What to Put on the Table[*]

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Abstract

This paper investigates the circumstances under which negotiating simultaneously over multiple issues or assets helps reduce inefficiencies due to the presence of asymmetric information. Consider the case where one agent controls all assets. One would expect that strong substitutabilities among assets, would help trade, as the seller would value the additional units less. We show that this intuition is true only if assets are heterogeneous. If assets are homogeneous, efficiency is never possible, irrespective of the degree of substitutability or complementarity among them. When ownership is dispersed, in the sense that different assets are owned by different agents, efficiency is actually more common when assets are homogeneous. When assets are heterogeneous, efficiency can be possible only when assets are complements. JEL classification codes: C72, D82, L14. Keywords: *efficient mechanism design, multiple units, complements, substitutes, ownership structure, partnerships.*

Many important economic and political decisions are determined through negotiations: They determine the terms of firm acquisitions,[1] of mergers, and of labor contracts and play a key role in international treaties, constitutional reforms, and dispute resolutions. There are usually multiple issues at stake and money often changes hands, as in the cases of M&As and of labor contracts. An important economic insight is that the presence of asymmetric information seriously hinders the ability of negotiating parties to achieve mutually beneficial agreements. The seminal paper by Myerson and Satterthwaite (1983) shows in a bilateral trading environment with double-sided asymmetric information that no feasible ex-post efficient negotiation procedure exists when gains from trade are uncertain. For this reason, asymmetric information is viewed as a serious form of transaction costs in Coase's tradition.

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[1]Recent empirical work by Boone and Mulherin (2007) suggests that about half of company sales are performed via negotiations.

In many situations, negotiating parties have the option to put more than one issue on the table: In multilateral trade negotiations, a large number of issues are discussed simultaneously. In complex mergers, the ownership of many assets is on the table at the same time. Another example is labor markets for professional sports players. There, the ownership of assets (players' rights) is determined via negotiations, which usually involve multiple players and cash. Each team's valuation for a particular player is private information; players are heterogeneous across multiple characteristics and, most importantly, present strong complementarities and substitutabilities with each other. All these facts make them sometimes expendable for one team, but critical to the success of another. Then, what forces determine whether they can be efficiently allocated given the presence of asymmetric information? Perhaps surprisingly, the economics literature so far has very little to say about such multi-issue negotiations, despite the fact that single-issue negotiations are more the exception than the rule.

The goal of this paper is to study multi-issue negotiations under the presence of asymmetric information, complementarities and substitutabilities among assets, and to ask under which circumstances efficiency is possible. In our formulation, an agent's payoff from a given settlement is a function of his private information (which can be multidimensional). In order to investigate whether there is any conceivable negotiation procedure that leads to efficiency, we use tools of the mechanism design literature. Formally, we ask: In which negotiation scenarios can we expect to find incentive-feasible mechanisms that satisfy voluntary participation without outside transfers? The answer to this question provides insight into how to design the agenda of negotiations (what to put on the table) if the goal is efficiency.

To understand how putting more issues on the table can help, let us first consider the case of two teams negotiating over a single player, whose rights are owned by one of them. From Myerson and Satterthwaite (1983), we know that ex-post efficient trade is impossible. But what if the seller owns, for instance, two forwards and has to bench one of them (so the utility of having two players is less than the sum of having each of them alone)? These log-jams are quite common. Consider, for example, Barcelona's 2007 soccer team and the situation involving Henry and Eto'O who played at the similar positions. Can inefficiencies be reduced if this team negotiates with another one over these players simultaneously? To what extent, and under which circumstances, do substitutabilities help reduce inefficiencies?

Our first result establishes that, when one agent controls all assets and they are homogeneous, efficiency is never possible, irrespective of whether assets are complements or substitutes. If, however, assets are heterogeneous, as in the case of the soccer players mentioned earlier, then efficiency can be feasible when they are substitutes. Now, consider the case where ownership is dispersed, in the sense that different assets (or issues) are owned (or controlled) by different agents. Then, efficiency for homogeneous assets can be feasible both when assets are complements or when they are substitutes. However, efficiency for heterogeneous assets, can be feasible only if assets are complements. The key difference between the cases of homogeneous and of heterogeneous assets is the dimensionality of private information.

These results suggest that the multi-dimensionality of private information reduces the information and participation costs when ownership is concentrated, while it increases them when ownership is dispersed. What can account for this difference? To answer this question, we identify two key forces that determine whether or not efficiency is feasible: the status quo allocation and the characteristics of the assets. The interplay of these two determines the surplus created from trade, as well as the agents' outside options, both of which are crucial for efficiency to be feasible. The level of participation costs depends on the agents' "critical types." These are the types where gains from trade are minimized, and agents are the most reluctant to participate. Let us see now how the effect of the status quo differs with the dimensionality of private information. Suppose that ownership is concentrated, so we can speak of the agent who owns everything as the seller, and that the assets are homogeneous. Then, the type of the seller where gains from trade are minimal is his highest valuation. This implies a very high outside option, and we get an impossibility, regardless of the complementarity or substitutability of the assets. This echoes the Myerson and Satterthwaite (1983) theorem. On the other hand, if private information is multi-dimensional, and assets are substitutes, along the dimension of the asset with the lower marginal utility, the critical type can be interior or even equal to the lowest valuation. This relaxes the participation costs and we get efficiency. Now, when ownership is dispersed, both agents can be sellers or buyers. With homogeneous assets, critical valuations can be interior, which reduces participation costs. However, if assets are heterogeneous, each owner of an asset is a seller just for that asset and a buyer for the other one, making the corresponding critical types the highest and the lowest valuation.

In the paper, we also study how the subsidy that a broker should provide in order to make efficiency possible under voluntary participation varies with the complementarity or substitutability of the assets. We see that the effect is complex and often non-monotonic. For example, in the case of concentrated ownership, it would be natural to expect that, as the degree of complementarity between assets increases, the deficit incurred also increases since the seller's bundle becomes more valuable to him. While this intuition is true for complements, it fails to hold for substitutes. Sometimes, as issues become less substitutes the deficit decreases. Why is this so? Because less substitutability also means a "bigger pie,"and it can increase the amount a buyer is willing to pay in an incentive-compatible mechanism.

Going back to our sports team example, our findings suggest that in the presence of a log-jam, teams are more likely to find an efficient parting with some of the players. Putting multiple players on the table helps to achieve efficiency when they are substitutes since players are heterogeneous. Moreover, negotiations between teams that own complementary players also help efficiency.

Related Literature This paper relates to the enormous literature on efficient mechanism design, which includes the seminal papers by Vickrey (1961), Clarke (1971) and Groves (1973). A significant fraction of this literature is concerned with the design of efficient *trading* mechanisms. The seminal contribution here is Myerson and Satterthwaite (1983). Important extensions, with methodological developments from which we borrow extensively, are in the papers by Makowski and Mezzetti (1993,1994), Williams (1999), Krishna and Perry (2000) and Schweizer (2006). None of these papers investigates the role of complementarities or substitutabilities vis-a-vis the status quo for efficiency. Recently, Segal and Whinston (2010) show that when the status quo is equal to the expectation of the efficient allocation efficiency is possible. We ask a different question: Given the characteristics of assets and the status quo, when is efficiency possible? We often identify possibility in cases where the status quo is different from the expectation of the efficient allocation.

Our results also differ in spirit from those of Fang and Norman (2006) and Jackson and Sonnenschein (2007). Those papers investigate the extent to which inefficiencies can be alleviated by linking together a large number of independent decisions. This can be done by exploiting the law of large numbers, and the ex-ante Pareto efficiency of the desired social choice function to achieve approximate efficiency. The idea of linking independent decisions, which is the main force behind those two papers (and some earlier works mentioned therein), is different from the forces in this paper. Here, we look at a small number of issues and investigate the joint role of their characteristics (whether they are complements or substitutes) and the initial ownership structure for efficiency.

We proceed to describe our model of negotiations.

1. A model of negotiations

There are I risk-neutral agents negotiating over k issues (or assets). An outcome $z \in Z$, where Z is finite, specifies how the issues are resolved. Agent i's payoff from outcome z is $\pi_i^z(v_i)$, where $v_i = (v_i^1, ..., v_i^k)$. Hence, types are multidimensional and values are private. For all $i \in I$, π_i^z is *decreasing*, *convex* and *differentiable* for all z. We impose no restrictions on how π_i depends on z. This formulation allows for many assets, which may be complements or substitutes. The vector v_i is distributed on $V_i = \times_{k \in K}[\underline{v}_i^k, \overline{v}_i^k]$ according to F_i, with $0 \leq \underline{v}_i^k \leq \overline{v}_i^k < \infty$ for all $k \in K$, and is independent from v_j. We use $F(v) = \Pi_{i \in I} F_i(v_i)$, where $v \in V = \times_{i \in I} V_i$, and $F_{-i}(v_{-i}) = \Pi_{j \neq i} F_j(v_j)$ where $v_{-i} \in V_{-i} = \times_{j \neq i} V_j$. We assume throughout that the distribution F_i has a continuous density function f_i that is strictly positive in its support. It is easy to see that this model contains, as special cases, the environments in Myerson and Satterthwaite (1983) and Cramton, Gibbons and Klemperer (1987).

Basic Definitions By the revelation principle, we know that any outcome that can be achieved by a bargaining procedure, arises at a truth-telling equilibrium of a direct revelation game. Therefore, we can, without loss of generality ,restrict attention to incentive-compatible direct revelation mechanisms. A *direct revelation mechanism* (*DRM*), $M = (p, x)$, consists of an *assignment rule* $p : V \longrightarrow \Delta(Z)$ and a *payment rule* $x : V \longrightarrow \mathbb{R}^I$.

The assignment rule specifies the probability of each outcome for a given vector of reports. We denote by $p^z(v)$ the probability that outcome z is implemented when the vector of reports is v. The payment rule

x specifies, for each vector of reports v, a vector of expected net transfers, one for each agent. The interim expected utility of an agent of type v_i when he participates and declares $\tilde{v}_i$ is

$$u_i(v_i, \tilde{v}_i; (p, x)) = E_{v_{-i}} \left[\sum_{z \in Z} [p^z(\tilde{v}_i, v_{-i}) \pi_i^z(v_i)] + x_i(\tilde{v}_i, v_{-i}) \right]. \tag{1}$$

At an incentive-compatible mechanism we have that $v_i \in \arg\max_{\tilde{v}_i} u_i(v_i, \tilde{v}_i; (p, x))$ for each $i \in I$ and $v_i \in V_i$, and we let

$$U_i(v_i) \equiv u_i(v_i, v_i; (p, x)),$$

or

$$U_i(v_i) = E_{v_{-i}} \sum_{z \in Z} [p^z(v_i, v_{-i}) \pi_i^z(v_i)] + X_i(v_i), \tag{2}$$

where $X_i(v_i) = E_{v_{-i}} [x_i(v_i, v_{-i})]$.

If negotiations break down because of agent i's unwillingness to participate, allocation $Q_i \in \Delta(Z)$ prevails. The payoff from non-participation is, then, given by

$$\underline{U}_i(v_i) = \sum_z Q_i^z \pi_i^z(v_i), \tag{3}$$

where Q_i^z denotes the probability assigned to outcome z by Q_i. Notice that non-participation payoffs may depend on i's type. If $Q_i \equiv Q$ for all i, we call Q the *status quo*.

The timing is as follows: At stage 0, the designer chooses mechanism (p, x). At stage 1, agents decide whether or not to participate. If all participate, they report their types and the mechanism determines the outcome of the negotiations and the payments. If agent i decides not to participate, Q_i determines the outcome. If two or more decide not to participate, some arbitrary $\{\tilde{Q}_i\}_{i \in I}$ is implemented.

We now provide a formal definition of what it entails for a direct revelation mechanism to be feasible.

Definition 1 *(Feasible Mechanisms) For given outside options $\{Q_i\}_{i \in I}$, we say that a mechanism (p, x) is feasible iff it satisfies:*

(IC) Incentive Constraints
$U_i(v_i) \geq u_i(v_i, \tilde{v}_i; (p, x))$ for all $v_i, \tilde{v}_i \in V_i$ and $i \in I$
(VP) Voluntary Participation Constraints
$U_i(v_i) \geq \underline{U}_i(v_i)$ for all $v_i \in V_i$, and $i \in I$
(RES) Resource Constraints
$\sum_{z \in Z} p^z(v) = 1$, $p^z(v) \geq 0$ for all $v \in V$
(BB) Budget Balance
$\sum_{i \in I} x_i(v) = 0$ for all $v \in V$

Summarizing, feasibility requires that p and x are such that (i) agents prefer to tell the truth about their valuation parameter; (ii) agents choose voluntarily to participate in the mechanism; (iii) p is a probability distribution over Z; and (iv) the mechanism does not generate any surplus or deficit.

Our objective is to investigate the forces that enable the existence of feasible mechanisms that are ex-post efficient.

Definition 2 *A mechanism (p, x) is ex-post efficient iff for all $v \in V$, $p^z(v) > 0$ implies that $z \in \arg\max_{z' \in Z} \sum_{i=1}^{I} \pi_i^{z'}(v_i)$.*

Simply put, an ex-post efficient assignment rule assigns positive probability only to outcomes that maximize the sum of agents' utilities. The *total social surplus* at an ex-post efficient assignment rule is given by:

$$W(v) = \max_{z \in Z} \sum_{i \in I} \pi_i^z(v_i).$$

We now investigate when feasible ex-post efficient mechanisms exist. Very similar conditions have been derived in different setups by McAfee (1991), Makowski and Mezzetti (1994), Williams (1999), Krishna and Perry (2000), and, more recently, by Schweizer (2006). The derivation here is included to facilitate the understanding behind the possibility and impossibility results that we will be establishing later.

From the revenue equivalence theorem,[2] we know that all incentive-compatible mechanisms that implement the same allocation rules generate the same expected payoff for each agent up to a constant. This is, of course, also true for efficient allocation rules. Therefore, the interim information rent of an agent is identical for all incentive-compatible and efficient mechanisms up to a constant. A simple way to calculate the rent is to use a particular class of mechanisms that satisfies these properties, such as the Vickrey-Clarke-Groves class (VCG). In other words, when one needs to investigate properties (such as interim voluntary participation, or ex-post budget balance) of incentive-compatible ex-post efficient mechanisms, the VCG class is a canonical class in that it describes all possible interim payoffs up to a constant. Making the constants large enough is an easy way to satisfy interim voluntary participation (VP), but may break the budget (violate BB). On the other hand, choosing the constants appropriately, one can guarantee BB, but then VP may fail. If both VP and BB are desirable, then it helps to know what are the smallest constants to add to the agents' allocation-dependent part of payoff to guarantee that VP is satisfied. If, at these constants, a surplus is possible, that is, if $\sum_{i \in I} x_i(v) \leq 0$, and assuming free disposal, then budget-balance is possible. In what follows, we formalize these ideas and show how to find the transfer-minimizing VCG, subject to voluntary participation.

[2] See Krishna and Perry (2000) for a general version allowing for multi-dimensional types.

The Transfer-Minimizing VCG As is well known, at a VCG mechanism, an agent's interim payoffs are equal to the expected gains from trade plus a constant; that is,

$$U_i(v_i) = E_{v_{-i}}[W(v)] + K_i. \tag{4}$$

From (4) and (2), it follows the well known fact that, at a VCG mechanism, agent i's expected payment is given by

$$X_i(v_i) = E_{v_{-i}} \sum_{\substack{j \in I \\ j \neq i}} \left[\sum_{z \in Z} [p^z(v)\pi_j^z(v_j)] \right] + K_i, \tag{5}$$

where p^z is an ex-post efficient assignment.

Voluntary participation requires that $U_i(v_i) \geq \underline{U}_i(v_i)$, which, with the help of (4), can be written as

$$K_i \geq \underline{U}_i(v_i) - E_{v_{-i}}[W(v)] \text{ for all } v_i.$$

The type(s) of agent i least eager to participate, is the one where the difference in i's payoffs at the status quo and the gains from trade are the largest that is,

$$v_i^* \in \arg\max_{v_i} \{\underline{U}_i(v_i) - E_{v_{-i}}[W(v)]\}. \tag{6}$$

We choose any of these types arbitrarily and call it the *critical type* of agent i, v_i^*.[3] If K_i is large enough to attract type v_i^*, it will also do so for all other types. Therefore, the *lowest* constant that ensures voluntary participation for all types of i, is

$$K_i^* = \underbrace{\left[\underline{U}_i(v_i^*) - E_{v_{-i}}[W(v_i^*, v_{-i})]\right]}_{\text{a constant from type } v_i\text{'s perspective}}, \tag{7}$$

with v_i^* given by (6). In fact, the VCG where the payment rule is given by (4) with $K_i = K_i^*$ is the VCG that minimizes the sum of transfers among all VCG (and, hence, among all efficient) mechanisms that satisfy VP because it makes the most reluctant type just indifferent between participating and not.[4]

[3] For our purposes, any element of the maximizers will do, because all we are interested in, is the maximal difference $\underline{U}_i(v_i) - E_{v-i}[W(v)]$, which is, by definition, the same for all candidate critical types and for all ex-post efficient assignments, when there is more than one.

[4] To see this, note that by substituting (5) with K_i^* from (7), into (2), we get that

$$\begin{aligned} U_i(v_i) &= E_{v_{-i}} \sum_{z \in Z} [p^z(v)\pi_i^z(v_i)] + X_i(v_i) \\ &= E_{v_{-i}} \sum_{z \in Z} [p^z(v)\pi_i^z(v_i)] + \sum_{\substack{j \in I \\ j \neq i}} \left[\sum_{z \in Z} [p^z(v)\pi_i^z(v_i)] \right] + \underline{U}_i(v_i^*) - E_{v_{-i}}[W(v_i^*, v_{-i})] \\ &= E_{v_{-i}}[W(v)] - E_{v_{-i}}[W(v_i^*, v_{-i}) + \underline{U}_i(v_i^*) \end{aligned}$$

which together with (6) establishes that at the lowest subsidy K_i^*, the critical type is just indifferent between participating and not; then, it is immediate that every other type participates.

By combining (5) with (7), and by adding over all agents, we find that the lowest possible transfers needed to guarantee voluntary participation are

$$\begin{aligned}\sum_{i\in I} X_i(v_i) &= \sum_{i\in I} E_{v_{-i}}\left[\sum_{\substack{j\in I\\ j\neq i}}\sum_{z\in Z}[p^z(v)\pi_i^z(v_i)] - W(v_i^*, v_{-i}) + \underline{U}_i(v_i^*)\right] \\ &= \sum_{i\in I}\underline{U}_i(v_i^*) + \sum_{i\in I} E\left[W(v) - E_{v_{-i}}W(v_i^*, v_{-i})\right] - E_{v_{-i}}W(v). \qquad (8)\end{aligned}$$

The VCG with the lowest possible transfers generates a surplus if the sum of transfers that the designer needs to make to the agents is negative- that is, if $\sum_{i\in I} E_v[x_i(v)] \leq 0$, which from (8), is equivalent to $E[S(v)] \geq 0$, where

$$S(v) \equiv \underbrace{W(v)}_{\text{pie}} \underbrace{-\sum_{i\in I}[W(v) - W(v_i^*, v_{-i})] - \sum_{i\in I}\underline{U}_i(v_i^*)}_{\text{total incentive costs+participation costs}}. \qquad (9)$$

We refer to $E[S(v)]$ as the expected surplus (or deficit): It is equal to the maximized sum of all agents' utilities minus the compensations that agents need to receive. This compensation is in the form of total information rents (incentive costs) and outside options (total participation costs). Using a procedure identical to that in Krishna and Perry (2000),[5] one can show a version of their Theorem 2 that states that there exists an efficient, incentive-compatible and individually rational mechanism that balances the budget ex-post iff the VCG mechanism that minimizes the sum of transfers satisfies $E[S(v)] \geq 0$.

As noted by Schweizer (2006), whenever critical types $\{v_i^*\}_{i\in I}$ are such, $S(v) \geq 0$ for all type profiles (and not only in expectation), the *possibility* result is strong in the following sense: For any distribution of types F that generates the critical types $\{v_i^*\}_{i\in I}$, there exists a feasible and ex-post efficient mechanism. If, on the other hand, $S(v) < 0$ for all type profiles we have a strong impossibility result. The distribution of types matters because, together with the assignment rule, they determine the shape of U_i, which, in turn, together with $\underline{U}_i$, determine the critical types, which are a crucial input of (9). In some specific environments, such as, the one in Myerson and Satterthwaite (1983), the critical types are the same for all distributions: The critical type for the seller is his highest possible valuation, and the critical type for the buyer is his lowest valuation. However, in general, different distributions F_i could induce different vectors of critical types v_i^*. In order to carry out the analysis in those cases, it seems that, a priori, it is impossible to avoid having to find out the critical types, which requires the computation of the expectation U_i. This can be especially tedious and cumbersome in multi-dimensional settings like the ones that we examine below. There, one needs to first find the ex-post efficient assignment for each region of valuations and to then integrate agents' payoffs over these different multi-dimensional regions.

[5] One can also use the more general construction from d'Aspremont, Crémer and Gérard-Varet (2004) or Borgers and Norman (2009).

In what follows, we show how, even in such a-priori seemingly intractable cases, we can employ (9) to answer the questions we posed in the introduction. In order to do so, we look at the simplest possible scenarios that allow us to investigate the interaction between the assets characteristics (that is, whether they are complements or substitutes, homogeneous or heterogeneous) and the initial ownership structure in achieving efficiency. To that end, we will calculate (9) for a number of special cases of the general environment presented in the first paragraph of this section. In some of these environments, agents' private information is one-dimensional; while in others it is multi-dimensional. The benefit of writing down the general model of this section is that it permits us to obtain the general version of (9), which we then adapt to each of the environments that we consider.

In Section 2. we analyze the cases of homogeneous assets and in Section 3. the case of heterogeneous assets. In each of these classes, we look at the cases where assets are complements or substitutes and at the cases where ownership is concentrated in the sense that all assets are owned by the same agent-or dispersed- in the sense that different assets are owned by different agents.

2. Negotiations under Exclusive Ownership I: Homogeneous Assets

This section studies negotiations over multiple homogeneous assets. There are two agents, 1 and 2, and two identical and indivisible assets. There are three possible allocations: Agent 1 gets both assets: allocation $z_1 = (2, 0)$; each agent ends up with one asset: allocation $z_2 = (1, 1)$; or agent 2 gets both assets: allocation $z_3 = (0, 2)$. The payoffs that accrue to agents 1 and 2 at each of these allocations are respectively given by:

$$\begin{array}{ll} \pi_1^{z_1}(v_1) = (1+\alpha)v_1 & \pi_2^{z_1}(v_2) = 0 \\ \pi_1^{z_2}(v_1) = v_1 & \pi_2^{z_2}(v_2) = v_2 \\ \pi_1^{z_3}(v_1) = 0 & \pi_2^{z_3}(v_2) = (1+\alpha)v_2 \end{array},$$

where v_i, $i \in \{1, 2\}$ is distributed according to F_i on $[\underline{v}_i, \bar{v}_i]$ with full support, and that gains from trade are uncertain, in the sense that $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$.[6]

When $\alpha < 1$, the marginal utility of owning the second asset is lower than the first, and the assets are substitutes. When $\alpha > 1$, the marginal utility of owning the second unit is higher than the first, and the assets are complements: The second unit is more useful at the margin for an agent who already owns one unit. The units are "unrelated" if $\alpha = 1$ since, in this case, the marginal utility of owning a unit of the asset is independent of the number of units owned.

[6] The condition that gains from trade are uncertain $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$, is a straightforward generalization of the Myerson and Satterthwaite (1983) condition $\underline{v}_2 < \bar{v}_1$. This modification is relevant for the cases where $\alpha < 1$, where the smallest degree of substitutability that makes the analysis non-trivial is $\alpha > \frac{\underline{v}_2}{\bar{v}_1}$.

2.1 Concentrated Ownership

We first examine the case of concentrated ownership, where both assets are owned by one agent, and, without loss, the status quo is given by allocation $(2, 0)$. Our first result is a strong impossibility result: It shows that if ownership is concentrated, and gains from trade are uncertain, then ex-post efficient negotiation procedures do not exist irrespective of the degree of substitutability or complementarity between the two assets. This could be viewed as surprising because if α is very small, the owner does not really care about the second unit, which implies an extremely small conflict of interest.

Theorem 1 *If ownership is concentrated, then there is no ex-post efficient, incentive-compatible and individually rational mechanism that balances the budget.*

The proof amounts to showing that the sum of transfers, as expressed in (9), is less than zero for all vectors of realized valuations. Let's consider, for example, the case where $\alpha < 1$ and efficiency dictates that each agent should own one object. In that case, $v_2 > \alpha v_1$ (which is very likely if α is small) and gains from trade are very big. Still, the sum of transfers is negative, since at a VCG mechanism, agent 1 (the seller) receives the marginal valuation of agent 2 $(-v_2)$, while agent 2 (the buyer) pays agent 1's marginal valuation (αv_1). But since $v_2 > \alpha v_1$, the sum of transfers $-v_2 + \alpha v_1$ is negative. The other cases follow the same logic and the details can be found in the Appendix.

Theorem 1 shows that irrespective of the substitutability or complementarity of the assets, which is indexed by α, there is a deficit ((9) is negative). A natural question to ask, is how α affects this deficit. In the current environment, the deficit can be parameterized by α as $\bar{S}(\alpha) = E_v[S(v, \alpha)]$, and can be viewed as a measure of inefficiency. Its magnitude equals the transfers that a broker should bring into the system in order to make efficiency under budget balance possible. The higher the subsidy needed, the higher the degree of inefficiency.

Proposition 1 *If ownership is concentrated, then the expected surplus $\bar{S}(\alpha)$ is decreasing in α for all $\alpha > 1$, whereas it can be non-monotonic in α for $\alpha \in [\frac{v_2}{\bar{v}_1}, 1]$.*

Proposition 1 shows that $\bar{S}$ is decreasing in α when the goods are complements $(\alpha > 1)$. This is intuitive, since then the agent with the highest valuation should end up with both assets. Therefore, we have a Myerson-Satterthwaite scenario with an asset that gives a higher marginal utility, and the subsidy needed is obviously increasing in α. However, when assets are substitutes $(\alpha < 1)$, the subsidy $\bar{S}(\alpha)$ may decrease as α increases. To understand this, let's consider the same situation as before, where efficiency dictates that each agent should own one object. In that case, we saw that the sum of transfers is $-v_2 + \alpha v_1$ which is increasing in α. In other regions, $S(v, \alpha)$ could be decreasing in α. Ultimately, the sign of $\bar{S}'(\alpha)$ depends on the relative size of the various regions. Examples can be found in Appendix B.

Summing up, when one agent owns all the (homogeneous) assets, efficiency cannot be achieved regardless of the degree of complementarity or substitutability. Not surprisingly, a higher degree of complementarity (a higher α) reduces the expected deficit only when $\alpha > 1$. However, more surprisingly, the opposite may be true when $\alpha < 1$. Do these results hold when each asset is owned by a different agent? This is addressed next.

2.2 Dispersed Ownership

We now examine the case of dispersed ownership, where each agent owns one asset and the status quo is given by allocation $(1, 1)$. We establish a possibility result when assets are complements ($\alpha \geq 1$) that extends the possibility result of Cramton, Gibbons and Klemperer (1987). We also show that even in the case where assets are substitutes, $\alpha < 1$, efficiency can be sometimes possible.

Theorem 2 *If ownership is dispersed, then, if the assets are complements ($\alpha \geq 1$), efficiency is possible for all cases where* $\max\{v_1^*, v_2^*\} \leq \alpha \min\{v_1^*, v_2^*\}$*. In particular, efficiency is possible for all symmetric environments. If assets are substitutes ($\alpha < 1$), then for any $\alpha < 1$, there exist environments (distributions F_1,F_2), such that efficiency is possible.*

As we mentioned in the case of concentrated ownership, when assets are complements ($\alpha > 1$) at the ex-post efficient assignment, the agent with the highest valuation should end up with both units. Hence, the situation is very similar to a single-asset scenario, with the difference that the marginal value of the asset is higher. Theorem 2 shows that efficiency requires that agents' payoffs at their critical types are close. In particular, if agents are ex-ante symmetric, efficiency is possible since critical types are the same.[7] Ceteris paribus, a higher α relaxes the condition $\max\{v_1^*, v_2^*\} \leq \alpha \min\{v_1^*, v_2^*\}$. This is because α increases the gains from trade, as the status quo is always inefficient, since both goods must be assigned to the agent with the highest valuation.

When assets are substitutes ($\alpha < 1$), ex-post efficiency requires sometimes that each agent owns one asset, in which case dispersed ownership implies that gains from trade are zero. As a consequence, equality of critical types across agents is not enough on its own to guarantee the existence of an efficient mechanism. Still, efficiency is possible in some special environments, as in the class we consider in the proof of Theorem 2. In those environments, gains from trade are big since it is very likely that one agent should own both assets.

We now investigate how α affects the expected surplus. Not surprisingly, when $\alpha < 1$, α has an ambivalent effect, because, on one hand, it positively affects the gains from trade, but on the other, it may increase information costs. In Appendix C, we provide examples to illustrate this effect. However, the

[7] This is related to Figueroa and Skreta (2011), which studies asymmetric partnerships and shows that the sum of transfers necessary for voluntary participation is minimized when critical types are equalized across agents.

following Proposition establishes that when agents are ex-ante symmetric expected surplus $\bar{S}(\alpha)$ is positive and increasing in α, for all $\alpha > 1$:

Proposition 2 *If ownership is dispersed, then, when agents are ex-ante symmetric, the expected surplus $\bar{S}(\alpha)$ is positive and increasing in α for all $\alpha > 1$. When $\alpha < 1$, for any distribution F there exists a cutoff α^*, such that, the expected surplus $\bar{S}(\alpha)$ is negative if $\alpha < \alpha^*$.*

Proposition 2 establishes that when ownership is dispersed and agents are ex-ante symmetric the expected surplus is increasing in α when $\alpha > 1$. This is the opposite to what happens in the case of concentrated ownership (Proposition 1) and intuitive: As goods become more complements, the gains from trade increase if ownership is dispersed, emphasizing the possibility forces in Cramton, Gibbons and Klemperer (1987), while the information rents increase if ownership is concentrated, emphasizing the impossibility forces behind Myerson and Satterthwaite (1983). The second part of the Proposition 2 shows that substitutabilities play an unequivocal role if they are extreme. For any environment, if substitutabilities are big ($\alpha < \alpha^*$), then efficient dissolution is impossible.

There is another difference between dispersed and concentrated ownership. With concentrated ownership, the sign of $\frac{\partial S(v,\alpha)}{\partial \alpha}$ is always negative, while with dispersed ownership, it changes with v. Additionally, the size of the regions of each sign change with the locations of the critical types, and critical types vary with α and the distribution F. For these reasons, we can sign $\frac{\partial S(v,\alpha)}{\partial \alpha}$ only in expectation.[8]

This concludes our analysis of negotiations of multiple homogeneous assets. For concentrated ownership, an impossibility result holds, regardless of α. For dispersed ownership, too much substitutabilities can create an impossibility, but as α grows, efficiency becomes possible.

In the following section, we examine negotiations over heterogeneous assets. In a model almost identical to this one, we show that the existence of multidimensional private information greatly affects the possibility or impossibility of efficiency.

3. Negotiations under Exclusive Ownership II: Heterogeneous Assets

Here, we look at two agents who negotiate over two heterogeneous assets A and B. An agent's type consists of two parameters: one for each asset-in other words, types are multidimensional. There are four possible allocations: agent 1 gets both assets, allocation $z_1 = (AB, 0)$; agent 1 gets asset A, whereas asset B goes to agent 2, allocation $z_2 = (A, B)$; agent 1 gets asset B, and agent 2 gets asset A, allocation $z_3 = (B, A)$; and, finally, agent 1 gets none of the assets, allocation $z_4 = (0, AB)$. The two agents' payoffs in each of these possible allocations are given by:

[8] This explains another key difference with the case of concentrated ownership, where surplus is monotonic in α, even in asymmetric environments. With dispersed ownership, this is no longer true. See Appendix D for an example.

$$
\begin{array}{ll}
\pi_1^{z_1}(v_1^A, v_1^B) = v_1^A + \alpha v_1^B & \pi_2^{z_1}(v_2^A, v_2^B) = 0 \\
\pi_1^{z_2}(v_1^A, v_1^B) = v_1^A & \pi_2^{z_2}(v_2^A, v_2^B) = v_2^B \\
\pi_1^{z_3}(v_1^A, v_1^B) = v_1^B & \pi_2^{z_3}(v_2^A, v_2^B) = v_2^A \\
\pi_1^{z_4}(v_1^A, v_1^B) = 0 & \pi_2^{z_4}(v_2^A, v_2^B) = \alpha v_2^A + v_2^B
\end{array}
. \tag{10}
$$

Exactly, as in the case of homogeneous assets, we call them substitutes if $\alpha < 1$, and complements if $\alpha > 1$. This payoff specification has the advantage that it is analogous to the one used for homogeneous assets and it allows for direct comparisons.[9] Valuations v_1 and v_2 are distributed according to F_1, and F_2 with full supports $[\underline{v}_1^A, \overline{v}_1^A] \times [\underline{v}_1^B, \overline{v}_1^B]$ and $[\underline{v}_2^A, \overline{v}_2^A] \times [\underline{v}_2^B, \overline{v}_2^B]$, that satisfy $0 < \underline{v}_i^j < \overline{v}_i^j < \infty$.

3.1 Concentrated Ownership

First, we examine whether or not efficiency is feasible when ownership is concentrated. In particular, we assume, without loss, that the status quo is given by allocation $(AB, 0)$.

Concentrated ownership implies high participation costs, which are the driving force of the Myerson-Satterthwaite impossibility theorem, as well as or our impossibility Theorem 1. Theorem 1 is even more negative, since it says that efficiency is impossible even if substitutability is very strong, in the sense that the seller puts minimal value on owning the second unit. Here we show that this result is no longer true when assets are heterogeneous: Efficient negotiations can be feasible where assets are substitutes $(\alpha < 1)$. However, we still get an impossibility when assets are complements, $(\alpha > 1)$:

Theorem 3 *If ownership is concentrated,* $\alpha > 1$, *and*

$$\underline{v}_2^B < \alpha \overline{v}_1^B \tag{11}$$

and

$$\underline{v}_2^A < \overline{v}_1^A \tag{12}$$

hold, then there is no feasible and ex-post efficient mechanism.

Together with Theorem 1, Theorem 3 shows that the impossibility of ex-post efficiency when goods are complements is a robust result: Impossibility holds regardless of the dimensionality of private information. The proof is analogous to the case of homogeneous assets and it amounts to establishing that the sign of S is negative over all valuations. Since it is quite lengthy, we relegate it to Appendix A. However, the

[9] With this formulation, when $\alpha < 1$, it may be the case that an agent is better-off by throwing away an asset. While this can be realistic in some situations (an unhappy player sitting on the bench can be an unwanted distraction) we must stress that our possibility result (Proposition 3) does not depend on this feature. In fact, the proof builds around a situation where no mass in put in a region of valuations where the an agent would rather have one asset. Of course there are other payoff specifications that one can use to capture complementarities and substitutabilities in a multi-dimensional framework. In a footnote below we summarize another plausible environment.

dimensionality of private information does matter when goods are substitutes ($\alpha < 1$): The impossibility result established in Theorem 1 fails when private information is two-dimensional.

Proposition 3 *If $\alpha < 1$ and ownership is concentrated, then, there exist distributions of types F_1, F_2 for which it is possible to design feasible and ex-post efficient mechanisms.*

Proof. Consider the status quo allocation $(AB, 0)$. We establish the Proposition by showing that there exist distributions F_1, F_2 with supports $[0,1]^2$ for which efficient trade is possible. In order to apply (9) we first need to determine the critical types. Since agent 2 does not own any of the assets, his outside payoff is 0, so irrespective of his expected payoff at an ex-post efficient assignment, the type vector where the participation constraint binds (the critical type) is $(0,0)$. Now, since agent 1 owns both assets, his payoff from non-participation is given by $v_1^A + \alpha v_1^B$. Therefore, along the dimension of asset A, agent 1's non-participation payoff has the highest possible slope, namely 1, and along the dimension of asset B, the slope is α. Then, regardless of the shape of the participation payoff determined by the ex-post efficient allocation and the distribution of types, $v_1^{A^*} = 1$, but along the dimension of asset B, it can be any type $v_1^{B*} \in [0,1]$.

Given this, the surplus becomes

$$\begin{aligned} S(v_1^A, v_1^B, v_2^A, v_2^B) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, v_1^{B*}, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 0) - (1 + \alpha v_1^{B*}) \\ &= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \\ &\quad + \max\{1 + \alpha v_1^{B*}, 1 + v_2^B, v_1^{B*} + v_2^A, \alpha v_2^A + v_2^B\} \\ &\quad + \max\{v_1^A + \alpha v_1^B, v_1^A, v_1^B, 0\} - (1 + \alpha v_1^{B*}) - 0 \end{aligned}$$

Consider distributions that put almost all probability mass in the region where

$$v_1^B + v_2^A = \max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\}. \tag{13}$$

In this region, at the ex-post efficient assignment agent 1 ends up with unit B with probability close to 1. This implies that the slope of agent 1's participation payoff along this dimension is almost 1. Recalling that the slope of his non-participation payoff along this dimension is $\alpha < 1$, implies that $v_1^{B*} = 0$. Then,

$$\max\{1 + \alpha v_1^{B*}, 1 + v_2^B, v_1^{B*} + v_2^A, \alpha v_2^A + v_2^B\} = \max\{1, 1 + v_2^B, v_2^A\} = 1 + v_2^B. \tag{14}$$

Suppose also that

$$v_1^A + \alpha v_1^B = \max\{v_1^A + \alpha v_1^B, v_1^A, v_1^B, 0\}, \tag{15}$$

then for this region we have that

$$S(v_1^A, v_1^B, v_2^A, v_2^B) = -v_1^B - v_2^A + 1 + v_2^B + v_1^A + \alpha v_1^B - 1 = -v_1^B - v_2^A + v_2^B + v_1^A + \alpha v_1^B.$$

If valuations are all equal to each other but strictly positive, that is $v_1^A = v_1^B = v_2^A = v_2^B > 0$, then because $\alpha < 1$, we are in the desired region where (13), (14) and (15) hold, moreover in this region $S(v_1^A, v_1^B, v_2^A, v_2^B) = \alpha v_1^B > 0$. Therefore, if the distributions F_1 and F_2 put enough weight on the region of valuations where $v_1^A \simeq v_1^B \simeq v_2^A \simeq v_2^B > 0 > 0$, then it is possible to design ex-post efficient negotiating procedures. ■

The proof illustrates quite well the forces behind the possibility result: A positive surplus is generated in a region where efficiency dictates that agent 1 keeps asset B and agent 2 obtains asset A. This works because, together with the fact that $\alpha < 1$, it implies that $v_1^{B*} = 0$, which lowers participation costs: even though agent one has both assets his participation rents are as if he only has one (asset A). Note that this would never happen if goods are homogeneous, where the critical type is always the best type. This is a crucial difference between singe- and multi-dimensional private information. By declaring his true valuation of asset A, agent 1 does not reveal at the same time his valuation of asset B, which he keeps (and gets a higher marginal valuation of 1 instead of α from it). If assets were homogeneous, an agent that declares a low valuation would be surrendering both assets with high probability, and would require accordingly a high compensation.[10]

It is interesting to note that the above result depends on the initial ownership structure being $(AB, 0)$. If, in the setup given by (10), the ownership structure is either (A, B) or (B, A), efficiency is not possible when $\alpha < 1$. This is established next in Theorem 4, and it contradicts the conventional wisdom, which suggests that it is easier to achieve efficiency if property rights are "more balanced," in the sense that both agents own some part of the total endowment.

[10] An alternative formulation is a model where payoffs are given by:

$$\begin{array}{ll} \pi_1^{AB} = v_1^{AB} & \pi_2^{AB} = v_2^{AB} \\ \pi_1^{A} = v_1^{A} & \pi_2^{A} = v_2^{A} \\ \pi_1^{B} = v_1^{B} & \pi_2^{B} = v_2^{B} \\ \pi_1^{0} = 0 & \pi_2^{0} = 0 \end{array} . \tag{16}$$

Suppose that the assets are substitutes in the sense that $v_i^{AB} < v_i^A + v_i^B$ for $i = 1, 2$ and that the status quo is given by allocation $(AB, 0)$ (or $(0, AB)$). Then, there exist distributions of types F_1, F_2 for which it is possible to design feasible and ex-post efficient mechanisms. When the status quo allocation is $(AB, 0)$ the critical vectors of valuations for agent 1 and agent 2 are $\left(0, 0, \bar{v}_1^{AB}\right)$ and $(0, 0, 0)$ respectively. Going through all the cases (details available from the authors upon request), establishes that efficiency in *only* possible, when

$$v_1^B \geq \bar{v}_1^{AB} \text{ or } v_1^A \geq \bar{v}_1^{AB}.$$

In other words, efficiency is possible only when the owner, agent 1, actually prefers to throw away one of the two assets, versus having one, even when the value of the bundle is at the highest possible level.

3.2 Dispersed Ownership

We now examine the case of dispersed ownership where, without loss, the status quo is (A, B). Theorem 2, shows that dispersed ownership in a world with single-dimensional information reduces participation costs and makes efficiency possible even if goods are substitutes. However, this is not case when information has the same dimension as the number of assets. In this case each agent is always a seller for his asset, and a buyer for the other, which makes total participation costs for the case of substitutes ($\alpha < 1$) higher vis-a-vis the case of concentrated ownership, resulting in an impossibility Theorem:

Theorem 4 *If ownership is dispersed, $\alpha < 1$, and*

$$\underline{v}_1^B \leq \overline{v}_2^B \tag{17}$$

and

$$\underline{v}_2^A \leq \overline{v}_1^A, \tag{18}$$

hold, then there is no feasible and ex-post efficient mechanism.

Theorem 4 establishes that when assets are substitutes, and each agent owns one asset, it is impossible to have ex-post efficient trade when gains from trade are uncertain (conditions (17) and (18)). The same forces that made efficiency possible when ownership rights were concentrated (Proposition 3) go against it when ownership is dispersed. An agent acquiring an asset must compensate the owner based on his marginal disutility of losing it, which is 1. But at the same time, acquiring it either has a lower marginal utility α, or it creates a lower marginal utility for the other asset. Combined, these effects make it impossible to adequately compensate the owner of an asset. Its proof is analogous to that of Theorem 3 and can be found in Appendix A.

We now turn to the case of complements ($\alpha > 1$). In this case, the marginal valuation of an asset by a potential buyer is bigger than the one of the potential seller since it "completes" a bundle. This makes the existence of efficient mechanisms sometimes possible, as we see in the next Proposition:

Proposition 4 *If $\alpha > 1$ and ownership is dispersed, then, there exist distributions of types F_1, F_2 for which a feasible and ex-post efficient mechanism exists.*

Proof. We establish the Proposition by showing that there exist distributions F_1, F_2 with supports $[0, 1]^2$ for which efficient trade is possible. Given status quo (A, B), it is easy to see that the critical types for agent 1 and 2 are given by $(v_1^A, v_1^B) = (1, 0)$ and $(v_2^A, v_2^B) = (0, 1)$. The expression (9) for each v_1, v_2 becomes:

$$\begin{aligned} S(v_1, v_2) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, 0, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 1) - 2 \\ &= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \\ &\quad + \max\{1, 1 + v_2^B, v_2^A, \alpha v_2^A + v_2^B\} \\ &\quad + \max\{v_1^A + \alpha v_1^B, v_1^A + 1, v_1^B, 1\} - 2. \end{aligned}$$

The sum of transfers $S(v_1, v_2)$ is positive whenever ex-post efficiency requires that either agent 1 or agent 2 should get both units, that is whenever $v_1^A + \alpha v_1^B = \max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\}$ or $\alpha v_2^A + v_2^B = \max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\}$ and the value of owning both units is at least 2, which is the highest possible sum of payoffs at the status quo. This second requirement implies that $\alpha v_2^A + v_2^B = \max\{1, 1 + v_2^B, v_2^A, \alpha v_2^A + v_2^B\}$ and $v_1^A + \alpha v_1^B = \max\{v_1^A + \alpha v_1^B, v_1^A + 1, v_1^B, 1\}$.

Consider the situation where $v_1^B = v_2^A = 1$; $v_1^A > v_2^B$ and $v_1^A > 2-a$ and $v_2^B = 2-\alpha$. It is straightforward to check that $S(v_1, v_2)$ is strictly positive. Then, if distributions F_1, F_2 put enough mass on a region around this point, efficiency is possible. ■

The proof of Proposition 4 shows that if valuations are such that one agent should end up with both objects and his payoff is greater than the sum of valuations at the status quo, a surplus is generated. In those regions of valuations, the potential buyer of an asset has a higher marginal valuation for it than the potential seller. The likelihood of these regions increases with α.[11]

These results show that in determining which assets to negotiate about simultaneously, one has to think about the nature of the assets (the α in our model), as well as the initial ownership structure. The nature of assets determines the "pie" that agents split at the ex-post efficient assignment, which together with the initial ownership structure it determine the information and participation costs. Our analysis highlighted why participation costs differ with the dimensionality of private information.

4. Conclusions

In this paper, we investigated the ownership structures under which the presence of complementarities and substitutabilities among assets for trade help alleviate the inefficiencies that arise from asymmetric information.

First, we analyzed the case where two agents negotiate over multiple homogeneous assets that they perceive either as substitutes or as complements. When assets are homogeneous, private information is one-dimensional. Then, when ownership is concentrated, in the sense that one agent owns all assets, we showed that efficiency is never possible regardless of whether the assets are complements or substitutes. We also showed that when assets are complements, simultaneous negotiation exacerbates the conflict: The subsidy needed to achieve efficiency is increasing in the degree of complementarity between the two

[11] One might be wondering what would happen in a model where payoffs are given by (16). Suppose that the assets are complements in the sense that $v_i^{AB} > v_i^A + v_i^B$ for $i = 1, 2$ and that the status quo is given by allocation (A, B) (or (B, A)). Then there exist distributions of types F_1, F_2 for which a feasible and ex-post efficient mechanism exists. When the status quo allocation is (A, B), the critical types agent 1 and 2 are respectively $(\bar{v}_1^A, 0, 0)$ and $(0, \bar{v}_2^B, 0)$. Going through all the cases (details available from the authors upon request), establishes that efficiency in *only* possible, when complementarity is very strong in the sense that

$$v_2^{AB} \geq \bar{v}_1^A + \bar{v}_2^B \text{ or } v_1^{AB} \geq \bar{v}_1^A + \bar{v}_2^B.$$

assets. When assets are substitutes, the effect of substitutability is ambiguous: As assets become more substitutable, this increases the surplus by reducing the seller's outside option on the one hand, but it decreases the buyer's payments on the other hand. When ownership is dispersed, in the sense that each asset is owned by a different agent, efficiency is often possible. There, the surplus is increasing in the degree of complementarity of the assets. The reason is that the higher the degree of complementarity, the higher the difference in marginal utilities between the agent acquiring the second unit and the agent selling away his asset. Again, when assets are substitutes, the effect of higher substitutability is ambiguous. As substitutability drops, trade opportunities emerge; however, payments generated may not be high enough to cover participation costs.

Our analysis suggests that when assets are homogeneous, the results echo the pre-existing impossibility (à la Myerson-Satterthwaite (1983)) and possibility (à la Cramton, Gibbons and Klemperer (1987)) results. This is no longer the case when assets are heterogeneous. There, private information is multi-dimensional, and this has an important effect on the inefficiencies created due to information and participation costs. In some cases, where we have impossibility with single-dimensional private information, we obtain efficiency in the multidimensional setting: For example, efficiency can be feasible for heterogeneous assets that are substitutes and are all owned by the same agent.

Our findings suggest that if agents have a choice of which issues to put on the table in order to make efficiency more likely, they would have to consider how similar these issues are. For heterogeneous issues, if one agent has control of all the assets (issues), he should put more issues that exhibit substitutabilities; if assets are controlled by different agents, more issues that exhibit complementarities should be put on the table.

The main lesson of our analysis is that in the presence of asymmetric information, negotiations over multiple issues cannot generally be viewed as a union of single-issue negotiations. If the objective is to design a negotiation procedure that maximizes gains from trade, one has to think carefully about what issues to put together on the table. The relevant variables are the issues' nature and the initial control of rights. We hope that our findings provide some guidelines for how one should go about doing so.

5. Appendix A: Proofs

Proof of Theorem 1

We establish the result for the case that the status quo allocation is $(2, 0)$. First, note that, given this status quo, it is easy to see that, regardless of the distributions of valuations, and of the exact form of the ex-post efficient assignment, the critical types for agent 1 and 2, are always given by $v_1^* = \bar{v}_1$ and $v_2^* = \underline{v}_2$, respectively. This is because the slope of the seller's payoff from non-participation is $1 + \alpha$, while the slope of his payoff from an ex-post efficient assignment is weakly less than $1 + \alpha$, for all $v_1 \in [\underline{v}_1, \bar{v}_1]$. Now, the slope of the buyer's non-participation payoff is 0, and the slope of his payoff from an ex-post efficient assignment is weakly greater than 0, for all $v_2 \in [\underline{v}_2, \bar{v}_2]$.

With these critical types, $S(v_1, v_2, \alpha)$ from (9) becomes

$$\begin{aligned} S(v_1, v_2, \alpha) &= -W(v_1, v_2) + W(\bar{v}_1, v_2) + W(v_1, \underline{v}_2) - (1+\alpha)\bar{v}_1 \\ &= -\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\} \\ &\quad + \max\{(1+\alpha)\bar{v}_1, \bar{v}_1 + v_2, (1+\alpha)v_2\} \\ &\quad + \max\{(1+\alpha)v_1, v_1 + \underline{v}_2, (1+\alpha)\underline{v}_2\} - (1+\alpha)\bar{v}_1. \end{aligned} \tag{19}$$

Notice that we explicitly note the dependence of the surplus on the degree of complementarity/substitutability of the assets α. In order to determine the sign of S, we examine the following scenarios:

If $(1+\alpha)\bar{v}_1 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1+v_2, (1+\alpha)v_2\}$, then the second and the last terms on the right-hand-side of (19) cancel out, and we immediately have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\} \\ &\quad + \max\{(1+\alpha)v_1, v_1 + \underline{v}_2, (1+\alpha)\underline{v}_2\} \leq 0. \end{aligned}$$

If $(1+\alpha)v_2 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1 + v_2, (1+\alpha)v_2\}$, then it follows that $(1+\alpha)v_2 = \max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\}$, which implies that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -(1+\alpha)\bar{v}_1 + \max\{(1+\alpha)v_1, v_1 + \underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{(1+\alpha)(v_1 - \bar{v}_1), (v_1 - \bar{v}_1) + (\underline{v}_2 - \alpha\bar{v}_1), (1+\alpha)(\underline{v}_2 - \bar{v}_1)\} \leq 0, \end{aligned}$$

where the last inequality follows from the fact that $\underline{v}_2 < \alpha\bar{v}_1$.

Finally, if $\bar{v}_1 + v_2 = \max\{(1+\alpha)\bar{v}_1, \bar{v}_1 + v_2, (1+\alpha)v_2\}$, this implies that $v_2 > \alpha\bar{v}_1$, so $\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\}$ is either $v_1 + v_2$ if $v_1 > \alpha v_2$, or $(1+\alpha)v_2$ otherwise. If $v_1 > \alpha v_2$, then $v_1 + v_2 = \max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -v_1 - \alpha\bar{v}_1 + \max\{(1+\alpha)v_1, v_1 + \underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{\alpha(v_1 - \bar{v}_1), \underline{v}_2 - \alpha\bar{v}_1, (\underline{v}_2 - \alpha\bar{v}_1) + (\alpha\underline{v}_2 - v_1)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 > \underline{v}_2$, and the fact that, in this case, $v_1 \geq \alpha v_2$. If $v_1 < \alpha v_2$, then $(1+\alpha)v_2 = \max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -\alpha v_2 - \alpha\bar{v}_1 + \max\{(1+\alpha)v_1, v_1 + \underline{v}_2, (1+\alpha)\underline{v}_2\} \\ &= \max\{(v_1 - \alpha v_2) + \alpha(v_1 - \bar{v}_1), (v_1 - \alpha v_2) + (\underline{v}_2 - \alpha\bar{v}_1), (\underline{v}_2 - \alpha\bar{v}_1) + \alpha(\underline{v}_2 - v_2)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 > \underline{v}_2$, and the fact that, in this case, $v_1 \leq \alpha v_2$.

Therefore, $S(v_1, v_2, \alpha)$ is less than zero for all v_1, v_2. Moreover, there are regions of types with a non-empty interior where $S(v_1, v_2, \alpha) < 0$. These two observations together establish that it is impossible to design ex-post efficient mechanisms. ■

Proof of Proposition 1

Consider $S(v, \alpha)$. By integrating over v, we get $\bar{S}(\alpha) = \sum_{\rho \in \mathcal{Q}} \int_{R_\rho(\alpha)} S_\rho(v, \alpha) dv$, where each region $R_\rho(\alpha)$ corresponds to a different realization of the maxima of (19), and $\mathcal{Q}$ stands for the number of regions where S takes a different expression. By looking at (19), it is easy to see that there are finitely many of these different regions.

Given that there exists K such that $|S_\rho(v, \alpha)| \leq K$, and $|\frac{\partial S_\rho(v,\alpha)}{\partial \alpha}| \leq K$, and that $S(\cdot, \alpha)$ and $R_\rho(\cdot)$ are continuous, we can write, by familiar envelope arguments, that

$$\bar{S}'(\alpha) = \sum_{\rho \in \mathcal{Q}} \int_{R_\rho(\alpha)} \frac{\partial S_\rho(v, \alpha)}{\partial \alpha} dv.$$

This observation tells us that the indirect effect of α on $\bar{S}'_\rho(\alpha)$ through the change of the regions of the maxima $R_\rho(\alpha)$ is zero, and it allows us to focus only on the direct effect of α on $\bar{S}(\alpha)$.

For $\alpha > 1$, we have the following:

- In the region R_ρ where $v_1 \geq v_2$, we have that $S_\rho(v, \alpha) = 0$.
- In the region R_ρ where $v_1 \leq v_2$ and $v_1 \geq \underline{v}_2$, we have that $S_\rho(v, \alpha) = (1 + \alpha)(v_1 - v_2)$.
- In the region R_ρ where $v_1 \leq v_2$, $v_1 \leq \underline{v}_2$ and $v_2 \leq \bar{v}_1$, we have that $S_\rho(v, \alpha) = (1 + \alpha)(\underline{v}_2 - v_2)$.
- In the region R_ρ where $v_1 \leq v_2$, $v_1 \leq \underline{v}_2$ and $v_2 \geq \bar{v}_1$, we have that $S_\rho(v, \alpha) = (1 + \alpha)(\underline{v}_2 - \bar{v}_1)$.

Since $\frac{\partial S_\rho(v,\alpha)}{\partial \alpha} \leq 0$ for all regions, and it is strictly negative in a region of positive measure, the result follows.

Now, to establish the non-monotonicity result for $\alpha < 1$, it is enough to show that there are regions where $\frac{\partial S_\rho(v,\alpha)}{\partial \alpha}$ is bigger or smaller than zero. If the distribution puts enough mass there, the surplus would be increasing or decreasing in α at a given level.

Consider, first, the region where $\frac{v_2}{\alpha} \geq v_1 \geq \alpha v_2$, $\alpha v_1 \geq \underline{v}_2$ and $\alpha \bar{v}_1 \geq v_2$, then $S_\rho(v, \alpha) = -v_2 + \alpha v_1$. In that case, $\frac{\partial S(v,\alpha)}{\partial \alpha} \geq 0$. Then, consider the region where $\frac{v_2}{\alpha} \geq v_1 \geq \alpha v_2$, $\alpha v_1 \geq \underline{v}_2$ and $\alpha \bar{v}_1 \leq v_2$, $S(v, \alpha) = \alpha(v_1 - \bar{v}_1)$. In that case, $\frac{\partial S(v,\alpha)}{\partial \alpha} \leq 0$. These two observations imply the non-monotonicity of the surplus in α whenever $\alpha < 1$. ■

Proof of Theorem 2

We first look at the case where $\alpha > 1 :$ In order to apply (9) we first need to investigate what the critical types would be. Here each agent owns an asset, so his payoff from non-participation is v_i which has obviously slope 1. Participation payoffs can have any slope between 0 and $(1+\alpha)$, therefore $i's$ critical type can be any type v_i^*.

We start by writing down (9) for the case under consideration.

$$\begin{aligned} S(v,\alpha) &= -\max\{(1+\alpha)v_1,(1+\alpha)v_2\} \\ &\quad +\max\{(1+\alpha)v_1^*,(1+\alpha)v_2\} \\ &\quad +\max\{(1+\alpha)v_2^*,(1+\alpha)v_1\} - v_1^* - v_2^*. \end{aligned} \tag{20}$$

This can be rewritten as

$$S(v,\alpha) = \begin{cases} (1+\alpha)(v_2 - v_1) + \alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 > v_2 \\ (1+\alpha)v_2 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 > v_2 \\ \alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 > v_2 \\ -(1+\alpha)v_1 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 > v_2 \\ \alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 < v_2 \\ (1+\alpha)v_1 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 < v_2 \\ (1+\alpha)(v_1 - v_2) + \alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 < v_2 \\ -(1+\alpha)v_2 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 < v_2 \end{cases}. \tag{21}$$

Consider the situation where $v_1^* \geq v_2^*$ (the other case is analogous). Then, the first case is impossible, and the second, third and fourth yield a nonnegative surplus. Moreover, if $\alpha v_2^* \geq v_1^*$, the fifth to eighth cases are also nonnegative.

We now examine the case when $\alpha < 1$. As before, in order to apply (9) we first need to investigate what the critical types would be. Here each agent owns an asset, so his payoff from non-participation is v_i which has obviously slope 1. Participation payoffs can have any slope between 0 and $(1+\alpha)$, therefore $i's$ critical type can be any type v_i^*.

We establish the possibility of efficiency for a symmetric environment. In such a case, (9) can be written as

$$\begin{aligned} S(v,\alpha) &= -\max\{(1+\alpha)v_1,(1+\alpha)v_2, v_1+v_2\} \\ &\quad +\max\{(1+\alpha)v^*,(1+\alpha)v_2, v^*+v_2\} \\ &\quad +\max\{(1+\alpha)v_1,(1+\alpha)v^*, v^*+v_1\} - 2v^*, \end{aligned} \tag{22}$$

where v^* is the critical type of both agents. At that type, participation and non-participation payoffs are tangent, which implies that v^* satisfies $\alpha F(\alpha v^*) + F(v^*/\alpha) = 1$.

Consider a distribution F and a point $\hat{v} \in [\underline{v}, \bar{v}]$ such that $F(\alpha\hat{v}) = \frac{1}{1+\alpha} - \epsilon$ and $F(\hat{v}/\alpha) = \frac{1}{1+\alpha} + \epsilon$. Then, $v^* \approx \hat{v}$ and, with probability close to 1, we have one of the following cases:

- $v_i \leq \alpha v^*$, in which case $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + 2\alpha v^* \geq 0$
- $v_i \geq \frac{v^*}{\alpha}$. Then, we immediately have that $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + (1+\alpha)v_1 + (1+\alpha)v_2 - 2v^* \geq 0$.
- $v_i \leq \alpha v^*$ and $v_j \geq \frac{v^*}{\alpha}$. In this case, it is easy to see that $S(v,\alpha) = v^*(\alpha-1) < 0$.

The integral over the third region is approximately equal to $\frac{2\alpha}{1+\alpha}v^*(\alpha-1)$, since the probability of a type being below αv^* is $F(\alpha v^*) \sim \frac{1}{1+\alpha}$, and the probability of a type being above $\frac{v^*}{\alpha}$ is approximately equal to $1-F(\frac{v*}{\alpha}) \sim \frac{\alpha}{1+\alpha}$. On the other hand, the first term happens with probability $\frac{1}{(1+\alpha)^2}$, and if almost all mass is concentrated at 0 over that region, the integral is approximately equal to $\frac{1}{(1+\alpha)^2}4\alpha v^*$. Putting these two terms together we obtain $\frac{2\alpha v^*}{(1+\alpha)} > 0$. Then since $S(v,\alpha)$ in the second region is positive, we obtain and overall positive expected surplus. ■

Proof of Proposition 2

We first show that $\bar{S}(\alpha)$ is increasing in a, whenever $\alpha > 1$ and agents are symmetric. First, we calculate the surplus for a given v_1 and then we integrate with respect of v_1 in order to get $\bar{S}(\alpha)$. In these calculations, we use the observation that at the critical type of agent 1, it must hold that $v_1^* = F^{-1}(\frac{1}{1+\alpha})$.

Now, for $v_1 \geq v_1^*$ we have that

$$\begin{aligned}
S(v_1) &= \int_{\underline{v}}^{v^*}(\alpha-1)v^* dF(v_2) + \int_{v^*}^{v_1}[(1+\alpha)v_2 - 2v^*]dF(v_2) + \int_{v_1}^{\overline{v}}[(1+\alpha)v_1 - 2v^*]dF(v_2) \\
&= (\alpha-1)v^*F(v^*) - 2v^*(1-F(v^*)) + (1+\alpha)[\int_{v^*}^{v_1} v_2 dF(v_2) + v_1(1-F(v_1))] \\
&= -v^* + (1+\alpha)\int_{v^*}^{v_1} v_2 dF(v_2) + (1+\alpha)v_1(1-F(v_1))
\end{aligned}$$

and for $v_1 \leq v_1^*$ we have that

$$
\begin{aligned}
S(v_1) &= \int_{\underline{v}}^{v_1}[2\alpha v^* - (1+\alpha)v_1]dF(v_2) + \int_{v_1}^{v^*}[2\alpha v^* - (1+\alpha)v_2]dF(v_2) + \int_{v^*}^{\overline{v}}(\alpha-1)v^* dF(v_2) \\
&= 2\alpha v^* F(v^*) - (1+\alpha)v_1 F(v_1) - \int_{v_1}^{v^*}(1+\alpha)v_2 dF(v_2) + (\alpha-1)v^*(1-F(v^*)) \\
&= (1+\alpha)v^* F(v^*) - (1+\alpha)[v_1 F(v_1) + \int_{v_1}^{v^*} v_2 dF(v_2)] + (\alpha-1)v^* \\
&= \alpha v^* - (1+\alpha)v_1 F(v_1) - (1+\alpha)\int_{v_1}^{v^*} v_2 dF(v_2).
\end{aligned}
$$

Using the two expressions derived previously, we get that $\bar{S} = \int S(v_1)dv_1$ can be written as

$$
\begin{aligned}
\bar{S} &= \alpha v^* F(v^*) - v^*(1-F(v^*)) \\
&\quad -(1+\alpha)\int_{\underline{v}}^{v^*} vF(v)dF(v) + (1+\alpha)\int_{v^*}^{\overline{v}} v(1-F(v))dF(v) \\
&\quad -(1+\alpha)\int_{\underline{v}}^{v^*}\int_{v_1}^{v^*} v_2 dF(v_2)dF(v_1) + (1+\alpha)\int_{v^*}^{\overline{v}}\int_{v^*}^{v_1} v_2 dF(v_2)dF(v_1)
\end{aligned}
$$

Recalling that $v^* = F^{-1}(\frac{1}{1+\alpha})$, we see that the terms in the first line cancel out, and using integration by parts for the terms in the third line, we get

$$
\bar{S} = -2(1+\alpha)\int_{\underline{v}}^{v^*} vF(v)dF(v) + 2(1+\alpha)\int_{v^*}^{\overline{v}} v(1-F(v))dF(v).
$$

We can then write

$$
\begin{aligned}
\frac{1}{2}\frac{\partial \bar{S}}{\partial \alpha} &= -\int_{\underline{v}}^{v^*} vF(v)dF(v) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) \\
&\quad -(1+\alpha)v^*F(v^*)dF(v^*)\frac{\partial v^*}{\partial \alpha} - (1+\alpha)v^*(1-F(v^*))dF(v^*)\frac{\partial v^*}{\partial \alpha} \\
&= -\int_{\underline{v}}^{v^*} vF(v)dF(v) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) + v^*F(v^*) \\
&\geq -v^*F(v^*) + \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) + v^*F(v^*) \\
&= \int_{v^*}^{\overline{v}} v(1-F(v))dF(v) \\
&\geq 0,
\end{aligned}
$$

from which we can conclude that the expected surplus is increasing in the degree of complementarity of assets α.

We now turn to establish the properties of the expected surplus when $\alpha < 1$. The first point is direct from the definition of $S(v, \alpha)$. For the second point, fix $\epsilon > 0$. Then, there exists α^* such that, for all $\alpha < \alpha^*$, there is a fraction bigger than $1-\epsilon$ in the region where $\alpha v_2^* \leq v_1 \leq \frac{v_2^*}{\alpha}$ and $\alpha v_1^* \leq v_2 \leq \frac{v_1^*}{\alpha}$. Finally, we note that in that region $S(v, \alpha)$ can take only three values, 0, $-\alpha v_1 + v_2$ and $-\alpha v_2 + v_1$, we get that $\frac{\partial S(v,\alpha)}{\partial \alpha} \leq 0$, with strict inequality in a set of positive measure, and the last result follows. ■

Proof of Theorem 3

In order to apply (9) we first need to investigate what the critical types would be. This task is immediate for agent 2: Since he does not own any of the assets, his outside payoff is 0, so irrespective of his expected payoff at an ex-post efficient assignment, the type vector where the participation constraint binds (the critical type) is $(\underline{v}_2^A, \underline{v}_2^B)$. Now, since agent 1 owns both assets, his payoff from non-participation depends on his type, and it is given by $v_1^A + \alpha v_1^B$, where $\alpha > 1$. Therefore, regardless of the shape of the participation payoff determined by an ex-post efficient allocation and the distribution of types, along the dimension of asset A, the critical type for agent 1 is $\overline{v}_1^A$, and along the dimension of asset B, it is $\overline{v}_1^B$.

The surplus equals:

$$
\begin{aligned}
S(v_1^A, v_1^B, v_2^A, v_2^B) \quad = \quad & -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} \text{ (A)} \\
& + \max\{\overline{v}_1^A + \alpha \overline{v}_1^B, \overline{v}_1^A + v_2^B, \overline{v}_1^B + v_2^A, \alpha v_2^A + v_2^B\} \text{ (B)} \\
& + \max\{v_1^A + \alpha v_1^B, v_1^A + \underline{v}_2^B, v_1^B + \underline{v}_2^A, \alpha \underline{v}_2^A + \underline{v}_2^B\} \text{ (C)} \\
& - \overline{v}_1^A - \alpha \overline{v}_1^B
\end{aligned}
$$

We now establish that under (11) and (12), the surplus S is negative: In order to do that, we have to examine a number of straightforward cases:

Case 1: $A = v_1^A + \alpha v_1^B$

In this case, C must be $v_1^A + \alpha v_1^B$, and, because $\alpha > 1$, $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. Then, the surplus equals to $S = 0$.

Case 2: $A = v_1^A + v_2^B$

This case implies that the following inequalities must be true:

$$
\begin{aligned}
v_1^A + v_2^B \quad & \geq \quad v_1^A + \alpha v_1^B \text{ } (2.i) \\
& \geq \quad v_1^B + v_2^A \text{ } (2.ii) \\
& \geq \quad \alpha v_2^A + v_2^B \text{ } (2.iii)
\end{aligned}
$$

With this information, we immediately have that $\overline{v}_1^A + v_2^B \geq \alpha v_2^A + v_2^B$ and $v_1^A + \underline{v}_2^B \geq \alpha \underline{v}_2^A + \underline{v}_2^B$. So, we have the following cases to consider:

Case 2.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. The surplus can be written as $S = -v_1^A - v_2^B + C = C - A$. Thus, S is always negative (this is implied by $2.i$, $2.ii$ and $2.iii$).

Case 2.1: $B = \overline{v}_1^A + v_2^B$. The surplus can be written as $S = -v_1^A - \alpha \overline{v}_1^B + C$. First, if $C = v_1^A + \alpha v_1^B$, then we have that $S = \alpha(v_1^B - \overline{v}_1^B) \leq 0$. If $C = v_1^A + \underline{v}_2^B$,then $S = \underline{v}_2^B - \alpha \overline{v}_1^B$, which is negative because of (12). And if $C = v_1^B + \underline{v}_2^A$, then $S = v_1^B - \alpha \overline{v}_1^B + \underline{v}_2^A - v_1^A$, which because of $2.iii$ becomes $S \leq v_1^B - \alpha \overline{v}_1^B < 0$.

Case 2.3: $B = \overline{v}_1^B + v_2^A$. The surplus can be then written as $S = -v_1^A - v_2^B + \overline{v}_1^B + v_2^A - \overline{v}_1^A - \alpha \overline{v}_1^B + C$. First, if $C = v_1^A + \alpha v_1^B$, $S = -v_1^A - v_2^B + v_1^A + \alpha v_1^B + \overline{v}_1^B + v_2^A - \overline{v}_1^A - \alpha \overline{v}_1^B \leq \overline{v}_1^B - \alpha \overline{v}_1^B + v_2^A - \overline{v}_1^A \leq \overline{v}_1^B - \alpha \overline{v}_1^B$. For the first inequality we used $2.i$, and for the second $2.iii$. If $C = v_1^A + \underline{v}_2^B$,then $S = v_2^A - \overline{v}_1^A + \overline{v}_1^B(1 - \alpha) + \underline{v}_2^B - v_2^B$. It is easy to see, using iii, that $S \leq \overline{v}_1^B(1-\alpha) + \underline{v}_2^B - v_2^B < 0$. Finally, if $C = v_1^B + \underline{v}_2^A$, $S = v_1^B - v_2^B + v_2^A - \overline{v}_1^A + \underline{v}_2^A - v_1^A + (1-\alpha)\overline{v}_1^B$. Using $2.i$ and $2.iii$, we have $S \leq (1-\alpha)\overline{v}_1^B < 0$.

Case 3: $A = \alpha v_2^A + v_2^B$

This case implies that the following inequalities hold:

$$
\begin{aligned}
\alpha v_2^A + v_2^B \quad & \geq \quad v_1^A + v_2^B \text{ } (3.i) \\
& \geq \quad v_1^B + v_2^A \text{ } (3.ii) \\
& \geq \quad v_1^A + \alpha v_1^B \text{ } (3.iii)
\end{aligned}
$$

Case 3.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. The surplus can be written as $S = C - A$. Thus S, is always negative (implied by $3.i$, $3.ii$ and $3.iii$).

Case 3.2: $B = \overline{v}_1^A + v_2^B$. The surplus can be written as $S = -\alpha v_2^A - \alpha \overline{v}_1^B + C$. If $C = v_1^A + \alpha v_1^B$, then $S = v_1^A - \alpha v_2^A + +\alpha(v_1^B - \overline{v}_1^B) < 0$ (implied by $3.i$). If $C = v_1^A + \underline{v}_2^B$, then $S = v_1^A - \alpha v_2^A + \underline{v}_2^B - \alpha \overline{v}_1^B \leq \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$. For the first inequality, we used $3.i$, and for the second (11). If $C = v_1^B + \underline{v}_2^A$, then $S = \underline{v}_2^A - \alpha v_2^A + v_1^B - \alpha \overline{v}_1^B < 0$. Finally, if $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, then $S = \alpha(\underline{v}_2^A - v_2^A) + \underline{v}_2^B - \alpha \overline{v}_1^B < \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$, which follows from (12).

Case 3.3: $B = \overline{v}_1^B + v_2^A$. Then, if $C = v_1^A + \alpha v_1^B$, $S = -\alpha v_2^A - v_2^B + \overline{v}_1^B + v_2^A + v_1^A + \alpha v_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B = -\alpha v_2^A - v_2^B + v_2^A + v_1^B - v_1^B + \overline{v}_1^B + v_1^A + \alpha v_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B \leq (\alpha - 1)(v_1^B - \overline{v}_1^B) + v_1^A - \overline{v}_1^A < 0$, where the second to last inequality follows from $(3.ii)$. If $C = v_1^A + \underline{v}_2^B$,then $S = \{v_2^A - \alpha v_2^A\} + \{\underline{v}_2^B - v_2^B\} + \{\overline{v}_1^B - \alpha \overline{v}_1^B\} + \{v_1^A - \overline{v}_1^A\} < 0$; this is easy to see given that $\alpha > 1$. Now, if $C = v_1^B + \underline{v}_2^A$, $S = -\alpha v_2^A - v_2^B + v_2^A + v_1^B + \underline{v}_2^A - \overline{v}_1^A + \overline{v}_1^B - \alpha \overline{v}_1^B$, then using $(3.ii)$, $S \leq \underline{v}_2^A - \overline{v}_1^A + \overline{v}_1^B - \alpha \overline{v}_1^B < \underline{v}_2^A - \overline{v}_1^A$. Thus, $S < 0$, because of (12). Finally, if $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, then $S = \{v_2^A - \alpha v_2^A\} + \{\underline{v}_2^B - v_2^B\} + \{\overline{v}_1^B - \alpha \overline{v}_1^B\} + \{\alpha \underline{v}_2^A - \overline{v}_1^A\}$, which is less than zero because of (11), (12) and $\alpha > 1$.

Case 3.4: $B = \alpha v_2^A + v_2^B$. The surplus can be written as $S = C - \overline{v}_1^A - \alpha \overline{v}_1^B$. If $C = v_1^A + \alpha v_1^B$, then $S = v_1^A - \overline{v}_1^A + \alpha(v_1^B - \overline{v}_1^B) < 0$. If $C = v_1^A + \underline{v}_2^B$,then $S = v_1^A - \overline{v}_1^A + \underline{v}_2^B - \alpha \overline{v}_1^B < 0$, which follows from (11). If $C = v_1^B + \underline{v}_2^A$, then $S = v_1^B - \alpha \overline{v}_1^B + \underline{v}_2^A - \overline{v}_1^A < \underline{v}_2^A - \overline{v}_1^A \leq 0$, which follows using (12). If $C = \alpha \underline{v}_2^A + \underline{v}_2^B$, $S = \alpha \underline{v}_2^A - \overline{v}_1^A + \underline{v}_2^B - \alpha \overline{v}_1^B \leq 0$ (11) and (12).

Case 4: $A = \alpha v_2^A + v_2^B$.

This case implies that the following inequalities hold:

$$\begin{aligned} v_1^B + v_2^A &\geq v_1^A + \alpha v_1^B \ (4.i) \\ &\geq v_1^A + v_2^B \ (4.ii) \\ &\geq \alpha v_2^A + v_2^B \ (4.iii) \end{aligned}$$

Using $(4.iii)$ and the fact that $\alpha > 1$, it is easy to show that $\overline{v}_1^B + v_2^A \geq \alpha v_2^A + v_2^B$ and $v_1^B + \underline{v}_2^A \geq \alpha \underline{v}_2^A + \underline{v}_2^B$. Moreover, from $(4.iii)$ is easy to check that $v_1^B > v_2^B$. Thus, $v_1^A + \alpha v_1^B > v_1^A + v_2^B$ and C can necessarily take only two values.

Case 4.1: $B = \overline{v}_1^A + \alpha \overline{v}_1^B$. Then, the surplus can be written as $S = C - A < 0$.

Case 4.2: $B = \overline{v}_1^A + v_2^B$. Then, v_2^B must be greater than $\alpha \overline{v}_1^B$. Thus, $\alpha v_2^A + v_2^B \geq \alpha v_2^A + \alpha \overline{v}_1^B > \alpha(v_1^B + v_2^A)$, which is a contradiction with $(4.iii)$, implying that this case is impossible.

Case 4.3: $B = \overline{v}_1^B + v_2^A$. In this case, $S = -v_1^B + \overline{v}_1^B - \overline{v}_1^A - \alpha \overline{v}_1^B + C$. If $C = v_1^A + \alpha v_1^B$, which implies that $S = (\alpha - 1)(v_1^B - \overline{v}_1^B) + v_1^A - \overline{v}_1^A < 0$. Finally, if $C = v_1^B + \underline{v}_2^A$, then $S = \overline{v}_1^B(1 - \alpha) + \underline{v}_2^A - \overline{v}_1^A < \underline{v}_2^A - \overline{v}_1^A$, which is negative because of (12). ■

Proof of Theorem 4

In order to apply (9) we first need to investigate what the critical types would be. Here agent 1 owns asset A and his payoff from non-participation is v_1^A, whereas agent 2 owns asset B, hence his payoff from non-participation is v_2^B. Then, it is immediate to see that the critical type for agent 1 is $\left(\overline{v}_1^A, \underline{v}_1^B\right)$ and the one for agent 2 is $\left(\underline{v}_2^A, \overline{v}_2^B\right)$.

In this case, the surplus can be written as:

$$
\begin{aligned}
S(v_1^A, v_1^B, v_2^A, v_2^B) \quad = \quad & -\max\{v_1^A+\alpha v_1^B, v_1^A+v_2^B, v_1^B+v_2^A, \alpha v_2^A+v_2^B\} \ (\tilde{\mathrm{A}}) \\
& +\max\{\overline{v}_1^A+\alpha\underline{v}_1^B, \overline{v}_1^A+v_2^B, \underline{v}_1^B+v_2^A, \alpha v_2^A+v_2^B\} \ (\tilde{\mathrm{B}}) \\
& +\max\{v_1^A+\alpha v_1^B, v_1^A+\overline{v}_2^B, v_1^B+\underline{v}_2^A, \alpha\underline{v}_2^A+\overline{v}_2^B\} \ (\tilde{\mathrm{C}}) \\
& -\overline{v}_1^A-\overline{v}_2^B
\end{aligned}
$$

We now establish that under (17) and (18), the surplus S is negative: In order to do that, we have to examine a number of straightforward cases:

Case 1: $\tilde{B}=\overline{v}_1^A+\alpha\underline{v}_1^B$. Then, when $\tilde{C}=v_1^A+\alpha v_1^B$ and $S=-\tilde{A}+v_1^A+\alpha v_1^B+\overline{v}_1^A+\alpha\underline{v}_1^B-\overline{v}_1^A-\overline{v}_2^B \leq \alpha\underline{v}_1^B-\overline{v}_2^B \leq 0$, by (17). If $\tilde{C}=v_1^A+\overline{v}_2^B$, then $S=-\tilde{A}+v_1^A+\overline{v}_2^B+\overline{v}_1^A+\alpha\underline{v}_1^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^A-\alpha v_1^B+v_1^A+\alpha\underline{v}_1^B \leq 0$. If $\tilde{C}=v_1^B+\underline{v}_2^A$ then $S=-\tilde{A}+v_1^B+\underline{v}_2^A+\overline{v}_1^A+\alpha\underline{v}_1^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+v_1^B+\underline{v}_2^A+\alpha\underline{v}_1^B-\overline{v}_2^B < \alpha\underline{v}_1^B-\overline{v}_2^B \leq 0$, where the last inequality stands due to (17). Finally, when $\tilde{C}=\alpha\underline{v}_2^A+\overline{v}_2^B$, then $S=-\tilde{A}+\alpha\underline{v}_2^A+\overline{v}_2^B+\overline{v}_1^A+\alpha\underline{v}_1^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+\alpha\underline{v}_1^B+\alpha\underline{v}_2^A < 0$.

Case 2: $\tilde{B}=\overline{v}_1^A+v_2^B$. Now, if $\tilde{C}=v_1^A+\alpha v_1^B$, then $S=-\tilde{A}+v_1^A+\alpha v_1^B+\overline{v}_1^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq v_2^B-\overline{v}_2^B \leq 0$, the equality holds when $v_2^B=\overline{v}_2^B$. If $\tilde{C}=v_1^A+\overline{v}_2^B$, then $S=-\tilde{A}+v_1^A+\overline{v}_2^B+\overline{v}_1^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B=v_1^A+v_2^B-\tilde{A} \leq 0$. If $\tilde{C}=v_1^B+\underline{v}_2^A$, then $S=-\tilde{A}+v_1^B+\underline{v}_2^A+\overline{v}_1^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+v_1^B+\underline{v}_2^A+v_2^B-\overline{v}_2^B \leq 0$, and finally if $\tilde{C}=\alpha\underline{v}_2^A+\overline{v}_2^B$, then we have that $S=-\tilde{A}+\alpha\underline{v}_2^A+\overline{v}_2^B+\overline{v}_1^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_2^B-\alpha v_2^A+\alpha\underline{v}_2^A+v_2^B \leq 0$.

Case 3: $\tilde{B}=\underline{v}_1^B+v_2^A$. In this case, if $\tilde{C}=v_1^A+\alpha v_1^B$, then $S=-\tilde{A}+v_1^A+\alpha v_1^B+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+v_1^A+\alpha v_1^B+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B=(\alpha-1)v_1^B+v_1^A-\overline{v}_1^A+\underline{v}_1^B-\overline{v}_2^B < \underline{v}_1^B-\overline{v}_2^B \leq 0$ by (17). If $\tilde{C}=v_1^A+\overline{v}_2^B$, then $S=-\tilde{A}+v_1^A+\overline{v}_2^B+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B \leq -v_2^A-v_1^B+v_1^A+\underline{v}_1^B+v_2^A-\overline{v}_1^A \leq 0$. Now, in the case that $\tilde{C}=v_1^B+\underline{v}_2^A$, we have that $S=-\tilde{A}+v_1^B+\underline{v}_2^A+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+v_1^B+\underline{v}_2^A+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B=\underline{v}_2^A-\overline{v}_1^A+\underline{v}_1^B-\overline{v}_2^B \leq 0$, which follows by (17) and (18). Finally, if $\tilde{C}=\alpha\underline{v}_2^A+\overline{v}_2^B$ then $S=-\tilde{A}+\alpha\underline{v}_2^A+\overline{v}_2^B+\underline{v}_1^B+v_2^A-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+\alpha\underline{v}_2^A+\underline{v}_1^B+v_2^A-\overline{v}_1^A=\underline{v}_1^B-v_1^B+\alpha\underline{v}_2^A-\overline{v}_1^A$, which is less than zero because of (18).

Case 4: $\tilde{B}=\alpha v_2^A+v_2^B$. In this case, if $\tilde{C}=v_1^A+\alpha v_1^B$, then we have that $S=-\tilde{A}+v_1^A+\alpha v_1^B+\alpha v_2^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq -v_1^B-v_2^A+v_1^A+\alpha v_1^B+\alpha v_2^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B=(\alpha-1)(v_1^B+v_2^A)+v_1^A-\overline{v}_1^A+v_2^B-\overline{v}_2^B < 0$. If $\tilde{C}=v_1^A+\overline{v}_2^B$, then we have that $S=-\tilde{A}+v_1^A+\overline{v}_2^B+\alpha v_2^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq -\alpha v_2^A-v_2^B+v_1^A+\alpha v_2^A+v_2^B-\overline{v}_1^A=v_1^A-\overline{v}_1^A \leq 0$, where equality holds whenever $v_1^A=\overline{v}_1^A$. Now if $\tilde{C}=v_1^B+\underline{v}_2^A$, then $S=-\tilde{A}+v_1^B+\underline{v}_2^A+\alpha v_2^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq \alpha v_2^A-v_2^A+\underline{v}_2^A-\overline{v}_1^A+v_2^B-\overline{v}_2^B < 0$, due to (18). Finally, if $\tilde{C}=\alpha\underline{v}_2^A+\overline{v}_2^B$, then $S=-\tilde{A}+\alpha\underline{v}_2^A+\overline{v}_2^B+\alpha v_2^A+v_2^B-\overline{v}_1^A-\overline{v}_2^B \leq \alpha\underline{v}_2^A-\overline{v}_1^A$, which is negative by (18)

From the analysis of the above cases, we saw that $S \leq 0$ always holds. We also saw that there is a region of types with a non-empty interior where $S < 0$. Hence, there is no feasible and ex-post efficient mechanism. If the status quo is given by (B, A), the analysis is analogous. ■

6. Appendix B: Homogeneous Assets, Concentrated Ownership:

For the case of uniform distribution, the expected surplus (deficit) turns out to be decreasing in α for all $\alpha > 0$: The expected deficit as a function of α is given by $\bar{S}(\alpha) = E_v[S(v_1, v_2, \alpha)] = \frac{2}{3}\alpha^2 - \frac{5}{6}\alpha - \frac{1}{6}\alpha^4$, and is depicted in Figure 1:

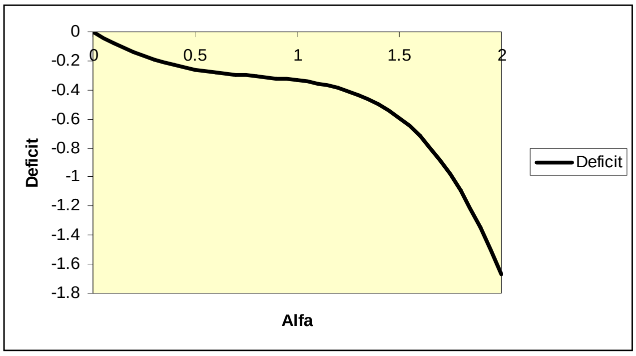


Figure 1: Surplus (Deficit) with Uniform Distribution

However, as our earlier discussion alluded to, in general, expected surplus can be non-monotonic when $\alpha < 1$. An example where this holds is as follows: Suppose that both agents' valuations are distributed according to

$$F(v) = \begin{cases} \frac{\epsilon}{\alpha^2 \bar{v}} v & \text{if } v \leq \alpha^2 \bar{v} \\ \frac{1-2\epsilon}{\alpha(1-\alpha)\bar{v}} v + \epsilon - \frac{(1-2\epsilon)\alpha}{1-\alpha} & \text{if } v \in [\alpha^2 \bar{v}, \alpha \bar{v}] \\ \frac{\epsilon}{(1-\alpha)\bar{v}} v + 1 - \frac{\epsilon}{1-\alpha} & \text{if } v \geq \alpha \bar{v} \end{cases} .$$

For this scenario the derivative of expected surplus with respect to α converges to $\bar{v}(1 - \frac{\alpha(1-\alpha)}{2} - (1-\alpha)) > 0$ as ϵ goes to 0.[12]

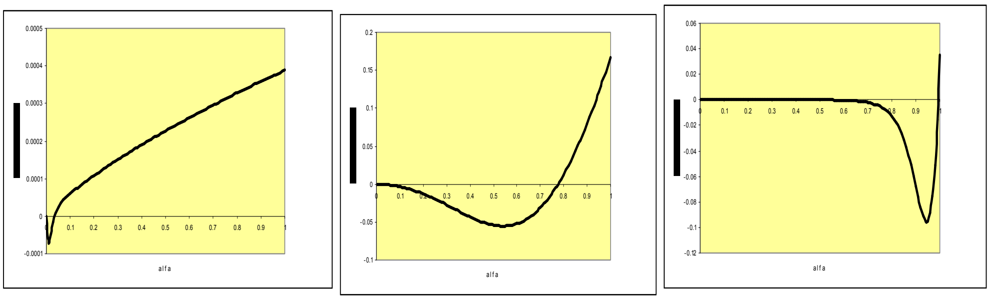
7. Appendix C: Homogeneous Assets, Dispersed Ownership: Substitutes

Here we show that the surplus (or deficit) can be non-monotonic even in symmetric environments. Suppose that both agents are symmetric and their valuations are distributed on $[0, 1]$ according to

$$F(v_i) = v_i^p, \text{ for } 0 < p < \infty. \tag{23}$$

[12] Details of the calculations for this example are available upon request.

We study how the parameter p (which determines how concentrated the distribution on small values is) affects the threshold level of α needed to achieve efficiency.[13] For this family, there is a cutoff value of α above which efficiency is possible, which is increasing in p. For example, for $p = 0.5$, this cutoff is 0.6, whereas for $p = 0.25$, the cut-off is 0.41. In the following figure, we graph the expected surplus for $p = 0.01$, and for $p = 1$ (the uniform case) and $p = 10$:



p=0.01 p=1 p=10

The graphs highlight the non-monotonicity of the surplus in α, whenever $\alpha \leq 1$. We can see that as p grows, and high valuations become more probable, there is a higher threshold value for α, above which efficiency is possible. Higher valuation, which makes an agent less likely to be willing to part with his object, makes efficiency more difficult. There a higher α increases $\bar{S}$ because it increases the willingness to pay for the asset.

8. Appendix D: Homogeneous Assets, Dispersed Ownership: Complements

Example: Expected Surplus (or Deficit) can be Non-Monotonic in Asymmetric Environments. Consider an environment with distributions $F_i(x) = x^{p_i}$ in $[0, 1]$. It is easy to see that $v_i^* = (\alpha^{2p_j+1} + 1)^{-1/p_j}$ and $\frac{\partial v_i^*}{\partial \alpha} = -(\alpha^{2p_j+1} + 1)^{-1/p_j-1} p_j^{-1}(2p_j + 1)\alpha^{2p_j}$.

Fix $\alpha > 1$, but close to 1, and take $p_1 \longrightarrow 0$ and $p_2 \longrightarrow +\infty$. Then, with probability close to 1, $v_2 \geq v_1$ and, moreover, $F_2(v_1^*) = 1/(\alpha^{2p_2+1} + 1) \sim 0$, but $F_1(v_2^*) \sim 1/2$. Therefore, only two cases appear with

[13] When both agents' valuations are distributed according to (23), the expected surplus is

$$
\begin{aligned}
\bar{S}(\alpha) &= 2\int_{\alpha v^*}^{\alpha} v dF - 2\alpha \int_0^1 v^{p+1}\alpha^p dF - 2\int_0^{\alpha} v^{p+1}\alpha^{-p} dF + \alpha\{1+\alpha^p\}\int_{v^*\alpha^{-1}}^{1} v dF \\
&= 2\frac{p}{p+1}\alpha^{p+1}(1 - v^{*p+1}) - 4\frac{p}{2p+1}\alpha^{p+1} + \alpha(1+\alpha^p)\frac{p}{p+1}(1 - \frac{v^{*p+1}}{\alpha^{p+1}}),
\end{aligned}
$$

where v^* satisfies $1 = \alpha(\alpha v^*)^p + (\frac{v^*}{\alpha})^p$.

probability bigger than ϵ, both of them close to $\frac{1}{2}$:

$$\begin{aligned} S(v_1, v_2, \alpha) &= (1+\alpha)v_1 - v_1^* - v_2^* \text{ if } v_1 \geq v_2^* \text{ and } v_1^* \leq v_2 \\ S(v_1, v_2, \alpha) &= \alpha v_2^* - v_1^* \text{ if } v_1 \leq v_2^* \text{ and } v_1^* \leq v_2. \end{aligned}$$

Then, we have

$$\frac{\partial \bar{S}(\alpha)}{\partial \alpha} = \int_{v_2^*}^{1} v_1 f_1(v_1) dv_1 - \frac{\partial v_1^*}{\partial \alpha} + \frac{v_2^*}{2} + \frac{\alpha - 1}{2} \frac{\partial v_2^*}{\partial \alpha}.$$

Noting that the first three terms are bounded by one, but that $\frac{\partial v_2^*}{\partial \alpha}$ goes to minus infinity, we see that $\frac{\partial \bar{S}(\alpha)}{\partial \alpha} < 0$.

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