Self Governance in Social Networks of Information Transmission

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Abstract

This paper studies the problem of self governance in a model in which information flows are governed by the community structure. In each round of an infinitely repeated game, an agent and an investor, who is selected from a finite set of investors, play a trust game. Investors receive information only from their own social contacts and may act upon after receiving news about opportunistic behavior. We explore the social networks leading to self governance and emphasize how the architecture of such networks is shaped by the technology with which surplus can be created in each round. We formally show how the details of the transactions determine the density and cohesiveness of the social network. An application to the interaction between formal and informal institutions of exchange is provided, and it is shown that markets may harm networked relationships by deteriorating the monitoring quality.

JEL classification numbers: D85, C73, L14, D82

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1 Introduction

Oftentimes parties to a transaction are exposed to opportunistic behavior (Williamson 1979). The use of informal mechanisms of governance, or self governance, to facilitate efficient economic transactions has been widely recognized and documented by economists (Greif 1993, Milgrom, North, and Weingast 1990), political scientists (Ostrom 1990, Fearon and Laitin 1996), sociologists (Coleman 1990), and legal scholars (Bernstein 1992). For instance, Greif (1993) discusses how informal ongoing relationships successfully fostered exchange in some pre modern European societies, even when parties were tempted to renege on their obligations. These implicit arrangements work on a multilateral basis as following inappropriate actions, not only does the cheated party refuse trade in subsequent encounters, but so do other community members who are connected to the miscreant.

Crucial to the operation of these informal arrangements is how information flows and is employed. As highlighted by Coleman (1990) and Dixit (2006), among others, the community structure, being a determinant of information flows, effectively constrains feasible social norms and the extent to which efficient economic transactions can be attained. In particular, in cohesive communities information flows are rich and thus self governance is a feasible outcome (Greif 1993, Clay 1997). However, whether efficient exchange can arise in imperfectly cohesive communities is not well understood. This paper explores which characteristics of a community are important for the emergence of cooperation and trust as a self enforcing phenomenon.

Several studies highlight the importance of the social context in establishing cooperative relationships, even in modern industrial societies with established legal systems. The internet has opened up opportunities to trade services and goods (Levin 2011). Internet platforms keep transactions faithful by implementing reputation systems (e.g. eBay) and facilitating social networking (e.g. Taobao and CouchSurfing). These information transmission mechanisms allow parties to exchange their trading experiences and moderate incentives to behave opportunistically. Cooperation and trust also allow firms to organize their procurement relationships, even when the threat of hold up can destroy any relationship specific joint value (Williamson 1979, Granovetter 1985). A nice example of relationship specific value creation comes from Japanese keiretsus (see McMillan 1995). In this system procuring firms are organized to exchange information about mischievous actions, and contracts are renewed only when implicit contractual terms are followed\footnote{These business practices not only impact the national supply chains, but can also affect the patterns of international trade (Rauch 2001, Head, Ries, and Spencer 2004).} Uzzi (1996) analyzes the apparel industry in New York.
and shows how firms repeatedly dealing with a single partner in a fully embedded relationship are more likely to survive. Rooks, Raub, and Tazelaar (2006) show how transaction outcomes in the Dutch information technology industry are molded by the ability of clients to spread news about unsatisfactory deliveries among other clients. All these examples show there is a variety of community structures in which self governance can arise, but so far no theoretical framework evaluates the impact of different network arrangements on transaction outcomes. This paper systematically studies how the details of the transactions determine the architecture of social networks conducive to efficient trade.

We consider an infinitely repeated game played by \( N \) investors and one agent. At each round \( t \geq 1 \), one out of the \( N \) investors is selected and plays a trust game with the agent. More specifically, the investor decides whether or not to invest; if he invests, then the agent chooses whether to cooperate or to defect. The socially desirable outcome is obtained when the investor invests and the agent cooperates. Yet, in a one shot interaction, the agent will behave opportunistically, and thus investment will not take place, as described by Williamson (1979). In our dynamic model, this temptation may be moderated by the existence of community sanctions governed by a social network of investors \( G \). We assume that if the agent misbehaves when facing investor \( i \), then \( i \) and all his connections in \( G \) (i.e., all those who are linked to \( i \)) become aware of that and act upon by refusing to invest in all subsequent rounds. This information does not disseminate through the network any further or, in other words, only victims of mischievous actions complain. We focus on sustainable networks, loosely defined as social networks of investors in which it is in the agent’s interest to cooperate in all encounters.

An important determinant of the architecture of social networks conducive to efficient transactions is the technology with which investment opportunities arise. In our model, the agent can interact with a single investor at any given round, and we explore how two polar technologies to match the agent to an investor shape the architecture of sustainable networks. More specifically, we study a random matching model in which the identity of the selected investor is randomly and uniformly determined across time, and a directed matching model in which the identity of the selected investor is chosen by the agent. The random matching model captures situations in which ex post not all investors have access to enough resources to undertake the investment in each period. In contrast in the directed matching model, each investor has the resources to undertake the investment in each period if both both the agent and the investor so choose. While these distinctions are immaterial in a complete information model, they play a key role in determining the architecture of sustainable networks when information is incomplete.
Our model yields several novel insights. When matching is directed, the set of sustainable networks coincides with the set of communities in which all trading investors are connected. More technically, the set of sustainable networks equals the set of complete components, though the size of the network is undetermined. This result is due to the fact that if two community members are unconnected, a deviation against one of them entails no continuation value loss to the agent, since the cheated agent can be easily substituted for in the continuation game by trading with investors unaware of the deviation. This property of sustainable networks provides a particularly strong force towards cohesiveness, a topic that has received considerable attention and that we discuss below.

We also explore the architecture of efficient networks, defined as networks that maximize the total sum of players’ benefits minus the total costs of maintaining links. Because in the directed matching model investors are substitutable, the total benefit of a sustainable network is independent of the network size, while the costs are strictly increasing on the number of links. As a result, efficient networks result in bilateral relationships in which the agent interacts repeatedly with a single investor. This result resonates well with Uzzi’s (1996) empirical findings on the stability and durability of fully embedded relationships.

When matching is random, the architecture of sustainable networks is richer than when matching is directed. Our first result shows that the size of the neighborhood of any member of a sustainable network must be sufficiently high. Otherwise, defecting against a barely connected investor does not entail a sufficiently significant loss of future trading opportunities for the agent, even when he and his connections cannot be subsequently substituted for. This lower bound on the size of each neighborhood belonging to a sustainable network is, in general, not a sufficient condition. The reason stands for the fact that upon a deviation, there might be opportunities to deviate against remaining uninformed investors. Yet, we identify conditions under which these deviating opportunities are unattractive and determining the stability of a social network reduces to checking the number of connections each investor has.

We also identify conditions in the random matching model under which sustainable networks exhibit varying degrees of cohesiveness. This rests on the double deviation logic discussed above, and follows after observing that if two unconnected investors have many connections in common, a deviation against one of them leaves the other barely connected in the after-deviation network; as a result, it is in the agent’s interest to deviate once more. This makes the first deviation attractive and ruling out these double deviations imposes upper bounds on the numbers of paths of length 2 that unconnected investors must have. We derive lower bounds
on the local clustering coefficient of members of sustainable networks as a function of the size of their neighborhoods. Moreover, there are restrictions on the game implying that the set of efficient networks coincides with the set of networks formed by complete components of a given size.

These results show that cohesiveness may be necessary for self governance as it creates local common knowledge of game histories that deters opportunistic behaviors. Yet, we also identify limits to the natural presumption that such common knowledge is essential for self governance. The importance of cohesiveness for the emergence of cooperative attitudes has been forcefully stressed by Coleman’s (1990).\textsuperscript{2} Some empirical studies have related closure to community size (Allcott, Karlan, Mobius, Rosenblat, and Szeidl 2007). Our work provides a new rationale for the importance of network closure, and the mechanisms at work emerge from a full fledged dynamic model of incomplete information.

We also provide an application of our set up to study the interaction between formal and informal institutions of exchange. In particular, we explore how the existence of an anonymous market through which standardized transactions can be successfully realized limits the feasibility of socially desirable relationship based trade. While there are many channels through which markets and relationships may interact, our imperfect monitoring setting is particularly appropriate to explore a new dimension of this interaction, namely, that the existence of market standards may make harder to monitor unfaithful behavior by providing new defection opportunities that mimic market outcomes. We modify our baseline model to incorporate the possibility of market based trade and show how markets can severely limit the feasibility of networked relationships.

Our work connects to the repeated games literature (Abreu, Pearce, and Stacchetti 1990, Kandori 2002). In particular, Kandori (1992), Ellison (1994), Harrington (1995), and Okuno-Fujiwara and Postlewaite (1995) explore how extreme assumptions on the information flows shape players’ incentives to cooperate in large communities in which agents interact infrequently.\textsuperscript{3} We contribute to this literature by providing a new model of a large community in which information flows are not governed by how trading opportunities arise—which in some of these paper makes feasible the use of the so called contagious strategies—but by how the social structure determines the flows of information.

\textsuperscript{2}Cohesiveness has been found to be important in other social settings, notably in collective action games (e.g. Chwe 2000)

\textsuperscript{3} Several papers have extended this line of research. Ghosh and Ray (1996) study a repeated game model with adverse selection in which players build relationships starting small (as in Watson 2002) and decide whether to continue the relationship at the end of each round. Takahashi (2010) proves a folk theorem for a community enforcement game with first order information. Deb (2008) proves a folk theorem in which players only observe their own interactions and exchange messages before each round of play.
There has been some interest in embedding repeated game models into social networks, and early antecedents come from work by Raub and Weesie (1990) and Bendor and Mookherjee (1990). The focus on social networks as conduits of information is relatively recent (Ahn and Suominen 2001, Balmaceda 2006, Lippert and Spagnolo 2010, Ali and Miller 2010). Ahn and Suominen (2001) is perhaps more closely related to our work as they also study a repeated game model in which players receive signals about past play from their neighbors. However, in their model, the social network is randomly and independently drawn across time. In particular, the substantive question of how the social structure and the matching technology limit the emergence of cooperative behaviors cannot be answered in their model. From a technical perspective, their assumption about a community wide observed randomization device allows simplifications that our model does not. Balmaceda (2006), Lippert and Spagnolo (2010) and Ali and Miller (2010) also study repeated game models in which the network is determined once and for all. Our model is distinctive in several aspects. First, the incentive problem here studied is one sided; as a result, our work immediately connects to the hold up literature, as initiated by (Williamson 1979), and to Greif’s (1993) agency relations problems. Second, this paper explores how feasible social norms are determined when the only source of information flows is given by communication, and communication itself is the result of the community structure. In contrast, Balmaceda (2006) and Lippert and Spagnolo (2010) study models in which information can be transmitted through both actions and communication, while in Ali and Miller (2010) the network creates trading opportunities and information about unfaithful behavior is transmitted through actions. Third, we explore how alternative matching models, interpreted as different technologies with which surplus can be created, impact the architecture of sustainable and efficient network; previous work has ignored this aspect.  

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4Bhaskar (1998) studies an OLG model in which each player only observes the play of the previous cohort. His model can be interpreted as a game in which both the order of moves and the information flows are governed by a social network \(G\), the set of nodes of the network being the naturals and the set of directed links being all pairs of the form \((t, t + 1)\), with \(t\) a natural numbers. Nodes in the network are seen as players, links are seen as determining the flows of information. While Bhaskar (1998) does not push the social network interpretation of his results, his paper connects to the current social network literature as it explores how particular networks of information flows constraint the outcomes of his game.

5Proposition 2 in Ahn and Suominen (2001) crucially depends on the existence of a randomization device. The technical problem a randomization device solves in their model is the simplification of the off-path play. Even if one could adapt their tools to study a set up as ours—in which the network does not change as time passes—such simplification would not allow us to understand the role of cohesiveness in self governance as the off-path effects present in our set up are absent in their framework. Proposition 1 in Ahn and Suominen (2001) is similar to part (i) of our Proposition.

6Mihm, Toth, and Lang (2009) and Kinateder (2010) establish folk theorems for games played on networks. Our focus is different as we fix the discount factor and derive properties of sustainable networks. For completeness, we present a folk theorem for the random matching model in the Appendix. There are also models of repeated games on networks with complete information. Haag and Lagunoff (2006) study a repeated game model in which each individual chooses a single action that determines the payoff with all his links (for example, the action of turning on the porch light at night affects all street neighbors, and each individual makes just one choice, to
The paper is organized as follows. Section 2 introduces and discusses the model and presents some definitions. Section 3 illustrates some of our results using a simple three-node model. Section 4 presents necessary and sufficient conditions for sustainability. Section 5 presents our results on the impact of markets on networked relationships. Section 6 presents some concluding comments. Supporting arguments, examples, and proofs are relegated to the Appendix.

2 Set Up

2.1 The Environment

We study a repeated game model in discrete time between \( N + 1 \) players, with \( N \geq 2 \). At each round \( t = 1, 2, \ldots \) an agent, hereinafter player 0, faces an investor \( i_t \in \{1, \ldots, N\} \) and they play a trust game. The way in which the investor \( i_t \) is selected to play the trust game may be random or directed. Each round of interaction generates information that is spread through a social network \( G \) of investors. Below we describe the details of the game.

2.1.1 Payoffs and Matching Technology

The investor selected at round \( t \geq 1 \), \( i_t \), and the agent play the trust game shown in Figure 1, with \( l, g > 0 \). Action \( I \) (resp. \( NI \)) stands for invest (resp. do not invest), while \( C \) (resp. \( D \)) stands for cooperate (resp. defect). We observe that the only Nash equilibrium of this stage game results in the inefficient payoff profile \((0, 0)\).

<table>
<thead>
<tr>
<th>( i_t )</th>
<th>( I )</th>
<th>( NI )</th>
</tr>
</thead>
<tbody>
<tr>
<td>agent</td>
<td>( C )</td>
<td>( D )</td>
</tr>
<tr>
<td>1,1</td>
<td>(-l, 1+g)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Figure 1: A trust game.

We assume that investors who are not selected to play the trust game obtain a period payoff equal to 0. All players discount period payoffs with a discount rate \( \delta \in ]0, 1[ \).

We will consider two different ways in which the investor \( i_t \) is selected in each round \( t \).

Random Matching Investor $i_t$ is uniform and independently drawn from $\{1, \ldots, N\}$ across time at the beginning of each round.

Directed Matching Investor $i_t$ is selected by the principal from $\{1, \ldots, N\}$ at the beginning of each round.

These two matching technologies capture different economic environments. In the random matching model, the only way in which surplus can be created in a given round $t$ is through a partnership between the agent and a randomly determined investor $i_t$; this can be seen as an environment in which ex post not all investors have enough resources to undertake the investment in each period. In contrast, in the directed matching model, all investors have access to enough resources and therefore, from the agent’s point of view, investors are alike in terms of their possibilities of undertaking the investment at any given round. The nature of the resources needed to carry out the investment will depend on the application, and can be the result of technological heterogeneity or financial constraints. For example, the random matching model can be seen as an environment in which investors may be subject to credit constraints and ex post only one of them will have financial slack to undertake the investment. The directed matching technology models an industry in which financial resources abound.

Our assumption that in each round the agent interacts with only one agent may seem more natural when the game takes place in continuous time and investment opportunities arise exponentially, as in Ali and Miller (2010) and Jackson, Rodriguez-Barraquer, and Tan (2010).

2.1.2 Social Networks, Information Flows and Histories

In our model, there is a social network of investors $G = (N^G, E^G)$, where $N^G \subseteq \{1, \ldots, N\}$ is the set of nodes and $E^G$ is the set of links between these nodes. We denote by $N^G(i) = \{j \in N^G \mid ij \in E^G\}$ the set of $i$’s neighbors in $G$ and $\tilde{N}^G(i) = N^G(i) \cup \{i\}$.

Information flows are as follows. Let $i_t \in \{1, \ldots, N\}$ be the investor chosen at round $t$. If the chosen investor $i_t$ picks $I$, then all investors $j \in N^G(i_t)$ become informed of that and observe whether the agent chooses $C$ or $D$. No other information is transmitted, which means that from the perspective of player $j \in N^G(i)$, whether $i$ was not selected or was selected but picked $NI$ are part of the same information set. Thus, a history of length $t$ for investor $i \in \{1, \ldots, N\}$ consists of the rounds at which he has faced the agent, what the outcomes of those stage games have been, and the observations he has received from each of his neighbors in $G$ who have been selected and have chosen $I$ in previous rounds. A history for the agent consists of the whole
history in the game as, by construction, he has been involved in all the stage games that have taken place. Recall is perfect and the details of the extensive form game (in particular, the social network of information transmission $G$) are common knowledge. This completes the description of our dynamic game of incomplete information.

In our model, the social network plays the most minimal role in terms of information transmission that one could think of. In particular, an investor aware of a mischievous action in an encounter involving one of his neighbors does not spread the news any further. Of course, even if he could, his continuation payoff is not determined by how continuation play evolves and thus it is not in his interest to further the news. Thus, the assumptions on the information transmission technology can be seen as the result of voluntary communication in a larger game. While the equilibrium of the larger game may seem arbitrary in that only the victim complains, we think it is an appropriate approximation as victims of opportunistic behavior have a higher tendency to complaining.

We think our assumptions on the information transmission technology are of interest in other settings too. It is possible to model a richer information flow protocol in which news about deviations travel $n \geq 1$ steps at once by elaborating on our results. On the other hand, if we were to allow for richer information flows, it would seem appropriate to assume that as information travels farther away from its initial source –the cheated investor– its quality deteriorates. While one may have a relatively accurate assessment of the interactions in which one’s friends have been involved, such estimate is likely to be distorted when assessing friends of friends’ social interactions. Our results apply to situations in which distant news are of low quality and thus ignored; we describe the social arrangements that are robust to all those information imperfections.

2.2 Interpretations

Our model naturally applies to situations in which there is room for opportunistic behavior (Williamson 1979), and a large group of players interact repeatedly. We provide some examples

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7 In a model in which communication is voluntary, the agent could condition his behavior on whether an investor communicates his trading experiences and by manipulating the messages an investor aware of some defection could strictly benefit by omitting some news. Lippert and Spagnolo (2010) are concerned with issues of this sort in a related context.

8 We could also think that the social network is not a network of information transmission, but a network in which players observe the play of their neighbors. For example, if we interpret the model as a game between a monopolist and a set of consumers (as discussed below), it seems conceivable that a consumer could observe the quality of the good bought by their neighbors, but is not able to observe or become informed of the trading experiences of distant connections. The network of information transmission can be thought of as a network of observational learning.
of our environment below.

**Specific investments and hold up** Investors \( i \in \{1, \ldots, N\} \) are suppliers, the agent is a firm. Value can be created through a partnership between a supplier and the firm. To create the value, the supplier \( i_t \) must make a specific investment (play \( I \)) and the firm may hold up the investment (play \( D \)) or share the returns (Klein, Crawford, and Alchian 1978, Grossman and Hart 1986). The network represents the business relationships among suppliers, and each of the suppliers can communicate to its business partners whether its investment was held up.

**Maghribi traders** Each investor \( i \in \{1, \ldots, N\} \) is a merchant, who may need an agent to handle their overseas operations. A merchant may hire or not the agent (play \( I \) or \( NI \)), and the agent, if hired, may or may not embezzle the merchant’s goods (play \( C \) or \( D \)). The social network represents all the social ties between merchants facilitating information exchange. Greif (1993) studies a coalition between a group of Medieval merchants, the Maghribus, that allowed full and rich information flows among their members. A coalition of merchants can be seen as a complete network, in which all potential merchants can fully exchange information. We introduce a friction in Greif’s (1993) model by embedding his framework into a social network model of imperfect information transmission.

**Experience goods and social networking** Investors are clients, the agent is a firm. The selected investor \( i_t \) may or may not buy the good (play \( I \) or \( NI \)), while the quality of the good produced by the firm can be either high or low (the firm may either cooperate or defect). The client can verify the quality of the good purchased only after he has made a sunk payment. A social network can be seen as modeling all the alternative sources of information on the firm’s past performance, ranging from online feedback systems and media coverage to plain word-of-mouth communication. Indeed, our social network model can accommodate overlapping internet platforms where some users participate in two or more forums. Our model also applies to social networking, where users share their purchase experiences with their contacts, as is the case in China’s Taobao (Guo, Wang, and Leskovec 2011).

### 2.3 Sustainable Networks

We are interested in characterizing social networks that are conducive to efficient transactions. Technically speaking, our repeated game model has private monitoring and, as is well

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9 See, for instance, Dellarocas (2003) and Levin (2011) for additional background
known, characterizing the whole set of equilibria is a nontrivial task in such games (Mailath and Samuelson 2006). We thus restrict attention to a family of simple stationary strategies for the investors and analyze the incentives the agent has to refrain from acting opportunistically. As will become clear soon, given social network $G$, our equilibrium strategies Pareto dominate any other equilibrium strategy if matching is random while when matching is directed, our equilibria maximizes total surplus."''

Formally, we define a profile of trigger strategies $\sigma^G = (\sigma^G_i)_{i=1}^N$. For $i \in N^G$ and a history $h^i_t$, investor $i \in N^G$ plays $NI$ unless $h^i_t$ is the null history or $h^i_t$ shows that the outcome in each of the preceding stage games in which $i$ has been involved was $(I,C)$ and he has not received any information showing the play of $(I,D)$. If $i \notin N^G$, then play $NI$. These strategies are a natural generalization of the well known trigger strategies to our social network setting (Mailath and Samuelson 2006). A player belonging to the social network trusts, unless the agent has misbehaved when facing him or one of his neighbors at some previous round.

Let $BR^G(\sigma)$ be the set of sequential best replies in behavior strategies for the agent when the social network is $G$ and the investors’ strategies are given by $\sigma = (\sigma_i)_{i=1}^N$. Take $\sigma_0 \in BR^G(\sigma^G)$, where $\sigma^G$ is the trigger strategy for investors $i \in \{1, \ldots, N\}$ defined above. Such $\sigma_0$ induces a probability distribution over the sequence of (random) networks $(G^t)_{t \geq 1}$ formed by investors who are willing to invest at the beginning of round $t$ (so that $G^1 = G$). Denote such probability distribution over sequences of networks as $\mathbb{P}[\cdot \mid G, \sigma_0]$.

**Definition 1** We say that the network $G$ is sustainable if there exists $\sigma_0 \in BR^G(\sigma^G)$ such that for all $t \geq 1$

$$\mathbb{P}[G^t = G \mid G, \sigma_0] = 1.$$  

When the condition above holds, we will also say that $G$ is $\sigma_0$-sustainable.

Our definition of sustainable networks imposes two key requirements on the social network $G$ and the agent’s strategy $\sigma_0$. First, given the information flows $G$, it must be sequentially rational for the agent to comply with $\sigma_0$. On the other hand, on the equilibrium path, $\sigma_0$ must leave the initial network $G$ unchanged. In other words, the outcome of the model is stationary as all possible cheating opportunities have been exhausted and the network of investors willing to trade is kept unchanged as time passes by. Thus, sustainable networks can be seen as the

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10 Observe that in this efficiency notions the network is fixed. In other words, we are characterizing points in the Pareto frontier of a repeated game in which the monitoring technology is fixed. Later on, we will also discuss optimal network design when forming links is costly.

11 What kind of functions the set of best replies contains depends on the matching technology.
result of a long run process, where the agent behaves in a sequentially rational way and there is no more room for on path unfaithful behavior.

Sustainability is a relatively mild requirement and stronger restrictions on strategies could seem appropriate. For example, instead of requiring $\sigma_0 \in \text{BR}^G(\sigma^G)$, one may want to impose a sequential equilibrium restriction on $(\sigma_0, \sigma^G)$ (Kreps and Wilson 1982). Our results obviously apply when more demanding requirements are imposed, but Lemma 1 in the Appendix shows the sequential equilibrium requirement turns out to be immaterial: if $\sigma_0$ and $\sigma^G$ are as in the definition above, then they form a sequential equilibrium.

We will also study efficient networks, defined as networks that maximize the sum of players’ payoffs when forming links is costly. Given a $\sigma_0$-sustainable network $G$, we denote the expected total payoff accruing to player $i \in \{0, \ldots, N\}$ by $u_i(G, \sigma_0)$. We assume that links are costly and that player $i \in \{1, \ldots, N\}$ must incur a cost of $c_i(|N^G(i)|)$, with $c_i : \mathbb{N} \to \mathbb{R}_+$ strictly increasing and $c_i(0) = 0$, to maintain his $|N^G(i)|$ connections.

**Definition 2** A $\sigma_0$-sustainable network $G$ is efficient if

$$
(G, \sigma_0) \in \arg \max \left\{ \sum_{i=0}^{N} u_i(G', \sigma'_0) - \sum_{i=1}^{N} c_i(|N^{G'}(i)|) \mid G' \text{ is } \sigma'_0\text{-sustainable} \right\}
$$

A network is efficient if it maximizes the sum of players’ payoffs, which consists of the benefits obtained in the repeated game minus the costs of forming the links. See Goyal (2007) and Jackson (2008) for additional discussion.

We also consider sustainable networks for which no link can be removed without impairing its sustainability.

**Definition 3** A sustainable network $G$ is minimally sustainable if for all $ij \in E^G$ the network $G' = (N^G, E^G \setminus \{ij\})$, obtained by deleting link $ij$, is not sustainable.

The idea behind the definition of minimally sustainable networks is that the network should solve the social dilemma of enforcing proper behavior in group $N^G$, but if any link is deleted, then the resulting arrangement is not sustainable. Minimal sustainability can be justified on several grounds. Efficient networks are minimally stable. So are Nash equilibrium networks of

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12 We will assume that the costs are incurred at $t = 0$, right before the play of the repeated game (which transpires across $t \geq 1$). When comparing benefits and costs of forming links, all expressions will be written from the perspective of period $t = 0$. 

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Figure 2: When $\alpha \leq 1$, the complete information network $G^1$ is sustainable regardless of the matching technology. Network $G^2$ is sustainable when (i) matching is directed and $\alpha < 1$ or (ii) matching is random and $\alpha < \frac{2}{3}$. Network $G^3$ cannot be sustainable when matching is directed. When matching is random, $G^3$ is sustainable when $\alpha \in \left[\frac{1}{3}, \frac{2}{3}\right]$ and $\delta$ is low enough.

the network formation game in which investors publicly announce whether they want to be a node in the network, which links they want to form, a link $ij$ is formed if and only if both $i$ and $j$ announce link $ij$, and the payoffs to $i$ are

$$U_i(G) = \begin{cases} u_i(G, \sigma_0(G)) - c_i(|N^G(i)|) & \text{if } G \text{ is sustainable}, \\ -c_i(|N^G(i)|) & \text{if not} \end{cases}$$

where $\sigma_0(G)$ is such that $G$ is $\sigma_0(G)$-sustainable. We thus see minimal sustainability as a mild restriction when links are instrumentally formed either through centralized or decentralized mechanisms.

3 An Example

Before presenting the main results, we illustrate some of the forces in our model by restricting attention to $N = 3$. For concreteness, we assume that $\delta$ and $g$ are such that $\alpha := \frac{1-\delta}{g} \in \left[\frac{1}{3}, \frac{2}{3}\right]$. We will study the sustainability properties of each of the three networks in Figure 2.

Take first the complete information network $G^1$. After an improper action of the agent, all investors become aware of that and act upon in all subsequent rounds. Thus, regardless of the matching technology, the agent’s continuation value after defection is equal to 0. The network will be sustainable if and only if

$$(1 - \delta) + \delta \geq (1 - \delta)(1 + g)$$

which can be equivalently written as $\alpha \leq 1$. In the complete information network $G^1$, whether
the matching process is random or directed is irrelevant to determine the sustainability of the social arrangement.

Consider now network $G^2$, which consists of two nodes – investors 1 and 2 – that are linked. Network $G^2$ is not a complete information network as investor 3 does not become aware of how play evolves when investors 1 and 2 are selected. Suppose first that matching is random. Observe that, by construction of the trigger strategies, investor 3 – who is not part of the network – never invests. On the contrary, investors 1 and 2, who are part of the network, are expected to invest. If the agent cooperates in all encounters with 1 and 2, then his period payoffs conditional on facing either 1 or 2 equals $(1 - \delta) + \delta \frac{2}{3}$. If he deviates when facing either investor 1 or 2, then both of them will refuse to trade in all subsequent rounds and the agent’s normalized total payoff would equal $(1 - \delta)(1 + g)$. This implies that the sustainability of $G^2$ is equivalent to

$$(1 - \delta) + \delta \frac{2}{3} \geq (1 - \delta)(1 + g).$$

This condition holds under our assumption that $\alpha < \frac{2}{3}$ and thus $G^2$ is sustainable when matching is random. But network $G^2$ is also sustainable when matching is directed. Indeed, when matching is directed, it is optimal for the agent to select either investor 1 or investor 2 and to cooperate after they invest. Such a strategy yields a normalized total payoff equal to 1, while the best deviation entails a current payoff of $(1 - \delta)(1 + g)$ followed by a continuation payoff of 0. Such a deviation cannot be optimal as $\alpha \leq 1$ and, therefore, $G^2$ is sustainable when matching is directed. We thus observe that determining the sustainability of networks $G^1$ and $G^2$ amounts to comparing $\alpha$ to a given threshold. The parameter $\alpha$ fully captures the severity of the incentive problem in models in which trading investors are fully connected and will play a key role throughout the paper.

Let us finally explore the sustainability properties of network $G^3$, in which all investors have at least one link but investors 1 and 3 are not connected. Social network $G^3$ is of interest as all investors are willing to invest, yet there are no information flows between investors 1 and 3. Consider first the model with directed matching. Observe that if the network is sustainable, then the normalized total payoff to the agent equals 1. Yet, the strategy of choosing investor 1 and defecting against him combined with the continuation strategy of selecting investor 3 – who does not become aware of the deviation against 1 – and cooperating yields a payoff equal to $(1 - \delta)(1 + g) + \delta > 1$. When matching is directed, network $G^3$ is not sustainable regardless of $\alpha$. This arises because the network allows the agent to deviate against one of the community members bearing no loss in continuation values. This makes such a deviation attractive and
breaks down the sustainability of the social arrangement.

The incentives the agent faces in network $G^3$ are different when matching is random. For $G^3$ to be sustainable, it must be that it is in the agent’s interest to choose the high action in all encounters. Now, if the agent deviates when facing investor 1, then investor 2 becomes informed of that and both agents refuse trade in all subsequent encounters. Investor 3, who is not connected to 1 and cannot distinguish between no trade and no investment by investor 2, does not become aware of the deviation against 1 and is still willing to trade when a subsequent trading opportunity arises. When such trading opportunity with investor 3 arises, however, the agent chooses the low action as $\frac{1-\delta}{3}g > \frac{1}{3}$. Thus, when matching is random, network $G^3$ is sustainable if and only if

$$1 \geq (1 - \delta)(1 + g) + \sum_{t=1}^{\infty} \delta^t \left( \frac{2}{3} \right)^{t-1} \frac{1}{3} (1 - \delta)(1 + g).$$

The incentive constraint ensuring the sustainability of $G^3$ can be written as $\alpha \leq \frac{2-\delta}{3}$. Keeping the severity of the incentive problem $\alpha \in \left[\frac{1}{3}, \frac{2}{3}\right]$ fixed, we observe that there exists $\delta$ such that the social network $G^3$ is sustainable if and only if $\delta \leq \delta^{13}$. To get an intuition why this result holds, note that after a deviation against 1 the agent is loosing all subsequent trading opportunities arising from 1 and 2. However, in contrast to network $G^2$, now the deviation against 1 creates one more deviating opportunity as investor 3 is still willing to invest, since he is not informed about the agent’s deviation against investor 1. If the agent’s utility function puts sufficiently high weight on future payoffs, that second deviation will cause the breakdown of incentives to cooperate when facing investors 1 and 3. This double deviation, first against 1 and then against 3, is unattractive only when $\delta$ is sufficiently low.

In sum, the matching technology plays a key role at determining the architecture of sustainable networks. When any investor can undertake the investment, as is the case when matching is directed, deterring opportunistic behavior is possible only when all network members are linked (or more technically, the set of sustainable networks coincides with the set of complete components). The size of the sustainable network is undetermined, but all investors willing to invest must be connected. Sustainable networks are thus fully clustered. In contrast, when matching is random, deviating against one of the members of the network entails losses in continuation.

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13Observe that network $G^3$ is formed as a tree union of network $G^2$ and the network formed by nodes 2 and 3 and a link between 2 and 3. The fact that $G^3$ need not be sustainable contrasts with Jackson, Rodriguez-Barraquer, and Tan’s (2010) results.

14In other words, the agent becomes contagious. Note that in contrast to previous papers in the community enforcement literature (Kandori 1992, Ellison 1994), in our model whether or not the agent becomes contagious is endogenously determined.
values to the agent. Such losses may or may not be significant enough to deter a deviation. When two network members are not connected, the possibility to double deviate may break down the incentives of the agent to cooperate. In the random matching model, sustainability imposes lower bounds on the minimum number of connections each investor must have as a function of $\alpha$, and when $\delta$ is high, sustainable networks must be complete information networks. In particular, linking an isolated node to a sustainable network may end up destroying self governance because it provides more deviating opportunities.

4 Analysis

We now go back to the general model presented in Section 2. In this section we explore properties of sustainable networks and discuss how the parameters of the game (including the matching technology) shape the architecture of sustainable networks. Section 4.1 studies the directed matching model, while Sections 4.2 and 4.3 are devoted to the study of the random matching model. In Section 4.4, we discuss and summarize our results. All proofs are presented in the Appendix.

Some new notation is needed. For each $i \in N^G$, we denote by $G \setminus \bar{N}^G(i)$ the network consisting of nodes $N^G \setminus \bar{N}^G(i)$ and links $\{jk \in E^G \mid \text{ there is no } l \in \bar{N}^G(i) \text{ with } l = j \text{ or } l = k\}$; that is, $G \setminus \bar{N}^G(i)$ is the network remaining after node $i$ and all his neighbors are deleted and the links between them and other nodes are also deleted. A network $G$ is said to be complete if all its members are linked: $\bar{N}^G(i) = N^G$ for all $i \in N^G$. A component of a network $G$ is a subnetwork $G'$ consisting of nodes $N^{G'} \subseteq N^G$ and links $E^{G'} \subseteq E^G$ such that any two nodes in $N^{G'}$ are connected through a path in $G'$ and if $i \in N^{G'}$ and $ij \in E^G$, then $ij \in E^{G'}$.

Consider a complete information model, in which the social network $G$ consists of all nodes $N^G = \{1, \ldots, N\}$ and all links $E^G = \{ij \mid i, j \in N^G\}$. Regardless of the matching technology, the network is sustainable if and only if $(1 - \delta) + \delta \geq (1 - \delta)(1 + g)$ or, equivalently, $\alpha := \frac{1 - \delta}{\delta} g \leq 1$. This condition is assumed throughout the paper.

Assumption 1 (Nontrivial model) $\alpha \leq 1$.

Parameter $\alpha$ is important as it fully captures the intensity of the incentive problem when the network is complete. Subsequent analysis will show that when this assumption does not hold, the set of sustainable networks is empty. Without loss, instead of using $\delta$ and $g$, we will sometimes use $\alpha$ and $g$ or $\alpha$ and $\delta$ as parameters of the model to state some results.
4.1 Directed Matching Model: Characterization

The following proposition characterizes sustainable networks in the directed matching model.

**Proposition 1** Suppose matching is directed. Then,

\[ G \text{ is sustainable if and only if } G \text{ is a complete network}. \]

To see the logic behind this result, it is important to observe that if the agent does not play \( C \) when facing \( i \in N^G \), then only \( i \)'s neighbors become informed of that. If some members of \( N^G \) are not connected to \( i \), then those investors remain willing to invest. Because matching is directed, the defection against \( i \) entails no loss in continuation values. Thus, if \( G \) is not complete, an attractive deviation opportunity arises and that makes \( G \) unstable.

**Corollary 1** Suppose matching is directed. Then, the following hold:

i. A network \( G \) is efficient if and only if \( |N^G| = 1 \);

ii. A network \( G \) is minimally sustainable if and only if \( G \) is sustainable.

The first part of the corollary shows that in a directed matching model, efficiency nails down the set of sustainable networks to one node networks. In other words, the efficient network is a bilateral relationship, where the agent and a single isolated investor interact throughout the whole game. This is driven by the fact that, since all investors are able to undertake the investment at any time, two or more nodes produce the same total expected payoffs in the repeated game as a single node, yet a network with two or more nodes must be a complete network and therefore it costs strictly more than a one-node network. The second part of the corollary shows that minimal sustainability does not narrow down the set of sustainable networks. This is due to the fact that if a network is sustainable and a link is removed, then the resulting network will not be complete and thus will not be sustainable.

4.2 Random Matching Model: Degree Properties

Let us start the analysis of the random matching model by considering a complete network formed by \( \kappa \geq 1 \) investors. It will be in the agent’s interest to cooperate when matched to one of the \( \kappa \) investors if and only if \( (1 - \delta) + \delta \frac{\kappa}{N} \geq (1 - \delta)(1 + g) \). This condition can be
equivalently written as $\kappa \geq \kappa^*$, where $\kappa^* = \lceil \alpha N \rceil$. Under Assumption $\square \; \kappa^* \leq N$ and thus the set of sustainable networks contains at least all the complete networks of $\kappa^*$ or more nodes.

Before presenting the main substantive results, we observe that the agents’ incentive problem can be formulated as a dynamic programming problem in which the state variable is the identity of the current trading partner $i_t$ and the network of investors who are still willing to trade. We denote the normalized expected sum of discounted per-period payoffs to the agent when facing $i$ and given the network $G$ by $v(i, G)$, and explore its main properties in the Appendix. This formulation allows us to manipulate the agent’s incentive constraints to derive necessary and sufficient conditions for sustainability.

The following result characterizes sustainability in terms of the sizes of the neighborhoods of each of the network members.

**Proposition 2** Suppose matching is random. Then, the following hold:

i. If $G$ is sustainable, then $|\bar{N}^G(i)| \geq \kappa^*$ for all $i \in N^G$;

ii. If $|\bar{N}^G(i)| \geq \frac{g}{1+g} |N^G| + \frac{1}{1+g} \alpha N$ for all $i \in N^G$, then $G$ is sustainable.

The first part of the proposition establishes that in any sustainable network $G$, each player has a number of links at least equal to the threshold $\kappa^*$. If this necessary condition fails, the after-deviation reduction in continuation values due to the loss of investors willing to invest is small and therefore, the agent prefers to defect. As shown in Section $\Box$, the converse need not hold.

In the example, $\kappa^* = \lceil \alpha N \rceil = 2$, but in network $G^3$ all of the nodes have at least one neighbor, yet $G^3$ is not sustainable when $\delta$ is high enough. The second part of the proposition presents a partial converse by showing that sufficiently dense networks are always sustainable. This is driven by the fact that in dense networks a deviation entails losses in trading opportunities that are large compared to what can be gained by exploiting subsequent deviating opportunities.

A simple, but important corollary follows.

**Corollary 2** Suppose matching is random. Let $G$ be such that $g(N^G - \kappa^*) \leq \kappa^* - \alpha N$. Then, the following hold:

i. $G$ is sustainable if and only if $|\bar{N}^G(i)| \geq \kappa^*$ for all $i \in N^G$.

ii. $G$ is minimally sustainable if and only if $\min_{k \in \bar{N}^G(i)} |\bar{N}^G(k)| = \kappa^*$ for all $i \in N^G$.
This result completely characterizes sustainable and minimally sustainable networks when, keeping the intensity of the incentive problem $\alpha$ fixed, the agent is impatient and the defection gains are low ($\delta$ and $g$ are low). The result shows that sustainability only imposes restrictions on the closed degrees of the network members when $g$ and $\delta$ are low enough. As illustrated in Section 3, this is so because when there are two or more network members who are not connected, and thus the network allows two or more deviations, the potential gains of future deviations are low when the agent is impatient and $g$ is low. Under the conditions of the proposition, establishing the sustainability of a network reduces to counting the number of connections each member has.

When the agent is impatient, any link between two nodes, each of them having $\kappa^*$ or more connections, can be removed without impairing sustainability. Thus, as established by the second part of the corollary, minimally sustainable networks can be easily found by checking that all neighborhoods have a member attaining the lower bound $\kappa^*$ on the closed degree. Under the conditions of the corollary and the additional assumption that $c_i(\kappa^* - 1) \leq \frac{\delta}{1 - \frac{1}{N}}$ for all $i$, the set of minimally sustainable networks coincides with the set of Nash equilibrium networks of the network formation game described in Section 2.3. Note that, in contrast to the directed matching model, there are sustainable networks that are not minimally sustainable.

4.3 Random Matching Model: Cohesiveness Properties and Efficient Networks

The results in the previous subsection show that when the agent is impatient, sustainability is purely a restriction on the number of connections each network member has. We now explore situations in which the agent is patient and show conditions under which sustainable networks have cohesiveness properties, to be defined later on.

Let $N^G_2(i)$ be the set of nodes within distance 2 of investor $i \in N^G$; that is

$$N^G_2(i) = \left\{ j \in N^G \mid \text{there exists } k \in N^G \setminus \{i, j\} \text{ with } ik, kj \in E^G \right\}.$$  

For instance, in Figure 2, $N^G_2(1) = \emptyset$ but $N^G_2(2) = \{3\}$.

The following result provides an upper bound on the number of common connections between an investor $i$ and any subset $Q$ of investors indirectly connected through a path of length 2 to $i$.

\footnote{Observe that when $\kappa^* = \alpha N$, the result is uninformative. The next Subsection provides results that apply when $\kappa^* = \alpha N$.}
Proposition 3  Suppose matching is random. Let $G$ be sustainable and $i \in N^G$. Then, for any subset $Q \subseteq N^G_2(i)$,

$$
\sum_{j \in Q} |\{k \in N^G \mid ik, kj \in E^G\}| \leq \left(\frac{N - \delta N}{\delta} + |Q|\right)(|\bar{N}^G(i)| - \alpha N) + \sum_{j \in Q} (|\bar{N}^G(j)| - \alpha N).
$$

This proposition shows that the number of connections in common that two unconnected members may have is bounded above. To grasp the intuition behind this result, note the following three important facts: (i) after the agent has defected when facing $i \in N^G$, players in $Q \subseteq N^G_2(i)$ are still willing to invest since they are not informed of the agent’s deviation; (ii) defecting when facing investors in $Q$ is more attractive the lower the number of neighbor investors $j \in Q$ have in the resulting after-deviation network $G \setminus \bar{N}^G(i)$; and (iii) if $G$ is sustainable, the continuation value after defecting against $i$ is low enough. The second observation implies that, after defecting against $i$, the value of continuation play $\frac{1}{N} \sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$ is bounded below by a nondecreasing function of the total number of neighbors in $G \setminus \bar{N}^G(i)$ of nodes $j \in Q$, and the third observation implies an upper bound for $\frac{1}{N} \sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$ as a function of the parameters of the model. It then follows that the number of common paths between investor $i$ and any subset $Q \subseteq N^G_2(i)$ cannot be too large, otherwise the agent can exploit the lack of information of players in $Q$ to double deviate, first against $i$, then against players in $Q$.

Proposition 3 has some implications that illustrate how the double deviation logic may impose cohesiveness restrictions on sustainable networks. Recall that for each network $G$ and $i \in N^G$, the individual clustering coefficient for node $i$ is defined as

$$
Cl^G(i) = \frac{2|\{jk \in E^G \mid j \neq k, j, k \in N^G(i)\}|}{|N^G(i)|(|N^G(i)| - 1)}.
$$

The clustering coefficient for node $i$ measures the number of links between $i$’s neighbors as a fraction of the maximum number of potential links. It gives a measure of how cohesive network $G$ is around $i$. While there are many measures of cohesiveness, the local clustering coefficient is easy to interpret and has been used in several studies (Goyal 2007, Jackson 2008).

Corollary 3  Suppose matching is random. Let $G$ be a sustainable network such that for some $\beta \in ]0, 1[$, $|\bar{N}^G(i)| = \beta N$, for all $i \in N^G$. Then,

$$
Cl^G(i) \geq 1 - \frac{\alpha/\gamma + 2(1 - \beta)}{(\beta - \frac{1}{N})(\beta - \frac{1}{N})}(\beta - \alpha),
$$

for all $i \in N^G$. 

20
An investor $i \in N^G$ in a sustainable network, in which each investor has the same number of neighbors, has a bounded number of connections within distance 2 and thus, most of $i$’s neighbors must have connections within $i$’s neighborhoods. This implies a lower bound on the number of links between members of $N^G(i)$ and thus a lower bound on the clustering coefficient. When the nodes have a degree close to $\alpha N$, it is harder to deter the previously described double deviation and, as a result, the bound on the clustering coefficient is close to 1. On the other hand, the lower bound on the clustering coefficient becomes less tight as the network becomes more dense and $g$ falls. Indeed, as we have already seen in Corollary 3, any sufficiently dense network is sustainable and thus, in general, a sustainable network needs not be clustered.

We finally investigate the architecture of efficient networks. To simplify the exposition, we assume that the costs of forming links are symmetric across players and equal to $c: \mathbb{N} \to \mathbb{R}_+$ (which, as discussed in Section 2, is assumed strictly increasing with $c(0) = 0$).

**Corollary 4** Suppose matching is random. Then, the following hold:

i. If $c(\kappa^* - 1) > \frac{2}{N} \frac{\delta}{1-\delta}$, then the only efficient network is the empty network.

ii. Suppose that

a. If $c(\kappa^* - 1) < \frac{2}{N} \frac{\delta}{1-\delta}$;

b. $\frac{N}{\kappa^*} \in \mathbb{N}$;

c. $(\kappa^* - \alpha N) \left( \frac{N}{\kappa^*} - \frac{\delta}{\delta} + 2 \right) < 1$.

Then, a network $G$ is efficient if and only if it consists of $\frac{N}{\kappa^*}$ complete components, each of them having $\kappa^*$ nodes.

This corollary provides conditions under which the architecture of efficient networks can be fully characterized. When the gains from cooperation are sufficiently low compared to the costs of maintaining links, any nonempty network is dominated by the empty one. On the other hand, when the costs of forming links are sufficiently low, the optimal network consists of separate complete components of size $\kappa^*$, provided such network is feasible and, more crucially, the agent is sufficiently patient. The result follows since efficiency pushes the number of links to the threshold $\kappa^*$, organizing several separate complete components of $\kappa^*$ members is feasible, and any other organization in which all the closed degrees equal $\kappa^*$ must be given by separate complete components, as otherwise the resulting network cannot be sustainable when the agent is patient.
Finally, Condition c in the corollary above may seem hard to satisfy in applications. In the Appendix, we enrich the model to allow for a variable project size (Ali and Miller 2010, Wolitzky 2011). In that model, when the investment size is chosen at the beginning of the game once and for all, Condition c is obtained for free as it is implied by Condition b. In other words, Corollary 3 immediately applies when a stronger efficiency requirement is imposed. With endogenous investments efficient networks trade off the following effects: while increasing the size of the project is socially desirable, enforcing cooperation requires larger (more costly) complete components. See Appendix A.2 for details.

4.4 Discussion and Summary

Our results provide necessary and sufficient conditions for sustainable networks. One of the properties sustainable networks have is their cohesiveness. When matching is directed, the cohesiveness of sustainable networks is particularly stark: sustainable networks are complete networks in which all network members are linked. In contrast, when matching is random the cohesiveness of sustainable networks depends on: (i) the network density; and (ii) holding the intensity of the incentive problem $\alpha$ fixed, on players’ patience. Results in Sections 4.1 and 4.3 identify conditions under which some level of common knowledge of part of the history of the game is essential for attaining self governance.

However, in the random matching model, self governance can be attained even in barely cohesive communities. Indeed, as Proposition 2 shows, any sufficiently dense network is sustainable and, when players are impatient, the sustainability of a social arrangement is purely a matter of degree. Since a sufficiently dense network is always cohesive, a consequence of Corollary 3 is that, for regular networks, the local clustering coefficient of sustainable networks, as a function of the degree, is bounded below by a U-shaped function when matching is random and players are patient.

Efficient networks exhibit sharper clustering properties. In the directed matching model, Corollary 1 shows that the set of efficient networks reduces to bilateral relationships in which the agent repeatedly trades with a single investor. When matching is random, Corollary 4 shows conditions under which efficient networks tend to be cohesive (e.g. several separate complete components). These results identify game theoretical forces that favor the formation of clustered relationships. In particular, a bilateral relationship between a firm and a supplier economizes on links and information transmission. When the gains from expanding the supplier base are low (as is the case of a firm whose production requires little update of inputs) a one-to-one relationship
between a firm and a supplier is uniquely optimal. As a result, bilateral relationships maximize the total surplus and are likely to be more stable and last longer, as empirically confirmed in the apparel industry by Uzzi (1996). When expanding the base of trading investors is beneficial, efficiency favors the formation of complete components in which several trading members fully exchange information about the history of transactions. This organization resembles a number of business associations, such as Japan’s keiretsus and Korea’s chaeobl (McMillan 1995).

We have identified conditions under which cohesiveness is a crucial ingredient to attain efficient economic transactions. However, we have also found limits to the natural presumption that cohesiveness is necessary for self governance. Case studies show how cohesive communities trade by means of community based sanctions (Milgrom, North, and Weingast 1990, Greif 1993, Bernstein 1992), yet evidence from social networking in internet platforms suggests there is a fair amount of trade even in the absence of perfect information dissemination. Our results provide a unified framework in which the cohesiveness of the social network is determined by the particulars of the transactions.

Our results have testable implications about the architecture of sustainable networks. In particular,

i. When matching is directed, sustainable networks must be complete components;

ii. When matching is random, a sustainable network must be sufficiently dense;

iii. When matching is random, the agent is impatient and defection gains are low (δ and g are low), networks are sustainable as long as each agent has a sufficiently high degree (i.e., $|N^G(i)| \geq \kappa^*$);

iv. When matching is random, either the agent is patient or defection gains are low, networks of intermediate density are sustainable as long as each agent’s clustering coefficient exceeds a lower bound

5 Application: Markets and Networked Relationships

The implicit mechanism of misconduct deterrence studied in this paper is one among many alternatives to organize exchange. In this section, we enrich the model to study how the existence of anonymous markets may affect self governance.

There are several ways in which formal and informal arrangements may interact, and recent literature has explored a few (Baker, Gibbons, and Murphy 1994, Attanasio and Rios-Rull 2000,
Dixit 2003). In particular, Kranton (1996) studies the tension between effective search and reciprocity in a model in which markets and cooperative relationships coexist. Spagnolo (2005) shows how cooperation through long run relationships can be hurt by an improvement in the functioning of financial markets. Fafchamps (2002) examines how relational contracting may foster the spontaneous emergence of markets. We add to this list the idea that the existence of an anonymous market through which standardized goods can be traded could hinder the detection of opportunistic behavior in relationship based exchange. In a nutshell, a seemingly innocuous market transaction may actually hide an unfaithful transaction in which one of the parties did not play according to the implicit agreement.

To be more concrete, consider the experience good interpretation of our model, discussed in Section 2.2. Now we assume that players who are not selected in a round can buy a standardized version of the good through an anonymous market. Such transaction yields period payoffs equal to 0. The selected player \( i_t \) may choose \( NI \) and buy the standardized good through the market, or may choose \( I \) and engage in a partnership with the agent, paying upfront a sufficiently high amount of money with the expectation of receiving an upgraded version of the experience good (in which case the period payoff to \( i_t \) equals 1). As in the model discussed in the previous section, the agent may choose to deliver not only a high- or low-quality good, which is observed by all of \( i_t \)'s neighbors, but also the standardized (market quality good) version of the good. When neighbors observe that the standardized quality of the good was delivered to an investor \( i \in \{1, \ldots, N\} \), they do not observe the channel through which the good was acquired (partnership vs. anonymous market). This induces a new and potentially attractive deviation opportunity to the agent. Mainly, this allows the agent to engage in “window-dressing” defined as the agent’s ability to undertake actions that are detrimental for the current partner and beneficial for himself without loss in continuation payoffs.

More formally, we assume that the investor selected at round \( t, i_t \), and the agent play a modified trust game:

\[
\begin{array}{c|cc|c}
\text{agent} & I & NI \\
\hline
C & M & D \\
\hline
\hline
0,0 & 1,1 & -\tilde{l}, 1 + \tilde{g} & -l, 1 + g
\end{array}
\]

Figure 3: A trust game in which the agent may play \( C, D, \) or \( M \).

We assume that \( \tilde{l} > 0 \) and \( \tilde{g} > 0 \) so that the only equilibrium of the stage game yields, once
again, the inefficient payoff vector \((0, 0)\).

Information flows are as follows. When the selected investor \(i_t\) plays \(I\), then \(i_t\)'s neighbors observe whether the agent chooses \(C\) or \(D\). However, if the agent chooses \(M\) and produces the standardized quality good, then \(i_t\)'s neighbors cannot determine whether that standardized version was produced by the agent or bought by \(i_t\) through the anonymous market. The new information problem that this model brings to the environment is that an investor \(i\)'s neighbors, by observing a market like experience, cannot determine whether the transaction was a market transaction or was the result of a mischievous action in which the investor \(i\) made the investment but only gets back a market quality good.

We extend trigger strategies by assuming that a player belonging to the social network \(G\) will invest, unless the investor observed that the agent behaved unfaithfully when facing him or one of his neighbors. We will say that a social network \(G\) is sustainable with partially observable deviations if there is a sequential best reply \(\sigma_0\) for the agent to the trigger strategies used by investors such that the agent plays \(C\) in all encounters occurring on the path of play. This definition is analogous to Definition 1, with the added twist that in this model there are more deviations as the agent may choose to deviate playing either \(M\) or \(D\).

Let us define \(\tilde{\kappa} := [\tilde{\alpha}N]\), where \(\tilde{\alpha} := \frac{1-\tilde{\delta}}{\tilde{\delta}}\tilde{\gamma} < 1\).

**Proposition 4** The following hold:

i. Suppose that matching is directed and let \(G\) be a nonempty network. Then,

\(G\) is sustainable with partially observable deviations if and only if \(|N^G| = 1\).

ii. Suppose that matching is random and \(\tilde{\kappa} > 1\). Then,

\(G\) is sustainable with partially observable deviations if and only if \(N^G = \emptyset\).

The first part of the proposition shows that when matching is directed, a network is conducive to efficient trade if and only if the network consists of a single node. Any network consisting of two or more nodes opens up the possibility for the agent to deviate playing \(M\) against one of them, without incurring in value losses in the continuation game. The only way in which such deviation can be deterred is by having a network consisting of a single trading investor. This is in contrast with the model without partially observable deviations. Recall that in that case
sustainability requires a complete components network architecture, while here only a long-term partnership makes trading in good terms sustainable. In addition, under partially observable deviations, the sustainable network is efficient. Thus, when the agent can pick his partner in each period, the possibility of *window dressing* makes socially embedded relationships go away. When matching is random and $\tilde{\kappa}$ is above one, it is always optimal for the agent to deviate by playing $M$ and, as a result, the only sustainable network is the empty one. Regardless of the network architecture, deviating playing $M$ entails losing the trading opportunities with a single agent (the cheated agent) and such deviation cannot be deterred if the severity of the incentive problem associated to that deviation, $\tilde{\kappa}$, is large enough.

There is some evidence suggesting the emergence of markets may be harmful for relationships. Bertrand (2004) documents the weakening of long term relationships in US firms that have been hit by tough import competition. Clay (1997) argues that during the 1830s, a coalition of merchants in Mexican California fostered trade expansion, but the subsequent annexation of California to the US –and the change in the legal order– caused the end of the coalition. Ensminger (1992) describes the century-long process through which changes in the environment finally triggered the Orma tribe in Kenya to move from a rule by a council of elders to the recognition of the authority of the modern Kenyan nation state. All of these works suggest that not only the architecture of the social network plays a crucial role at determining the feasibility of self governance, but also market conditions matter. That the existence of markets may deteriorate the quality of monitoring in networked relationships seems plausible, but clearly this mechanism is one of many tentative explanations for the decline of relationships as markets expand.

6 Concluding Remarks

This paper studies the problem of self governance in a model in which information flows are governed by the community structure. Our results show that the way in which trading opportunities arise is a crucial determinant of the architecture of social networks conducive to efficient trade. How easily it is to substitute potential trading partners has a nontrivial effect on the architecture of social networks conducive to efficient trade. In particular, when matching is directed and investors are easily substitutable, efficient trade is attainable only in complete networks. In contrast, when matching is random and investors cannot be substituted to produce surplus, there are conditions under which self governance can emerge even in barely cohesive communities. Efficient networks maximize the total surplus created and we have identified con-
ditions under which their architecture reduces to one or more complete components. Our model yields testable implications relating the fundamentals of the game to the architecture of social networks and can be accommodated to explore the interaction between formal and informal institutions.

Several variations seem worth exploring. First, whether our framework could be extended to study two-sided incentive problems (Kandori 1992) seems an interesting question. Second, the social network $G$ need not be known to the agent. For example, in transactions of experience goods, a firm is unlikely to know the social ties a client has. Third, one could study alternative matching technologies in which investors could be selected randomly according to a Markov process of recognition. It seems important to understand how this process (which can be seen as a network of emerging trading opportunities) restricts sustainable networks. In practice, it is likely to be the case that the network of information transmission is related to the network of trading opportunities induced by the Markov chain. Fourth, investors could be asymmetrically informed about the type of the agent, and they could learn the type by observing neighbors’ play. We leave all these questions for future work.

\footnote{Fainmesser (2010) studies a repeated game model in which the social network is not common knowledge.}
APPENDIX

This Appendix consists of four sections. Section A presents some additional results. Section B presents the dynamic programming formulation of the random matching model. Section C presents omitted proofs. Section D presents some examples.

A Additional Results

This Section provides some additional results. Section A.1 shows that the definition of sustainability is consistent with sequential equilibrium. Section A.2 extends the model to allow for endogenous investments. Section A.3 presents a simple but new folk theorem.

A.1 Sustainable Networks and Sequential Equilibrium

The following result connects sustainable networks to sequential equilibria.

Lemma 1 Let $G$ be a $\sigma_0$-sustainable network. Then, $(\sigma^0, \sigma^G)$ is a sequential equilibrium of the game.

Proof. Suppose first that matching is random, fix $\sigma_0$ and $\sigma_G$ and take any consistent system of beliefs. When $i \in N^G$ knows the agent has played $D$ when facing him or one of his neighbors, it is common knowledge between the agent and $i$ that the agent will play $D$ against $i$ in all subsequent rounds. Thus, it is in player $i$’s interest to play $NI$. It follows that the prescribed strategies are sequentially rational. When matching is directed, the result follows noting that $G$ is a complete network. Therefore, if an investor is selected off-path, the investor knows the continuation strategy of the agent and, given that continuation strategy it is optimal to play as mandated by the trigger strategy. ■

It is perhaps of interest to contrast the above construction when matching is random to Kandori’s (1992). In our random matching model, after player $i_t$ has observed some off path behavior, regardless of his belief about when the deviation occurred, it is optimal for him to play $NI$ in all subsequent rounds. In Kandori’s (1992) model, after an off-path observation, a player’s optimal continuation strategy will, in general, depend on his belief about when the off-path phase was triggered. That makes Kandori’s (1992) construction much more involved.
A.2 A Model with Endogenous Investments

We extend the model studied in the text and assume that the size of the investment, denoted $S$, is set at the beginning of the game once and for all. The trust game is now presented below:

<table>
<thead>
<tr>
<th></th>
<th>$I$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$D$</td>
<td>0,0</td>
</tr>
<tr>
<td>$S,S$</td>
<td>$-I + g(S)$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: A trust game with investment $S$.

Observe that in order to characterize sustainability, given a size $S$, we can always normalize payoffs and apply the results in the main text. Given $S$, the intensity parameter $\alpha(S) = \frac{1-\delta}{\delta} g(S)$ and the threshold $\kappa^*(S) = \lceil \alpha(S) N \rceil$ are now functions of $S$. We will study efficient networks allowing the size $S \geq 0$ to be centrally determined at time 0, when matching is random.

**Proposition 5** Suppose matching is random. Assume that $\frac{g(S)}{S}$ is increasing. Let $S^* > 0$ be an efficient project size satisfying condition b in Corollary 4. Then, condition c in Corollary 4 holds.

**Proof.** Observe that the value of the efficient design must equal the total social payoffs obtained by forming $\frac{N}{\kappa^*(S^*)}$ components, each of them consisting of $\kappa^*(S^*)$ nodes. If condition c does not hold, the total social value can be increased by raising the project size to $\tilde{S} > S^*$, keeping the threshold $\kappa^*(\tilde{S}) = \kappa^*(S^*)$ fixed, and forming $\frac{N}{\kappa^*(S^*)}$ complete components. But this contradicts the efficiency of the original arrangement. Therefore, $S^*$ must satisfy condition c. ■

The assumption that $\frac{g(S)}{S}$ is increasing implies that the larger the project $S$, the higher the temptation to renege. This condition is a force that makes increasing the project size more costly in terms of incentives. As a result, the only way to achieve an efficient design is by economizing on links and forming several complete components. A similar monotonicity condition is also imposed by Ali and Miller (2010), but in that model determining the project size helps to establish existence of contagious equilibria (Kandori 1992).

Finally, in a model with endogenous investments, the larger the project size the more connections each player must have. The proposition above assumes the existence of a solution $S^*$, but

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17We have assumed that $I$ does not depend on $S$ just to simplify the exposition.
such solution need not exist. A condition ensuring the existence of solution is \( \lim_{S \to \infty} \alpha(S) > 1 \), meaning that if the project is too large the complete network cannot is not sustainable.

A.3 A Folk Theorem

In contrast to most work in repeated games (e.g., Fudenberg and Maskin 1986, Fudenberg, Levine, and Maskin 1994), we do not restrict attention to \( \delta \) arbitrarily close to 1. On the contrary, we fix players’ preferences and ask what properties sustainable networks exhibit. Just for the record, though, we offer a folk-theorem-like result.

**Proposition 6** Suppose matching is random. Fix \( g > 0 \), \( N \in \mathbb{N} \) and a social network \( G \). Then, there exists \( \bar{\delta} < 1 \) such that for all \( \delta > \bar{\delta} \), \( G \) is a sustainable network.

**Proof.** Let \( \bar{\delta} = \frac{g}{g + 1} \). Fix \( \delta > \bar{\delta} \) and note that \( (1 - \delta) + \frac{\delta}{N} \geq (1 - \delta)(1 + g) \). This means that \( \kappa^* = 1 \) and thus any network with an empty set of links is sustainable. For each \( n \geq 0 \), let \( \mathcal{G}^n = \{G = (N^G, E^G) \mid \vert N^G \vert \leq n\} \). We have already argued that any \( G \in \mathcal{G}^0 \) is sustainable. Assume that all networks in \( \mathcal{G}^{n-1} \) are sustainable. Let \( G \in \mathcal{G}^n \). Then, for all \( i \in N^G \), \( \tilde{N}^G(i) \geq 1 = \kappa^* \). Moreover, if \( i \in N^G \), then \( G \setminus \tilde{N}^G(i) \in \mathcal{G}^{n-1} \) is sustainable. Proposition 7 implies that \( G \) is sustainable. 

Our game model is a private monitoring game –a player \( i \in \{1, \ldots, N\} \) cannot observe all players’ actions and how continuation play evolves is not common knowledge–. Though our game is extremely simple, existent general folk theorems (Mailath and Samuelson 2006) do not apply.

B Dynamic Programming Formulation of the RM Model

In this Section, we study the agent’s optimization problem in the random matching model. Suppose the agent is facing \( i \in \{1, \ldots, N\} \) at the first round of play \( t = 1 \). If \( i \notin N^G \), then \( i \) plays \( NI \). This implies that regardless of player 0’s action no information is transmitted and thus it is in player 0’s interest to choose \( C \). If \( i \in N^G \), player \( i \) chooses \( I \). If the agent chooses \( D \), then his period payoff will be \( 1 + g \). In the next round \( t = 2 \), player \( i \)’s neighbors get informed and, as mandated by the strategy profile \( (\sigma_j^G)_{j \in \tilde{N}^G(i)} \), they do not to trade. Thus, none of the links involving nodes in \( \tilde{N}^G(i) \) transmits any information. Thus, at round \( t = 2 \), the agent is effectively facing a network in which all nodes in \( \tilde{N}^G(i) \) and their links have been removed.
other words, the problem the producer will face is similar to the one faced at \( t = 1 \), but with the smaller network \( G \setminus \tilde{N}^G(i) \) replacing \( G \). When facing investor \( i \) at \( t = 1 \), the agent can also choose the high action, get a period payoff of 1, and keep the network \( G \) unchanged.

We can then formulate the decision problem faced by agent as a dynamic programming problem. Denoting by \( v(i, G') \) the expected discounted sum of normalized payoffs when the agent faces investor \( i \), given a network \( G' \), it follows that

\[
v(i, G') = \begin{cases} 
(1 - \delta) + \frac{\delta}{N} \sum_{j=1}^{N} v(j, G'), (1 - \delta)(1 + g) + \frac{\delta}{N} \sum_{j=1}^{N} v(j, G' \setminus \tilde{N}^G(i)) & \text{if } i \in N^G' \\
\frac{\delta}{N} \sum_{j=1}^{N} v(j, G') & \text{if not.}
\end{cases}
\]

Equation (B1) is a Bellman equation and standard arguments show the existence and uniqueness of the value function \((v(i, G'))_{i, G'}\) (Stokey and Lucas 1989).

It will be in general hard to find closed form solutions to continuation values. Yet, the following lemma provides bounds that will be useful in the sequel.

**Lemma 2** The following statements hold:

i. For all \( G \), \( \sum_{j=1}^{N} v(j, G) \geq |N^G| \). The equality holds provided \( G \) is sustainable.

ii. For all \( G \), \( \sum_{j=1}^{N} v(j, G) \leq (1 + g)|N^G| \).

**Proof.** Let us prove the first part of the lemma. By definition, for all \( i \in N^G \),

\[
v(i, G) \geq (1 - \delta) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G)
\]

while for \( i \notin G \), \( v(i, G) = \frac{\delta}{N} \sum_{k=1}^{N} v(k, G) \). Adding up and solving for \( \sum_{k=1}^{N} v(k, G) \), it follows that \( \sum_{k=1}^{N} v(k, G) \geq |N^G| \). Note also that if \( G \) is sustainable then (B2) holds with equality, this completes the first part of the lemma.

To prove the second part, note that the statement holds when \( N^G = \emptyset \). Assume that the statement holds for all networks \( G' \) with \( |N^{G'}| \leq n - 1 \) and let us prove the result when \( G \) is such that \( |N^G| = n \). Note first that for all \( j \in N^G \)

\[
v(j, G) \leq (1 - \delta)(1 + g) + \frac{\delta}{N} \max \left\{ (1 + g) \left( |N^G| - |\tilde{N}^G(j)| \right), \sum_{k=1}^{N} v(k, G) \right\}
\]
where we use that $N^G \setminus \bar{N}^G(i) \leq n - 1$. Let $P \subseteq N^G$ be the set of all $j$ such that $(1 + g)\left(|N^G| - |\bar{N}^G(j)|\right) \geq \sum_{k=1}^{N} v(k, G)$. If $P$ is empty, the result follows immediately. If not, we deduce that

$$
\sum_{j=1}^{N} v(j, G) \leq (1 - \delta)(1 + g)|N^G| + \frac{\delta}{N}\left(1 + g\right)\sum_{j \in P} \left(|N^G| - |\bar{N}^G(j)|\right) + (N - |P|)\sum_{k=1}^{N} v(k, G)
$$

and thus

$$
\sum_{j=1}^{N} v(j, G) \leq \frac{(1 - \delta)(1 + g)|N^G|}{1 - \delta\left(\frac{N - |P|}{N}\right)} + \frac{\delta(1 + g)}{1 - \delta\left(\frac{N - |P|}{N}\right)} \sum_{j \in P} \frac{|N^G| - |\bar{N}^G(j)|}{N}
$$

$$
\leq \frac{(1 - \delta)(1 + g)|N^G|}{1 - \delta\left(\frac{N - |P|}{N}\right)} + \frac{\delta(1 + g)}{1 - \delta\left(\frac{N - |P|}{N}\right)} |P| \frac{|N^G|}{N}
$$

$$
\leq (1 + g)|N^G| \left\{ \frac{(1 - \delta) + \delta \frac{|P|}{N}}{1 - \delta\left(\frac{N - |P|}{N}\right)} \right\}
$$

$$
=(1 + g)|N^G|
$$

which proves the result.

This lemma provides upper and lower bounds on the agent’s continuation values for any network $G$. The lower bound states that the agent’s continuation value is at least what he gets if he complies in all possible encounters when facing members of the network $G$, while the upper bound says that the continuation value cannot be greater than what the agent could get if he systematically chooses the low action in all encounters, but he does not lose any trading opportunity.

C Omitted Proofs

This Section provides proofs that have been omitted.

C.1 Proof of Proposition 1

If $G$ is a complete network, then any $\sigma_0 \in BR^G(\sigma^G)$ is such that the outcome of the game has only members of $N^G$ chosen, the chosen investors invest, and the agent cooperates. Thus, $G$ is sustainable.

Take now a nonempty sustainable network $G$ and assume it is not a complete network. Take $\sigma_0 \in BR^G(\sigma^G)$ such that on path the network is kept unchanged. Then, the expected discounted
sum of normalized payoffs for the agent at the beginning of \( t = 1 \) is equal to 1. Since \( G \) is not a complete network, there exist \( i, j \in N^G \) such that \( j \notin \tilde{N}^G(i) \). Consider the strategy \( \tilde{\sigma}_0 \) for the agent: at \( t = 1 \) choose \( i_1 = i \) and defect, in the continuation game starting at \( t = 2 \) play a best reply given the network \( G \setminus \tilde{N}^G(i) \). Note that \( j \in G \setminus \tilde{N}^G(i) \) and thus the normalized continuation value starting at \( t = 2 \) is at least 1. It follows that \( \tilde{\sigma}_0 \) yields a normalized payoff to the agent greater than or equal to \( (1 - \delta)(1 + g) + \delta > 1 \) and thus \( \sigma_0 \) cannot be a sequential best reply. This implies that \( G \) is not sustainable.

### C.2 Proofs of Section 4.2

#### Proof of Proposition 2

**Part (i)** Since \( G \) is sustainable, \( \sum_{j=1}^{N} v(j, G) = |N^G| \). For \( i \in N^G \),

\[
(1 - \delta) + \frac{1}{N} |N^G| \geq (1 - \delta)(1 + g) + \frac{1}{N} \sum_{j=1}^{N} v(j, G \setminus \tilde{N}^G(i)) \geq (1 - \delta)(1 + g) + \delta \left| \frac{|N^G| - |\tilde{N}^G(i)|}{N} \right|,
\]

where the first inequality is by definition of sustainability and the second one follows from Lemma 2. This in turn implies \( (1 - \delta) + \delta \left| \frac{|N^G| - |\tilde{N}^G(i)|}{N} \right| \geq (1 - \delta)(1 + g) \). By definition of \( \kappa^* \), \( |\tilde{N}^G(i)| \geq \kappa^* \).

**Part (ii)** Assume that \( G \) is not sustainable. Then, there exists \( i \in N^G \) such that

\[
(1 - \delta)(1 + g) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i)) > (1 - \delta) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G).
\]

Using both parts of Lemma 2 it follows that \( (1 - \delta)(1 + g) + (1 + g) \frac{\delta}{N} (|N^G| - |\tilde{N}^G(i)|) > (1 - \delta) + \delta \frac{|N^G|}{N} \). The result follows rearranging terms. \( \blacksquare \)

#### Proof of Corollary 2

**Part (i)** If \( \tilde{N}^G(i) \geq \kappa^* \) for all \( i \in N^G \), then

\[
\tilde{N}^G(i) \geq \kappa^* \geq \kappa^* + \frac{g}{1 + g}(|N^G| - \kappa^*) - \frac{1}{1 + g} (\kappa^* - \frac{1 - \delta}{\delta} gN) = \frac{g}{1 + g} |N^G| + \frac{1}{1 + g} \frac{1 - \delta}{\delta} gN
\]

and the sustainability of \( G \) is deduced from Proposition 2. The converse is immediate.

**Part (ii)** If \( G \) is minimally sustainable, then \( i \in N^G \), \( \min_{k \in \tilde{N}^G(i)} |\tilde{N}^G(k)| \geq \kappa^* \) from Part (i). Moreover, Proposition 2 there must be \( k \in \tilde{N}^G(i) \) such that

\[
|\tilde{N}^G(k)| < \kappa^* + 1 + \frac{1}{1 + g} \left( g(|N^G| - \kappa^*) + \frac{1 - \delta}{\delta} gN - \kappa^* \right) \leq \kappa^* + 1
\]

and thus \( |\tilde{N}^G(k)| = \kappa^* \). Conversely, let \( G \) be such that for all \( i \in N^G \), \( \min_{k \in \tilde{N}^G(i)} |\tilde{N}^G(k)| = \kappa^* \).
Then, Part (i) implies that $G$ is sustainable. To see $G$ is minimally sustainable, note that no link can be removed without impairing sustainability as a consequence of Proposition 2.

C.3 Proofs of Section 4.3

Proof of Proposition 3. Partition the set of nodes as $\{1, \ldots, N\} = Q \cup \left[ N^G \setminus (Q \cup \tilde{N}^G(i)) \right] \cup \left[ (\{1, \ldots, N\} \setminus N^G) \cup \tilde{N}^G(i) \right]$. First note that for all $j \in Q$

$$v(j, G \setminus \tilde{N}^G(i)) \geq (1 - \delta)(1 + g) + \frac{\delta}{N} \left( \sum_{k=1}^{N} v(k, [G \setminus \tilde{N}^G(i)] \setminus \tilde{N}^G \setminus N^G(j)) \right)$$

$$\geq (1 - \delta)(1 + g) + \frac{\delta}{N} \left( |N^G| - |\tilde{N}^G(i)| - |\tilde{N}^G \setminus N^G(j)) \right)$$

while for $j \in N^G \setminus (Q \cup \tilde{N}^G(i))$

$$v(j, G \setminus \tilde{N}^G(i)) \geq (1 - \delta) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i))$$

and for $j \in (\{1, \ldots, N\} \setminus N^G) \cup \tilde{N}^G(i)$

$$v(j, G \setminus \tilde{N}^G(i)) = \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i)).$$

It then follows that

$$\sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i)) \geq |Q| \left( (1 - \delta)(1 + g) + \frac{\delta}{N} \left( |N^G| - |\tilde{N}^G(i)| \right) \right) - \frac{\delta}{N} \sum_{j \in Q} \left( |\tilde{N}^G \setminus N^G(i)) \right)$$

$$+ \left( |N^G| - |Q| - |\tilde{N}^G(i)| \right) \left( (1 - \delta) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i)) \right)$$

$$+ (N - |N^G| + |\tilde{N}^G(i)]) \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i))$$

and solving for $\sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i))$

$$\frac{1}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}^G(i)) \geq \frac{1}{N(1 - \delta) + \delta |Q|} \left\{ (1 - \delta)(|N^G| - |\tilde{N}^G(i)|) + (1 - \delta)g|Q| \right.$$

$$+ \frac{\delta}{N} |Q||N^G| - \frac{\delta}{N} \sum_{j \in Q} \left( |\tilde{N}^G \setminus N^G(i)) \right)$$

$$\left. + \frac{\delta}{N} \sum_{j \in Q} \left( |\tilde{N}^G \setminus N^G(i)) \right) \right\}.$$
Since $G$ is sustainable, $(1 - \delta) + \delta \frac{|NG|}{N} \geq (1 - \delta)(1 + g) + \frac{\delta}{N} \sum_{k=1}^{N} v(k, G \setminus NG(i))$. Plugging in the inequality above for $\frac{1}{N} \sum_{k=1}^{N} v(k, G \setminus \tilde{N}G(i))$, we deduce that

$$\frac{N(1 - \delta)}{N(1 - \delta) + \delta|Q|} \left\{ 2g|Q| - |\tilde{N}G(i)| + \frac{1 - \delta}{\delta} gN \right\} \leq \frac{\delta}{N(1 - \delta) + \delta|Q|} \sum_{j \in Q} (|\tilde{N}G \setminus N\tilde{G}(i)(j)| + |\tilde{N}G(i)|).$$

Now, note that for all $j \in Q \subseteq N_G(i)$, $|\tilde{N}G \setminus N\tilde{G}(i)(j)| = |\tilde{N}G(j)| - |\{k \in N_G | ik, kj \in E^G\}|$ and arrange terms to obtain the desired results.

**Proof of Corollary 3** Take a sustainable network $G$ and $i \in N_G$. Proposition 3 implies that

$$\sum_{k \in N^G(i)} |N^G(k) \setminus \tilde{N}G(i)| = \sum_{j \in N_G^G(i)} |\{k \in N_G | ik, kj \in E^G\}|$$

$$\leq \left( \frac{N(1 - \delta)}{\delta} + 2|N_G^G(i)| \right) (K_G(i) - \frac{1 - \delta}{\delta} gN)$$

where $K_G(i) = \max\{|\tilde{N}G(j)| | j \in \tilde{N}G(i) \cup N_G^G(i)\}$. Thus, the number of links between nodes in $N^G(i)$ is at least

$$\frac{1}{2} \left\{ \sum_{k \in N^G(i)} \left( |N^G(k)| - 1 \right) - \sum_{k \in N^G(i)} |N^G(k) \setminus \tilde{N}G(i)| \right\}$$

$$\geq \frac{1}{2} \left\{ \sum_{k \in N^G(i)} \left( |N^G(k)| - 1 \right) - \left( \frac{N(1 - \delta)}{\delta} + 2|N_G^G(i)| \right) (K_G(i) - \frac{1 - \delta}{\delta} gN) \right\}$$

and thus the clustering coefficient of node $i$ can be bounded below by

$$\text{CL}^G(i) \geq \frac{\sum_{k \in N^G(i)} \left( |N^G(k)| - 1 \right) - \left( \frac{N(1 - \delta)}{\delta} + 2|N_G^G(i)| \right) (K_G(i) - \frac{1 - \delta}{\delta} gN)}{|N^G(i)|(|N^G(i)| - 1)}. \quad (C1)$$

Since for all $i \in N_G$, $|\tilde{N}G(i)| = \beta N$, (C1) reduces to the inequality in the corollary.

**Proof of Corollary 4** Part (i) Suppose that a nonempty network $G$ is efficient. Proposition 2 implies that for all $i \in N_G$, $|N^G(i)| \geq \kappa^* - 1$ and thus $\sum_{i=1}^{N} c(|N^G(i)|) \geq |N_G|c(\kappa^* - 1)$. On the other hand, the sum across all players of total expected payoffs in the repeated game equals $\frac{|N_G|}{N} \cdot \frac{2g}{1 - \delta} < |N_G|c(\kappa^* - 1) \leq \sum_{i=1}^{N} c(|N^G(i)|)$ and therefore the empty network, yielding a total expected payoff of 0, strictly dominates $G$.

Part (ii) Consider a sustainable network $G$ such that $|N^G| = m\kappa^* + n$, where $m, n \geq 1$ and $n \leq \kappa^* - 1$. At least one component of network $G$ has $M \geq \kappa^* + 1$ nodes. Condition $c$ in the statement of the result together with Proposition 3 imply there exists a node $i$ in such
component such that $\bar{N}^G(i) \geq \kappa^* + 1$. Since $G$ is sustainable, all remaining nodes must have closed degree at least equal to $\kappa^*$. The objective function defining efficient networks evaluated at $G$ is at most

$$\frac{m\kappa^* + n}{N} \frac{2\delta}{1 - \delta} - c(\kappa^*) - (m\kappa^* + n - 1)c(\kappa^* - 1).$$

Form now a new network $\bar{G}$ with $|\bar{N}^G|$ consisting of $m + 1$ complete components, each having $\kappa^*$ nodes. Such network can always be formed, as a result of Condition b, and is sustainable. The objective function (2.1) at $\bar{G}$ equals

$$\frac{(m + 1)\kappa^*}{N} \frac{2\delta}{1 - \delta} - (m + 1)\kappa^* c(\kappa^* - 1).$$

The difference between the term above and (C2) equals

$$(\kappa^* - n) \left( \frac{2\delta}{1 - \delta} \frac{1}{N} - c(\kappa^* - 1) \right) + \left( c(\kappa^*) - c(\kappa^* - 1) \right),$$

which is strictly positive. It then follows that any efficient network $G$ is such that $\frac{|N^G|}{\kappa^*} \in \mathbb{N}$. For any such network size $|N^G|$, a new application of Proposition 3 and Condition c implies that the least total number of links can only be attained when the $|N^G|$ nodes are arranged in $\frac{|N^G|}{\kappa^*}$ components. Condition a implies that the value of each component is strictly positive and the result follows.

\section{Examples of Sustainable Networks}

This section provides a few examples of sustainable networks. The following property turns out to be useful in some applications.

\textbf{Proposition 7} Suppose that matching is random and let $G$ be such that $\bar{N}^G(i) \geq \kappa^*$ and $G \setminus \bar{N}^G(i)$ is sustainable, for all $i \in N^G$. Then, $G$ is sustainable.

\textbf{Proof.} Since $G \setminus \bar{N}^G(i)$ is sustainable, $\sum_{j=1}^{N} v(j, G \setminus \bar{N}^G(i)) = |G \setminus \bar{N}^G(i)| = |N^G| - |\bar{N}^G(i)|$. Sustainability of $G$ requires for $i \in N^G$ that

$$(1 - \delta) + \frac{\delta}{N} \sum_{j=1}^{N} v(j, G) \geq (1 - \delta)(1 + g) + \frac{\delta}{N} \sum_{j=1}^{N} v(j, G \setminus \bar{N}^G(i))$$

$$= (1 - \delta)(1 + g) + \delta \frac{|N^G| - |\bar{N}^G(i)|}{N}. $$
Because of Lemma 2, the inequality is satisfied if \((1 - \delta) + \frac{\delta}{N}|N^G| \geq (1 - \delta)(1 + g) + \frac{\delta}{N}(|N^G| - |\bar{N}^G(i)|)\) or equivalently, \(|\bar{N}^G(i)| \geq \frac{1 - \delta}{\delta}Ng\). But \(|\bar{N}^G(i)| \geq \kappa^*\), and thus the result follows.

This proposition shows that a network is sustainable provided each investor \(i\) has at least \(\kappa^*\) links and deleting any node \(i\) and all the investors connected to \(i\) results in a smaller sustainable network. Such a network \(G\) can also be seen as robust in the sense that an off-path deviation against any \(i \in N^G\) does not lead the agent to commit additional mischievous actions.

**Stars**

We now derive necessary and sufficient conditions for a star to be sustainable, and explore how our general results compare to these conditions. This exercise can also be seen as a generalization of the study of the incomplete network \(G^3\) in Section 3.

Consider the model studied in Section 2. Let \(G\) be a star consisting of \(k + 1 \geq N\) nodes, with \(k \geq 2\). Recall that \(G\) is a star if there exists a node \(i \in N^G\) such that for every node \(j \in N^G\), \(N^G(j) = \{i\}\). The question we ask is whether network \(G\) is sustainable.

There are simple conditions ruling out the sustainability of the star. When matching is directed, Proposition 1 implies that only complete networks can be sustainable and as result, the star, being an incomplete network, cannot be sustainable. When matching is random, Proposition 2 implies that members of sustainable networks must have closed degrees greater than or equal to \(\kappa^*\). Therefore, when matching is random and \(\kappa^* \geq 3\), the star cannot be sustainable. On the other hand, when matching is random and \(\kappa^* = 1\), the star is sustainable as can be seen from by applying Proposition 7.

The only case left to study is when matching is random and \(\kappa^* = 2\).

**Proposition 8** Suppose that matching is random, \(\kappa^* = 2\), and let \(\bar{k} = 1 + \frac{\kappa^* - \alpha N}{N} \left(N(1 - \delta) + \delta\right)\). Then, \(G\) is sustainable if and only if \(k \leq \bar{k}\).

**Proof.** The social network \(G\) is sustainable if and only if

\[(1 - \delta) + \delta \frac{k + 1}{N} \geq (1 - \delta)(1 + g) + \delta v_{k-1}\]

where \(v_{k-1}\) is the ex ante continuation value accruing to the agent when he is facing a set of \(k - 1\) unconnected nodes. Now, note that \((v_k)_{k=0}^N\) satisfies the following recursion

\[v_k = \frac{k}{N} \left((1 - \delta)(1 + g) + \delta v_{k-1}\right) + \frac{N - k}{N} \delta v_k\]
with \( v_0 = 0 \). The recursion can be written as

\[
v_k = \frac{k}{N(1-\delta)+\delta k} \left( (1-\delta)(1+g) + \delta v_{k-1} \right)
\]

and is solved by

\[
v_k = \frac{k(1-\delta)(1+g)}{N(1-\delta)+\delta}.
\]

Thus the star \( G \) composed of \( k + 1 \) nodes is sustainable if and only if

\[
\frac{1}{N} \left( 2 - \frac{1-\delta}{\delta} gN \right) \geq (k-1) \left( \frac{(1-\delta)(1+g)}{N(1-\delta)+\delta} - \frac{1}{N} \right) \tag{D1}
\]

which can be equivalently written as

\[
k \leq 1 + \frac{\kappa^* - \frac{1-\delta}{\delta} gN}{\frac{1-\delta}{\delta} gN} \left( \frac{N}{N(1-\delta)+\delta} \right)
\]

which proves the result. \( \blacksquare \)

This result shows that the star will be sustainable if and only if its size is sufficiently low. If the network is too large, then there are attractive opportunities to double deviate by first defecting against a peripheral investor and then taking advantage of uninformed investors. This implies that sustainable stars cannot have too many members.

We can also use our general results to derive necessary and sufficient conditions for sustainability. Corollary 2 implies that if \( k \leq 1 + \frac{\kappa^* - \alpha N}{g} \), then the star is sustainable. Proposition 3 allows us to derive the following necessary condition for sustainability when \( \kappa^* - \alpha N < \frac{1}{2} \):

\[
k \leq 1 + \frac{N(1-\delta)}{\kappa^* - \alpha N} \left( \frac{1-\delta}{\delta} gN - 1 \right) \left( N(1-\delta) + \delta \right)
\]

These bounds are not tight, but they restrict the parameters of the game in a meaningful way and have qualitative implications consistent with the sharp characterization of Proposition 8.

**Unions of Complete Components** We now restrict attention to the random matching model. Complete networks consisting of \( \kappa^* \) or more nodes are always sustainable, regardless of the matching technology. The question we ask is whether unions of complete components can result in an sustainable network.

Let \( G^1 \) and \( G^2 \) be two disjoint complete components, with \( N^{G^n} \geq \kappa^* \) for \( n = 1, 2 \). Take the graph \( G = (N^G, V^G) \), with \( N^G = N^{G^1} \cup N^{G^2} \) and \( V^G = V^{G^1} \cup V^{G^2} \cup \{ lm \} \), where \( l \in N^{G^1} \) and \( m \in N^{G^2} \). Network \( G \) is the union of networks \( G^1 \) and \( G^2 \), and a bridge connecting both of them.

**Proposition 9** The following hold:
i. If $|N^{G_n}| \geq \kappa^* + 1$ for $n = 1, 2$, then $G$ is sustainable.

ii. If $|N^{G_n}| = \kappa^*$ for some $n$ and $\kappa^* - \alpha N < \frac{1}{N^\frac{1-\delta}{\delta} + 2}$, then $G$ is not sustainable.

**Proof.** The first statement follows from Proposition 7 by noting that for all $i \in N^G$, $\bar{N}^G(i) \geq \kappa^*$ and $G \setminus \bar{N}^G(i) \geq \kappa^*$ is always stable. The second part follows by applying Proposition 3 to a node in $N^{G_n}$ that is not a bridge. ■

It is also of interest to study whether tree unions of networks can be sustainable. Assume that $G^1$ and $G^2$ have a single node in common and let $G$ be the tree union of $G^1$ and $G^2$ defined as $N^G = N^{G^1} \cup N^{G^2}$ and $E^G = E^{G^1} \cup E^{G^2}$. The three union operation has been shown to result in equilibrium networks (or, using our terminology, sustainable networks) in a repeated game model of complete information (Jackson, Rodriguez-Barraquer, and Tan 2010). As the following Proposition shows, this need not be the case in our model of incomplete information.

**Proposition 10** The following hold:

i. If $|N^{G_n}| \geq \kappa^* + 1$ for $n = 1, 2$, then $G$ is sustainable.

ii. If $|N^{G_n}| = \kappa^*$ for $n = 1, 2$ and $(\kappa^* - \alpha N) < \frac{\kappa^* - 1}{N^\frac{1-\delta}{\delta} + 1}$, then $G$ is not sustainable.

**Proof.** The first part follows from Proposition 7. The second part follows by taking any node which is not in $N^{G^1} \cap N^{G^2}$ and noting that a necessary condition for stability, as implies by Proposition 3 is

$$\kappa^* - 1 \leq (N^\frac{1-\delta}{\delta} + 2)(\kappa^* - \alpha N).$$

The result follows by noting this necessary condition is violated. ■

As already illustrated in Section 3, tree unions of cliques of a minimal size $\kappa^*$ need not be sustainable as, after a deviation against one of the network members, it may still be possible to exploit further deviating opportunities. The same mechanism is at work in the general model, and this contrasts with the model of Jackson, Rodriguez-Barraquer, and Tan (2010).

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