ON THE OPTIMALITY OF ONE-SIZE-FITS-ALL CONTRACTS: THE LIMITED LIABILITY CASE

FELIPE BALMACEDA
On the Optimality of One-size-fits-all Contracts: The Limited Liability Case

Felipe Balmaceda
Centro de Economía Aplicada
Industrial Engineering Department
University of Chile

August 6, 2012

1I would like to thank participants in the regular seminar at the Economics Department of University of Chile, the Latin American Meeting of the Econometric Society in Bs. As-Argentina, the Latin American Theory workshop at IMPA, Rio de Janeiro and the Latin American Econometric Society Theory workshop at PUC, Rio de Janeiro. Financial support was provided by Fondecyt through research grant #1100267. Any error however remains my own responsibility.

2República 701, Santiago, 6521122, Chile fbalmace@dii.uchile.cl, http://www.dii.uchile.cl/economia/profe.htm
Abstract

In this paper I study a multi-task principal agent model with a risk-neutral principal and a risk-neutral agent subject to limited liability in an environment with adverse selection and moral hazard. The main results are as follows: (1) the optimal contracts in each possible case is a bonus-type contract that pays a bonus only when the highest signal is realized; (2) the informational rent as well as the limited liability rent are not independent; and (3) under moral and adverse selection the bonus contract exhibits a one-size-fits-all property; that is, in equilibrium all agents are offered the same contract. Under this contract more talented agents work harder and have a higher expected payoff and are on average more productive than less talented agents. This provides a rationale for the absence of menu of contracts in many different settings such as sales contracts, debt contracts, farming workers and optimal regulation.

Keywords: Moral Hazard, Adverse Selection, Multiple Tasks, Limited Liability.

JEL-Classification: D82, D86, J33.
Consider a retail company facing a pool of applicants in need of a sales representative to sell a complex good that requires multidimensional effort such as effort to learn about the product and a different effort to communicate the features of the good to customers in an attractive way. The intuition tell us that the firm will offer a menu of contracts where some contracts will have a low fixed wage and a large piece-rate or bonus, while others will have a high fixed wage and low piece rate or bonus. The reason is that this will induce each sales representative to put his money where his mouth is; that is, more talented employers will choose contracts with a low fixed payment and a high bonus, while the less talented ones will choose contracts with a higher fixed wage and a lower bonus. However, empirical and casual observation (see, for instance, Baker, Gibbs, and Holmstrom (1994) and Sugato and Lafontaine (1995)) show that this is not usually the case. When hiring workers for a job such as the one already described, most firms will offer the same contract to all workers and in most cases it will be a simple contract such as a liner or bonus-type contract. Casual observation also suggests that moral hazard could be of practical importance. In fact, most sales workers are paid according to a fixed wage and either a bonus paid when a certain sales target is achieved or a commission rate over total sales. Franchisees are also motivated by contracts that entail a fixed payment and an agreement about how to share profits or sales. Additionally, managerial contracts usually consist of a combination of fixed wages and payments that are conditioned on performance. In short, linear and bonus contracts are ubiquitous. In fact, Salanié (2005, p. 474) concludes the following: “The recent literature provides very strong evidence that contractual forms have large effects on behavior. As the notion that "incentive matters" is one of the central tenets of economists of every persuasion, this should be comforting to the community. On the other hand, it raises an old puzzle: if contractual form matters so much, why do we observe such a prevalence of fairly simple contracts?”

The theory of incentives predicts that optimal contracts are highly complex and that their terms should respond to differences in agents’ characteristics. Thus, the observed prevalence of simple contracts remains a challenge to incentive theory when one does not resort to things such as legal restrictions, transaction costs and/or fairness considerations. This paper’s goal is to provide a framework to understand why is it that most firms offer, even for complex jobs, simple contracts in the sense that contracts are not customized to workers’ unknown ability and they are of the bonus type. This paper shows that under moral, adverse selection and limited liability, the second-
best optimal menu of contracts has one contract only and this is a bonus-type contract. In other words, the second-best contracting world exhibits the one-size-fits-all property; that is, all workers are offered the same contract regardless of their type. Despite this simplicity, workers’ equilibrium behavior is such that more talented workers work harder, are more productive and have a higher average compensation than less talented agents. This is consistent with the evidence in, for instance, Brown (1992), Lazear (2000a) and Paarsch and Shearer (1999, 2000).

This paper considers a model where two tasks must be performed, there is an aggregated performance measure that can take more than two possible outcomes and the probability of each outcome occurring depends positively on the agent’s effort in each task and his or her type. In addition, the probability that each outcome occurs is assumed to be (weakly) supermodular in effort and ability and satisfies the monotone likelihood ratio property (MLRP) in each input (i.e., effort and ability). The principal and the agent are risk neutral, the agent is subject to a non-negative limited liability constraint and she learns her type before signing the contract. In addition, the cost of effort function as well as her reservation utility are independent of the agent’s type. In short, this paper studies optimal contracts in a multi-task environment with adverse selection, moral hazard, risk neutrality and limited liability for the case in which there is an aggregated performance measure.

I derive the optimal contract under two different informational frictions: (i) moral hazard with regard to the agent’s effort; and (ii) moral hazard with regard to effort and asymmetric information with respect to the agent’s ability. The results are as follows: (i) regardless of the informational friction considered, the optimal contract is a bonus contract of the pass/fail type that pays a bonus only when the outcome with the highest likelihood ratio is realized; (ii) when there is moral hazard only, the second-best optimal contract is customized to the agent’s ability; and (iii) when there is both moral hazard and asymmetric information, the second-best optimal contract is independent of the agent’s type; that is, it is optimal to use a one-size-fits-all contract.

The intuition is as follows. Suppose for the time being that the principal knows the agent’s type and there is one task only. Then under MLRP, Kim (1997) and Demougin and Fluet (1998) show that the optimal contract entails a bonus only when the outcome is higher than a given threshold and a fixed wage equal to the limited liability for all outcomes lower than the threshold. Thus, the principal leaves a rent to the agent, known as the limited-liability rent. The effort required is downward distorted with respect to the first-best efficient effort in order to reduce the limited-liability rent. The bonus will rise with the agent’s ability level as well as the equilibrium effort when effort and ability are complements. This intuition extends to the case of multiple tasks when
effort in one task does not decrease the marginal productivity of effort in the other tasks (i.e., efforts are complements), and for any given effort profile, the distribution of output satisfies the MLRP with respect to the effort in each task. It is still optimal from the principal’s viewpoint to pay a bonus only when the highest outcome is observed since this is the most likely outcome to come from a distribution in which both efforts are high. Furthermore, MLRP, together with the assumption that the upper cumulative probability distributions are increasing and strictly concave in the effort profile, are sufficient to show that the first-order approach is valid. Thus, under (weak) complementarity and an aggregated performance measure, the one task result can be extended to more than one task.

At first glance the extension to two tasks seems trivial, but it is not. The reason is twofold. First, the assumptions ensuring that the first-order approach is valid in the one task case are not enough to ensure its validity when there is more than one task and agents are compensated according to an aggregated performance measure. Second, there is a multitasking problem which is of a different nature from that in Holmström and Milgrom (1991). Mainly, the multitasking problem here is one of implementation. That is, the set of effort pairs that is implementable is restricted to effort pairs that yield the same marginal utility of effort. The reason for this result is twofold: first, the performance measure is aggregated and confounds the efforts of two non-conflicting tasks; and second, the performance measure and the agents’ costs are such that for any incentive intensity, the agent has no preferences over either task. Because the least costly contract compensate the agent only after the highest outcome is realized and actions are complements, the incentive compatibility constraint for each task implies that if it is optimal for the principal to induce the agent to exert effort in one task, then it is optimal to induce him to exert effort in both tasks.

When the principal does not observe the agent’s ability before contracting, he would like to offer a menu of contracts where contracts with higher bonuses have lower fixed wages in order to keep total compensation as small as possible and to achieve self-selection. Because of MLRP, the principal also wants to assign the highest wage at the upper end of return distribution since this provides the agent with the strongest incentives at the lowest possible cost for the principal. However, the limited liability constraint places a lower bound to the other wages. This makes self-selection through contracts terms impossible. To see this lets suppose that there are two ability types, high and low, and for a given effort profile, the high type is more likely to produce a higher output. Then the menu should have two different contracts one intended for the high type and the other for the low type such that each agent self-select. In order for this to happen the contract for
the highest type must offer a steeper wage profile than that for the lowest type. Because MLRP holds for each dimension and the agent is subject to limited liability, the optimal wage profile for any outcome must set the wage for any outcome equal to the limited liability \( L \geq 0 \), except for the highest outcome, which should be set equal to \( L \) plus a positive bonus. Yet, because agents are risk-neutral and the cost of effort is the same for each ability type, if the highest type chooses this contract, the low type will also choose the same contract and each type will choose the effort profile accordingly. If the principal offers the higher type a contract that pays more than the limited liability for any outcome different from the highest, this contract will also be more attractive for the lower type. Thus the only menu of contracts that induce truth telling is the one in which all contracts are identical and belong to the pass/fail family. Furthermore, because high ability agents are more productive and effort and ability are complements, high ability agents will choose a higher effort profile and will have a higher average productivity.

The provision of incentives in organizations is at the crux of the organizational design problem. A long standing literature studies the question of how optimal incentive contracts look like in different situations.\(^1\) One strand of this literature focuses on noisy performance measures and risk averse workers. The cornerstone of this literature is the existence of a negative trade-off between risk and incentives; that is, the more noisy the performance measure, the less power the incentives. However, as Prendergast (2002a) and Prendergast (2002b) point out, the data do not confirm the existence of such trade-off.

Another strand focuses on the issue of how to reward a given task without harming other tasks and how the number of tasks affects the risk and incentives trade-off within the effort substitution approach. Understanding this dimension of incentive contracting is at the core of the multi-tasking literature as developed by Holmstrom and Milgrom (1987), Holmström and Milgrom (1991), Baker (2002), Itoh (1991, 1992, 1994), Dewatripont, Jewitt, and Tirole (1999) and MacDonald and Marx (2001). The main result of this literature is that firms reduce the power of incentives to avoid rewarding the wrong behavior. The driving force behind this is the existence of noisy or distorted performance measures coupled with effort substitution that result in distorted incentives.

In terms of the type of multitasking problem studied here, this article is more closely related to the contributions of Ratto and Schnedler (2008) and Dewatripont and Tirole (1999). Ratto and Schnedler (2008) study a situation where production requires two non-conflicting tasks, and the manager wants to direct production to achieve a preferred allocation of effort across tasks.

\(^1\)See Gibbons (1998) for an extensive review of the incentive contracting literature.
However, aggregated production is the only indicator of agent activity. The main result is that the principal cannot implement the preferred allocation with a single agent, yet he is able to do so by inducing a game among two agents. These results complement those in Dewatripont and Tirole (1999), who also found that under multitasking there are implementation problems, but based on direct conflicts between tasks. They show that it always better to split the task of finding evidence in favor and against a decision between two agents. The reason stands for the fact that the optimal compensation can be based only on an aggregated measure of the task and this is increasing in the outcome of one task and decreasing in the outcome of the other task. This implies that it is impossible to induce one agent to exert more effort in both tasks and thus it is optimal to split the task between two different agents to avoid conflict of interest in job design.²

This is also related to the strand that studies incentive contracting under adverse selection. Lazear (1986) and Matutes and Rockett (1994) show that the least productive workers self-select into firms offering straight salaries and the most productive choose firms offering piece rates. Balmaceda (2009b) provides a rationale for the emergence of pay-for-performance contracts and self-selection in a competitive labor market setting, where workers are risk averse, firms are risk neutral and unaware of workers’ abilities. He shows that under certain parameterization the second-best menu has more than one contract, and under others the menu contains one contract only. He shows that the prevalence of pay for performance rises and the pay-for-performance sensitivity falls as environmental uncertainty increases. This empirical regularity is unaccounted for alternative models such as the standard agency model. Lazear (2005) focuses on pay for performance as a way to extract worker’s private information with regard to firm’s prospects and retaining them. Arya and (2005) show that stock options are granted to distinguish workers. Together, the two papers show that a manager confident of his contribution to a firm’s value is willing to accept a contract loaded with stock options. Cadenillas and Zapatero (2005) consider the problem of a risk-neutral firm that tries to hire a risk-averse executive of unknown ability. Executives affect stock price dynamics through the choice of volatility, and by applying costly effort. In this setting, they show that the use of options discourage low-ability executives from applying to the firm since the implicit risk of an option will make its value lower than low-ability executive’s reservation wage. Moen and Rosén (2005) analyze a multi-tasking model with risk neutral agents, a distorted performance measure, and private information about the agent’s cost of effort. They show that in equilibrium

²Conflicting tasks also provides incentives for sabotage and collusion is unavoidable, which is not the case with non-conflicting tasks.
pay-for-performance contracts are too high powered as they can result in a misallocation of effort across tasks. They also show that pay for performance induces too much effort while it imposes compensation risk on risk averse workers.

This paper adds to the incentive contracting literature by focusing on an issue that has been somehow overlooked in the literature, which is the simplicity of observed incentives contracts and the lack of customization of them to agents. McAfee and McMillan (1986), Sugato and Lafontaine (1995) and Holmstrom and Milgrom (1987) among others have notice the prevalence of simple linear incentive contracts and have rationalized them in frameworks considering moral hazard only. However, I am aware of only one paper that deals with the commonly observe one-size-fits-all property of optimal contracts, which is Koch and Peyrache (2008). They show in a model with moral hazard and career concerns in which workers frequently change employers and complete (long-term) contracts are not feasible, that one-size-fits-all contracts could be optimal under certain parametrization. Mainly, when the difference in skills between the two types considered by them is sufficiently large, one-size-fits-all is optimal. On the one hand, lack of customization lowers total incentives, on the other hand, it reduces information transmission to the market and increases the probability of retention. The retention effect dominates the incentive effect when difference in skills between the two types is sufficiently large.

The rest of the paper is organized as follows. In Section 2, I introduce the model and the main assumptions. Section 3 presents the two benchmarks. The full information case and the pure moral hazard case. Conditions for the validity of the first-order approach are stated in this section. In Section 4, the optimal contract with adverse selection and moral hazard is derived. Section 5 concludes with some remarks. The proofs that are not central to the discussion can be found in the appendix.

2 The Model

Consider a relationship between a principal (P) and an agent (A). Both parties are risk neutral and the agent faces a limited liability constraint. The agent is characterized by his type (e.g., ability level) indexed by $\theta$ that is privately learnt by the agent before signing a contract. There is a continuum of types; namely, $\theta \in \Theta \equiv [\underline{\theta}, \bar{\theta}]$, and the principal knows that types are distributed according to the twice continuously differentiable distribution function $F(\theta)$ with associated density function $f(\theta)$. As is standard in the literature, the inverse of the hazard function $H(\theta) \equiv \frac{1-F(\theta)}{f(\theta)}$
is assumed to be bounded and monotonic. The principal hires the agent to work in a project that requires two actions: a non-observable strategy, denoted by \( s \in S \equiv [0, \bar{s}] \), and non-observable effort, denoted by \( e \in E \equiv [0, \bar{e}] \).

The project’s return is denoted by \( y_h \) for \( h = 1, \ldots, n \) with \( y_{h+1} > y_h \) for all \( h \). The probability that outcome \( y_h \) occurs depends on the agent’s action profile \( a = (e, s) \in A \equiv E \times S \) and the agent’s ability \( \theta \) in the following way: \( \Pr(R = y_h | \theta, a) \equiv p_h(\theta, a) \) for \( h = 1, \ldots, n \), with \( \sum_{h=1}^{n} p_h(\theta, a) = 1 \) for all \( (a, \theta) \in A \times \Theta \).

The agent’s private cost of action profile \( a \in A \) is \( c(a) = c \cdot (e + s) \). Thus, the agent’s private cost of actions is the same regardless of the agent’s type and actions are neither substitutes nor complements in the cost function.\(^3\)

I shall define the following likelihood ratios: (i) \( \ell_{hs}(\theta) \equiv \frac{p_{hs}(\theta, a)}{p_h(\theta, a)} \); (ii) \( \ell_{he}(\theta) \equiv \frac{p_{he}(\theta, a)}{p_h(\theta, a)} \); and (iii) \( \ell_{h\theta}(a) \equiv \frac{p_{h\theta}(\theta, a)}{p_h(\theta, a)} \).

The first two likelihood ratios are those with respect to the agent’s actions and the third is the likelihood ratio with respect to the agent’s type and it is the equivalent to the likelihood ratio for a one dimension moral hazard problem, but for the ability parameter \( \theta \).

Hereinafter I assume the following:

**[MLRP]** (i) \( \ell_{hs}(\theta, a) \) is increasing in \( h \) for all \( (a, \theta) \in A \times \Theta \) and bounded above; (ii) \( \ell_{he}(\theta, a) \) is increasing in \( h \) for all \( (a, \theta) \in A \times \Theta \) and bounded above; and (iii) \( \ell_{h\theta}(\theta, a) \) is increasing in \( h \) for all \( (a, \theta) \in A \times \Theta \) and bounded above.

Parts (i) and (ii) are the standard MLRP for moral hazard problems. Part (iii) is the MLRP with respect to the agent’s type guaranteeing that higher outcomes are more likely to come from distributions parameterized by a higher ability.\(^4\) Assuming MLRP with respect to the agent’s type implies that for any given effort level, the distribution of output for a more talented agent first-order stochastic dominates (FSD) that for a less talented agent.\(^5\) Thus, MLRP ensures that any

---

\(^3\)The results holds if: (i) \( c(\cdot) \) is a twice continuously differentiable and strictly convex function; and (ii) \( c_e(\theta) = 0 \) and \( c_s(a) \leq 0 \) for any \( a \in A \). Yet, the expression for the optimal contract becomes more cumbersome and the algebra messier without further gain in intuition.

\(^4\)See, Balmaceda (2009a) for a use of this MLRP used to derive the optimal contract under pure adverse selection, risk aversion and labor market competition.

\(^5\)The expectation of \( \ell_{h\theta}(\theta, a) \) is zero, and because \( \ell_{h\theta}(\theta, a) \) is increasing in \( h \), the covariance between \( \ell_{h\theta}(\theta, a) \) and \( y_h \) is positive; hence

\[
\sum_h p_h(\theta, a) \ell_{h\theta} y_h > 0 \text{ for all } e \in E.
\]
principal who has a payoff function that is increasing in the project’s outcome prefers the stochastic distribution of returns induced by higher effort, higher ability and a better strategy. MLRP with regard to the agent’s ability together with a non-decreasing wage contract ensure that a higher-ability agent’s expected utility single-crosses a lower ability agent’s expected utility in the space \((w_{h+1}, w_h)\) for all \(h = 1, \ldots, n - 1\).

The next assumption, together with MLRP validates the use of the first-order approach.

[Cp] (i) The upper cumulative probability distributions \(\sum_{h'>h} p_h(\theta, a)\) are increasing and strictly concave in \((a, \theta) \in A \times \Theta\) for all \(h'\); (ii) \(p_h(\theta, a)\) is thrice continuously differentiable for all \((a, \theta) \in A \times \Theta\), and \(p_h(\theta, a) > 0\) for all \((a, \theta) \in A \times \Theta\); and (iii) \(\lim_{s \to 0} \sum_{h'>h} p_{hs}(\theta, a)y_h \to \infty\)
and \(\lim_{e \to 0} \sum_{h'>h} p_{he}(\theta, a)y_h \to \infty\).

The full support assumption ensures that all outcomes can be realized no matter what the agent’s type is. This is meant to avoid the inference that for some outputs, the principal will be able to rule out some types after observing a given outcome. Part (iii) ensures the first-best efficient effort belongs to the interior of \(A\).

Finally, I will assume the following single-crossing property of the cumulative distribution functions in \(A \times \Theta\).

[Sc] (i) \(\sum_{h'>h} p_{h\theta a}(\theta, a) \geq 0\) for all \(h'\); and (ii) \(\sum_{h'>h} p_{hes}(\theta, a) \geq 0\) for all \((a, \theta) \in A \times \Theta\) and for all \(h'\).

Part (i) says that the distribution of marginal return to actions rise in the sense of FOSD with the agent’s ability. In short, ability and actions are complements. Part (ii) says that the distribution of marginal return to \(e\) rises in the sense of FOSD with \(s\). In short, effort and strategy are complements.

Since there are \(n\) possible contractible outcomes to the principal, a contract is given by a wage schedule: \(W = \{w_h\}_{h=1}^n\), where \(w_h\) is the wage paid when outcome \(h\) is realized. A direct consequence of the SC property is that for any non-negative wage schedule, greater actions lead to a higher expected wage the greater the agent’s ability.

Contracts are restricted by a limited liability constraint that prevents the principal from paying the agent a wage in any state lower than \(L \geq 0\):\(^6\)

\[ [Ll] \ w_h \geq L, \ \forall h. \quad (1) \]

\(^6\)When \(L = 0\), this constraint can be thought of as the case in which the agent owns no assets at the time of contracting.
Finally, each ability type has an outside option given by $U_\theta$, with $U_\theta = U \leq L$ for all $\theta \in \Theta$. That is, the outside option is type independent. Furthermore, $U \leq L$ implies that I am focusing on the case in which the first-best cannot be implemented at no extra cost.

3 Benchmarks

3.1 Complete Information Case

In this section I consider the case in which actions are contractible and the agent’s type is known to both the agent and the principal before signing the contract.

The action profile $a$ that maximizes joint welfare is given as the solution to the following problem

$$\max_{a \in A} S(\theta, a)$$

where

$$S(\theta, a) \equiv \sum_h p_h(\theta, a) y_h - c(a) - U.$$ 

The first-order conditions that determine the efficient action profile are:

$$e : e^*(\theta) \left( \sum_h p_h(e(\theta, a^*(\theta)) y_h - c(a) \right) = 0, \tag{2}$$

$$s : s^*(\theta) \left( \sum_h p_h(s(\theta, a^*(\theta)) y_h - c(a) \right) = 0. \tag{3}$$

Because of assumption $CP$, the first-order conditions are necessary and sufficient. Thus, the first-best surplus is given by $S(\theta, a^*(\theta))$, which is assumed to be positive.

Assumption $SC$ implies that for $\theta' > \theta$, $a^*(\theta') \geq a^*(\theta)$ and $S(\theta', a^*(\theta')) \geq S(\theta, a^*(\theta))$. Thus, the first-best efficient action profile is non-decreasing with the agent’s type and total welfare rises with the agent’s type.

3.2 The Pure Moral Hazard Case.

In this section, I analyze the case in which the agent’s ability is known to the principal and the agent before signing the contract, but the action profile is unobserved by the principal and thus non-contractible.

Lets suppose that an agent of ability $\theta \in \Theta$ is faced with contract $\{w_h\}_{h=1}^n$ and chooses the action profile $a \in A$, then his expected utility is given by:

$$U(W, a, \theta) \equiv \sum_h p_h(\theta, a) w_h - c(a). \tag{4}$$
Observe that the linearity of the agent’s payoff function with respect to wages implies that if the principal could offer a contract in which payments are unbounded, the first-best could be achieved by selling the company to the agent. This however is prevented here by the limited-liability constraint.

It readily follows from the agent’s payoff that a contract $W$ satisfying the limited liability constraint $\text{LL}$ induces an agent with ability $\theta$ to choose the action profile $a \in A$ if and only if the following incentive-compatibility constraint holds

$$U(W,a,\theta) \geq U(W,a',\theta), \quad \forall a' \in A$$  \hspace{1cm} (5)

and it induces the agent to participate if the following individually rationality constraint holds,

$$U(W,a,\theta) \geq U.$$  \hspace{1cm} (6)

Thus the principal’s problem consists on choosing a contract $(W,a)$ that maximizes his expected payoff subject to these constraints and the $\text{LL}$ constraint. Thus, the principal must solve the following unrelaxed program

$$\max_{(W,a) \in \mathbb{R}^n \times A} \sum_h p_h(\theta,a)(y_h - w_h)$$  \hspace{1cm} \text{OP}

subject to

$$\text{(1), (5) and (6)}.$$

Let the solution to this program be $(W^m, a^m)$.

**Assumption 1**

i) A solution to the unrelaxed program exists with $e \in (0,\bar{e})$ and $s \in (0,\bar{s})$.

ii) $\sum_h p_h(\theta,a^m)(y_h - w^m_h) > 0$.

Making use of the first-order approach, which entails to enlarging the constraint set, the incentive compatibility constraints in equation (5) can be replaced by the first- and second-order conditions for the agent’s problem, which are:

$$\sum_h p_{he}(\theta,a)w_h - e \begin{cases} 
\leq 0 & \text{if } e = 0 \\
= 0 & \text{if } e \in (0,\bar{e}) \\
\geq 0 & \text{if } e = \bar{e}
\end{cases}$$  \hspace{1cm} (7)
\[
\sum_h p_{hs}(\theta, a) w_h - c \begin{cases} 
\leq 0 & \text{if } s = 0 \\
= 0 & \text{if } s \in (0, \bar{s}) \\
\geq 0 & \text{if } s = \bar{s}
\end{cases}
\] (8)

and

\[
\sum_h p_{hes}(\theta, a) w_h \sum_h p_{he}(\theta, a) w_h - \left( \sum_h p_{hes}(\theta, a) w_h \right)^2 \geq 0 \text{ for any } a \in A.
\] (9)

Then the principal’s relaxed problem becomes

\[
\max_{(W, a) \in \mathbb{R}^n \times A} \sum_h p_h(\theta, a) (y_h - w_h) 
\] (RP)

subject to

(1), (6), (7), (8) and (9).

The Lagrangean for this problem is as follows

\[
\mathcal{L}(W, \mu_e, \mu_s, \gamma, \lambda) = \sum_h p_h(\theta, a) (y_h - w_h) + \mu_e \left( \sum_h p_{he}(\theta, a) w_h - c \right) + \\
\mu_s \left( \sum_h p_{hs}(\theta, a) w_h - c \right) + \gamma \left( \sum_h p_h(\theta, a) w_h - c(a) - U \right) + \sum_h \lambda_h (w_h - L),
\] (10)

where \( \mu_a \) is the Lagrange multiplier for the first-order condition with regard to action \( a \in \{e, s\} \), \( \gamma \) is the multiplier for the agent’s participation constraint and \( \lambda_h \) is the multiplier for the limited liability constraint with regard to the compensation when state \( h \) is realized.

The following is proven in the appendix.

**Proposition 1** Suppose that MLRP, CP and SC hold and \((W, a)\) is a solution to the relaxed program. Then, \((W, a)\) is a solution to the unrelaxed program.

The next result characterizes the optimal contract and the optimal effort.

**Proposition 2 (Pure Moral Hazard)**

i) The optimal contract has the following shape: for all \( h < n \),

\[
w^m_h(\theta) = L
\] (11)

and

\[
w^m_n(\theta) = \frac{c}{p_{ne}(\theta, a^m(\theta))} + L = \frac{c}{p_{ns}(\theta, a^m(\theta))} + L
\]
ii) The optimal action profile for a type $\theta$ agent $a^m(\theta)$ is determined by the unique solution to the following equations

$$
\sum_h p_{he}(\theta, a(\theta)) y_h - c = \frac{p_n(\theta, a(\theta))}{p_{ne}(\theta, a(\theta))} \left( \frac{p_{nee}(\theta, a(\theta)) p_{nss}(\theta, a) - p_{nse}(\theta, a(\theta))}{2 p_{ne}(\theta, a) - p_{nee}(\theta, a) - p_{nss}(\theta, a)} \right) c
$$

(12)

and

$$
\sum_h p_{hs}(\theta, a(\theta)) y_h - c = \frac{p_n(\theta, a(\theta))}{p_{ns}(\theta, a(\theta))} \left( \frac{p_{nee}(\theta, a(\theta)) p_{nss}(\theta, a(\theta)) - p_{nse}(\theta, a(\theta))}{2 p_{ne}(\theta, a) - p_{nee}(\theta, a) - p_{nss}(\theta, a)} \right) c
$$

(13)

The optimal contract is a bonus contract of the pass/fail type. It can be described by a non-contingent transfer $w^m(\theta)$ and a bonus equal to $w^m(\theta) - w^m(\theta)$ paid to the agent when $y_h = y_n$ is observed. The value of the non-contingent transfer $w^m(\theta)$ is fully determined by the limited liability constraint and therefore $w^m(\theta) = L$.\footnote{If outside option were to positive and large, the non-contingent transfer $w^m(\theta)$ would be determined by the outside option.}

The cost to the principal is that the agent earns a limited liability rent.\footnote{If $U$ were such that the limited liability does not binds, then $w^m(\theta)$ would be set to satisfy the participation constraint. In this case the contract leaves no rent to the agent.}

MLRP implies that the partition of the set of signals on which payments are conditioned does not depend on the action implemented (nor does it depend on the agent’s liability limit or on his cost function). This says that from the principal’s viewpoint most of the relevant information provided by the performance measure is irrelevant in the sense that the optimal contract would be feasible even if the principal were able to observe only a binary performance measure.\footnote{Demougin and Fluet (1998) shows that having multiple signals has no added value in the single-task case. However, given the results here and those in Holmström and Milgrom (1991), I conjecture that result does not hold for the multi-task case since the principal could use one signal to provide more incentives in one task and the other to do the same in the other task. Thus, having more signals would in general increase the set of implementable action profiles.}

This result is similar to the one derived by Demougin and Fluet (1998) in the one action moral hazard model with a risk-neutral principal and agent and limited liability and to the one obtained by Laux (2001) in the multiple action moral hazard model with limited liability.\footnote{Laux (2001) assumes that the agent’s reservation utility is non-binding, there is an observable outcome for each possible task and the effort level in each task is binary.}

Observe that the action profile $a^m(\theta)$ satisfies the following

$$
\frac{p_{ne}(\theta, a^m(\theta))}{p_{ns}(\theta, a^m(\theta))} = 1.
$$
The reason for this result is twofold: first, the performance measure is aggregated and confounds the efforts of two non-conflicting tasks; and second, the performance measure and the agents’ costs are such that for any incentive intensity, the agent has no preferences over either task. Because the least costly contract compensate the agent only after the highest outcome is realized and actions are complements, the incentive compatibility constraint for each task implies that if it is optimal for the principal to induce the agent to exert higher effort in one task, then it is optimal to induce him to exert higher effort in both tasks. This leads to the following result.

**Corollary 3** Suppose the agent’s type is common knowledge. Then, \( a^m(\theta) \leq a^*(\theta), \forall \theta \in \Theta. \)

It is interesting to note that the distortion of the action profile with respect to the first-best is not due to incongruence of the information system with the contribution of the agent’s action to the expected firm value \( \sum_h p_h(\theta, a(\theta)) y_h. \) It is the result of the following things: (i) the use of an aggregated performance measure (i.e., the one-signal-per-task assumption made by Holmstrom and Milgrom (1987) is not satisfied); (ii) the principal’s incentive to reduce the agent’s limited liability rent by mean of making an efficient use of the information system; (iii) the effort complementarity across tasks; and (iv) MLRP holds for both actions and therefore the wage profile implementing any positive action profile must be increasing.

### 4 Moral Hazard and Adverse Selection

In this section, I consider the case in which at the time of contracting the principal does not know the agent’s type and does not observe the actions undertaken by the agent, but the agent knows his own type. The principal’s goal then is two-fold: on the one hand, she has to provide the agent with incentives to choose the principal’s preferred action profile, and on the other hand, she has to induce the agent to truthfully reveal his or her type. The problem faced by the principal here is complicated by the fact that it is not enough to request an effort level, but also it must be optimal for each type to choose the actions requested by the firm. In short any offer \((W(\theta), a(\theta))\) must be incentive compatible if it induces truthful revelation of types (i.e., a \(\theta\)-type agent prefers contract \(W(\theta)\) and obedience (i.e., an agent with type \(\theta\) chooses the action profile prescribed by the principal for an agent of type \(\theta\)).

In light of the extended revelation principle,\(^{11}\) truthful revelation of information and obedience

Proposition 4  Suppose the first-order approach is valid. Then, the following conditions are sufficient for a differentiable contract \((W(\theta), a(\theta))\) to be implementable:

\[
\sum_h p_h(\theta, a(\theta)) \frac{\partial w_h(\theta)}{\partial \theta} = 0 \text{ for all } \theta \in \Theta. 
\]

Equation (15) and (16) holding for \(\theta' = \theta\) constitute necessary conditions for implementability.

Let's denote \(U(W(\theta), a(\theta), \theta)\) by \(U(\theta)\). Then note that

\[
w_n(\theta) = \frac{U(\theta) + c(a(\theta))}{p_n(\theta, a(\theta))} - \sum_{h \neq n} \frac{p_h(\theta, a(\theta))}{p_n(\theta, a(\theta))} w_h(\theta). 
\]

Then solving for the wage when outcome \(y_n\) is realized as functions of all other wages and the agent’s rent, and substituting the payment \(w_n(\theta)\) away in the principal’s objective and the corresponding constraints, the principal’s optimization program can be written as

\[
\max_{W(\theta) \in \mathbb{R}^{n-1}, U(\theta)} \int (S(\theta, a(\theta)) - U(\theta)) f(\theta) d\theta 
\]

subject to

\[
U'(\theta) = \sum_{h \neq n} [\ell_{h\theta}(a(\theta)) - \ell_{n\theta}(a(\theta))] p_h(\theta, a(\theta)) w_h(\theta) + \ell_{n\theta}(a(\theta))(U(\theta) + c(a(\theta)));
\]

\[
U(\theta) \geq \max\{U, L\} \text{ for all } \theta \in \Theta
\]

\[
w_h(\theta) \geq L \text{ for all } h \text{ and } \theta \in \Theta.
\]

In this optimization problem the control variables are the payments, the state variable is \(U\), the co-state variable is \(\mu\), the Hamiltonian is

\[
H(U, \mu, a, W) = [S(\theta, a(\theta)) - U(\theta)] f(\theta) + \mu(\theta) \left( \sum_{h \neq n} [\ell_{h\theta}(a(\theta)) - \ell_{n\theta}(a(\theta))] p_h(\theta, a(\theta)) w_h(\theta) + \ell_{n\theta}(a(\theta))(U(\theta) + c(a(\theta))) \right)
\]
and the Lagrangean is
\[
\mathcal{L} = H(U, \mu, a, W) + \gamma(\theta)(U(\theta) - U) + \sum_h \lambda_h(\theta)(w_h(\theta) - L),
\]
(24)

where \(\gamma(\theta)\) is the multiplier for the participation constraint.

The following is proven in the appendix.

**Proposition 5**

i) The principal’s relaxed problem ASMHPR has a unique solution given by: \(w_h(\theta) = L\) for all \(h < n\) and \(\theta \in \Theta\) and
\[
w_{am}^n(\theta) = \frac{c(a(\theta))}{p_n(\theta, a(\theta))} + L, \ \forall \theta \in \Theta,
\]
(25)

ii) The optimal efforts satisfy the following:
\[
p_{ne}(\theta, a(\theta)) \frac{c(a(\theta))}{p_n(\theta, a(\theta))} - c = 0,
\]
and
\[
p_{ns}(\theta, a(\theta)) \frac{c(a(\theta))}{p_n(\theta, a(\theta))} - c = 0.
\]

iii) \(a(\theta)\) increases with \(\theta\).

As with the pure moral hazard case, the optimal contract is a bonus contract of the pass/fail type. It can be described by a non-contingent transfer \(w_{am}(\theta)\) and a bonus equal to \(w_{am}^n(\theta) - w_{am}^h(\theta)\) paid to the agent when \(y_h = y_n\) is observed. The value of the non-contingent transfer \(w_{am}^h(\theta)\) for all \(h < n\) is fully determined by the limited liability constraint. Thus, the bonus is independent of the agent’s type and therefore, the optimal contract exhibits the one-size-fits-all property. However, this does not imply that the expected output is the same for each type. The second-best optimal action profile rises with the agent’s type \(\theta\) and therefore the greater the type, the greater the expected output and the agent’s expected compensation.

The two benchmarks studied before (the first-best and pure moral hazard case) suggest that the optimal wage schedule should be customized to the agent’s characteristics. In particular, the greater the agent’s type, the higher the effort requested and the higher the bonus. This suggests that the optimal contract should exhibit the same feature here, yet it does not. The intuition is as follows. If effort were contractible, we know from a pure-adverse selection case that the principal will distort
the action profile away from the first-best level in order to minimize the informational rents since the "punishment" are bounded below by the limited-liability constraint. When the principal cannot observe the action profile, this cannot be done since all agents will have an incentive to claim to be the highest type since this strategy will provide them with the highest reward in the case that the highest outcome is observed and the same punishment when any other outcome is observed. It turns out that this strategy yields a higher expected payoff to all types. The reason is twofold: first, each type can adjust his effort according to the optimal contract for the highest type; and second, an agent that chooses the action profile that maximizes his utility when he truthfully reveals his type, but claims to be the highest type possible obtain a higher expected payoff since the bonus when the highest outcome is realized is higher when he lies than when he is honest. The principal aware of this and restricted in the use of penalties for low outputs by the limited liability constraint, is forced to offer the same contract to each type so that it can induce the agent to truthfully reveal his type and ask for different effort to different types. Namely, the second-best optimal effort rises with the agent’s type.

5 Conclusions

This paper shows that optimal contracts in the presence of limited liability, risk neutrality, moral hazard and adverse selection exhibit three highly empirically observed properties: (i) are bonus type; (ii) satisfy the one-size-fits-all property; that is, contracts are not customized to the agent’s ability or type; and (iii) better workers work harder and have a higher productivity (see, for instance, Lazear (2000a), Paarsch and Shearer (2000) and Seiler (1984)). Thus, the theoretical predictions of the model here are consistent with casual observation about the shape of real-life contracts and the fact that contract menus are uncommon and they are also consistent with formal empirical evidence.

The result in this paper may be used to understand why many financial institutions offer one-size-fits-all contracts. The economic intuition suggests that a bank’s optimal strategy should be to offer a menu of contracts where each borrower choose the one that is customized for him. However, here I show that this intuition is wrong since doing so entails a greater costs in terms of the limited liability rent needed to induce higher types to choose the higher efforts and it would achieve truth-telling.

The results of the model also suggest a reason why one does not usually observe regulation contracts of the form predicted by the optimal regulation theory. Mainly, that regulatory agencies
should offer regulated firms a menu of contracts that induce them to self-select. While the model does not capture the subtleties of regulation, the intuition here can be extended to study optimal regulation contracts when firms have limited liability and their revenues are stochastic. In fact, Gary-Bobo and Spiegel (2006) consider an optimal regulation model in which the regulated firm’s production cost is subject to random and publicly observable shocks. The distribution of these shocks is correlated with the firm’s cost type, which is private information. The regulator designs an incentive-compatible regulatory scheme, which adjusts itself automatically ex post given the realization of the cost shock. They derive the optimal scheme, assuming that there is an upper bound on the financial losses that the firm can sustain in any given state. They show that it is optimal to offer a menu of contracts and that the first-best effort is implementable under certain parameterizations. The reason why that holds in their paper is that they do not consider moral hazard, they just consider only incomplete information regarding the firm’s type (i.e., efficiency parameter).  

---

12The same happens here.
Appendix

Proof of Proposition 1.

Lemma 1 If \((W,a)\) solves the relaxed program, there exists real numbers \(\gamma, \mu_e, \mu_s\) and \(\lambda_h, h = 1, \ldots, n\) such that

\[
\begin{align*}
& w_h : -p_h(\theta,a) + \mu_e p_{he}(\theta,a) + \mu_s p_{hs}(\theta,a) + \gamma p_h(\theta,a) + \lambda_h = 0, \quad (A1) \\
& e : \sum_h p_{he}(\theta,a)(y_h - w_h) + \mu_e \sum_h p_{he}(\theta,a)w_h + \\
& \mu_s \sum_h p_{hs}(\theta,a)w_h + \gamma \left( \sum_h p_{he}(\theta,a)w_h - c \right) \begin{cases} 
  \leq 0 & \text{if } e = 0 \\
  = 0 & \text{if } e \in (0, \bar{e}) \\
  \geq 0 & \text{if } e = \bar{e} 
\end{cases} \\
& s : \sum_h p_{hs}(\theta,a)(y_h - w_h) + \mu_e \sum_h p_{hs}(\theta,a)w_h + \\
& \mu_s \sum_h p_{hs}(\theta,a)w_h + \gamma \left( \sum_h p_{hs}(\theta,a)w_h - c \right) \begin{cases} 
  \leq 0 & \text{if } s = 0 \\
  = 0 & \text{if } s \in (0, \bar{s}) \\
  \geq 0 & \text{if } s = \bar{s} 
\end{cases}
\end{align*}
\]

and

\[
\mu_e \geq 0, \quad \mu_s \leq 0, \quad \lambda_h \geq 0 \quad \text{and} \quad \gamma \geq 0. \quad (A4)
\]

Proof. These are the Khun-Tucker necessary conditions for optimality. \(\blacksquare\)

Lemma 2 If \((W,a)\) solves the relaxed program and \(a \in (0, \bar{a}]\) for \(a = e, s\), then \(w_h \geq L, \forall h = 1, \ldots, n - 1\) and \(w_n > L\). If \(a = 0\), then \(w_n = L\).

Proof. Summing up the first-order conditions in equation (5) over \(h\), one gets that \(\sum_h \lambda_h = 1 - \gamma \geq 0\), since \(\sum_h p_{ha}(\theta,a) = 0\) for all \(a\). Thus, if the participation constraint does not bind, the LL constraint binds for at least one outcome. Multiplying both sides of the FOC by \(w_h(\theta) - L\) and summing over \(h\) one gets that

\[
- \sum_h p_h(\theta,a)(w_h - L) + \mu_e \sum_h p_{he}(\theta,a)(w_h - L) + \mu_s \sum_h p_{hs}(\theta,a)(w_h - L) + \\
\gamma \sum_h p_h(\theta,a)(w_h - L) + \sum_h \lambda_h(w_h - L) = 0.
\]
Using the incentive constraint for actions, the complementarity slackness constraint for the agent’s participation decision and complementarity slackness constraint for \( w_h \), this can be written as follows

\[
\sum_h p_h(\theta, a)(w_h - L) = \mu_e \sum_h p_{he}(\theta, a)(w_h - L) + \mu_s \sum_h p_{hs}(\theta, a)(w_h - L) + \gamma (U + c(a) - L). \tag{A5}
\]

In what follows I will assume that the participation constraint does not bind and then I will show that at the optimal contract this is always satisfied.

Suppose that \( \mu_e \) and \( \mu_s \) are such that \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) is strictly decreasing in \( h \). Then it follows from the FOC in equation (5) that if the wage is greater than \( L \) for two different outcomes, say \( y_h \) and \( y_{h'} \), then \( \lambda_h = \lambda_{h'} = 0 \). Because \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) falls with \( h \), it follows from the FOC in equation (5) that

\[
\gamma - 1 + \ell_e p_{he}(\theta, a) + \ell_s p_{hs}(\theta, a) = \gamma - 1 + \ell_e p_{he}(\theta, a) + \ell_s p_{hs}(\theta, a) \tag{A6}
\]

which contradicts the hypothesis that \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) falls with \( h \). It follows from this that \( \lambda_h \geq 0 \) for \( h = 1 \) and \( \lambda_h > 0 \) for all \( h > 1 \).

This implies that \( w_h \geq w_{h+1} \) for all \( h = 1, \ldots, n - 1 \) and thus the agent’s first order condition for effort \( e \) and that for \( s \) in equations (7) and (8) are respectively such that \( U_e(W, a, \theta) < 0 \) and \( U_s(W, a, \theta) < 0 \). This implies that \( e = 0 \) and \( s = 0 \) and thus the optimal contract cannot implement a positive effort in either task.

Suppose now \( \mu_e = 0 \) and \( \mu_s = 0 \) and therefore \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) = 0 \) for all \( h = 1, \ldots, n \). Then it follows from that \( \lambda_h > 0 \), \( \forall h \) and therefore \( w_h = L \), \( \forall h \). This implies that \( U_e(W, a, \theta) < 0 \) and \( U_s(W, a, \theta) < 0 \). This implies that \( e = 0 \) and \( s = 0 \) and thus the optimal contract cannot implement a positive effort in either task. Thus, \( \mu_e \) or \( \mu_s \) must be such that \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) rises with \( h \).

Let define the set \( \Psi \) as the set of outcomes for which the LL constraint is non-binding; i.e., \( \Psi = \{ h \in \{1, \ldots, n\} \mid \lambda_h = 0 \} \). This implies that \( w_h = L \) for all \( h \in \Psi^c \), where \( \Psi^c \) is the complement of \( \Psi \).

The first-order condition for all \( h \in \Psi \) becomes

\[
-p_h(\theta, a) + \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) + \gamma p_h(\theta, a) = 0. \tag{A7}
\]

Let the wage be greater than \( L \) for two different outcomes, say \( y_h \) and \( y_{h'} \). This implies that \( \lambda_h = \lambda_{h'} = 0 \) and therefore the following must hold,

\[
\mu_e \ell_{he} + \mu_s \ell_{hs} + \gamma - 1 = \mu_e \ell_{h'e} + \mu_s \ell_{h's} + \gamma - 1. \tag{A8}
\]
Then because \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) rises with \( h \), this results in a contradiction. Thus, the payment after only one outcome, say outcome \( h \), is greater than \( L \). In particular, for this outcome \( \lambda_h = 0 \) and thus the following holds
\[
\gamma + \mu_e \ell_{he} + \mu_s \ell_{hs} = 1.
\]
Because \( \mu_e p_{he}(\theta, a) + \mu_s p_{hs}(\theta, a) \) rises with \( h \), if this holds for any \( h' < n \), then \( \lambda_h < 0 \) for all \( h > h' \), which is a contradiction, since \( \lambda_h \geq 0 \) for all \( h \). Thus, only the wage when the highest outcome is realized is greater than \( L \).

It follows now from the fact that \( w_h = L \) for all \( h \leq n - 1 \) and \( w_n > L \), \( U_e(W, a, \theta) \geq 0 \) and \( U_o(W, a, \theta) \geq 0 \) and that \( U < L \) that the participation constraint is always satisfied.

\[\square\]

**Lemma 3** If \((W, a)\) solve the relaxed program, then the agent’s expected utility at \( W \) is concave in the action profile \( a \) for any \( a > 0 \).

**Proof.**

Recall that
\[
\sum_{h=0}^{n} f_h g_h = f_0 \sum_{h=0}^{n} g_h + \sum_{j=0}^{n-1} (f_{j+1} - f_j) \sum_{h=j+1}^{n} g_h. \tag{A9}
\]

Let \( P_h(\theta, a) \) be \( \sum_{h' \geq h} p_{h'}(\theta, a) \) and \( \Delta w_h \) be \( w'_m - w_{h-1}' \) for \( h \geq 1 \) and \( \Delta w_1 = w'_1 \) for \( h = 1 \). Then note that applying the summation by parts formula in equation (A9)
\[
U(W, a, \theta) = \sum_h \Delta w_h P_h(\theta, a) - c(a).
\]
Hence,
\[
U_{aa}(W, a, \theta) = \sum_h \Delta w_h P_{haa}(\theta, a) \text{ for } a \in \{e, s\}.
\]
and
\[
U_{es}(W, a, \theta) = \sum_h \Delta w_h P_{hes}(\theta, a).
\]

Note that the first equation is negative by \( \text{CP} \) and \( \Delta w_h \geq 0 \) for all \( h \). Thus, the agent’s second order condition is satisfied if and only if for all \( a \in A \)
\[
\left( \sum_h \Delta w_h P_{hee}(\theta, a) \right) \left( \sum_h \Delta w_h P_{hss}(\theta, a) \right) - \left( \sum_h \Delta w_h P_{hes}(\theta, a) \right)^2 \geq 0.
\]
Assumption \( \text{CP} \) together with the fact \( \Delta w_h \geq 0 \) guarantees that this term is positive. To see this note that
\[
\sum_h \Delta w_h P_{hee}(\theta, a) \sum_h \Delta w_h P_{hss}(\theta, a) - \left( \sum_h \Delta w_h P_{hes}(\theta, a) \right)^2 \geq \left( \sum_h \Delta w_h \sqrt{P_{hee}(\theta, a) / P_{hss}(\theta, a)} \right)^2 - \left( \sum_h \Delta w_h P_{hes}(\theta, a) \right)^2
\]
20
where the inequality follows from the Cauchy-Schwartz inequality. It follows then that the agent’s utility function is concave in $a$ for any $a \in A$ if
\[
\sum_h \triangle w_h \left( \sqrt{P_{hee}(\theta, a)} \sqrt{P_{hss}(\theta, a)} - P_{hes}(\theta, a) \right) \geq 0.
\]
Observe that assumption CP ensures that the term in square brackets is positive for all $h$. This together with the fact that $\triangle w_h \geq 0$ for all $h$ yields the desired result. Thus, $U(W, a, \theta)$ is concave in $a$ for any $a \in A$ and this proves that condition (9) holds.

Part (i) follows from 3. If $(W, a)$ solves the relaxed program, then the following holds
\[
U_{aa}(W, a, \theta) \leq 0, \forall a \in [0, \bar{a}] \text{ for } a = e, s,
\]
\[
U_a(W, a, \theta) \geq 0, \forall a \in (0, \bar{a}] \text{ for } a = e, s
\]
These conditions imply that the agent’s action profile is global maximum and $(W, a)$ is an element of the unrelaxed constraint set. Since $(W, a)$ maximize the principal’s utility over the relaxed constraint set, it maximizes the principal’s utility over the unrelaxed constraint set since it was assume that the unrelaxed program has a positive solution (assumption 1).

Proof of Proposition 2. Let $\hat{w}_h$ be $w_h - L$. Then, using the first-order conditions for action $a$, the agent’s incentive constraint, the FOC for $e$ can be written as follows
\[
e : \sum_h p_{he}(\theta, a)y_h - c + \mu_e p_{nee}(\theta, a)\hat{w}_n + \mu_s p_{nse}(\theta, a)\hat{w}_n = 0
\]
and that for $s$
\[
s : \sum_h p_{hs}(\theta, a)y_h - c + \mu_e p_{hes}(\theta, a)\hat{w}_n + \mu_s p_{nss}(\theta, a)\hat{w}_n = 0.
\]
Furthermore, the first-order condition for $w_n$ is given by:
\[
\mu_e \ell_{ne} + \mu_s \ell_{ns} = 1
\]
Adding equations (A10) and (A10) and solving for $\mu_s$ and $\mu_e$ from this new equation and equation (A12), one gets that
\[
\mu_s = \frac{\ell_e \hat{w}_n \sum_h \left( p_{hs}(\theta, a) + p_{he}(\theta, a) \right) y_h - 2\ell_e c + p_{nee} + p_{nse}}{\ell_s (p_{nee} + p_{nse}) - \ell_e (p_{nss} + p_{nse})}
\]
and
\[
\mu_e = -\frac{(\ell_e/\hat{w}_n) \sum_h (p_{hs}(\theta, a) + p_{he}(\theta, a)) y_h - 2\ell_e c + p_{nss} + p_{nse}}{\ell_s(p_{nse} + p_{nse}) - \ell_e(p_{nss} + p_{nse})}.
\]  
(A14)

Using the complementary slackness in equation (A5) and the FOC for the agent’s effort choice, one gets the optimal wage when state \(n\) is realized
\[
w_n = \frac{c}{p_{ne}(\theta, a)} + L = \frac{c}{p_{ns}(\theta, a(\theta))} + L.
\]  
(A15)

It follows from this that the equilibrium effort profile must be such that:
\[
\ell_{ne} = \ell_{ns}.
\]

Finally, it readily follows from this and the first-order conditions in equations (A10) and (A11) that the effort profile must satisfy the following
\[
\sum_h p_{he}(\theta, a) y_h - c = \frac{p_n(\theta, a(\theta))}{p_{ne}^2(\theta, a(\theta))} \left( \frac{p_{nss}(\theta, a) - p_{nse}(\theta, a)}{2p_{nse}(\theta, a) - p_{nss}(\theta, a) - p_{nss}(\theta, a)} \right) c
\]
and
\[
\sum_h p_{hs}(\theta, a) y_h - c = \frac{p_n(\theta, a(\theta))}{p_{ns}^2(\theta, a(\theta))} \left( \frac{p_{nss}(\theta, a) - p_{nse}(\theta, a)}{2p_{nse}(\theta, a) - p_{nss}(\theta, a) - p_{nss}(\theta, a)} \right) c
\]

It readily follows from this that \(\sum_h p_{hs} y_h - c > 0\) or \(\sum_h p_{he} y_h - c > 0\) and because \(P_{nse}(\theta, a) \geq 0\), \(a^m(\theta) \leq a^*(\theta)\) and \(a^m(\theta) \neq a^*(\theta), \forall \theta \in \Theta\).

**Proof of Proposition 5.** The agent’s problem in terms of his revelation of type \(\theta'\), or equivalent in terms of his choice from the menu of contracts offered by the principal, is
\[
\max_{\theta' \in \Theta} U(W(\theta'), a(\theta', \theta'), \theta).
\]
The first-order condition is given by
\[
\sum_h p_h(\theta, a(\theta', \theta')) \frac{\partial w_h(\theta')}{\partial \theta'} + \sum_a \left\{ \sum_h p_{ha}(\theta, a(\theta', \theta')) w_h(\theta') - c_a(a(\theta', \theta')) \right\} \frac{\partial a(\theta', \theta')}{\partial \theta'} = 0 \text{ for all } \theta \in \Theta.
\]
Using the first-order condition for effort the second term vanishes and thus the condition boils down to the first term only. This shows that
\[
\sum_h p_h(\theta, a(\theta)) \frac{\partial w_h(\theta)}{\partial \theta} + \sum_a \left\{ \sum_h p_{ha}(\theta, a(\theta)) w_h(\theta) - c \right\} \frac{\partial a(\theta)}{\partial \theta} = 0 \text{ for all } \theta \in \Theta,
\]  
(A16)
is necessary for implementability.

The result follows from the fact that the agent’s first-order condition for action \(a \in A\) ensures that the second term vanishes.
Solving the differential equation for $\mu$ because the second-order condition requires the first term to be non-positive, this implies that
\[ \sum_h p_h(\theta, a(\theta)) \frac{\partial w_h(\theta, \theta')}{\partial \theta'} + \sum_a \left\{ \sum_h p_h a(\theta, a(\theta')) w_h(\theta') \frac{\partial a(\theta, \theta')}{\partial \theta'} \right\} \geq 0. \]
evaluated at $\theta' = \theta$ is necessary for the second-order condition to be satisfied.

The same arguments show that equations
\[ \sum_h p_h(\theta, a(\theta)) \frac{\partial w_h(\theta)}{\partial \theta} + \sum_a \left\{ \sum_h p_h a(\theta) w_h(\theta) - c \right\} \frac{\partial a(\theta)}{\partial \theta} = 0 \text{ for all } \theta \in \Theta, \tag{A17} \]
\[ \sum_h p_h(\theta, a(\theta)) \frac{\partial w_h(\theta')}{\partial \theta'} + \sum_a \left\{ \sum_h p_h a(\theta') w_h(\theta') \frac{\partial a(\theta')}{\partial \theta'} \right\} \geq 0 \text{ for all } \theta, \theta' \in \Theta, \tag{A18} \]
holding for all $\theta, \theta' \in \Theta$ are sufficient conditions for implementability. The result follows from the fact that the agent’s first-order condition for action $a \in A$ ensures that the second term in equation (A17) vanishes.

**Proof of Proposition 4.** The first-order conditions for the Hamiltonian in equation (22) are as follows
\[ \mu'(\theta) = -\frac{\partial L}{\partial U} = f(\theta) - \gamma(\theta) - \mu(\theta) \ell_{n\theta}(\theta, a(\theta)) \tag{A19} \]
\[ \mu(\theta) \left[ \ell_{\theta}(\theta, a(\theta)) - \ell_{n\theta}(\theta, a(\theta)) \right] p_h(\theta, a(\theta)) + \lambda_h(\theta) + \frac{\partial a}{\partial w_h} \sum_a \left[ S_a(\theta, a(\theta)) + \right. \]
\[ \mu(\theta) \left( \sum_{h \neq n} \frac{\partial}{\partial a} \left[ \ell_{\theta}(\theta, a(\theta)) - \ell_{n\theta}(\theta, a(\theta)) \right] p_h(\theta, a(\theta)) w_h(\theta) + \right. \]
\[ \left. \ell_{n\theta}(\theta, a(\theta)) c_a(a(\theta)) + \frac{\partial}{\partial a} \ell_{n\theta}(\theta, a(\theta))[U(\theta) + c(a(\theta))] \right\} = 0 \tag{A20} \]
\[ \gamma(\theta)(U(\theta) - U) = 0, \gamma(\theta) \geq 0, U(\theta) \geq U \tag{A21} \]
\[ \mu(\theta)(U(\theta) - \max\{U, L\}) = 0, \mu(\theta) \leq 0, \mu(\theta)(U(\theta) - \max\{U, L\}) = 0, \mu(\theta) \geq 0 \tag{A22} \]

Solving the differential equation for $\mu'(\theta)$, because $U(\theta)$ is free I can use the transversality condition $\mu(\theta) = 0$, and assuming that $\gamma(\theta) = 0$ for all $\theta$, one gets
\[ \mu(\theta) = \frac{1}{p_n(\theta, a(\theta))} \left[ \int_\theta^\theta p_n(x, a(x)) f(x) dx - \int_\theta^\theta p_n(x, a(x)) f(x) dx \right]. \]
This can be written as
\[ \mu(\theta) = \frac{1 - F(\theta | n)}{f(\theta | n)} f(\theta). \]
where
\[ F(\theta \mid n) = \frac{\int_{\theta}^{\psi} p_n(x, a(x)) f(x) \, dx}{\int_{\theta}^{\psi} p_n(x, a(x)) f(x) \, dx} \]
and
\[ f(\theta \mid n) = \frac{p_n(\theta, a(\theta)) f(\theta)}{\int_{\theta}^{\psi} p_n(x, a(x)) f(x) \, dx}. \]

Multiplying both side of the first-order condition for \( w_h(\theta) \) by \( w_h(\theta) - L \) and using the complementary-slackness condition for the limited liability constraint and actions’ incentive compatibility constraints, I get that
\[
(w_h(\theta) - L) \left( \mu(\theta) \left( \ell_{h\theta}(\theta, a(\theta)) - \ell_{n\theta}(\theta, a(\theta)) \right) p_h(\theta, a(\theta)) + \frac{\partial a}{\partial w_h} \sum_a \left( S_a(\theta, a(\theta)) f(\theta) + \mu(\theta) \frac{\partial U'(\theta)}{\partial a} \right) \right) = 0.
\]
where
\[
\frac{\partial U'(\theta)}{\partial a} = \mu(\theta) \left( \sum_{h \neq n} \frac{\partial}{\partial a} [\ell_{h\theta}(\theta, a(\theta)) - \ell_{n\theta}(\theta, a(\theta))] p_h(\theta, a(\theta)) w_h(\theta) + \ell_{n\theta}(\theta, a(\theta)) c_a(a(\theta)) + \frac{\partial}{\partial a} \ell_{n\theta}(\theta, a(\theta)) [U(\theta) + c(a(\theta))] \right).
\]

Because \( \mu(\theta) < 0 \) for all \( \theta \in \Theta, \frac{\partial a}{\partial w_h} \neq 0 \) and MLRP implies that \( \ell_{h\theta}(\theta, a(\theta)) < \ell_{n\theta}(\theta, a(\theta)) \) for all \( h, w_h(\theta) = L \) for all \( h < n \) and \( \theta \in \Theta \).

Differentiating \( w_n(\theta) \) in equation (17) with respect to \( \theta \), one gets that
\[
\frac{\partial w_n(\theta)}{\partial \theta} = -\frac{1}{p_n(\theta, a(\theta))} w_n(\theta) + \frac{1}{p_n(\theta, a(\theta))} \left( U'(\theta) + p_{n\theta}(\theta, a(\theta)) L \right) + \frac{1}{p_n(\theta, a(\theta))} \sum_a \frac{\partial a}{\partial \theta} \left( p_{na}(\theta, a(\theta))(w_n(\theta) - L) - c \right) = 0
\]
where the inequality is due to the agent’s first-order condition for effort and the fact that
\[ U'(\theta) = p_{n\theta}(\theta, a(\theta))(w_n(\theta) - L). \]
Thus, \( \frac{\partial w_n(\theta)}{\partial \theta} = 0. \)

Integrating \( U'(\theta) \), I get that
\[ U(\theta) = U(\theta) + \int_{\theta}^{\theta} p_{n\theta}(x, a(x))(w_n(x) - L) \, dx. \]
Substituting this into \( w_n(\theta) \) in equation (17), one gets that
\[
w_n(\theta) = \frac{1}{p_n(\theta, a(\theta))} \left( U(\theta) + \int_{\theta}^{\theta} p_{n\theta}(x, a(x))(w_n(x) - L) \, dx + c(a(\theta)) - L \right) + L. \quad (A23)
\]
Using the complementary slackness condition for \( \theta \), one gets that \( U(\theta) = L \) (since \( L \geq U \) and \( \mu(\theta) < 0 \)). Thus,

\[
w_n(\theta) = \frac{1}{p_n(\theta, a(\theta))} \left( \int_{\theta}^{\theta} p_n(\theta, a(\theta))(w_n(x) - L)dx + c(a(\theta)) \right) + L. \tag{A24}
\]

It readily follows from this that

\[
w_n(\theta) = \frac{c(a(\theta))}{p_n(\theta, a(\theta))} + L. \tag{A25}
\]

Because \( \frac{\partial w_n(\theta)}{\partial \theta} = 0 \), this together with \( w_n(\theta) \) yields the result in equation (25).

Before ending the proof I need to check that this solution satisfies the second-order condition in equation (16), which requires the following

\[
p_n(\theta, a(\theta)) \frac{\partial w_h(\theta')}{\partial \theta'} + \sum_a p_{na}(\theta, a(\theta'))[w_n(\theta') - L] \frac{\partial a(\theta')}{\partial \theta'} \geq 0 \quad \text{for all } \theta, \theta' \in \Theta.
\]

Because \( \frac{\partial w_n(\theta')}{\partial \theta} = 0 \) for all \( \theta' \in \Theta \), then \( a(\theta, \theta') = a(\theta) \) for all \( \theta' \in \Theta \). This together with the fact that \( w_n(\theta') \geq L \) for all \( \theta' \in \Theta \), the global condition becomes

\[
\sum_a \left\{ \sum_h p_{na}(\theta, a(\theta'))w_h(\theta') \frac{\partial}{\partial \theta'} a(\theta') \right\} \geq 0 \quad \text{for all } \theta, \theta' \in \Theta.
\]

Observe that this holds since \( p_{na}(\theta, a(\theta')) \geq 0 \) for all \( h \), \( w_h(\theta') \geq L \geq 0 \) for all \( h \) and \( \frac{\partial}{\partial \theta'} a(\theta') > 0 \) for \( a = e, s \). The latter follows from the fact that \( \frac{\partial w_n(\theta)}{\partial \theta} = 0 \) and from partially differentiating the agent’s first-order conditions

\[
p_{ne}(\theta, a(\theta)) \frac{c(a(\theta))}{p_n(\theta, a(\theta))} - c = 0,
\]

\[
p_{ns}(\theta, a(\theta)) \frac{c(a(\theta))}{p_n(\theta, a(\theta))} - c = 0.
\]

This leads to

\[
\frac{\partial e(\theta)}{\partial \theta} = - \frac{p_{ns}(\theta, a(\theta))p_{ne}(\theta, a) + p_{nes}(\theta, a)p_{ne}(\theta, a)}{p_{ns}(\theta, a)p_{ne}(\theta, a) - p_{nes}(\theta, a)^2} \frac{c(a(\theta))}{p_n(\theta, a(\theta))} > 0
\]

and

\[
\frac{\partial s(\theta)}{\partial \theta} = - \frac{p_{ne}(\theta, a)p_{ne}(\theta, a) + p_{nes}(\theta, a)p_{ne}(\theta, a)}{p_{ns}(\theta, a)p_{ne}(\theta, a) - p_{nes}(\theta, a)^2} \frac{c(a(\theta))}{p_n(\theta, a(\theta))} > 0
\]

Finally, one can show, following the steps in the proof of proposition 2, that the first-order approach is valid because of contract monotonicity and condition \( \text{CP} \).
References


2012

291. On the Optimality of One-size-fits-all Contracts: The Limited Liability Case
Felipe Balmaceda

290. Self Governance in Social Networks of Information Transmission
Felipe Balmaceda y Juan F. Escobar

289. Efficiency in Games with Markovian Private Information
Juan F. Escobar y Juuso Toikka

288. EPL and Capital-Labor Ratios
Alexandre Janiaka y Etienne Wasmer

Sofía Bauducco y Alexandre Janiak

2011

286. Comments on Donahue and Zeckhauser: Collaborative Governance
Ronald Fischer

Benjamín Villena-Rodán y Cecilia Ríos-Aguilar

284. Towards a Quantitative Theory of Automatic Stabilizers: The Role of Demographics
Alexandre Janiak y Paulo Santos Monteiro

283. Investment and Environmental Regulation: Evidence on the Role of Cash Flow
Evangelina Dardati y Julio Riutort

282. Teachers’ Salaries in Latin America. How Much are They (under or over) Paid?
Alejandra Mizala y Hugo Ñopo

281. Acyclicity and Singleton Cores in Matching Markets
Antonio Romero-Medina y Matteo Triossi

280. Games with Capacity Manipulation: Incentives and Nash Equilibria
Antonio Romero-Medina y Matteo Triossi

279. Job Design and Incentives
Felipe Balmaceda

278. Unemployment, Participation and Worker Flows Over the Life Cycle
Sekyu Choi - Alexandre Janiak - Benjamín Villena-Roldán

277. Public-Private Partnerships and Infrastructure Provision in the United States
Eduardo Engel, Ronald Fischer y Alexander Galetovic
2010


274. Structural Unemployment and the Regulation of Product Market Alexandre Janiak

273. Non-revelation Mechanisms in Many-to-One Markets Antonio Romero-Medina y Matteo Triossi

272. Labor force heterogeneity: implications for the relation between aggregate volatility and government size Alexandre Janiak y Paulo Santos Monteiro

271. Aggregate Implications of Employer Search and Recruiting Selection Benjamin Villena Roldán

270. Wage dispersion and Recruiting Selection Benjamin Villena Roldán

269. Parental decisions in a choice based school system: Analyzing the transition between primary and secondary school Mattia Makovec, Alejandra Mizala y Andrés Barrera


267. Costly information acquisition. Better to toss a coin? Matteo Triossi

266. Firm-Provided Training and Labor Market Institutions Felipe Balmaceda

2009

265. Soft budgets and Renegotiations in Public-Private Partnerships Eduardo Engel, Ronald Fischer y Alexander Galetovic

264. Information Asymmetries and an Endogenous Productivity Reversion Mechanism Nicolás Figueroa y Oksana Leukhina

262. Renegociación de concesiones en Chile
Eduardo Engel, Ronald Fischer, Alexander Galetovic y Manuel Hermosilla

261. Inflation and welfare in long-run equilibrium with firm dynamics
Alexandre Janiak y Paulo Santos Monteiro

260. Conflict Resolution in the Electricity Sector - The Experts Panel of Chile
R. Fischer, R. Palma-Behnke y J. Guevara-Cedeño

259. Economic Performance, creditor protection and labor inflexibility
Felipe Balmaceda y Ronald Fischer

258. Effective Schools for Low Income Children: a Study of Chile’s Sociedad de Instrucción Primaria
Francisco Henríquez, Alejandra Mizala y Andrea Repetto

257. Public-Private Partnerships: when and how
Eduardo Engel, Ronald Fischer y Alexander Galetovic

2008

256. Pricing with markups in industries with increasing marginal costs
José R. Correa, Nicolás Figueroa y Nicolás E. Stier-Moses

255. Implementation with renegotiation when preferences and feasible sets are state dependent
Luis Corchón y Matteo Triossi

254. Evaluación de estrategias de desarrollo para alcanzar los objetivos del Milenio en América Latina. El caso de Chile
Raúl O’Ryan, Carlos J. de Miguel y Camilo Lagos

253. Welfare in models of trade with heterogeneous firms
Alexandre Janiak

252. Firm-Provided Training and Labor Market Policies
Felipe Balmaceda

251. Emerging Markets Variance Shocks: Local or International in Origin?
Viviana Fernández y Brian M. Lucey

250. Economic performance, creditor protection and labor inflexibility
Ronald Fischer

249. Loyalty inducing programs and competition with homogeneous goods
N. Figueroa, R. Fischer y S. Infante

248. Local social capital and geographical mobility: A theory
Quentin David, Alexandre Janiak y Etienne Wasmer
247. On the planner’s loss due to lack of information in bayesian mechanism design
José R. Correa y Nicolás Figueroa

246. Política comercial estratégica en el mercado aéreo chileno
Ronald Fischer

245. A large firm model of the labor market with entry, exit and search frictions
Alexandre Janiak

244. Optimal resource extraction contracts under threat of expropriation
Eduardo Engel y Ronald Fischer

2007

243. The behavior of stock returns in the Asia-Pacific mining industry following the Iraq war
Viviana Fernandez

242. Multi-period hedge ratios for a multi-asset portfolio when accounting for returns comovement
Viviana Fernandez

241. Competition with asymmetric switching costs
S. Infante, N. Figueroa y R. Fischer

240. A Note on the Comparative Statics of Optimal Procurement Auctions
Gonzalo Cisternas y Nicolás Figueroa

239. Parental choice and school markets: The impact of information approximating school effectiveness
Alejandra Mizala y Miguel Urquiola

238. Marginal Cost Pricing in Hydro-Thermal Power Industries: Is a Capacity Charge Always Needed?
M. Soledad Arellano and Pablo Serra

237. What to put on the table
Nicolas Figueroa y Vasiliki Skreta

236. Estimating Discount Functions with Consumption Choices over the Lifecycle
David Laibson, Andrea Repetto y Jeremy Tobacman

235. La economía política de la reforma educacional en Chile
Alejandra Mizala

234. The Basic Public Finance of Public-Private Partnerships
(Por aparecer en J. of the European Economic Association)
Eduardo Engel, Ronald Fischer y Alexander Galatovic

* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.