CENTRO DE ECONOMÍA APLICADA

INGENIERÍA INDUSTRIAL · UNIVERSIDAD DE CHILE

DOCUMENTOS DE TRABAJO
Serie Economía

Nº 287

MINIMUM WAGES STRIKE BACK: THE EFFECTS ON CAPITAL AND LABOR DEMANDS IN A LARGE-FIRM FRAMEWORK

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Minimum wages strike back: the effects on capital and labor demands in a large-firm framework *

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May 2, 2012

Abstract

We study the effect of a binding minimum wage on labor market outcomes, the accumulation of capital and welfare. We consider a large firm that invests in physical capital and hires several types of workers. Labor markets are characterized by search and matching frictions, while incomplete wage contracts allow workers to expropriate part of the return on each factor. Absent a minimum wage, the model generates levels of capital and employment that are inefficiently too low or too high. We show that the introduction of a binding minimum wage has ambiguous effects on employment, capital and welfare, and depends on the ability of the minimum wage to deter rent appropriation by workers.

Our model is able to generate situations where i) a minimum wage increases the level of employment through an increase in labor demand, or has a small impact on employment, at the same time as ii) it increases the aggregate stock of capital. The first outcome is in line with the empirical literature and holds even in partial equilibrium. The second result offers an explanation for the higher capital-to-output ratios observed in Continental Europe as compared to the US.

Keywords: Minimum wage; Employment; Capital; Search; Large firm; Hold up.

JEL codes: E24; J63; J68; L20.

*Alexandre Janiak thanks Fondecyt for financial support (project no 11080251). We would like to thank Felipe Balmaceda for useful conversations. All errors are our own.

1 Introduction

Modern labor economists hardly see minimum wages as being responsible for the observed differences in capital and employment among developed economies. Two reasons support this statement. First, although common wisdom suggests that a minimum wage should have a negative effect over employment, there is an extensive empirical literature on this topic that suggests a weak—or even a positive—relation between the two variables. Card and Krueger (1994) is the seminal contribution that lead to a rapid expansion of this literature, which mainly challenges the neoclassical view that increases in the regulatory minimum level necessarily lead to a decrease in employment (Stigler, 1946).[1]

Second, it is sometimes argued that labor market institutions may be a possible explanation for the higher capital-output ratios observed in Continental Europe as compared to the US.[2] Standard explanations rely on capital-labor substitution: an increase in the relative cost of labor forces firms to substitute away from labor and towards capital, as argued in Caballero and Hammour (1998). In this context, a mechanism based on minimum wages can hardly account for an increase in capital if it has to generate a small or positive impact on employment simultaneously.

In this paper, we show that the presence of a binding minimum wage can be consistent with a higher capital stock and a response of employment in line with the empirical literature mentioned before. To do so, we introduce a binding minimum wage into a "large-firm" framework in which labor markets are characterized by search frictions. Our model is able to generate situations where a minimum wage has a very small impact on the employment level of workers earning the minimum or even increases the demand for those workers. Moreover, this negligible (or positive) impact on employment is attained at the same time as the aggregate capital stock is increased.[3] Hence,

[1] Card and Krueger (1994) exploit a difference-in-difference approach to study the impact of an increase in the New Jersey's minimum wage on the employment level of fast-food restaurants by using establishments of the same industry in Pennsylvania as a control group. Their conclusion is that the rise in the minimum wage did not seem to reduce employment. Similar conclusions have been reached in other studies for the United States, such as Katz and Krueger (1992), Card (1992b) and Card (1992a), while Machin and Manning (1994), Machin and Manning (1996), Stewart (2004b) and Stewart (2004a) obtain similar results for the United Kingdom, and Dolado et al. (1996) for France, the Netherlands and Spain. Negative effects over employment may sometimes be found by some authors, especially in the case of young workers (e.g. Linneman (1982), Currie and Fallick (1996), Dolado et al. (1996), Burkhauser et al. (2000) and Portugal and Cardoso (2006)) or vulnerable sectors (e.g. Machin et al. (2003) and Machin and Wilson (2004)).

[2] Evidence on higher capital-output ratios in Continental Europe as compared to the US can be found in Caselli and Feyrer (2007) and Hall and Jones (1999).

[3] The literature has stressed the importance of minimum wages for wage inequality. See for instance Lee (1999), Teulings (2000), Manning (2003) and Autor et al. (2010), among others. In particular, this literature emphasizes the presence of spill-over effects on the wage of workers not directly affected by minimum wages.

minimum wages may serve as a potential explanation for the higher capital-output ratios observed in Continental Europe.[4]

Our setup relies on contract incompleteness under wage negotiation. Stole and Zwiebel (1996b) and Stole and Zwiebel (1996a) have shown that, in this context, firms *ex ante* choose a specific organizational structure in order to influence the outcome of the bargaining process *ex post*. This may occur because firms aim at limiting the effect of expropriation by workers as in Grout (1984) for instance—the so called "holdup" problem—or simply because firms wish to reduce the size of the rent that workers obtain when bargaining over wages. We model these inefficiencies in a framework with search and matching frictions as in Bertola and Caballero (1994), Smith (1999), Bertola and Garibaldi (2001) and Cahuc et al. (2008), among others. In our model, a representative firm may hire several types of labor and capital, generating a rich set of strategic interactions between workers and the firm under wage negotiation. We ask how labor and capital demands react to the introduction of a binding minimum wage in this setup. We find that, when capital and labor are complements,[5] capital utilization always increases, but the impact of a minimum wage on labor demand is ambiguous: it may be negative, as in the neoclassical framework, but the impact may also be positive.

Our model economy is characterized by a standard holdup problem, yielding lower incentives to invest in physical capital when capital and labor are complements. When the minimum wage is binding, part of the disincentives to accumulate capital are eliminated, since the firm no longer underinvests as a way to reduce the wage of workers earning the minimum regulatory level. This mechanism explains why the introduction

Our model is able to generate these effects through an increase in the demand for high-wage workers. See Section 4.

[4]Data from Caselli and Feyrer (2007) shows that the capital-output ratio is 36% larger in France than in the US, 29% in Spain and 22% in the Netherlands. See also Hall and Jones (1999). At the same time, those countries are also characterized by higher minimum wages. Data from the OECD reveals that the ratio of minimum to median wage of full-time workers is 36% in the US, while it is 56% in France, 43% in Spain and 51% in the Netherlands. In other countries such as Germany and Italy, there is no minimum wage regulation for most sectors of the economy. Instead, wages are determined through collective bargaining. In our model, as long as collective bargaining imply that individual firms take the stream of future wages as given, it should generate similar results on employment and capital accumulation as a minimum wage. According to the OECD, the proportion of workers covered by collective agreements is 92% in Germany, 82% in Italy and 18% in the US.

[5]The assumption that capital and labor are complements is in line with US data. Krusell et al. (2000) estimate a production function for the US that includes two types of capital, structures and equipment, and two types of labor, skilled and unskilled. They find that, although the elasticity of substitution is higher between capital equipment and unskilled labor than between capital equipment and skilled labor, capital equipment is a complement of both types of labor. This is also true for capital structures.

of a binding minimum wage increases the aggregate stock of capital in the model.[6]

When capital and labor are complements, the increase in capital demand also tends to increase labor demand. Moreover, we also show that an increase in the demand of workers earning the minimum wage is possible in our model. This comparative static becomes more likely when the different types of labor considered are substitutes. The reason is the following. Cahuc et al. (2008) have shown that, absent a minimum wage, the firm chooses to "overemploy" one type of labor in order to reduce the wage of the other type: because the equilibrium wage depends on the marginal product of labor under Nash bargaining, overemployment allows to reduce the other type's wage by decreasing its marginal product. Unfortunately for the firm, this leads to an additional appropriation problem as the overemployed workers claim part of the decrease in the other type's wage rate. As a result, the firm ends up paying higher wages and making lower profits. In a context where the wage of one type of workers is fixed by regulation, rent appropriation by those workers is no longer possible. Thus, overemploying these workers becomes a more attractive option for the firm to affect the wage of the other type.

The lack of clear-cut evidence supporting a negative effect of minimum wages on employment has favored the popularity of theoretical models that predict a positive effect as an alternative to the neo-classical framework. An example is the monopsony model, where employment is increased through an increase in labor supply without labor demand being necessarily affected.[7] In our paper, we move further away from conventional modelling since we find that the introduction of a binding minimum wage may increase labor demand, even at the *microeconomic* level. We insist on the term "microeconomic" here because this result does not rely on any general-equilibrium mechanism. Moreover, the rise in labor demand may occur for any labor input, including the labor type for which the minimum wage is binding. In this sense, this result represents a step further away from conventional models.[8]

[6] Because a minimum wage helps deter rent appropriation by workers, it may also generate an increase in measures of aggregate welfare. Our paper is not the first to show welfare-improving properties of minimum wages. See Flinn (2006) and the references therein.

[7] In the monopsony model, the firm has some market power over workers, which allows it to fix wages below the competitive level. Because the slope of the labor supply curve is positive, the introduction of a binding minimum wage increases employment by enhancing the incentives to supply more labor. The marginal impact on employment remains positive as long as the minimum regulatory level does not go beyond the competitive wage. The consequences of a minimum wage are similar in the oligopoly extension of the basic monopsony model and in its version with search frictions. See e.g. Boal and Ransom (1997), Burdett and Mortensen (1998), and Manning (2003).

[8] An exception is the paper by Cahuc et al. (2001). They study the effect of a binding minimum wage in a context where firms negotiate wages with trade unions. They show that, when skilled and unskilled labor are substitutes, an increase in the minimum wage may increase the employment level of both groups.

Our analysis is closely related to the work of Smith (1999). This paper also studies the effect of a minimum wage in the context of a large-firm search model. In his model, firms hire one type of labor and the production function displays decreasing returns to scale. Those assumptions generate two effects. First, firms tend to overemploy workers in order to decrease wages by decreasing their marginal product. Second, overemployment creates a negative externality that forces some firms out of the industry by increasing the labor-market search cost. As a result, there are too few firms in the economy and those are inefficiently too large. By introducing a minimum wage, it is possible to eliminate this inefficiency: firms stop to overemploy, which increases firm entry and aggregate employment. Our work improves on Smith's contribution in two ways. First, the seminal study of Card and Krueger (1994) shows a positive effect of the minimum wage on firm size and no significant effect on the number of firms. Our results are consistent with Card and Kruger's findings. Second, while Smith (1999) shows a positive effect on aggregate labor demand, we show that labor demand may also increase at the microeconomic level.

Our paper is also related to the literature that discusses the effect of a minimum wage on capital accumulation. In a context with search frictions, Acemoglu and Shimer (1999) show that the holdup problem is avoided if the wage rate (as a function of the capital stock) is constant in the neighborhood of the efficient capital stock. In our model, introducing a minimum wage allows to fulfill this necessary condition. Acemoglu (2001) builds a model where firms open too few capital-intensive jobs because workers expropriate part of the return on capital. The introduction of a binding minimum wage helps correct for this externality and enhances the creation of capital-intensive jobs. However, an increase in the minimum wage always results in an increase in unemployment in his model, while the effect is ambiguous in the context of our model. Moreover, our model considers a richer set of strategic interactions between workers, the firm and capital. Similarly, Kaas and Madden (2008) illustrate the beneficial effects of a minimum wage for capital investment in the context of an oligopsonistic model, but they do not obtain a positive effect on employment.

The rest of the paper is organized as follows. In Section 2 we describe the model. The effect of a minimum wage on employment and capital in partial equilibrium is analyzed in Section 3. General equilibrium effects are studied in Section 4, along with the implications of a minimum wage for welfare and wage dispersion. An extension of the model is analysed in Section 5. Finally Section 6 concludes.

2 The model

We consider an economy in steady state, where time is continuous and discounted at a rate r and agents are risk neutral. For notational simplicity, we suppress the time indices t while describing the economy and analyzing the equilibrium, while we denote by primes variables evaluated in period $(t+dt)$, where dt is an arbitrarily small interval of time.

2.1 A representative firm

Output is produced by a representative firm. This firm hires two types of workers in quantities n_h and n_l and owns capital in quantity k.[9] The production function f is increasing and concave in each argument.[10] Standard Inada conditions are assumed such that an equilibrium exists on all markets for inputs, independently of the flow value of being unemployed[11].

Without loss of generality, we assume that the first type of labor is the one paid the highest wage. We refer hereafter to the two types of labor as "high wage" and "low wage" workers respectively, hence the subscripts h and l. We interpret these types as corresponding both to unskilled labor (and consequently, subject to binding minimum wages) but with varying job characteristics (tasks, experience, training...).

2.2 Labor

There are two labor markets corresponding to the two types of labor the firm hires. The mass of high-wage workers is normalized to one and ς denotes the mass of low-wage workers in the economy. Thus, ς is also the ratio of low-wage to high-wage workers. A high-wage worker cannot hold a low-wage job and *vice versa*.

Workers on each market can be either employed or unemployed. The presence of search and matching frictions explains the existence of unemployment on the two

[9]We introduce two types of labor in the analysis because the effects of a minimum wage over employment are ambiguous in this case, and depend on the assumptions about complementarity or substitutability between production factors. Cahuc et al. (2008) show that, with several types of labor, strategic interactions between workers appear. The presence of these strategic interactions is necessary to generate incentives for the firm to demand more labor when a minimum wage is introduced.

[10]We do not restrict the production function to be constant returns to scale. A reason is because firm size is undetermined in partial equilibrium under constant returns to scale. Thus our simulations in partial equilibrium consider a production function with decreasing returns to scale (Section 3). Both constant and decreasing returns to scale are allowed in general equilibrium (Section 4).

[11] $\lim_{n_i \to 0} \frac{\partial f(n_h,n_l,k)}{\partial n_i} = \infty$, $\lim_{n_i \to \infty} \frac{\partial f(n_h,n_l,k)}{\partial n_i} = 0$, $\forall i \in \{h,l\}$, $\forall k > 0$, $\lim_{k \to 0} \frac{\partial f(n_h,n_l,k)}{\partial k} = \infty$, $\lim_{k \to \infty} \frac{\partial f(n_h,n_l,k)}{\partial k} = 0$, $\forall n_k > 0$, $\forall n_l > 0$.

markets (in quantities u_h and u_l respectively). Firms post vacancies at a flow cost c in order to hire workers. We denote by v_i, $i \in \{h, l\}$, the mass of posted vacancies by the firm on each labor market. Vacancies on market i, $i \in \{h, l\}$, are filled at a rate $q(\theta_i)$ that depends negatively on the labor market tightness $\theta_i \equiv \frac{v_i}{u_i}$, i.e. the vacancy-unemployment ratio. This rate is derived from a matching function $m(u_i, v_i)$ with constant returns to scale, increasing in both its arguments, concave and satisfying the property $m(u_i, 0) = m(0, v_i) = 0$, implying that $q(\theta_i) = \frac{m(u_i, v_i)}{v_i} = m(\theta_i^{-1}, 1)$. Separations occur at an exogenous rate s.[12]

2.3 Prices

We denote by $w_i(n_h, n_l, k, \chi_h, \chi_l)$ the wage paid to a given type of labor $i \in \{h, l\}$. Wages can be either set equal to a minimum wage ω or, because there are search and matching frictions on both labor markets, negotiated between workers and the firm. Call $\tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l)$ the wage determined under Nash bargaining, with $\beta \in (0, 1)$ denoting the bargaining power of workers. Then, χ_h and χ_l are indicator functions such that:

$$\chi_i = \begin{cases} 1 & \text{if } w_i(n_h, n_l, k, \chi_h, \chi_l) = \tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) \\ 0 & \text{if } w_i(n_h, n_l, k, \chi_h, \chi_l) = \omega \end{cases} \tag{1}$$

for $i = h, l$. If $\chi_i = 1$, wages of group i are continuously renegotiated.

Notice that our notation for wages explicitly emphasizes their dependence on the employment levels n_h and n_l and the capital stock k. The reason for this is that, if $w_i(\cdot) = \tilde{w}_i(\cdot)$, the firm may choose a particular level of employment or capital before wages are negotiated in order to influence the outcome of the bargaining process *ex post*.[13] For example, Smith (1999) shows that when the production function is concave in each factor, the firm may choose to overemploy in order to reduce wages through a reduction of the marginal product of labor[14]. Cahuc and Wasmer (2001a) and Cahuc et al. (2008) show that the complementarity (substitutability) between different types

[12]For notational simplicity we assume that the parameters s, b, c and the function m is common across labor groups. However, all the results presented in the next sections go through when those parameters and functions are allowed to differ across groups.

[13]We assume that the firm acquires capital *before* wages are bargained, and that the capital stock of the firm cannot be adjusted while wage negotiation occurs. In the case, capital is predetermined. If capital could be freely adjusted, the holdup problem would not be present, see Cahuc and Wasmer (2001a) for a discussion.

[14]We refer to a situation of *overemployment* when (in partial equilibrium) the firm hires a quantity of labor larger than the level that would prevail under a situation where the firm takes the stream of future wages as given. Similarly, when employment is below that level, we refer to a situation of *underemployment*.

of labor may induce the firm to underemploy (overemploy) one type of labor in order to reduce the wage of other workers.

The purchase of a unit of capital is priced one unit of final good and depreciates at a rate δ.

2.4 Value functions

The present-discounted value of profits for the representative firm is

$$\Pi(n_h, n_l, k) = \max_{\{v_h, v_l, a\}} \tag{2}$$
$$\frac{1}{1+rdt}\left(\left[f(n_h, n_l, k) - \sum_{j=\{h,l\}} [w_j(n_h, n_l, k, \chi_h, \chi_l)n_j + v_j c] - a\right] dt + \Pi(n_h', n_l', k')\right),$$

subject to the constraints

$$n_i' = (1 - sdt)n_i + q(\theta_i)v_i dt, \quad \forall i \in \{h, l\} \tag{3}$$

and

$$k' = (1 - \delta dt)k + adt, \tag{4}$$

where a denotes investment in physical capital and dt is an arbitrarily small interval of time. We specifically consider the case where dt tends to zero.

The values of being unemployed and employed follow a standard formulation and respectively write in steady state as

$$rU_i = b + \theta_i q(\theta_i) [W_i - U_i] \tag{5}$$

and

$$rW_i = w_i(n_h, n_l, k, \chi_h, \chi_l) + s [U_i - W_i] \tag{6}$$

with b the flow utility of being unemployed.

3 Wage determination in partial equilibrium

We first illustrate the effect of a binding minimum wage in partial equilibrium in order to show that its possible positive impact on employment actually holds at the microeconomic level and does not depend on general equilibrium effects: it is due to the strategic interactions between the representative firm and workers in bargaining. We define a partial equilibrium as an equilibrium where all aggregate variables are exogenous, that is, where the labor-market tightness θ_i and the present discounted values of being unemployed U_i are exogenous $\forall i \in \{h, l\}$.

3.1 Equilibrium wages

In equilibrium, the wage effectively paid to an l-type worker is $\tilde{w}_l(n_k, n_l, k, 1, 1)$ if this wage is higher than the minimum wage, and ω otherwise:

$$w_l(n_h, n_l, k, \chi_h, \chi_l) = \max\{\tilde{w}_l(n_h, n_l, k, 1, 1), \omega\}. \tag{7}$$

Given our assumption that wages under Nash bargaining are always higher for h-type workers, the equilibrium wage of this group is determined according to

$$w_h(n_h, n_l, k, \chi_h, \chi_l) = \begin{cases} \tilde{w}_h(n_h, n_l, k, 1, 1) & \text{if } \chi_l = 1 \\ \max\{\tilde{w}_h(n_h, n_l, k, 1, 0), \omega\} & \text{if } \chi_l = 0. \end{cases} \tag{8}$$

3.2 First-order conditions

The first-order conditions for vacancy posting and capital investment are, respectively,

$$\frac{c}{q(\theta_i)} = \frac{\frac{\partial f(n_h, n_l, k)}{\partial n_i} - w_i(n_h, n_l, k, \chi_h, \chi_l) - \sum_{j \in \{h,l\}} \chi_j \frac{\partial w_j(n_h, n_l, k, \chi_h, \chi_l)}{\partial n_i} n_j}{r + s}, \quad \forall i \in \{h, l\} \tag{9}$$

and

$$r + \delta = \frac{\partial f(n_h, n_l, k)}{\partial k} - \sum_{i \in \{h,l\}} \chi_i \frac{\partial w_i(n_h, n_l, k, \chi_h, \chi_l)}{\partial k} n_i. \tag{10}$$

The vacancy-posting conditions (9) equate the expected search cost of hiring a worker of type i to the discounted sum of profits that the marginal worker brings to the firm after being hired. They differ from the condition of the standard model with one worker per firm (Pissarides, 1985) through two strategic effects. First, the employment level of group i may affect the wage of that group. Incentives to overemploy may appear when the production function is concave in n_i (Smith, 1999). Second, the employment level of group i may affect the wage of the other group $j \neq i$. Incentives to overemploy may appear when factors are substitutes and underemployment may result from complementarity between factors (Cahuc et al., 2008). These strategic effects may of course counteract each other. At the same time, some of these effects may disappear when the wage is set to the minimum wage level for a particular group i because, in this case, the equilibrium wage of this group cannot be affected by the employment levels n_h or n_l. This occurs when $\chi_i = 0$.

The capital investment condition (10) equates the opportunity cost of capital to the marginal income of capital. The latter differs from its neoclassical counterpart through the effect on wages: depending on the complementarity/substitutability of capital with labor, the representative firm may choose to underinvest/overinvest in order to reduce wages.

3.3 Intrafirm bargaining and the minimum wage

The equilibrium wage rates under Nash bargaining $\tilde{w}_i(\cdot)$ are determined following a standard Nash bargaining rule. In this case, they solve the following equation:

$$\beta \frac{\partial \Pi(n_h, n_l, k)}{\partial n_i} = (1-\beta)\left[W_i - U_i\right], \quad \forall i \in \{h, l\}, \tag{11}$$

where W_i is defined in (6) and the firm's surplus $\frac{\partial \Pi(n_h, n_l, k)}{\partial n_i}$ is calculated by applying the envelope theorem to (2).

The solution to this equation reads:

$$\begin{aligned} \tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) = \quad & \beta \frac{\partial f(n_h, n_l, k)}{\partial n_i} + (1-\beta) r U_i \\ & -\beta \textstyle\sum_{j \in \{h,l\}} \chi_j \frac{\partial w_j(n_h, n_l, k, \chi_h, \chi_l)}{\partial n_i} n_j, \quad \forall i \in \{h, l\}. \end{aligned} \tag{12}$$

The wage equation (12) differs from the standard equation from Pissarides (1985) through the last term in the equation. Its presence can be explained as follows. When the firm bargains with a worker, its surplus is composed by two elements: the marginal product of labor and the effect of this worker's hire on the wages of other workers. The first element is present in the standard one-worker-per-firm model, while the latter is due to the large-firm aspect of our model. Under Nash bargaining, workers can appropriate part of the decrease in wages of the other workers. This explains the difference between the wage equation (12) and the standard one.

By plugging the solution for wages into (9) and (10), the vacancy-posting conditions can be rewritten as

$$\frac{c}{q(\theta_i)} = \frac{\Omega_i\left(\chi_h, \chi_l\right) \frac{\partial f(n_h, n_l, k)}{\partial n_i} - w_i(n_h, n_l, k, \chi_h, \chi_l)}{r+s}, \quad \forall i \in \{h, l\}, \tag{13}$$

where

$$\tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) = \beta\Omega_i\left(\chi_h, \chi_l\right) \frac{\partial f(n_h, n_l, k)}{\partial n_i} + (1-\beta)rU_i, \quad \forall i \in \{h, l\}, \tag{14}$$

while the capital-investment condition takes the form

$$r + \delta = \Omega_k\left(\chi_h, \chi_l\right) \frac{\partial f(n_h, n_l, k)}{\partial k}. \tag{15}$$

Notice the presence of the *front-load factors* $\Omega_i\left(\chi_h, \chi_l\right) > 0$ in equations (13) and (15), the value of which may differ depending on the fact that the minimum wage may bind or not. Their presence is the outcome of the strategic interactions between workers and the representative firm in bargaining. When the Ω_i of a particular factor i takes value larger than one, we refer to this situation as a situation of *overemployment* in the sense that the firm employs a quantity of type-i workers larger than in the case where the firm considers future wages as given. Similarly, underemployment of factor i appears when the respective factor is lower than one.

The following Proposition identifies the values taken by each Ω_i depending on the values of the χ_i's:

Proposition 1 *The front-load factors can be written as follows.*

- *In the absence of a binding minimum wage,*

$$\Omega_h\left(1,1\right) = \frac{\int_0^1 \frac{\partial f(n_h z, n_l z, k)}{\partial (n_h z)} \varphi(z) dz}{\frac{\partial f(n_h, n_l, k)}{\partial n_h}}, \quad \Omega_l\left(1,1\right) = \frac{\int_0^1 \frac{\partial f(n_h z, n_l z, k)}{\partial (n_l z)} \varphi(z) dz}{\frac{\partial f(n_h, n_l, k)}{\partial n_l}}$$

and

$$\Omega_k\left(1,1\right) = \frac{\int_0^1 \frac{\partial F(n_h z, n_l z, k)\varphi'(z)}{\partial k} dz}{\frac{\partial F(n_h, n_l, k)}{\partial k}}.$$

where $\varphi(z) = \frac{1}{\beta} z^{\frac{1-\beta}{\beta}}$ *and* $\varphi'(z) = \frac{1-\beta}{\beta} z^{\frac{1-2\beta}{\beta}}$ *are weights assigned to the infra-marginal products of labor and capital, with* $\int_0^1 \varphi(z)dz = \int_0^1 \varphi'(z)dz = 1$.

- *If low-wage workers are paid the minimum wage, but not high-wage workers,*

$$\Omega_h\left(1,0\right) = \frac{\int_0^1 \frac{\partial f(n_h z, n_l, k)}{\partial (n_h z)} \varphi(z) dz}{\frac{\partial f(n_h, n_l, k)}{\partial n_h}}, \quad \Omega_l\left(1,0\right) = \frac{\int_0^1 \frac{\partial f(n_h z, n_l, k)}{\partial n_l} \varphi'(z) dz}{\frac{\partial f(n_h, n_l, k)}{\partial n_l}}$$

and

$$\Omega_k\left(1,0\right) = \frac{\int_0^1 \frac{\partial F(n_h z, n_l, k)}{\partial k} \varphi'(z) dz}{\frac{\partial F(n_h, n_l, k)}{\partial k}}.$$

- *If the minimum wage is binding for all workers,*

$$\Omega_i\,(0,0) = 1, \quad \forall i \in \{h,l,k\}.$$

The Proposition does not illustrate the case where $\chi_h = 0$ and $\chi_l = 1$ because we assume without loss of generality that, absent a minimum wage, the h-type workers earn more than the l-type workers. Thus, if high-wage workers are paid the minimum legal level, low-wage workers are necessarily paid that amount too. Hence, the Proposition illustrates three situations: i) one where the minimum wage never binds, ii) one where it binds only in the case of the l-type workers and iii) one where it binds for both labor groups.

When the minimum wage is binding, the set of strategic tools workers and the firm can use is reduced. The firm cannot affect the wage of workers for which the minimum wage is binding—i.e. some χ's take value zero in equation (9)—and workers cannot obtain higher wages by claiming they help reduce the wage of others—i.e. the same χ's are forced to take value zero in equation (12). These additional restrictions in the bargaining process explain the differences between the front-load factors given in Proposition 1.

For example, consider the value of Ω_h in the absence of a binding minimum wage. Ω_h is the ratio of two elements: its denominator is the marginal product of labor, while its numerator is a weighted average of the infra-marginal products of labor. The weights in the latter are given by the parameters $\varphi(z)$. Notice that $\Omega_h = 1$ when these weights are zero for all $z < 1$ and one for $z = 1$, since the numerator is equal to the denominator in this case. Ω_h is also equal to one when the marginal product of labor is independent of n_h and n_l. For other values of $\varphi(z)$ and with a non-linear production function, Ω_h may differ from one. Three effects may take the value of the front-load factor away from one.[15] First, the concavity in n_h of the production function tends to increase its value: the more concave the production function, the larger are the incentives for the firm to overemploy h-type workers in order to reduce their wage. Second, the substitutability (complementarity) with l-type workers tends to increase (decrease) the value of Ω_h: overemployment (underemployment) allows to decrease the wage of l-type workers by decreasing their marginal product. Third, the shape of the function $\varphi(z)$ also allows to affect the value of Ω_h by weighting the different infra-marginal products of labor at a different intensity. Specifically, when the bargaining power of workers is large, the representative firm has more incentives to reduce wages.

Now observe how the value of the front-load factor Ω_h compares to the case when the minimum wage binds for low-wage workers only. In this situation, h-type work-

[15]See Cahuc et al. (2008) and Cahuc and Wasmer (2001a) for more details.

ers cannot obtain higher wages by claiming that they help reduce the wage of l-type workers. This explains why the z variable does not appear next to the level of n_l in the numerator of Ω_h in the case where the minimum wage binds for low-wage workers only, while it is present in the absence of a binding minimum wage. Hence, holding constant the levels of n_h, n_l and k, Proposition 1 suggests that introducing a minimum wage that marginally binds for low-wage workers increases the incentives to hire h-type workers (larger Ω_h) when they are a complement of low-wage workers, while it suggests lower incentives when they are substitutes.

It is possible to understand the differences in the values of Ω_l in a similar way. Two differences can be noticed. First, the concavity in n_l, that generates incentives to overemploy absent a minimum wage, does not operate when a binding minimum wage is introduced. Thus, holding constant the levels of n_h, n_l and k, this effect suggests lower incentives to hire l-type workers (lower Ω_l) when a binding minimum wage is introduced. Second, it turns out that the weighting function $\varphi(z)$ is also affected by the introduction of the minimum wage. In particular, the weighting function $\varphi'(z)$ (that operates under a binding minimum wage) assigns larger weights to low values of z than the weighting function $\varphi(z)$ (which prevails in the absence of a binding minimum wage). Intuitively, when the minimum wage binds for l-type workers, these workers cannot expropriate part of the decrease in the wage of h-type workers. Varying the employment level of l-type workers thus becomes a more attractive tool for the representative firm to affect the wage of h-type workers. This makes the demand for l-type workers more elastic. This additional effect suggests higher incentives to hire l-type workers when the two types of labor are substitutes and lower incentives when they are complements.

Under Nash bargaining, the negotiated wage is a function of the capital stock. Moreover, in the absence of binding wage contracts, workers do not share in the cost of ex ante investments. This leads to underinvestment (overinvestment) when capital is complementary (substitutable) to labor because the representative firm anticipates that investing more (less) in physical capital amounts to bargaining to a higher wage. The values of Ω_k illustrate how investment by the representative firm responds to the introduction of a binding minimum wage in this context. Specifically, the minimum wage legislation helps deter rent appropriation by workers since the wage, as a function of the capital stock, becomes constant for values of k below the equilibrium capital stock that would prevail absent a binding minimum wage. This implies that the value of Ω_k gets closer to one when the minimum wage becomes binding.

Finally, if the minimum wage binds for all labor groups, then the front-load factors are all equal to one because the firm cannot influence strategically any wage rate.

3.4 Equilibrium

As explained before, we define a partial equilibrium as an equilibrium where all aggregate variables are exogenous.

Definition 1 *A steady-state partial equilibrium is a set of indicator functions χ_i, firm size indicators n_h, n_l and k and wage rates w_h and w_l such that the definitions (1), the first-order conditions (9) and (10) and the wage equations (7), (8) and (14) are satisfied, given exogenous values for θ_i and U_i and a minimum wage ω, $\forall i \in \{h, l\}$.*

We restrict our attention to equilibria that are *internally consistent*. An equilibrium in which the minimum wage is binding for l-type workers is said to be internally consistent if, at any point in time, no l-type worker has incentives to renegotiate its wage with the firm, given that he earns ω. Formally,

Definition 2 *An equilibrium in which the minimum wage is binding for l-type workers is internally consistent if*

$$w_l^{ic} \leq \omega,$$

where

$$w_l^{ic} = \beta \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta) r U_l - \beta \frac{\partial w_h(n_h, n_l, k, \chi_h, 0)}{\partial n_l} n_h. \tag{16}$$

The previous definition states w_l^{ic} as the wage that a marginal l-type worker would obtain if he renegotiated the wage with the firm, given that all other l-type workers earn the minimum wage. Notice that w_l^{ic} acknowledges the fact that, by employing a marginal l-type worker, the firm is able to influence wages of h-type workers. By imposing that the equilibria we analyze satisfy internal consistency, we require that l-type workers have no incentives to deviate from the strategy of providing labor to the firm at wage ω.

The following proposition characterizes the wage w_l^{ic} that a marginal l-type worker would negotiate with the firm, if he deviated from the strategy of earning the minimum wage ω.

Proposition 2 *The wage w_l^{ic} that a marginal l-type worker would obtain if he renegotiated the wage with the firm, given that all other l-type workers earn the minimum wage, is*

$$w_l^{ic} = \beta \Omega_l(1, 0) \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta) r U_l. \tag{17}$$

Table 1: Parameter values

η	0.5
β	0.5
m_0	0.7
r	0.004
s	0.036
b	0.71
c	0.356
δ	0.01
ς	1.5
α	2/3
ν if complements	0.9α
ν if substitutes	1.3α
γ	0.3

The next proposition shows that a steady-state partial equilibrium in which the minimum wage is binding is always internally consistent.

Proposition 3 *A steady-state partial equilibrium in which $w_l = \omega$, is always internally consistent.*

Proofs are available in the Appendix.

3.5 A numerical example

The discussion above has emphasized how the introduction of a binding minimum wage affects the incentives to overemploy or underemploy by analyzing how the front-load factors change with the policy. Nevertheless our comments on the variation in the Ω_i's are made in a context where all the levels of n_h, n_l and k are held constant. It is tempting to extrapolate these results to the whole incentives to open up vacancies and believe that, if a front-load factor increases (holding constant n_h, n_l and k), then the marginal income of the respective factor increases too. However, we know that this may not be true because of the possible complementarities or substitutabilities between factors in the production process.

We now illustrate through a numerical example how the results given in Proposition 1 can be extended to a context where the levels of n_h, n_l and k are allowed to change.

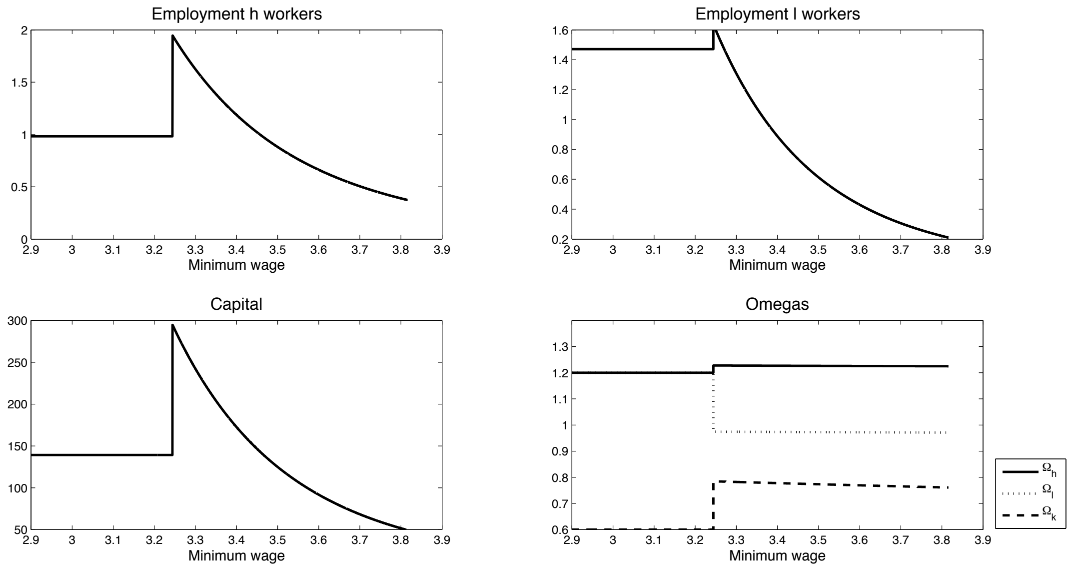

Figure 1: Factor utilization and front-load factors - n_h and n_l as complements

For the purpose of the exercise, we need to specify a production function and a matching function and parametrize the model. We use a Cobb-Douglas matching function of the form

$$m(1-n, v) = m_0(1-n)^{\eta} v^{1-\eta}, \tag{18}$$

and a production function represented by

$$f(n_h, n_l, k) = [{n_h}^{\nu} + {n_l}^{\nu}]^{\frac{\alpha}{\nu}} \, k^{\gamma}. \tag{19}$$

We parametrize the model following Pissarides (2009).[16] The parameter values used for the numerical exercise are reported in Table 1.

We perform the following exercise: we first compute the equilibrium values of factor utilization, front-load factors and labor market tightness θ_h and θ_l when there is no minimum wage regulation in place. The integrals in the expression for the front-load factors in Proposition (1) are approximated using the Gauss-Legendre algorithm. Keeping θ_h and θ_l fixed, we then set a minimum wage ω and compute the new equilibrium values of factor utilization and front-load factors that arise from the maximization problem of the firm and the wage bargaining process.

Figure 1 shows how the equilibrium values of labor and capital utilization change

[16]See also Janiak (2010).

for different values of the minimum wage, when both types of labor are complements. Consider the equilibrium quantities when the minimum wage is marginally binding, with respect to the case in which it is not binding. As explained before, the front-load factor of high-wage workers, Ω_h, increases in this case, because the firm can no longer reduce the wages of low-wage workers through a reduction of their marginal productivity associated to a decrease in n_h. By a similar reasoning, the decrease in Ω_l is due to the fact that, given that the minimum wage is binding for l-type workers, the firm strategically uses low-wage workers only to decrease the marginal productivity of h-type workers. Since both labor inputs are complements, this is achieved through a reduction in the incentives to hire low wage workers.

Notice that Ω_k is always lower than 1 and that, after the minimum wage becomes binding, it increases. This is due to the holdup problem previously described: since workers are able to seize part of the rent from capital, the firm underinvests, which translates into $\Omega_k < 1$. When the minimum wage becomes binding for n_l workers, they can no longer extract this rent and, consequently, the holdup problem is alleviated, which encourages the firm to invest more. Consequently, Ω_k increases. The effects of the binding minimum wage over n_h, n_l and k can be easily understood in light of this result. Since capital is complementary to both labor inputs, the increase in capital leads to an increase in the marginal productivities of h-type workers and l-type workers and, as a consequence, to an increase in n_h and n_l. For l-type workers, there are two opposing effects that take place simultaneously: on the one hand, the firm would like to decrease the use of such workers to reduce the wage of h-type workers, but on the other hand, the increase in the capital stock implies a higher marginal productivity of l-type workers. The latter effect dominates and n_l increases, although much less than n_h.

Figure 2 shows how the equilibrium values of labor and capital utilization change for different values of the minimum wage, when n_h and n_l are substitutes. The analysis is similar to the previous case. In this case, the front-load factor of high wage workers, Ω_h decreases, because the firm can no longer reduce the wages of low wage workers through a decrease of their marginal productivity associated to an increase of n_h. By a similar reasoning, the decrease of Ω_l is due to the fact that, given that the minimum wage is binding for l-type workers, the firm can no longer strategically use low wage workers to decrease their marginal productivity.

The effect of the minimum wage over Ω_k is completely analogous to the previous case: the fact that low wage workers cannot appropriate part of the rents of capital when the minimum wage is binding for them increases the incentives to invest of the firm. This translates into a higher Ω_k and, finally, higher k. The increase in the capital stock exerts a positive effect over the marginal productivities of high-wage and

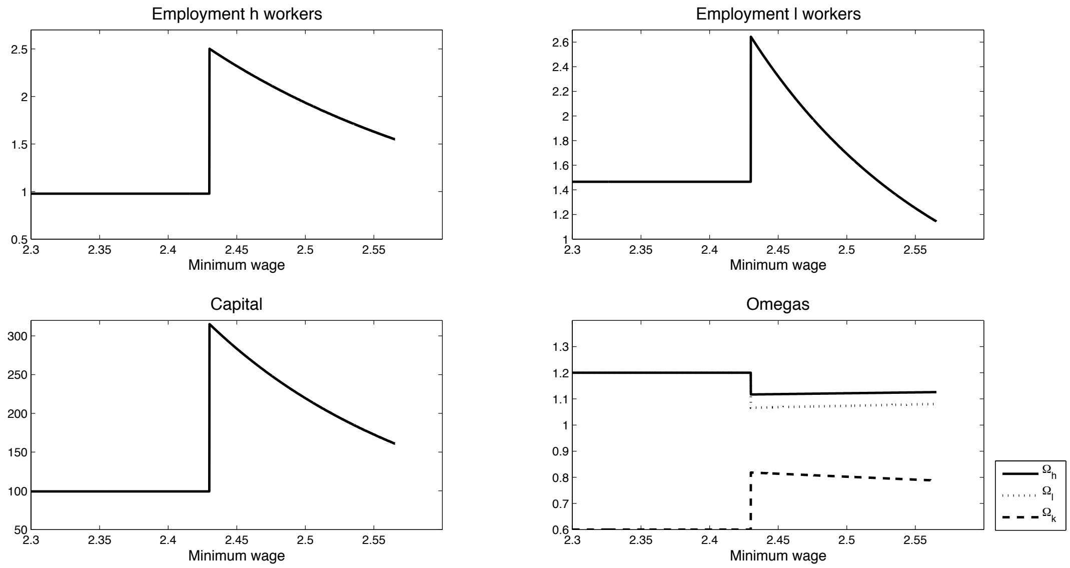

Figure 2: Evolution of factor utilization and front-load factors - n_h and n_l as substitutes

low-wage workers which, in turn, translates into higher utilization of both labor inputs.

As the minimum wage increases, Ω_l, Ω_h and Ω_k remain practically unchanged. This is due to the fact that the main change in the front-load factors occurs when the minimum wage starts binding, because of how such wage changes the strategic decisions of the firm. For higher values of the minimum wage, the labor costs of the firm increase and, consequently, profits decrease. This leads the firm to reduce production by decreasing the use of all factors of production.

4 General equilibrium

4.1 Equilibrium concept

In general equilibrium, the present-discounted values of being unemployed U_i and the labor-market tightness θ_i are endogenous. The former is determined through equation (5), making wages to respond to labor-market tightness as follows:

$$\tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) = \beta\Omega_i(\chi_h, \chi_l)\frac{\partial f(n_h, n_l, k)}{\partial n_i} + (1-\beta)b_i + \beta\theta_i c, \quad \forall i \in \{h, l\}, \tag{20}$$

while the latter is obtained in a standard way, by equating the flows in and out of employment and leading to the Beveridge relations

$$n_h = \frac{\theta_h q(\theta_h)}{s + \theta_h q(\theta_h)} \quad \text{and} \quad n_l = x\frac{\theta_l q(\theta_l)}{s + \theta_l q(\theta_l)}. \tag{21}$$

This yields the following definition of equilibrium:

Definition 3 *A steady-state general equilibrium is a set of indicator functions χ_i, employment levels n_h and n_l, a capital stock k, wage rates w_h and w_l and labor-market tightness θ_h and θ_l such that the definitions* (1)*, the first-order conditions* (9) *and* (10)*, the wage equations* (7)*,* (8) *and* (20) *and the Beveridge relations* (21) *are satisfied, given a minimum wage ω and indicator functions χ_i, $\forall i \in \{h, l\}$.*

As in the partial equilibrium case, we are interested in equilibria that are internally consistent. The definition of an internally consistent equilibrium is analogous to Definition 2. Proposition 2, which characterizes the wage w_l^{ic} that a marginal l-type worker would negotiate with the firm if he deviated from the strategy of providing labor at wage ω, also holds in general equilibrium.

In partial equilibrium, Proposition 3 states that the equilibrium is always internally consistent. In general equilibrium, however, the value of the outside option of workers rU^i is no longer exogenous and depends positively on θ_i. If the labor market tightness of l-type workers increases, then it is possible that the wage w_l^{ic} that a marginal l-type worker would obtain if he renegotiated with the firm is higher than the minimum wage ω, thus rendering the equilibrium internally inconsistent.

In the following section, we define a notion of *robust equilibrium* such that the equilibrium is always internally consistent.

4.2 Robust Equilibrium

As explained before, in general equilibrium Definition 3 may yield internally inconsistent equilibria, for a given minimum wage ω. If this is the case, the equilibrium obtained can be regarded as unstable, in the sense that workers have incentives to deviate from the strategy in which they earn the minimum wage by renegotiating the terms of the contract with the firm.

To overcome this difficulty, we assume an alternative setup such that, when the minimum wage is binding for l-type workers, the firm negotiates the wage by Nash bargaining with a fraction x of low-wage workers, and pays the minimum wage ω to the remaining $(1 - x)$ fraction of workers. The value of x is determined in equilibrium such that all wages of l-type workers are equal.

The representative firm takes the fraction x as given. Hence, the present-discounted value of profits can be restated as

$$\Pi(n_h, n_l, k) = \max_{\{v_h, v_l, a\}} \frac{1}{1+rdt}([f(n_h, n_l, k) - \tilde{w}_h(n_h, n_l, k, 1, x)n_h - (x\tilde{w}_l(n_h, n_l, k, 1, x) + (1-x)\omega)n_l + (v_h + v_l)c - a]dt + \Pi(n_h', n_l', k')), \tag{22}$$

subject to the constraints (3) and (4).

Notice that the indicator function χ_l now assumes the value x, which is the proportion of l-type workers that negotiate their wage with the firm.

We solve for wages contracted between the firm, h-type workers and a fraction x of l-type workers.[17]

The vacancy-posting conditions can be rewritten as

$$\frac{c}{q(\theta_h)} = \frac{\Omega_h(1, x) \frac{\partial f(n_h, n_l, k)}{\partial n_h} - \tilde{w}_h(n_h, n_l, k, 1, x)}{r + s}, \tag{23}$$

$$\frac{c}{q(\theta_l)} = \frac{\Omega_l(1, x) \frac{\partial f(n_h, n_l, k)}{\partial n_l} - (x\tilde{w}_l(n_h, n_l, k, 1, x) + (1-x)\omega)}{r + s}, \tag{24}$$

where

$$\tilde{w}_h(n_h, n_l, k, 1, x) = \beta\Omega_h(1, x) \frac{\partial f(n_h, n_l, k)}{\partial n_h} + (1-\beta)b + \theta_h\beta c, \tag{25}$$

$$\tilde{w}_l(n_h, n_l, k, 1, x) = \beta\Omega_l(1, x) \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta)b + \theta_l\beta q(\theta_l)\left(\frac{c}{q(\theta_l)} + \frac{(1-x)(\omega - \tilde{w}_l(n_h, n_l, k, 1, x))}{r+s}\right). \tag{26}$$

Notice that in expressions (25) and (26) we have used the fact that the value of the outside option rU_i is determined by equation (5). Finally, the capital-investment condition takes the form

$$r + \delta = \Omega_k(1, x) \frac{\partial f(n_h, n_l, k)}{\partial k}. \tag{27}$$

The following Proposition identifies the values taken by each $\Omega_i(1, x)$:

[17] The details of the derivation are contained in the appendix.

Proposition 4 *The front-load factors can be written as follows:*

$$\Omega_h(1,x) = \frac{\int_0^1 \varphi(z) \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial(n_h z)} dz}{\frac{\partial f(n_h,n_l,k)}{\partial n_h}}, \tag{28}$$

$$\Omega_l(1,x) = \frac{\int_0^1 \varphi_x(z) \partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right) / \partial\left(n_l z^{\frac{(1-\beta)x}{1-\beta x}}\right) dz}{\frac{\partial f(n_h,n_l,k)}{\partial n_l}}, \tag{29}$$

$$\Omega_k(1,x) = \frac{\int_0^1 \varphi'(z) \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial k} dz}{\frac{\partial f(n_h,n_l,k)}{\partial k}}, \tag{30}$$

with $\varphi(z) = \frac{z^{\frac{1-\beta}{\beta}}}{\beta}$, $\varphi'(z) = \frac{1-\beta}{\beta} z^{\frac{1}{\beta}-2}$ *and* $\varphi_x(z) = \frac{1-\beta}{\beta} \frac{z^{\frac{1-\beta}{\beta} - \frac{1-x}{1-\beta x}}}{1-\beta x}$.

Next, we provide a definition of a robust equilibrium:

Definition 4 *A steady-state robust general equilibrium is a set of employment levels* n_h *and* n_l*, a capital stock* k*, wage rates* $\tilde{w}_h$ *and* $\tilde{w}_l$*, a share* x *and labor-market tightness* θ_h *and* θ_l *such that the first-order conditions* (23)*,* (24) *and* (27)*, the wage equations* (25)*,* (26) *and the Beveridge relations* (21) *are satisfied, given a minimum wage* ω*. Moreover,*

$$x = \begin{cases} 1 & \text{if } \tilde{w}_l(n_h, n_l, k, 1, 1) > \omega, \\ 0 & \text{if } w_l^{ic} < \omega, \\ \text{is such that } \tilde{w}_l(n_h, n_l, k, 1, x) = \omega & \text{otherwise.} \end{cases}$$

It is clear from the previous definition that, by requiring that x is such that $\tilde{w}_l(n_h, n_l, k, 1, x) = \omega$, we are guaranteeing that l-type workers will be indifferent between renegotiating their wage with the firm and earning ω. Thus, a steady-state robust general equilibrium is, by definition, internally consistent.

4.3 Welfare

Our model is characterized by two types of inefficiencies. First, congestion externalities are not necessarily internalized by the Nash bargaining rule as in Hosios (1990). Second, appropriability distorts employment and capital decisions, as in Grout (1984) and Cahuc et al. (2008) among others. Of course both inefficiencies may partly compensate each other. For instance, the social losses of a large bargaining power, which leads to too few vacancies in the standard model with one worker per firm, may be reduced by an overemploying representative firm.

We now illustrate these ideas in the context of a centralized equilibrium.[18] The value function characterizing the social planner's solving problem is

$$V(n_h, n_l, k) = \max_{\{v_h, v_l, a\}} \frac{1}{1+rdt} \left(\left[f(n_h, n_l, k) + b(1 + x - n_h - n_l) - \sum_{j=\{h,l\}} v_j c - a \right] dt + V(n'_h, n'_l, k') \right), \tag{31}$$

subject to the constraints (3), (4), $\theta_h = \frac{v_h}{1-n_h}$ and $\theta_l = \frac{v_l}{x-n_l}$.

The first-order conditions for a maximum are, in steady state,

$$\frac{c}{q(\theta_i)} = \frac{(1-\eta(\theta_i))\left(\frac{\partial f(n_h,n_l,k)}{\partial n_i} - b\right) - \eta(\theta_i)\theta_i c}{r+s}, \quad \forall i \in \{h, l\}, \tag{32}$$

and

$$r + \delta = \frac{\partial f(n_h, n_l, k)}{\partial k}, \tag{33}$$

where $\eta(\theta_i) \equiv -\frac{\theta_i q'(\theta_i)}{q(\theta_i)}$.

These optimality conditions can be compared to the vacancy-posting and capital-investment conditions of the representative firm in the context of the steady-state equilibrium given in Definition 3. This allows us to establish the following result on the efficiency of the equilibrium:

Proposition 5 *A centralized equilibrium is a set of employment levels n_h and n_l, a capital stock k and labor-market tightness θ_h and θ_l such that the optimality conditions (32) and (33) and the Beveridge relations (21) are satisfied.*

Hence, a steady-state equilibrium is efficient if, $\forall i \in \{h, l\}$,

$$\beta = \eta(\theta_i) + (1-\eta(\theta_i))(\Omega_i - 1)\frac{\frac{\partial f(n_h,n_l,k)}{\partial n_i}}{\Omega_i \frac{\partial f(n_h,n_l,k)}{\partial n_i} - b + \theta c} \tag{34}$$

when $\chi_i = 1$,

$$\omega = (\eta(\theta_i) + \Omega_i - 1)\frac{\partial f(n_h, n_l, k)}{\partial n_i} + [1 - \eta(\theta_i)]b + \eta(\theta_i)\theta_i c \tag{35}$$

[18] See also Smith (1999) and Cahuc and Wasmer (2001b).

when $\chi_i = 0$ *and*

$$\Omega_k = 1. \tag{36}$$

Condition (36) is a standard condition for an efficient capital allocation, while condition (34) is an augmented Hosios-Pissarides condition, with (35) being its counterpart in presence of a binding minimum wage. Both reduce to the standard condition of a model with one worker per firm when $\Omega_i = 1$, with $\beta = \eta(\theta_i)$ when $\chi_i = 1$ and $\omega = \eta(\theta_i)\frac{\partial f(n_h,n_l,k)}{\partial n_i} + [1-\eta(\theta_i)]b + \eta(\theta_i)\theta_i c$ when $\chi_i = 0$. When $\Omega_i > 1$, a value for β larger than $\eta(\theta_i)$ is required in order to compensate for overemployment by the representative firm. The opposite occurs when $\Omega_i < 1$.

A minimum wage can fill one of the efficiency conditions given in Proposition 5. For example, condition (35) may be satisfied when the decentralized equilibrium wage is inefficiently too low absent a minimum wage legislation. This happens when the representative firm has incentives to overemploy ($\Omega_i > 1$) or when the share it obtains under wage bargaining is too high, generating congestion on the vacancy side. Similarly, condition (36) is satisfied when the minimum wage is binding for both labor groups. In this case, intrafirm bargaining cannot take place, which alleviates the holdup problem.

However, these efficiency conditions are rarely satisfied together. Moreover, the fulfilment of a subset of them does not necessarily produce an improvement in welfare. It may be the case that reaching optimality on one market leads to augmented inefficiencies on another market. For example, implementing (35) in the case of low-wage workers may induce the front-load factor for high-wage workers to deviate even more from its social optimum. Similarly, implementing (36) on the capital market or (35) on the market for high-wage workers may be too expensive if it requires increasing the wage cost of l-type workers.

4.4 Numerical illustration

In this section we show, by means of an example, how the utilization of productive factors n_h, n_l and k and front-load factors Ω_h, Ω_l and Ω_k vary with the minimum wage. Moreover, we also report the profile of wages, profits of the firm and aggregate welfare for different values of the minimum wage. We first show results for the case in which both labor factors are complements. One can think that this case reflects the situation in which the two types of labor correspond, for example, to skilled and unskilled labor. We then proceed to show results for the case in which the labor factors are substitutes. As commented before, we can associate this case to one in which both types of labor are unskilled but they differ in tenure or experience.

Throughout this section, we use the same parameter values and functional forms used in Section 3.5 in order to make them comparable. In the Appendix, we show that

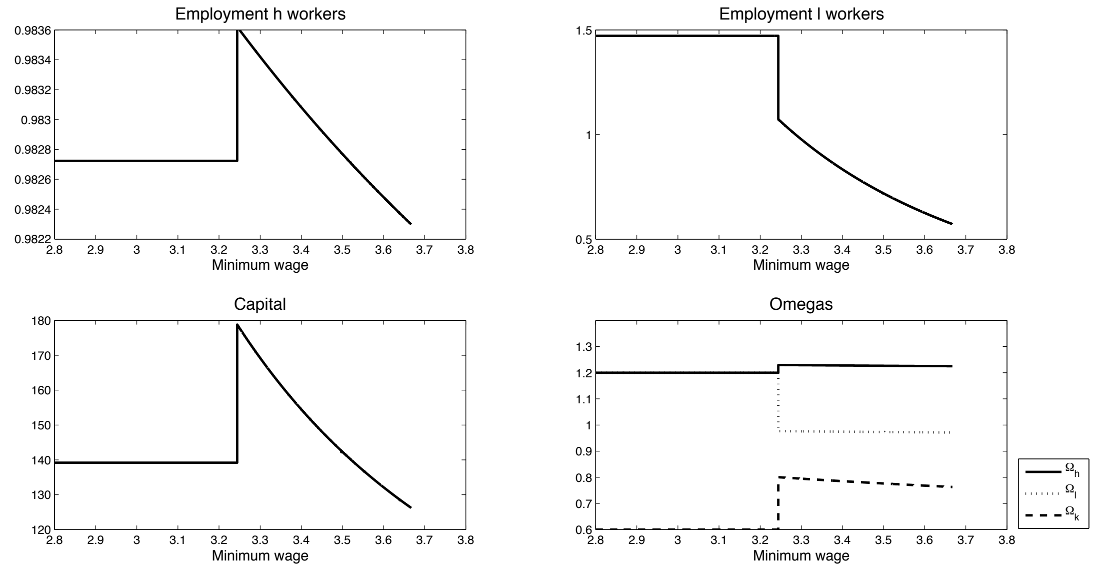

Figure 3: Factor utilization and front-load factors in general equilibrium- n_h and n_l as complements

the results are barely similar under constant returns to scale (with $\gamma = 1 - \alpha$).

4.4.1 h-type workers and l-type workers as complements

Figure 3 shows how the equilibrium values of labor and capital utilization change for different values of the minimum wage, when both types of labor are complements and θ_i and U_i adjust so as to equate the search cost that firms must face to the expected future profits from posting a vacancy.

Notice first that the behavior of the front-load factors with respect to the binding minimum wage is very similar to the case in which θ_i and U_i were kept constant (partial equilibrium). Consequently, the intuition provided in Section 3.5 still holds in the current case. The difference now arises because the search cost of the firm varies with θ_i. In particular, given the high demand of h-type workers from the firm, θ_h increases substantially and, consequently, so does the search costs of this type of workers. This translates into a subtle increase in n_h utilized in the production process. Since n_h and k are complements, this reduces the marginal productivity of capital (with respect to the partial equilibrium case) and, therefore, of capital acquired for production.

The lower use of k and n_h causes a decrease in the marginal productivity of l-type workers. As explained in Section 3.5, the demand for low-wage workers is influenced by two forces: on the one hand, the firm no longer has incentives to increase n_l in

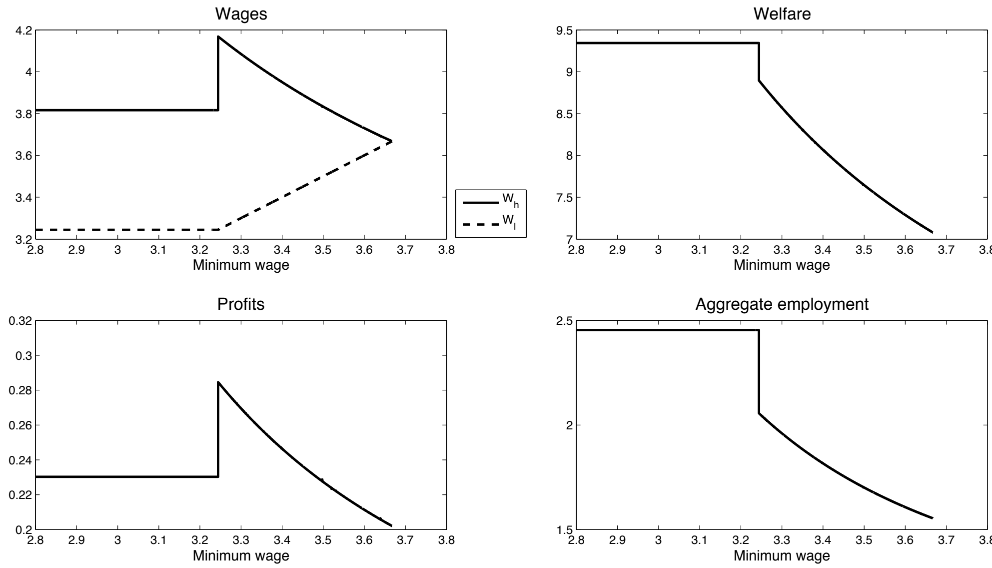

Figure 4: Wages, profits and welfare in general equilibrium - n_h and n_l as complements

order to reduce their wage, and this hinders the demand for such labor input. On the other hand, the mitigation of the holdup problem and the increase in the demand of h-type workers lead to an increase in the marginal product of l-type workers and, consequently, on its demand. In partial equilibrium, the latter effect dominates and n_l increases, though not substantially. In general equilibrium, however, the positive effects on n_l are much milder and the negative effect prevails.

As in the partial equilibrium case, when the minimum wage is high, the increase in the costs of production and the consequent reduction in profits imply that firms reduce their demand for all productive factors, thus offsetting the effects due to the alleviation of the holdup problem. For a high value of ω, n_h, n_l and k are lower than they would be were the minimum wage absent.

Figure 4 shows the equilibrium wages of h-type and l-type workers, and the profiles of welfare and firm profits for different values of the minimum wage. For a marginally binding minimum wage for low-wage workers, the wage of high-wage workers increases. This is due to the fact, given the high demand for these workers, the labor market tightness θ_h increases, causing a further increase in wages. Notice that, despite the increase in wages, profits of the firm increase as well.

Finally, the flow of aggregate welfare, computed as the flow in equation (31), decreases when the minimum wage becomes binding and thereafter. This is so because, despite the fact that the minimum wage partially corrects for the holdup problem, em-

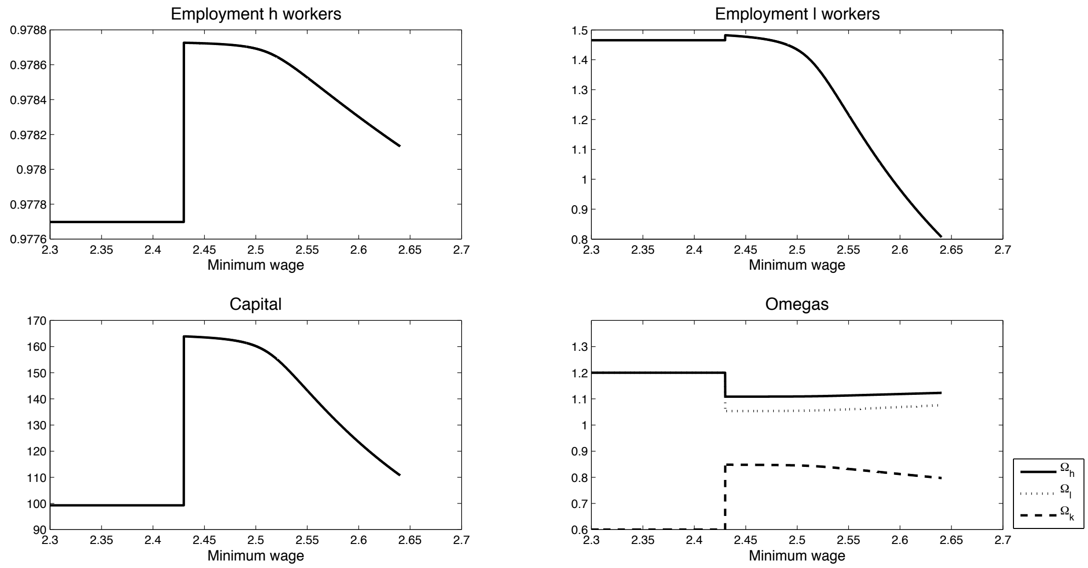

Figure 5: Factor utilization and front-load factors in general equilibrium- n_h and n_l as substitutes

ployment for l-type workers decreases and the search costs for h-type workers increase.

It can be easily shown that the equilibrium in this case is always internally consistent. This is the case because labor market tightness of l-type workers, θ_l decreases and, consequently, the value of the outside option rU_l for these workers decreases as well.

4.4.2 h-type workers and l-type workers as substitutes

Figure 5 shows how the equilibrium values of labor and capital utilization change for different values of the minimum wage, when both types of labor are substitutes and θ_i and U_i adjust so as to equate the search cost that firms must face to the expected future profits from posting a vacancy.

Once again, the effects of the minimum wage over the front-load factors are similar to the case of partial equilibrium, so they will not be repeated here. It is still the case, as in the previous section, that the increase in the demand of h-type workers causes the search cost associated to this type of workers to increase through a higher θ_h. This has a detrimental effect on the marginal productivity of capital and, as a consequence, on its demand, lowering k compared to the situation in partial equilibrium. In the present case, however, h-type and l-type workers are substitutes. The relative decrease in the demand of h-type workers translates into a relative increase in the marginal

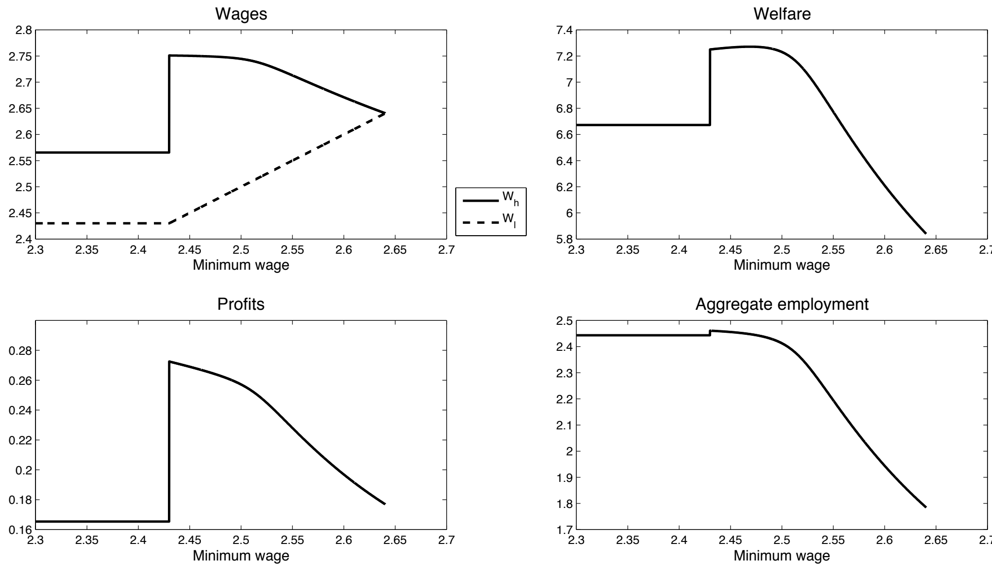

Figure 6: Wages, profits and welfare in general equilibrium - n_h and n_l as substitutes

productivity of l-type workers which, in turn, translates into a higher demand for l-type workers.

Figure 6 shows the equilibrium wages of high-wage and low-wage workers, and the profiles of welfare and firm profits for different values of the minimum wage. As in the previous case, for a marginally binding minimum wage for l-type workers, the wage of h-type workers increases due to the high demand for these workers that increases θ_h. It is still the case that profits of the firm increase even as wages increase, because workers are able to seize less rents of capital, reducing the holdup problem.

In the current example, the alleviation of the holdup problem is sufficiently strong to increase aggregate welfare, as shown in the upper right panel of Figure 6. We can understand this result in light of Proposition 5. In our parameterization, we have set $\beta = \eta = 0.5$ which, given the Cobb-Douglas matching function we use, assures that $\beta = \eta(\theta_i)$. Then, according to expression (34), the efficient steady state entails $\Omega_i = 1$ $\forall i$. Since the introduction of the minimum wage drives all the front-load factors closer to 1, the equilibrium moves toward the efficient one, thus increasing welfare.

For a particular range of the minimum wage considered, the equilibrium depicted in Figures 5 and 6 is not internally consistent.[19] This is due to the fact that, in this

[19] This can be easily checked by computing, for every possible value of the binding minimum wage, the wage rate that a marginal l-type worker would obtain if he renegotiated the wage with the firm, given by equation (17), and comparing these two wage rates. If $w_l^{ic} > \omega$, the equilibrium is not internally consistent.

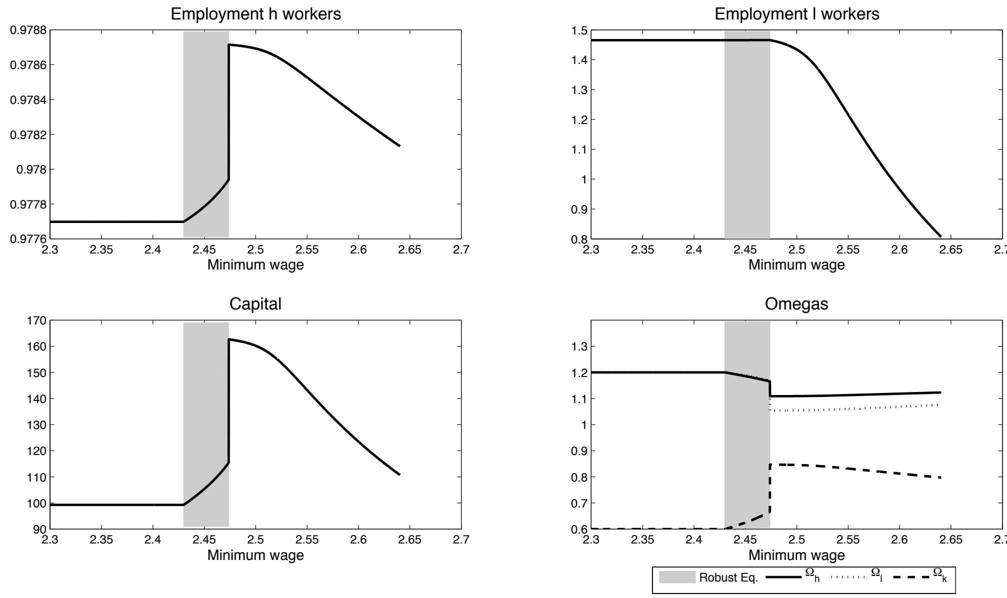

Figure 7: Factor utilization and front-load factors in robust general equilibrium - n_h and n_l as substitutes

range, the labor market tightness of l-type workers, θ_l, increases. Therefore, the value of the outside option of low-wage workers rU_l increases, rendering w_l^{ic} higher than the minimum wage ω. To overcome this problem, we follow Section 4.2 and compute the robust equilibrium when this is the case.

Figure 7 is analogous to Figure 5 and shows the equilibrium values of labor and capital utilization, and the front-load factors of the three production factors. The shaded area corresponds to the range of the minimum wage for which we compute the robust equilibrium. Notice that, in this case, employment of h-type workers and capital increase gradually after the minimum wage starts binding, and the corresponding front-load factors change gradually as well. This behavior of allocations is due to the fact that, in the robust equilibrium, only a *fraction* of l-type workers receive the minimum wage, while the remaining fraction negotiates the wage rate with the firm. As the minimum wage increases, this fraction increases, until a point in which the original equilibrium is internally consistent and all l-type workers perceive the minimum wage. [20] Then, the alleviation of the holdup problem is only gradual, and increases with ω. Notice that, when the minimum wage is marginally binding, employment of low-wage workers no longer increases (as in Figure 5) but rather stays constant, since the fraction

[20]Figure 9 in the appendix shows the evolution of the proportion of l-type workers that negotiate their wage with the firm, for every possible value of the minimum wage.

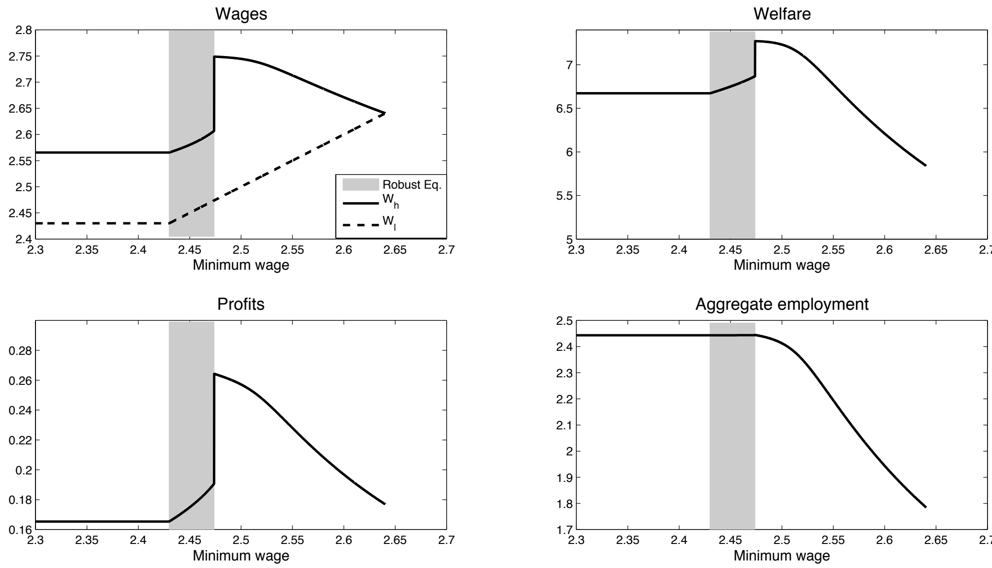

Figure 8: Wages, profits and welfare in robust general equilibrium - n_h and n_l as substitutes

of workers that negotiate their wage rate with the firm is close to one.

Figure 8 shows that, in a robust equilibrium, the evolution of wages, welfare, profits and aggregate employment also present smoother transitions when ω starts being binding. The qualitative behavior of wages, welfare and profits remains the same as in Figure 6. Aggregate employment increases when the minimum wage becomes binding, although this change is very close to zero. This is in line with some of the empirical evidence on the effect of the minimum wage over aggregate employment that have found such effect to be insignificant.

5 Extension: non-credible threats in bargaining

Our analysis in the previous sections has shown that our model can produce situations where the introduction of a binding minimum wage enhances the accumulation of capital without necessarily decreasing the aggregate stock of employment. This result potentially gives an explanation for the higher capital-output ratios observed in Continental Europe as compared to the US and is consistent with the empirical literature that describes an ambiguous impact of minimum wages on employment. Nevertheless, several papers, including the seminal contribution of Card and Krueger (1994), suggest that the ambiguity of the impact does not concern aggregate employment only, but also particular employment groups, including those directly affected by the minimum wage legislation. While our simulations show that under substitutability the demand for low-wage workers increase when a binding minimum wage is introduced,[21] the step from partial to general equilibrium actually smooths the increase in the employment of low-wage workers out.[22] This is intuitive: if the increase in demand pushes the outside option of those workers upwards, then those workers choose not to be paid the minimum wage anymore and have incentives to negotiate higher wages.

In this Section, we suggest a possible extension of our model that may generate situations where the employment of low-wage workers increase even in general equilibrium. The idea is to limit the influence of the outside option on the wage bargain. This prevents low-wage workers from negotiating wages above the minimum when the outside option goes up.

Since the contributions by Diamond (1982) and Mortensen (1982), it has become standard practice to use the axiomatic Nash solution to wage bargaining in search models. However, some authors such as Hall and Milgrom (2008), argue that this solution is based on unrealistic assumptions about bargaining threats: once a worker meets an employer, a threat to walk away, permanently terminating the bargain, is not credible. These authors thus propose a modelling framework where the threats are to extend bargaining rather than to terminate it. As a result, the outside option rU_i looses importance in the solution for wages as compared to the standard Nash solution. If both parties actually have incentives to extend the bargaining process forever, the $(1-\beta)rU_i$ term in equation (12) even disappear. This is for instance the case in some papers from the money search literature such as Trejos and Wright (1995),[23] yielding the following solution instead of (11):

$$\beta \frac{\partial \Pi(n_h, n_l, k)}{\partial n_i} = (1-\beta) W_i, \quad \forall i \in \{h, l\},$$

with the following constraint required for a solution to exist:

$$W_i \geq U_i, \quad \forall i \in \{h, l\}.$$

Under this wage rule, equations (12) and (14) read instead as

$$\tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) = \beta \left(\frac{\partial f(n_h, n_l, k)}{\partial n_i} - \sum_{j \in \{h,l\}} \chi_j \frac{\partial w_j(n_h, n_l, k, \chi_h, \chi_l)}{\partial n_i} n_j \right), \quad \forall i \in \{h, l\},$$

[21]See Figure 2.
[22]See Figure 6.
[23]See also the papers by Shi (1995) and Coles and Wright (1998).

and

$$\tilde{w}_i(n_h, n_l, k, \chi_h, \chi_l) = \beta\Omega_i(\chi_h, \chi_l)\frac{\partial f(n_h, n_l, k)}{\partial n_i}, \quad \forall i \in \{h, l\},$$

where the constant terms $(1-\beta)rU_i$ disappear.[24]

With this wage rule the general equilibrium is always internally consistent and there is no need for the robust specification described in Section 4. Moreover, the simulations in Figure 2 in the partial-equilibrium case extend to a general-equilibrium context. More precisely, the impact on the employment level of low-wage workers in Figure 6 is still positive.

6 Conclusions

Common wisdom among economists suggests that a minimum wage should have a negative effect over employment. The rationale for this is simple: a minimum wage acts as a price floor that reduces the demand for labor by firms because of the increase in its cost. However, a large body of literature that has tried to assess this effect in the data has found either that the introduction (or increase) of a minimum wage has an insignificant effect over employment, or even that this effect is positive.

In line with the previous argument, it is reasonable to think that a minimum wage encourages investment, by pulling down the relative price of capital. Acemoglu and Shimer (1999) and Acemoglu (2001) reinforce this idea by showing that a minimum wage may alleviate the holdup problem that firms face when workers, by negotiating wages with the firm, expropriate part of the returns on capital. The effect of a minimum wage on labor is still negative and, consequently, contrary to the evidence on the subject.

In this paper we study the effects of a minimum wage on labor and capital demand in a standard large firm model with search frictions in the labor market. We are able to generate an increase in the capital stock of the firm following the introduction of a minimum wage, and at the same time a null or slightly positive effect on employment through an increase in the demand for labor of firms. Moreover, the mechanism described in Acemoglu (2001) by which a minimum wage deters rent appropriation by workers and fosters investment is embedded in our model, and implies that, under some parameter configurations, the introduction of a minimum wage is welfare-improving.

Given the correspondence between the empirical findings mentioned before and the

[24] Of course, if the worker earns the flow utility b while negotiating, the negotiated wage would depend on b as well.

qualitative results of our model, we believe this type of model proves useful to quantitatively analyze the effects of a minimum wage over factor utilization under specific parameterizations. The model is also well suited to study issues of redistribution between types of workers and efficiency in the allocation of production factors. We leave these exercises for future research.

A Appendix: proofs

A.1 Front-load factors

In this section, we show how to obtain the expressions for the front-load factors in Proposition 1.

Absent a binding minimum wage, our economy reduces to a particular case of the models studied in Cahuc et al. (2008), with two types of labor and capital as factors of production. These authors use spherical coordinates to solve the system of differential equations given by equation (12). This leads to the wage expression (14) and the corresponding front-load factors $\Omega_h(1,1)$, and $\Omega_l(1,1)$.

To obtain the value of $\Omega_k(1,1)$, we first need to derive (14) with respect to k:

$$\frac{\partial \tilde{w}_j(n_h, n_l, k, \chi_h, \chi_l)}{\partial k} = \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial^2 f(n_h z, n_l z, k)}{\partial (n_j z) \partial k} dz, \quad \forall j \in \{h, l\}.$$

Given the expression above, we can integrate by parts $\sum_{j\in\{h,l\}} n_j \frac{\partial \tilde{w}_j(n_h,n_l,k,\chi_h,\chi_l)}{\partial k}$ as

$$\sum_{j\in\{h,l\}} n_j \frac{\partial \tilde{w}_j(n_h, n_l, k, \chi_h, \chi_l)}{\partial k} = \frac{\partial f(n_h, n_l, k)}{\partial k} - \frac{1-\beta}{\beta} \int_0^1 z^{\frac{1-2\beta}{\beta}} \frac{\partial f(n_h z, n_l z, k)}{\partial k} dz.$$

Plugging this expression in the capital-investment condition yields (15) and the respective expression for $\Omega_k(1,1)$.

When the minimum wage binds only in the case of l-type workers, one can calculate the wage of h-type workers by solving one differential equation given by (12). This differential equation can be solved as in Cahuc and Wasmer (2001a), where the variables n_l and k can be considered as parameters. This produces the expression for $\Omega_h(1,0)$ given in Proposition 1.

The values of $\Omega_l(1,0)$ and $\Omega_k(1,0)$ can be obtained in a similar way. We first need to derive (14) with respect to the relevant factor (n_l or k):

$$\frac{\partial \tilde{w}_h(n_h, n_l, k, \chi_h, \chi_l)}{\partial \gimel} = \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial^2 f(n_h z, n_l, k)}{\partial (n_h z) \partial \gimel} dz, \quad \forall \gimel = n_l, k,$$

where $\gimel$ refers to the relevant factor.

Given the expression above, we can integrate by parts

$$n_h \frac{\partial \tilde{w}_h(n_h, n_l, k, \chi_h, \chi_l)}{\partial \gimel} = \frac{\partial f(n_h, n_l, k)}{\partial \gimel} - \frac{1-\beta}{\beta} \int_0^1 z^{\frac{1-2\beta}{\beta}} \frac{\partial f(n_h z, n_l, k)}{\partial \gimel} dz, \quad \forall \gimel = n_l, k.$$

Plugging this expression in the capital-investment condition in the case of $\gimel = k$ yields (15), while by plugging it in the vacancy-posting condition in the case of $\gimel = n_l$ we get (13). The respective expressions for $\Omega_k(1,0)$ and $\Omega_l(1,0)$ are then deduced.

Finally the result $\Omega_i(0,0) = 1$, $\forall i \in \{h, l\}$, is straightforward.

A.2 Internal consistency

In this section we prove Propositions 2 and 3. We begin by obtaining w_l^{ic}, which is the wage that a marginal l-type worker would negotiate with the firm, if he were to deviate from the strategy of earning the minimum wage ω. From (16):

$$w_l^{ic} = \beta \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta) r U_l - \beta \frac{\partial w_h(n_h, n_l, k, 1, 0)}{\partial n_l} n_h.$$

Given expression (14) for w_h,

$$w_l^{ic} = \beta \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta) r U_l - \beta \int_0^1 \frac{\partial^2 f(n_h z, n_l, k)}{\partial n_h z \, \partial n_l} z^{\frac{1-\beta}{\beta}} n_h dz. \tag{37}$$

Integrating by parts the integral in the right hand side of the last expression, we obtain

$$\int_0^1 \frac{\partial^2 f(n_h z, n_l, k)}{\partial n_h z \, \partial n_l} z^{\frac{1-\beta}{\beta}} n_h dz = (1 - \Omega_l(1,0)) \frac{\partial f(n_h, n_l, k)}{\partial n_l}.$$

Plugging this in (37) we obtain

$$w_l^{ic} = \beta \Omega_l(1,0) \frac{\partial f(n_h, n_l, k)}{\partial n_l} + (1-\beta) r U_l.$$

In order to prove Proposition 3, consider the case in which $\omega = \tilde{w}_l(n_h, n_l, k, 1, 1) + \epsilon$. Then, if $\epsilon < 0$, equation (13) reads

$$(r+s)\frac{c}{q(\theta_l)} = \Omega_l(1,1) \frac{\partial f(n_h^{\tilde{w}}, n_l^{\tilde{w}}, k^{\tilde{w}})}{\partial n_l} - \tilde{w}_l(n_h^{\tilde{w}}, n_l^{\tilde{w}}, k^{\tilde{w}}, 1, 1). \tag{38}$$

Similarly, if $\epsilon > 0$,

$$(r+s)\frac{c}{q(\theta_l)} = \Omega_l(1,0) \frac{\partial f(n_h^{\omega}, n_l^{\omega}, k^{\omega})}{\partial n_l} - \omega. \tag{39}$$

For $\epsilon \to 0$, $\omega = \tilde{w}_l(n_h, n_l, k, 1, 1)$ and, from equations (38) and (39),

$$\Omega_l(1,1) \frac{\partial f(n_h^{\tilde{w}}, n_l^{\tilde{w}}, k^{\tilde{w}})}{\partial n_l} = \Omega_l(1,0) \frac{\partial f(n_h^{\omega}, n_l^{\omega}, k^{\omega})}{\partial n_l}.$$

From expression (17) for w_l^{ic},

$$
\begin{aligned}
w_l^{ic} &= \beta\Omega_l(1,0)\frac{\partial f(n_h^\omega, n_l^\omega, k^\omega)}{\partial n_l} + (1-\beta)rU_l^\omega,\\
&= \beta\Omega_l(1,1)\frac{\partial f(n_h^{\tilde{w}}, n_l^{\tilde{w}}, k^{\tilde{w}})}{\partial n_l} + (1-\beta)rU_l^\omega,\\
&= \tilde{w}_l(n_h^{\tilde{w}}, n_l^{\tilde{w}}, k^{\tilde{w}}, 1, 1),\\
&= \omega,
\end{aligned}
$$

since, in partial equilibrium, $U^{\tilde{w}} = U^\omega$. Then, when the minimum wage becomes binding and $\omega = \tilde{w}_l(n_h, n_l, k, 1, 1)$, the equilibrium is internally consistent. Now assume $\epsilon > 0$. From (39),

$$
\frac{\partial\left(\Omega_l(1,0)\frac{\partial f(n_h^\omega, n_l^\omega, k^\omega)}{\partial n_l}\right)}{\partial\omega} = 1. \tag{40}
$$

From the expression for w_l^{ic} (equation (37)), and using the previous result

$$
\frac{\partial w_l^{ic}}{\partial\omega} = \beta\frac{\partial\left(\Omega_l(1,0)\frac{\partial f(n_h^\omega, n_l^\omega, k^\omega)}{\partial n_l}\right)}{\partial\omega} = \beta \leq 1.
$$

Then, as ω increases, the wage that a worker would be able to negotiate with the firm if he deviated from the strategy of earning the minimum wage, increases less than proportional to the increase in the minimum wage. Consequently, the partial equilibrium is always internally consistent.

A.3 Robust equilibrium

Solving the problem stated in (22), the first-order conditions for vacancy posting and capital are, respectively,

$$
\frac{c}{q(\theta_h)} = \frac{\frac{\partial f(n_h,n_l,k)}{\partial n_h} - \tilde{w}_h(n_h,n_l,k,1,x) - \frac{\partial\tilde{w}_h(n_h,n_l,k,1,x}{\partial n_h}n_h - x\frac{\partial\tilde{w}_l(n_h,n_l,k,1,x}{\partial n_h}n_l}{r+s}, \tag{41}
$$

$$
\frac{c}{q(\theta_l)} = \frac{\frac{\partial f(n_h,n_l,k)}{\partial n_l} - (x\tilde{w}_l(n_h,n_l,k,1,x) + (1-x)\omega) - \frac{\partial\tilde{w}_h(n_h,n_l,k,1,x}{\partial n_l}n_h - x\frac{\partial\tilde{w}_l(n_h,n_l,k,1,x}{\partial n_l}n_l}{r+s}, \tag{42}
$$

$$
r+\delta = \frac{\partial f(n_h,n_l,k)}{\partial k} - \frac{\partial\tilde{w}_h(n_h,n_l,k,1,x)}{\partial k}n_h - x\frac{\partial\tilde{w}_l(n_h,n_l,k,1,x)}{\partial k}n_l. \tag{43}
$$

The equilibrium wage rates under Nash bargaining $\tilde{w}_i(\cdot)$ solve the following equations:

$$\beta \frac{\partial \Pi(n_h, n_l, k)}{\partial n_h} = (1-\beta)\left[W_h - U_h\right], \quad \forall i \in \{h, l\}, \tag{44}$$

$$\beta \frac{\partial \Pi(n_h, n_l, k)}{\partial x n_l} = (1-\beta)\left[W_l - U_l\right], \quad \forall i \in \{h, l\}, \tag{45}$$

where W_i is defined in (6) and the firm's surpluses $\partial\Pi(n_h, n_l, k)/\partial n_h$, $\partial\Pi(n_h, n_l, k)/\partial x n_l$ are calculated from the profits function of the firm. Notice that, in order to compute the marginal profit of the firm from hiring an additional l-type worker, one has to differentiate the profits function of the firm with respect to the amount of l-type workers that negotiate the wage by Nash bargaining, that is, xn_l, because these are the relevant actors in the negotiation. The marginal value of a worker $i \in \{h, l\}$ is, then,[25]

$$(r+s)J_i = \frac{\partial f(n_h, n_l, k)}{\partial n_i} - \frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial n_i} n_h - x \frac{\partial \tilde{w}_l(n_h, n_l, k, 1, x)}{\partial n_i} n_l.$$

This leads to the following system of differential equations to be solved:

$$\begin{cases} \tilde{w}_h(n_h, n_l, k, 1, x) = (1-\beta) r U_h + \beta \frac{\partial f(n_h, n_l, k)}{\partial n_h} - \beta \left(\frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial n_h} n_h + \frac{\partial \tilde{w}_l(n_h, n_l, k, 1, x)}{\partial n_h} x n_l \right) \\ \tilde{w}_l(n_h, n_l, k, 1, x) = (1-\beta) r U_l + \beta \frac{\partial f(n_h, n_l, k)}{\partial n_l} - \beta \left(\frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial n_l} n_h + \frac{\partial \tilde{w}_l(n_h, n_l, k, 1, x)}{\partial n_l} x n_l \right) \end{cases}$$

This system can be re-expressed as in Cahuc et al. (2008) by applying the change of variable

$$\hat{w}_l(n_h, n_l, k, 1, x) = x \tilde{w}_l(n_h, n_l, k, 1, x), \tag{46}$$

which yields

$$\begin{cases} \tilde{w}_h(n_h, n_l, k, 1, x) = (1-\beta) r U_h + \beta \frac{\partial f(n_h, n_l, k)}{\partial n_h} - \beta \left(\frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial n_h} n_h + \frac{\partial \hat{w}_l(n_h, n_l, k, 1, x)}{\partial n_h} n_l \right), \\ \hat{w}_l(n_h, n_l, k, 1, x) = (1-\beta) x r U_l + \beta x \frac{\partial f(n_h, n_l, k)}{\partial n_l} - \beta x \left(\frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial n_l} n_h + \frac{\partial \hat{w}_l(n_h, n_l, k, 1, x)}{\partial n_l} n_l \right). \end{cases}$$

The solution to this system corresponds to the one presented in Cahuc et al. (2008) in the case where workers are characterized by different bargaining powers. Specifically, the two bargaining powers to be considered are equal to β and βx, respectively. Hence,

[25] Notice that, strictly speaking, we should write $\partial f(n_h, n_l, k)/\partial x n_l$ and $\partial \tilde{w}_i(n_h, n_l, k, 1, x)/\partial x n_l$ in the equation for $\tilde{w}_l$. However, since at this stage the firm takes xn_l as a specific factor of production, it is the case that

$$\frac{\partial f(n_h, n_l, k)}{\partial n_l} = \frac{\partial f(n_h, n_l, k)}{\partial x n_l} \quad \text{and} \quad \frac{\partial \tilde{w}_i(n_h, n_l, k, 1, x)}{\partial n_l} = \frac{\partial \tilde{w}_i(n_h, n_l, k, 1, x)}{\partial x n_l}.$$

the solution to this system is

$$\begin{cases} \tilde{w}_h(n_h, n_l, k, 1, x) = (1-\beta)rU_h + \beta \int_0^1 \varphi(z) \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial (n_h z)} dz \\ \hat{w}_l(n_h, n_l, k, 1, x) = (1-\beta)xrU_l + \beta \int_0^1 \frac{z^{\frac{1-\beta x}{\beta x}}}{\beta} \frac{\partial f\left(n_h z^{\frac{1-\beta x}{(1-\beta)x}}, n_l z, k\right)}{\partial (n_l z)} dz, \end{cases} \tag{47}$$

from which the solution for $\tilde{w}_l$ can be deduced:

$$\tilde{w}_l(n_h, n_l, k, 1, x) = (1-\beta)rU_l + \beta \int_0^1 \frac{z^{\frac{1-\beta x}{\beta x}}}{\beta x} \frac{\partial f\left(n_h z^{\frac{1-\beta x}{(1-\beta)x}}, n_l z, k\right)}{\partial (n_l z)} dz.$$

After applying the change of variable $\zeta = z^{\frac{1-\beta x}{(1-\beta)x}} \Rightarrow dz = \frac{(1-\beta)x}{1-\beta x} \zeta^{-\frac{1-x}{1-\beta x}} d\zeta$ to the above equation, we obtain:

$$\tilde{w}_l(n_h, n_l, k, 1, x) = (1-\beta)rU_l + \beta \Omega_l(1, x) \frac{\partial f(n_h, n_l, k)}{\partial n_l}. \tag{48}$$

Using equation (5) to substitute rU_i in (47) and (48), we obtain expression (25) and (26) in the text. Expressions (23) and (24) are computed by plugging in the previous expressions for wages, and their correspondent derivatives, into the first order conditions of the firm (41) and (42).[26]

To obtain the capital front-load factor, we first need to compute

$$\begin{aligned} & n_h \frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial k} + x n_l \frac{\partial \tilde{w}_l(n_h, n_l, k, 1, x)}{\partial k} = \\ & \int_0^1 z^{\frac{1-\beta}{\beta}} \left[n_h \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial (n_h z) \partial k} + n_l \frac{(1-\beta)x}{1-\beta x} z^{-\frac{1-x}{1-\beta x}} \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial \left(n_l z^{\frac{(1-\beta)x}{1-\beta x}}\right) \partial k} \right] dz. \end{aligned}$$

Integrating by part the above expression yields

$$\begin{aligned} & n_h \frac{\partial \tilde{w}_h(n_h, n_l, k, 1, x)}{\partial k} + x n_l \frac{\partial \tilde{w}_l(n_h, n_l, k, 1, x)}{\partial k} = \\ & \qquad \frac{\partial f(n_h, n_l, k)}{\partial k} - \int_0^1 \varphi'(z) \frac{\partial f\left(n_h z, n_l z^{\frac{(1-\beta)x}{1-\beta x}}, k\right)}{\partial k} dz. \end{aligned}$$

By plugging this expression into the first-order condition for capital, we obtain the value for $\Omega_k(1, x)$.

[26] To obtain equations (23) and (24), it is necessary to carry out an integration by parts, as in Section A.1.

This completes the proof of Proposition 4.

A.4 Welfare

This section is dedicated to the proof of Proposition 5. More specifically, we show below how to obtain equations (32) and (33). Once these equations are obtained, it is straightforward to derive conditions (34)-(36) from the vacancy-posting and capital-investment conditions. In particular, equations (34)-(36) are obtained by comparing (32) and (33) with the first-order conditions in the case of the decentralized equilibrium. As a reminder, they correspond to the following conditions:

$$\frac{c}{q(\theta_i)} = \frac{(1-\beta)\Omega_i \frac{\partial f(n_h,n_l,k)}{\partial n_i} - \beta\theta_i c - (1-\beta)b}{r+s} \quad \text{when } \chi_i = 1, \quad \forall i \in \{h,l\},$$

$$\frac{c}{q(\theta_i)} = \frac{\Omega_i \frac{\partial f(n_h,n_l,k)}{\partial n_i} - \omega}{r+s} \quad \text{when } \chi_i = 0, \quad \forall i \in \{h,l\},$$

and

$$r + \delta = \Omega_k \frac{\partial f(n_h,n_l,k)}{\partial k}.$$

To obtain (32) and (33), first define $p(\theta) = \theta q(\theta)$. The first-order conditions of the program in (31) are

$$c = \frac{\partial V(n_h,n_l,k)}{\partial n_i} p'(\theta_i), \quad \forall i \in \{h,l\},$$

and

$$\frac{\partial V(n_h,n_l,k)}{\partial k} = 1.$$

By applying the envelope theorem, we get

$$(r + s - \theta_i^2 q'(\theta_i)) \frac{\partial V(n_h,n_l,k)}{\partial n_i} = \frac{\partial f(n_h,n_l,k)}{\partial n_i} - b, \quad \forall i \in \{h,l\},$$

and

$$(r+\delta)\frac{\partial V(n_h,n_l,k)}{\partial k} = \frac{\partial f(n_h,n_l,k)}{\partial k}.$$

Plugging these two equations in the first-order conditions yields equation (33) and

$$\frac{c}{p'(\theta_i)} = \frac{\frac{\partial f(n_h,n_l,k)}{\partial n_i} - b}{r + s - \theta_i^2 q'(\theta_i)}, \quad \forall i \in \{h,l\}.$$

Notice that $p'(\theta_i) = q(\theta_i)(1 - \eta(\theta_i))$. Hence,

$$c\left(r + s - \theta_i^2 q'(\theta_i)\right) = \left(\frac{\partial f(n_h, n_l, k)}{\partial n_i} - b\right) q(\theta_i)(1 - \eta(\theta_i)), \quad \forall i \in \{h, l\}.$$

By rearranging this equation, one can obtain equation (32).

B Appendix: numerical algorithms

We briefly describe here the numerical algorithms to solve the examples depicted in the main text. We first describe how to solve the example of Section 3.5 in partial equilibrium, and then proceed to describe how to obtain the results in general equilibrium discussed in Section 4.4.

B.1 Partial equilibrium

1. Specify the parameter values and functional forms according to Table 1 and expressions (18) and (19).

2. Suppose the minimum wage is not binding, therefore $\chi_h = \chi_l = 1$. Obtain the equilibrium values of θ_h, θ_l, n_h, n_l and k such that equations (13), (14), (15) and (21) are satisfied. In order to compute the integrals in $\Omega_i(1,1)$, $i = l, h, k$, use a Gauss-Legendre quadrature, suited to compute the area under a curve.[27]

3. Set the fixed values of θ_h and θ_l to the values found in 2.[28]

4. Compute the wages determined by Nash bargaining $\tilde{w}_i(n_h, n_l, k, 1, 1)$, $i = h, l$.

5. Construct a grid for the minimum wage such that $\omega \in [\omega_{min}; \omega_{max}]$. For every value of ω^j in the grid, check whether $\tilde{w}_l(\cdot) \lesseqgtr \omega^j$:
   (a) If $\tilde{w}_l(\cdot) \geq \omega^j$, then it must be the case that $\tilde{w}_h(\cdot) > \omega^j$. Set n_h^j, n_l^j, k^j to the values obtained in 2.
   (b) If $\tilde{w}_l(\cdot) < \omega^j$, set $\chi_l^j = 0$ and compute the new values of n_h^j, n_l^j, k^j such that equations (13), (14) and (15) are satisfied, for $w_h^j(\cdot) = \tilde{w}_h^j(\cdot)$ and $w_l^j(\cdot) = \omega^j$.
   (c) If $\chi_l^j = 0$, check whether $\tilde{w}_h^j \lesseqgtr \omega^j$. If $\tilde{w}_h^j \geq \omega^j$, the allocations and wages are the ones obtained in 5b. If, on the other hand, $\tilde{w}_h^j < \omega^j$, set $\chi_h^j = 0$ and compute the new values of n_h^j, n_l^j, k^j such that equations (13) and (15) are satisfied, for $w_h^j(\cdot) = \omega^j$ and $w_l^j(\cdot) = \omega^j$.

B.2 General equilibrium

1. Specify the parameter values and functional forms according to Table 1 and expressions (18) and (19).

[27]To compute the integral, we use 100 quadrature nodes.

[28]These values will be kept fixed throughout the exercise, since we are analyzing the partial equilibrium case.

2. Suppose the minimum wage is not binding, therefore $\chi_h = \chi_l = 1$. Obtain the equilibrium values of θ_h, θ_l, n_h, n_l and k such that equations (13), (14), (15) and (21) are satisfied.

3. Compute the wages determined by Nash bargaining $\tilde{w}_i(n_h, n_l, k, 1, 1)$, $i = h, l$.

4. Construct a grid for the minimum wage such that $\omega \in [\omega_{min}; \omega_{max}]$. For every value of ω^j in the grid, check whether $\tilde{w}_l(\cdot) \lesseqgtr \omega^j$:
   (a) If $\tilde{w}_l(\cdot) \geq \omega^j$, then it must be the case that $\tilde{w}_h(\cdot) > \omega^j$. Set n_h^j, n_l^j, k^j, θ_h^j, θ_l^j to the values obtained in 2.
   (b) If $\tilde{w}_l(\cdot) < \omega^j$, set $\chi_l^j = 0$ and compute the new values of n_h^j, n_l^j, k^j, θ_h^j, θ_l^j such that equations (13), (14), (15) and (21) are satisfied, for $w_h^j(\cdot) = \tilde{w}_h^j(\cdot)$ and $w_l^j(\cdot) = \omega^j$.
   (c) If $\chi_l^j = 0$, check whether $\tilde{w}_h^j \lesseqgtr \omega^j$. If $\tilde{w}_h^j \geq \omega^j$, the allocations and wages are the ones obtained in 4b. If, on the other hand, $\tilde{w}_h^j < \omega^j$, set $\chi_h^j = 0$ and compute the new values of n_h^j, n_l^j, k^j, θ_h^j, θ_l^j such that equations (13), (15) and (21) are satisfied, for $w_h^j(\cdot) = \omega^j$ and $w_l^j(\cdot) = \omega^j$.
   (d) Check whether the equilibrium is internally consistent. To this purpose, compute w_l^{ic} from equation (17). If $\omega^j \geq w_l^{ic}$ the equilibrium is internally consistent.
   (e) If $\omega^j < w_l^{ic}$, compute the robust allocations such that equations (23)-(27) are satisfied, for $w_l^r = \omega^j$.

C Figures

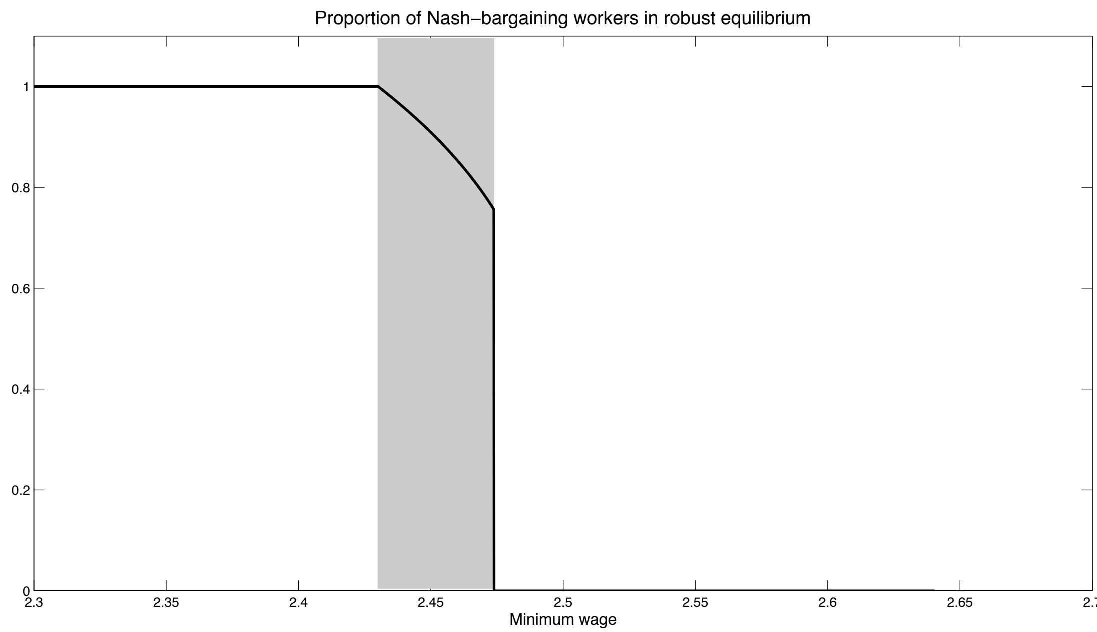

Figure 9: Proportion of l-type workers that negotiate their wage with the firm in robust general equilibrium - n_h and n_l as substitutes

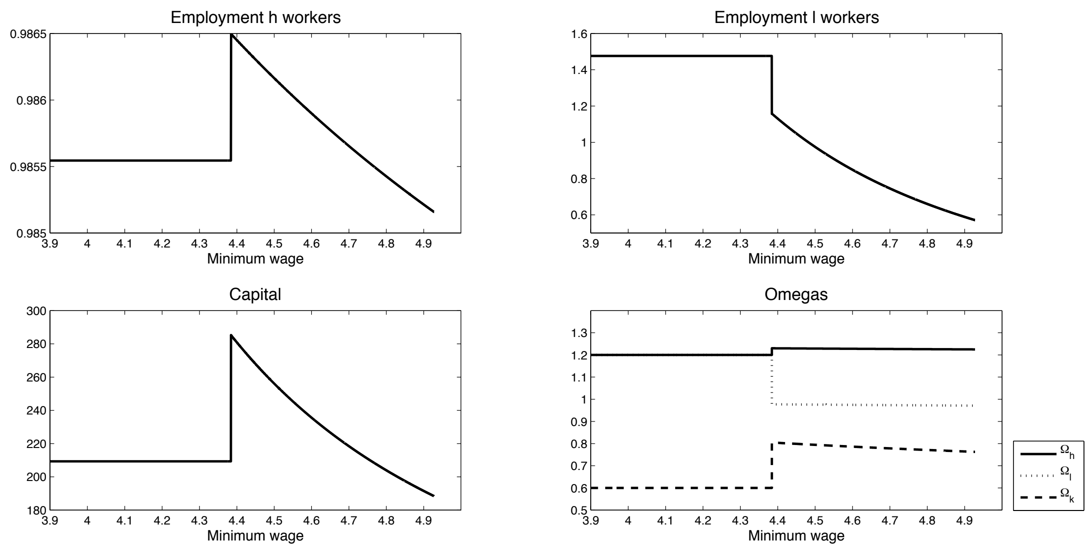

Figure 10: Factor utilization and front-load factors in general equilibrium- n_h and n_l as complements - CRS production function

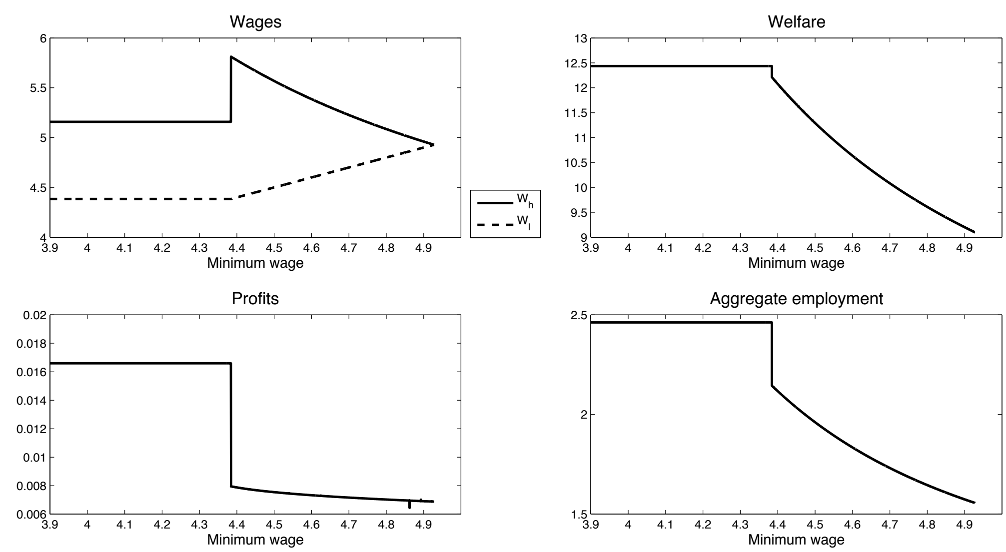

Figure 11: Wages, profits and welfare in general equilibrium - n_h and n_l as complements - CRS production function

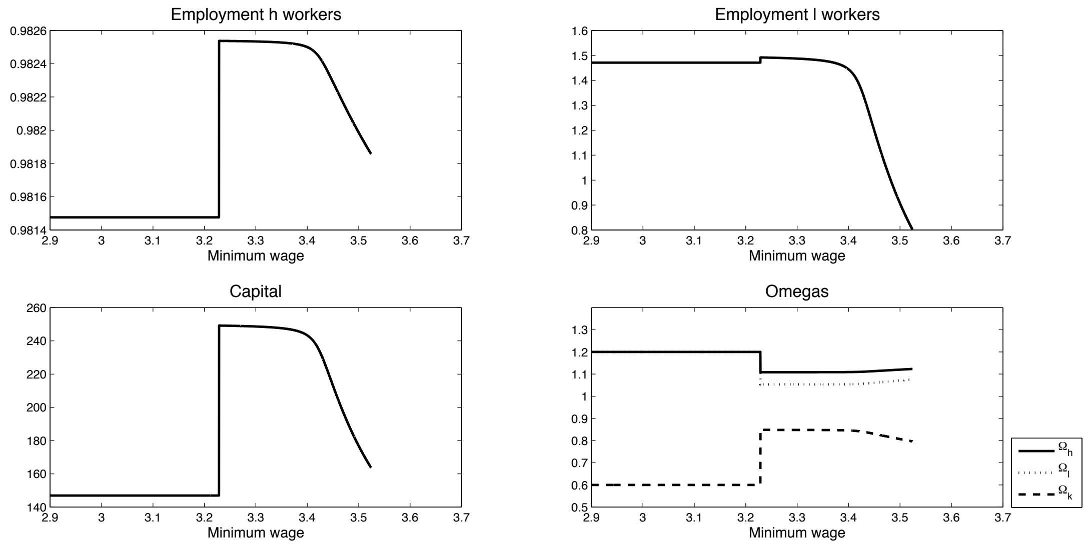

Figure 12: Factor utilization and front-load factors in general equilibrium- n_h and n_l as substitutes - CRS production function

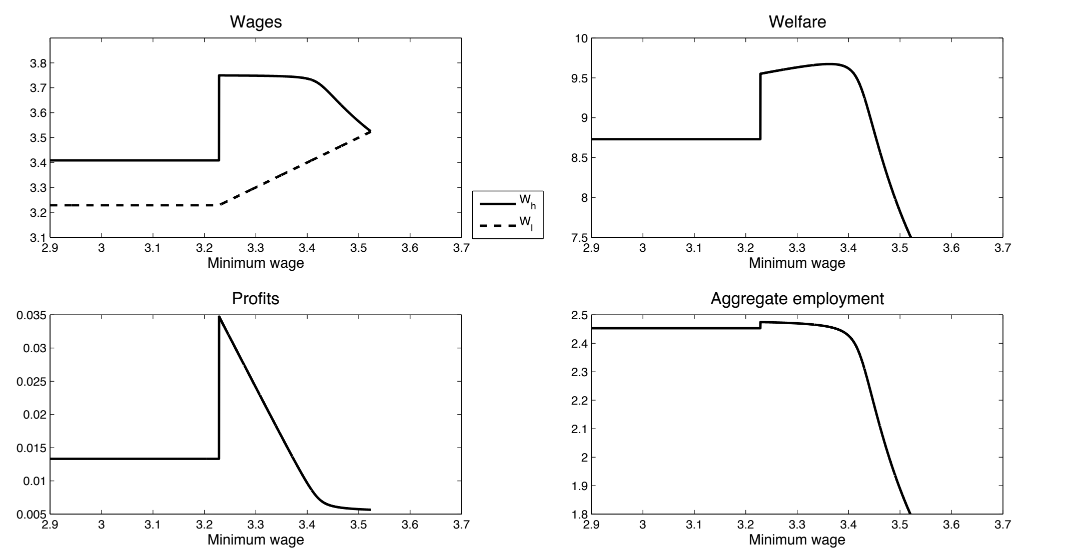

Figure 13: Wages, profits and welfare in general equilibrium - n_h and n_l as substitutes - CRS production function

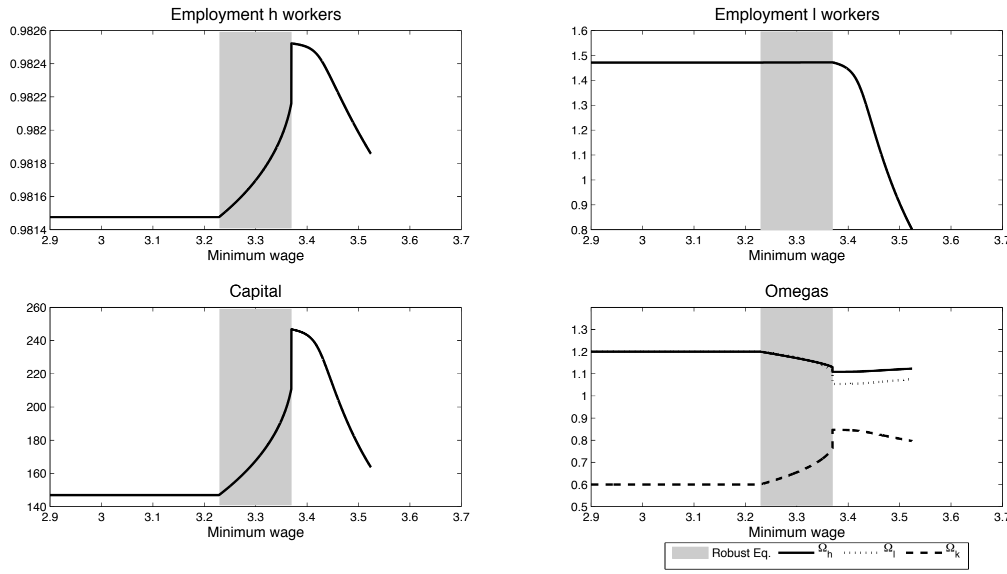

Figure 14: Factor utilization and front-load factors in robust general equilibrium - n_h and n_l as substitutes - CRS production function

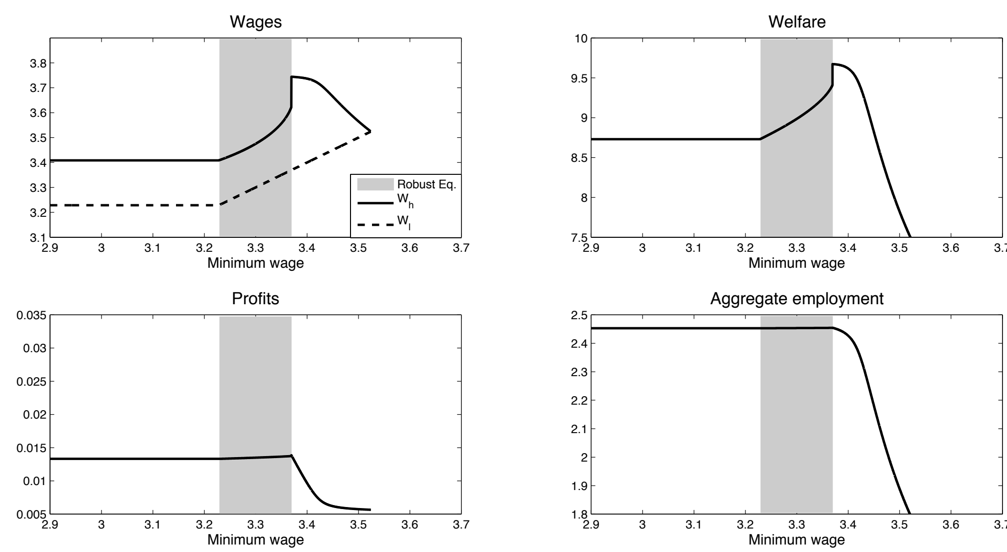

Figure 15: Wages, profits and welfare in robust general equilibrium - n_h and n_l as substitutes - CRS production function

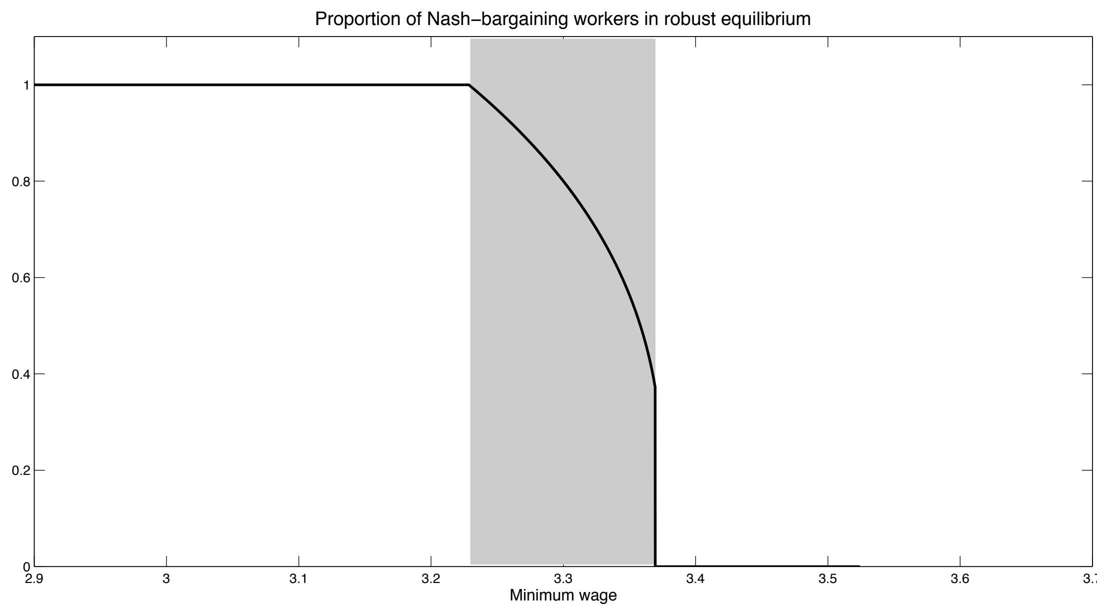

Figure 16: Proportion of l-type workers that negotiate their wage with the firm in robust general equilibrium - n_h and n_l as substitutes - CRS production function

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