Asymmetric Partnerships*

Nicolás Figueroa, Universidad de Chile † **Vasiliki Skreta**, NYU‡

February 14, 2011

Abstract

We study asymmetric partnerships and show that the surplus (gains from trade minus informational rents) is maximized if all agents' payoffs at their valuations where gains of trade are minimal are equal. Moreover, we show that such property rights exist and allow efficient dissolution. For this to hold, the agent that most likely has the highest valuation for an asset should initially own a bigger share of it. We discuss implications of these findings for the design of negotiation agendas, partnerships and joint ventures. JEL classification codes: C72, D82, L14. Keywords: *efficient mechanism design, ownership structure, partnerships.*

*We are grateful to Mariagiovanna Baccara, Heski Bar-Isaac, Luis Cabral, Barbara Katz, Roger Myerson, Phil Reny and Ennio Stacchetti for useful discussions and comments and to Jorge Catepillán for excellent research assistance. We also benefited from comments of the audiences at Northwestern University, and at the University of Chicago.

† *Centro de Economía Aplicada, Universidad de Chile, República 701, Santiago, Chile.; nicolasf@dii.uchile.cl;*

‡ *Leonard Stern School of Business, Kaufman Management Center, 44 West 4th Street, KMC 7-64, New York, NY 10012, Email:vskreta@stern.nyu.edu*

1. Introduction

An important economic insight is that the presence of asymmetric information seriously hinders the ability of negotiating parties to achieve mutually beneficial agreements. The seminal paper by Myerson and Satterthwaite (1983) considers a bilateral trading environment with double-sided asymmetric information and shows that no feasible ex-post efficient negotiation procedure exists when gains from trade are uncertain. For this reason, asymmetric information is viewed as a serious form of transaction costs in Coase's tradition. On the other hand, the work by Cramton, Gibbons and Klemperer (1987) shows that if a group of ex-ante identical agents jointly own an asset in equal (or close to equal) shares, then it is possible to give complete control to the partner with the highest ex-post valuation. They conclude that similar property rights are a key factor in determining whether or not efficiency is achievable.

In this paper, we consider a partnership environment where partners' valuations are private information but are drawn from different distributions. We investigate which ownership structures make efficient dissolution possible, why they do so, and what is the relationship between the degree of asymmetries across partners and the ownership structures that make efficient dissolution possible. Along the way, we shed new light to the impossibility result of Myerson and Satterthwaite (1983) and the possibility one of Cramton, Gibbons and Klemperer (1987).

We show that efficient dissolution is possible if all agents' payoffs are equal at their valuations where gains of trade are minimal ($\pi_i(v_i^*) = \pi_j(v_j^*)$). For the standard case of linear valuations and symmetric distributions (as in Cramton, Gibbons and Klemperer (1987)), the condition $\pi_i(v_i^*) = \pi_j(v_j^*)$ holds with equal property rights. However, for asymmetric environments, we show that the property rights that guarantee $\pi_i(v_i^*) = \pi_j(v_j^*)$ can be extremely unequal. Moreover, we show that agents who most likely have the highest valuation, must initially own a bigger share of the asset.

The analysis proceeds as follows: First, we show that an agent's payoff at the critical type is the marginal cost of increasing his ownership share while maintaining voluntary participation. Therefore, if at an ownership structure it holds that $\pi_i(v_i^*) = \pi_j(v_j^*)$ for any pair of agents, then this ownership structure is transfer-minimizing. We then show that the transfer-minimizing partnership is efficiently dissolvable.

Our results can shed light on the problem of efficient allocation of new technologies among various firms. In a patent race, initial property rights can be seen as the probability that each agent will win the race. Then, our findings indicate that to have efficient trade, the firms that are better at inventing (that is, have the bigger "initial share") should also have a higher capacity for developing applications *after* the technology is discovered (a better distribution of valuations).[1] This could provide a rationale for the

[1] This is often not true, as can be seen in the case of the technology used in BlackBerry mobile devices. Research in Motion (RIM), the developer of BlackBerry, did not own the rights to the technology and fought a costly litigation for more than three years with NTP. NTP owned the rights to the technology, but it is primarily a patent-owning company, with no ability to directly develop products and profit from the patents. There is extensive press coverage of this lawsuit. For a sample, see "Detractors of BlackBerry See Trouble Past Patents," *The New York Times*, March 6, 2006.

integration of research departments into big firms. Integration helps avoid lost profits due to transaction costs associated with incomplete information.

2. Trading Mechanisms with Co-ownership (Partnerships)

There are I risk-neutral agents. Agent i's payoff from owning a fraction r of the asset is $r \cdot \pi_i(v_i)$,where π_i is strictly increasing and convex in v_i. Types v_i are independently distributed according to F_i on $V_i = [\underline{v}_i, \overline{v}_i]_i$, with $0 \leq \underline{v}_i \leq \overline{v}_i < \infty$. The partnership is characterized by the initial property rights, $Q = (r_1, ..., r_I)$ with $\Sigma_{i \in I} r_i = 1$. Agent i's payoff at the status quo $Q = (r_1, ..., r_I)$, is $\underline{U}_i(v_i) = r_i \cdot \pi_i(v_i)$.

At an ex-post efficient assignment the agent with the highest valuation is awarded exclusive ownership of the asset and the *total social surplus is* $W(v) = \max_i \pi_i(v_i)$. Then, the expected payoff for agent i, when his valuation is v_i is $U_i(v_i) = \prod\limits_{j \neq i} F_j(\pi_j^{-1}(\pi_i(v_i))) \cdot \pi_i(v_i) + E_{v_{-i}}[x_i(v)]$, where $\prod\limits_{j \neq i} F_j(\pi_j^{-1}(\pi_i(v_i)))$ is the probability that he has the highest valuation, given type v_i. Voluntary participation requires that $U_i(v_i) \geq r \cdot \pi_i(v_i)$.

We ask when can we expect to find efficient and incentive-feasible mechanisms that satisfy voluntary participation without outside transfers?

By the revelation principle, it is without any loss to restrict attention to incentive-compatible direct mechanisms. From the revenue equivalence theorem, we know that all incentive-compatible mechanisms that implement the same allocation rules generate the same expected payoff for each agent up to a constant. Therefore, the interim information rent of an agent is identical for all incentive-compatible and efficient mechanisms up to a constant. As is well known, a Vickrey-Clarke-Groves (VCG) mechanism is efficient and incentive-compatible. Hence, when we are interested in properties of efficient mechanisms we can, without loss, restrict attention to VCG mechanisms: Moreover, at a VCG mechanism, an agent's interim payoffs are equal to the expected gains from trade plus a constant; that is,

$$U_i(v_i) = E_{v_{-i}}[W(v)] + K_i. \tag{1}$$

The transfer-minimizing ownership structure Now, for any agent i, consider $K_i^*(r_i) = \max_{v_i}[r_i \pi_i(v_i) - E_{v_{-i}} W(v_i, v_{-i})]$ and let v_i^* be a maximizer. $K_i^*(r_i)$ is the smallest constant that must be added to the standard VCG mechanism so that the participation constraints are satisfied. Then, finding the ownership structure that minimizes the sum of transfers necessary to guarantee the agents' voluntary participation, is equivalent to solving

$$\min_{\{r_i\}_{i \in I}} \Sigma_{i \in I} K_i^*(r_i) \text{ subject to } \Sigma_{i \in I} r_i = 1. \tag{2}$$

In some sense, designing property rights that enable the efficient dissolution of the partnership can be seen as a problem of optimally allocating resources: The more property rights are given to an agent, the more he would have to be paid later in the mechanism. The next result, which comes from an envelope condition, derives the marginal cost (in terms of transfers) of increasing the property rights of an agent.

Lemma 1 *Suppose that the ownership share of agent i is marginally increased. Then, the marginal increase in the transfers necessary to guarantee voluntary participation is*

$$\frac{dK_i^*(r_i)}{dr_i} = \pi_i(v_i^*).$$

Proof. The marginal increase in the transfers necessary to guarantee voluntary participation is

$$\begin{aligned}\frac{dK_i(r_i)}{dr_i} &= \pi_i(v_i^*) + r_i \frac{\partial \pi_i(v_i^*)}{\partial v_i}\frac{\partial v_i^*}{\partial r_i} - \frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i}\frac{\partial v_i^*}{\partial r_i} \\ &= \pi_i(v_i^*) - \frac{\partial v_i^*}{\partial r_i} \cdot \left(\frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i} - r_i \frac{\partial \pi_i(v_i^*)}{\partial v_i} \right) \\ &= \pi_i(v_i^*), \text{ for all } i\end{aligned}$$

where the last inequality follows from the fact that at an (interior) critical type, we have $\frac{\partial E_{v_{-i}} W(v_i^*, v_{-i})}{\partial v_i} - r_i \frac{\partial \pi_i(v_i^*)}{\partial v_i} = 0.$ ■

With this, we can shed light on the impossibility result in Myerson and Satterthwaite (1983) and the possibility result in Cramton, Gibbons and Klemperer (1987). We show that at the transfer-minimizing ownership structure, we have that $\pi_i(v_i^*) = \pi_j(v_j^*)$ for all $i, j \in I$. This is because the marginal transfer that must be paid to an agent i if his property rights are increased, is his valuation at the "critical type" $\pi_i(v_i^*)$. Then, if $\pi_i(v_i^*) \neq \pi_j(v_j^*)$, there is an obvious way to reduce the transfers: by reducing the property rights of an agent with a high valuation at the critical type, and redistributing it to an agent with a lower one. Note, however, that this may imply property rights that are extremely unequal, as in Example 1.

Proposition 1 *Consider a partnership and suppose that π_i is strictly increasing in v_i, for all agents. If all valuation functions have the same range ($\pi_i(\underline{v}_i) = \underline{\pi}$, and $\pi_i(\bar{v}_i) = \bar{\pi}$), then, the transfer-minimizing property rights $(r_1, ..., r_I)$ are such that $\pi_i(v_i^*) = \pi_j(v_j^*)$ for all $i, j \in I$. Moreover, such property rights exist.*

Proof. The first part is direct, since the first-order conditions of (2) imply $\frac{dK_i(r_i)}{dr_i} = \frac{dK_j(r_j)}{dr_j}$, which from Lemma 1 becomes $\pi_i(v_i^*) = \pi_j(v_j^*)$.

Now, we establish that for any distributions F_i, $i \in I$, there exists an initial ownership structure $(r_1, ..., r_I)$ that guarantees that $\pi_i(v_i^*) = \pi_j(v_j^*)$ holds.

First, note that at interior critical types, participation and non-participation payoffs must be tangent, implying that

$$\prod_{\substack{j \in I \\ j \neq i}} F_j(\pi_j^{-1}(\pi_i(v_i^*)))\pi_i'(v_i^*) = r_i \cdot \pi_i'(v_i^*). \tag{3}$$

Defining $G_i(s) = \prod_{\substack{j\in I \\ j\neq i}} (F_j \circ \pi_j^{-1})(s)$, and noticing that it is invertible (since it is strictly increasing), (3) can be rewritten as

$$\pi_i(v_i^*) = G_i^{-1}(r_i). \tag{4}$$

Therefore, for $\pi_i(v_i^*) = \pi_j(v_j^*)$ to be true, we must have

$$G_i(G_j^{-1}(r_j)) = r_i \text{ for all } i, \tag{5}$$

which, for a given r_i determines r_j. Now, we only have to check the consistency requirement that $\sum_{1=1}^{I} r_i = 1$, which using (5) becomes

$$r_1 + \Sigma_{i=2}^{I} G_i \left(G_1^{-1}(r_1)\right) = 1. \tag{6}$$

Equation (6) has a solution since the LHS is equal to 0 at $r_1 = 0$, it is greater than 1 at $r_1 = 1$ and it is a continuous function of r_1. ■

In a symmetric linear environment (like the one in Cramton, Gibbons and Klemperer (1987)), equality of payoffs at the critical types is equivalent to equality of property rights r_i . This is one way to explain what is going on behind the possibility result of Cramton, Gibbons and Klemperer (1987): In a symmetric environment, equal property rights maximize the expected surplus of a mechanism by equating critical types.[2] On the other hand, extreme property rights, as in Myerson and Satterthwaite (1983), imply that the critical type for the seller is his highest valuation, while the critical type for the buyer is his lowest valuation, thus maximizing the difference in agents' payoffs at their critical types.

From Proposition 1, it follows directly that in order to design efficiently dissolvable partnerships, one should aim for environments where the valuations of critical types are the same across agents. Our next Proposition shows that when this is true, efficiency is indeed feasible.

Proposition 2 *Consider a partnership. If property rights* $(r_1, ..., r_I)$ *are such that*

$$\pi_i(v_i^*) = \pi_j(v_j^*), \text{ for all } i \text{ and } j, \tag{7}$$

then there exists a feasible and ex-post efficient mechanism.

Proof. From the analysis of Schweizer (2006) we know that it is possible to design an ex-post efficient mechanism iff the maximized sum of all agents' utilities minus information rents (incentive costs): $\max_{i\in I}\{\pi_i(v_i)\}] - \sum_{i\in I} E\left[\max_{i\in I}\{\pi_i(v_i)\} - \max\{\pi_i(v_i^*), \pi_{-i}(v_{-i})\right]$ and outside options (participation costs):

[2] In Proposition 1, we considered the case in which the range of valuation functions is the same. When this is not true, we may be at a 'corner' solution where the marginal costs of participation of each agent are not equal.

$\sum_{i\in I} r_i \cdot \pi_i(v_i^*)$ is non-negative:

$$E\left[-(I-1)\max_{i\in I}\{\pi_i(v_i)\} + \sum_{i\in I}\max\{\pi_i(v_i^*), \pi_{-i}(v_{-i})\}\right] - \sum_{i\in I} r_i \cdot \pi_i(v_i^*) \geq 0. \tag{8}$$

A sufficient condition for (8) is that

$$-(I-1)\max_{i\in I}\{\pi_i(v_i)\} + \sum_{i\in I}\max\{\pi_i(v_i^*), \pi_{-i}(v_{-i})\} \geq \sum_{i\in I} r_i \cdot \pi_i(v_i^*). \tag{9}$$

We just need to verify that (9) is satisfied whenever (7) is satisfied.

Suppose that, for some vector of valuations v, we have that

$$\pi_k(v_k) = \max_{i\in I}\{\pi_i(v_i)\}.$$

Then, for this vector of valuations (9) holds, if the following holds:

$$-(I-1)\pi_k(v_k) + (I-1)\pi_k(v_k) + \max\{\pi_k(v_k^*), \pi_{-k}(v_{-k})\} \geq \sum_{i\in I} r_i \pi_i(v_i^*),$$

which always does whenever $\pi_i(v_i^*) = \pi_j(v_j^*)$. Since this holds for any v, (9) always holds. ■

Proposition 2 is related to Proposition 2 in Schweizer (2006), which states that if the social surplus is a linear function of the information profile and a convex function of the collective decision, there exists a default option such that efficient negotiations are possible. In relation to Schweizer (2006), we not only state that there exist property rights that guarantee the efficient dissolution of an asymmetric partnership, but we also identify that the critical condition is $\pi_i(v_i^*) = \pi_j(v_j^*)$. More importantly, we discover (cf. Lemma 1) *why* it is crucial: An agent's payoff at the critical type is the marginal cost of increasing his ownership share while maintaining voluntary participation. Finally, below we also explain what are the economic consequences of that condition: ex'ante property rights have to be well aligned with the distribution of the ex-post realized valuation of the asset.

For the linear case where $\pi_i(v_i) = v_i$, (7) reduces to $v_i^* = v_j^*$.[3] If, moreover, all types are distributed according to the same distribution F, we have that $v_i^* = v_j^*$ is satisfied if $r_i \equiv \frac{1}{I}$, exactly as in Cramton, Gibbons and Klemperer (1987). However, when distributions are asymmetric, the property rights that guarantee the condition in Proposition 2 can be extremely unequal:

Example 1 *Consider a partnership with two agents, $\pi_i(v_i) = v_i$, $F_1(v_1) = v_1^n$ and $F_2(v_2) = v_2^{\frac{1}{n}}$. Then, it is easy to see that $v_1^* = F_2^{-1}(r_1) = r_1^n$ and $v_2^* = F_1^{-1}(r_2) = r_2^{\frac{1}{n}}$. From Proposition 2, we know that if $v_1^* = v_2^*$, efficient dissolution is possible. For this example, this condition is equivalent to $r_1^n = r_2^{\frac{1}{n}}$. Recalling that $r_1 + r_2 = 1$, this reduces to*

$$r_1^n = (1-r_1)^{\frac{1}{n}}.$$

[3] This condition was discovered co-currently and independently by Che (2006).

For $n = 3$ we obtain $r_1 = 0.8243$ and $r_2 = 0.1757$, which give us that $v_1^* = v_2^* = 0.56009$. Moreover, a simple calculation shows that for these distributions, with property rights of $r_i = \frac{1}{2}$, there is no possibility of efficient dissolution. For $n = 99$, optimal property rights are even more extreme: $r_1 = 0.99926$ and $r_2 = 0.00074$. For this case, the corresponding critical types are $v_1^* = v_2^* = 0.92933$.

Example 1 shows that, for certain distributions, very extreme property rights are needed in order to have efficient dissolution of the partnership, quite contrary to the intuition one gets from the discussion of symmetric environments. In fact, for the very extreme case of $n = 99$, property rights very close to $(1, 0)$ are needed. However, from Myerson and Satterthwaite (1983), we know that $(r_1, r_2) = (1, 0)$ will never allow an efficient dissolution. In fact, our example shows that arbitrarily close-to-extreme property rights can be needed for efficient dissolution, but property rights that *are* extreme will never allow it. Moreover, this example suggests that agent 1, whose valuation is more likely to be higher, must own a higher proportion of the asset and vice versa. This turns out to be a general result with interesting economic consequences:

Corollary 1 *Let us suppose that*

$$F_1 \circ \pi_1^{-1}(\cdot) \geq F_2 \circ \pi_2^{-1}(\cdot) \geq \ldots \geq F_I \circ \pi_I^{-1}(\cdot).^4 \tag{10}$$

Then, the property rights that guarantee the possibility of efficient dissolution satisfy $r_1 \leq r_2 \leq \ldots r_I$.

Proof. We know that at the critical types $\prod_{j \neq i} F_j(\pi_j^{-1}(\pi_i(v_i^*))) = r_i$. Moreover, for the property rights that guarantee dissolution, we have $\pi(v_i^*) \equiv x$ for all $i \in I$. Therefore, we have $r_i = \prod_{j \neq i} [F_j \circ \pi_j^{-1}](x)$, from which we obtain that $\frac{r_i}{r_j} = \frac{[F_j \circ \pi_j^{-1}](x)}{[F_i \circ \pi_i^{-1}](x)} > 1$. ■

Our findings generalize straightforwardly to the case where a partnership owns multiple assets.[5]

References

[1] Che, Y. K. (2006): "Beyond the Coasian Irrelevance: Asymmetric Information." Unpublished Notes.

[2] Cramton, P., R. Gibbons, and P. Klemperer(1987): "Dissolving a Partnership Efficiently," *Econometrica*, 55, 615-632.

[3] Myerson R. B. and M. A. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29(2):p. 265–281.

[4] The conditions in (10) are equivalent to the distributions of valuations being ordered according to FOSD (first-order stochastic dominance) since $F_i \circ \pi_i^{-1}(x) = P(\pi_i(v_i) \leq x)$.

[5] Details are available from the authors upon request.

[4] Schweizer, U. (2006): "Universal Possibility and Impossibility Results," *Games and Economic Behavior*, 57, 73-85.