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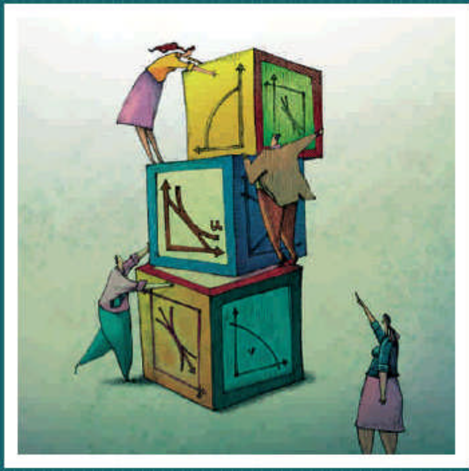

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ASSESSING THE EXTENT OF DEMOCRATIC FAILURES. A 99%-CONDORCET'S JURY THEOREM

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Assessing the extent of democratic failures.

A 99%-Condorcet's Jury Theorem.

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Abstract

This paper determines the probability a large electorate will take take the correct decision under qualified majority rules. The model allows the competence of each elector to vary with the size of the electorate, thus the results represent a connection between "naive" and "strategic" Condorcet's Jury Theorems.

Keywords Condorcet's Jury theorem · probability of success · naive-strategic connection

JEL Classification D72 · H11 · P16

1 Introduction

According to the Condorcet's Jury Theorem (Condorcet 1785), as long as each voter is more likely to be right than wrong, the probability that an election which uses the majority rule will result in the best possible outcome approaches one as the number of voters grows large (see, for instance Grofman et al. (1983) and Miller, 1986 for proofs of the result).[1] The result provides an epistemic justification for a democratic form of government (see Landemore, 2012 for a discussion and detailed references).

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[1]Another version of the theorem claims that a majority of a group is more likely than a single individual to choose the better of two alternatives (see for instance Black, 1958; Ben-Yashar and Paroush, 2000; Berend and Sapir, 2005 and 2007). Here, we deal only with the asymptotical version of the theorem.

Classical Condorcet's Jury theorems also reveal when large elections fail to aggregate information (see, for instance, Berend and Paroush, 1998 and Paroush, 1998). However, they cannot assess the extent of such democratic failures. And while we ought to be concerned if elections yielded the correct decision less than half of the times, we would probably be satisfied with elections yielding the correct decision 99% of the times. This paper tackles this issue and goes beyond classical formulations of the Condorcet's Jury theorem. Our main result, Theorem 1, determines the probability that a large electorate will take the right the decision when a given qualified majority rule is used.[2]

Furthermore, we allow the probability that each electors vote for the correct alternative to vary with size of the electorate.[3] This feature allows the model to account for network effects, externalities and economies of scale. More importantly, our result constitutes a connection between "naive" Condorcet's Jury theorems and "strategic" ones, despite the fact that we assume that electors vote truthfully. When agents act strategically, the marginal utility of a vote depends on the probability each agent is pivotal, thus on the number of agents. In many cases voting according to private information is not be part of equilibrium behavior (see Austen-Smith and Banks, 1996).[4] When this happens, equilibrium behavior involves the use of mixed strategy and, in this case, the probability each agent vote for the correct alternative depends on the number of agents (see Feddersen and Pesendorfer, 1998). Even when sincere voting is optimal, the probability each electors votes for the correct alternative depends on the number of electors when information acquisition is endogenous (see, for instance Martinelli, 2006 and 2007; Oliveros, 2013; Triossi, 2013). Thus, our findings can be used as a tool by scholars interested in strategic voting. In the appendix we present a simple application to strategic voting and prove that the Condorcet's Jury theorem holds even in a strategic setup. The result provides an alternative proof to the findings of Feddersen and Pesendorfer (1998) and of Wit (1998).

Finally, our results can be helpful to understand the effect of the rational ignorance hypothesis (Schumpeter, 1950; Downs, 1957). According to this hypothesis, electors have little information since information acquisition is costly and each elector has little probability of being decisive. This is consistent with empirical evidence (see, for instance, Delli Carpini and Keeter, 1996; Nannestad and Paldam, 2000; Caplan

[2]Other papers dealing with qualified majority rules are Nitzan and Paroush (1984), Ben-Yashar and Nitzan (1997) and Fey (2003).

[3]We also allow for heterogeneity, but maintain the assumption of independent voting. For analysis of the dependent case, see Boland (1989); Ladha (1992; 1993); Berg, (1993); Berend and Sapir (2007); Peleg and Zemir (2012).

[4]However, Ben-Yashar (2006) presents an argument in favor of sincere voting.

2007) and could have important implications for the quality of democratic deliberations. According to this hypothesis, we should expect individual competence to be low and to decrease with the number of voters (which decreases pivotal probabilities). Our main result implies that the Condorcet's Jury Theorem holds in majoritarian elections as long as the average competence of the electorate approaches one half at a rate that is slower than one over the square root of the number of electors. Thus, it implies hat even a rational ignorant electorate could be able to take the correct decision with very high probability (see also Martinelli, 2006 and 2007; Triossi, 2013).[5]

The structure of the paper is the following: the next section introduces the model, Section 3 presents the results and Section 4 concludes. In the Appendix, the interested reader will find an application of our results to a strategic setup.

2 The Model

There is an universe of voters $N = \{1, ..., n\}$ and they have to decide between two alternatives A and B by a qualified majority voting. We assume that A is the correct alternative. Let $\beta \in (0, 1]$ be the fraction of the electorate required to choose an alternative. That is, at least $\lfloor \beta n \rfloor + 1$ electors are required to elect alternative A.[6] If only $\lfloor \beta n \rfloor$ electors vote for A, A is selected with probability $\alpha_n \in [0, 1]$. Such a qualified majority rule is known as a β-rule. The unanimity rule corresponds to $\beta = 1$ and the majority rule corresponds to $\beta = \frac{1}{2}$. Electors vote independently, but the probability each elector votes for an a alternative depends on her identity and on the number of agents. For every $n \geq 1$ and every $i \leq n$, let p_{in}, be the probability elector i votes for alternative A when there are n agents. We will call p_{in}, agent i's *competence*. Let $\bar{p}_n$ be the average of $\{p_{in}\}_{i \leq n}$: $\bar{p}_n = \frac{1}{n} \sum_{i=1}^{n} p_{in}$.

Differently than most literature about the Condorcet's Jury theorem, we do not assume that electors are more likely to be right than wrong, but simply that their competence is uniformly bounded below by a small positive number. Formally, we assume that there exists $\varepsilon \in \left(0, \frac{1}{2}\right)$ and $\bar{m}$ such that $p_{in} \geq \varepsilon$ for all $i \leq n$ and for all $n \geq \bar{m}$.

Let $P_\beta \left(\{p_{in}\}_{i \leq n} \right)$ be the probability that the elections will select the correct alternative when a β-rule is

[5] For a different approach to rational ignorance see Ben-Yashar and Zaavi (2011)

[6] For every $x \in \mathbb{R}$, $\lfloor x \rfloor$ denotes the largest integer lower than x, $\lfloor x \rfloor = \max \{n \in \mathbb{N} \mid n \leq x\}$.

used, when there are n electors and each agent votes for A with probability p_{in}, for all $i \leq n$. We will say that the *Condorcet's Jury Theorem* holds if $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = 1$.

Finally, let Φ be the standard normal distribution: $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{x^2}{2}}dx$.

3 The Results

First, we prove our main result.

Theorem 1 *Let* $t_n = 2\sqrt{n}\left(\bar{p}_n - \beta\right)$*. Assume that* $t = \lim_{n\to\infty} t_n$ *exists. Then* $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = \Phi(t)$*.*

Proof. The proof of the statement is in two steps.

(i) Let $z_n = \frac{\left(\sum_{i=1}^{n} p_{in}\right)-\beta n}{\left[\sum_{i=1}^{n} p_{in}(1-p_{in})\right]^{\frac{1}{2}}}$. Assume that $z = \lim_{n\to\infty} z_n$ exists. First, we prove that $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = \Phi(z)$. For every n, let $\sigma_n^2 = \sum_{i=1}^{2n+1} p_{in}\left(1-p_{in}\right)$.

(a) We first consider the case where $\lim_{n\to\infty}\sigma_n = \infty$. Set

$$X_{in} = \begin{cases} 1 - p_{in} & \text{if voter } i \text{votes for } A \\ -p_{in} & \text{if voter } i \text{votes for } B \end{cases}$$

We have: $E\left[X_{in}\right] = 0$, $E\left[X_{in}^2\right] = p_{in}\left(1-p_{in}\right)$, $\rho_{in} = \left|E\left[X_{in}^3\right]\right| = p_{in}\left(1-p_{in}\right)\left|2p_{in}-1\right|$.

Define

$$S_n = \frac{\sum_{i=1}^{n} X_{in}}{\sigma_n}$$

A wins the elections with certainty if $S_n \geq \frac{\lfloor\beta n\rfloor+1-\sum_{i=1}^{n} p_{in}}{\sigma_n}$ and wins the elections with probability α_n if $S_n = -z_n$.

Thus, the probability that A wins the election is

$$P\left(S_n \geq \frac{\lfloor\beta n\rfloor + 1 - \sum_{i=1}^{n} p_{in}}{\sigma_n}\right) + \alpha_n\left[P\left(S_n \leq \frac{\lfloor\beta n\rfloor + 1 - \sum_{i=1}^{n} p_{in}}{\sigma_n}\right) - P\left(S_n \leq -z_n\right)\right]$$

From the Berry-Esseen theorem (see Chow and Teicher, 1997) it follows that there exists C, such that

$$|P(S_n \leq x) - \Phi(x)| \leq D \frac{\sum_{i=1}^{n} p_{in}(1-p_{in})|2p_{in}-1|}{\sigma_n^3}$$

for every x and for every n. Since $\lim_{n\to\infty} \sigma_n = \infty$ and $\sum_{i=1}^{n} p_{in}(1-p_{in})|2p_{in}-1| < \sigma_n^2$, we have $\lim_{n\to\infty} |P(S_n \leq x) - \Phi(x)| = 0$ for every x.

The claim follows by observing that $\frac{\lfloor \beta n \rfloor + 1 - \sum_{i=1}^{n} p_{in}}{\sigma_n} \approx -z_n$ as $n \to \infty$.

(b) Now assume that $\lim_{n\to\infty} \sum_{i=1}^{n} p_{in}(1-p_{in}) = \sigma^2 < \infty$. Let $r > 0$ such that $\{1-\varepsilon, \beta\} < r < 1$. First, we prove that the number of agents i voting for A with probability lower than s is uniformly bounded. Formally, we prove that there exists $S > 0$ such that $S_n = \sharp\{i \mid p_{in} < r\} \leq S$, for all n. Since $\lim_{n\to\infty} \sum_{i=1}^{n} p_{in}(1-p_{in}) = \sigma^2$, there exists m such that $\sum_{i=1}^{n} p_{in}(1-p_{in}) < \sigma^2 + 1$ for all $n > m$. Then $\sigma^2 + 1 > \sum_{i=1}^{n} p_{in}(1-p_{in}) > S_n r(1-r)$, for all $n > m$. Thus, $S_n \leq \max\left\{\max\{S_k \mid k \leq m\}, \frac{\sigma^2+1}{r(1-r)}\right\} = S$ for all n. It follows that $\lim_{n\to\infty} z_n = \infty$, thus we have to prove that $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = 1$.

Consider the following ancillary model where $\hat{p}_{in} = 1-\varepsilon$ if $p_{in} \geq 1-\varepsilon$ and $\hat{p}_{in} = \varepsilon \leq p_{in}$ otherwise. We have n, $\sum_{i=1}^{n} \hat{p}_{in} \approx n(1-\varepsilon)$ and $\sum_{i=1}^{n} \hat{p}_{in}(1-\hat{p}_{in}) \approx n(1-\varepsilon)$ as $n \to \infty$. Thus, $\lim_{n\to\infty} \frac{\sum_{i=1}^{n} \hat{p}_{in}(1-\hat{p}_{in})|2\hat{p}_{in}-1|}{\left[\sum_{i=1}^{n} \hat{p}_{in}(1-\hat{p}_{in})\right]^{\frac{3}{2}}} = 0$ and $\lim_{n\to\infty} \frac{\left(\sum_{i=1}^{n} \hat{p}_{in}\right) - \beta n - 1}{\left[\sum_{i=1}^{n} \hat{p}_{in}(1-\hat{p}_{in})\right]^{\frac{1}{2}}} = \infty$. The Berry-Esseen implies that the probability that probability that at least $\lfloor \beta n \rfloor + 1$ agents vote for A approaches one in this ancillary model, when $n \to \infty$. Now observe that the probability that at least $\lfloor \beta n \rfloor + 1$ agents vote for A can be written as $p_{in}\left(P_{\lfloor \beta n \rfloor} - P_{\lfloor \beta n \rfloor + 1}\right) + P_{\lfloor \beta n \rfloor + 1}$, where $P_{\lfloor \beta n \rfloor} = \sum_{k \geq \lfloor \beta n \rfloor, \{i_1,\ldots,i_k\} \cup \{i_{k+1},\ldots,i_n\} = N \setminus \{i\}} \prod_{r=1}^{k} p_{i_r} \prod_{r=k+1}^{n} (1-p_{i_s})$ and
$P_{\lfloor \beta n \rfloor + 1} = \sum_{k \geq \lfloor \beta n \rfloor + 1, \{i_1,\ldots,i_k\} \cup \{i_{k+1},\ldots,i_n\} = N \setminus \{i\}} \prod_{r=1}^{k} p_{i_r} \prod_{r=k+1}^{n} (1-p_{i_s})$ do not depend on p_{in} and $P_{\lfloor \beta n \rfloor + 1} \leq P_{\lfloor \beta n \rfloor}$. Since $\hat{p}_{in} \leq p_{in}$ for all $i \leq n$, the probability probability that at least $\lfloor \beta n \rfloor + 1$ agents vote for A approaches one when $n \to \infty$ in the original model as well.

(ii) Finally, we prove that l exists if and only if t exists and that they coincide. We have $z_n = \frac{\sum_{i=1}^{n}(p_{in} - \beta n)}{\sqrt{n}\left[\frac{1}{4} - \frac{1}{n}\sum_{i=1}^{n}\left(p_{in} - \frac{1}{2}\right)^2\right]^{\frac{1}{2}}} = \frac{\sum_{i=1}^{n} x_{in}}{\sqrt{n}\left[\frac{1}{4} - \frac{1}{n}\sum_{i=1}^{n} x_{in}^2\right]^{\frac{1}{2}}}$, where $x_{in} = p_{in} - \beta$, for all $i \leq n$. Let $x_n = \sum_{i=1}^{n} x_{in}$. Assume that $x_n > 0$ for large enough n. We have $z_n \leq t_n$, for large n. Let $\{y_i\}_{i\geq 1}$ be a sequence of real numbers, we have $\left(\sum_{i=1}^{n} y_i\right)^2 = \sum_{i=1}^{n} y_i^2 + \sum_{i,j=1, i\neq j}^{n} y_i y_j \leq \sum_{i=1}^{n} y_i^2 + \frac{1}{2}\sum_{i,j=1,i\neq j}^{n}\left(y_i^2 + y_j^2\right) = n\sum_{i=1}^{n} y_i^2$. It follows that $\frac{t_n}{\left[1 - \frac{4}{n^2} x_n^2\right]^{\frac{1}{2}}} \leq z_n \leq t_n$. First, consider the case where $\lim_{n\to\infty} \frac{x_n}{n^2} = 0$, in this case $\lim_{n\to\infty} \frac{z_n}{t_n} = 1$, which implies the claim. Now, consider the case where $\lim_{k\to\infty} \frac{x_{n_k}^2}{n_k^2} = c$ for some subsequence $\{n_k\}_{k\geq 1}$. It follows that $\lim \lim_{k\to\infty} \frac{x_{n_k}}{\sqrt{n_k}} = \infty$, thus $\lim_{k\to\infty} z_{n_k} = \lim_{k\to\infty} t_{n_k} = \infty$, which implies the claim. If $x_n < 0$ for large enough n, the proof the claim is specular, taking in account that now $t_n \leq z_n \leq \frac{t_n}{\left[1-\frac{4}{n^2}\left(\sum_{i=1}^n x_{in}\right)^2\right]^{\frac{1}{2}}}$. In order to prove the general case, it suffices to apply the previous findings to the subsequences $\{n_{1k}\}_{k\geq 1}$ and $\{n_{2k}\}_{k\geq 1}$, where $x_{n_{1k}} > 0$ and $x_{in_{2k}} < 0$ for all k, respectively. ■

In particular, we have:

Corollary 1 *The Condorcet's Jury Theorem holds if and only if* $\lim_{n\to\infty} \sqrt{n}\,(\bar{p}_n - \beta) = \infty$.

Notice that Corollary 1 coincides with Theorem 1 in Berend and Paroush (1998) when there exists $\{p_i\}_{i\geq 1}$ such that $p_{in} = p_i$ for all $i \leq n$.

Now consider majoritarian elections which is assume $\beta = \frac{1}{2}$. Theorem 1 implies that even a poorly informed electorate can take accurate decision with high probability. For instance, assume that each elector votes for the correct alternative with probability $\frac{1}{2} + \frac{0.65}{\sqrt{n}}$. Even if every elector is almost as equally likely to vote for the right and for the wrong alternative, elections will yield the correct decision over 99% of the times, when there are enough electors.

Theorem 1 cannot be extended to the unanimity rule, which is to the case where $\beta = 1$. Indeed, Part (i), (b) of the proof of Theorem 1 does not work in this case.

Example 1 *(i) Let* $p_{in} = 1 - \frac{1}{n^2}$ *for all* $i \leq n$. *Then* $\sum_{i=1}^n p_{in}\,(1 - p_{in}) = \frac{1}{n}\left(1 - \frac{1}{n^2}\right) \to 0$ *as* $n \to \infty$ *and* $t = \infty$. *Then probability of taking the right decision when there are* n *voters is* $\left(1 - \frac{1}{n^2}\right)^n \to 1$ *as* $n \to \infty$.

(ii) Let $p_{in} = 1 - \frac{1}{n}$ *for all* $i \leq n$. *Then* $\sum_{i=1}^n p_{in}\,(1 - p_{in}) = \left(1 - \frac{1}{n}\right) \to 1$ *as* $n \to \infty$ *and* $t = \infty$. *However, the probability of taking the right decision when there are* n *voters is* $\left(1 - \frac{1}{n}\right)^n \to \frac{1}{e}$ *as* $n \to \infty$.

However, when $\sum_{i=1}^n p_{in}\,(1 - p_{in}) = \infty$, part (i), (a) and part (ii) of the proof of Theorem 1 holds for the unanimity rule as well. Thus, we have:

Proposition 1 *Let* $t_n = 2\sqrt{n}\,(\bar{p}_n - 1)$. *Assume that* $\sum_{i=1}^n p_{in}\,(1 - p_{in}) = \infty$ *and that* $t = \lim_{n\to\infty} t_n$ *exists. Then,* $\lim_{n\to\infty} P_1\left(\{p_{in}\}_{i\leq n}\right) = \Phi(t)$. *Furthermore* $\lim_{n\to\infty} \Phi\left(2\sqrt{n}\,(\bar{p}_n - 1)\right) = \lim_{n\to\infty} \prod_{i=1}^n \log p_{in}$.

For instance, this result apply when the electorate is not too smart not too stupid, which is when there exists $\delta > 0$ and $\bar{n}$ such that, for all $n > \bar{n}$, $\delta \leq p_{in} \leq 1 - \delta$ for all $i \leq n$.

From now on, we exclude the unanimity rule from our analysis, thus we assume that $\beta < 1$. An informal statement of the Condorcet's Jury theorem is that the result holds if the average competence of the electorate is greater than $\frac{1}{2}$. When a β-rule is used, one should expect a similar result if the average competence of the electorate is is greater than β. However, the statement is false (see also Paroush, 1998).

Example 2 *Let $\{x_i\}_{i\geq 1}$ be a sequence of positive numbers such that $\sum_{i=1}^{n} x_i$ converges. Note that $\sum_{i=1}^{n} x_i^2$ converges as well. Set $p_{in} = \beta + x_i + \frac{1}{n} > \frac{1}{2}$ for all $i \leq n$. We have $\sqrt{n}\,(\bar{p}_n - \beta) = \frac{1+\sum_{i=1}^{n} x_i}{\sqrt{n}}$ which converges to zero. In this case, Theorem 1 implies that the the probability of taking the right decision approaches $\frac{1}{2}$.*

Instead, the Condorcet's Jury Theorem holds if the average competence stays *boundedly* above β when a β-majority rule is used.

Corollary 2 *Let Assume that there exists $\delta > 0$, such that $\bar{p}_n > \beta + \delta$ for large enough n, then the Condorcet's Jury theorem holds.*

However, the condition determined in Corollary 2 is not necessary for information aggregation.

Example 3 *Let $p_{in} = \beta + n^{-\frac{1}{4}}$ for all $i \leq n$. Notice that, the hypothesis of Corollary 2 do not hold in this case, but according to Theorem 1 the Condorcet's Jury theorem holds.*

We conclude this section by providing a converse of Theorem 1. We prove that for any precision level p there is an electorate that will take the correct decision with probability arbitrarily close to p.

Proposition 2 *Let $\beta \in (0,1)$, $p \in [0,1]$. Then, there exists $\left\{\{p_{in}\}_{i\leq n}\right\}_{n\geq 1}$, such that $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = p$.*

Proof. Let $p \in (0,1)$. Let $t = \frac{1}{2}\Phi^{-1}(p)$. For every $n \geq 1$, set $p_{in} = \beta + \frac{t}{\sqrt{n}}$ for all $i \leq n$. From Theorem 1, we have $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = \Phi(2t) = p$.
Now, let $p = 0$ and let $\varepsilon \in (0,\beta)$. For every $n \geq 1$, set $p_{in} = \varepsilon$ for all $i \leq n$. From Theorem 1, we have $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = 0$.
Now, let $p = 0$ and let $\rho \in (\beta,1)$. For every $n \geq 1$, set $p_{in} = \rho$ for all $i \leq n$. From Theorem 1, we have $\lim_{n\to\infty} P_\beta\left(\{p_{in}\}_{i\leq n}\right) = 1$. ■

4 Conclusions

In this paper we extend the Condorcet's Jury Theorem by deriving the probability that an electorate using a qualified majority rule will reach the correct decision. Since individual competence varies with the number of electors in our model, the work also connects "naive" Condorcet's Jury Theorems and "strategic" ones.

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Appendix: A simple strategic application

Here, we present a strategic application of Theorem 1. We provide an alternative proof of the results discovered independently by Feddersen and Pesendorfer (1997) and Wit (1998). Assume that there are two alternatives, A and B and two states of the world a and b. Elections are decided by simply majority rule. Without loss of generality we assume that the utility of choosing the correct alternative is one and the utility of choosing the wrong alternative is zero. Formally, $U(A,a) = U(B,b) = 1 = 1 - U(B,a) = 1 - U(A,b)$. Voters act strategically and before voting every agent receives a signal in $\{A, B\}$ about which alternative is the correct one. We have $p(A \mid a) = p \in \left(\frac{1}{2}, 1\right)$, and $p(B \mid b) = r \in \left(\frac{1}{2}, 1\right)$. After receiving the signal, every citizen has to cast his vote for one of the two alternatives. The ex ante probability of each state is $q \in \left[\frac{1}{2}, 1\right)$. We furthermore assume that there is an odd number of electors (the case of an even number of electors is similar). We will verify the Condorcet's Jury theorem in this setup for symmetric Bayesian equilibria. We exclude from our analysis *trivial equilibria* where everybody votes independently on the signal. If τ_n be a

profile of symmetric strategies for the game when there are $2n+1$ voters. Let $P_\omega(\tau_n)$ the probability that the electorate chooses the correct alternative at state $\omega \in \{a,b\}$, if electors vote according to τ_n.
Next we prove that non-trivial symmetric Bayesian equilibria exist when there are many voters and that along any sequence of non-trivial symmetric equilibrium the Condorcet Jury Theorem holds.

Proposition 3 *For every p, r and q and for n large enough there exists a non-trivial symmetric Bayesian equilibrium. Let $\{\sigma_n\}_{n\geq 1}$ a sequence of symmetric Bayesian equilibria such that, σ_n is non-trivial for large enough n. Then* $\lim_{n\to\infty} P_a(\sigma_n) = \lim_{n\to\infty} P_b(\sigma_n) = 1$.

Proof. Let p_ω be the probability of a tie at state $\omega \in \{a,b\}$. Using Bayes' rule we have $p(a \mid A) = \frac{pq}{pq+(1-r)(1-q)}$ and $p(b \mid B) = \frac{r(1-q)}{r(1-q)+(1-p)q}$. The proof of the result is in two steps.
(i) We start by considering pure strategy symmetric equilibria. Notice that voting against both signals is never an equilibrium. Voting according to both signals is an equilibrium if and only if $p(a \mid A)p_a \geq p(b \mid A)p_b$ and $p(b \mid B)p_a \geq p(a \mid B)p_b$. In such a case $p_a = \binom{2n}{n}p^n(1-p)^n$ and $p_b = \binom{2n}{n}r^n(1-r)^n$. Thus, equilibria where agents follow the signal exist if and only if $pqp^n(1-p)^n \geq (1-r)(1-q)r^n(1-r)^n$ and $r(1-q)r^n(1-r)^n \geq (1-r)qp^n(1-p)^n$. Notice that, when $n$is large, such inequalities hold if and only $p=r$ and $p \geq q$.
(ii) Now, we check for mixed strategy symmetric equilibria. There are two classes of such equilibria.
(a) First, we study equilibria where agents mix when they receive signal A. In such equilibria, if any, $p(a \mid A)p_a = p(b \mid A)p_b$. Then $p(b \mid B)p_b = \frac{p(b|B)}{p(b|A)}p(b \mid A)p_b = \frac{p(b|B)}{p(b|A)}p(a \mid A)p_a = \frac{p(b|B)}{p(b|A)}\frac{p(a|A)}{p(a|B)}p(a \mid B)p_a > p(a \mid B)p_a$. Thus agents follow signal B. Let λ_n be the probability an agent vote for A when she receives signal A. We have $p_a = \binom{2n}{n}p^n\lambda_n^n[1-p\lambda_n]^n$ and $p_b = \binom{2n}{n}(1-r)^n\lambda_n^n[1-(1-r)\lambda_n]^n$. Thus, if such an equilibrium exists we have $pqp^n[1-p\lambda_n]^n = (1-r)(1-q)(1-r)^n[1-(1-r)\lambda_n]^n$. Simple algebra yields $\lambda_n = \frac{c_np-(1-r)}{c_np^2-(1-r)^2}$, where $c_n = \left[\frac{pq}{(1-r)(1-q)}\right]^{\frac{1}{n}}$. We have $\lambda_n \geq 0$ for every n since $p > \frac{1}{2}$ and $r > \frac{1}{2}$. We have $\lambda_n \leq 1$ if and only if $c_np-(1-r) \leq c_np^2-(1-r)^2$ which is $c_np(1-p) \leq (1-r)r$. If $p<r$, then, $r(1-r) < p(1-p)$. Since $\lim_{n\to\infty} c_n = 1$, for large n we have $r(1-r) < c_np(1-p)$. Thus, such a strategy does not constitute a symmetric Bayesian equilibrium of the game when n is large. If $r=p$, this is equivalent to $c_n \leq 1$ which is never true since $p > \frac{1}{2}$ and $q \geq \frac{1}{2}$. If $r<p$ then, $p(1-p) < r(1-r)$. Since $\lim_{n\to\infty} c_n = 1$, we have $c_np(1-p) \leq r(1-r)$ for large enough n . Thus, such a strategy constitutes a symmetric Bayesian equilibrium of the game for large n. In such an equilibrium the probability an agent votes for B at state b is $r > \frac{1}{2}$ and the probability an agents votes for A at state a is $p\lambda_n$. Notice that $\lim_{n\to\infty} p\lambda_n = \frac{p}{1+p-r} > \frac{1}{2}$. Thus, Corollary 2 implies that the probability of taking the right decision approaches one when there are many agents along sequences of such mixed strategy equilibria.

(b) Finally, we consider equilibria where agents mix when they receive signal B. In such equilibria, if any $p(a \mid B)p_a = p(b \mid B)p_b$. Then $p(a \mid A)p_a = \frac{p(a|A)}{p(a|B)}p(a \mid B)p_a = \frac{p(a|A)}{p(a|B)}p(b \mid B)p_b = \frac{p(a|A)}{p(a|B)}\frac{p(b|B)}{p(a|B)}p(a \mid B)p_a > p(b \mid A)p_b$. Thus agents follow signal A. Let λ_n be the probability an agent vote for B when she receives signal B. We have $p_a = \binom{2n}{n}(1-p)^n\lambda_n^n[1-(1-p)\lambda_n]^n$ and $p_b = \binom{2n}{n}r^n\lambda_n^n[1-r\lambda_n]^n$. Thus, if such an equilibrium exists we have $(1-p)q(1-p)^n[1-(1-p)\lambda_n]^n = r(1-q)r^n[1-r\lambda_n]^n$. Simple algebra yields $\lambda_n = \frac{r-c_n(1-p)}{r^2-c_n(1-p)^2}$, where $c_n = \left[\frac{(1-p)q}{r(1-q)}\right]^{\frac{1}{n}}$. We have $\lambda_n \geq 0$ for every n since $p > \frac{1}{2}$ and $r > \frac{1}{2}$. We have $\lambda_n \leq 1$ if and only if $r(1-r) \leq c_np(1-p)$. If $r<p$ then $p(1-p) < r(1-r)$. Since $\lim_{n\to\infty} c_n = 1$, for large n we have $c_np(1-p) < r(1-r)$. Thus, such a strategy does not constitute a symmetric Bayesian equilibrium of the game, for large n. If $r=p$, $r(1-r) \leq c_np(1-p)$ is equivalent to $c_n \geq 1$ which is $p \leq q$. Finally, consider the case where $r>p$. Then, $r(1-r) < p(1-p)$. Since $\lim_{n\to\infty} c_n = 1$, we have $r(1-r) \leq c_np(1-p)$ for large n. Thus, such a strategy constitutes a symmetric Bayesian equilibrium of the game for large n. In such an equilibrium the probability an agent votes for B at state b is $r\lambda_n$ and the probability an agents votes for A at state A is p. Notice that $\lim_{n\to\infty} r\lambda_n = \frac{r}{1+r-p} > \frac{1}{2}$. Then, Corollary 2 implies that the probability of taking the right decision approaches one when there are many agents along sequences of such mixed strategy equilibria. This completes the proof of the claim. ■

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