LABOR FORCE HETEROGENEITY: IMPLICATIONS FOR THE RELATION BETWEEN AGGREGATE VOLATILITY AND GOVERNMENT SIZE

ALEXANDRE JANIAK - PAULO SANTOS MONTEIRO
Labor force heterogeneity: implications for the relation between aggregate volatility and government size∗

Alexandre Janiak\textsuperscript{a} and Paulo Santos Monteiro\textsuperscript{b}

\textsuperscript{a}University of Chile  
\textsuperscript{b}University of Warwick  
December 20, 2010

Abstract

There is substantial evidence of a negative correlation between government size and output volatility. We put forward the hypothesis that large governments stabilize output fluctuations because in economies with high tax rates the share of total market hours supplied by demographic groups exhibiting a more volatile labor supply is lower. This hypothesis is motivated by the observation that employment volatility is larger for young workers than for prime aged workers, and that the share of hours worked by the young workers is lower in countries with high tax rates. This paper illustrates these empirical facts and assesses in a calibrated model their quantitative importance for the relation between government size and macroeconomic stability.

∗We are grateful (in alphabetical order) to Henrique Basso, Sofía Bauducco, Antonio Fatás, Jordi Galí, Juan Pablo Nicolini and Varinia Tromben as well as seminar participants at the Latin America Meetings of the Econometric Society in Río de Janeiro, WPEG workshop, the Simposio de Analisis Economico in Madrid, University of Manchester, University of Warwick, University of Loughborough, Catholic University of Chile, Central Bank of Chile, University of Chile, USACH, Universidad Autonoma de Madrid, for helpful comments. Alexandre Janiak thanks Fondecyt for financial support (Project No 11080251).
1 Introduction

The motivation for this paper consists of two simple observations. The first motivating observation is that there is substantial evidence that countries or regions with large governments display business cycle fluctuations which are less volatile, as shown in Galí (1994), Rodrick (1998) and Fatás and Mihov (2001). The second motivating observation, which is documented by Clark and Summers (1981), Ríos-Rull (1996), and Gomme et al. (2004) is that fluctuations in hours of market work over the business cycle vary quite dramatically across different demographic groups of the population. In particular, the young experience much greater volatility of employment and hours worked than the prime-aged over the business cycle. Moreover, in a recent paper Jaimovich and Siu (2009) find that changes in the age composition of the labor force account for a significant fraction of the variation in business cycle volatility observed in the U.S. and other G7 economies. Hence, in this article we pose the following question: can the relation between the size of the government and macroeconomic stability be explained by changes in the demographic characteristics of the labor force which take place as the scope of the government in the economy changes?

The hypothesis we put forward is that large governments stabilize output fluctuations because in economies with high tax rates the share of total market hours that is supplied by the young workers is smaller. In turn, this change in the age composition of the labor force stabilizes output fluctuations since a larger fraction of total aggregate hours is supplied by the prime aged workers who have a more stable labor supply.

The suggestion that time devoted to market work is affected by changes in tax and transfer policies is one which has received considerable attention. For instance, recent work by Prescott (2004), Rogerson (2006, 2008) and Ohanian et al. (2008) argues that differences in tax and transfer policies can account for a large share of the difference in the amount of hours spent working in Europe and in the U.S. In particular, Rogerson and Wallenius (2009) document that the differences in employment rates between Europe and the U.S. are due almost exclusively to differences in the employment rates for young and old workers. They argue that differences in market hours that result from variation in tax and transfer policies are dominated by differences among young and old individuals. This observation offers further motivation for the work we develop in this paper.
Galí (1994) examines whether income taxes and government purchases behave as automatic stabilizers in the basic, technology shock-driven, real business cycle model. He finds that the relation between government size and macroeconomic stability implied by the standard model is very weak and often counterfactual. In this paper we incorporate labor force heterogeneity within the real business cycle framework, along the lines of Kydland (1984) and Jaimovich et al. (2010). In our model the stand-in household is composed of different types of individuals, which we interpret as different demographic groups. Heterogeneity is introduced by making the different demographic groups vary in their labor supply elasticities. These differences are calibrated to match the differences in the volatility of market work across age groups which have been documented in previous literature.\(^1\)

The mechanism whereby changes in the scope of the government affects macroeconomic stability has to do with the heterogeneity in the labor force. Specifically, we represent preferences using the Greenwood, Hercowitz and Huffman (GHH) utility function which eliminates wealth effects in the individual’s labor supply choice.\(^2\) Because the stand-in household has GHH preferences, in an economy with high tax rates all individuals spend less time working independently of the way in which the government uses the proceeds from taxation.\(^3\) Since the intratemporal substitution effect is stronger for the demographic groups that have higher labor supply elasticity, the share of hours worked by the volatile group of the population is reduced as the tax rate increases. Therefore, changes in the scope of the government affect macroeconomic stability.

The model is able to explain the relation between government size and macroeconomic

---

\(^1\)Jaimovich et al. (2010) consider an alternative explanation for the differences in the volatility of hours worked by different demographic groups. Specifically, they consider differences in the cyclical labor demand volatility. Our explanation for the relation between the government size and aggregate volatility applies independently of the mechanism explaining the life-cycle profile of employment volatility. We opted for modeling preferences heterogeneity for simplicity.

\(^2\)See Greenwood et al. (1988). Jaimovich and Rebelo (2009) find that, in order for some business cycle properties to be robust to the timing and nature of the technology shocks (both contemporaneous and news shocks), the short-run wealth effects on the labor supply must be weak.

\(^3\)Guo and Harrison (2006) show that, when the utility function is such that income effects on labor supply are strong, an increase in fiscal transfers to household tend to increase employment volatility. This is because the steady-state supply of hours worked decreases through the standard income effect. Because the marginal utility of consumption is larger when labor supply is lower, hours worked respond more to fluctuations in aggregate productivity. This explains the positive correlation between government size and macroeconomic volatility found in Galí (1994).
Figure 1: Government size and aggregate volatility

(a) Gov. size and output volatility

(b) Gov. size and consumption volatility

(c) Gov. size and investment volatility

(d) Gov. size and hours volatility

Note: All variables (but government size) are log deviations from an HP trend with smoothing parameter 6.25. Annual data on tax to GDP ratios, output, consumption and investment is from the OECD outlook database and data on hours worked is from the Conference Board Total Economy database, for the 1970-2007 sample period and the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

stability which is observed in the data along several dimensions. As can be seen from Figure 1, the negative correlation between government size and aggregate volatility is present in the data for output but also for each components of private aggregate expenditure taken separately (private consumption and investment), and also for the level of aggregate hours worked. Therefore, the stabilizing effect of the government goes beyond a simple compositional effect whereby government consumption is an increasing share of output. The mechanism in our model is centered around the relation between the size of the government and the share of hours supplied by the volatile demographic group. As a result, the model generates a negative correlation between aggregate hours worked and the size of the government. In turn, this also implies a negative correlation between consumption and investment volatility and the size of the government.
The ability of the model to generate a negative correlation between the scope of the government and the volatility of the private components of aggregate expenditure is an important contribution of our paper. Indeed, in a recent paper AndréS et al. (2008) study how alternative models of the business cycle can replicate the relation between government size and macroeconomic stability. Their analysis shows that adding nominal rigidities and costs of capital adjustment to an otherwise standard RBC model can generate a negative correlation between government size and the volatility of output, but the stabilizing effect is only due to a composition effect and it is not present if the analysis is restricted to the private components of aggregate expenditure. They suggest introducing rule-of-thumb consumers to replicate the negative correlation between government size and the volatility of consumption. Our framework instead focuses on the role of labor force heterogeneity in an otherwise standard RBC model.

To our knowledge, we are the first to assess quantitatively the ability of the RBC framework to replicate the relation between government size and macroeconomic stability in the OECD countries. Earlier contributions mostly focus on the sign of the relation between government size and macroeconomic stability. We calibrate the model to the U.S. economy by using aggregate annual time-series data and information on the relative level and volatility of market hours for each demographic groups. We then follow standard practice in development accounting. We vary the parameters describing the fiscal profile of the economies as they vary in the data. This allows us to generate a sample of simulated OECD economies. Those economies differ from the benchmark calibrated economy only in the fiscal policy parameters. For each of the simulated economies we are able to compute the size of the government and measures of aggregate volatilities. The implied relation between government size and macroeconomic stability can then be compared with the one that appears on Figure 1.

Quantitatively, we find that our benchmark model is able to explain up to 56 percent of the empirical relation between aggregate hours volatility and government size; it explains about 25 percent of the relation between government size and consumption volatility and about 6 percent of the relation between government size and investment volatility. However, the model only explains 4 percent of the relation between output volatility and the
government size, suggesting that other factors help to explain the stabilizing role of the government.

The paper is organized as follows. In Section 2 we present empirical evidence motivating our theoretical model. In Section 3, we describe our model with labor force heterogeneity within an otherwise standard RBC framework. Section 4, examines the relation between government size and the demographic composition of the labor force implied by the model. In Section 5, we examine the quantitative implications of the model. Finally, Section 6 offers some concluding remarks.

2 Labor force structure and output volatility: some data correlations

The hypothesis put forward in this paper is that large governments stabilize output fluctuations because in economies with high tax rates the share of total market hours which is supplied by the demographic groups exhibiting a more volatile labor supply is smaller. In this Section we present some correlations in the data that motivate our framework. Using panel-regression methods, our results suggest that accounting for the demographic composition of the labor force is empirically relevant to explain the volatility of the cyclical component of output. Furthermore, we document the relation between the size of the government—measured by the ratio between total tax revenue and GDP—and the demographic structure of the labor force.

We begin by documenting a well established relation between employment volatility and age: The employment volatility for young and old workers is larger than the employment volatility for prime-age workers. Indeed, several studies have illustrated that the labor market behavior of the young and the old differs from the behavior of prime-aged workers. For instance, Pencavel (1986), Killingworth and Heckman (1986) and Blundell and MaCurdy (1999) provide microeconometric evidence that the elasticity of labor supply is larger for the younger and the older workers. Here, to illustrate this fact, we follow the approach of Gomme et al. (2004), and Jaimovich and Siu (2009), who report cyclical
employment volatility for various age groups.\(^4\) In particular, we use annual data on employment by age group from the OECD outlook database for an unbalanced panel of 25 countries over the period 1970–2007.\(^5\) We build seven categories: workers aged between 15 and 19 years old, 20–24, 25–29, 30–39, 40–49, 50–59 and 60–64 years old. For each of these categories, we extract the business-cycle component of employment by applying the Hodrick-Prescott filter to the logged series with smoothing parameter equal to 6.25 as suggested by Ravn and Uhlig (2002), and we calculate the standard deviation. Figure 2 displays the results for the cross-section of countries, where volatility (i.e. the standard

\(^4\)Janiak and Wasmer (2008) also estimate a series of VAR models with European data where the endogenous variables are employment, unemployment and labor market participation. They distinguish between three age groups and show that employment impulse responses for young and old workers are larger in magnitude than middle-age workers.

\(^5\)We exclude Mexico from the sample because there are too many missing observations, impeding the use of the H–P filter.
deviation of the business-cycle component) is normalized by the volatility of the group aged between 40 and 49. As one can observe, there is a clear U-shaped relation between age and employment volatility at business cycle frequencies.\textsuperscript{6} In particular, volatility is much higher for the workers aged between 15 and 19 and for those aged between 60 and 64. The employment volatility of the youngest workers is on average four times that of the workers in the 40–49 age group (for France, this ratio is as high as 10). The 60–64 age group also displays large volatility and on average this volatility is three times the volatility of the 40–49 age group (this ratio is as high as 11 for Austria).

As Jaimovich and Siu (2009) suggest, those patterns are important to understand the business-cycle fluctuations. For this reason, we also look at gross domestic product (GDP) and how the standard deviation of its business-cycle component is correlated with the age structure of the labor force. Table 1 considers the relation between output volatility, the government scope, and the age structure of the labor force. To produce this Table, we use data for the period 1970–2007 from the OECD outlook database and the OECD Labor Force Statistics.\textsuperscript{7} In particular, we use data on GDP, the tax to GDP ratio and the share of workers in the labor force aged 15–29, 30–39, 40–49, 50–59 and 60–64 years old. GDP data is quarterly and filtered using the Hodrick-Prescott (1997) procedure with smoothing parameter 1,600 applied to the logged series. Following Jaimovich and Siu (2009), in order to calculate cyclical volatility in quarter \( t \), we use the standard deviation of the filtered real GDP during a 41-quarter (10-year) window centered around quarter \( t \). Finally, the resulting quarterly volatility measured is averaged to produce a yearly measure.

Each column in Table 1 reports results for a random-effect regression where the dependent variable is GDP volatility, and we consider alternative explanatory variables. The first column documents the well known negative relation between the government scope

\textsuperscript{6}Not reported here, we also used data at the US-state level (for both employment and hours volatility), which we constructed from the Current Population Survey. Results are qualitatively similar. Quantitatively, the volatility ratio of the 15–19 years old is lower with an average equal to 2. The 60–64 age group displays similar volatility. For the US state-level data, the identity of the group displaying the lowest volatility is more heterogeneous. The lowest volatility age group is either the 30–39, the 40–49 or the 50–59 group.

\textsuperscript{7}The countries included are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, and the United States.
Table 1: Random-effect regressions for output volatility, OECD

<table>
<thead>
<tr>
<th></th>
<th>(1) Std Y</th>
<th>(2) Std Y</th>
<th>(3) Std Y</th>
<th>(4) Std Y</th>
<th>(5) Std Y</th>
<th>(6) Std Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax/GDP</td>
<td>−0.060***</td>
<td>−0.028***</td>
<td>−0.023***</td>
<td>−0.021***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 15-29</td>
<td>0.005***</td>
<td>0.004***</td>
<td>0.003***</td>
<td>0.003***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 30-39</td>
<td>−0.002</td>
<td>−0.000</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 40-49</td>
<td>0.000</td>
<td>−0.000</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 60-64</td>
<td>0.013***</td>
<td>0.009**</td>
<td>0.007</td>
<td>0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.037***</td>
<td>0.004**</td>
<td>0.014***</td>
<td>0.029***</td>
<td>0.011***</td>
<td>0.017***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Δ Fiscal Coefficient</td>
<td>−53%</td>
<td></td>
<td></td>
<td>−8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time dummies</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>811</td>
<td>563</td>
<td>563</td>
<td>811</td>
<td>563</td>
<td>563</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01

and output volatility, already reported in, for instance, Galí (1994) and Fatás and Mihov (2001). The size of the government is measured by the tax to GDP ratio. Consistent with the evidence in the literature, there is a significant negative correlation between this variable and the cyclical volatility of output. The value of the regression coefficient is large and significant at the 1 percent level. An increase by 10 percentage points in the tax to GDP ratio lowers cyclical GDP volatility by 0.6 percentage points.

The second column in Table 1 considers the relation between the cyclical volatility of output and the age structure of the labor force. In particular, the explanatory variables are the ratios between the number of workers in each age-group and the number of prime-aged...
workers (the prime-aged group are the workers aged 50–59). Clearly, the data displays a U-shaped relation between volatility and age. In particular, as the number of workers aged 15–29 increases relative to the number of prime-aged workers, the cyclical volatility of output significantly increases. Similarly, an increase in the number of workers older than 60 relative to the number of prime aged workers raises significantly the cyclical volatility of output. An increase by 10 percentage points in the share of workers aged 15–29 raises the cyclical GDP volatility by 0.05 percentage points, and a similar increase in the share of workers aged 60–64 raises the cyclical GDP volatility by 0.13 percentage points. Those are large values given the average volatility is 1.74 percent in our sample of countries.

The result of interest for our study is what occurs when both sets of variables are included in the regression. In particular, we are interested in what happens to the coefficient associated with the tax to GDP ratio once we control for the demographic structure of the labor force. If the hypothesis put forward in this paper is valid, we would expect the absolute size of this coefficient to fall. The third column of Table 1 reports our results. The findings confirm the importance of controlling for the structure of the labor force. The tax to GDP coefficient falls by 55 percent once we control for the structure of the labor force.\(^9\) Moreover, the difference between the two coefficients is clearly statistically significant.\(^10\)

Columns four to six illustrate the results when we add time dummies to the set of controls. For these set of regressions, introducing time dummies reduces the significance of both government size and the demographic structure of the labor force. However, the U-shape volatility profile is a robust finding. Moreover, controlling for the demographic structure lowers the coefficient associated with the tax to GDP ratio but the difference is small and not statistically significant.

Finally, Table 2 considers the relation between the size of the government and the ratio of

\(^9\)An issue related to the use of the tax to GDP ratio as a measure for government size is that it is affected by cyclical conditions. For instance, if the elasticity of taxes relative to changes in output is bellow one, this ratio should fall in recessions. This is a problem when running panel data regressions. For this reason, we also ran random-effect regressions where government size corresponds to the average of the tax to GDP ratio over ten years. The results, available upon request, are qualitatively similar.

\(^10\)We performed other regressions (not reported), where government size is measured by either: i) total government receipts (excluding gross interest receipts); ii) total government disbursements (excluding gross interest payments). The fall in the coefficient associated with government size is, respectively, equal to 44% and 75%.
Table 2: Government size and labor force structure

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio of young workers</td>
<td>Ratio of young workers</td>
</tr>
<tr>
<td></td>
<td>to prime age</td>
<td>to prime age</td>
</tr>
<tr>
<td>Gov. Size (tax)</td>
<td>$-1.139^{***}$</td>
<td>$-0.157^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.095)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Constant</td>
<td>$0.858^{***}$</td>
<td>$0.633^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Time dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>563</td>
<td>563</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$

young to prime-age workers. In this Table, we define young workers as those aged between 15 and 29 and prime age workers as those aged between 30 and 64.\textsuperscript{11} The findings are consistent with the view put forward in this paper. As the size of the government increases, the share of young workers decreases. This effect is large and statistically significant. Moreover, the finding are robust to the inclusion of time effects in the regression equation.

In summary, in this Section we have documented the three following facts: i) the employment of young and older individuals fluctuates much more over the business cycle than that of prime-aged individuals; ii) there is a negative relation between the size of the government and the cyclical volatility of output, but controlling for the demographic structure of the population attenuates this relation; iii) across OECD countries, the share of young workers in the labor force declines as the size of the government increases. In the next Section we propose a theoretical model based on these three facts.

\textsuperscript{11} Jaimovich and Siu (2009), consider how the “volatile-aged labor force share” is correlated with aggregate volatility. In addition to the workers aged 15–29, their volatile-aged labor force includes workers aged 60–64. We only consider young workers in Table 2 to make the empirical analysis compatible with the study in Section 5.4.3. However, the findings are robust to changes in the definition of the volatile age group.
3 The model

The economy is inhabited by a large number (unit measure) of identical and infinitely lived families. Each family is composed of a unit mass of individuals of different ages with each individual living a maximum of $Q$ periods. Ages are denoted by $i \in I \equiv \{1, \ldots, Q\}$. Within each family the mass of individuals aged $i$ is $a_i$, with $\sum_{i=1}^{Q} a_i = 1$. Whether individuals live $Q$ periods with certainty or, instead, may die earlier is irrelevant since there is perfect risk-sharing within each family. All individuals are endowed with one unit of time each period. An age $i$ individual’s unit of time can be transformed into $e_i$ efficient units of labor.

3.1 Preferences

Within each family, individuals’ period utility function is age dependent and we assume that it has the form introduced by Greenwood et al. (1988):

$$u(c, n; i) = \ln \left( c - \lambda_i n^{1+\theta_i} \frac{1}{1+\theta_i} \right), \quad (1)$$

where $c$ and $n$ are consumption and time spent working, respectively. The parameter $\theta_i$ is the inverse of the Frisch elasticity of labor supply and is age dependent. Notice that the choice of utility function excludes intertemporal substitution effects on labor supply. Rather than being a drawback, this implication of the utility function has the advantage of emphasizing the importance of age-specific labor supply elasticities and is instrumental in the calibration exercise.

The preferences of the representative family are given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{Q} a_i u(c_{it}, n_{it}; i) \right], \quad (2)$$

where $c_{it}$ and $n_{it}$ are, respectively, consumption and time spent working by family members aged $i$, in period $t$. 

12
3.2 Technology

We consider a one-sector model economy where the single good produced serves two purposes: consumption and investment. Output is produced by a representative firm operating a constant returns to scale Cobb-Douglas production function:

\[ Y_t = \exp(z_t) K_t^\alpha H_t^{1-\alpha}, \]  

(3)

where \( K_t \) and \( H_t \equiv \sum_{i=1}^Q e_i N_{it} \) are, respectively, accumulated capital and efficiency units of labor services in period \( t \), and \( N_{it} = a_i n_{it} \) is aggregate labor effort by individuals aged \( i \) in period \( t \). Capital depreciates at a positive rate \( \delta \). Fluctuations are driven by random transitory movements in total factor productivity:

\[ z_t = \rho z_{t-1} + \sigma \epsilon_t, \quad 0 < \rho < 1, \]  

(4)

where \( \epsilon_t \) is identically and independently standard normal distributed.

The first-order conditions for the firm’s profit maximization yield the following functions for the wage rate and rental rate of capital:

\[ w(z_t, K_t, H_t) = (1 - \alpha) \exp(z_t) K_t^\alpha H_t^{-\alpha}, \]  

(5)

\[ r(z_t, K_t, H_t) = \alpha \exp(z_t) K_t^{\alpha-1} H_t^{1-\alpha}. \]  

(6)

3.3 The government sector

The government taxes capital income, labor income and consumption expenditure, at the rates \( \tau_k, \tau_h \) and \( \tau_c \), respectively.\(^{12}\) From the expenditure side, the government spends \( G_t \) as government consumption and provides lump-sum transfers denoted \( T_t \). The government

\(^{12}\)Notice that the government taxes net capital income. This contrasts with the economies described in Gali (1994) and Andres et al. (2008). We consider taxes on net capital income because we find this assumption more realistic. Moreover, this is in line with the literature on optimal taxation such as e.g. Chari et al. (1994). Considering taxes on gross income has implications for aggregate volatility. In Section 5.4.4, we illustrate how our quantitative results are affected by changing this assumption.
is assumed to run a balanced budget each period, so that:

\[ R_t = \tau_k (r_t - \delta) K_t + \tau_h w_t H_t + \tau_c C_t = G_t + T_t, \]  

(7)

where \( R_t \) is government revenue in period \( t \). We take government lump-sum transfers as being exogenous and, in particular, constant over time \( T_t \equiv T \). Therefore, government consumption adjusts each period so that \( G_t = R_t - T \).

For simplicity, we do not model how agents benefit from government consumption. This framework is appropriate because we are not interested in examining the potential automatic-stabilizing role of budget-deficits but instead we want to investigate the impact that changes in the fiscal policy parameters has on macroeconomic stability.

### 3.4 Competitive equilibrium

The representative family chooses each member’s consumption and labor supply, and how much to invest, to maximize (2) subject to the sequence of budget constraints:

\[ x_t + (1 + \tau_c) \sum_{i=1}^{Q} a_i c_i t \leq (1 - \tau_k) r_t k_t + \tau_k \delta k_t + (1 - \tau_h) w_t \sum_{i=1}^{Q} a_i e_i n_i t + T_t, \]  

(8)

where \( x_t \) is the family’s investment in period \( t \). The family’s capital stock obeys the following law of motion:

\[ k_{t+1} = (1 - \delta) k_t + x_t, \quad 0 \leq \delta \leq 1. \]  

(9)

When elaborating the optimal plan, each family takes the factors’ prices and the economy’s aggregates, \( K_t, X_{t+1} \) and \( H_t \), as given. The representative family’s dynamic programming problem is stationary and, hence, can be cast formally as:

\[ V (k; K, z) = \max_{\{(c_i, n_i)\}_{i=1}^{Q}, x} \left\{ \sum_{i=1}^{Q} a_i u (c_i, n_i; i) + \beta \int V (k'; K', z') d\Psi (z' | z) \right\}, \]  

(10)
subject to

\[ x + (1 + \tau_c) \sum_{i=1}^{Q} a_i c_i = (1 - \tau_k) r(z, K, H) k + \tau_k \delta k \]

\[ + (1 - \tau_h) w(z, K, H) \sum_{i=1}^{Q} a_i e_i n_i + T, \]

(11)

\[ k' = (1 - \delta) k + x, \]

(12)

\[ K' = (1 - \delta) K + X, \]

(13)

\[ z' = \rho z + \epsilon \]

(14)

and \( c_i, k \) non-negative, and \( 0 \leq n_i \leq 1 \), for all \( i \). In addition, \( X \) and \( H \) are given functions of \((z, K)\). Assuming an interior solution, the upshot of this optimization problem is summarized by the following set of efficiency conditions—in addition to (8):

\[ \frac{1}{(1 + \theta_i)} c_i - \lambda_i n_i^{1+\theta_i} - \lambda_i n_1^{1+\theta_1} = \beta \int \frac{1 + (1 - \tau_k) [r(z', K', H') - \delta]}{(1 + \theta_i) c_i' - \lambda_i n_i^{1+\theta_i}} d\Psi (z' | z), \]

(15)

\[ c_i = c_1 + \frac{\lambda_i n_i^{1+\theta_i}}{1 + \theta_i} - \frac{\lambda_i n_1^{1+\theta_1}}{1 + \theta_1}, \]

(16)

\[ n_i = \left[ \frac{(1 - \tau_h) w(z, K, H) e_i}{(1 + \tau_c) \lambda_i} \right]^{\frac{1}{\theta_i}}, \]

(17)

for \( i = 1, \ldots, Q \) and where \( c_1 \) and \( n_1 \) are, consumption and time spent working by agents aged \( i = 1 \).

The first equation, equation (15), is the standard Euler condition for intertemporal efficiency. The next equation, equation (16), is a static optimality condition requiring the marginal utility of consumption to be equalized across all family members. This equation describes two effects of labor supply on consumption. First, there is substitution between leisure and consumption within each demographic group, implying that an increase in the
supply of labor of a group raises its consumption too. Second, there is complementarity between leisure and consumption between each demographic group, implying that an increase in labor supply for a particular group (holding consumption constant for that group) generates a reduction in consumption for other groups. In Section 5 we refer to these two effects to explain the evolution of consumption volatility over the life cycle. Finally, equation (17) is a static optimality condition governing the choice of labor effort by each family member. This equation shows the absence of wealth effect on labor supply as no term in consumption appears.

The following formally defines a competitive equilibrium for the economy:

**Definition 1.** A competitive equilibrium for this economy consists of a set of family’s decision rules \( \{c_i(s)\}_{i=1}^Q, \{n_i(s)\}_{i=1}^Q, x(s) \) (where \( s \equiv (k,K,z) \) is the family’s vector of relevant states), a set of aggregate laws of motion \( X(S), \{N_i(S)\}_{i=1}^Q, H(S) \) and \( G(S) \) (where \( S \equiv (K,z) \) is the vector of aggregate level states) and a value function \( V(s) \), such that:

i. the functions \( V, X \) and \( H \) satisfy (10) – (14) and \( \{c_i\}_{i=1}^Q, \{n_i\}_{i=1}^Q \) and \( x \) are the associated set of family decision rules;

ii. \( k = K, x = X \) and \( \sum_{i=1}^Q a_in_i = N_i \) for all \( i \);

iii. the government budget balances, \( G(S) = R(S) - T \); and

iv. the functions \( c(s) \) and \( x(s) \) satisfy \( c(s) + x(s) + G(S) = Y(S) \) for all \( s \).

4 Government size and labor force heterogeneity

In this section, we examine two aspects of the model. First, we illustrate the differences in hours volatility across demographic groups in the model. Second, we focus on the steady-state of the economy and ask how the share of hours worked by each age group varies as the size of the government is changed.

The following Proposition compares hours volatility across age groups.
Proposition 1. Denote by $\sigma_i$ the standard deviation of the logarithm of hours worked by individuals aged $i$ and $\sigma_w$ the standard deviation of the logarithm of the wage rate. It follows that

$$\sigma_i = \eta_i \sigma_w,$$

where $\eta_i \equiv 1/\theta_i$ is the Frisch labor supply elasticity.

The proposition follows from equation (17). It shows that age groups with large labor supply elasticity display more volatile labor effort. This simple result is one of the two elements behind the mechanism that explains the relation between the government size and macroeconomic stability in the model we study. In particular, if the share of hours worked by the high volatility group decreases, the volatility of aggregate hours worked also decreases.

To show how the share of hours worked by each age group varies as the size of the government is changed, we characterize the steady state for the certainty version of the model. We denote the steady-state variables by the variable’s symbol with a hat over it and we call this equilibrium a stationary competitive equilibrium. In steady state, consumption and labor effort by individuals aged $i \in \mathcal{I}$ are constant over time and equation (15) can be transformed into:

$$(1 - \tau_k) \left[ r \left( 0, \hat{K}, \hat{H} \right) - \delta \right] = \frac{1}{\beta} - 1,$$  

(19)

In turn, by making use of equation (19), it is possible to solve for the steady state capital-labor ratio:

$$\frac{\hat{K}}{\hat{H}} = \left[ \frac{(1 - \tau_k) \alpha}{\beta - 1 + \delta (1 - \tau_k)} \right]^{\frac{1}{1 - \alpha}},$$  

(20)

Next, by combining conditions (5) and (17), the amount of time spent working, in steady state, by individuals aged $i$, is found to satisfy:

$$\hat{n}_i = \left[ \frac{e_i (1 - \tau_h) (1 - \alpha)}{(1 + \tau_c) \lambda_i} \right]^{\eta_i} \left[ \frac{(1 - \tau_k) \alpha}{\beta - 1 + \delta (1 - \tau_k)} \right]^{\frac{1}{1 - \alpha} \eta_i},$$

(21)

where $\eta_i$ is the Frisch labor supply elasticity for individuals aged $i$. Notice that, because of the form chosen for the utility function, each family member’s labor effort is determined.
independently of the intertemporal consumption-saving choice. Thus, as the size of the government increases, the time spent working by individuals with high labor supply elasticity (high \( \eta_i \)) falls relatively to the time spent working by individuals with low labor supply elasticity (low \( \eta_i \)). These relative changes alter the labor workforce composition toward individuals with less elastic labor supplies. When analyzing how changes in the size of the government, as controlled by \( \tau_h \), \( \tau_k \) and \( \tau_c \), affect labor supply volatility, our framework stresses changes in the workforce composition brought about by differences in the elasticity of labor supply across individuals in different stages of their life-cycle.

**Proposition 2.** Consider the stationary competitive equilibrium. The elasticity of labor effort to changes in the labor income tax rate, \( \tau_h \), for individuals aged \( i \) is

\[
\frac{dn_i \tau_h}{d\tau_h n_i} = - \frac{\tau_h}{1 - \tau_h} \eta_i. \tag{22}
\]

The elasticity of labor effort to changes in the consumption tax rate, \( \tau_c \), for individuals aged \( i \) is,

\[
\frac{dn_i \tau_c}{d\tau_c n_i} = - \frac{\tau_c}{1 + \tau_c} \eta_i. \tag{23}
\]

Finally, the elasticity of labor effort to changes in the capital income tax rate, \( \tau_k \), for individuals aged \( i \) is

\[
\frac{dn_i \tau_k}{d\tau_k n_i} = - \frac{\alpha}{1 - \alpha} \frac{\tau_k}{1 + \tau_k} \eta_i. \tag{24}
\]

The proof of Proposition 2 follows immediately from the inspection of equation (21).

Thus, an increase in any of the three tax-rates, leads to a change in the composition of the aggregate labor supply toward the less volatile individuals and, from Proposition 1, a decrease in the aggregate labor supply volatility. In what follows, we examine the quantitative properties of the model and, in particular, we investigate whether the model is capable of replicating the stabilizing role of the government, as observed in the data.
5 Quantitative analysis

We use the model economy just described to study how changes in the government scope change the economy’s business cycle properties. In particular, we want to investigate whether the model implies that economies with large governments—understood as large tax rates, \( \tau \)—have less volatile business cycles, in a way that is consistent with the data. Before proceeding to the results we describe carefully how the model is solved and calibrated.

5.1 Solution method

Because there are tax distortions in the economic environment described above, the competitive equilibrium is not Pareto optimal. Therefore, the social planner problem cannot be solved instead of the decentralized equilibrium problem and the latter has to be solved directly. To achieve this we use the method described in Greenwood and Huffman (1991), which consists of solving the representative family’s problem (10), by iterating on Bellman’s equation, requiring that the family’s individual choices be consistent with the aggregate laws of motion, as specified in the competitive equilibrium’s definition—item (ii). In the remainder of this section, we outline this procedure in more detail.

We begin by noticing that, combining (16) and (17), it is possible to eliminate \( c_i \), for all \( i \neq 1 \), and \( n_i \), for \( i \in \mathcal{I} \), from the budget constraint (11). The resulting family’s budget constraint is:

\[
x + (1 + \tau_c) c_1 - \frac{[(1-\tau_k)w(z,K,H)e_i]^{1+\theta_i}}{[1+\tau_c}\lambda_i]\frac{1}{\pi_i^{1+\theta_i}} + \sum_{i=1}^{Q} a_i \frac{[(1-\tau_h)w(z,K,H)e_i]^{1+\theta_i}}{[(1+\tau_c)\lambda_i]\pi_i^{1+\theta_i}} - T = (1-\tau_k) r(z,K,H) k_t + \tau_k \delta k + (1-\tau_h) w(z,K,H) \sum_{i=1}^{Q} a_i e_i \left[ \frac{[(1-\tau_h)w(z,K,H)e_i]}{[1+\tau_c}\lambda_i] \right]^{\frac{1}{\pi_i^{1+\theta_i}}}.
\]  

The following step is to eliminate \( H \) from (10) and (25) by combining the market clearing condition \( N_i = \sum_{i=1}^{Q} a_i n_i \) with condition (17). Noticing that \( H \equiv \sum_{i=1}^{Q} e_i N_i \), this
yields:

\[ B(H, K, z) \equiv \sum_{i=1}^{Q} a_i e_i \left[ \frac{(1 - \tau_h) (1 - \alpha) \exp(z) K^\alpha H^{-\alpha} e_i}{(1 + \tau_c) \lambda_i} \right]^{\frac{1}{\gamma}} - H = 0. \]  

(26)

By combining the constraints (25) and (26) with equations (16) and (17), it is possible to eliminate \( c_i \) and \( n_i \), for all \( i \in I \), and \( H \) from the family’s instantaneous utility function. The resulting representative family’s dynamic programming problem is now given by the following expression:

\[ V(k; K, z) = \max_{k'} \left\{ \sum_{i=1}^{Q} a_i \hat{u}(k, k', i; K, z) + \beta \int V(k'; K', z') d\Psi(z' \mid z) \right\}. \]  

(27)

In order to initiate the iterative procedure, an initial guess is made for both the value function on the right-hand side of (27) and the equilibrium law of motion for the capital stock. Denote these guesses by \( V_0(k'; K', z') \) and \( K'_0(K, z) \), respectively. Next, problem (27) is solved using these guesses. The optimized value of the maximand, which represents the left-hand side of the functional equation, is used as a revised guess for the value function, or \( V_1(k', K', z') \). As part of the solution to this problem, the individual’s decision rule for capital accumulation is obtained; it has the form \( k' = k'_0(k; K, z) \). Since in equilibrium capital accumulation at the individual and aggregate levels must coincide, or \( k = K \), this decision rule forms the basis for the revised guess for the law of motion for the aggregate capital stock \( K'_1(K, z) \). Specifically, \( K'_1(K, z) = k'_0(K; K, z) \). These revised guesses for \( V(K'; K', z') \) and \( K'(K, z) \) are used as the foundation for the next round in the iterative scheme, the procedure being repeated until the decision rule has converged.

To operationalize the iterative scheme discussed above the aggregate states for the economy and the individual states are constrained to be elements of finite time-invariant sets.

\[ \text{\textsuperscript{13}} \text{To establish that a unique level of aggregate effective labor } H \text{ corresponds to each state space element } S \equiv (K, z) \text{ notice that: the function } B(H, S) \text{ is continuous differentiable and its partial derivative with respect to } H \text{ is negative; the limit of } B(H, S) \text{ as } H \text{ goes to } 0 \text{ is } +\infty \text{ and the limit as } H \text{ goes to } +\infty \text{ is } -\infty. \text{ Therefore, for each } S, \text{ there is a unique } H \text{ satisfying } B(H, S) = 0. \text{ Notice that, because } N_i \leq 1 \forall i \in I \text{ only } H \leq Q \text{ are feasible. We parametrize the model so that for each } S \text{ in the admissible state space, corresponds a feasible value for } H. \]
Table 3: Distribution of hours and relative volatilities by age

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Share of hours</th>
<th>Relative volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 – 19</td>
<td>0.032</td>
<td>5.144</td>
</tr>
<tr>
<td>20 – 24</td>
<td>0.103</td>
<td>2.529</td>
</tr>
<tr>
<td>25 – 29</td>
<td>0.129</td>
<td>1.753</td>
</tr>
<tr>
<td>30 – 39</td>
<td>0.254</td>
<td>1.348</td>
</tr>
<tr>
<td>40 – 49</td>
<td>0.233</td>
<td>1.000</td>
</tr>
<tr>
<td>50 – 59</td>
<td>0.172</td>
<td>0.938</td>
</tr>
<tr>
<td>60 – 64</td>
<td>0.048</td>
<td>0.942</td>
</tr>
</tbody>
</table>

Note: The relative volatilities represent the relative standard deviation of the logarithm of hours worked and are computed based on HP filtered data as reported in Jaimovich and Siu (2009). The distribution of hours by age is obtained from the same source.

Thus, the stochastic shock, $z$, is constrained to follow a first-order Markov chain specification with states $z \in \mathcal{Z} \equiv (z_1, \ldots, z_m)$. The Markov process’s transition probabilities are chosen to approximate well the continuous-valued Gaussian autoregressive process (4) following the method described by Tauchen (1986) and, in particular, so that $E(z) = 0$ and $E(z^2) = \frac{\sigma^2}{1 - \rho^2}$. In turn, the economy’s aggregate capital stock is constrained to take values in $\mathcal{K} = (K_1, \ldots, K_j)$. Hence, the aggregate state space of the economy, $\mathcal{S} = \mathcal{K} \times \mathcal{Z}$, is discrete.

### 5.2 Calibration

A steady state for the deterministic version of the model economy is its rest point when the variance of the shocks is zero. The purpose of the calibration is to choose the parameter values for which the steady state values of the model aggregates are approximately equal to their empirical averages. We set a period length to be one year. Two types of data are used to calibrate the model, aggregate annual time-series data for the U.S. economy and cross-sectional information on the wage profile and on the relative level and volatility of market hours across age groups, also for the U.S. economy.

In addition, regarding the fiscal policy variables, we choose the tax rates on capital income, labor income and consumption based on evidence documented in Carey and Rabesona (2002) who have produced series for the average effective tax rates on capital income, labor income and consumption for the OECD countries based on the methodology proposed by Mendoza et al. (1994). We will make use of these cross-country data for examining the relation between fiscal policy and aggregate volatility. However, for the purpose of the calibration, we simply use the tax rates which are reported by these authors for the U.S.
economy. The values chosen for each tax rate are shown in Table 4. Finally, the calibration of the public finance parameters is concluded by choosing a value for $T$, the value of transfers. We choose $T$ so that in steady state the ratio of government consumption to output is equal to 19.1 percent, which corresponds to the ratio of final government consumption expenditure to GDP for the U.S. measured from the OECD national accounts data. This implies a value for $T$ which represents 11 percent of steady state output.

The investment to output ratio is measured at 13.3 percent, using the National Income and Product Accounts (NIPA). The steady-state investment-output ratio is given by

$$\left(\frac{\hat{X}}{\hat{Y}}\right) = \delta \left(\frac{\hat{K}}{\hat{H}}\right)^{1-\alpha}$$

which, making use of equation (20) can be expressed as follows

$$\frac{\hat{X}}{\hat{Y}} = \delta \left[\frac{(1 - \tau_k)^\alpha}{\frac{1}{\sigma} - 1 + \delta (1 - \tau_k)}\right].$$

We set $\delta = 0.10$, implying an annual depreciation rate of 10 percent, which is consistent with evidence in Gomme and Rupert (2007). The capital income share $\alpha$, is set equal to 0.283 based on the value implied by the NIPA. These choices imply that the value chosen for $\beta$ is equal to 0.949, in order to match the target for the investment-output ratio.

To choose values for the stochastic process for the technology shock we use the estimates from Gomme et al (2004). These authors construct a series for the Solow residual over the period 1954-2000 using annual data and then estimate an $AR(1)$ process assuming a
polynomial time trend. The estimated value for $\rho$ is 0.895 and for $\sigma$ is 0.153.

We now describe the aspects of the calibration which have to do with the demographic structure of the workforce. This is the most important part of the calibration because it determines the relation between the demographic composition of the workforce and aggregate volatility. We assume that the stand-in family is composed of seven distinct demographic groups, whose members have ages comprised between 15 and 64. The partition into the seven demographic groups is as illustrated in Table 3. The targets which are used for the purpose of calibration are the share of total hours worked by each age group and the relative volatility of hours worked by each age group. We take as the reference age group, the group which is composed of individuals aged between 40 and 49.

From Proposition 1 it follows that the standard deviation of the logarithm of hours worked by individuals in the age group $i$ relative to the volatility of the logarithm of hours worked by individuals in the reference age group 40–49 is given by

$$\frac{\sigma_i}{\sigma_{40-49}} = \frac{\eta_i}{\eta_{40-49}}.$$  \hspace{1cm} (29)

Therefore, given a value for the Frisch labor supply elasticity of the reference group, $\eta_{40-49}$, the Frisch elasticities of the other age groups are chosen so that for each age group $i$, the ratio $\eta_i/\eta_{40-49}$ equals the relative volatility of that group as shown in Table 3. We are left with only the reference age group labor supply elasticity undetermined. There is a voluminous literature that has estimated the Frisch elasticities for prime aged workers (e.g., see Blundell and MaCurdy, 1999). For instance, for adult males, MaCurdy (1981) obtained estimates of about 0.3. From Heckman and MaCurdy (1980) the corresponding value for females is about 2.2. We choose to set $\eta_{40-49} = 1$ which is certainly in the middle range of the existing estimates.

Making use of equation (21), it follows that the hours worked in steady state by the individuals in the age group $i$ are given by

$$\hat{N}_i = a_i \left[ \frac{e_i (1 - \tau_h) (1 - \alpha)}{(1 + \tau_c) \lambda_i} \right]^\eta_i \left[ \frac{1}{\beta} - 1 + \delta (1 - \tau_k) \right]^\frac{\eta_i}{\alpha},$$  \hspace{1cm} (30)

The shares of individuals in each age group $i$, $a_i$, are derived from the OECD population
Table 4: Baseline calibration: summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.283</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.010</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.949</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.895</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.153</td>
</tr>
<tr>
<td>$\tau_h$</td>
<td>0.256</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.371</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>0.053</td>
</tr>
<tr>
<td>$T/\hat{Y}$</td>
<td>0.114</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\eta_{15-19}$</th>
<th>$\eta_{20-24}$</th>
<th>$\eta_{25-29}$</th>
<th>$\eta_{30-39}$</th>
<th>$\eta_{40-49}$</th>
<th>$\eta_{50-59}$</th>
<th>$\eta_{60-64}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.144</td>
<td>2.529</td>
<td>1.753</td>
<td>1.348</td>
<td>1.000</td>
<td>0.938</td>
<td>0.942</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda_{15-19}$</th>
<th>$\lambda_{20-24}$</th>
<th>$\lambda_{25-29}$</th>
<th>$\lambda_{30-39}$</th>
<th>$\lambda_{40-49}$</th>
<th>$\lambda_{50-59}$</th>
<th>$\lambda_{60-64}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.862</td>
<td>1.345</td>
<td>1.949</td>
<td>2.871</td>
<td>4.171</td>
<td>4.605</td>
<td>6.232</td>
</tr>
</tbody>
</table>

statistics. The efficient labor units for each age group, $e_i$, are set to match the life-cycle profile of hourly earnings implied by the Panel Survey of Income Dynamics (PSID), which collects household level earnings data from a representative sample of the U.S. population. The resulting profile of efficiency units by age group and the shares of individuals in each age group are both shown in Figure 3. The only remaining parameters from equation (30) are the $\lambda_i$, which control the disutility of work for individuals in each age group $i$. Given a value for the reference’s age group disutility parameter, $\lambda_{40-49}$, the remaining $\lambda_i$’s are chosen to match the relative shares of total hours worked by each age group which are shown.
in Table 3. Finally, \( \lambda_{40-49} \) is chosen so that in steady state the stand-in family spends 25.5 percent of its endowment of time working, based on Gomme and Rupert (2007), who interpret evidence from the American Time-use Survey.

5.3 Model evaluation

In the following subsection we study the behavior of the model economy under the benchmark calibration. We first discuss the aggregate properties of the model. We then examine the implications of the model for the life-cycle.

5.3.1 Aggregate volatilities

Table 5 displays relevant aggregate statistics for the theoretical economy under the benchmark calibration. It shows the properties of output, consumption, investment, government spending and hours worked in both the data and the model, as described by the volatility of their cyclical components and the correlation of the cyclical components with the cyclical component of output. In this table, annual data on hours worked is from the Conference Board Total Economy Database for the sample period 1970–2007, while the rest of the variables are taken from the OECD Outlook database. Cyclical components are found by applying the Hodrick and Prescott (HP) filter to the logged series with a smoothing parameter equal to 6.25, as recommended in Ravn and Uhlig (2002).

Volatilities in the model are similar to those obtained in a standard RBC model. For the Frisch elasticities we have chosen, it generates an output volatility similar to the one observed in the data. Volatilities of consumption and investment are also comparable to their empirical counterparts. The model also suffers from the same drawbacks as in the standard RBC model: the volatility of hours worked is approximately two thirds of output’s, while it is higher in the data.\(^{14}\) Additionally, the volatility of government spending in the theoretical economy is larger than in the data because of our assumption that the government budget is always balanced. Furthermore, in the absence of budget deficits, government spending is necessarily procyclical in the theoretical economy, while it is coun-

Table 5: U.S. business cycle statistics, 1970-2007: model vs. data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.38</td>
<td>1.57</td>
<td>1.00</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.15</td>
<td>0.90</td>
<td>0.90</td>
<td>0.97</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>Investment</td>
<td>4.17</td>
<td>3.77</td>
<td>0.97</td>
<td>0.95</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>Government spending</td>
<td>0.84</td>
<td>2.57</td>
<td>-0.26</td>
<td>1.00</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>Hours</td>
<td>1.14</td>
<td>0.98</td>
<td>0.91</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: Data on GDP, consumption, investment and government spending is from the OECD Outlook database. Data on hours worked is from the Conference Board Total Economy Database. Cyclical parts are reported in logs as deviations from an HP trend with smoothing parameter 6.25. Model’s reported values correspond to the average over 500 simulations.

The government revenue is perfectly correlated with output. Since the government budget is balanced each period and the volatility of transfers is zero, it follows that the government spending is perfectly correlated with output and more volatile than output.

Finally, the shares of consumption, investment and government spending in output are the same as the ones found in the data because of the restrictions imposed by our calibration strategy.

5.3.2 Life-cycle patterns

We now examine the ability of the model to replicate the volatility of hours worked and consumption for each age group. The purpose of the calibration exercise is to replicate
Figure 4: Consumption and hours volatility in the calibrated model by age group

Note: All variables are reported in logs as deviations from an HP trend with smoothing parameter 6.25. Reported values for the model correspond to the average of the standard deviation of the filtered series over 500 simulations. Data on hours worked volatility by age group is from Jaimovich and Siu (2009).

the way in which labor supply volatility varies across demographic groups. This allows us to study whether this dimension of heterogeneity in the labor force contributes to explain the relation between government size and macroeconomic stability. Figure 4 depicts the standard deviation of the cyclical parts of consumption and hours worked by age group in the calibrated model. Moreover the lower-panel in the Figure contrasts the cyclical volatility across age groups in the theoretical model and in the data.

In the theoretical economy, the volatility of hours displayed in the second panel decreases over the life-cycle because the Frisch elasticity decreases with age too. Moreover the second panel reveals that the calibrated model displays volatilities of hours worked across age groups that are very similar to the data. This is because the calibration exercise chooses Frisch elasticities in order to reproduce these volatilities.

The upper-panel in Figure 4 shows the life-cycle profile of the volatility of consumption. There are two opposing forces which determine this profile. On the one hand, given the choice of utility function, leisure and consumption are substitutes. Therefore, an increase in labor supply is accompanied by an increase in consumption—see equation (16). Hence, when hours worked are volatile, consumption is also volatile. We call this first effect the “within-group substitutability” effect. Second, there is a “between-group complementarity”
effect whereby, when a group raises its supply of labor—therefore, raising its marginal utility of consumption—the marginal utility of consumption of the other demographic groups increases too. This second effect reduces the within-group wage-elasticity of consumption offsetting the “within-group substitutability” effect.

The relative strength of these two opposing forces depends on the relative hours worked by each age group. In particular, when the hours worked by an age group are high relative to the hours worked by the other age groups, the “within-group substitutability” effect dominates the “between-group complementarity” effect.\textsuperscript{17} If the “within-group substitutability” effect dominates, high relative volatility of hours worked is associated with a high relative volatility of consumption. If instead, the “between-group complementarity” effect dominates high relative volatility of hours worked is associated with a low relative volatility of consumption.

From the upper-panel of Figure 4 we see that, overall, the “within-group substitutability” effect dominates. In particular, starting from the 20–24 age category, the volatility is decreasing over the life cycle, varying from roughly 1 percent for the 20–24 age group to 0.7 percent for the 60–64 age group. However, the first category (15–19) displays lower volatility relative to most other age group—slightly less than 0.9 percent. Thus, the “between-group complementarity” effect is strong between the age groups 15–19 and the remaining groups. The reason is that in our calibration the relative hours supplied by the 15–19 age group are very low.

\textsuperscript{17}To see this formally, we consider the log-linear approximation of equation (16). Based on this approximation we express the variance of log consumption for the first age group—corresponding to the 15–19 demographic group—as follows

\[
\hat{c}_1^2 \sigma _{\hat{c}_1} = \hat{c}_1^2 \sigma _{\hat{c}_1} + \left( \lambda_1 \hat{n}_1^{1+\theta_1} \right)^2 \sigma _{\hat{n}_1}^2 + \left( \lambda_1 \hat{n}_1^{1+\theta_1} \right)^2 \sigma _{\hat{n}_i}^2 - 2 \lambda_1 \hat{n}_1^{1+\theta_1} \lambda_i \hat{n}_i^{1+\theta_i} \sigma _{\hat{n}_1, \hat{n}_i} + 2 \lambda_1 \hat{n}_1^{1+\theta_1} \lambda_i \hat{n}_i^{1+\theta_i} \sigma _{\hat{c}_1, \hat{c}_i} - 2 \lambda_1 \hat{n}_1^{1+\theta_1} \lambda_i \hat{n}_i^{1+\theta_i} \hat{c}_i \sigma _{\hat{n}_1, \hat{c}_i}, \tag{31}
\]

where \( i \neq 1 \) refers to an age group other than the 15–19 group; variables with ‘ denote log deviations from steady state and variables with hats are steady-state values; \( \sigma _x^2 \) is the variance of a particular variable \( x \) and \( \sigma _x,y \) is the covariance between \( x \) and \( y \). The negative covariance terms follow from the complementarity between leisure and consumption between groups. The weight given to the negative covariance terms is increasing in the steady-state labor supply of the age group \( i \). Moreover, the weight given to the positive covariance term increases in the level of hours worked by the reference group. Hence, all else equal, if \( n^*_i \) is high relative to \( n^*_1 \), the relative consumption volatility of group 1 will be low.
Figure 5 shows the volatility of consumption expenditure across age groups in the data for the United States over the 1984–2006 sample period. The data used to calculate these volatility measures is provided by the Bureau of Labor Statistics, and are produced from the CEX survey. Although the use of this data has been criticized in a life-cycle context because it only gives expenditures and not actual quantities consumed, this is not a concern if one wishes to compare volatilities across age groups. Notice that the age groups in this Figure do not correspond to the groups in Figure 4. Nonetheless, despite of the imperfect mapping, some information can be extracted from Figure 5 in order to assess the ability of the theoretical economy to replicate the volatility of consumption over the life cycle. Figure 5 shows that the volatility of consumption is large for individuals aged below 25 with a value of 1.4 percent and then remains more or less constant at 0.8 percent for individuals aged older than 25. Hence, the calibrated model is able to reproduce the fact that consumption volatility is higher among the young than among the old. However,\footnote{The age reported corresponds to the age of the reference person. The CEX survey defines the reference person as the person or one of the persons who owns or rents the home and can therefore be interpreted as the head of the household. The variable consumption corresponds to the total household expenditure over a given period.}

\footnote{See Aguiar and Hurst (2005, 2007) who analyze the evolution of consumed quantities over the life cycle together with home production and the fact that some age groups choose to consume goods characterized by lower prices, especially among the old.}
although it does well for age groups above 25 years old, more volatility is warranted among the young (1 percent in the model vs. 1.4 percent in the data).

We see the decrease in consumption volatility that our calibrated model generates as a nice feature of our model. Indeed, standard life-cycle models with incomplete markets such as Gomme et al. (2004) usually predict an upward sloping trend in consumption volatility. The reason has to do with how agents perceive wage shocks in those models: old workers interpret shocks as rather “permanent” because they do not have many periods of life remaining to smooth fluctuations in income, while young agents perceive them as “transitory” as they have plenty of time left to smooth shocks. Hence, from permanent income theory, it follows that consumption volatility is larger for old agents. This contradicts the data as Figure 5 shows. Instead, our model is able to reproduce this decreasing pattern because there is perfect risk-sharing within family.

5.4 The stabilizing role of the government

We now address the question of whether our model is able to reproduce a negative correlation between the government’s scope—as measured by the tax-revenue to output ratio—and aggregate volatility. Beyond the simple qualitative relation between the fiscal policy variables and macroeconomic stability, we are also interested in the quantitative implications. In particular, we compare the magnitudes of the effect of government size on macroeconomic volatility implied by the model with the magnitudes in the data. We proceed in two steps: first we study the model comparative statics—i.e., we examine what happens to the volatility of the macroeconomic aggregates as we change each fiscal policy parameter individually; second, we feed into the model different combinations of values for the tax rates and for the government spending as a share of GDP, with each combination chosen to mimic the fiscal policy parameters of a particular OECD country. By following this procedure, we make sure that the size of the government is varied endogenously, in a way which is dictated by the changes in fiscal policy parameters across OECD countries.

A possible way to obtain high consumption volatility among the young in life-cycle models with incomplete markets is to introduce borrowing constraints, which are particularly binding for those age groups.
This allows us to investigate whether we are able to replicate quantitatively the relation between government size and macroeconomic stability across OECD countries.

5.4.1 Comparative statics

In Figures 6 to 8, we perform the following exercise. We consider the model with the parameters calibrated as previously. Then we change each of the three tax rates ($\tau_h$, $\tau_k$ and $\tau_c$, respectively) one at a time and observe how aggregate volatilities vary. In particular, the figures illustrate the impact on the volatility of output, consumption, investment and aggregate hours worked.

Figure 6 shows that increasing the tax rate on labor income stabilizes output, consumption and investment, and also the aggregate hours worked. Output is stabilized when the tax rate increases because the share in aggregate hours of demographic groups characterized by large labor supply elasticities decreases, as Proposition 2 establishes. The change in the composition of the labor force stabilizes aggregate hours because the relative share
Note: All variables (but government size) are reported in logs as deviations from an HP trend with smoothing parameter 6.25. Reported values correspond to the average of the standard deviation of the filtered series over 500 simulations.

of aggregate hours which is supplied by the young (volatile) workers falls. Therefore, both aggregate hours and aggregate output volatility are reduced.

The volatility of consumption also decreases as the labor income tax rate increases. This has to do with the two effects we have previously described. First, the substitutability between consumption and leisure within demographic groups implies that lower hours volatility generates lower consumption volatility. Second, the complementarity between consumption and leisure across demographic groups also affects consumption volatility, especially among the young. The relative decrease in hours worked by the young amplifies the complementarity effect between consumption and leisure across demographic groups.\textsuperscript{21}

In Section 5.4.3 we describe in more detail how the two effects affect consumption volatility across demographic groups.

Finally, the volatility of investment is also lower at high labor income tax rates. This

\textsuperscript{21}See equation (31).
Figure 8: Aggregate volatility and consumption tax rate

![Graphs showing the relationship between consumption tax rate and volatility](image)

Note: All variables (but government size) are reported in logs as deviations from an HP trend with smoothing parameter 6.25. Reported values correspond to the average of the standard deviation of the filtered series over 500 simulations.

Figure 7 illustrates the response of the same statistics to changes in the capital income tax. We see that the volatility of output and consumption are practically not affected by changes in this parameter. Indeed, since the demographic composition of the labor force is not affected by changes in the capital income tax rate, the volatility of aggregate hours worked is also not affected. Hence, the volatility of output and consumption stay the same.

The volatility of investment is lower at higher capital income tax rates. This contrasts with the effect described in Galí (1994), where the volatility of investment increases with the tax rate. The effect described in Galí’s happens because higher tax rates on capital income reduce the capital-output ratio. Given that the marginal product of capital is larger when the stock of capital is low, investment responds more to fluctuations in aggregate productivity. The reason why the capital income tax rate has a different impact in our
model is because taxes apply to the net capital income, while Galí (1994) considers taxes on gross capital income. As a result, an increase in the capital income tax has an ambiguous effect on investment volatility. The negative effect on investment volatility dominates for our calibration.

Finally, Figure 8 illustrates that the effects of changes in the consumption tax rate are qualitatively the same as the effects of changes in the labor income tax but that the magnitude of the effect is much smaller. The smaller effect follows from Proposition 2. Given our calibration, Proposition 2 implies that the elasticity of the labor supply to changes in the labor income tax rate is about seven times larger than the elasticity of the labor supply to changes in the consumption tax rate.

5.4.2 How well does the theoretical economy reproduce the data?

We now examine whether the theoretical economy is able to reproduce quantitatively the negative correlation between the size of the government and the volatility of hours, and output and its components. For this purpose, we feed into the model different combinations of fiscal parameters. The fiscal parameters that need to be chosen for each simulated economy are the three tax rates, and the transfer parameter $T$. Each set of tax rates are chosen to mimic the tax rates for a given OECD country, as estimated by Carey and Rabesona (2002). In addition, the size of the transfers in each economy $T$ is chosen so that the steady state government spending to output ratio matches the historical average for the same country as reported in the OECD national accounts.

For each parameter set mimicking an OECD country, we simulate 500 artificial time series and compute the standard deviation of the macroeconomic aggregates of interest.

---

22 Not reported here, we reproduced Figure 7 with taxes on gross capital income. In that exercise, results are in lines with Galí (1994) findings: The tax rate on capital income tends to increase the volatility of investment. See also Section 5.4.4 where we illustrate how the quantitative exercises described in Section 5.4.2 are affected when we consider taxes on gross capital income.

23 The countries considered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Ireland, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. We consider only a subset of the countries included in the empirical evidence documented in Section 2 because some tax rates are not available in Carey and Rabesona (2002). In particular, we do not simulate observations for the fiscal profiles of Germany, Greece, Iceland, Luxembourg, Mexico and Turkey.
Table 6: Fiscal policy parameters

<table>
<thead>
<tr>
<th></th>
<th>$\tau_h$</th>
<th>$\tau_k$</th>
<th>$\tau_c$</th>
<th>$\hat{G}/\hat{Y}$</th>
<th>$R/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.226</td>
<td>0.389</td>
<td>0.095</td>
<td>0.181</td>
<td>0.283</td>
</tr>
<tr>
<td>Austria</td>
<td>0.442</td>
<td>0.259</td>
<td>0.212</td>
<td>0.197</td>
<td>0.489</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.456</td>
<td>0.418</td>
<td>0.169</td>
<td>0.243</td>
<td>0.495</td>
</tr>
<tr>
<td>Canada</td>
<td>0.272</td>
<td>0.450</td>
<td>0.122</td>
<td>0.226</td>
<td>0.343</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.412</td>
<td>0.486</td>
<td>0.333</td>
<td>0.264</td>
<td>0.576</td>
</tr>
<tr>
<td>Finland</td>
<td>0.413</td>
<td>0.378</td>
<td>0.269</td>
<td>0.223</td>
<td>0.523</td>
</tr>
<tr>
<td>France</td>
<td>0.431</td>
<td>0.318</td>
<td>0.200</td>
<td>0.230</td>
<td>0.477</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.252</td>
<td>0.223</td>
<td>0.234</td>
<td>0.223</td>
<td>0.355</td>
</tr>
<tr>
<td>Italy</td>
<td>0.398</td>
<td>0.288</td>
<td>0.135</td>
<td>0.206</td>
<td>0.412</td>
</tr>
<tr>
<td>Japan</td>
<td>0.254</td>
<td>0.384</td>
<td>0.053</td>
<td>0.158</td>
<td>0.275</td>
</tr>
<tr>
<td>Korea</td>
<td>0.063</td>
<td>0.153</td>
<td>0.116</td>
<td>0.169</td>
<td>0.142</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.482</td>
<td>0.311</td>
<td>0.176</td>
<td>0.239</td>
<td>0.497</td>
</tr>
<tr>
<td>Norway</td>
<td>0.381</td>
<td>0.339</td>
<td>0.324</td>
<td>0.199</td>
<td>0.534</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.222</td>
<td>0.113</td>
<td>0.158</td>
<td>0.176</td>
<td>0.277</td>
</tr>
<tr>
<td>Spain</td>
<td>0.334</td>
<td>0.205</td>
<td>0.099</td>
<td>0.159</td>
<td>0.333</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.523</td>
<td>0.527</td>
<td>0.222</td>
<td>0.281</td>
<td>0.595</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.307</td>
<td>0.300</td>
<td>0.079</td>
<td>0.109</td>
<td>0.320</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.258</td>
<td>0.580</td>
<td>0.157</td>
<td>0.247</td>
<td>0.386</td>
</tr>
<tr>
<td>United States</td>
<td>0.256</td>
<td>0.371</td>
<td>0.053</td>
<td>0.191</td>
<td>0.272</td>
</tr>
</tbody>
</table>

Note: Data on tax rates are from Carey and Rabesona (2002). The transfer parameter $T$ is calibrated such that the steady-state share of government spending in GDP (column 5) corresponds to the average ratio observed in the OECD Outlook Database. The last column of the table shows the average ratio of tax revenue to output for 500 simulations.

and the implied size of the government. The implied size of the government, which is an endogenous outcome, is measured by the ratio of the total tax-revenue to output. Using the simulated time-series we reproduce the cross-country regressions which are performed in Fatás and Mihov (2001) by regressing the volatility of each macroeconomic aggregate on the government size. Table 6 shows the fiscal policy parameters used to calibrate each simulated economy and the implied tax to output ratio.

Figure 9 reproduces Figure 1 for the simulated economies. It depicts the cross-country relation between government size and the volatility of output, consumption, investment and hours worked. The line drawn for each of the four graphs in Figure 9 corresponds to

---

24 We calculate the standard deviation of the cyclical component of the logarithm of each aggregate. To extract the cyclical part, we apply the Hodrick-Prescott filter with a smoothing parameter equal to 6.25.

25 The cross-country correlation between the implied tax to output ratio in the simulated economies (shown in column six of Table 6) and the tax to output ratio in the data is 0.98.
the OLS regression of volatility on government size. The slope of this line is negative in all graphs, confirming the stabilizing role of government size in our model.

Crucially, our model also predicts a negative correlation between the private components of aggregate expenditure and the size of the government. The fact that our model generates these negative relations is important given previous findings by Andrés et al. (2008). These authors assess to what extent several families of macroeconomic models can replicate the negative correlation between government size and macroeconomic volatility. Their analysis shows that adding nominal rigidities à la Calvo and costs of capital adjustment to an otherwise standard RBC model can generate a negative correlation between government size and the volatility of output. But they find that the stabilizing effect is only due to a composition effect: the volatility of aggregate output decreases because the share in output of government spending increases. However, their model is unable to explain the negative relation between the private components of aggregate expenditure and the size of the government unless additional frictions are introduced.26

Table 7 assesses to what extend the negative relations depicted in Figure 9 are quantitatively comparable to the data. We compare the estimated coefficients using the true data

---

26In particular, Andrés et al. (2008) suggest the introduction of rule of thumb consumers.
Table 7: Regressions of volatility over government size: model vs. data

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data: median regressions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>0.029</td>
<td>0.034</td>
<td>0.090</td>
<td>0.014</td>
</tr>
<tr>
<td>slope</td>
<td>-0.046</td>
<td>-0.056</td>
<td>-0.131</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data: OLS regressions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>0.026</td>
<td>0.031</td>
<td>0.090</td>
<td>0.016</td>
</tr>
<tr>
<td>slope</td>
<td>-0.035</td>
<td>-0.046</td>
<td>-0.132</td>
<td>-0.013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>0.016</td>
<td>0.013</td>
<td>0.041</td>
<td>0.011</td>
</tr>
<tr>
<td>slope</td>
<td>-0.002</td>
<td>-0.013</td>
<td>-0.008</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratio of slopes: model vs. data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median regressions</td>
<td>4.3%</td>
<td>23.2%</td>
<td>61.1%</td>
<td>55.6%</td>
</tr>
<tr>
<td>OLS regressions</td>
<td>5.7%</td>
<td>28.3%</td>
<td>61.1%</td>
<td>38.5%</td>
</tr>
</tbody>
</table>

Note: This table gives results for series of regressions where the dependent variables respectively are output volatility, consumption volatility, investment volatility and hours worked volatility and the explanatory variable is the tax to output ratio. All variables (but government size) are log deviations from an HP trend with smoothing parameter 6.25. Model’s volatilities are the average over 500 simulations for each country.

to the estimated coefficients using the simulated time series. This allows us to interpret our results from a quantitative perspective: the last two rows in Table 7 give the ratio of the estimated slope in the model to the estimated slope in the data. Notice that, in the case of the true data, we report the results of median regressions. This is because estimates are sensitive to the set of countries considered in the regression: we want to report robust estimates. For comparison, we also report the OLS estimates.

The model replicates well the negative relation between the volatility of hours worked and government size since it reproduces 55.6 percent of the relation between government size and aggregate hours volatility. Previous literature has suggested that cross-country differences in taxes are important to explain differences in hours worked. For instance, Prescott (2004), Rogerson (2006, 2008) and Ohanian et al. (2008) have shown that differences in taxes on labor income over time and across countries in the OECD account for a large share of differences in hours worked. Rogerson and Wallenius (2009) emphasize
the impact taxes have on cross-country variations in hours worked along the life cycle. Our results suggest that cross-country differences in taxes are also important to explain differences in the volatility of hours worked between OECD countries.

However, although the model reproduces the negative correlation between government size and output volatility, the effect is much weaker than the one observed in the data. The slope associated with the regression of output volatility on government size corresponds to 4.3 percent of the empirical counterpart. The quantitative results are also weak for the private components of aggregate expenditure. In the case of investment volatility, the model reproduces 6.1 percent of the empirical relation with government size, while results are better in the case of consumption volatility, where 23.2 percent of the relation is reproduced. Overall, the model accounts for 16 percent of the empirical relation between the private components of aggregate expenditure and the size of the government.

5.4.3 Life cycle implications of government size

Beyond aggregate statistics, it is interesting to analyze the impact of our policy experiment on the volatility of hours worked and consumption over the life cycle. These are displayed in Figures 10 and 11, respectively. Each line corresponds to the evolution of the volatility of hours worked (in Figure 10) and consumption (in Figure 11) over the life cycle for a given set of fiscal parameters. For example, the upper light blue line in Figure 11 gives the evolution of the volatility of consumption over the life cycle in the theoretical economy where the fiscal parameters correspond to the fiscal profile of Korea.

An inspection of Figure 10 reveals that as we change the tax parameters in the theoretical economy to mimic the fiscal policy profile of each OECD country in our sample, the volatility of hours worked by each demographic group does not change much. This implies that the change in the volatility of aggregate hours worked is accounted for by the change in the share of hours worked by the volatile workers and not by changes in the volatility of hours worked by each age group.\textsuperscript{27}

\textsuperscript{27}This property of the model is consistent with the data. We run cross-country regressions where the dependent variable is employment volatility of each age group and the explanatory variable is the tax to GDP ratio. With the only exception of the 24–29 age group, the coefficient associated with government size is very small and not significantly different from zero. However, these findings need to be interpreted
Section 2 already illustrated that in countries with large governments the share of employment of the young workers is lower. Because this composition effect is important to explain the cross-country dispersion in the aggregate volatility of hours worked, it is relevant to ask how this effect in the model compares to the data. Unfortunately, when addressing this question, we are unable to look at cross-country data on hours worked disaggregated by demographic group and have to rely on employment data. Nonetheless, in the spirit of the statistics presented in Table 7, we run cross-country regressions where the dependent variable is the average ratio of the employment of workers aged 15–29 to the employment of workers aged 30–64, and the explanatory variable is the average ratio of tax revenue to output. We do the same exercise for the simulated economies and compare the resulting regression coefficients. The ratio of the two slope parameters is about 80 percent. In the data, the slope of the median regression is equal to −0.79, while it is −0.63 in the case of the hours worked by young in the model. These findings are supportive of results in the literature on the importance of taxes to explain cross-country differences in hours worked over the life cycle, as in Rogerson and Wallenius (2009) for instance.

Figure 11 displays the volatility of consumption over the life cycle for several combi-

Note: All variables are reported in logs as deviations from an HP trend with smoothing parameter 6.25. Reported values correspond to the average of the standard deviation of the filtered series over 500 simulations for each country.

---

28 In the case of an OLS regression, the slope is equal to −0.64.
nations of fiscal parameters. The pattern is different from Figure 10. As one moves from economies with smaller governments to economies with larger governments, the overall volatility of consumption over the life cycle decreases. This is particularly true among the young age categories. For instance, compare a high-tax economy—Sweden—with a low-tax economy—Korea. Consumption volatility among the 15–19 age group is about six times larger in the case of Korea than in the case of Sweden. It is about 3.5 times larger among the 20–24 group and 2.5 times among the 25–29 group. This volatility ratio diminishes over the life-cycle and for the prime-age workers (40–49 age group) it is about 2/3.

The drop in consumption volatility is more pronounced among the young because there is perfect risk sharing within the representative family. Perfect risk sharing implies that the marginal utility of consumption has to be equal across demographic groups, generating substitutability between consumption and leisure within an age group and complementarity across groups. As government size increases, labor supplied by young agents decreases faster than labor supplied by prime aged, which tends to strengthen the complementarity and moderate consumption volatility among the young.

29See Section 5.3.2.
Table 8: Regressions of volatility over government size according to several model specifications: model vs. data (ratio of slopes)

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original calibration</td>
<td>4.3%</td>
<td>23.2%</td>
<td>6.1%</td>
<td>55.6%</td>
</tr>
<tr>
<td>Homogeneous population</td>
<td>−0.1%</td>
<td>19.6%</td>
<td>5.5%</td>
<td>−0.3%</td>
</tr>
<tr>
<td>No transfers</td>
<td>3.9%</td>
<td>6.3%</td>
<td>14.0%</td>
<td>50.0%</td>
</tr>
<tr>
<td>Gross capital income taxation</td>
<td>3.9%</td>
<td>22.4%</td>
<td>−8.4%</td>
<td>48.7%</td>
</tr>
<tr>
<td>Cross-country demographic</td>
<td>4.6%</td>
<td>24.3%</td>
<td>6.5%</td>
<td>55.6%</td>
</tr>
</tbody>
</table>

Note: This table gives results for series of regressions where the dependent variables, respectively, are output volatility, consumption volatility, investment volatility and hours worked volatility and the explanatory variable is the tax to output ratio. All variables (but government size) are log deviations from an HP trend with smoothing parameter 6.25. Model’s volatilities are the average over 500 simulations for each country.

5.4.4 Some additional experiments

In this section we consider several quantitative experiments to understand the robustness of our findings. First we compare our benchmark specification to an alternative model with homogeneous population (where there are no differences across demographic groups). This allows us to identify the role played by labor force heterogeneity. Next, we discuss the role played by two important features of the model, transfer payments and the treatment of capital income taxation. Finally, we also consider the robustness of our results to changes in the calibration of the demographic structure of the population. If there are differences in the age structure of the population across OECD countries, and if these differences are correlated with the differences in the tax to GDP ratio across countries, the model’s correlation between the tax to output ratio and aggregate volatility may be strengthened or weakened.

Table 8 summarizes the quantitative findings for each model specification. Each row shows the ratio between the slope coefficients from the median regressions of the simulated data and the true data. The first row of the table considers the ability of the benchmark model to replicate the data (this information is also available in Table 7). The other rows consider alternative specifications. We first look at the model’s properties when there is no labor force heterogeneity. In this case, the composition of the labor force is irrelevant and the volatility of aggregate hours worked is not affected by changes in the government size. Consequently, the implied slope coefficient in the regression between output volatility and
the tax to output ratio is almost zero. However, the tax to output ratio is still negatively correlated to the consumption volatility. The reason is that in countries where the tax to output ratio is high, the size of transfers is also high. Transfers stabilize consumption because they reduce the volatility of permanent income. The implied slope coefficient for consumption volatility is 19.6 percent of the empirical coefficient (almost the same as in the benchmark model). It follows that the stabilization of consumption is explained almost entirely by the increase in transfers when the tax to output ratio increases.

The third row considers the model without transfers \((T = 0)\). An advantage of representing preferences using the GHH utility function is that the results of the paper about the reaction of the labor supply to changes in tax rates are robust to changes in the assumptions about the government sector. In particular, the elasticity of the labor supply of each age group is invariant to changes in the level of transfer payments of the government. Thus, the aggregate hours worked stabilize by roughly the same amount as in the benchmark model when the tax to output ratio increases. In particular, the model explains 50 percent of the relation between government size and the cyclical volatility of hours. In the model without transfers, consumption volatility is less affected by changes in the tax to output ratio. However, the correlation is still negative. This is because two effects remain. First, as the size of the government increases, the young workers (whose consumption is more volatile) enjoy a smaller share of aggregate consumption. Second, their own consumption also becomes less volatile as the complementarity between consumption and leisure across demographic groups is strengthened and the substitutability within groups is weakened. Therefore, the volatility of consumption is reduced as the government size increases.

The fourth row considers the case of gross capital income taxation. This specification is considered in Galí (1994). In this case, increases in the tax to output ratio destabilize investment. This is because increases in the size of the government lower the capital to output ratio. At lower capital to output ratio levels, the response of investment to technology shocks is stronger. Instead, for the case of net income taxation the deductibility of capital depreciation implies a subsidy to investment that is stabilizing as the size of the government increases.

The final row of Table 8 considers an alternative calibration where the demographic
structure of the population varies across countries. The reason for considering this exercise is that the demographic structure of the population may be correlated with the tax to output ratio. For example, in countries with an aging population it is possible that the tax to GDP ratio is high, and given there demographic structure these countries also have a more stable labor force. Comparing the final row of Table 8 to the original calibration we see that the results are almost unaffected. The only change is that the negative correlations between the tax to GDP ratio and output, and the tax to GDP ratio and consumption are slightly stronger.

6 Conclusion

Two empirical facts serve as the principal motivation for this paper. The first is that there is a strong negative correlation between government size and the volatility of business cycles across OECD countries. This feature of the data is difficult to explain using the standard real business cycle model. The second empirical fact is that there is substantial heterogeneity across demographic groups in terms of the cyclical volatility of employment and hours worked. Taken together, these two empirical facts suggest a mechanism whereby changes in the size of the government are associated with changes in the demographic composition of the labor force that in turn determine the volatility of business cycle fluctuations. When the size of the government is small, the associated low tax rates increase the share of aggregate hours which are supplied by the volatile demographic groups. As a consequence, the volatility of aggregate hours and output increases.

We calibrate the model using several aggregate data and also data about the cross-section of volatility of market hours across demographic groups and we use the theoretical economy to investigate the relation between the size of the government and the volatility over the business cycle of hours worked, output and the several components of aggregate expenditure. We find that the model is able to explain a substantial part of the negative correlation between government size and the business cycle volatility of aggregate hours worked. The model is also able to replicate about one fourth of the relation between the tax to output ratio and the volatility of aggregate consumption. Our results suggest
that modeling labor force heterogeneity and, in particular, differences across demographic groups is important to explain quantitatively some important features of the business cycle.

Despite the relative success to explain the relation between fluctuation in hours and the size of the government, our benchmark model is only able to explain 6 percent of the relation between the tax to output ratio and investment volatility. This implies that only 4 percent of the relation with output volatility is generated by the model. These results suggest that future research proposing alternative economic mechanisms should give importance to the investment channel and how it relates to the government sector.
References


2010

272. Labor force heterogeneity: implications for the relation between aggregate volatility and government size
Alexandre Janiak and Paulo Santos Monteiro

271. Aggregate Implications of Employer Search and Recruiting Selection
Benjamín VillenaRoldán

270. Wage dispersion and Recruiting Selection
Benjamín VillenaRoldán

269. Parental decisions in a choice based school system: Analyzing the transition between primary and secondary school
Mattia Makovec, Alejandra Mizala y Andrés Barrera

Alejandra Mizala, Pilar Romagueray Sebastián Gallegos

267. Costly information acquisition. Better to toss a coin?
Matteo Triossi

266. Firm-Provided Training and Labor Market Institutions
Felipe Balmaceda

2009

265. Soft budgets and Renegotiations in Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

264. Information Asymmetries and an Endogenous Productivity Reversion Mechanism
Nicolás Figueroa y Oksana Leukhina

263. The Effectiveness of Private Voucher Education: Evidence from Structural School Switches
Bernardo Lara, Alejandra Mizala y Andrea Repetto

262. Renegociación de concesiones en Chile
Eduardo Engel, Ronald Fischer, Alexander Galetovic y Manuel Hermosilla
261. Inflation and welfare in long-run equilibrium with firm dynamics
Alexandre Janiak y Paulo Santos Monteiro

260. Conflict Resolution in the Electricity Sector - The Experts Panel of Chile
R. Fischer, R. Palma-Behnke y J. Guevara-Cedeño

259. Economic Performance, creditor protection and labor inflexibility
Felipe Balmaceda y Ronald Fischer

258. Effective Schools for Low Income Children: a Study of Chile’s Sociedad de Instrucción Primaria
Francisco Henríquez, Alejandra Mizala y Andrea Repetto

257. Public-Private Partnerships: when and how
Eduardo Engel, Ronald Fischer y Alexander Galetovic

2008

256. Pricing with markups in industries with increasing marginal costs
José R. Correa, Nicolás Figueroa y Nicolás E. Stier-Moses

255. Implementation with renegotiation when preferences and feasible sets are state dependent
Luis Corchón y Matteo Triossi

254. Evaluación de estrategias de desarrollo para alcanzar los objetivos del Milenio en América Latina. El caso de Chile
Raúl O’Ryan, Carlos J. de Miguel y Camilo Lagos

253. Welfare in models of trade with heterogeneous firms
Alexandre Janiak

252. Firm-Provided Training and Labor Market Policies
Felipe Balmaceda

251. Emerging Markets Variance Shocks: Local or International in Origin?
Viviana Fernández y Brian M. Lucey

250. Economic performance, creditor protection and labor inflexibility
Ronald Fischer

249. Loyalty inducing programs and competition with homogeneous goods
N. Figueroa, R. Fischer y S. Infante
248. Local social capital and geographical mobility. A theory
Quentin David, Alexandre Janiak y Etienne Wasmer

247. On the planner's loss due to lack of information in bayesian mechanism design
José R. Correa y Nicolás Figueroa

246. Política comercial estratégica en el mercado aéreo chileno
Ronald Fischer

245. A large firm model of the labor market with entry, exit and search frictions
Alexandre Janiak

244. Optimal resource extraction contracts under threat of expropriation
Eduardo Engel y Ronald Fischer

2007

243. The behavior of stock returns in the Asia-Pacific mining industry following the Iraq war
Viviana Fernandez

242. Multi-period hedge ratios for a multi-asset portfolio when accounting for returns
comovement
Viviana Fernández

241. Competition with asymmetric switching costs
S. Infante, N. Figueroa y R. Fischer

240. A Note on the Comparative Statics of Optimal Procurement Auctions
Gonzalo Cisternas y Nicolás Figueroa

239. Parental choice and school markets: The impact of information approximating school
effectiveness
Alejandra Mizala y Miguel Urquiola

238. Marginal Cost Pricing in Hydro-Thermal Power Industries: Is a Capacity Charge Always
Needed?
M. Soledad Arellano and Pablo Serra

237. What to put on the table
Nicolas Figueroa y Vasiliki Skreta

236. Estimating Discount Functions with Consumption Choices over the Lifecycle
David Laibson, Andrea Repetto y Jeremy Tobacman
235. La economía política de la reforma educacional en Chile
Alejandra Mizala

234. The Basic Public Finance of Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

233. Sustitución entre Telefonía Fija y Móvil en Chile
M. Soledad Arellano y José Miguel Benavente

232. Note on Optimal Auctions
Nicolás Figueroa y Vasiliki Skreta.

231. The Role of Outside Options in Auction Design
Nicolás Figueroa y Vasiliki Skreta.

230. Sequential Procurement Auctions and Their Effect on Investment Decisions
Gonzalo Cisternas y Nicolás Figueroa

2006

229. Forecasting crude oil and natural gas spot prices by classification methods
Viviana Fernández

228. Copula-based measures of dependence structure in assets returns
Viviana Fernández

227. Un Análisis Econométrico del Consumo Mundial de Celulosa
José Ignacio Sémbler, Patricio Meller y Joaquín Vial

M. Soledad Arellano

225. Socioeconomic status or noise? Tradeoffs in the generation of school quality information (Por aparecer en el Journal of Development Economics).
Alejandra Mizala, Pilar Romaguera y Miguel Urquiola.

224. Mergers and CEO power
Felipe Balmaceda

123. Task-Specific Training and Job Design
Felipe Balmaceda

* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.