There are two channels for this effect. First, financial liberalization leads to enhanced stock market liquidity, which raises productivity growth. Second, the entry of more efficient foreign banks leads to improvements in productivity.

Even though foreign banks may be more efficient, later researchers noted that they are at a disadvantage when lending to small and medium sized enterprises (SME) which are informationally opaque, and which require relationship lending. Berger, Klapper, and Udell (2001) examine a large set of loans in Argentina and show that these firms tend to receive fewer loans from foreign banks. We have already mentioned the evidence to the same effect that appears in Mian (2006), Gormley (2010) and Detriagache et al. (2008) showing the disadvantages facing informationally opaque SMEs vis-à-vis foreign bank lending.

More ambiguously, Giannetti and Ongena (2009) uses a panel of 60,000 firm-year observations from Eastern Europe to show that foreign bank presence has a beneficial impact on all firms, excepting those that are connected to domestic banks ("crony capitalism") or to the government. However, the positive effects are smaller for SMEs. Other contradictory evidence comes from Clarke, Cull, and Peria (2005) who use data from four Latin American countries to show that while foreign banks in general lend less to SMEs, this is not necessarily the case for foreign banks which have a large presence in the economy.

Dell'Ariccia and Marquez (2004) provide a theoretical explanation for the difficulties faced by foreign banks in dealing with small informationally opaque firms. They show that first, information about the quality of informationally opaque lenders cannot be credibly transferred to outsiders, so opaque firms are captured by informed banks. The strength of the capture effect increases with the opaqueness of information. Thus, when foreign banks have a lower cost of funds, but domestic lenders are better informed about the quality of the firms, the equilibria features market segmentation, with more opaque firms served by domestic banks, while foreign banks compete for firms with lower informational disadvantages.[3]

There is also conflicting evidence on the effects of banking competition on lending. Zarutskie (2006) uses corporate tax files from the United States, to analyze the effects of increased banking competition following deregulation in 1994. She shows that firms less than five years of age and therefore observationally more opaque than older, larger firms are more credit constrained, invest less and grow slower. Since the reverse is true for firms 16 years or older, the total impact of financial liberalization is ambiguous, but probably positive, according to the author. Bonaccorsi di Patti and Dell'Ariccia (2004) use a panel of 22 industries in 103 provincial credit markets in Italy to show that increased bank market power is relatively more beneficial to firm creation in opaque industries. They argue that their results are consistent with models of asymmetric information between borrowers and lenders, and that these results are inconsistent with the traditional competitive paradigm.

Finally, regarding the effects of banking competition, the empirical evidence in Zarutskie (2006) is consistent with the theoretical model of Petersen and Rajan (1995). In this two period model, which is itself an adaptation of Stiglitz and Weiss (1981), the less competitive the banking sector, the more profits a bank can extract once it discovers that the borrower is a good entrepreneur. Hence banks operating in less competitive environments are more willing to lend to riskier, usually younger, firms. The authors test this model using the National Survey of Small Business Finances and proxy for the competitiveness of markets by using the Herfindahl index of deposits in local credit markets. Their results are consistent with the model.[4]

The next section describes the basic elements of the model and proves the conditions for potential entrepreneurs with different levels of wealth to reveal their wealth levels. Next, we examine the closed economy, beginning by the analysis of the competitive case, followed by the case with a banking monopoly. We show that a monopolist can be inefficient and not lend all available loanable finds. In section 4 we

[3]See also the models of Detriagache et al. (2008) and Gormley (2011), already mentioned.

[4]An alternative explanation is Marquez (2002), who shows that increased competition after entry can increase inefficiencies because information about the quality of borrowers is lost when a firm changes lenders, and this may translate into higher interest rates.

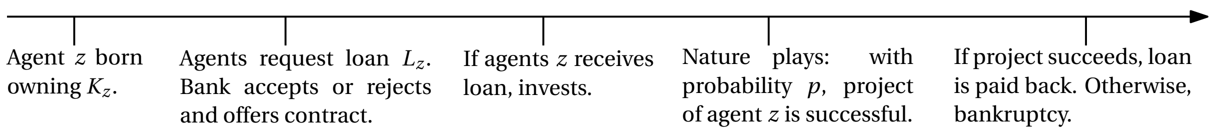

Figure 1: Time line of the model

consider the economy after financial liberalization and we analyze the cases of ex ante competition and ex ante monopoly. In section 5 we examine the interaction between the effects of liberalization and changes in the distribution of wealth. The last sections sets out the conclusions.

2 The model

We consider a one-period economy with two types of risk neutral agents: entrepreneurs and domestic bankers. Entrepreneurs are protected by limited liability. We divide the single period into four stages. In the first stage, a continuum of agents indexed by $z \in [0,1]$ are born, each endowed with one unit of inalienable specific capital (an idea, a project or an ability) and different amounts of wealth, K_z, which is privately observed. The wealth distribution is common knowledge and given by $G(\cdot)$, which has a continuous density $g(\cdot)$ and full support in $[0,1]$. We impose the following condition on the wealth distribution:

Assumption 1 *$G(x)$ is log-concave and therefore* $\frac{\partial}{\partial x}\left[\frac{g(x)}{1-G(x)}\right] > 0$

In order to carry out a project, entrepreneur z must invest a verifiable fixed amount $I > 1$, so even the wealthiest entrepreneur needs financing.

In the second stage bankers intermediate between agents, who are simultaneously owners of wealth and prospective entrepreneurs who require external funds to carry out their projects. They are the sole source of external financing in this economy. We will consider two different credit markets: a fully competitive market and a double-sided banking monopoly. The contract terms stipulates the sharing of returns between the bank and the entrepreneur and a contribution by the entrepreneur to the financing of the project.

Bank financing entails an opportunity cost of funds per dollar equal to $1 + \rho$ We assume that the cost of funds are sufficiently small so that the net present value (NPV) of the projects is positive, i.e.,

Assumption 2 $pR - (1 + \rho)I > 0$

In the third stage, if an entrepreneur receives financing, then he can invest in his risky project which succeeds with probability $p \in (0,1)$ and fails with probability $1 - p$. After success, it returns R, while it returns 0 after failure. Returns are publicly observable but partially verifiable in the following sense. Entrepreneur z can enjoy a fraction ϕ of the return, with $\phi < 1$, as private benefits.[5] The remaining share $1 - \phi$ is verifiable and thus contractible. The parameter ϕ is meant to capture the quality of the law, or the ability of banks to restrict entrepreneurial rents. Thus, the parameter ϕ vary across country-bank pairs. In order for this reduced-form problem to be interesting we will assume that entrepreneurial rents are large enough that they eliminate any possibilities of external funding when an entrepreneur requires full funding; if ρ is the cost of funds for banks,[6]

Assumption 3 $p(1 - \phi)R - (1 + \rho)I < 0.$

2.1 Preliminaries

Let K_z be the wealth of entrepreneur z and $\hat{K}_z$ be the wealth she declares to the bank. A contract for an entrepreneur who declares herself to be of type K_z is a pair $(R_e(K_z), I_e(K_z))$, where the first entry is the return that the entrepreneur receives in the event of success and the second entry is the contribution that she makes to financing of the project. Given limited liability, entrepreneur z cannot provide a higher contribution $I_e(\hat{K}_z)$ than her net worth K_z. Because of the revelation principle, we will focus on direct revelation mechanisms of the form $\{R_e(K_z), I_e(K_z)\}_{z\in[0,1]}$.

Given this mechanism, entrepreneur z's expected utility when she declares wealth $\hat{K}_z$ is

$$U(\hat{K}_z, K_z) \equiv pR_e(\hat{K}_z) + (1 + \rho)(K_z - I_e(\hat{K}_z)) - (1 + \rho)K_z, \tag{1}$$

where the first term is the expected return obtained by the entrepreneur in the case of success, the second term is the gain obtained by depositing excess funds $(K_z - I_e(\hat{K}_z)) > 0$ in the banking system (possibly via an associate, to avoid detection), and the third term is the opportunity cost of the capital K_z. Then, entrepreneur z's problem is:

$$\max_{\hat{K}_z \geq 0} U(\hat{K}_z, K_z) \tag{2}$$

That is, entrepreneur z chooses an announcement $\hat{K}_z$ so as to maximize U. All the formal proofs are

[5]Holstrom and Tirole (2011) defines them as entrepreneurial rents or limited pledgeability of the project returns. The authors describe several sources for the origin of these rents.

[6]As we show below, this assumption will lead to credit rationing in equilibrium

relegated to the appendix.

Lemma 1 *The following condition is sufficient for a contract* $\{R_e(K_z), I_e(K_z)\}_{z\in[0,1]}$ *to be implementable:*

$$pR_e'(\hat{K}_z) - (1+\rho)I_e'(\hat{K}_z) = 0 \tag{3}$$

and this holding for $\hat{K}_z = K_z$ *constitute necessary conditions for implementability.*

Observe that truthful revelation requires that each entrepreneur gets the same expected utility from each possible contract she accepts, so they are all equivalent to her. This follows from the fact that the marginal utility from any declaration $\hat{K}_z$ is independent of the true wealth level K_z and therefore different entrepreneurs have the same incentives to lie. Technically speaking, the single-crossing property does not hold.

It is useful to observe that non-publicly observable wealth plays an important role when we study the case of a non-competitive financial market before liberalization. The reason is that if wealth were observable, a monopoly bank would charge a higher rate to wealthier entrepreneurs. This is at odds with the empirical and casual evidence showing that the interest rate falls with the wealth of an entrepreneur. This will have consequences on the type of entrepreneurs that are excluded from the market.

3 The closed economy

Initially the economy is closed, so there is no access to international capital nor foreign banks. The only possible source of funds for entrepreneurs is the domestic banking system. We begin by examining the equilibrium under perfect competition and then under a monopoly bank.

3.1 The Competitive Case

In this case, there is free entry of banks and we have to define a competitive equilibrium. The most natural equilibrium concept in this setting is the one described by Rothschild and Stiglitz (1976). Under this concept, a menu of contracts $\{R_e(K_z), I_e(K_z)\}_{z\in[0,1]}$ is an equilibrium if, holding this menu of contracts unchanged, no firm can enter offering a different menu and making positive profits, and no firm prefers to withdraw from the market or change its menu of contracts. In other words, the competitive equilibrium is a Nash equilibrium in a game among banks where the pure strategies of the banks are menus of contracts.

In equilibrium, banks must make zero expected profits; that is,

$$p(R - R_e) + (1 + \rho)(I - \lambda I_e) = 0$$

where $\lambda \in (0,1)$ is a parameter which is introduced to create a gap between the agent's valuation of his contribution to the project and the valuation of this investment by the bank. This parameter can be interpreted as a transaction cost: a cost of evaluating the real value of the agent's contribution to the project, or when the entrepreneurs investment consists of a real asset –real estate– a legal cost of transferring this guarantee to the bank. The usefulness of this parameter is that it eliminates the continuum of payoff equivalent optimal contracts that arises when $\lambda = 1$.[7] As argued below the elimination of this type of multiplicity has no consequences for the qualitative results of the paper, but it simplifies the presentation of the results.

To see why in equilibrium zero expected profits are optimal, suppose that in equilibrium there exists a bank that offers a menu of contracts in which at least the contract designed for the entrepreneur of type K_z makes positive profits for the bank, i.e., $p(R - R_e(K_z)) - (1 + \rho)(I - \lambda I_e(K_z)) > 0$. Consider now an entrant that offers a degenerate menu with the contract $\{R_e(K_z) + \epsilon, I_e(K_z)\}$, with $\epsilon > 0$. It is clear that entrepreneurs who claim to be of type K_z in the absence of the entrant will switch to this new contract (and maybe other entrepreneurs will also switch). Because the bank's payoff is continuous in $R_e(K_z)$, there is an $\epsilon > 0$ such that this new contract is profitable, which destabilizes the candidate equilibrium. Therefore, in equilibrium there cannot be a menu of contracts that makes positive profits to the bank. That is why in a competitive equilibrium, a bank must offer the most advantageous menu of contracts for entrepreneurs subject to the constraint that that the bank makes non-negative profits.

Because only a share $1 - \phi$ is pledgeable to the bank, $R - R_e(K_z) \leq (1 - \phi)R$, which implies the following limited pleadgability constraint: $R_e(K_z) \geq \phi R$. Thus, the problem faced by a competitive bank is:

$$\underset{R_e(K_z), A, I_e(K_z)}{\text{Max}} \int_A^1 [pR_e(K_z) - (1 + \rho)I_e(K_z)] dG_z \tag{PCCl}$$

subject to $\forall K_z \in [A, 1]$

[7]We thank an anonymous referee for this suggestion.

$$\int_A^1 [p(R - R_e(K_z)) - (1+\rho)(I - \lambda I_e(K_z))]dG_z \geq 0, \tag{IRB}$$

$$R_e(K_z) \geq \phi R, \tag{LPC}$$

$$R \geq R_e(K_z), \tag{LL1}$$

$$pR_e(K_z) + (1+\rho)(K_z - I_e(K_z)) \geq (1+\rho)K_z, \tag{IRE}$$

$$pR_e'(K_z) - (1+\rho)I_e'(K_z) = 0, \tag{TTE}$$

$$K_z \geq I_e(K_z), \tag{LL2}$$

where the decision variable A represents a level of wealth such that only entrepreneurs with wealth $K_z \geq A$ receive financing.

Condition (IRB) is the bank's participation constraint. Condition (LPC) is the limited pledgeability constraint. Condition (LL1) captures that there is a limit to the maximum repayment in case of success of the project. Condition (IRE) is the entrepreneur's participation constraint, while condition (TTE) is the truth-revealing equation. Finally, condition (LL2) limits the contribution to the project of an entrepreneur to at most, her own wealth.

We need an additional assumption guaranteing that the transaction costs are such that in equilibrium there will be entrepreneurs that obtain financing from domestic banks. For this we require that[8]

Assumption 4 $pR(1-\phi) + pR\phi\lambda - (1+\rho)I > 0$

The following result records the key features of the solution.

Proposition 1 *Suppose that the cost of capital is fixed at ρ. Then, in a closed competitive economy,*

i) Only entrepreneurs with wealth

$$K_z \geq K(\rho) \equiv \frac{1}{\lambda}\left[I - \frac{pR(1-\phi)}{1+\rho}\right] \tag{4}$$

obtain loans.

[8]See the proof of proposition 1 for the need for the condition. Note that the LHS in the condition increases as the transaction costs fall; in the limit it implies that the project is welfare improving: when $\lambda = 1$ the condition becomes $pR - (1+\rho)I > 0$.

ii) For the entrepreneur z, there exist a unique contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \{\phi R, K(\rho)\}.$$

Assumption 3 on page 10 implies $K(\rho) > 0$. Therefore, there will be credit rationing in equilibrium, in the sense that not all the viable projects will be funded (in particular, those of entrepreneurs with less wealth). This is the most efficient allocation of scarce capital in the economy, yielding the highest aggregate surplus.

Observe that there is only one incentive compatible contract for all agents that have access to loans, and that the contract provides the same return in the event of success $R_e(K_z) = \phi R$ and the same required contribution to the project $I_e(K_z) = K(\rho)$. In other words, there is a pooling equilibrium in the credit market. This result is a special case of the contracts that appear when we assume $\lambda = 1$, in which case there exist a continuum of payoff equivalent contracts for each entrepreneur with wealth $K_z \geq K(\rho)$, and for each bank. The intuition is that when $\lambda = 1$ there is no deadweight loss associated with the entrepreneur's contribution to the project so an increase in $I_e(K_z)$ can be always be compensated with an equivalent increase in $R_e(K_z)$ such that both the bank and the entrepreneur are indifferent. The introduction of a small transaction cost eliminates this multiplicity without qualitatively affecting the results.

To ensure that the problem is interesting, we must assume that not all entrepreneurs are credit constrained, i.e., $K(\rho) < 1$. This requires

Assumption 5 $pR(1 - \phi) - (1 + \rho)(I - \lambda I) > 0$.

Now, let us consider the credit market equilibrium in this economy, by equating the supply of loanable funds from agents to the demand for funds. In the closed economy, the supply of loanable funds is independent of the interest rate and given by:[9]

$$K_S \equiv \int_0^1 K_z dG \tag{5}$$

Letting the superscript "*bc*" on a variable stand for banking competition, the demand for loans is the sum

[9]It is slightly more complex to have the supply of loanable funds be an increasing function of the interest rate. However, for the purposes of this paper, nothing is gained by adding this complication.

of the investments made by all entrepreneurs that are funded:

$$K_D^{bc} \equiv I\left[1 - G(K(\rho))\right] \tag{6}$$

Since $\frac{\partial K_D^{bc}}{\partial \rho} = -Ig(K(\rho))\frac{\partial K(\rho)}{\partial \rho} < 0$, the demand curve for capital is downwards sloping. We assume that at the minimum interest rate of zero, there is excess demand for capital, and therefore there is a unique and positive competitive interest rate ρ_{bc} that equates supply and demand. Hence,

Proposition 2 *Suppose that assumptions 2, 3, 4 and 5 hold. Then, in a closed competitive economy, only entrepreneurs with wealth $K_z \geq K(\rho_{bc})$ receive loans.*

Note that the competitive equilibrium just studied is efficient in the sense that all the capital in the economy is employed in socially productive projects.

It is interesting to examine the effects of changes in the various parameters of the economy on the resulting credit market equilibria. As in the closed competitive economy all the capital is allocated to productive projects, the mass of firms receiving loans does not change in response to changes in the parameters of the model. This happens because the supply of loanable funds is completely inelastic in the interest rate, so the only effect of a change in a parameter that shifts the demand curve for capital is to change the equilibrium interest rate ρ_{bc}.[10] We have the following result:

Corollary 1 *In a closed competitive economy, we have that $\frac{\partial K(\rho_{bc})}{\partial \phi} = 0$ and $\frac{\partial K(\rho_{bc})}{\partial \lambda} = 0$.*

3.2 The Monopoly Case

In order to simplify the analysis we consider the benchmark case of a banking monopoly.[11] We assume that the bank operates as a double-sided monopoly: it is a monopoly in the loan market and a monopsony in the deposit market. It is the only linkage between these two markets. The two sided monopoly will exploit both sides by pricing appropriately: a low price to savers and a high price to borrowers.[12]

[10]As observed by an anonymous referee, an increase in pledgeability $(1 - \phi)$ raises interest rates, which appears surprising at first sight. The effect occurs because the increase in pledgeability increases the number of potential borrowers at the original interest rate, i.e., it shifts up the demand for loans. Qualitatively, this effect does not depend on the elasticity of the supply curve for loans. However, if the increase in pledgeability were to increase the supply of capital directly, i.e., by shifting the supply curve directly, increased pledgeability might lead to a reduction in the domestic equilibrium interest rate. For a formal analysis of this point, see the appendix. Note that our main results do not depend on the comparative statics of the increase in pledgeability.

[11]Due to the logic that drives our results, the results extend to the case of imperfect competition, and in particular, with no change whatsoever, to a cartel of banks, since it replicates monopoly behavior.

[12]We have noted in the introduction that there is empirical evidence that in non-competitive markets the rate paid on deposits is lower than the rate under competitive conditions.

The problem faced by the monopoly bank is:

$$\underset{\{\rho, R_e(K_z), A, I_e(K_z)\}}{\text{Max}} \int_A^1 p(R - R_e(K_z)) - (1+\rho)(I - \lambda I_e(K_z))dG_z \tag{PMCl}$$

subject to $\forall K_z \in [A, 1]$

$$R_e(K_z) \geq \phi R, \tag{LPC}$$

$$R \geq R_e(K_z), \tag{LL1}$$

$$pR_e(K_z) + (1+\rho)(K_z - I_e(K_z)) \geq (1+\rho)K_z, \tag{IRE}$$

$$pR_e'(K_z) - (1+\rho)I_e'(K_z) = 0, \tag{TTE}$$

$$K_z \geq I_e(K_z), \tag{LL2}$$

$$K_S \geq I(1 - G(A)) \tag{CA}$$

The objective function is the expected utility of the bank when it grants credit to all entrepreneurs with wealth levels equal to or higher than A. Condition (LPC) is the limited pledgeability constraint, while equation (LL1) is the maximum repayment constraint. Condition (IRE) is the entrepreneurs' participation constraint. Condition (TTE) is the truth-revealing equation, while condition (LL2) ensures that entrepreneurs cannot contribute more than their wealth. Finally, condition (CA) reflects the fact that the total investment cannot be greater than the economy's aggregate capital stock.

We define the auxiliary function $\varphi(x) \equiv x - \frac{1-G(x)}{g(x)}$, which will play an important role in the following. Assumption 1 on page 9 allows us to characterize the truth-revealing equilibrium:

Proposition 3 *Suppose that assumptions 1, 2, 3, 4 and 5 hold. Then, in a closed economy with a monopoly bank,*

i) The monopoly bank sets the cost of funds at $\rho_M = 0$.

ii) Only entrepreneurs with wealth $K_z \geq K_M$ *obtain funding, where* K_M *is determined from:*

a) If $\varphi(K(\rho_{bc})) \geq K(0)$*, then the monopolist is efficient, in the sense that* $K_M = K(\rho_{bc})$.

b) If $\varphi(\min\{1, p\phi R\}) > K(0) > \varphi(K(\rho_{bc}))$*, the monopolist is inefficient, with* $K_M \in (K(\rho_{bc}), \min\{1, p\phi R\})$*, determined from* $\varphi(K_M) = K(0)$.

c) If $K(0) \geq \varphi(\min\{1, p\phi R\})$*, then the monopolist is inefficient, and* $K_M = \min\{1, p\phi R\}$.

iii) For the entrepreneur z, there exist a unique contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \{\phi R, K_M\}.$$

As expected, the monopoly bank sets the deposit rate at its lowest level, since it enjoys monopsony power over savers and the supply of funds is completely inelastic in the deposit rate. Also, note that in the second and third cases (of the second point) of proposition 3 the monopolist is inefficient in the sense that it provides less credit to the economy than under a competitive banking system. Essentially, the bank restricts the supply of funds (even though they have zero cost) because in this model, the truth-revealing condition plays a role that is analogous to that of the marginal revenue curve of a standard monopoly. The comparative statics analysis leads to:

Corollary 2 *In a closed economy with a monopoly bank,*

i) $\frac{\partial K_M}{\partial \lambda} < 0$ *if* $\varphi(K(\rho_{bc})) < K(0) \leq \varphi(\min\{1, p\phi R\})$ *and* $\frac{\partial K_M}{\partial \lambda} = 0$ *otherwise.*

ii) If $p\phi R \geq 1, \frac{\partial K_M}{\partial \phi} > 0$ *if* $\varphi(K(\rho_{bc})) \leq K(0) < \varphi(1)$ *and* $\frac{\partial K_M}{\partial \phi} = 0$ *otherwise.*

iii) If $p\phi R < 1, \frac{\partial K_M}{\partial \phi} > 0$ *if* $\varphi(K(\rho_{bc})) \leq K(0)$ *and* $\frac{\partial K_M}{\partial \phi} = 0$ *otherwise.*

Therefore we have that in the case of closed economies where the monopoly bank is inefficient[13] an increase in the cost of transaction ($\lambda \downarrow$) lowers the efficiency of the economy, since fewer loans are provided. Similarly, a reduction in credit protection (higher ϕ) leads to more inefficiency.

[Place Figure 2 here]

Figure 2 shows the effect of a variation in the structural variables: the cost of collateral and of the level of entrepreneurial rent. In the vertical axis we have the wealth K_z of the last agent that receives a loan. On the abscissae we have the parameters $1 - \lambda$ and ϕ. Under competition these have no effect. Under monopoly, however, there is point at which the economy becomes inefficient because the monopoly bank ceases to provide all available capital to entrepreneurs.

[13]But not too inefficient, $K(0) > \varphi(\min\{1, p\phi R\})$.

4 The Open Economy

We now proceed to the open economy case. Again here we consider the case of domestic competition and monopoly. In our definition, financial liberalization implies that the banking system is no longer limited by the internal capital stock. We assume that the banking system has access to capital from international financial markets at a fixed rate ρ^*. Similarly, entrepreneurs as providers of funds can lend their wealth at the rate ρ^*, so that the cost of funds for the monopoly will increase.[14] We assume that entrepreneurs can borrow only from a domestic or a foreign bank (they cannot get a credit directly from international financial markets at the rate ρ^*).

Foreign banks enter the domestic market by taking over existing local banks or by setting up new subsidiaries. We assume that foreign banks are risk neutral and competitive. Compared to domestic banks, foreign banks face more adverse information constraints when they provide credit to domestic agents. For instance, these constraints may include higher costs to provide monitoring services or more effort needed to identify potentially good borrowers. [15]

As we mentioned in the introduction, there is ample evidence showing that foreign banks are handicapped in providing services that require a close association with domestic firms. Mian (2006) suggests that when a foreign bank opens a branch in another country, it faces relatively higher costs in providing loans that require close knowledge of domestic borrowers. More precisely, the evidence shows that foreign banks face difficulties in lending to small and medium enterprizes given the opacity of their financial conditions. Berger et al. (2001) argue that the distance between the CEO of the foreign bank and the domestic firms, the different local commercial cultures or institutional barriers, constrain the contacts between foreign banks and the domestic firms. In addition, Detriagache et al. (2008), and Gormley (2010) suggest that domestic banks have better knowledge of local conditions and therefore can forge closer associations with domestic firms.

[14]The fact that the rate paid on savings is equal to the international rate is a simplification. All that is required for our results is that the rate paid on domestic savings increases after liberalization, reflecting the increased competition for deposits by foreign banks.

[15]A foreign bank may buy a domestic bank to try to obtain local talent, connections and knowledge. Thus, one may think that this particular mode of entry may be able to transfer the local market know-how. However, this strategy is not totally successful; for instance, Detriagache et al. (2008) write that:

> *"Even when foreign banks enter by purchasing local banks, local market knowledge and relationships with customers may be lost as distant managers need to impose formal accountability to monitor local loan officers".*

.

To incorporate this evidence into the model in a simple manner, we assume that the ex-post moral hazard problem of entrepreneurs is more severe when they receive financing from a foreign bank. In that case, the entrepreneur is guaranteed a fraction ϕ^* of the return of the project, with $\phi^* > \phi$.

Given this disadvantage, foreign banks must have a countervailing advantage to be able to operate in the domestic market. One possibility is that foreign banks are more efficient than domestic banks. The evidence appears to confirm this intuition. Fathi (2010) uses observations of 1770 active banks in 54 developing countries during the period 1993-2001, to find that foreign banks are more efficient than domestic banks, where the level of banking efficiency is estimated using the stochastic frontier approach. This evidence will be incorporated into the model assuming that foreign banks enjoy a lower cost of funds than domestic banks, as in Gormley (2011). In particular, we assume that while the cost of funds for foreign banks is ρ^*, the cost of funds for domestic banks is higher: $\rho^* + \Delta^*$. The spread $\Delta^* > 0$ is incurred by domestic banks (i.e., both domestic banks and foreign banks pay the same rate ρ^* to depositors) and is a measure of lower competitiveness. The higher cost of funds of domestic banks may represent both technological and managerial differences between foreign and domestic banks as well as a risk premium due to more uncertainty about the financial condition of domestic banks.[16]

Given these differences between domestic and foreign banks, we study the effects of the entry of foreign banks. As foreign banks have lower cost of funds than domestic banks, they can outcompete domestic banks by charging a lower interest rate on loans. Since foreign banks are assumed to be competitive, we can use the same argument of section 3.1 to argue that in the equilibrium the profits of foreign banks cannot be positive. This means that entrepreneurs always prefer to be financed by foreign banks. Formally, the problem facing a foreign bank is (an asterisk on a variable indicates that it refers to a foreign bank):

$$\underset{R_e^*(K_z), A^*, I_e^*(K_z)}{\text{Max}} \int_{A^*}^{1} [pR_e^*(K_z) - (1+\rho^*)I_e^*(K_z)]dG_z \tag{PFB}$$

subject to $\forall K_z \in [A^*, 1]$

$$\int_{A^*}^{1} [p(R - R_e^*(K_z)) - (1+\rho^*)(I - \lambda I_e^*(K_z))]dG_z \geq 0 \tag{IRB}$$

$$R_e^*(K_z) \geq \phi^* R, \tag{LPC}$$

[16]The results in this paper can also be obtained when the difference between domestic and foreign banks is a lower fixed cost of loan origination, without requiring a difference in funding costs (see the working paper version). This shows the robustness of the economic argument for the results.

$$R \geq R_e^*(K_z), \tag{LL1}$$

$$pR_e^*(K_z) + (1+\rho^*)(K_z - I_e^*(K_z)) \geq (1+\rho^*)K_z, \tag{IRE}$$

$$pR_e^{*'}(K_z) - (1+\rho^*)I_e^{*'}(K_z) = 0, \tag{TTE}$$

$$K_z \geq I_e^*(K_z), \tag{LL2}$$

Solving this problem, we obtain the following result:

Proposition 4 *After liberalization,*

i) Only entrepreneurs with wealth $K_z \geq K^* \equiv \frac{1}{\lambda}\left[I - \frac{pR(1-\phi^*)}{1+\rho^*}\right]$ *obtain loans from foreign banks.*

ii) For the entrepreneur z, there exist a unique contract of the form

$$\{R_e^*(K_z), I_e^*(K_z)\} = \{\phi^* R, K^*\}.$$

As it is cheaper to obtain a loan from foreign banks, an immediate effect of financial liberalization is that entrepreneurs with sufficient capital will switch to foreign banks. Agents with fewer assets will continue to operate with domestic banks, because these institutions are better able to contain and handle the agency problems of domestic agents, by limiting the extent of entrepreneurial rents. We now examine the effects of liberalization considering the different initial market structures.

4.1 *Ex-ante* competition

In this case, financial liberalization does not change the competitive situation in the banking system. This means that we can use the result obtained in Proposition 1 (but now using $\rho^* + \Delta^*$ as the cost of capital), to show that entrepreneurs with wealth $K_z \leq K(\rho^*) \equiv \frac{1}{\lambda}\left[I - \frac{pR(1-\phi)}{1+\rho^*+\Delta^*}\right]$ will not receive loans. To make the problem interesting, the following assumption ensures that there is a range of agents who only have access to loans from domestic banks in the open economy (otherwise the market share of domestic banks would go to zero after liberalization):

Assumption 6

$$K^* > K(\rho^*) \Leftrightarrow \frac{(\phi^* - \phi)(1+\rho^*)}{1-\phi^*} > \Delta^*$$

Hence we have the next result:

Proposition 5 *Suppose that assumptions 2, 3, 4, 5, and 6 hold. With* ex ante *financial competition, financial liberalization means that entrepreneurs with wealth $K_z \geq K^*$ operate with foreign banks. Entrepreneurs with wealth in the range $K^* > K_z \geq K(\rho^*)$ must operate with domestic banks.*

4.1.1 Comparative statics

We now examine the effect of changes in the parameters of the open economy, when we have *ex ante* perfect competition. It can be shown that:

Lemma 2 $\frac{\partial K(\rho^*)}{\partial \rho^*} > 0$, $\quad , \frac{\partial K(\rho^*)}{\partial \phi} > 0$, $\quad \frac{\partial K(\rho^*)}{\partial \Delta^*} > 0$*, and* $\frac{\partial K(\rho^*)}{\partial \phi^*} = 0$*. and* $\frac{\partial K(\rho^*)}{\partial \lambda} = 0$.
Moreover: $\frac{\partial K^*}{\partial \rho^*} > 0$*; and* $\frac{\partial K^*}{\partial \phi^*} > 0$.

Thus we have that in open economies where the domestic banking system is competitive, increases in the cost of funds of domestic banks (ρ^* or $\Delta^* \uparrow$), the cost of transaction ($\lambda \downarrow$), or an increase in the rate of entrepreneurial rents from domestic bank funded projects ($\phi \uparrow$), will lead to a less efficient economy, since fewer entrepreneurs obtain loans. On the other hand, a change in the rate of entrepreneurial rents from foreign bank funded projects does not change the number of entrepreneurs that receive loans. The reason is that changes in this parameter affects only the number of entrepreneurs that obtain loans from foreign banks. An increase in the entrepreneurial rents affecting foreign banks raises the value of K^*, and therefore the source of funding of entrepreneurs shifts towards more expensive local banks, but does not change the access to funding. Observe that a change on the international interest rate ρ^* decreases access to loans as well as the number of entrepreneurs with access to less expensive foreign bank loans.

4.2 *Ex ante* Monopoly

This is the interesting case, in the sense that it may lead to the counterintuitive result that financial liberalization may lead to a decrease in credit market penetration, as agents with less wealth are excluded from credit. As we have mentioned, this has been empirically observed by several authors, among others Gormley (2010), Rashid (2011). Similar effects were observed in the US after financial liberalization and the entry of foreign banks (Berger et al., 1995).

More specifically related to the effects of entry in markets with *ex ante* imperfect competition, McCall and Peterson (1977) study the effects of entry in rural financial markets in the US with fewer than three banks. They find large decreases in the profits of established banks (as expected), but also a higher rate paid on deposits, and therefore an increase in the cost of funds of incumbent banks. One year after entry they observed a large raise in the interest rate paid on time deposits and a change in lending policies, with a larger fraction of assets directed at commercial loans and a reduction on the amount of consumer loans. Consumer loans are often used by small businesses as a source of credit.[17] More recently, in their review of the deregulation of the US banking industry, Berger et al. (1995) conclude that banks lost their monopsony power over depositors, leading to a higher cost of funds and that this led to a reduction in commercial and industrial lending to both small and medium sized businesses in the first half of the 1990s.

Therefore, it appears that liberalization and entry into a concentrated financial market (or alternatively, a market constrained by regulation from competing, as in the case of *financial repression*) raises the cost of funds for the incumbents, and reduces credit penetration for entrepreneurs with fewer assets. Both these observations are verified by our model. Before financial liberalization, the single bank operates as a monopsony in the funds market and therefore sets a low rate on deposits. As domestic agents can deposit their funds in competitive foreign banks after liberalization, the deposit rate for the monopoly bank rises to the international level plus the spread $\rho^* + \Delta^*$. Thus, financial liberalization implies a rise in the cost of funds for the monopoly bank.

Consider now the effects on borrowers. Wealthy entrepreneurs, as in the previous subsection, will switch to foreign banks because they offer lower lending rates (see Proposition 4). Hence, the domestic bank loses the customers with wealth $K_z \geq K^*$. However, the domestic bank has an advantage over entrepreneurs with less capital, as it faces a lower level of entrepreneurial rents (has better knowledge of local conditions and better monitoring skills) than foreign banks. As a consequence, entrepreneurs in the range $K^* > K_z \geq K(\rho^*)$ can only be financed by the local monopoly bank. The behavior of the domestic bank is described by the solution to the following problem, noting that the domestic bank continues to operate as a monopoly over agents that do not have access to loans from the foreign banks:

[17] Kneiding and Kritikos (2008) show that households with no dependent workers use consumer loans to finance businesses, while Trumbull (2010) estimates that 30% of consumer loans are used to finance entrepreneurial projects.

$$\underset{\{A, R_e(K_z), I_e(K_z)\}}{\text{Max}} \int_A^{K^*} \left[p(R - R_e(K_z)) - (1 + \rho^* + \Delta^*)(I - \lambda I_e(K_z))\right] dG_z \tag{PMO}$$

subject to $\forall K_z \in [A^*, K^*]$

$$R_e(K_z) \geq \phi R, \tag{LPC}$$

$$R \geq R_e(K_z), \tag{LL1}$$

$$pR_e(K_z) + (1 + \rho^*)(K_z - I_e(K_z)) \geq (1 + \rho^*)K_z, \tag{IRE}$$

$$pR_e'(K_z) - (1 + \rho^*)I_e'(K_z) = 0, \tag{TTE}$$

$$K_z \geq I_e(K_z), \tag{LL2}$$

$$K^* \geq A \geq K(\rho^*) \tag{LIM}$$

Define the function $\widehat{\varphi}(x) \equiv x - \frac{G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(x)}{g(x)}$. The following result describes the equilibrium:

Proposition 6 *Suppose that assumptions 1, 2, 3, 4, 5, and 6 hold. If there is an ex ante monopoly, financial liberalization means that*

i) If $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$*, then.*

a) The domestic bank only finances entrepreneurs with wealth in the range $K_z \in [K_M^*, K^*)$*, where* $K_M^* > K(\rho^*)$*. The limit capital* K_M^* *is determined from*

$$\widehat{\varphi}(K_M^*) = K(\rho^*)$$

b) For entrepreneur z, there exist a unique contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \left\{\phi R, K_M^*\right\}.$$

ii) If $\frac{(1+\rho^*)(1-\lambda)}{\lambda} < \Delta^*$*, then*

a) The domestic bank only finances entrepreneurs with wealth in the range $K_z \in [K_M^*, K^*)$*, where*

$K_M^* > K(\rho^*)$. The limit capital K_M^* is determined from

$$\widehat{\varphi}(K_M^*) = K(\rho^*)\left[\frac{(1+\rho^*+\Delta^*)\lambda}{1+\rho^*}\right] + \left[\frac{\Delta^*\lambda - (1+\rho^*)(1-\lambda)}{1+\rho^*}\right] K_M^*$$

b) For the entrepreneur z, there exists a unique contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \left\{\phi R + \frac{(1+\rho^*)(K_z - K_M^*)}{p}, K_z\right\}.$$

The results in this proposition are rather different from those derived previously, because now the optimal contract can be wealth dependant. The reason for this change is that domestic banks face a spread Δ^* per dollar raised in the external capital markets. A dollar deposited in the domestic bank returns $1+\rho^*$, but the cost to the bank is $1+\rho^*+\Delta^*$.[18] Hence, in the absence of any other cost (i.e., when $\lambda = 1$), the bank prefers to minimize the amount that each entrepreneur deposits in the bank. This is achieved by mean of a contract that requires each entrepreneur to invest as much as possible into the project; that is, $I_e(K_z) = K_z$. In other words, the local bank offers a menu of contracts that achieves perfect sorting among its clients. If we combine this with the transaction cost λ, the bank faces a trade-off, a lower investment $I_e(K_z)$ saves the bank this transaction cost, but it costs Δ^* per dollar not invested. When λ is close to 1 and Δ is relatively large, the second effect dominates and the optimal contracts are such that perfect sorting is achieved, while when λ is small, the first effect dominates and therefore the banks prefer to pool all clients under the same contract. This explains the two different cases in the proposition above.

We now examine the effect of changes in the parameters of the open economy, when we have *ex ante* monopoly.

Lemma 3 *We have:*

i) $\frac{\partial K_M^*}{\partial \Delta^*} > 0$ *and* $\frac{\partial K_M^*}{\partial \phi} > 0$.

ii) Suppose $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$. *Then:*

a) If $\frac{p\phi R}{1+\rho^*} > K^*$, *then* $\frac{\partial K_M^*}{\partial \rho^*} > 0$, $\frac{\partial K_M^*}{\partial \phi^*} > 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$.

b) If $\frac{p\phi R}{1+\rho^*} < K^*$, *then* $\frac{\partial K_M^*}{\partial \phi^*} = 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$. *The sign of* $\frac{\partial K_M^*}{\partial \rho^*}$ *is ambiguous.*

[18] Assuming that foreign banks are less risky than domestic banks, the spread represents the additional amount the domestic bank must pay for a saver to receive the same risk-adjusted return as when it deposits in the foreign bank.

iii) Suppose $\frac{(1+\rho^*)(1-\lambda)}{\lambda} < \Delta^*$. *Then:*

a) If $\frac{p\phi R}{1+\rho^*} > K^*$, *then* $\frac{\partial K_M^*}{\partial \phi^*} > 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$. *The sign of* $\frac{\partial K_M^*}{\partial \rho^*}$ *is ambiguous.*

b) If $\frac{p\phi R}{1+\rho^*} < K^*$, *then* $\frac{\partial K_M^*}{\partial \phi^*} = 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} = 0$. *The sign of* $\frac{\partial K_M^*}{\partial \rho^*}$ *is ambiguous.*

iv) If $(1-\phi)(1+\rho^*)^2 > \phi\lambda(1+\rho^*+\Delta^*)^2 \frac{g\left(\min\left\{K^*, \frac{\phi p R}{1+\rho^*}\right\}\right)}{g(K_M^*(\rho^*))}$, *then* $\frac{\partial K_M^*}{\partial \rho^*} > 0$

The proposition implies that increases in the cost of funds for domestic banks, represented by the spread Δ^*, as well as increases in the rate of entrepreneurial rents will (weakly) increase the number of entrepreneurs that do not receive credit in an open economy with an initial banking monopoly. More interestingly, and in contrast to the competitive case, an increase in entrepreneurial rents facing foreign banks, ϕ^*, will (weakly) reduce the access to loans from the monopoly bank. The explanation is that foreign banks are weaker competitors and the domestic banks has more monopoly power, but still faces a higher cost of funds than in the closed economy. Hence, when liberalizing an initially imperfectly competitive financial system, it is vital to reduce any disadvantages (legal, or otherwise) facing foreign competitors of the domestic system.

In what follows we focus on ϕ that satisfy sufficient condition *iv)* in lemma 3, which implies that an increase in the foreign interest rate unambiguously reduces access to loans of less wealthy entrepreneurs, for all (λ, ϕ). Observe that the condition holds for high degrees of pledgeability (low ϕ). With the results of lemmata 2 and 3 we are able to establish the following proposition:

Proposition 7 *When there is* ex ante *competition, the benefits of liberalization are larger in economies with lower rates of entrepreneurial rents* ($\phi\downarrow$) *and lower costs of collateral* ($\lambda\uparrow$) *of domestic banks. These results do not necessarily hold in the case of* ex ante *monopoly, but conditions improve if the higher entrepreneurial rents facing foreign banks are reduced* ($\phi^*\downarrow$).

4.3 Core Results: Effects of financial liberalization under different market structures

Recalling proposition 5, under *ex ante* competition, financial liberalization leads to the exclusion from loans of previously funded entrepreneurs only if $K(\rho^*) > K(\rho_{bc})$. Therefore, unless we assume that international interest rates are higher than the competitive equilibrium rate of the closed economy, i.e., $\rho_{bc} < \rho^*$, the number of entrepreneurs that have access to loans must increase. The opposite case would mean that the closed economy is relatively abundant in capital, which is unlikely for a developing economy. Assuming therefore that the cost of funds for banks falls after liberalization ($\rho^* < \rho_{bc}$), we have the expected result that:

Proposition 8 *If there is* ex ante *competition, financial liberalization produces an inflow of capital, leading to an increase in the number of entrepreneurs that receive financing.*

This is the result expected by those who promote financial liberalization for poor countries. Compare this with the results in case of *ex ante* monopoly in the domestic financial market. Recalling proposition 6, we note the difference between the effects of liberalization depending on the competitive conditions in the *ex ante* market: if the market was competitive *ex ante*, the number of entrepreneurs that receive financing after liberalization is larger than if the market was a monopoly: $K_M^* > K(\rho^*)$. In other words, the monopoly domestic bank is always inefficient in the open economy, an outcome that is not always true in a closed economy (see proposition 3). Moreover, competition from foreign banks does not eliminate profits for the domestic bank, though it limits their size by rasing the cost of funds and reducing its market share.

Recall that under a closed economy monopoly, the bank was a monopsony in the market for deposits, and paid a rate of zero on loanable funds. When the economy opens up, the monopoly bank faces competition for deposits and the rate it has to pay for deposits rises to the international rate. Hence, there is the additional perverse possibility that some entrepreneurs that were granted loans under closed financial markets are excluded from access to finance after liberalization. The condition for the open economy with a monopoly bank to have fewer entrepreneurs with access to funds than before liberalization is that:

$$K_M^*(\rho^*) > K_M \Longleftrightarrow \rho^* > K_M^{*-1}(K_M) \tag{7}$$

Hence, the main result of this paper (which requires the condition derived in item *iv)* of lemma 3):

Proposition 9 *If the cost of funds for domestic bank after liberalization is higher than $K_M^{*-1}(K_M)$, then there will be a group of entrepreneurs with low wealth who are excluded from access to loans after financial liberalization.*

The reason for this result is that the increase in the cost of funds for the domestic monopoly bank, coupled to the level of entrepreneurial rents in the economy, may no longer leave a margin for lending to these agents.[19] As we mentioned in the introduction, these results represent an alternative explanation for the conflicting results of financial liberalization on credit penetration to those provided by models that postulate a switch from a pooling to a separating financial equilibrium post liberalization as in Gormley (2011) and Detriagache et al. (2008).

4.4 Implication of the model for the optimum form of financial liberalization

The model described above can be used to discuss the optimal path of financial liberalization. If the banking system is initially competitive, the answer is obvious: there should be free entry of banks, and they should be allowed to raise funds locally, while domestic banks should be able to access international funds.[20] The reduction in the cost of funds for domestic banks will increase the access of borrowers to capital, and banks will no longer be constrained by the internal capital market. Strong borrowers will access the less expensive foreign banks, with their better technology. Lenders will be worse off, because the return on their deposits will be lower, but their higher previous return was due to a distortion in the capital markets that has been removed, and this should increase aggregate national welfare.

Things are different in the case of imperfect capital markets prior to financial liberalization. If the objective is to increase lending to entrepreneurs, even if it is at the expense of depositors, then foreign banks should be allowed to operate, but not to raise funds locally, and depositors should not be allowed to send their savings abroad. The benefit of this approach is that the domestic banks continue to be oligopsonic in their home markets, and therefore their cost of funds does not rise. A smaller (than if foreign banks are allowed to raise finds locally) group of wealthy entrepreneurs will get funded by foreign banks. Since there is a reduced demand for capital by these firms, there are more savings available to fund projects of less wealthy entrepreneurs. Thus, unless the equilibrium is inefficient in the sense of proposition 3, there is increased access to funding for entrepreneurs. This seems to be the policy used by China, where domestic savers face few options and the financial markets are repressed, leading to high rates of investment and easy access to capital by entrepreneurs.[21] However, this is an optimal policy only

[19]Note that if the government improves financial legislation and its enforcement, it can reduce entrepreneurial rents, and therefore could potentially eliminate this perverse case.

[20]Allowing depositors to have access to international financial markets has no additional effect in this setting, if foreign banks are competitive.

[21]See, for instance, Yukon Huang, "Time for China to give up financial repression", *Financial Times*, May 2, 2012:

> "Many critics blame 'financial repression'–keeping interest rates paid to savers below the rate of inflation, therefore discouraging consumption; encouraging investment; and distorting growth.[...] What makes China's form of financial repression unique is the limited investment alternatives for household savings, a situation rein-

if we do not consider the welfare of savers and if we believe that the increased investments will not be inefficient.

Finally consider the option of allowing capital outflows while not allowing foreign bank entry. This seems to be a bad approach, since it raises the cost of funds of the monopoly bank, reducing the access of capital-poor entrepreneurs to loans, while still keeping the inefficiency of the monopoly bank (no access to new low cost technologies, and inefficient lending). On the other hand savers are better off, but no better than if foreign bank entry were allowed, which provides access to less expensive finance for some firms.

5 Implications of the model regarding changes in wealth distribution

The most interesting results on the effect of changes in the wealth distribution are those that are not contaminated with changes in the aggregate wealth of the economy (first order stochastic dominance), but are pure changes in the wealth distribution, keeping aggregate wealth constant. The results of financial liberalization depend on the wealth of the economy. We define (Balmaceda and Fischer, 2010) an economy to be *capital constrained* (unconstrained), when $K(\rho^*) > \widetilde{K}$ (<). This means that a country is capital constrained when the aggregate wealth of the economy is low (high), so that the last agent that could have been financed at the international interest rate, in the closed economy has above (below) average wealth. Our main result is that if under ex ante competition, the wealth distribution of two countries is separated by an MPS, the benefits of financial liberalization are higher in the more unequal country when it is credit constrained. Conversely, liberalization is better for the more equal country if it is not credit constrained. There is no equivalent result for an ex ante monopoly.

6 Conclusions

One of the puzzles of the recent experience with financial liberalization in developing countries has been the observation that in some countries access to credit falls rather than increases, as would have been expected from the increased competition and better lending technologies of multinational banks. This paper presents an explanation for this observation based on the notion that the effects of liberalization depend on the *ex ante* financial market structure in the country.

forced by capital controls.”

In particular, we show that when the country has an initially competitive banking structure, financial liberalization is unambiguously beneficial, in terms of access to lower international rates, increased availability of capital, lower loan costs for some firms and increased access to credit. In the case of initially imperfect competition, access among weaker firms may decrease, because the cost of funds for domestic banks rises, as a result of increased competition for funds by foreign banks (or because some savers now have access to international markets). The increased cost of funds coupled to reduction in lending to weaker firms are consistent with the observations of the effects of financial deregulation in the US (Berger et al., 1995), and there is recent evidence of these effects in other countries after financial liberalization (Rashid, 2011). Finally, observe that it is possible to test the model by including, in the models that examine the effects of financial liberalization on market access, a variable representing the competitiveness of the financial system previous to liberalization.

The logic of our results is driven by the fact that under imperfect competition, the cost of funds for banks is lower than under competition. Therefore, once the financial system in liberalized, there is a rise in the cost of funds for banks, and increased competition for wealthier entrepreneurs, because of the entry of more efficient foreign banks. The end result is a possible reduction in lending to weaker entrepreneurs and lower credit penetration. The reader may question the relevance of our results, since it is unusual for the banking industry in a country to be controlled by a single firm. However, it is straightforward to show that the results extend without any changes to the case of a banking cartel, which is not uncommon in developing countries with closed financial markets. Moreover, the main results should continue to hold in more general models of imperfect competition, but it would make the algebra much more complicated without further gain in intuition beyond that obtained in the monopoly case.

Nonetheless, it is possible to speculate on possible models of imperfect competition to tackle the issues studied here. One possibility would be to extend our model to the double Bertrand Competition model; i.e., having competition on both loans and deposits, in the absence of an infinitely elastic supply of funds as in Freixas and Rochet (1997, p. 63-67). In these models, there is an equilibrium in which banks make zero profits, there is one active bank and the profit margin (net of fixed costs) is positive. A second approach is to have a differentiated products Salop-type model with n banks equidistantly distributed on the unit circle and where, at each location there is a mass 1 of entrepreneurs and the wealth distribution is the same at each location. Again, this model would predict that the profit margin is positive and falls as the number of banks increases, suggesting that our results would continue to hold when there are a small number of banks prior to liberalization.

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A Appendix: Proofs

Lemma 1 *The following condition is sufficient for a contract* $\{R_e(K_z), I_e(K_z)\}_{z\in[0,1]}$ *to be implementable:*

$$pR'_e(\hat{K}_z) - (1+\rho)I'_e(\hat{K}_z) = 0 \tag{8}$$

and this holding for $\hat{K}_z = K_z$ *constitute necessary conditions for implementability.*

Proof: Given this mechanism, entrepreneur z's expected utility when she declares wealth $\hat{K}_z$ is

$$U(\hat{K}_z, K_z) \equiv pR_e(\hat{K}_z) + (1+\rho)(K_z - I_e(\hat{K}_z)) - (1+\rho)K_z, \tag{9}$$

where the first term is the expected return obtained by the entrepreneur in the case of success, the second term is the gain obtained by depositing excess funds $(K_z - I_e(\hat{K}_z)) > 0$ in the banking system (possibly via an associate, to avoid detection), and the third term is the opportunity cost of the capital K_z. Then, entrepreneur z's problem is:

$$\max_{\hat{K}_z \geq 0} U(\hat{K}_z, K_z) \tag{10}$$

That is, entrepreneur z chooses an announcement $\hat{K}_z$ so as to maximize U. The first-order condition with respect to $\hat{K}_z$ is given by

$$\frac{\partial U(\hat{K}_z, K_z)}{\partial \hat{K}_z} = 0, \ \forall K_z \in [0,1] \tag{11}$$

Because this holds as an identity in K_z, the expression can be totally differentiated:

$$\frac{\partial^2 U(\hat{K}_z, K_z)}{\partial \hat{K}_z^2} + \frac{\partial^2 U(\hat{K}_z, K_z)}{\partial \hat{K}_z \partial K_z} = 0 \tag{12}$$

Because the second-order condition of (10) requires that the first term to be non-positive (i.e., it requires global concavity with respect to $\hat{K}_z$), this implies that

$$\frac{\partial^2 U(\hat{K}_z, K_z)}{\partial \hat{K}_z \partial K_z} \geq 0 \tag{13}$$

evaluated at $\hat{K}_z = K_z$ is a necessary condition for the second-order condition to be satisfied. The same arguments shows that equations (11) and (13) holding for all $\hat{K}_z, K_z \in [0,1]$ are sufficient conditions for implementability. But calculating equation (13) directly from (9), we obtain that it is 0 for all $\hat{K}_z, K_z \in [0,1]$. Hence the only required condition for truthful revelation of wealth is (11) and we have proved the desired result.

∎

Proposition 1 *Suppose that the cost of capital is fixed at ρ. Then, in a closed competitive economy,*

- *Only entrepreneurs with wealth*

$$K_z \geq K(\rho) \equiv \frac{1}{\lambda}\left[I - \frac{pR(1-\phi)}{1+\rho}\right] \tag{14}$$

  obtain loans.

- *For the entrepreneur z, there exist a unique contract of the form*

$$\{R_e(K_z), I_e(K_z)\} = \{\phi R, K(\rho)\}.$$

Proof: First, as it has been argued, in equilibrium banks make zero expected profits. So, the bank's participation constraint (IRB) will be binding.

First, consider the truth-revealing condition (TTE). Solving this first-order ordinary differential equation for $R_e(K_z)$ and $I_e(K_z)$, we obtain:

$$pR_e(K_z) = (1+\rho)I_e(K_z) + r_0$$

where r_0 is the constant of integration. As the bank can choose optimally the value of the constant r_0, we can express $I_e(K_z)$ as a function of r_0 and $R_e(K_z)$, and therefore $I_e(K_z)$ can be eliminated of the maximization problem.

Conditions (LL1) and (LL2) can be grouped into a single restriction, $\min\left\{(1+\rho)K_z + r_0, pR\right\} \geq pR_e(K_z)$. Also, conditions (LPC) and (IRE) can be rewritten as $pR_e(K_z) \geq p\phi R$ and $r_0 \geq 0$, respectively.

Then, the maximization problem is now:

$$\underset{R_e(K_z),A,r_0}{\text{Max}} \int_A^1 r_0 dG_z$$

subject to

$$\int_A^1 [pR - pR_e(K_z)(1-\lambda) - (1+\rho)I - \lambda r_0] dG_z = 0$$

$$\min\left\{(1+\rho)K_z + r_0, pR\right\} \geq pR_e(K_z), \qquad \forall K_z \in [A,1]$$

$$pR_e(K_z) \geq p\phi R, \qquad \forall K_z \in [A,1]$$

$$r_0 \geq 0$$

For the moment assume that $\min\left\{(1+\rho)K_z + r_0, pR\right\} = (1+\rho)K_z + r_0$ (we will back to this later). Therefore, the second restriction now is $(1+\rho)K_z + r_0 \geq pR_e(K_z), \forall K_z \in [A,1]$. Also, since the objective function is increasing in r_0, then the third restriction must be binding $\forall K_z \in [A,1]$. To see this, suppose that in equilibrium $\exists K_z \in [A,1]$ such that $pR_e(K_z) > p\phi R$. If we simultaneously reduce $R_e(K_z)$ by a small $\epsilon > 0$ and increase r_0 by $p\epsilon(1-\lambda)/\lambda$, then the left hand side of the first restriction remains unchanged, but the objective function increases (note that by doing this we do not have problems with others restrictions). So, we have that $R_e(K_z) = \phi R$, $\forall K_z \in [A,1]$. Now the problem is:

$$\underset{A,r_0}{\text{Max}} \int_A^1 r_0 dG_z$$

$$\int_A^1 [pR(1-\phi) + pR\phi\lambda - (1+\rho)I - \lambda r_0] dG_z = 0$$

$$(1+\rho)K_z + r_0 \geq p\phi R, \qquad \forall K_z \in [A,1]$$

$$r_0 \geq 0$$

Given that the term inside the integral in the first restriction is a constant, we obtain that $r_0 = (pR(1-\phi) + pR\phi\lambda - (1+\rho)I)/\lambda$, which is positive by Assumption 4 on page 13 (now it is clear that if this assumption was not held, then financing would not exist in equilibrium, i.e., $A = 1$). Then, with the second restriction we can obtain the minimum capital required to obtain the credit, $K_z \geq K(\rho) \equiv \frac{1}{\lambda}\left[I - \frac{pR(1-\phi)}{1+\rho}\right]$. Finally, with the optimal value of r_0, it is easy to show that $\min\left\{(1+\rho)K(\rho) + r_0, pR\right\} = (1+\rho)K(\rho) + r_0$, and this ends the proof.

∎

Proposition 3 *Suppose that assumptions 1, 2, 3, 4 and 5 hold. Then, in a closed economy with a monopoly bank,*

i) The monopoly bank sets the cost of funds at $\rho_M = 0$.

ii) Only entrepreneurs with wealth $K_z \geq K_M$ *obtain funding, where* K_M *is determined from:*

a) If $\varphi(K(\rho_{bc})) \geq K(0)$*, then the monopolist is efficient, in the sense that* $K_M = K(\rho_{bc})$.

b) If $\varphi(\min\{1, p\phi R\}) > K(0) > \varphi(K(\rho_{bc}))$*, the monopolist is inefficient, with* $K_M \in (K(\rho_{bc}), \min\{1, p\phi R\})$*, determined from* $\varphi(K_M) = K(0)$.

c) If $K(0) \geq \varphi(\min\{1, p\phi R\})$*, then the monopolist is inefficient, and* $K_M = \min\{1, p\phi R\}$.

iii) For the entrepreneur z, there exist a unique of contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \{\phi R, K_M\}.$$

Proof: First, consider the truth-revealing condition (TTE). Solving this first-order ordinary differential equation for $R_e(K_z)$ and $I_e(K_z)$, we obtain:

$$pR_e(K_z) = (1 + \rho)I_e(K_z) + r_0$$

where r_0 is the constant of integration. As the bank can choose optimally the value of the constant r_0, we can express $I_e(K_z)$ as a function of r_0 and $R_e(K_z)$, and therefore $I_e(K_z)$ can be eliminated of the maximization problem.

For the moment we ignore condition (LL1), since it will not be binding in equilibrium (we will check this later). Also, conditions (LPC) and (IRE) can be rewritten as $pR_e(K_z) \geq p\phi R$ and $r_0 \geq 0$, respectively. Finally, condition (CA) is the same as $A \geq K(\rho_{bc})$, because at $K(\rho_{bc})$ all capital is used for loans. Then, the maximization problem is now:

$$\underset{\rho, R_e(K_z), A, r_0}{\text{Max}} \int_A^1 [pR - pR_e(K_z)(1 - \lambda) - (1 + \rho)I - \lambda r_0] dG_z$$

subject to

$$(1+\rho)K_z + r_0 \geq pR_e(K_z), \qquad \forall K_z \in [A,1]$$

$$pR_e(K_z) \geq p\phi R, \qquad \forall K_z \in [A,1]$$

$$r_0 \geq 0$$

$$A \geq K(\rho_{bc})$$

First we will show that at the optimum it holds that $\rho = 0$. To see this, look the left hand side of the first restriction. If we simultaneously reduce ρ by a small $\epsilon > 0$ and increase r_0 by ϵK_z, then the left hand side of the first restriction remains unchanged, but the objective function increases because $I > \lambda K_z$, $\forall z$. By this argument, it is clear that $\rho = 0$ at the optimum.

Observe that $R_e(K_z)$ must be determined by the first and the second restriction. In fact, by these conditions we have that $K_z + r_0 \geq pR_e(K_z) \geq p\phi R$. This, together with the fact that the objective function is decreasing $R_e(K_z)$, allows us to set the values of this variable so that the following holds: $K_z + r_0 \geq pR_e(K_z) = p\phi R$ (note here that with this value of $R_e(K_z)$ condition (LL1) will not be binding). As this equation must hold $\forall K_z \in [A,1]$, then it must hold that $r_0 \geq p\phi R - A$. As the objective function is decreasing in r_0, we have that $r_0 = \max\{0, p\phi R - A\}$. Furthermore, observe that the last two constraints cannot be binding simultaneously, because if this were the case then $r_0 = 0$ and $A = K(\rho_{bc}) < p\phi R$, where the last inequality follows from the definition of $K(\rho_{bc})$ in equation (14) and assumption 2. This inequality violates the constraints emerging from the first and the second constraint since if $r_0 = 0$, these require that $A \geq pR_e(A) \geq p\phi R > K(\rho_{bc}) = A$. These allows us to re-write the maximization problem as follows:

$$\operatorname*{Max}_{A}[1 - G(A)][pR(1-\phi) + pR\phi\lambda - (1+\rho)I - \lambda\max\{0, p\phi R - A\}]$$

subject to

$$1 \geq A$$

$$A \geq K(\rho_{bc})$$

If $1 \geq p\phi R$, then the objective function would be decreasing in A in all the interval $1 \geq A \geq p\phi R$, as $\max\{0, p\phi R - A\} = 0$. Then, in this case, the optimum would necessarily be in the interval $[K(\rho_{bc}), p\phi R]$.

However, if $p\phi R \geq 1$, then the solution would be in the interval $[K(\rho_{bc}), 1]$. This analysis can be synthesized in the followings restrictions: $\min\left\{1, p\phi R\right\} \geq A$ and $A \geq K(\rho_{bc})$. Recalling the definitions of $K(0)$ and $K(\rho_{bc})$, the objective function now is (normalized by $1/\lambda$):

$$[A - K(0)]\,[1 - G(A)]$$

Taking the natural logarithm of the objective function (this does not change the optimum value of the optimization problem) and replacing the expresion $r_0 = p\phi R - A$ in the third condition, we can rewrite the problem as follows:

$$\underset{A}{Max}\ \ln\left(1 - G(A)\right) + \ln\left(A - K(0)\right)$$

subject to

$$\min\left\{1, p\phi R\right\} \geq A$$

$$A \geq K(\rho_{bc})$$

The constraint set is nonempty, convex, and compact, and the objective function is continuous, so a solution exists. Additionally, by Assumption 1 on page 9, the objective function is concave, so the solution is unique. Letting μ_1 and μ_2 be the Lagrange multipliers associated to the first and second restriction respectively, the Lagrange function is:

$$\mathcal{L} = \ln\left(1 - G(A)\right) + \ln\left(A - K(0)\right) + \mu_1(A - K(\rho_{bc})) + \mu_2(\min\left\{1, p\phi R\right\} - A)$$

The resulting Kuhn-Tucker conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial A} &= -\frac{g(A)}{1 - G(A)} + \frac{1}{A - K(0)} + \mu_1 - \mu_2 = 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_1} &= A - K(\rho_{bc}) \geq 0,\ \mu_1 \frac{\partial \mathcal{L}}{\partial \mu_1} = 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_2} &= \min\left\{1, p\phi R\right\} - A \geq 0,\ \mu_2 \frac{\partial \mathcal{L}}{\partial \mu_2} = 0 \end{aligned}$$

Denote K_M as the solution. We analyze the four cases in turn:

Case 4: $\mu_1 > 0$ and $\mu_2 > 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = \frac{\partial \mathcal{L}}{\partial \mu_2} = 0$, which is the same as $K_M = \min\left\{1, p\phi R\right\} =$

$K(\rho_{bc})$. But this cannot be true for the definition of $K(\rho_{bc})$. In particular, we know that min $\left\{1, p\phi R\right\} >$ $K(\rho_{bc})$, so this case can be dropped.

Case 3: $\mu_1 > 0$ and $\mu_2 = 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = 0$. The solution is

$$(K_M, \mu_1, \mu_2) = \left(K(\rho_{bc}), \frac{g(K(\rho_{bc}))}{1 - G(K(\rho_{bc}))} - \frac{1}{K(\rho_{bc}) - K(0)}, 0\right)$$

only if $\mu_1 > 0$.

Case 2: $\mu_1 = 0$ and $\mu_2 > 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_2} = 0$. The solution is

$$(K_M, \mu_1, \mu_2) = \left(\min\left\{1, p\phi R\right\}, 0, -\frac{g(\min\left\{1, p\phi R\right\})}{1 - G(\min\left\{1, p\phi R\right\})} + \frac{1}{\min\left\{1, p\phi R\right\} - K(0)}\right)$$

only if $\mu_2 > 0$.

Case 1: $\mu_1 = 0$ and $\mu_2 = 0$. The solution K_M is determined from $\frac{g(K_M)}{1 - G(K_M)} = \frac{1}{K_M - K(0)}$, where it must hold that min $\left\{1, p\phi R\right\} \geq K_M \geq K(\rho_{bc})$.

Cases 1 to 3 correspond to the alternatives in proposition 3.

∎

Corollary 2 *In a closed economy with a monopoly bank,*

- $\frac{\partial K_M}{\partial \lambda} < 0$ *if* $\varphi(K(\rho_{bc})) < K(0) \leq \varphi(\min\{1, p\phi R\})$ *and* $\frac{\partial K_M}{\partial \lambda} = 0$ *otherwise.*
- *If* $p\phi R \geq 1$,
  - ★ $\frac{\partial K_M}{\partial \phi} > 0$ *if* $\varphi(K(\rho_{bc})) \leq K(0) < \varphi(1)$ *and* $\frac{\partial K_M}{\partial \phi} = 0$ *otherwise.*
- *If* $p\phi R < 1$,
  - ★ $\frac{\partial K_M}{\partial \phi} > 0$ *if* $\varphi(K(\rho_{bc})) \leq K(0)$ *and* $\frac{\partial K_M}{\partial \phi} = 0$ *otherwise.*

Proof: We will prove only the first case, as the other cases are similar. When $\varphi(K(\rho_{bc})) \geq K(0))$, then $K_M = K(\rho_{bc})$ and the monopoly is efficient. As all capital is already in use, a marginal increase of λ does not change K_M. On the other hand, when $K(0) > \varphi(\min\{1, p\phi R\})$ we know that $K_M = \min\{1, p\phi R\}$. Then, it is direct that a marginal increase of λ does not change K_M. Therefore, we need to carry out a comparative-static analysis when $\varphi(K(\rho_{bc})) < K(0) \leq \varphi(\min\{1, p\phi R\})$, i.e, when the bank is inefficient and K_M is determined from $\varphi(K_M) = K(0)$.

By applying implicit differentiation of equation $\varphi(K_M) = K(0)$, we have that:

$$\frac{\partial K_M}{\partial \lambda} = \frac{\frac{\partial K(0)}{\partial \lambda}}{2 + \left(\frac{1-G(K_M)}{g(K_M)}\right) g'(K_M)}$$

By assumption 1 on page 9, the denominator is always positive. The sign of the numerator will depend on the sign of the partial derivative $\frac{\partial K(0)}{\partial \lambda}$. From the definition of $K(\rho)$ (equation 14 on page 35)), this term is negative. Then, $\frac{\partial K_M}{\partial \lambda} < 0$.

∎

Proposition 6 *Suppose that assumptions 1, 2, 3, 4, 5, and 6 hold. If there is an ex ante monopoly, financial liberalization means that,*

- *If* $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$, *then*

  i) The domestic bank only finances entrepreneurs with wealth in the range $K_z \in [K_M^*, K^*)$, *where* $K_M^* > K(\rho^*)$. *The limit capital* K_M^* *is determined from*

  $$\widehat{\varphi}(K_M^*) = K(\rho^*)$$

  ii) For the entrepreneur z, there exist a unique contract of the form

  $$\{R_e(K_z), I_e(K_z)\} = \left\{\phi R, K_M^*\right\}$$

  Each of these contracts is implementable.

- *If* $\frac{(1+\rho^*)(1-\lambda)}{\lambda} < \Delta^*$, *then*

  i) The domestic bank only finances entrepreneurs with wealth in the range $K_z \in [K_M^*, K^*)$, *where* $K_M^* > K(\rho^*)$. *The limit capital* K_M^* *is determined from*

  $$\widehat{\varphi}(K_M^*) = K(\rho^*)\left[\frac{(1+\rho^*+\Delta^*)\lambda}{1+\rho^*}\right] + \left[\frac{\Delta^*\lambda - (1+\rho^*)(1-\lambda)}{1+\rho^*}\right] K_M^*$$

ii) For the entrepreneur z, there exist a unique contract of the form

$$\{R_e(K_z), I_e(K_z)\} = \left\{\phi R + \frac{(1+\rho^*)(K_z - K_M^*)}{p}, K_z\right\}$$

.

Proof: First, consider the truth-revealing condition (TTE). Solving this first-order ordinary differential equation for $R_e(K_z)$ and $I_e(K_z)$, we obtain:

$$pR_e(K_z) = (1+\rho^*)I_e(K_z) + r_0$$

where r_0 is the constant of integration. As the bank can choose optimally the value of the constant r_0, we can express $I_e(K_z)$ as a function of r_0 and $R_e(K_z)$, and therefore $I_e(K_z)$ can be eliminated of the objective function in equation (PMO). This allows us to re-write the maximization problem as follows:

$$\underset{R_e(K_z), A, r_0}{\text{Max}} \int_A^{K^*} \left\{pR - pR_e(K_z)\left[\frac{(1+\rho^*)(1-\lambda) - \Delta^*\lambda}{1+\rho^*}\right] - (1+\rho^* + \Delta^*)I - r_0\left[\frac{(1+\rho^*+\Delta^*)\lambda}{1+\rho^*}\right]\right\} dG_z$$

subject to

$(1+\rho^*)K_z + r_0 \geq pR_e(K_z), \qquad \forall K_z \in [A, K^*]$

$pR_e(K_z) \geq p\phi R, \qquad \forall K_z \in [A, K^*]$

$r_0 \geq 0$

$K^* \geq A$

$A \geq K(\rho^*)$

First, note that the fourth condition cannot be binding, because the expected utility of the bank would be zero. Also note that there is an important difference between this proof and the proof of Proposition 3: now the objective function can be decreasing or increasing in $R_e(K_z)$, depending on the sign of the term $(1+\rho^*)(1-\lambda) - \Delta^*\lambda$. As a consequence, we must divide the analysis in two cases.

We begin with the case when the objective function is decreasing in $R_e(K_z)$. This holds when $(1+\rho^*)(1-\lambda) - \Delta^*\lambda > 0$, which is the same as $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$. Now, using the first and the second restriction we have that $(1+\rho^*)K_z + r_0 \geq pR_e(K_z) \geq p\phi R$. This, together with the fact that the objective function is decreasing $R_e(K_z)$, allows us to set the values of this variable so that the following holds: $(1+\rho^*)K_z + r_0 \geq pR_e(K_z) = p\phi R$. As this equation must hold $\forall K_z \in [A, K^*]$, then it must hold that $r_0 \geq p\phi R - (1+\rho^*)A$. As the objective function is decreasing in r_0, we have that $r_0 = \max\{0, p\phi R - (1+\rho^*)A\}$. Next we follow the same steps as those in the the proof of Proposition 3, but now noting that unlike the proof of Proposition 3 where the cost of fund is equal to zero now the cost of fund is $\rho^* + \Delta^* > 0$. The problem of the monopolist is as follows (we have normalized the objective function by $\frac{1}{(1+\rho^*+\Delta^*)\lambda}$):

$$
\begin{aligned}
&\underset{A}{Max}\ \ln\left(G(K^*) - G(A)\right) + \ln\left(A - K(\rho^*)\right) \\
&\text{s.t.} \quad \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} \geq A \\
&\qquad\quad A \geq K(\rho^*)
\end{aligned}
$$

Since the constraint set is nonempty, convex, and compact, and the objective function is continuous, a solution exists, and since by assumption 1 on page 9 the objective function is concave, so it has a unique global optimum.

Letting μ_1 and μ_2 be the Lagrange multipliers associated to the first and second restriction respectively, the Lagrange function equals:

$$
\mathcal{L} = \ln\left(G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(A)\right) + \ln\left(A - K(\rho^*)\right) + \mu_1(A - K(\rho^*)) + \mu_2(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} - A)
$$

The resulting Kuhn-Tucker conditions are:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial A} = -\frac{g(A)}{G(\min\left\{K^*, \frac{p\phi R}{1+\rho p\phi R}\right\}) - G(A)} + \frac{1}{A - K(\rho^*)} + \mu_1 - \mu_2 = 0 \\
&\frac{\partial \mathcal{L}}{\partial \mu_1} = A - K(\rho^*) \geq 0,\ \mu_1 \frac{\partial \mathcal{L}}{\partial \mu_1} = 0 \\
&\frac{\partial \mathcal{L}}{\partial \mu_2} = \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} - A \geq 0,\ \mu_2 \frac{\partial \mathcal{L}}{\partial \mu_2} = 0
\end{aligned}
$$

Denote K_M^* as the solution. We study the four possible cases,

Case 4: $\mu_1 > 0$ and $\mu_2 > 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = \frac{\partial \mathcal{L}}{\partial \mu_2} = 0$, which implies $K_M^* = \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} = K(\rho^*)$. This is a contradiction because $\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} > K(\rho^*)$.

Case 3: $\mu_1 > 0$ and $\mu_2 = 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = 0$. In this case, the solution is $K_M^* = K(\rho^*)$ only if it holds that $-\frac{g(K(\rho^*))}{G(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\})-G(K(\rho^*))}+\frac{1}{K(\rho^*)-K(\rho^*)}+\mu_1=0$. Clearly, this is not possible.

Case 2: $\mu_1=0$ and $\mu_2>0$. It must hold that $\frac{\partial\mathcal{L}}{\partial\mu_2}=0$. The solution would be $K_M^*=\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}$ if $-\frac{g(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\})}{G(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\})-G(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\})}+\frac{1}{\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}-K(\rho^*)}-\mu_2=0$ holds. Also, this is not possible.

Case 1: $\mu_1=0$ and $\mu_2=0$. The solution K_M^* is determined from $\frac{g(K_M^*)}{G(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\})-G(K_M^*)}=\frac{1}{K_M^*-K(\rho^*)}$, where it must hold that $\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}\geq K_M\geq K(\rho^*)$. Note that necessarily it holds that $K_M^*>K(\rho^*)$.

Now we study the second case, i.e., when the objective function of the initial problem is increasing in $R_e(K_z)$. This holds when $\frac{(1+\rho^*)(1-\lambda)}{\lambda}<\Delta^*$. Using the first and the second restriction we have that $(1+\rho^*)K_z+r_0\geq pR_e(K_z)\geq p\phi R$. This, together with the fact that the objective function is increasing in $R_e(K_z)$, allows us to set the values of this variable so that the following holds: $(1+\rho^*)K_z+r_0=pR_e(K_z)\geq p\phi R$. As this equation must hold $\forall K_z\in[A,K^*]$, then it must hold that $r_0\geq p\phi R-(1+\rho^*)A$. Since the objective function is decreasing in r_0, we have that $r_0=\max\{0,p\phi R-(1+\rho^*)A\}$. As before, we follow the same steps as those in the the proof of Proposition 3, and the maximization problem now is:

$$\underset{A}{\text{Max}}\int_A^{K^*}\left\{(K_z-K(\rho^*))(1+\rho^*+\Delta^*)\lambda-(K_z-A)(1+\rho^*)\right\}dG_z$$

subject to

$$\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}\geq A$$

$$A\geq K(\rho^*)$$

Since the constraint set is nonempty, convex, and compact, and the objective function is continuous, a solution exists. We assume that the objective function is concave, i.e., $-(1+\rho^*)-(1+\rho^*+\Delta^*)\lambda-(A-K(\rho^*))(1+\rho^*+\Delta^*)\lambda\frac{g'(A)}{g(A)}<0,\forall A\in[K(\rho^*),K^*]$, so it has a unique global optimum.

Letting μ_1 and μ_2 be the Lagrange multipliers associated to the first and second restriction respectively, the Lagrange function equals:

$$\mathcal{L}=\int_A^{\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}}\left\{(K_z-K(\rho^*))(1+\rho^*+\Delta^*)\lambda-(K_z-A)(1+\rho^*)\right\}dG_z$$
$$+\mu_1(A-K(\rho^*))+\mu_2(\min\left\{K^*,\frac{p\phi R}{1+\rho^*}\right\}-A)$$

Using Leibniz integral rule, the resulting Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial A} = (1+\rho^*)(G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(A)) - (A - K(\rho^*))(1+\rho^*+\Delta^*)\lambda g(A) + \mu_1 - \mu_2 = 0$$
$$\frac{\partial \mathcal{L}}{\partial \mu_1} = A - K(\rho^*) \geq 0,\ \mu_1 \frac{\partial \mathcal{L}}{\partial \mu_1} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \mu_2} = \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} - A \geq 0,\ \mu_2 \frac{\partial \mathcal{L}}{\partial \mu_2} = 0$$

Denote K_M^* as the solution. We study the four possible cases,

Case 4: $\mu_1 > 0$ and $\mu_2 > 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = \frac{\partial \mathcal{L}}{\partial \mu_2} = 0$, which implies $K_M^* = \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} = K(\rho^*)$. This is a contradiction because $\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} > K(\rho^*)$.

Case 3: $\mu_1 > 0$ and $\mu_2 = 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_1} = 0$. In this case, the solution is $K_M^* = K(\rho^*)$ only if it holds that $(1+\rho^*)(G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(K(\rho^*))) + \mu_1 = 0$. Clearly, this is not possible since the first and the second term are positive.

Case 2: $\mu_1 = 0$ and $\mu_2 > 0$. It must hold that $\frac{\partial \mathcal{L}}{\partial \mu_2} = 0$. The solution would be $K_M^* = \min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}$ if $-(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} - K(\rho^*))(1+\rho^*+\Delta^*)\lambda g(A) - \mu_2 = 0$ holds. Also, this is not possible, since both terms are negative.

Case 1: $\mu_1 = 0$ and $\mu_2 = 0$. The solution K_M^* is determined from $\frac{g(K_M^*)}{G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(K_M^*)} = \frac{1+\rho^*}{(1+\rho^*+\Delta^*)\lambda}\left[\frac{1}{K_M^* - K(\rho^*)}\right]$, where it must hold that $\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\} \geq K_M \geq K(\rho^*)$. Note that necessarily it holds that $K_M^* > K(\rho^*)$. This ends the proof.

∎

Lemma 3 *We have:*

i) $\frac{\partial K_M^*}{\partial \Delta *} > 0$ *and* $\frac{\partial K_M^*}{\partial \phi} > 0$.

ii) Suppose $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$. *Then:*

a) If $\frac{p\phi R}{1+\rho^*} > K^*$, *then* $\frac{\partial K_M^*}{\partial \rho^*} > 0$, $\frac{\partial K_M^*}{\partial \phi^*} > 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$.

b) If $\frac{p\phi R}{1+\rho^*} < K^*$, *then* $\frac{\partial K_M^*}{\partial \phi^*} = 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$. *The sign of* $\frac{\partial K_M^*}{\partial \rho^*}$ *is ambiguous.*

iii) Suppose $\frac{(1+\rho^*)(1-\lambda)}{\lambda} < \Delta^*$. *Then:*

a) If $\frac{p\phi R}{1+\rho^*} > K^*$, *then* $\frac{\partial K_M^*}{\partial \phi^*} > 0$ *and* $\frac{\partial K_M^*}{\partial \lambda} < 0$. *The sign of* $\frac{\partial K_M^*}{\partial \rho^*}$ *is ambiguous.*

b) If $\frac{p\phi R}{1+\rho^*} < K^*$, then $\frac{\partial K_M^*}{\partial \phi^*} = 0$ and $\frac{\partial K_M^*}{\partial \lambda} = 0$. The sign of $\frac{\partial K_M^*}{\partial \rho^*}$ is ambiguous.

iv) If $(1-\phi)(1+\rho^*)^2 > \phi\lambda(1+\rho^*+\Delta^*)^2 \frac{g\left(\min\left\{K^*, \frac{\phi p R}{1+\rho^*}\right\}\right)}{g(K_M^*(\rho^*))}$, then $\frac{\partial K_M^*}{\partial \rho^*} > 0$

Proof: We begin with the case when $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$. By implicitly differentiating equation $\widehat{\varphi}(K_M^*) = K(\rho^*)$ in proposition 6 with respect to $\omega \in \{\rho^*, \Delta^*, \phi, \phi^*, \lambda\}$, we have that

$$\frac{\partial K_M^*}{\partial \omega} = \frac{\frac{\partial K(\rho^*)}{\partial \omega} + \frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}}{2 + \left(\frac{G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(K_M^*)}{g(K_M^*)}\right) g'(K_M^*)}$$

It is easy to show that by Assumption 1 on page 9, the denominator is always positive. The sign of the numerator will depend on the signs of the terms $\frac{\partial K(\rho^*)}{\partial \omega}$ and $\frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}$. The term $\frac{\partial K(\rho^*)}{\partial \omega}$ is positive when $\omega \in \{\rho^*, \Delta^*, \phi\}$, negative when $\omega = \lambda$, and 0 when $\omega = \phi^*$.
However, to define the sign of the second term, $\frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}$, we have to study two cases. The first case is when $K^* < \frac{p\phi R}{1+\rho^*}$. In this case we have that $\frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}$ is positive when $\omega \in \{\rho^*, \phi^*\}$, negative when $\omega = \lambda$, and 0 when $\omega \in \{\Delta^*, \phi\}$. That completes the analysis of the case $K^* < \frac{p\phi R}{1+\rho^*}$.
The other case is when $K^* > \frac{p\phi R}{1+\rho^*}$, and we have that $\frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}$ is positive when $\omega = \phi$, negative when $\omega = \rho^*$, and 0 when $\omega \in \{\Delta^*, \phi^*, \lambda\}$. The final result is that the sign of the numerator is positive when $\omega \in \{\Delta^*, \phi\}$, negative when $\omega = \lambda$, 0 when $\omega \in \{\phi^*\}$, and ambiguous when $\omega = \rho^*$.

Now we study the case when $\frac{(1+\rho^*)(1-\lambda)}{\lambda} < \Delta^*$. Equation $\widehat{\varphi}(K_M^*) = K(\rho^*)\left[\frac{(1+\rho^*+\Delta^*)\lambda}{1+\rho^*}\right] + \left[\frac{\Delta^*\lambda - (1+\rho^*)(1-\lambda)}{1+\rho^*}\right] K_M^*$ in proposition 6 can be rewritten as $K_M^* + \widehat{\varphi}(K_M^*) = (K(\rho^*) + K_M^*)A$, with $A = \frac{(1+\rho^*+\Delta^*)\lambda}{1+\rho^*}$.

By implicitly differentiating equation with respect to $\omega \in \{\rho^*, \Delta^*, \phi, \phi^*, \lambda\}$, we have that

$$\frac{\partial K_M^*}{\partial \omega} = \frac{\frac{\partial [K(\rho^*)A]}{\partial \omega} + \frac{\partial G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\})}{\partial \omega}\frac{1}{g(K_M^*)}}{3 + \left(\frac{G(\min\left\{K^*, \frac{p\phi R}{1+\rho^*}\right\}) - G(K_M^*)}{g(K_M^*)}\right) g'(K_M^*) - A}$$

Note that $A < 2$, so it is easy to show that by Assumption 1 on page 10 the denominator is always positive. To study the sign of the numerator, first we study the sign of $\frac{\partial [K(\rho^*)A]}{\partial \omega}$. This term is positive when $\omega \in \{\Delta^*, \phi\}$, 0 when $\omega \in \{\lambda, \phi^*\}$ and ambiguous when $\omega = \rho^*$. The analysis of the second term in the numerator is equal to the case when $\frac{(1+\rho^*)(1-\lambda)}{\lambda} > \Delta^*$ (see above).

It is straightforward to check that the following is a sufficient condition for $K_M^*(\rho^*)$ to be increasing with ρ^*

$$(1-\phi)(1+\rho^*)^2 > \phi\lambda(1+\rho^*+\Delta^*)^2 \frac{g(\min\{K^*, \frac{\phi p R}{1+\rho^*}\})}{g(K_M^*(\rho^*))}.$$

and this concludes the proof.

∎

A.1 Effects of increased pledgeability

In footnote 10 we promised a formal analysis of the effects of increases of plegeability on interest rates (following a suggestion by a referee) . We pointed out there that the increase in pledgeability $(1-\phi)$ raises interest rates because higher pledgeability leads to an increase in the number of potential borrowers at the original interest rate, i.e., it shifts up the demand for loans. Qualitatively, this effect does not depend on the elasticity of the supply curve for loans. However, if the increase in pledgeability were to increase the supply of capital directly, i.e., by shifting the supply curve directly, higher pledgeability might lead to a reduction in the domestic equilibrium interest rate. Formally, assume $K^S = K^S(\rho;\phi)$, with $\partial K^S/\partial\phi < 0$. From the equilibrium condition $K^D = K^S$ and the definition of K^d we have:

$$\frac{\partial K^S}{\partial\rho}\cdot\frac{d\rho}{d\phi} + \frac{\partial K^s}{\partial\phi} = -Ig(K^d(\rho))\left\{\frac{\partial K^d}{\partial\rho}\cdot\frac{\partial\rho}{\partial\phi} + \frac{pR}{1+\rho}\right\}$$

Clearing:

$$\frac{d\rho}{d\phi}\left\{\frac{\partial K^S}{\partial\rho} + \frac{\partial K^d}{\partial\rho}g(K(\rho))I\right\} = -\left(\frac{\partial K^s}{\partial\phi} + g(K(\rho))I\frac{pR}{1+\rho}\right)$$

There are two cases depending on the sign of the RHS. When negative (positive), $d\rho/d\phi < 0$ requires that the expression in brackets be positive (negative), i.e. that $\epsilon^S + \epsilon^d g(K(\rho)I) > (<)0$, where ϵ^S, ϵ^d are the elasticities of supply and demand for capital, respectively. Thus the condition required for $d\rho/d\phi < 0$ depends on the magnitude of the impact of a reduction in pledgeability on supply relative to the other (positive) term in the RHS.

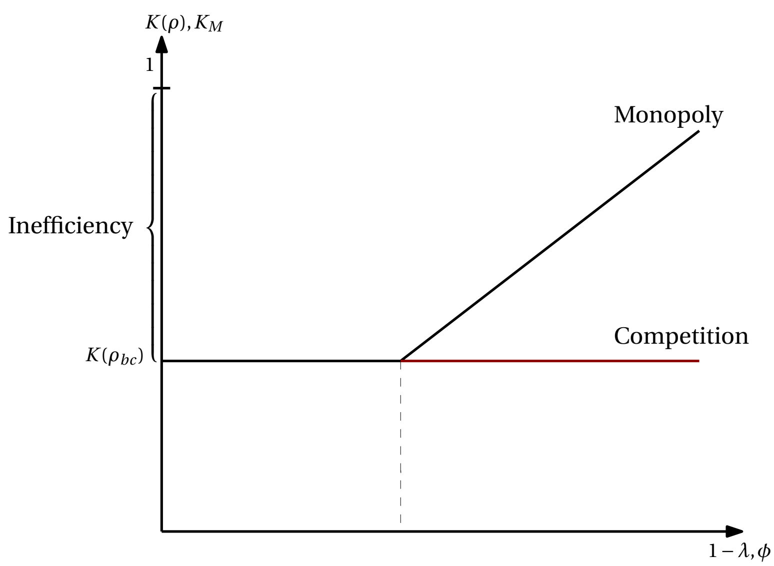

Figure 2: Comparative static effect of changes in the various structural parameters