Sequential Procurement Auctions and Their Effect on Investment Decisions*

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Abstract

In this paper we characterize the optimal procurement mechanism and the investment level for an environment where two projects must be adjudicated sequentially, and the winner of the first project has the opportunity to invest in a distributional upgrade for its costs in the second project. We study 4 cases, based on the commitment level of the seller and the observability of the investment decision. We find that with commitment, the second period mechanism gives an advantage to the first period winner, and induces an investment level that is larger than the efficient one. With non-commitment, the second period mechanism gives a disadvantage to the first period winner, and induces an investment level that is smaller than the efficient one. Observability is irrelevant in the commitment case, but makes the effects more pronounced in the non-commitment case. Finally, we extend the model to allow for investment by a first period loser. Keywords: *Procurement Auctions, Sequential Mechanisms, Mechanism Design, Cost Reducing Investment JEL D44, C7, C72.*

1. Introduction

During the last decades procurement auctions have been widely used as mechanisms to assign high cost projects concerning goods and services. By the year 1998, the sum of all governments expenditures in procurements (excluding defense and labor compensation) was estimated as 7.1% of the worldwide GDP[1]. The repeated utilization of procurement auctions by specific by governments and private institutions, which contract with the same pool of firms over time, has made the study of cost reduction investment by these firms specially relevant.

The objective of this paper is twofold. First, to characterize the cost-minimizing procurement mechanisms in an environment with repeated interaction between a buyer and multiple sellers, where investment can be undertaken as a cost-reduction device, and also as an strategic action devised to obtain advantages in future procurement auctions. Second, to analyze the effects these cost-minimizing mechanisms have on the sellers' investment decisions. This is particularly relevant if the relative size of expected expenditures

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[1]Source: OECD, *The Size of Government Procurement Markets*, Journal of Budgeting Vol.1, No.4, March 2002, ISBN 9264196307.

is big compared with the value of just one particular purchase, since in that case the use of tools that may reduce future costs, even at the cost of raising today's expenditures, can be particularly profitable.

We consider a buyer who wants to procure two consecutive projects and faces $n \geq 2$ potential suppliers. Costs for performing both tasks are distributed independently across time and competitors, and they are private information to each firm. We assume, though, that the first procurement winner (whose identity is public information for the second procurement) acquires an intrinsic advantage characterized by a *distributional upgrade*. This reflects a knowledge acquired concerning the task performed, to which the rest of the firms have no access. Moreover, this upgrade can be influenced by a costly investment that the winner may carry out between the two procurements. In such a context, second period rules, through the degree of advantage they give to a first period winner, affect the benefits of such an investment, and therefore determine the investment carried out by the first period winner. As a consequence, the second period cost distribution of a first period winner, and the asymmetry between him and the first period losers, are determined endogenously.

We first consider the ex-post efficient benchmark: each period the project is assigned to the lowest cost supplier, and the investment level, I^e is such that the marginal cost of investment equals the marginal benefit (in expected terms) of cost reduction. We show that, independent of investment observability, such an incentive compatible mechanism exists and can be implemented through two second price sealed bid procurements.

We then use the mechanism design approach to characterize the cost-minimizing mechanism, which we determine under the assumptions of full commitment and non-commitment of the buyer. In the first case, we assume that the rules for both mechanisms are decided at t=0, before any cost realization occurs. We show that the optimal mechanism is independent of investment observability, and gives an *advantage gap* to the first period winner in the second period. That is, he can win the second procurement auction even if his cost is higher than the minimum of the other competitors. Moreover, in this context, the investment level induced, I^*, is higher than the efficient level I^e, so *over-investment* occurs. This result can seem counterintuitive, since an advantage gap can make the first period winner "relax", knowing he owns a big advantage over competitors and is likely to win anyway. However, there is a second effect that dominates: investment, by decreasing costs, increases the expected profits in case the firm gets the contract. Since with a bigger advantage gap this happens more often, the expected benefits due to cost reduction are higher, thus investment becomes more profitable.

In the non-commitment case, the second period rules are determined after the winner of the first period is announced and he has invested in cost reduction. We find that it is optimal for the buyer to give a *disadvantage* to the first period winner, who holds a distributional advantage over its competitors. Now, a first period winner does not necessarily win the second period procurement even if he has the lowest cost among all competitors. This fact is anticipated by the first period winner, making investment less attractive and therefore leading to investment levels of cost-reduction below the efficient one. Here, the observability of investment makes a difference. When investment is not observable, the interaction between the buyer and the first period winner is equivalent to both agents simultaneously choosing mechanisms and investment levels, respectively. On the contrary, when investment can be monitored, the buyer *optimally reacts* with more disadvantageous mechanisms as the first period winner distribution improves. Thus, the first period winner invests less in this last case. Denoting by $\widehat{I}$ and $\widehat{\widehat{I}}$ the investment levels when investment is observable and non-observable we get

$$\widehat{I} \leq \widehat{\widehat{I}} < I^e < I^*$$

These results have important implications. First, cost-minimizing buyers, with the ability to commit, and working in a dynamic environment, induce investment levels above the efficient level. This is because the second period advantages are not only introduced to encourage investment, but also to increase competition in the first period. In this respect, the buyer introduces inefficiencies to increase competition, in a similar way a static buyer introduces inefficiencies setting up a reservation cost. However, a buyer's lack of commitment induces under-investment, since the disadvantages to first period winners make cost reduction investment less attractive. Second, investment observability does not play a role in the case of full commitment, neither for efficiency nor for cost minimization. Therefore, the intuition from the moral hazard literature, with risk-neutral agents, the first best level of investment (effort) can be induced, can be extended to the case of revenue maximization. This result is quite interesting since it does not hold for any mechanism: we show that, in the full commitment case, a second period mechanism with different rules leads to a seller's investment decision different from the one the buyer would have chosen. To conclude the analysis when the first period winner is the only agent allowed to invest, we derive some comparative statics with respect to the number of players. As it increases, all the investment levels mentioned above decrease, but weakly preserving the order among them. For a number of firms sufficiently large, the investment level in all environments collapses to zero: if the number of competitors is large enough, inducing investment is too expensive for the buyer relative to the marginal benefit in cost reduction. This is so because the added probability of the first period winner getting a cost lower than the minimum of *all other competitors*, decreases with the number of competitors.

Finally, we also examine the robustness of our results to the possibility of investment by a first period loser as well. If investment is observable, and depending on the shape of the cost distribution, two possibilities appear. Either the buyer will use the same mechanism as the one when only the winner can invest and induce no loser's investment, or it will give less advantage to the winner, inducing a positive investment by the loser and a smaller investment by the winner. Without observability, the problem is much more complex. With some restrictions on the set of available mechanisms, we prove that the buyer will optimally choose the same mechanism as when only the winner can invest, but this will induce a positive investment by the loser and a smaller investment by the winner.

Our work is related to the literature in several ways. With the methodology of mechanism design, Pesendorfer and Jofre-Bonnet [9], derived the cost minimizing mechanism for an exogenous level of complementarity (which is understood as an exogenous distributional change between both procurement auctions), full commitment and two players. We extend this result to allow for investment (and therefore endogenous complementarity), non-commitment and multiple players. This extension is not obvious, in particular, because of the interesting features that arise as the number of competitors increase: under full commitment, although the advantage given to the first period winner at the last competition also decreases with the number of firms, it never disappears. This occurs because introducing sequentiality enables the buyer to distribute incentives inter-temporally in such a way that more competition is induced in the first procurement auction, which out-weights the second period inefficiencies regardless of the number of players. In a non-commitment setting, Luton and McAfee [5] determined the optimal mechanism for a particular type of exogenous distributional upgrade.

Investment as a cost reduction device has been analyzed mainly as a stage prior to a one-shot procurement auction. In this context, the possibility of achieving efficiency was studied by Piccione and Tan [10] when firms are ex-ante symmetric and they can all invest in a technology that presents diminishing returns to scale. They show that the full information solution can be implemented if the buyer can fully commit to the mechanism in advance. We extend this result to the case of revenue maximization in a sequential auction setting. We also show that, surprisingly, this result cannot be extended to other objective functions: for other advantage levels required in the second stage the incentives of the buyer and the sellers do not coincide, so investment observability is crucial for implementation. Finally, several under-investment results can be found in the literature. In particular, Piccione and Tan [10] show that the buyer's lack of commitment induces an investment level below the efficient one. Dasgupta in a similar model [2], obtains the same result and shows that commitment rises the investment level, but always below efficiency. Arozamena and Cantillon in [1], analyze the effects of allowing only one firm to invest before a first price sealed bid procurement auction, and make this action observable by competitors, finding that *under-investment* occurs as a response to the more aggressive bidding of the competitors, which reduces the investment incentives. In our paper, we also get under-investment, but for a reason that goes in line with [10]: under non-commitment, it is the mechanism designer (the buyer) that changes behavior, giving an advantage to worse firms, and therefore decreasing the incentives to invest. Nevertheless, we introduce the new idea that the buyer's commitment in a multi-period context can increase investment above efficiency.

2. The Model

2.1 The Environment

Consider a risk-neutral buyer who wants to procure two projects, one at $t = 1$ and the other at $t = 2$. The set of competing sellers is $N = \{1, ..., n\}$, $n \geq 2$, all of which are risk-neutral and live for the two periods. The buyer is compelled to procure the two goods or services[2]. In each period, the cost of undertaking the project for a seller is drawn from the interval $C = [\underline{c}, \bar{c}]$, and it is private information. At $t = 1$, these costs are independently distributed according to a distribution $F(\cdot)$, differentiable, that satisfies $f(c) \equiv F'(c) > 0$ if $c \in C$ (so sellers are *ex-ante* symmetric).

At $t = 2$, the competitors costs are drawn independently from those drawn in period 1, and independently across sellers as well. The costs of the first period losers are taken from the same distribution $F(\cdot)$. Instead, the winner of the first procurement (from now on *the winner*) has the option of investing an amount I between auctions, at a monetary cost $\Psi(I)$, and changes his distribution to $G(\cdot, I)$, differentiable with $\frac{\partial G}{\partial c}(c, I) > 0$ and with the same support. Assumption 1 below implies a distribution improvement for the winner as a function of investment: investment implies an increase in the chance of obtaining lower costs relative to higher ones. As a consequence, higher investment induces a "better" cost distribution in the usual sense of first order stochastic dominance. The formal result is in lemma 9 in the next section.

Assumption 1 *$G \in C^2(C \times I\!R_+)$. For all $0 \leq I' < I \in I\!R$ and $c' < c \in C$,*

$$\frac{f(c')}{f(c)} \leq \frac{\frac{\partial G}{\partial c}(c', I')}{\frac{\partial G}{\partial c}(c, I')} < \frac{\frac{\partial G}{\partial c}(c', I)}{\frac{\partial G}{\partial c}(c, I)}$$

The first inequality, when evaluated at $I' = 0$ indicates that there can exist an exogenous improvement for the first period winner. He acquires a knowledge concerning the task performed (the *know-how*), which is not available to the losers. The second inequality shows that the final degree of complementarity will depend on the amount the seller invests in developing this *know-how*. We also impose that the marginal benefit of investment is decreasing:

Assumption 2 *For all $I \in I\!R_+$, $\frac{\partial^2 G}{\partial I^2}(\cdot, I) < 0$ in $(\underline{c}, \bar{c})$.*

[2] This is equivalent to assume that the cost for the buyer of carrying out the project himself is $C_0 = +\infty$.

We now state two technical assumptions, the first one is the increasing hazard rate, and the second is a condition needed for integrability.

Assumption 3 $\frac{F(c)}{f(c)}$, $\frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ *are increasing in c. Also,* $\frac{F(c)}{f(c)}$ *is differentiable (in particular, F is twice differentiable).*

Assumption 4 *There exists* $h \in L^1(I\!R)$ *such that*

$$\left|\frac{\partial G}{\partial I}(c,I)\right| = \frac{\partial G}{\partial I}(c,I) < h(c),\ \forall\ I \in I\!R$$

Finally, for the investment technology we assume, as usual, that is increasing and convex.

Assumption 5 $\Psi(\cdot)$ *is twice differentiable and satisfies* $\Psi'(\cdot) > 0$, $\Psi''(\cdot) \geq 0$.

The previous assumptions are not hard to satisfy. For example, the family of distributions introduced in Piccione and Tan [10], which they argue is a way of modeling investment in cost reduction of R&D technologies, satisfies them.

Example 6 *Suppose that* $F(\cdot)$ *is a twice differentiable concave distribution. Then, it is straightforward that verifies the* regularity *assumption. The family of distributions given by* $G(c,0) = F(c)^{\eta}$ *with* $0 < \eta < 1$ *and* $G(c,I) = 1 - (1 - G(c,0))^{\gamma I + 1}$ *with* $\gamma > 0$ *satisfies assumptions 1 through 4 (see Appendix for a proof).*

For notation purposes, we denote the joint density of the first-period distribution as $f^n(c) = \prod_{j=1}^{n} f(c_j)$ and $f^{n-1}(c_{-i}) = \prod_{j \neq i} f(c_j)$, with $c_{-i} = (c_1, ..., c_{i-1}, c_{i+1}, ..., c_n)$.

2.2 The Mechanisms

The fact that costs are drawn independently across time enables the buyer to pay attention only to incentive compatible mechanisms because the revelation principle holds. We will focus on two types of environments: *full commitment* and *non-commitment* of the buyer. In the first case, the buyer can commit to the first and second period mechanisms before any cost realization takes place. In the second, he cannot, and he chooses the second period rules after the identity of the first period winner is known and the investment level has been selected.

In each case, we analyze the cases when investment is observable and when it is not. The difference is that in the observable case, the investment, and therefore the winner's cost distribution for the second period is public information. Thus, in this setting the mechanisms used by the buyer at $t = 2$ may depend on the level of investment chosen by the winner of the first procurement auction.

We consider second period procurement mechanisms that are history dependent only to the extent that they depend on the identity of the first period winner, but not on the cost realization (revelation) of that first period. Moreover, we do not allow the buyer to exclude sellers in the second period if they do not participate in the first. Notice, however, that more general mechanisms can decrease the seller's expected cost. An extreme example of this is the threat of non-participation in the second period. With this strategy, the first period payments can be increased in the amount of the expected utility in the second period mechanism, allowing the seller to fully extract the second period surplus. Such mechanisms seem unrealistic and seldom used in practice, probably because of the inability of sellers to commit to such a strong and ex-post inefficient punishments.

From now on, we define $\Delta_n = \left\{(x_1,...,x_n) \in I\!R^n_+ | \sum_{i=1}^{n} x_i = 1\right\}$ the unit simplex. If investment is observable and the amount of it chosen by the first period winner is I, we will use subscript (w, I) when we refer to this player at $t = 2$. Analogously, we will use subscript (l, I, i) at $t = 2$ if player $i \in N$ was a first period loser (if investment can't be observed, we drop the subscript I). Using the previous notations, we have:

Definition 7 *A direct mechanism, when investment is not observable, is given by the tuple*

$$\Gamma_{no} = (t^1, q^1, t^2_w, q^2_w, t^2_l, q^2_l)$$

where $t^1 : C^n \longrightarrow I\!R^n$, $q^1 : C^n \longrightarrow \Delta_n$, $t^2_w : C^n \longrightarrow I\!R$, $q^2_w : C^n \longrightarrow [0,1]$, $t^2_l : C^n \longrightarrow I\!R^{n-1}$, $q^2_l : C^n \longrightarrow [0,1]^{n-1}$, *such that* $q^2_w(c) + \sum_{i \neq w} q^2_{l,i}(c) = 1$ *for all* $c \in C^n$.

Definition 8 *A direct mechanism, when investment is observable, is given by the tuple*

$$\Gamma = (t^1, q^1, \{t^2_{w,I}\}_{I\geq 0}, \{q^2_{w,I}\}_{I\geq 0}, \{t^2_{l,I}\}_{I\geq 0}, \{q^2_{l,I}\}_{I\geq 0})$$

where $t^1 : C^n \longrightarrow I\!R^n$, $q^1 : C^n \longrightarrow \Delta_n$, $t^2_{w,I} : C^n \longrightarrow I\!R$, $q^2_{w,I} : C^n \longrightarrow [0,1]$, $t^2_{l,I} : C^n \longrightarrow I\!R^{n-1}$, $q^2_{l,I} : C^n \longrightarrow [0,1]^{n-1}$, *such that* $q^2_{w,I}(c) + \sum_{i \neq w} q^2_{l,I,i}(c) = 1$ *for all* $c \in C^n$ *and* $I \geq 0$.

When investment is not observable, $t^s(c) = (t^s_1(c), ..., t^s_n(c))$, and $t^s_i(c)$ corresponds to the payment to seller $i \in N$ at time $s = 1, 2$, conditional on the report $c = (c_1, ..., c_n)$. Analogously, $q^s(c) = (q^s_1(c), ..., q^s_n(c))$, with $q^s_i(c)$ the probability that competitor i wins the procurement auction at time $s = 1, 2$ conditional on the report. Finally, when investment can be monitored, the functions are the same, but now the second period rules may depend on the investment level carried out by the first period winner.

3. Preliminary Results

We now state a well-known result involving some consequences of assumption 1. In particular, that the monotone likelihood ratio property implies first order stochastic dominance as the investment level decreases (see Milgrom [6]).

Lemma 9 *Suppose that assumption 1 holds, then:*

(i)

$$\frac{\frac{\partial G}{\partial c}(c, I)}{1 - G(c, I)} < \frac{\frac{\partial G}{\partial c}(c, I')}{1 - G(c, I')}, \forall\, c \in C,\ 0 \leq I < I'.$$

(ii)

$$\frac{G(c, I)}{\frac{\partial G}{\partial c}(c, I)} < \frac{G(c, I')}{\frac{\partial G}{\partial c}(c, I')}, \forall\, c \in C,\ 0 \leq I < I'.$$

(iii) *For every* $c \in C$, *the function* $G(c, \cdot)$ *is increasing. This is equivalent to* first order stochastic dominance *as* I *decreases in the family of distributions* $\{G(\cdot, I) |\ I \geq 0\}$.

Proof: Standard.

□

We denote by $Q_i^1(c_i')$, player i's expected probability of winning the first period procurement auction if his announcement is c_i' and the other players are telling the truth. Analogously we denote by $T_i^1(c_i')$, player i's expected transfer if his announcement is c_i' and the other players are telling the truth:

$$Q_i^1(c_i') = \int_C q_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i}, \ i \in N. \tag{1}$$

$$T_i^1(c_i') = \int_C t_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i}, \ i \in N. \tag{2}$$

In the case of investment observability, expected probabilities, if the investment level carried out by the first period winner is I, are given by:

$$Q_{w,I}^2(c_w') = \int_{C^{n-1}} q_{w,I}^2(c_w', c_{-w}) f^{n-1}(c_{-w}) dc_{-w} \tag{3}$$

$$Q_{l,I,i}^2(c_i') = \int_{C^{n-1}} q_{l,I,i}^2(c_i', c_{-i}) f^{n-2}(c_{-w,i}) \frac{\partial G}{\partial c}(c_w, I) dc_{-i}, \ i \neq w. \tag{4}$$

with expected transfers $T_{w,I}^2$, $T_{l,I,i}^2$ defined analogously.

We denote by $\Pi_{w,I}^2(c_w, c_w')$ the expected utility at $t = 2$ of a first period winner with real cost c_w that declares c_w'. Analogously, $\Pi_{l,I,i}^2(c_i, c_i')$ is the expected utility of a first period loser with real costs c_i and declaration c_i':

$$\Pi_{w,I}^2(c_w, c_w') = T_{w,I}^2(c_w') - c_w Q_{w,I}^2(c_w') - \Psi(I) \tag{5}$$

$$\Pi_{l,I,i}^2(c_i, c_i') = T_{l,I,i}^2(c_i') - c_i Q_{l,I,i}^2(c_i'), \ i \neq w \tag{6}$$

Finally, we denote by $\Pi_{i,I}^1(c_i, c_i')$ the discounted expected utility at $t = 1$ for seller i with cost c_i that declares c_i', conditional on truth-telling at $t = 2$.

$$\Pi_{i,I}^1(c_i, c_i') = T_i^1(c_i') - c_i Q_i^1(c_i') + \beta Q_i^1(c_i') \int_C \Pi_{w,I}^2(c, c) \frac{\partial G}{\partial c}(c, I) dc + \beta[1 - Q_i^1(c_i')] \int_C \Pi_{l,I,i}^2(c, c) f(c) dc. \tag{7}$$

This last expression consists in expected payments and costs of the first procurement (the first two terms), and the ones related to the second period expected utility (which depends on being the winner or a loser in the first period).

As we said before, we can restrict the analysis to direct mechanisms. Truth-telling in the second period can be written as:

$$IC_o^2 \begin{cases} \Pi_{w,I}^2(c_w, c_w) \geq \Pi_{w,I}^2(c_w, c_w'), \ \forall \ c_w, c_w' \in C, \ \forall I \geq 0. \\ \\ \Pi_{l,I,i}^2(c_i, c_i) \geq \Pi_{l,I,i}^2(c_i, c_i'), \ \forall \ c_i, c_i' \in C, \ \forall \ i \neq w, \forall I \geq 0 \end{cases}$$

And for the first period it can be written as:

$$IC_o^1\text{: } \forall\ i \in N \text{ and } I \geq 0,\ \Pi^1_{i,I}(c_i, c_i) \geq \Pi^1_{i,I}(c'_i, c_i),\ \forall\ c_i, c'_i \in C.$$

Observation: We have defined second period rules (transfers and probabilities) and incentive compatibility constraints for any possible investment level $I \geq 0$. However, the only relevant investment level is the one the buyer wants to induce. The rules for any other I can be set-up such that those investment levels are never selected, and IC is satisfied trivially. This allows us to see mechanisms when investment is observable as a tuple (Γ, I), where I is the investment level "forced" upon the first period winner and Γ specifies second period rules for that particular I (and also first period rules).

When investment is not observable, second period rules can't depend on investment level I. In this setting, the rules $(q^2_w, q^2_{l,i}, t^2_w, t^2_{l,i})$ omit the variable I and all the above expressions can be obtained with this notational change. Nevertheless, incentive compatibly implies that the investment level I (chosen by the seller) and both period rules must simultaneously satisfy

$$IC_{no} \begin{cases} I \in arg \max\limits_{K \geq 0} \int\limits_C \Pi^2_w(c, c) \frac{\partial G}{\partial c}(c, K) dc \\ \\ \Pi^2_w(c_w, c_w) \geq \Pi^2_w(c_w, c'_w),\ \forall\ c_w, c'_w \in C \\ \\ \Pi^2_{l,i}(c_i, c_i) \geq \Pi^2_{l,i}(c_i, c'_i),\ \forall\ c_i, c'_i \in C,\ \forall\ i \neq w \\ \\ \Pi^1_i(c_i, c_i) \geq \Pi^1_i(c'_i, c_i),\ \forall c_i, c'_i \in C,\ \forall i \in N. \end{cases}$$

The first condition requires that the the investment level chosen by the first period winner must maximize his utility level conditional on second period rules. Although we do not write subscript I in all the inequalities, it implicitly appears in the second period expected utility of the first period losers and in the first period utility of all players, through the function $G(c, I)$. We omit the mentioned variable just to emphasize that the buyer can make no use of this more general mechanisms when investment is non-observable.

In the next lemma we state the usual characterization of incentive compatible mechanisms:

Lemma 10 ***(Incentive Compatibility):*** *Γ is incentive compatible if and only if*

(i) *For all $i \in N$ and $I \geq 0$,*

- *$Q^1_i(\cdot)$ is non increasing*
- *$\Pi^1_{i,I}(c_i, c_i) = \Pi^1_{i,I}(\bar{c}, \bar{c}) + \int\limits_{c_i}^{\bar{c}} Q^1_i(s) ds$ for all $c_i \in C$*

(ii) *For all $I \geq 0$,*

- *$Q^t_k(\cdot)$ is non increasing, $k = (w, I), (l, I, i)$, $i \neq w$, $i \in N$.*
- *$\Pi^2_k(c_k, c_k) = \Pi^2_k(\bar{c}, \bar{c}) + \int\limits_{c_k}^{\bar{c}} Q^2_k(s) ds$ for all $c_k \in C$, $k = (w, I), (l, I, i)$, $\forall\ i \neq w$, $i \in N$.*

Proof: See Appendix.

□

Participation constraints depend on the level of commitment and investment observability analyzed, and will be discussed in the sections where optimal mechanisms are characterized.

4. Efficiency

As a benchmark we find an ex-post efficient mechanism, that is, in each period the project is assigned to the lowest cost supplier and induces an investment level such that the marginal cost of investment equals the marginal benefit (in expected terms) of cost reduction. In this section we characterize this investment level, which we will use as a benchmark, and quickly establish the possibility of implementing such an investment, regardless of investment observability. The key point is that for mechanisms that assign to the lowest cost supplier, the investment level (in the non-observable case) is automatically defined (see equation (9), and this coincides with the buyer's desired level.[3]

Since the efficient mechanism, that we denote by Γ^e, must assign each project to the competitor with the lowest cost, the assignment rules are given by:

$$q_i^{t,e}(c) = \begin{cases} 1 & c_i < c_j,\ \forall j \in N \\ 0 & \sim \end{cases} \tag{8}$$

for $t = 1, 2,\ i \in N$. Given this rules, and if the first period winner invests a quantity I, the expected social cost is

$$\begin{aligned} \mathcal{C}(\Gamma^e, I) &= \int\limits_{C^n} \left[\sum_{i=1}^n c_i q_i^{1,e}(c) \right] f^n(c)dc + \beta \int\limits_{C^n} \left[c_w q_w^{2,e}(c) + \sum_{i \neq w} c_i q_i^{2,e}(c) \right] f^{n-1}(c_{-w}) \frac{\partial G}{\partial c_w}(c_w, I)dc + \beta\Psi(I) \\ &= n \int\limits_C c[1-F(c)]^{n-1} f(c)dc + \beta \int\limits_C c[1-F(c)]^{n-1} \frac{\partial G}{\partial c}(c,I)dc \\ &\quad + \beta(n-1) \int\limits_C c[1-F(c)]^{n-2}[1-G(c,I)]f(c)dc + \beta\Psi(I) \end{aligned}$$

Therefore, the efficient investment level, I^e, is the solution $\min\limits_{I \geq 0}\ \mathcal{C}(\Gamma^e, I)$. In the next result we characterize I^e and find a mechanism that achieves efficiency regardless of investment observability.

Proposition 11 The efficient investment level I^e is the solution to

$$\min_{I \geq 0} \Psi(I) - \int\limits_C [1-F(c)]^{n-1} G(c,I)dc \tag{9}$$

Moreover, it can be induced regardless of investment observability using second price sealed bid procurement auctions.

Proof: See Appendix.

□

In equation (9) we can see the costs and benefits associated to investment. In the first term, we have the cost of investment, and in the second the expected cost reduction due to one competitor having a better distribution[4].

To conclude this section, note that the irrelevance of investment observability is due to risk neutrality. From this perspective, the investment level decision correspond to the effort level chosen by the agent in a risk neutral principal-agent model, where it is known that the first best solution can be achieved when effort is not observable.

[3] In a slightly different setup, with ex-ante and ex-post investment, the same result is derived in Piccione and Tan [10].
[4] Recall that for each $c \in C$, $I \mapsto G(c, I)$ is increasing, thanks to Lemma 9, (iii)

5. Revenue Maximization Under Full Commitment

In this section we assume the existence of institutions that may enforce the contracts established by the buyer. He commits, before any cost realization, to rules for the first and second period procurement mechanisms. Second period rules can give different conditions to the first period winner and losers, and can depend on the investment level when it is observable. However, if a seller decides not to participate in the first period, we assume the buyer does not have enough commitment power to guarantee he will not have access to the second period mechanism and treated just as a first period loser. This last point is important (as it will be explained when we discuss the participation constraints), and very plausible. Even if a buyer could commit to this rule, a first period buyer could change the name of his firm and ask for acceptance in the second period, making it impossible for a seller to leave him out without being accused of partiality.

We first characterize the cost minimizing mechanism with *investment observability.* We find that it gives an advantage to the first period winner and that this advantage is independent of the level of investment required. This, in turn, induces over-investment by the first period winner. Then we turn to the case of non-observable investment. We find that the optimal mechanism remains unchanged and, moreover, the level of investment chosen by the first period winner is the same the buyer would have chosen if he could observe it. As a consequence, the irrelevance of investment observability found for the efficient mechanism can be extended to the case of cost minimization. This is quite interesting since it only holds for the cost-minimizing mechanism: in section 7 we show that if the buyer commits to second period rules that give less advantage to the first period winner than the cost minimizing mechanism, the amount of investment selected by the former agent is larger than the one carried out by the latter.

5.1 Investment Observability and Full Commitment

In this context, since investment is observable, the buyer can induce any amount of investment he wants. This can be done by setting transfers low enough (even payments to the buyer) so that any other level chosen by the first period winner is unprofitable for him. Therefore, we assume the buyer chooses the investment level $I \geq 0$.

Participation in the second period is ensured by

$$PC_o^2(I) \begin{cases} \Pi^2_{w,I}(c_w, c_w) \geq 0, \ \forall c_w \in C \\ \\ \Pi^2_{l,I,i}(c_i, c_i) \geq 0, \ \forall c_i \in C, \ i \neq w. \end{cases}$$

At $t = 1$, we require participation in both procurements auctions to be more profitable (in expected terms) than doing it only in the last one (as in Pesendorfer and Jofre-Bonet [9]). The justification for this comes from the fact the buyer cannot commit to exclude a player who did not participate in the first period mechanism from the second period one.[5] Therefore, the first period participation constraint can be written as[6]:

$$PC_o^1(I): \ \Pi^1_{i,I}(c_i, c_i) \geq \beta \int_C \Pi^2_{l,I,i}(c, c) f(c) dc, \ \forall \ c_i \in C, \ \forall \ i \in N$$

[5] If the seller could do it, he could extract all the expected seller's rent from the second period. He could ask for them as extra payments in the first period and threaten with the impossibility of participation in the second period for sellers who are not willing to comply.

[6] By subscript "o" we denote the investment observability setting.

We are now ready to state the optimization problem faced by the mechanism designer. Denote by $\mathcal{C}(\Gamma, I)$ the expected procurement cost when the buyer chooses a mechanism Γ and an investment level I. It corresponds to:

$$\mathcal{C} = \sum_{i=1}^{n} \int_C T_i^1(c) f(c) dc + \beta \left[\int_C T_{w,I}^2(c) \frac{\partial G}{\partial c}(c, I) dc + \sum_{j \neq w} \int_C T_{l,I,j}^2(c) f(c) dc \right] \tag{10}$$

Therefore, this buyer solves:

$$\mathcal{P}_o \begin{cases} \min_{\Gamma, I} \ \mathcal{C}(\Gamma, I) \\ s.t. \\ IC_o^1, \ IC_o^2 \\ PC_o^1(I), \ PC_o^2(I) \end{cases}$$

The following result shows that the cost minimizing mechanism under investment observability satisfies: (i) In the first period it assigns the contract to the lowest virtual cost seller, (ii) Second period rules give an advantage to the first period winner, but that advantage does not depend on the investment level chosen by the buyer.

Proposition 12 *Under full commitment and investment observability, the cost minimizing mechanism, Γ^*, does not depend on the investment level chosen by the buyer, and it is characterized by the probability functions*

$$q_i^{1*}(c_1, ..., c_n) = \begin{cases} 1 & \text{if } c_i + \frac{F(c_i)}{f(c_i)} < c_j + \frac{F(c_j)}{f(c_j)} \quad \forall j \neq i \\ 0 & \sim \end{cases} \tag{11}$$

$$q_w^{2*}(c_w, c_{-w}) = \begin{cases} 1 & \text{if } c_w < k(c_i) \quad \forall i \neq w \\ 0 & \sim \end{cases} \tag{12}$$

$$q_{l,i}^{2*}(c_i, c_{-i}) = \begin{cases} 1 & \text{if } k(c_i) = min\{c_w, k(c_j); \quad \forall j \neq w\} \\ 0 & \sim \end{cases} \tag{13}$$

and transfers

$$t_w^{2*}(c_1, ..., c_n) = \begin{cases} min\{k(c_i); i \neq w\} & \text{if } c_w < k(c_i) \quad \forall i \neq w \\ 0 & \sim \end{cases} \tag{14}$$

$$t_{l,i}^{2*}(c_i, c_{-i}) = \begin{cases} min\{c_w, k(c_j); j \neq w, i\} & \text{if } k(c_i) = min\{c_w, k(c_j); \forall j \neq w\} \\ 0 & \sim \end{cases} \tag{15}$$

$$t_i^{1*}(c_i, c_{-i}) = \begin{cases} min\{c_i; j \neq i\} - \beta[\Pi_{w,I}^2 - \Pi_{l,I,i}^2] & \text{if } c_i = min\{c_j, \forall j \neq i\} \\ 0 & \sim \end{cases} \tag{16}$$

with $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$, $\Pi_{w,I}^2 = \int_C \Pi_{w,I}(c,c) \frac{\partial G}{\partial c}(c, I) dc$ *and* $\Pi_{l,I,i}^2 = \int_C \Pi_l(c,c) f(c) dc$

Proof: See Appendix.

□

At $t = 1$ the optimal rules correspond to the ones derived by Myerson in [8], and are efficient because of the ex-ante symmetry of competitors and assumption 3. However, the first period winner obtains an *advantage gap* for the second procurement, that is, he can obtain the second contract even when some rivals have lower costs, so the cost minimizing rule in the second period sacrifices efficiency to reduce expected costs. This advantage gap decreases with the number of sellers: as the number of competitors increases, giving an advantage to the first period winner is more expensive, since it is more likely that one of the first period losers has the lowest cost. Nevertheless, this gap never disappears, showing that sequentiality introduces *memory* in the optimal contract, expressed in the aforementioned advantage gap.

As it can be seen from the characterization, the optimal mechanism in the second period is quite simple. It corresponds to a modified second price auction, where the offer of a first period loser of type c_i is treated as an offer $k(c_i)$. This is particularly advantageous to a first period winner, since it increases its probability of winning and also the transfers when it wins in the second period. But why is the buyer interested in this? The answer lies in the first period transfers: the buyer gets back the expected marginal benefit in the second period of winning in the first period.

Observation: Since these rules do not depend on the level of investment I, they are also feasible when this variable is not observable. In fact, the optimal second period mechanism does not rely on the *ex-ante* degree of complementarity (given by $G(c, 0)$) nor on the *ex-post* degree of complementarity (given by $G(c, I)$). The optimal way to inter-temporally distribute incentives across time, gives an advantage to the winner based on the loser's cost distribution. In order to increase the competition in the first period, rules for the second period are modified, generating a cost reduction in the first period. The tradeoff between this first period effect, and the inefficiencies introduced by an advantage in the second period give rise to the optimal advantage level.

To complete our characterization of the cost-minimizing mechanism, we now characterize the investment level chosen by the buyer, I^*.

Proposition 13 *Under full commitment of the buyer and investment observability, the cost minimizing investment level I^* is the solution to*

$$\min_{I \geq 0} \Psi(I) - \int_C [1 - F(k^{-1}(c))]^{n-1} G(c, I) dc \tag{17}$$

Proof: See Appendix.

□

In the previous expression we can see the the three effects of investment. The first term is the cost of investment (at $t = 2$ the buyer must ensure participation for the first period winner), the second one reflects the advantages of it: as investment goes up $G(c, I)$ goes up, since more weight is put in low-cost realizations of the first period winner. However, when compared to the efficiency case (equation 9), this last effect is stronger. Because now the first period winner is granted an advantage gap at the second procurement, it happens that $k^{-1}(c) < c$ which implies that $[1 - F(k^{-1}(c))]^{n-1} > [1 - F(c)]^{n-1}$. The benefits of investment for the buyer are now larger, since the first period winner is assigned the contract more often, due to the advantage gap. From now on we assume that the first order condition in (17) is satisfied.

5.2 Investment Non-Observability and Full Commitment

As we said before, in this case rules cannot be functions of the investment level chosen by the first period winner. He decides, knowing the second period rules, how much to invest. Therefore, the second period rules chosen by the buyer influence the investment decision, and this effect must be taken into account by the buyer. With all this in mind, we can write the buyer's problem as[7]:

$$\mathcal{P}_{no} \begin{cases} \min\limits_{\Gamma_{no}, I} \ \mathcal{C}(\Gamma_{no}, I) \\ s.t. \\ I \in arg \max\limits_{J \geq 0} \int\limits_{C} \Pi_w^2(c,c) \frac{\partial G}{\partial c}(c,J) dc \\ IC_{no}^1, \ IC_{no}^2 \\ PC_{no}^1(I), \ PC_{no}^2(I) \end{cases}$$

The first restriction must be added because the investment level is chosen by the first period winner. It specifies that the cost minimizing investment level that the buyer wishes to be implemented must be profit maximizing under mechanism Γ_{no} for the first period winner.

Denote by $(\Gamma_{no}^*, I_{no}^*)$ the solution to $\mathcal{P}_{no}$. It is clear that

$$\mathcal{C}(\Gamma^*, I^*) \leq \mathcal{C}(\Gamma_{no}^*, I_{no}^*)$$

due to the additional restrictions in $\mathcal{P}_{no}$. The next result establishes that the mechanism (Γ^*, I^*), optimal when investment is observable, is also feasible when investment is non-observable. This is so not only because Γ^* has second period rules that do not depend on investment, but also because I^*, under this rules, is profit maximizing for the first period winner. So, it satisfies IC_{no}.

Proposition 14 *Under full commitment of the buyer and investment non-observability, the solution to* $\mathcal{P}_{no}$ *is* $\Gamma_{no}^* = \Gamma^*$, $I_{no}^* = I^*$, *with* (Γ^*, I^*), *the solution with investment observability.*

Proof: See Appendix.

□

Therefore, as (Γ^*, I^*) is the solution in both settings, observable and non-observable investment, we call it the full commitment cost-minimizing solution. We can see that when the buyer set rules according to Γ^*, he provides the right incentives to induce the first period winner to invest I^*, the same level that the buyer would have chosen himself. This strengthens the result which establishes the possibility of efficiency regardless of investment observability, showing that this is also the case when the buyer's objective is cost minimization. Later on we prove that this is not the case for *any* mechanism, but a special case of these two. For other advantage levels in the second period mechanism, the buyer and the seller's desired levels of investment differ.

5.3 Over-Investment

Under full commitment, the buyer gives an advantage to the first period winner in the second period. Surprisingly, this induces an investment level that is above the efficient one, as shown in the next result.

[7] By subscript "no" we denote the investment non-observability environment. Recall that in this case the rules chosen by the buyer are not functions of investment.

Proposition 15 *Assume that I^e and I^* satisfy the first order conditions of problems (9) and (17), respectively. Thus, if the buyer is fully committed over-investment occurs, that is, $I^e < I^*$.*

Proof: See Appendix.

□

The intuition for this result depends on the degree of investment observability. If investment is non-observable, two effects are present. On the one hand the second period advantage gap gives the first period winner incentives to *relax*, and to invest less in a distributional upgrade, since he does not really need a low cost to win and investment is costly. On the other hand, investment generates lower expected costs, increasing his profit margin in case of winning. As the advantage gap increases, winning becomes more likely and therefore investment becomes more attractive. This second effect dominates the first.

To see this, we can rewrite the conditions that define I^e and I^* ((9) and (17)) as

$$\begin{aligned} I^e &\in arg\max_{I\geq 0}\int_C [1-F(c)]^{n-1}G(c,I)dc - \Psi(I) \\ I^* &\in arg\max_{I\geq 0}\int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc - \Psi(I) \end{aligned}$$

The first term in both equations corresponds to the fraction of the first period winner's expected payoff in the second period that depends on investment. In both cases, this term increases with I, because $G(c,\cdot)$ is increasing. But since $k^{-1}(c) < c$, this effect is bigger in the full-commitment cost-minimizing case. This shows how the incentives to relax are dominated by the incentives to invest and increase the profit margins. Moreover, this effect is quite strong. Consider two mechanisms $\tilde{\Gamma}$ and $\bar{\Gamma}$, not depending on investment, such that the corresponding second period expected winning probability functions for the first period winner satisfy $\tilde{Q}^2_w(c) \geq \bar{Q}^2_w(c)$, $\forall c \in C$, with strict inequality on a positive-measure subset of C. That is, $\tilde{\Gamma}$ gives more advantage to the first period winner at $t = 2$ than $\bar{\Gamma}$. Therefore,

$$\int_C \tilde{Q}^2_w(c)G(c,I)dc - \Psi(I) > \int_C \bar{Q}^2_w(c)G(c,I)dc - \Psi(I)$$

which allow us to conclude that $\tilde{I} > \bar{I}$, with $\tilde{I}, \bar{I}$ the corresponding induced investment levels (assuming that the first order conditions hold). This shows that the most inefficient mechanism, $\tilde{Q}^2_w(c) \equiv 1$, induces the largest amount of investment possible.

If investment is observable, and thus chosen by the buyer, the result is the same but the intuition is different. If the first period winner is going to have an advantage in the second period, and therefore he is likely to win anyway, it is good for the buyer to increase the likelihood of him having low costs. This effect makes investment more attractive than in the case of efficient second period rules (which give no advantage), and therefore over-investment occurs.

6. Revenue Maximization Under Non-Commitment

In this section we assume that the buyer cannot commit to the second period rules before the investment stage, and this is known by the sellers. This fact induces, through a second period mechanism that is disadvantageous to the first period winner, an investment level below the efficient one. Also, in this setting, the observability of investment will make a difference (unlike the commitment case).

To begin with, suppose that the first period winner invests an amount I before the second procurement. Now, the buyer has incentives to change rules based on the investment level, and considers the investment expenditures, $\Psi(I)$, as sunk costs.

We start with the case of observable investment.

6.1 Investment Observability and Non-Commitment

Because investment is observable, the buyer can make use of mechanisms of the form

$$\Gamma^2 = (\{t^2_{w,I}\}_{I\geq 0}, \{q^2_{w,I}\}_{I\geq 0}, \{t^2_{l,I}\}_{I\geq 0}, \{q^2_{l,I}\}_{I\geq 0})$$

It is worth to emphasize that because of the buyer's inability to commit to mechanisms, he cannot decide the investment level even though it is observable.

Since participation in the second period must give non-negative profits, it must be the case that $T^2_{w,I}(c) - cQ^2_{w,I}(c) \geq 0,\ \forall c \in C$. Also, because the second period expected utility for the first period winner is expressed by $\Pi^2_{w,I}(c,c) = T^2_{w,I}(c) - cQ^2_{w,I}(c) - \Psi(I)$, the participation constraint can be written as:

$$(PC^2_{nc}) \qquad \Pi^2_{w,I}(c,c) \geq -\Psi(I),\ \forall c \in C. \tag{18}$$

Finally, as in the previous sections, the second procurement expected cost corresponds to

$$\mathcal{C}^2(I) = \int_C T^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \sum_{i\neq w}\int_C T^2_{l,I,i}(c)f(c)dc$$

If the first period winner has already chosen an investment level I (observable), sequential rationality implies that the buyer solves, for the second period:

$$\widehat{\mathcal{P}}_o(I)\left\{\begin{array}{lc} & \min\limits_{\Gamma^2}\ \mathcal{C}^2(I) \\ \\ s.t & \Pi^2_{w,I}(c,c) \geq -\Psi(I),\ \forall c \in C \\ \\ & \Pi^2_{l,I,i}(c,c) \geq 0,\ \forall c \in C,\ \forall i \neq w,\ i \in N \\ \\ & IC^2_o \end{array}\right.$$

The following result shows that, given any investment level carried out by the first period winner, the buyer gives *disadvantage* to this agent at $t = 2$. Since the buyer cannot commit to contracts and the first period winner has improved his distribution, the buyer has no incentive to continue giving the mentioned *advantage gap* of the full commitment solution. As usual, informational asymmetries harm the competitor with the best distribution in one-shot auctions.

Lemma 16 *In the absence of buyer's commitment, the second period cost minimizing mechanism $\widehat{\Gamma}^2(I)$, when the investment observed is I, corresponds to*

$$\widehat{q}^2_{w,I}(c_w, c_{-w}) = \begin{cases} 1 & c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)} < \min\limits_{j\neq w}\left\{c_j + \frac{F(c_j)}{f(c_j)}\right\} \\ 0 & \sim \end{cases} \tag{19}$$

$$\widehat{q}^2_{l,I,i}(c_i, c_{-i}) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < \min\left\{c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)}, \min\limits_{j\neq i,w}\left\{c_j + \frac{F(c_j)}{f(c_j)}\right\}\right\} \\ 0 & \sim \end{cases} \tag{20}$$

Proof : See Appendix.

□

The second period mechanism gives a disadvantage to the first period winner, since $c + \frac{F(c)}{f(c)} < c + \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$, $\forall I \geq 0$ (Lemma 9, (ii)). Thus, it can occur that the first period winner loses even if he has the lowest cost. We can see how an opportunistic behavior appears, exactly as in a *hold-up* situation. The first period winner commits to a sunk cost investment to improve his cost distribution, allowing the seller to take advantage of this specific investment. The buyer does so by changing the rules against the first period winner, and therefore extracting more rent from him.

The next result characterizes the optimal first period mechanism under non-commitment. The assumption of sequential rationality directly implies that these rules must be optimal in a one-shot procurement auction setting. Therefore, the buyer should compare virtual costs in order to assign the first period project.

Corollary 17 *In the absence of buyer's commitment, the first period cost minimizing assignment rules correspond to*

$$q_i^{1*}(c_1, ..., c_n) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < c_j + \frac{F(c_j)}{f(c_j)} \quad \forall j \neq i \\ 0 & \sim \end{cases} \tag{21}$$

Proof : Direct.

□

We now turn to the investment level induced in this environment. A first period winner, anticipating the buyer's behavior in the second period (the selection of mechanisms of the form $\widehat{\Gamma}^2(I)$), determines how much to invest. This allows us to fully characterize the investment level:

Proposition 18 *With non-commitment and investment observability, the investment level chosen by the first period winner, $\widehat{I}$, solves*

$$\max_{I \geq 0} V(I) \equiv \int_C [1 - F(J^{-1}(J_I(c))]^{n-1} G(c, I) dc - \Psi(I) \tag{22}$$

with $J_I(c) = c + \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ and $J(c) = c + \frac{F(c)}{f(c)}$. Thus, the optimal mechanism for the buyer is $\widehat{\Gamma}^2(\widehat{I})$.

Proof : See Appendix.

□

We can see how investment affects the first period winner's utility $V(\cdot)$ through three channels. The first term in the integral, $[1 - F(J^{-1}(J_I(c))]^{n-1}$, decreases with I, since as investment increases, the second period mechanism gives a bigger disadvantage to the first period winner. However, the second term in the integral, $G(c, I)$ is increasing in I for each $c \in C$, and it reflects the increased probability of winning due to a better cost distribution. The last term, $-\Psi(I)$, also reflects a negative effect of investment, since it is costly.

If we compare condition (22) with condition (9), that defines the efficient level of investment, we observe that the positive effect of investment (given by an increase in $G(c, I)$) is weaker in the case of non-commitment, since [8]

$$[1 - F(J^{-1}(J_I(c))]^{n-1} < [1 - F(c)]^{n-1}$$

[8]. Using assumption 1, lemma 9 (ii), and assumption 3 we have that $J^{-1}(J_I(c)) > c$

Intuitively, in an ex-post efficient mechanism, the probability of a first period winner succeeding at $t = 2$ is the probability that all first period losers have higher costs ($[1 - F(c)]^{n-1}$). On the other hand, his probability of winning in a non-commitment environment is the probability that all losers have higher *virtual* costs than him ($[1-F(J^{-1}(J_I(c))]^{n-1}$). This last probability is lower since virtual costs are higher when the cost distribution is improved.

The fact that the absence of commitment reduces the advantage granted to the first period winner at the second procurement (when compared to the efficient mechanism) will induce a lower investment level chosen by this agent, as shown in the next result:

Proposition 19 *Assume that I^e, I^* and $\widehat{I}$ satisfy the first order conditions of problems, (9), (17)and (22), respectively. Then, under non-commitment of the buyer and investment observability, investment falls below efficiency, that is $\widehat{I} < I^e < I^*$.*

Proof : See Appendix.

□

Here, a hold-up intuition appears: anticipating that the rules will be biased against him in the second procurement mechanism, and knowing this effect will be more pronounced the more he invests, the seller invests less than the efficient level. In other words, he does not internalize all the benefits from investment, since part of it is appropriated by the seller as rent extraction.

6.2 Investment Non-Observability and Non-Commitment

We now consider the case when investment is not observable. If the buyer could monitor the investment level chosen by the first period winner, I, he would react optimally imposing $\widehat{\Gamma}^2(I)$, as in the previous subsection. Now, since investment cannot be observed, the buyer cannot choose a contingent plan with one mechanism for each level I selected by the first period winner.

We model this situation as a simultaneous-move game between the first period winner and the buyer where the former chooses an investment level and the latter a second period mechanism. In such a game, the buyer chooses just one mechanism, which is a best response to the investment level chosen by the first period winner. Symmetrically, the investment level is a best response to the mechanism chosen by the buyer. Then, we define the action space for the first period winner to be $A_w = [0, +\infty)$. For the buyer, the strategy space could be the set of incentive compatible mechanisms that satisfy participation constraints. However, by a rationalizability argument, we consider only the mechanisms that are a best response for some investment level of the first period winner, that is $A_b = \{\widehat{\Gamma}(I) | \ I \geq 0\}$ where $\widehat{\Gamma}(I)$ is the sequentially optimal mechanism given an investment level I defined in lemma 16. In this context we can define a pure strategy equilibrium:[9]

Definition 20 *A pure strategy equilibrium under non-commitment and investment non-observability is a tuple $(\Gamma, I) \in A_b \times A_w$ such that*

(i) $\Gamma = \widehat{\Gamma}(I)$

(ii) $I \in arg \max\limits_{K \geq 0} \int\limits_C \Pi^2_{w,\Gamma}(c,c) \frac{\partial G}{\partial c}(c,K) dc - \Psi(K)$

where $\Pi^2_{w,\Gamma}(c,c)$ is the second period expected utility of a first period winner with cost c that reveals truthfully under the mechanism Γ.

[9] Piccione and Tan [10] use a similar construction in a slightly different environment.

Condition (i) implies that the mechanism chosen by the buyer is a best response to the investment level chosen by the first period winner. Condition (ii), respectively, implies that given the mechanism Γ, the first period winner is choosing his investment level optimally. We now state a simple lemma that allows to proceed with the characterization of the investment level and the optimal mechanism.

Lemma 21 *A pure strategy Nash equilibrium is given by* $(\widehat{\widehat{I}}, \widehat{\Gamma}(\widehat{\widehat{I}}))$ *where*

$$\widehat{\widehat{I}} \in arg \max_{K \geq 0} U(K, \widehat{\widehat{I}}) \equiv \int_C [1 - F(J^{-1}(J_{\widehat{\widehat{I}}}(c)))]^{n-1} G(c, K) dc - \Psi(K) \tag{23}$$

Proof : See Appendix.

□

The function $U(K, I)$ corresponds to the second period expected utility for the first period winner if he invests an amount K and faces the mechanism $\widehat{\Gamma}^2(I)$. If we compare the function $U(\cdot, I)$ with the function $V(\cdot)$ defined in proposition 18, which characterizes investment for the case of observability, we can see that the negative effect of investment due to more disadvantageous mechanisms is fixed. The reason is that the buyer cannot observe the investment level chosen by the first period winner. However, the positive effect corresponding to an increased probability of winning and the negative effect corresponding to the cost of investment are still present. Since one of the negative effects of investment is fixed, the incentives to invest are now bigger than in the investment observability case.

The following proposition ensures the existence of an equilibrium, under the assumption that the marginal benefit due to a distributional improvement disappears when the investment level carried out is large enough. It also states that under non-commitment of the buyer, the possibility to monitor investment induces levels below the ones chosen when investment cannot be observed. This occurs because, when investment is observable, the buyer can react optimally with more disadvantageous mechanisms as investment increases.

Proposition 22 *Suppose that the following condition holds*[10]*:*

$$\lim_{I \to \infty} [1 - F(J^{-1}(J_I(c))]^{n-1} \frac{\partial G}{\partial I}(c, I) = 0 \tag{24}$$

Then, a unique equilibrium exists and corresponds to the tuple $(\widehat{\widehat{I}}, \widehat{\Gamma}(\widehat{\widehat{I}}))$ *where the investment level* $\widehat{\widehat{I}}$ *is given by condition (23).*

Also, $\widehat{I} \leq \widehat{\widehat{I}}$*, with strict inequality if* $\widehat{I}$ *satisfies the first order condition of problem (22).*

Proof : See Appendix.

□

We can also compare this investment level with the efficient one.

Corollary 23 *If* I^e *satisfies the first order condition of problem (9), then,*

$$\widehat{I} \leq \widehat{\widehat{I}} < I^e$$

Proof : See Appendix.

[10] The family of distributional upgrades introduced in example 1 satisfies this condition.

□

The intuition of the result is the following: disadvantageous mechanisms disincentive cost-reduction investment. This is so because the marginal benefit of investment (which comes from the expected cost reduction in the second period) decreases when the second period rules become less attractive (since they give a disadvantage to the first period winner). This occurs in both the case of investment observability and non-observability. However, this effect is less marked when the buyer cannot observe the investment level and must choose a mechanism "guessing" the first period winner's decision. If he can observe the investment level, he can react optimally and choose even more disadvantageous mechanisms in the second period. This fact is anticipated by the first period winner, which results in lower investment levels.

To conclude, note that the assumption of sequential rationality implies that the optimal first period rules in this environment correspond to the ones defined in Corollary 17. As before, the first period transfers are set-up such that participation is ensured for all sellers at both competitions. As a consequence, the cost minimizing mechanisms under non-commitment (for observable and non-observable investment) are feasible for a fully committed buyer. Therefore, the expected cost in an environment without commitment is necessarily higher than in an environment with commitment

Corollary 24 *The expected cost of both procurements is lower under full commitment than under non-commitment of the buyer (whether investment is observable or not).*

Proof : See Appendix.

□

7. Discussion

7.1 Investment Observability "Irrelevancy"

The fact that investment observability cannot improve the buyer's ability to reduce expected costs (in the case of full commitment) is interesting. This strengthens the result (also found in Piccione and Tan [10]), that this is also the case when the buyer's objective is ex-post efficiency. A natural question that arises is whether this is true for any mechanism used by the buyer. The answer is no, as shown in the next result. When the buyer sets, for instance, a mechanism that gives less advantage to the first period winner than the one in the cost-minimizing mechanism, the investment level chosen by the buyer (in the observable case) is higher than the one chosen by the first period winner (in the non-observable case). The intuition is as follows: with a smaller second period advantage, second period rules are closer to a sequentially optimal mechanism. Therefore the buyer captures more of the surplus generated by a cost distribution improvement[11] and, consequently has a bigger marginal benefit from investment. It is not surprising, then, that if he chooses investment, he will select a level above the one a first period winner would choose.

Proposition 25 *Consider $n = 2$ and assume that the buyer wants to implement the following incentive compatible mechanism:*

$$\tilde{q}^2_{w,I}(c_w, c_l) = \begin{cases} 1 & c_w < g(c_l) \\ 0 & \sim \end{cases} \tag{25}$$

[11] Which is also bigger, since such a mechanism is also closer to an efficient one.

with $g(\cdot)$ an increasing function that satisfies $g(\underline{c}) = \underline{c}$ and $g(c) \leq k_2(c) = c + 2\frac{F(c)}{f(c)}$, $\forall c \in C$, with strict inequality on a subset of C with non-zero measure. Denote by $\tilde{I}_b$ and $\tilde{I}_w$ the investment levels chosen by the buyer and first period winner respectively when facing this mechanism. Then, $\tilde{I}_b > \tilde{I}_w$

Proof: See Appendix.

□

7.2 Number of Competitors

Let $n \in I\!N$ and $I^*(n)$ the full commitment investment level when there are n competitors and $C^*(n)$ the corresponding expected cost of both procurement auctions. The following proposition shows two results. First, as the number of competitors increases, the investment level decreases to 0 as n goes to infinity. Second, the expected cost of both procurement auctions tend to $2\underline{c}$ as n goes to infinity.

Proposition 26 *Suppose that $I^*(n)$ is the full commitment investment level when there are n competitors. Then, $I^*(n) \searrow 0$ as $n \to \infty$. As a consequence, $I^e(n), \widehat{I}(n)$ and $\widehat{\widehat{I}}(n)$ go to zero as $n \to \infty$. Finally, if we denote by $C^*(n)$ the expected cost of both procurement auctions under full commitment, we have that $C^*(n) \to 2\underline{c}$ as $n \to \infty$.*

Proof: See Appendix.

□

The intuition behind the first result is the following: as the number of competitors grows, giving an advantage in the second period mechanism becomes more costly. This is so because there is a higher probability that the lowest cost supplier will not win the procurement, and this increases the expected cost of an advantage. If investment is observable (and therefore chosen by the buyer), this smaller advantage reduces the incentives to induce cost reduction, because it is less likely that the first period winner will also win the second procurement. If investment is non-observable (and therefore chosen by the first period winner), then a reduced advantage in the second period decreases his incentives to invest. This effect is compounded with the fact that any advantage level becomes less significant as the number players grows. Even if the first period winner has an advantage, the probability that another seller draws a significantly lower cost and still beats him becomes more important when the number of competitors grows. Such and advantage exists even when n goes to infinity, (see section 5), but then it is just nominal. As the number of competitors goes to infinity, the probability that the first period winner is assigned the second contract despite a higher cost converges to 0.

As we can see, when the intensity of competition increases, the role of mechanisms as tools to induce investment and intertemporally redistribute incentives disappears. The only significant role played by the cost minimizing mechanism is to assign efficiently in each period.

The second result is direct from the fact that, as the number of sellers increases, it is more likely that some of the competitors reports a cost close to $\underline{c}$ in each procurement auction. Moreover, the probability that the lowest cost supplier is not assigned the project goes to 0, and therefore competition drives the cost of each project to $\underline{c}$.

8. Investment by the Loser

We now extend the model to the case where the loser can also invest, with a potentially different technology, to improve its cost distribution. In particular, we assume that the if the loser invests I, at a cost $\Psi_l(\cdot)$ ($\Psi_w(\cdot)$ for the winner and both functions increasing and convex) its cost distribution for the second period will be $F(\cdot, I)$, with $F(\cdot, 0) = F(\cdot)$. The buyer may now want to induce (or mandate, if investment is observable) investment by the loser, since it increases competitiveness in the second period. However, this comes at a cost, since winning in the first period becomes less attractive. In what follows, we characterize the optimal mechanism with an without observability, and compare the investment levels to the ones obtained when only the winner can invest. Technically, the analysis is more intricate, since the optimal second period mechanism depends on the *loser*'s distribution, which is now endogenous. To keep analytical tractability we restrict attention to the case of two players.

8.1 Investment Observability

As before, this corresponds to the case where the buyer can actually choose the investment levels, and a second period mechanism which is optimal given these levels. The next lemma provides a characterization of the buyer's problem.

Lemma 27 *Assume that investment is observable. The problem of the buyer corresponds to choose investment levels according to:*

$$\max_{I^w, I^l \geq 0} \int_C F(J_{I^l}^{-1}(c), I^l)dc + \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc - \Psi_w(I^w) - \Psi_l(I^l) \tag{26}$$

and a second period mechanism characterized by

$$q_w^{2*}(c_w, c_l) = \begin{cases} 1 & c_w < J_{I^l}(c_l) \\ 0 & \sim \end{cases} \tag{27}$$

with $J_{I^l}(c) = c + 2\frac{F(c,I^l)}{\frac{\partial F}{\partial c}(c,I^l)}$, $c \in C$. *Moreover,* I^{w*} *and* I^{l*}, *are characterized by:*

$$\begin{aligned} I^{w*} &\in arg \max_{I^w \geq 0} \int_C [1 - F(J_{I^{l*}}^{-1}(c), I^{l*})]G(c, I^w)dc - \Psi_w(I^w) & (28) \\ I^{l*} &\in arg \max_{I^l \geq 0} \int_C [1 - G(c, I^{w*}))]F(J_{I^l}^{-1}(c), I^l)dc - \Psi_l(I^l) & (29) \end{aligned}$$

Proof: See Appendix.

□

Note that I has two effects on the function $F(J_I^{-1}(c), I)$, which is the probability of the first period loser winning in the second period. On the one hand, I improves the loser's distribution, and therefore its winning probability ($\frac{\partial F}{\partial I}(c, I) \geq 0$). On the other, I induces a mechanism which is more disadvantageous for the loser ($J_I(c)$ is increasing in I). The composite effect is critical for the comparison of investment levels, and we concentrate in two polar cases:

(A1) For every $c \in C$, $I \mapsto F(J_I^{-1}(c), I)$ is decreasing.

(A2) For every $c \in C$, $I \mapsto F(J_I^{-1}(c), I)$ is increasing.[12]

[12]It is not difficult to see that, at $c = \underline{c}$ we get $\frac{\partial}{\partial I}(F(J_I^{-1}(c), I))\Big|_{c=\underline{c}} = -\frac{\partial F}{\partial c}(J_I^{-1}(\underline{c}), I)\frac{\partial}{\partial I}(J_I^{-1}(\underline{c})) \geq 0$, where the last inequality comes from $J_I^{-1}(\underline{c}) = \underline{c}$ and $\frac{\partial}{\partial I}(J_I^{-1}(c)) \leq 0$. Nevertheless, both (A1) and (A2) are possible, and they represent two polar cases of the the tradeoff that a loser faces when investment is observable: improving her distribution implies more disadvantageous rules.

From the optimality conditions for I^{w*} and I^{l*} in lemma 27, one could interpret the buyer's problem as an (artificial) investment game between the sellers, and define its associated best response functions, $BR^o_w(\cdot)$ and $BR^o_l(\cdot)$, for the first period winner and loser. They satisfy:

$$BR^o_w(I^l) = arg \max_{I^w \geq 0} \int_C [1 - F(J^{-1}_{I^l}(c), I^l)]G(c, I^w)dc - \Psi_w(I^w) \tag{30}$$

$$BR^o_l(I^w) = arg \max_{I^l \geq 0} \int_C [1 - G(c, I^w))]F(J^{-1}_{I^l}(c), I^l)dc - \Psi_l(I^l) \tag{31}$$

Note that if *(A1)* is satisfied, $BR^o_w(\cdot)$ is increasing and $BR^o_l(\cdot) \equiv 0$. On the other hand, if *(A2)* satisfied, both best response functions are decreasing. We obtain the following result:

Proposition 28 **(i)** *Assume that (A1) holds. Then, $I^{l*} = 0$ and $I^{w*} = I^*$, where I^* is the same level found in section 5, and corresponds to the solution of*

$$\max_{I^w \geq 0} \int_C [1 - F(J^{-1}_0(c), 0)]G(c, I^w)dc - \Psi_w(I^w)$$

(ii) *Assume that (A2) holds. Then, if*

$$\frac{\partial}{\partial I^l} \left(\int_C [1 - G(c, BR^o_w(0))]F(J^{-1}_{I^l}(c), I^l)dc - \Psi_l(I^l) \right) \Bigg|_{I^l=0} > 0 \tag{32}$$

$$\frac{\partial}{\partial I^w} \left(\int_C [1 - F(J^{-1}_{BR^o_l(0)}(c), BR^o_l(0))]G(c, I^w)dc - \Psi_w(I^w) \right) \Bigg|_{I^w=0} > 0 \tag{33}$$

$I^{w*}, I^{l*} > 0$ and they are characterized as in lemma 27. Moreover, $I^{w*} < I^*$.

Proof: See Appendix.

□

Throughout this section we will distinguish both cases *(A1)* and *(A2)* in order to establish rankings among mechanisms chosen by the buyer and the investment levels carried out by the sellers across different environments. Whenever we refer to case *(A2)* we will implicitly assume that the buyer's problem has an interior solution (i.e, $I^w, I^l > 0$).

8.2 Investment Non-Observability

When the investment decisions are not observable for the buyer, the problem becomes much more complex. The buyer must choose a mechanism among the class of incentive compatible mechanism such that participation is ensured for any cost realization of the sellers at both stages, which we denote by $\mathcal{B}$. By choosing a mechanism, the buyer is able to influence investment levels chosen by the sellers. Denoting by $\Pi^2_k(\Gamma, I^w, I^l)$ the second-period expected utility for agent $k \in \{w, l\}$ when the buyer sets a mechanism Γ

and the investment level profile carried out is (I^w, I^l), the buyer's problem can be written as

$$\begin{array}{lll} \min\limits_{\Gamma, I^w, I^l} & & \mathcal{C}(\Gamma, I^w, I^l) \\ s.t. & I^w & \in arg\max\limits_{\tilde{I}^w \geq 0} \Pi_w^2(\Gamma, \tilde{I}^w, I^l) \\ & I^l & \in arg\max\limits_{\tilde{I}^l \geq 0} \Pi_w^2(\Gamma, I^w, \tilde{I}^l) \\ & \Gamma & \in \mathcal{B} \end{array}$$

which we denote by $\mathcal{P}^{no}(\mathcal{B})$. Note that a mechanism $\Gamma \in \mathcal{B}$ induces investment levels $(I^w(\Gamma), I^l(\Gamma))$ as the equilibrium of a second-stage investment game played by the first period winner and loser, so the buyer's problem $\mathcal{P}^{no}(\mathcal{B})$ reduces to $\min\limits_{\Gamma \in \mathcal{B}} \quad \mathcal{C}(\Gamma, I^w(\Gamma), I^l(\Gamma))$. This formulation is intractable due to the size $\mathcal{B}$ coupled with the intrinsic nonlinearity of the investment levels induced. To make the problem tractable, we restrict the buyer's feasible set to the class of optimal mechanisms in the investment-observability case, that is $\mathcal{B}^* \equiv \{\Gamma^*(I^l) \mid I^l \geq 0\}$ and study the solution to $\mathcal{P}^{no}(\mathcal{B}^*)$. The next lemma characterizes the solution to this problem:

Lemma 29 *Define the functions $I \mapsto I^{w,no}(I)$ and $I \mapsto I^{l,no}(I)$ as*

$$I^{w,no}(I) \quad \in \quad arg\max_{I^w \geq 0} \int\limits_C [1 - F(J_I^{-1}(c), I^{l,no}(I))] G(c, I^w) dc - \Psi_w(I^w) \tag{34}$$

$$I^{l,no}(I) \quad \in \quad arg\max_{I^l \geq 0} \int\limits_{\underline{c}}^{J_I^{-1}(\bar{c})} [1 - G(J_I(c), I^{w,no}(I))] F(c, I^l) dc - \Psi_l(I^l) \tag{35}$$

The tuple $(I^{no}, I^{l,no}, I^{w,no})$ is a solution to $\mathcal{P}^{no}(\mathcal{B}^*)$ if and only if

(i) *I^{no} solves*

$$\max_{I \geq 0} \int\limits_C F(c, I^{l,no}(I)) dc + \int\limits_C [1 - F(J_I^{-1}(c), I^{l,no}(I))] G(c, I^{w,no}(I)) dc - \Psi_w(I^{w,no}(I)) - \Psi_l(I^{l,no}(I)) \tag{36}$$

(ii) *$I^{l,no} \equiv I^{l,no}(I^{no})$ and $I^{w,no} \equiv I^{w,no}(I^{no})$*

Proof: Obvious.

□

Condition (i) establishes that the buyer, anticipating the equilibrium investment levels of the second stage game, chooses I^{no} (and consequently $\Gamma^*(I^{no})$) by minimizing the expected cost of procurement. Condition (ii) establishes that in the ensuing investment game, sellers play Nash-equilibrium strategies. From now on, we assume that the solutions to both the buyer's and the sellers' problems are characterized by the first order conditions.

Fixing a mechanism $\Gamma^*(I) \in \mathcal{B}^*$, we define $BR_w^{no}(I^l|I)$ and $BR_l^{no}(I^w|I)$ as the best response function of the first period winner and loser, in the second stage game played by the sellers. They satisfy:

$$BR_w^{no}(I^l|I) \in arg\max_{I^w \geq 0} \int\limits_C [1 - F(J_I^{-1}(c), I^l)] G(c, I^w) dc - \Psi_w(I^w)$$

and

$$BR_l^{no}(I^w|I) \in arg\max_{I^l \geq 0} \int\limits_{\underline{c}}^{J_I^{-1}(\bar{c})} [1 - G(J_I(c), I^w)]F(c, I^l)dc - \Psi_l(I^l)$$

We obtain the following result:

Lemma 30 **(i)** *For any fixed mechanism $\Gamma^*(I^b)$, $BR_w^{no}(\cdot|I^b)$ and $BR_l^{no}(\cdot|I^b)$ are decreasing functions.*

(ii) *$I^{w,no}(I^b)$ is strictly increasing in I^b, and $I^{l,no}(I^b)$ is strictly decreasing in I^b.*

Proof: See Appendix.

□

Statement (i) comes from the fact that an agent's expected probability of winning decreases when his rival invests more in cost reduction and becomes more competitive. This reduces the investment incentives, leading to lower investment. The intuition for (ii) is that, as there is more advantage in the second period (a larger I^b in $\Gamma^*(I^b)$), the investment incentives for the first period winner increase, but decrease for the first period loser.

The next result states that, when investment is not observable, the buyer treats the loser as the worst possible competitor, that is, she chooses a mechanism indexed by a zero investment level.

Lemma 31 *Suppose both $I^{w,no}(\cdot)$ and $I^{l,no}(\cdot)$ are differentiable. Then, $I^{no} = 0$.*

Proof: See Appendix.

□

This result shows the robustness of the mechanism found in the full-commitment environment when the loser is not allowed to invest in cost reduction. If investment by the loser is possible and the buyer is restricted to choose mechanisms in $\mathcal{B}^*$, the optimal mechanism when the loser cannot invest is still optimal. In some sense, if the buyer is mistaken about the loser's possibility of investment, he incurs in no losses due to suboptimal mechanisms. This is not obvious since there is a trade-off in the incentives set to the competitors: by imposing a mechanism indexed by a higher investment level, on the one hand, the winner would invest more (more advantage is granted) which results in lower expected cost and, on the other, the loser's investment incentives are reduced. This result says that the latter effect always dominates the former.

With this result, when case *(A1)* holds, we are able to rank the investment levels obtained in both environments of investment observability and non-observability:

Proposition 32 *Suppose that (A1) holds, that is, for every $c \in C$, $I \mapsto F(J_I^{-1}(c), I)$ is decreasing. Then*

$$0 = I^{l*} \leq I^{l,no} \tag{37}$$

$$I^{w,no} \leq I^{w*} \tag{38}$$

where the last inequality is strict if and only if $I^{l,no} > 0$. As a consequence, $I^{w,no} \leq I^*$, therefore, the loser's possibility to invest reduces the investment carried out by the first period winner.

Proof: See Appendix.

□

When investment is not observable, the negative effect on the first period loser due to disadvantageous mechanisms (the $J_0^{-1}(\cdot)$ term) does not depend on the loser's investment. This explains the first inequality. The last one is simply the fact that, as long as the loser invests more, the expected probability of winning decreases (recall that in the investment game stage the mechanism is fixed), reducing the investment incentives for the first period winner.

We restrict from now on to the case *(A2)*, that is, for every $c \in C$, $I \mapsto F(J_I^{-1}(c), I)$ is increasing. In order to rank the investment levels induces in this setting we assume the following condition:

Assumption 33 (B) *For all* $c \in C$, $\frac{\partial}{\partial I}(F(J_I^{-1}(c), I)) < \frac{\partial F}{\partial I}(J_0^{-1}(c), I)\frac{d}{dc}(J_0^{-1}(c))$

This condition states that the loser's marginal benefit from investment (in distributional terms) decreases with investment observability. If investment is observable, investment improves the distribution but it also has a negative effect due to disadvantageous rules to the loser ($J_I^{-1}(c)$ decreases with I). This negative effect is absent when the mechanism is fixed with the implicit advantage $J_0(c)$. However, the expression $F(J_0^{-1}(c), I)$ has to be corrected by the term $\frac{d}{dc}(J_0^{-1}(c))$ reflecting the fact that for high cost draws, the loser is not assigned the project (the relevant costs for the loser lie in the interval $[\underline{c}, J_0^{-1}(\bar{c})]$).

Proposition 34 *Suppose that* (A2) *and* (B) *holds. Then, we obtain the same raking among investment levels:*

$$I^{l*} < I^{l,no} \tag{39}$$
$$I^{w,no} < I^{w*} \tag{40}$$

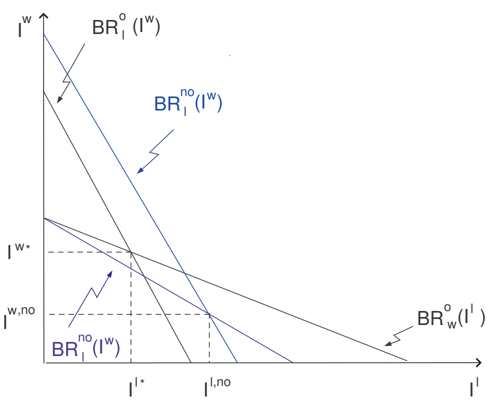
If (B) *does not holds, it is not clear what is the relation among the above variables.*

Proof: See Appendix.

□

Assumption (B) implies that the investment incentives for the loser are higher when investment is not observable, explaining the first inequality, while the last one can be deduced using the same reasoning argued in the previous proposition. The following figure illustrates the result.

Figure 1:



9. Conclusions

In this paper we characterized the cost minimizing mechanism for a buyer that must procure two projects and faces the same group of competitors over time with the winner of the first period being able to invest in cost reduction for the second competition. We considered the cases of full-commitment and non-commitment to second period rules, and observable and non-observable investment carried out by the first period winner. Moreover, we characterized the investment level induced in each of these environments.

We first defined, as a benchmark, an ex-post efficient mechanism, which always assigns the projects to the lowest-cost seller, and induces an investment level such that the social cost of investment is equal to the social gain of it.

We proved that, if the buyer can commit to second period mechanisms, he will choose to give an advantage to the first period winner in order to increase the intensity of competition in the first period. This advantage, which is ex-post inefficient, decreases with the number of competitors but never disappears. This is so because, for small advantages, the gains from the intertemporal distribution of incentives are always bigger than the losses due to ex-post inefficiency. This pressure to increase competition in the first period and its consequence, the advantage in the second period for first period winner, induces investment levels above the socially efficient one. We also showed that in the case of full-commitment, the observability of investment is irrelevant and the optimal mechanism remains the same.

In the case of a buyer who lacks commitment, he chooses second period mechanisms that are disadvantageous to the first period winner. This, in turn, induces investment levels below efficiency. Here, investment observability makes a difference, since the buyer chooses more disadvantageous rules as for bigger investment levels. If he can observe investment, and react optimally, this effect is stronger and anticipated by the first period winner, therefore inducing even lower investment levels than in the non-observable case, where he must anticipate the choice of the first period winner.

We extended the model to allow for simultaneous investment and analyze the implications of investment observability in a full commitment environment. In a two-competitors setting, we found that when investment is observable, if cost reduction harms the first period loser, the buyer chooses the same mechanism and investment levels as if the first period loser were not able to invest. To the contrary, if investment is profitable for the loser, the buyer imposes positive investment for the loser, chooses second period rules according to this level of investment and the degree of cost reduction carried out by the first period winner is lower than the one induced when the loser is not allowed to invest. Finally, when investment is not observable we showed that the mechanism chosen by the buyer is robust to the loser's investment possibilities, and the level of cost reduction chosen by the first period loser (winner) is higher (lower) than when investment cannot be monitored.

In this paper mechanisms help the buyer to reduce costs through three channels. They allow rent extraction in a single period, they allow the provider to intertemporally distribute incentives (through advantages) and they give incentives to investment. Without commitment, since only a *disadvantage* is credible in the second period, the second channel disappears and, moreover, investment incentives are small. As a result of this, commitment helps the buyer to reduce costs, since it gives him more tools for rent extraction and, by allowing him to induce higher levels of investment, it reduces the total expected cost of the project.

The present work gives interesting normative implications for an agency that must procure projects repeatedly over time. It stresses the value of commitment and the role of second period advantages as incentives for cost-reduction investment. The extension to a general multi-period model is not direct, however. In such a model, there is not only one "winner", since many sellers could have won a previous procurement auction. A mechanism should specify how a relative advantage is calculated, and how previously won auctions influence this advantage. Moreover, it is critical to decide who can invest at each point in time. We leave this as a direction for future work in the topic.

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10. Appendix: Proofs

Example 6: Assume that $F(\cdot)$ is a concave twice differentiable distribution. Define $G(c,0) = F(c)^{\eta}$, $0 < \eta < 1$ and $G(c,I) = 1-(1-G(c,0))^{\gamma I+1}$, $\gamma > 0$. It is straightforward that the first inequality in assumption 1 is checked when $I' = 0$. Also, it is clear that $\frac{\partial G}{\partial c}(c,I) = (\gamma I+1)(1-G(c,0))^{\gamma I}\frac{\partial G}{\partial c}(c,0)$. Now, setting $0 \leq I' < I$ and $c' < c$, simple algebra shows that the second inequality in assumption 1 is equivalent to $(1-G(c,0))^{\gamma(I-I')} < (1-G(c',0))^{\gamma(I-I')}$ which is obviously true since $c' < c$.

Assumption 2 holds since $\frac{\partial^2 G}{\partial I^2}(c,I) = -\gamma^2(1-G(c,0))^{\gamma I+1}log^2(1-G(c,0)) < 0$.

Assumption 4 holds because $\left|\frac{\partial G}{\partial I}(c,I)\right| = \gamma(1-G(c,0))^{\gamma I+1}|log(1-G(c,0))| < \gamma(1-G(c,0))|log(1-G(c,0))| \in L^1(I\!R)$.

It remains to show that $c \mapsto \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ is increasing in c, which is equivalent to $\left(\frac{\partial G}{\partial c}(c,I)\right)^2 - G(c,I)\frac{\partial^2 G}{\partial c^2}(c,I) > 0$. So, it suffices to show that $\frac{\partial^2 G}{\partial c^2}(c,I) < 0$. Since,

$$\frac{\partial^2 G}{\partial c^2}(c,I) = -(\gamma I+1)\gamma I(1-G(c,0))^{\gamma I-1}\left(\frac{\partial G}{\partial c}(c,0)\right)^2 + (\gamma I+1)(1-G(c,0))^{\gamma I}\frac{\partial^2 G}{\partial c^2}(c,0)$$

$\eta < 1$, $F'' \leq 0$ and $\frac{\partial^2 G}{\partial c^2}(c,0) = \eta(\eta-1)F^{\eta-2}(c)(F'(c))^2 + \eta F'(c)^{\eta-1}F''(c) < 0$, the result follows.

□

Proof Lemma 10: We define $V_i^k(c_i) = \max_{c_i'} \Pi_i^k(c_i, c_i')$. Since $V_i^k(c_i)$ is a maximum of linear functions, it follows that it is convex, and therefore differentiable a.e. It is easy to see that the following are equivalent: (a) The mechanism is incentive compatible (b) $V_i^k(c_i) = \Pi_i^k(c_i, c_i)$ for all i and $k = 1, 2$, (c) $-Q_i^k(c_i) \in \partial V_i^k(c_i)$ for all i and $k = 1, 2$.

We now prove the two implications required. For the sufficiency, notice that $-Q_i^k(c_i)$ is non-decreasing since it is a selection from the subdifferential of a convex function (see [11]). Since $-Q_i^k(c_i) \in \partial V_i^k(c_i)$ and $V_i^k(c_i)$ is convex we have that $V_i^k(c_i) = V_i^k(\overline{c}) + \int_{c_i}^{\overline{c}} Q_i^k(s)ds$. For the necessity, notice that $V_i^k(c_i) = \int_{c_i}^{c_i'} Q_i^k(s)ds \leq V_i^k(c_i') + Q_i^k(c_i')(c_i' - c_i)$. Therefore $Q_i^k(c_i) \in \partial V_i^k(c_i)$ and the mechanism is incentive compatible.

□

Proof Proposition 11: First note that the efficient mechanism plus the ex-ante complementarity assumption ensure the participation for all players at both procurement auctions. Participation at $t = 2$ is induced by scaling-up transfers as usual. For the first competition, assume the investment level chosen by the buyer is $I \geq 0$. Thus, the following must hold: $\forall i \in N$, $\forall c \in C$

$$T_{i,I}^{1,e}(c) - cQ_{i,I}^{1,e}(c) + \beta Q_{i,I}^{1,e}(c)\int_C \Pi_{w,I}^{2,e}(c,c)\frac{\partial G}{\partial c}(c,I)dc + \beta[1-Q_{i,I}^{1,e}(c)]\int_C \Pi_{l,I,i}^{2,e}(c,c)f(c)dc$$

$$\geq \beta\int_C \Pi_{l,I,i}^{2,e}(c,c)f(c)dc$$

However, under a distributional improvement, this mechanism increases the second period expected utility for a first period winner. Therefore, for every $I \geq 0$ we have $\int_C \Pi_{w,I}^{2,e}(c,c)\frac{\partial G}{\partial c}(c,I)dc \geq \int_C \Pi_{l,I,i}^{2,e}(c,c)f(c)dc$, which allows to impose (in some cases sub-optimally) that $T_{i,I}^{1,e}(c) - cQ_{i,I}^{1,e}(c) \geq 0$, $\forall c \in C$, as in a static procurement auction.

Now, recall that I^e solves $\min_{I\geq 0} \mathcal{C}(\Gamma^e, I)$ with

$$\begin{aligned}\mathcal{C}(\Gamma^e, I) &= n\int_C c[1-F(c)]^{n-1}f(c)dc + \beta\int_C c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc \\ &\quad +\beta(n-1)\int_C c[1-F(c)]^{n-2}[1-G(c,I)]f(c)dc + \beta\Psi(I) \qquad (41)\end{aligned}$$

Then I^e is the solution to

$$\min_{I\geq 0}\int_C c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc + (n-1)\int_C c[1-F(c)]^{n-2}[1-G(c,I)]f(c)dc + \Psi(I) \qquad (42)$$

Integrating by parts,

$$\int\limits_C c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc \;=\; c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)\Big|_{\underline{c}}^{\bar{c}} + (n-1)\int\limits_C c[1-F(c)]^{n-2}f(c)G(c,I)dc - \int\limits_C [1-F(c)]^{n-1}G(c,I)dc \tag{43}$$

The first term vanishes. Replacing this expression in the problem just mentioned we obtain

$$\min_{I\geq 0}\int\limits_C c[1-F(c)]^{n-2}f(c)dc - \int\limits_C [1-F(c)]^{n-1}G(c,I)dc + \Psi(I)$$

and because the first term does not depends on I we conclude that I^e is the solution to $\min\limits_{I\geq 0}\Psi(I) - \int\limits_C [1-F(c)]^{n-1}G(c,I)dc$ concluding the first part. For the last one, recall that under a second price sealed bid auction there are no incentive compatibility problems since truth-telling is a dominant strategy. In the first period, transfers are scaled-up by a constant such that participation in both competitions is ensured for all players (note that incentive compatibility holds). Since each project is assigned to the least-cost competitor, in the second period the expected probability function for the first period winner is $Q_w^{2,e}(c) = [1-F(c)]^{n-1}$. Therefore, if investment is not observable, the first period winner will solve

$$\max_{I\geq 0}\int\limits_C \Pi_w^{2,e}(c)G(c,I)dc - \Psi(I) = \int\limits_C Q_w^{2,e}(c)G(c,I)dc - \Psi(I) = \int\limits_C [1-F(c)]^{n-1}G(c,I)dc - \Psi(I)$$

and, as a consequence, I^e will be selected as well. Therefore, I^e is achieved in a context of full commitment regardless of investment observability.

□

Proof Theorem 12: Assume that the buyer wants to induce an investment level I and that $\Gamma(I)$ is incentive compatible. Rearranging terms in (5), (6), (7) and using Lemma 10 we obtain:

$$T_{w,I}^2(c) = \Pi_{w,I}^2(\bar{c},\bar{c}) + \int_c^{\bar{c}} Q_{w,I}^2(s)ds + cQ_{w,I}^2(c) + \Psi(I) \tag{44}$$

$$T_{l,I,i}^2(c) = \Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_c^{\bar{c}} Q_{l,I,i}^2(s)ds + cQ_{l,I,i}^2(c),\ i\neq w \tag{45}$$

$$T_i^1(c) = \Pi_{i,I}^1(\bar{c},\bar{c}) + \int_c^{\bar{c}} Q_i^1(s)ds + cQ_i^1(c) - \beta Q_i^1(c)\int\limits_C \Pi_{w,I}^2(s,s)\frac{\partial G}{\partial s}(s,I)ds - \beta[1-Q_i^1(c)]\int\limits_C \Pi_{l,I,i}^2(s,s)f(s)dc. \tag{46}$$

Then, integrating by parts

$$\int\limits_C T^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc = \Pi^2_{w,I}(\bar{c},\bar{c}) + \int\limits_C Q^2_{w,I}(c)G(c,I)dc + \int\limits_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) \quad (47)$$

$$\int\limits_C T^2_{l,I,i}(c)f(c)dc = \Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc + \int\limits_C cQ^2_{l,I,i}(c)f(c)dc, \ i \neq w \quad (48)$$

$$\begin{aligned}\int\limits_C T^1_i(c)f(c)dc &= \Pi^1_{i,I}(\bar{c},\bar{c}) + \int\limits_C Q^1_i(c)F(c)dc + \int\limits_C cQ^1_i(c)f(c)dc - \beta Q^1_i \int\limits_C \Pi^2_{w,I}(c,c)\frac{\partial G}{\partial c}(c,I)dc \\ &\quad -\beta[1-Q^1_i]\int\limits_C \Pi^2_{l,I,i}(c,c)f(c)dc. \end{aligned} \quad (49)$$

with $Q^1_i = \int\limits_C Q^1_i(c)f(c)dc$ (observe that $\sum\limits_{i=1}^n Q^1_i = 1$). Replacing these last expressions in (10) yields a procurement cost of

$$\begin{aligned}\mathcal{C} &= \sum_{i=1}^n \left[\Pi^1_{i,I}(\bar{c},\bar{c}) + \int\limits_C Q^1_i(c)F(c)dc + \int\limits_C cQ^1_i(c)f(c)dc\right] - \beta\left[\Pi^2_{w,I}(\bar{c},\bar{c}) + \int\limits_C Q^2_{w,I}(c,c)G(c,I)dc\right] \\ &\quad -\beta\sum_{i=1}^n\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] + \beta\sum_{i=1}^n Q^1_i\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] \\ &\quad +\beta\left[\Pi^2_{w,I}(\bar{c},\bar{c}) + \int\limits_C Q^2_{w,I}(c)G(c,I)dc\right] + \beta\int\limits_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \beta\Psi(I) \\ &\quad +\beta\sum_{i\neq w}\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] + \beta\sum_{i\neq w}\int\limits_C cQ^2_{l,I,i}(c)f(c)dc \end{aligned} \quad (50)$$

Since the second and fifth term cancel each other we get

$$\begin{aligned}\mathcal{C} &= \sum_{i=1}^n \left[\Pi^1_{i,I}(\bar{c},\bar{c}) + \int\limits_C Q^1_i(c)F(c)dc + \int\limits_C cQ^1_i(c)f(c)dc\right] \\ &\quad +\beta\left[\int\limits_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) + \sum_{i\neq w}\int\limits_C cQ^2_{l,I,i}(c)f(c)dc\right] \\ &\quad -\beta\sum_{i=1}^n\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] + \beta\sum_{i\neq w}\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] \\ &\quad +\beta\sum_{i=1}^n Q^1_i\left[\Pi^2_{l,I,i}(\bar{c},\bar{c}) + \int\limits_C Q^2_{l,I,i}(c)F(c)dc\right] \end{aligned} \quad (51)$$

Without loss of generality, because of the ex-ante symmetry across players, the buyer pay attention only to mechanisms that satisfy $Q^2_{l,I,i}(\cdot) \equiv Q^2_{l,I}(\cdot)$ and $T^2_{l,I,i}(\cdot) \equiv T^2_{l,I}(\cdot)$. Therefore, the total expected cost reduces to

$$\mathcal{C} = \sum_{i=1}^{n}\left[\Pi_{i,I}^{1}(\bar{c},\bar{c}) + \int_{C} Q_i^1(c)F(c)dc + \int_{C} cQ_i^1(c)f(c)dc\right]$$
$$+\beta\left[\int_{C} cQ_{w,I}^2(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) + (n-1)\int_{C} cQ_{l,I}^2(c)f(c)dc\right] \quad (52)$$

Note that at the optimum $\beta\sum_{i=1}^{n}\int_{C}\Pi_{l,I}^{2}(c,c)f(c)dc = \beta\sum_{i\neq w}\left(1+\frac{1}{n-1}\right)\left[\Pi_{l,I}^{2}(\bar{c},\bar{c}) + \int_{C} Q_{l,I}^{2}(c)F(c)dc\right]$ Now, by adding and subtracting the last and first term respectively we obtain

$$\mathcal{C} = \sum_{i=1}^{n}\left[\int_{C} Q_i^1(c)F(c)dc + \int_{C} cQ_i^1(c)f(c)dc\right] + \sum_{i=1}^{n}\left[\Pi_{i,I}^{1}(\bar{c},\bar{c}) - \beta\int_{C}\Pi_{l,I}^{2}(c,c)f(c)dc\right]$$
$$+\beta\int_{C} cQ_{w,I}^2(c)\frac{\partial G}{\partial c}(c,I)dc + \beta\Psi(I) + \beta\sum_{i=1}^{n}\Pi_{l,I}^{2}(\bar{c},\bar{c})$$
$$+\beta\sum_{i\neq w}\left[\int_{C} cQ_{l,I}^2(c)f(c)dc + \left(1+\frac{1}{n-1}\right)\int_{C} Q_{l,I}^2(c)F(c)dc\right] \quad (53)$$

Since in an optimal mechanism, $T_{l,I}^{2}(\bar{c})$ will be set up such that $\Pi_{l,I}^{2}(\bar{c},\bar{c}) = 0$ and $T_{i,I}^{1}(\bar{c})$ such that $\Pi_{i,I}^{1}(\bar{c},\bar{c}) = \beta\int_{C}\Pi_{l,I}^{2}(c,c)f(c)dc$ we have

$$\mathcal{C} = \sum_{i=1}^{n}\left[\int_{C} Q_i^1(c)F(c)dc + \int_{C} cQ_i^1(c)f(c)dc\right] + \beta\Psi(I)$$
$$+\beta\left[\int_{C} cQ_{w,I}^2(c)\frac{\partial G}{\partial c}(c,I)dc + (n-1)\int_{C} cQ_{l,I}^2(c)f(c)dc + n\int_{C} Q_{l,I}^2(c)F(c)dc\right] \quad (54)$$
$$= \int_{C^n}\sum_{i=1}^{n}\left[c_i + \frac{F(c_i)}{f(c_i)}\right]q_i^1(c)f^n(c)dc + \beta\Psi(I)$$
$$+\beta\int_{C^n}\left[c_w q_{w,I}^2(c) + \sum_{i\neq w}\left[c_i + \left(1+\frac{1}{n-1}\right)\frac{F(c_i)}{f(c_i)}\right]q_{l,I,i}^2(c)\right]f^{n-1}(c_{-w})\frac{\partial G}{\partial c_w}(c_w,I)dc \quad (55)$$

Pointwise maximization yields the following rules

$$q_i^{1*}(c_1,...,c_n) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < c_j + \frac{F(c_j)}{f(c_j)} \quad \forall j \neq i \\ 0 & \sim \end{cases}$$
$$q_w^{2*}(c_w,c_{-w}) = \begin{cases} 1 & c_w < k(c_i) \quad \forall i \neq w \\ 0 & \sim \end{cases}$$
$$q_{l,i}^{2*}(c_i,c_{-i}) = \begin{cases} 1 & k(c_i) = min\{c_w, k(c_j); \quad \forall j \neq w\} \\ 0 & \sim \end{cases}$$

with $k(c) = c+\left(1+\frac{1}{n-1}\right)\frac{F(c)}{f(c)}$. Finally, because this last function and $c+\frac{F(c)}{f(c)}$ are increasing (assumption 3), the expected probability functions in each period are non increasing. Finally, if transfers are computed according the following rules (which imply Lemma 10)

$$\begin{aligned} t_w^{2*} &= q_w^{2*}(c_w, c_{-w})c_w + \int\limits_{c_w}^{\bar{c}} q_w^{2*}(s, c_{-w})ds \\ t_{l,i}^{2*} &= q_{l,i}^{2*}(c_i, c_{-i})c_i + \int\limits_{c_i}^{\bar{c}} q_{l,i}^{2*}(s, c_{-i})ds \\ t_i^{1*} &= q_i^{1*}(c_i, c_{-i})c_i + \int\limits_{c_i}^{\bar{c}} q_i^{1*}(s, c_{-i})ds - \beta[\Pi_{w,I}^2 - \Pi_{l,I,i}^2] \end{aligned}$$

and use the probability functions derived above, we obtain the expressions stated in the proposition. This concludes the proof.

□

Proof Proposition 13: Recall that under full commitment and investment observability the rules defined by Γ^* minimize the expected cost of both projects for any level of investment $I \geq 0$ chosen by the first period winner. Thus, in this case, to obtain the optimal level of investment for the buyer, I^*, we replace the mentioned mechanism in the procurement cost expression and minimize with respect to I. Remember that, at the optimum of this problem, the expected procurement cost can be written as (see expression (54) in the proof of theorem 12)

$$\begin{aligned} \mathcal{C} &= \sum_{i=1}^{n} \left[\int\limits_{C} cQ_i^{1*}(c)f(c)dc + \int\limits_{C} Q_i^{1*}(c)F(c)dc \right] + \beta\Psi(I) \\ &\quad + \beta \left[\int\limits_{C} cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c, I)dc + n\int\limits_{C} Q_l^{2*}(c)F(c)dc + (n-1)\int\limits_{C} cQ_l^{2*}(c)f(c)dc \right] \\ &= \mathcal{K}(Q^{1*}) + \beta\mathcal{T}(Q^{2*}, I) + \beta\Psi(I) \end{aligned} \tag{56}$$

with $\mathcal{K}(Q^{1*}) = \int\limits_{C} cQ_i^{1*}(c)f(c)dc + \int\limits_{C} Q_i^{1*}(c)F(c)dc$ and $\mathcal{T}(Q^{2*}, I) = \int\limits_{C} cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c, I)dc + n\int\limits_{C} Q_l^{2*}(c)F(c)dc + (n-1)\int\limits_{C} cQ_l^{2*}(c)f(c)dc$. Then, in order to obtain the optimal investment level, the buyer solves $\min\limits_{I \geq 0} \mathcal{T}(Q^{2*}, I) + \Psi(I)$. It can be easily checked that $Q_w^{2*}(c) = [1-F(k^{-1}(c))]^{n-1}$ for $c \in C$ and $Q_l^{2*}(c) = (1-G(k(c), I))(1-F(c))^{n-2}$ for $c \in [\underline{c}, k^{-1}(\bar{c}]$ (and otherwise zero), are the expected probabilities of winning the second project for a winner and any loser respectively. Integrating by parts,

$$\begin{aligned} \int\limits_{C} cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c, I)dc &= \bar{c}[1 - F(k^{-1}(\bar{c}))]^{n-1} - \int\limits_{C} [1 - F(k^{-1}(c))]^{n-1} G(c, I)dc \\ &\quad + (n-1)\int\limits_{C} c[1 - F(k^{-1}(c))]^{n-2} f(k^{-1}(c))\frac{G(c, I)}{k'(k^{-1}(c))}dc \end{aligned}$$

Using $t = k^{-1}(c)$ in the last integral and replacing $k(c) = c + \left(1 + \frac{1}{n-1}\right)\frac{F(c)}{f(c)}$ in the same term we get

$$\int_C cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = \bar{c}[1-F(k^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc$$
$$+(n-1)\int_{\underline{c}}^{l^{-1}(\bar{c})} c[1-F(c)]^{n-2}f(c)G(k(c),I)dc + n\int_{\underline{c}}^{k^{-1}(\bar{c})} [1-F(c)]^{n-2}G(k(c),I)F(c)dc$$

Also, we have that

$$\begin{aligned} n\int_C Q_l^{2*}(c)F(c)dc &= n\int_{\underline{c}}^{k^{-1}(\bar{c})} [1-F(c)]^{n-2}[1-G(k(c),I)]F(c)dc \\ (n-1)\int_C cQ_l^{2*}(c)f(c)dc &= (n-1)\int_{\underline{c}}^{k^{-1}(\bar{c})} c[1-F(c)]^{n-2}[1-G(k(c),I)]f(c)dc \end{aligned}$$

Therefore

$$\begin{aligned} \mathcal{T}(Q^{2*},I) &= \bar{c}[1-F(k^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc \\ &\quad + n\int_{\underline{c}}^{k^{-1}(\bar{c})} [1-F(c)]^{n-2}F(c)dc + (n-1)\int_{\underline{c}}^{k^{-1}(\bar{c})} c[1-F(c)]^{n-2}f(c)dc \end{aligned}$$

concluding that the level of investment that minimizes the expected cost under full observability and commitment is the solution to $\min_{I\geq 0} \Psi(I) - \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc$.

□

Proof Proposition 14: Since (Γ^*, I^*), defined by expressions (11), (27) and (13), does not depend on the level of investment I, the rules in Γ^* are feasible when the investment level is not observable. When the winner of the first procurement faces the rules in Γ^*, he decides his level of investment by solving $\max_{I\geq 0} \int_C \Pi_w^2(c,c)\frac{\partial G}{\partial c}(c,I)dc - \Psi(I)$. Since $\int_C \Pi_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = T_w^{2*}(\bar{c}) - \bar{c}Q_w^{2*}(\bar{c}) + \int_C Q_w^{2*}(c)G(c,I)dc - \Psi(I)$, this is equivalent to solve $\max_{I\geq 0} \int_C Q_w^{2*}(c)G(c,I)dc - \Psi(I) = \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc - \Psi(I)$ using the definition of $Q_w^{2*}(\cdot)$. Since I^* solves the same optimization problem, we conclude that $(\Gamma_{no}^*, I_{no}^*)$ is incentive compatible, and therefore feasible for the non-observable case. Finally, it is obvious that $\mathcal{C}(\Gamma^*, I^*) \leq \mathcal{C}(\Gamma_{no}^*, I_{no}^*)$, so the feasibility of (Γ^*, I^*) for the non-observable case implies that $\mathcal{C}(\Gamma_{no}^*, I_{no}^*) = \mathcal{C}(\Gamma^*, I^*)$.

□

Proof Proposition 15: We assume that I^e and I^* satisfy their corresponding first order conditions. Since $l(\cdot)$ is increasing and $l(c) > c$, we obtain $[1-F(k^{-1}(c))]^{n-1} > [1-F(c)]^{n-1}$. Thus, $\int_C [1-F(k^{-1}(c))]^{n-1}\frac{\partial G}{\partial I}(c,I)dc - \Psi'(I) > \int_C [1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc - \Psi'(I)$, which yields $I^e < I^*$ using $\frac{\partial^2 G}{\partial I^2}(c,I) < 0$ and Ψ's convexity.

□

Proof Proposition 16: Suppose the first period winner had invested an amount I. It is easy to see that the second procurement expected cost corresponds to

$$\mathcal{C}^2(I) = \int\limits_{C^n} \left[\left(c_w + \frac{G(c_w, I)}{\frac{\partial G}{\partial c}(c_w, I)} \right) q^2_{w,I}(c^n) + \sum_{j \neq w} \left(c_j + \frac{F(c_j)}{f(c_j)} \right) q^2_j(c^n) \right] f^{n-1}(c_{-w}) \frac{\partial G}{\partial c}(c_w, I) dc^n \qquad (57)$$

Therefore, the second period cost-minimizing mechanism, call it $\widehat{\Gamma}^2(I)$, is defined by

$$\widehat{q}^2_{w,I}(c_w, c_{-w}) = \begin{cases} 1 & c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)} < \min\limits_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \right\} \\ 0 & \sim \end{cases}$$

$$\widehat{q}^2_{l,I,i}(c_i, c_{-i}) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < \min \left\{ c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)}, c_j + \frac{F(c_j)}{f(c_j)} \,\middle|\, j \neq i, w \right\} \\ 0 & \sim \end{cases}$$

This mechanism is incentive compatible due to the regularity conditions imposed in assumption 3.

□

Proof Proposition 18: Anticipating the selection of $\widehat{\Gamma}^2(I)$, a first period winner will solve $\max\limits_{I \geq 0} \int\limits_C \widehat{Q}^2_{w,I}(c) G(c, I) dc - \Psi(I)$ with $\widehat{Q}^2_{w,I}(c) = \int\limits_{C^{n-1}} \widehat{q}^2_{w,I}(c_w, c_{-w}) f^{n-1}(c_{-w}) dc_{-w}$. If we define $J_I(c) = c + \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ and $J(c) = c + \frac{F(c)}{f(c)}$ we have that $\widehat{q}^2_{w,I}(c_w, c_{-w}) = 1 \Leftrightarrow J^{-1}(J_I(c_w)) < c_i, \forall i \neq w$. Thanks to assumption 3, $J^{-1}(\cdot)$ exists, and because of Lemma 9, $J_I(\cdot) > J(\cdot)$, thus $J^{-1}(J_I(c)) > \underline{c}$ for all $I > 0$. Hence, we obtain $\widehat{Q}^2_{w,I}(c) = [1 - F(J^{-1}(J_I(c)))]^{n-1}$ Therefore, in absence of investment observability and buyer's commitment, the level of investment carried out by the first period winner, $\widehat{I}$, is the solution to $\max\limits_{I \geq 0} \int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1} G(c, I) dc - \Psi(I)$ and the optimal mechanism chosen by the buyer corresponds to $\widehat{\Gamma}^2(\widehat{I})$.

□

Proof Proposition 19: Remember the function $V(I) = \int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1} G(c, I) dc - \Psi(I)$. As before, assume that $\widehat{I}$ satisfies the corresponding first order condition, that is $V'(\widehat{I}) = 0$. Since for every fixed $c \in C$ the function $I \mapsto J_I(c)$ is strictly increasing (Lemma 9, (ii)) we have that $V'(I) < \int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$. Moreover, because $J_I(c) > J(c)$ for all $c \in C$ and $I \geq 0$ (Lemma 9, (ii)), the regularity assumption implies $[1 - F(J^{-1}(J_I(c)))]^{n-1} < [1 - F(c)]^{n-1}$ which leads to $V'(I) < \int\limits_C [1 - F(c)]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$. Recalling that I^e is such that $\int\limits_C [1 - F(c)]^{n-1} \frac{\partial G}{\partial I}(c, I^e) dc - \Psi'(I^e) = 0$, we conclude that $\widehat{I} < I^e$ using that $\frac{\partial^2 G}{\partial I^2}(c, I) < 0$ and Ψ's convexity.

□

Proof Lemma 21: In this environment, the definition of an equilibrium implies that the mechanism chosen by the buyer must be of the form $\widehat{\Gamma}(I)$, which was introduced in Lemma 16. As before, given this mechanism, $\widehat{Q}^2_{w,I}(c) = [1 - F(J^{-1}(J_I(c)))]^{n-1}$ Therefore, using the definition of an equilibrium, we conclude that the tuple $(\widehat{\widehat{I}}, \widehat{\Gamma}(\widehat{\widehat{I}}))$ is an equilibrium if it satisfies

$$\widehat{\widehat{I}} \in arg \max_{K \geq 0} U(K, \widehat{\widehat{I}}) \equiv \int_C [1 - F(J^{-1}(J_{\widehat{\widehat{I}}}(c)))]^{n-1} G(c, K)dc - \Psi(K)$$

□

Proof Proposition 22: Define $L(I) \equiv \frac{\partial U_w}{\partial K}(K, I)\Big|_{K=I} = \int_C [1 - F(J^{-1}(J_I(c)))]^{n-1} \frac{\partial G}{\partial I}(c, I)dc - \Psi'(I)$. Note that it is continuous because $[1 - F(J^{-1}(J_I(c)))]^{n-1} \frac{\partial G}{\partial I}(c, I)$ is continuous in I for each c and, $\left|[1 - F(J^{-1}(J_I(c)))]^{n-1} \frac{\partial G}{\partial I}(c, I)\right| < \frac{\partial G}{\partial I}(c, I) < h(c) \in L^1(I\!R)$. Moreover, $L(\cdot)$ is strictly decreasing because of assumptions 2, 5 and since $[1 - F(J^{-1}(J_I(c)))]^{n-1}$ strictly decreases with I. Finally, condition (24), implies that

$$\lim_{I \to \infty} L(I) = \lim_{I \to \infty} \int_C [1 - F(J^{-1}(J_I(c))]^{n-1} \frac{\partial G}{\partial I}(c, I)dc - \Psi'(I) = \lim_{I \to \infty} -\Psi'(I) < 0$$

which ensures the existence of $\tilde{I}$ such that $\forall I > \tilde{I}$, $L(I) < 0$.

In order to prove the proposition, we distinguish two cases regarding whether (a) $\widehat{I}$ (the investment level induced under non-commitment and investment observability) satisfies the first order condition of problem (22) or b) not.

As we have discussed, if the investment level chosen by the first period winner was I, the buyer has as a dominant strategy to impose $\widehat{\Gamma}(I)$, with $\widehat{Q}^2_{w,I}(c) = [1 - F(J^{-1}(J_I(c))]^{n-1}$.

Points like $(I, \widehat{\Gamma}(I))$ with $0 \leq I \leq \widehat{I}$ cannot be equilibria: since $V'(I) \geq 0$ in this zone (with equality only if $I = \widehat{I}$), for all $0 \leq \bar{I} \leq \widehat{I}$, $L(I) = \frac{\partial U}{\partial K}(K, I)\Big|_{K=I} > 0$ thanks to the absence in U of the negative effect due to $[1 - F(J^{-1}(J_I(c)))]^{n-1}$, which is present in V. As a consequence, the first period has a profitable deviation by investing more. Since $L(\widehat{I}) = \frac{\partial U}{\partial K}(K, \widehat{I})\Big|_{K=\widehat{I}} > 0$ and $L(I) < 0$ if $I > \tilde{I}$, because of $L(\cdot)$'s continuity and monotonicity, there exists a unique $\widehat{\widehat{I}} \in (\widehat{I}, \tilde{I})$ such that $L(\widehat{\widehat{I}}) = \frac{\partial U}{\partial I}(I, \widehat{\widehat{I}})\Big|_{I=\widehat{\widehat{I}}} = 0$. This is a maximum point since the function $K \mapsto U(\widehat{\widehat{I}}, K)$ is strictly concave in I (assumption 2), hence, we have proved that $(\widehat{\widehat{I}}, \widehat{\Gamma}(\widehat{\widehat{I}})$ is the unique pure strategy equilibrium in an interior solution of (22).

On the other hand, assume that (b) holds, that is, no investment level satisfies the first order condition of (22). The the previous reasonings and assumptions allow us to conclude that $V(\cdot)$ always decreases and, as a direct consequence, the investment level induced under *non-commitment* and investment observability is zero ($\widehat{I} = 0$). Thus, we can no longer ensure the existence of a point $\widehat{I}$ such that $L(\widehat{I}) > 0$, which was the main argument to show the existence of $\widehat{\widehat{I}}$. Therefore, we have two cases:

(i) Exists $\widehat{I} \geq 0$ such that $L(\widehat{I}) > 0$. In this case the existence and uniqueness of $\widehat{\widehat{I}}$ is ensured using the same arguments as before. Also we have $\widehat{I} < \widehat{\widehat{I}}$ because $L(\cdot)$ is strictly decreasing.

(ii) There is no $I > 0$ such that $L(I) = 0$. Because of (24), continuity and monotonicity of $L(\cdot)$ (decreasing), it must be that $L(I) < 0$ if $I > 0$, hence, $L(0) \leq 0$. In this case $(0, \widehat{\Gamma}(0))$ is an equilibrium: when the winner makes no investment, the buyer has no incentive to deviate from $\widehat{\Gamma}(0)$. On the other side, when facing this last mechanism, the first period winner has no incentive to invest because his marginal utility under this mechanism, $L(0)$, is always negative. It is unique because L's monotonicity.

Note that in all the previous cases, $\widehat{I} \leq \widehat{\widehat{I}}$, and strict inequality can be ensured if $\widehat{I}$ satisfies the first order condition of (22). This concludes the proof.

$\square$

Proof Corollary 23: We only need to show that $\widehat{\widehat{I}} < I^e$. Assume that $\widehat{\widehat{I}}$ satisfies the first order condition of (23). It has been argued that $\forall I \geq 0$, $[1 - F(J^{-1}(J_I(c)))]^{n-1} < [1 - F(c)]^{n-1}$, therefore, $L(I) = \int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1} \frac{\partial G}{\partial I}(c, I)dc - \Psi'(I) < \int\limits_C [1 - F(c)]^{n-1} \frac{\partial G}{\partial I}(c, I)dc - \Psi'(I)$. Since $\widehat{\widehat{I}}$ is such that $L(\widehat{\widehat{I}}) = 0$, applying the same arguments used in the previous results we can conclude that $\widehat{\widehat{I}} < I^e$. Finally, if $\widehat{\widehat{I}}$ does not satisfy the first order condition of (23), then, $\widehat{\widehat{I}} = 0$, and the result remains unchanged.

$\square$

Proof Corollary 24: It is direct that the mechanisms under non-commitment (observable and non-observable investment) are incentive compatible and ensure participation at $t = 2$ for all sellers. Also, the first period transfers are set-up such that participation at both competitions are ensured for all players, that is, under these mechanisms, $\Pi^1_{i,I}(c_i, c_i) \geq \beta \int\limits_C \Pi^2_{l,I,i}(c, c) \frac{\partial G}{\partial c}(c, I)dc$, $\forall c_i \in C$ is satisfied for all $i \in N$. Therefore, the optimal mechanisms under non-commitment (observable and non-observable investment) are feasible mechanisms under full commitment and investment observability. Thus, by optimality, the cost of both procurements in the case of full commitment is lower than the case of non-commitment (whether investment is observable or not).

$\square$

Proof Proposition 25: For any investment level I, total expected costs corresponds to

$$\begin{aligned} \mathcal{C}(I) &= \int\limits_C \int\limits_C \left[\sum_{i=1,2} (c_i + \frac{F(c_i)}{f(c_i)}) q^1_i(c_1, c_2) \right] f(c_1) f(c_2) dc_1 dc_2 \\ &+ \beta \int\limits_C \int\limits_C \left[c_w q^2_{w,I}(c_w, c_l) + \left(c_l + 2\frac{F(c_l)}{f(c_l)} \right) q^2_l(c_w, c_l) \right] \frac{\partial G}{\partial c_w}(c_w, I) f(c_l) dc_w dc_l + \beta\Psi(I) \end{aligned}$$

and the optimal second period rule is given by

$$q^2_{w,I}(c_w, c_l) = \begin{cases} 1 & c_w < c_l + 2\frac{F(c_l)}{f(c_l)} \\ 0 & \sim \end{cases}$$

Define $k_2(c) = c + 2\frac{F(c)}{f(c)}$ and consider the following incentive compatible mechanism:

$$\tilde{q}^2_{w,I}(c_w, c_l) = \begin{cases} 1 & c_w < g(c_l) \\ 0 & \sim \end{cases} \tag{58}$$

with $g(\cdot)$ an increasing function such that $g(\underline{c}) = \underline{c}$. Given an amount of investment carried out I, the second period expected cost can be written as $\mathcal{C}^2(I) = \int\limits_C f(c_l) \left(\int\limits_C h(c_l, c_w) \frac{\partial G}{\partial c_w}(c_w, I) dc_w \right) dc_l$, with

$$h(c_l, c_w) = \begin{cases} k_2(c_l) & g(c_l) < c_w \\ c_w & \sim \end{cases}$$

Therefore, under this mechanism, the buyer chooses the investment level by solving $\min_{I\geq 0}\ \mathcal{C}^2(I)+\Psi(I)$.

Now, given $c_l \in C$, $\int_C h(c_l,c_w)\frac{\partial G}{\partial c_w}(c_w,I)dc_w = \int_{g(c_l)}^{\bar{c}} k_2(c_l)\frac{\partial G}{\partial c_w}(c_w,I)dc_w + \int_{\underline{c}}^{g(c_l)} c_w\frac{\partial G}{\partial c_w}(c_w,I)dc_w$, but,

$$\begin{aligned}
\int_{g(c_l)}^{\bar{c}} k_2(c_l)\frac{\partial G}{\partial c}(c_w,I)dc_w &= \begin{cases} k_2(c_l)(1-G(g(c_l),I)) & g(c_l)<\bar{c} \\ 0 & \sim \end{cases} \\
\int_{\underline{c}}^{g(c_l)} c_w\frac{\partial G}{\partial c_w}(c_w,I)dc_w &= \begin{cases} g(c_l)G(g(c_l),I) - \int_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w & g(c_l)<\bar{c} \\ \bar{c} - \int_C G(c_w,I)dc_w & \sim \end{cases}
\end{aligned}$$

and, as a consequence,

$$\int_C h(c_l,c_w)\frac{\partial G}{\partial c_w}(c_w,I)dc_w = \begin{cases} k_2(c_l) + [g(c_l)-k_2(c_l)]G(g(c_l),I) - \int_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w & g(c_l)<\bar{c} \\ \bar{c} - \int_C G(c_w,I)dc_w & \sim \end{cases}$$

Replacing this last expression into the buyer's problem and considering only the terms that depend on I, we observe that the buyer minimizes

$$\int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l)-k_2(c_l))G(g(c_l),I)f(c_l)dc_l - \int_{\underline{c}}^{g^{-1}(\bar{c})}\int_{\underline{c}}^{g(c_l)} G(c_w,I)f(c_l)dc_w dc_l$$

$$-\int_{g^{-1}(\bar{c})}^{\bar{c}}\int_C G(c_w,I)f(c_l)dc_w dc_l + \Psi(I)$$

This last expression can be written as

$$\int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l)-k_2(c_l))G(g(c_l),I)f(c_l)dc_l - \int_C\left(\int_{\underline{c}}^{min\{g(c_l),\bar{c}\}} G(c_w,I)dc_w\right)f(c_l)dc_l + \Psi(I) \tag{59}$$

Defining $\mathcal{H}(I) = \int_C\left(\int_{\underline{c}}^{min\{g(c_l),\bar{c}\}} G(c_w,I)dc_w\right)f(c_l)dc_l$, the buyer solves

$$\max_{I\geq 0}\ \mathcal{H}(I) - \Psi(I) - \int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l)-k_2(c_l))G(g(c_l),I)f(c_l)dc_l \tag{60}$$

We denote the solution to this problem $\tilde{I}_b$. Also note that

$$\int_{\underline{c}}^{min\{\bar{c},g(c_l)\}} G(c_w,I)dc_w = \begin{cases} \int_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w & c_l < g^{-1}(\bar{c}) \\ \int_C G(c_w,I)dc_w & \sim \end{cases}$$

and therefore

$$\begin{aligned}\mathcal{H}(I) &= \int\limits_C \left(\int\limits_{\underline{c}}^{min\{\bar{c},g(c_l)\}} G(c_w,I)dc_w \right) f(c_l)dc_l \\ &= \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} \left(\int\limits_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w \right) f(c_l)dc_l + [1-F(g^{-1}(\bar{c}))] \int\limits_C G(c_w,I)dc_w \end{aligned} \tag{61}$$

Integrating by parts, we get

$$\int\limits_{\underline{c}}^{g^{-1}(\bar{c})} \left(\int\limits_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w \right) f(c_l)dc_l = F(g^{-1}(\bar{c})) \int\limits_C G(c_w,I)dc_w - \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} F(c_l)G(g(c_l),I)g'(c_l)dc_l$$

This leads to $\mathcal{H}(I) = \int\limits_C G(c,I)dc - \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} F(c)G(g(c),I)g'(c)dc = \int\limits_C [1-F(g^{-1}(t))]G(t,I)dt$ where we used $t = g(c)$ in the last equality. On the other hand, when facing the mechanism defined by (58) the first period winner solves $\max\limits_{I\geq 0} \int\limits_C \tilde{Q}^2_w(c)G(c,I)dc - \Psi(I)$. Now, since $g(\underline{c}) = \underline{c}$, we have that, $\tilde{Q}^2_w(c) = \int\limits_C \tilde{q}^2_w(c,c_l)f(c_l)dc_l = \int\limits_{g^{-1}(c)}^{\bar{c}} f(c_l)dc_l = 1-F(g^{-1}(c))$. So, it is straightforward that $\int\limits_C \tilde{Q}^2_w(c)G(c,I)dc = \int\limits_C [1-F(g^{-1}(c))]G(c,I)dc = \mathcal{H}(I)$, which implies that the first period winner solves $\max\limits_{I\geq 0} \mathcal{H}(I) - \Psi(I)$. Call the solution to this problem $\tilde{I}_w$. Finally, assume that the first order conditions for both problems are satisfied. As a consequence, it must be that $\mathcal{H}'(\tilde{I}_b) - \Psi'(\tilde{I}_b) + \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} (k_2(c_l)-g(c_l))\frac{\partial G}{\partial I}(g(c_l),\tilde{I}_b)f(c_l)dc_l = 0$ and $\mathcal{H}'(\tilde{I}_w) - \Psi'(\tilde{I}_w) = 0$. Assumptions 2 and 5 ensure that $\mathcal{H}'(\cdot)$ and $-\Psi'(\cdot)$ are decreasing. Since $k_2 \geq g$ with strict inequality on a set of positive measure, we conclude that $\tilde{I}_w < \tilde{I}_b$ using the fact that $\frac{\partial^2 G}{\partial I^2} < 0$.

□

Proof Corollary 24: Let $n < m$ and $I^*(n)$ and $I^*(m)$ the investment levels induced under full commitment when there are n and m competitors respectively. Consider the family of functions $k_n(c) = c + \left(1 + \frac{1}{n-1}\right)\frac{F(c)}{f(c)}$ with $n \in \mathbb{N}$, and recall that $k_n(\cdot)$ is exactly the function $k(\cdot)$ used in the full commitment section for the case of n competitors. Because these functions are increasing in c and $k_n(\cdot) > l_m(\cdot)$, it is easy to see that $Q^{2*}_{w,n}(c) = \left[1-F(k_n^{-1}(c))\right]^{n-1} > \left[1-F(k_m^{-1}(c))\right]^{m-1} = Q^{2*}_{w,m}(c)$. Thus, for any fixed investment level I we have $\int\limits_C Q^{2*}_{w,n}(c)\frac{\partial G}{\partial I}(c,I)dc - \Psi'(I) \geq \int\limits_C Q^{2*}_{w,m}(c)\frac{\partial G}{\partial I}(c,I)dc - \Psi'(I)$, with $Q^{2*}_{w,n}(c)$ the expected probability of winning at $t=2$ for the first period winner under the cost minimizing mechanism when there are n competitors. Using Ψ's convexity and the fact that the distributional marginal benefit of investment decreases with I (assumption 2), we obtain that that $I^*(n) > I^*(m)$. Since $\lim\limits_{n\to\infty} Q^{2*}_{w,n}(c) = 0$, $\forall c \in C$, we conclude that $I^*(n) \searrow 0$ as $n \to \infty$.

Now, denote $\widehat{I}(n)$, $\widehat{\widehat{I}}(n)$, $I^e(n)$ the investment levels induced under non-commitment and investment observability, non-commitment and investment non-observability and efficiency, when there are n competitors, respectively. The inequality $0 \leq \widehat{I}(n) \leq \widehat{\widehat{I}}(n) < I^e(n) < I^*(n)$, shows that they all collapse to zero when $n \to \infty$.

Finally, notice also that $C^*(n)$ converges to $2\underline{c}$, since the optimal mechanism induces costs below a double second-price procurement auction, whose cost monotonically converges to $2\underline{c}$ as $n \to \infty$.

$\square$

Proof of lemma 27: Define $J_I'(c) \equiv \frac{\partial J_I(c)}{\partial c}$. Recall that

$$\begin{aligned}\mathcal{T}^2(I^w, I^l) &= \bar{c}[1 - F(J_{I^l}^{-1}(\bar{c}), I^l)] - \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc + 2\int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} F(c, I^l)dc \\ &+ \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} c\frac{\partial F}{\partial c}(c, I^l)dc + \Psi_w(I^w) + \Psi_l(I^l)\end{aligned}$$

and note that $\bar{c}[1 - F(J_{I^l}^{-1}(\bar{c}), I^l)] = \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} \frac{d}{dc}\left[J_{I^l}(c)[1 - F(c, I^l)]\right] dc = \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)[1 - F(c, I^l)]dc - \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}(c)\frac{\partial F}{\partial c}(c, I^l)dc$. Then,

$$\begin{aligned}\mathcal{T}^2(I^w, I^l) &= \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)[1 - F(c, I^l)]dc - \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc + 2\int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} F(c, I^l)dc \\ &+ \left(\int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} c\frac{\partial F}{\partial c}(c, I^l)dc - \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}(c)\frac{\partial F}{\partial c}(c, I^l)dc\right) + \Psi_w(I^w) + \Psi_l(I^l)\end{aligned}$$

The term in brackets satisfy

$$L \equiv \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} c\frac{\partial F}{\partial c}(c, I^l)dc - \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}(c)\frac{\partial F}{\partial c}(c, I^l)dc = \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} [c - J_{I^l}(c)]\frac{\partial F}{\partial c}(c, I^l)dc$$

But, $[c - J_{I^l}(c)]\frac{\partial F}{\partial c}(c, I^l) = -2\frac{F(c,I^l)}{\frac{\partial F}{\partial c}(c,I^l)}\frac{\partial F}{\partial c}(c, I^l) = -2F(c, I^l)$, thus, it vanishes with $2\int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} F(c, I^l)dc$. As a consequence,

$$\mathcal{T}^2(I^w, I^l) = \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)[1 - F(c, I^l)]dc - \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc + \Psi_w(I^w) + \Psi_l(I^l)$$

To conclude, observe that $\int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)[1 - F(c, I^l)]dc = \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)dc - \int_{\underline{c}}^{J_{I^l}^{-1}(\bar{c})} J_{I^l}'(c)F(c, I^l)dc = \bar{c} - \int_C F(J_{I^l}^{-1}(c), I^l)dc$, where in the last integral we used the change of variables $t = J_{I^l}(c)$. Therefore,

$$\mathcal{T}^2(I^w, I^l) = \bar{c} - \int_C F(J_{I^l}^{-1}(c), I^l)dc - \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc + \Psi_w(I^w) + \Psi_l(I^l)$$

and the problem of the buyer reduces to:

$$\max_{I^w, I^l \geq 0} \int_C F(J_{I^l}^{-1}(c), I^l)dc + \int_C [1 - F(J_{I^l}^{-1}(c), I^l)]G(c, I^w)dc - \Psi_w(I^w) - \Psi_l(I^l) \tag{62}$$

Suppose this problem satisfies first and second order conditions, then, $(I^{w*}, I^{l*}) \geq 0$ is optimal must satisfy

$$D\mathcal{T}^2(I^{w*}, I^{l*}) \equiv \left[\frac{\partial \mathcal{T}^2}{\partial I^l}(I^{w*}, I^{l*}), \frac{\partial \mathcal{T}^2}{\partial I^l}(I^{w*}, I^{l*})\right] = 0$$

which can be written as

$$\begin{aligned} I^{w*} &\in arg \max_{I^w \geq 0} \int_C [1 - F(J_{I^{l*}}^{-1}(c), I^{l*})]G(c, I^w)dc - \Psi_w(I^w) \\ I^{l*} &\in arg \max_{I^l \geq 0} \int_C [1 - G(c, I^{w*}))]F(J_{I^l}^{-1}(c), I^l)dc - \Psi_l(I^l) \end{aligned}$$

□

Proof of Proposition 28: Assume that investment is observable and that *(A1)* holds. Then, $BR_l^o(\cdot) \equiv 0$. As a consequence, I^{w*} is optimal for the buyer if and only if

$$I^{w*} \in BR_w^o(0) \equiv arg \max_{I^w \geq 0} \int_C [1 - F(J_0^{-1}(c), 0)]G(c, I^w)dc - \Psi_w(I^w)$$

To the contrary, suppose that *(A2)* holds. Since $\frac{\partial}{\partial I^l}\left(\int_C [1 - G(c, BR_w^o(0))]F(J_{I^l}^{-1}(c), I^l)dc - \Psi_l(I^l)\right)\Big|_{I^l=0} > 0$, we have that $BR_l^o(BR_w^o(0)) > 0$, so $I^l = 0$ is never optimal for the buyer. The reasoning to show that $I^{w*} > 0$ is analogous. To conclude, since the buyer imposes $I^{l*} > 0$ and $BR_w^o(I^l)$ decreases with I^l (thanks to the concavity assumption over the distributional upgrades), we have that $I^{w*} = BR_w^o(I^{l*}) < BR_w^o(0)$, concluding the proof.

□

Proof of Proposition 30: Fix a mechanism $\Gamma^*(I)$ and an investment level I^w carried out by the first period winner. Then, the first period loser solves $\max_{I^l \geq 0} \int_{\underline{c}}^{J_I^{-1}(\bar{c})} [1 - G(J_I(c), I^w)]F(c, I^l)dc - \Psi_l(I^l)$. Then, if $I^{w'} < I^{w''}$ we have that $G(J_I(c), I^{w'}) < G(J_I(c), I^{w''})$ yielding

$$\int_{\underline{c}}^{J_I^{-1}(\bar{c})} [1 - G(J_I(c), I^{w'})]\frac{\partial F}{\partial I^l}(c, I^l)dc - \Psi_l'(I^l) > \int_{\underline{c}}^{J_I^{-1}(\bar{c})} [1 - G(J_I(c), I^{w''})]\frac{\partial F}{\partial I^l}(c, I^l)dc - \Psi_l'(I^l)$$

Since both functions are decreasing in I^l (the associated problems are concave), we have that $BR_l^{no}(I^{w'}|I) > BR_l^s(I^{w''}|I)$. The reasoning to show that $BR_w^{no}(\cdot|I)$ is decreasing is the same.

Now, fix I^w. Then, under enough regularity, we can write:

$$\frac{\partial}{\partial I}\left(\int\limits_{\underline{c}}^{J_I^{-1}(\bar{c})}[1-G(J_I(c),I^w)]\frac{\partial F}{\partial I^l}(c,I^l)dc-\Psi_l'(I^l)\right)$$

$$=[1-G(\bar{c},I^w)]\frac{\partial F}{\partial I^l}(J_I^{-1}(\bar{c}),I^l)\frac{\partial}{\partial I}(J_I^{-1}(\bar{c}))-\int\limits_{\underline{c}}^{J_I^{-1}(\bar{c})}\frac{\partial G}{\partial c}(J_I(c),I^w)\frac{\partial}{\partial I}(J_I(c))\frac{\partial F}{\partial I^l}(c,I^l)dc-\Psi_l(I^l)<0$$

since $\frac{\partial}{\partial I}(J_I(c))>0$ and the first vanishes ($G(\bar{c},I^w)=1$ for all $I^w\geq 0$). Taking $I^{'}$ and $I^{''}$ such that $I^{'}<I^{''}$ we obtain that:

$$\int\limits_{\underline{c}}^{J_{I'}^{-1}(\bar{c})}[1-G(J_{I'}(c),I^w)]\frac{\partial F}{\partial I^l}(c,I^l)dc-\Psi_l'(I^l)>\int\limits_{\underline{c}}^{J_{I''}^{-1}(\bar{c})}[1-G(J_{I''}(c),I^w)]\frac{\partial F}{\partial I^l}(c,I^l)dc-\Psi_l'(I^l) \tag{63}$$

implying that $BR_l^{no}(I^w|I^{'})>BR_l^{no}(I^w|I^{''})$. Using the same reasoning, it can be shown that the contrary holds in the case of the first period winner, that is, $BR_w^{no}(I^l|I^{'})<BR_w^{no}(I^l|I^{''})$ when $I^{'}<I^{''}$. Therefore, the equilibrium $(I^{l,no}(I),I^{w,no}(I))$ of the investment stage satisfies that $I^{l,no}(I)$ is decreasing in I and that $I^{w,no}(I)$ is increasing in I, concluding the proof.

□

Proof of Proposition 31: In the Stackelberg game the buyer solves

$$\max_{I\geq 0}\int\limits_C F(J_I^{-1}(c),I^{l,no}(I))dc+\int\limits_C[1-F(J_I^{-1}(c),I^{l,no}(I))]G(c,I^{w,no}(I))dc-\Psi_w(I^{w,no}(I))-\Psi_l(I^{l,no}(I))$$

$$s.t.\quad I^{w,no}(I)\in arg\max_{I^w\geq 0}\int\limits_C[1-F(J_I^{-1}(c),I^{l,no}(I))]G(c,I^w)dc-\Psi_w(I^w)$$

$$I^{l,no}(I)\in arg\max_{I^l\geq 0}\int\limits_{\underline{c}}^{J_I^{-1}(\bar{c})}[1-G(J_I(c),I^{w,no}(I))]F(c,I^l)dc-\Psi_l(I^l)$$

Assume that $I^{l,no}(\cdot)$ and $I^{w,no}(\cdot)$ are differentiable and define

$$\mathcal{K}(I)\equiv\frac{\partial}{\partial I}\left(\int\limits_C F(c,I^{l,no}(I))dc+\int\limits_C[1-F(J_I^{-1}(c),I^{l,no}(I))]G(c,I^{w,no}(I))dc-\Psi_w(I^{w,no}(I))-\Psi_l(I^{l,no}(I))\right) \tag{64}$$

It is straightforward that

$$\begin{aligned}\mathcal{K}(I) &= (I^{w,no})'(I)\left[\int\limits_C[1-F(J_I^{-1}(c),I^{l,no}(I))]\frac{\partial G}{\partial I^w}(c,I^{w,no}(I))dc-\Psi_w'(I^{w,no}(I))\right]\\ &\quad (I^{l,no})'(I)\left[\int\limits_C[1-G(c,I^{w,no}(I))]\frac{\partial F}{\partial I^l}(J_I^{-1}(c),I^{l,no}(I))dc-\Psi_l'(I^{l,no}(I))\right]\\ &\quad +\int\limits_C[1-G(c,I^{w,no}(I))]\frac{\partial F}{\partial c}(J_I^{-1}(c),I^{l,no}(I))\frac{\partial}{\partial I}(J_I^{-1}(c))dc\end{aligned}$$

Note that the first term in brackets is zero since it corresponds to the $I^{w,no}(I)$ is a best response to $I^{l,no}(I)$ given the mechanism (indexed by I) chosen by the buyer. Hence, recalling that $\frac{\partial}{\partial I}(J_I^{-1}(c)) < 0$, we obtain:

$$\begin{aligned}
\mathcal{K}(I) &< (I^{l,no})'(I)\left[\int_C [1-G(c,I^{w,no}(I))]\frac{\partial F}{\partial I^l}(J_I^{-1}(c),I^{l,no}(I))dc - \Psi_l'(I^{l,no}(I))\right] \\
&= (I^{l,no})'(I)\left[\int_{\underline{c}}^{J_I^{-1}(\bar{c})} [1-G(J_I(c),I^{w,no}(I))]\frac{\partial F}{\partial I^l}(c,I^{l,no}(I))[J_I'(c)]dc - \Psi_l'(I^l(I,no))\right]
\end{aligned}$$

where in the last equality we used the change of variables $t = J_I^{-1}(c)$. To conclude, since $J_I'(c) > 1$ for all $c \in C$ and $I \geq 0$, and $(I^{l,no})'(I) < 0$ we have that

$$\mathcal{K}(I) < (I^{l,no})'(I)\left[\int_{\underline{c}}^{J_I^{-1}(\bar{c})} [1-G(J_I(c),I^{w,no}(I))]\frac{\partial F}{\partial I^l}(c,I^{l,no}(I))dc - \Psi_l'(I^l(I,no))\right]$$

Nevertheless, the term in brackets is zero because $I^{l,no}(I)$ is a best response to $I^{w,no}(I)$ given the mechanism (indexed by I) chosen by the buyer. This leads to $\forall I \geq 0$, $\mathcal{K}(I) < 0$, implying $I^s = 0$.

□

Proof of Proposition 32: Suppose that *(A1)* is true. That $0 = I^{l*} \leq I^{l,no}$ is trivial. Also, for $I^{l,no} \geq 0$

$$\begin{aligned}
0 &= \int_C [1-F(J_0^{-1}(c),0)]\frac{\partial G}{\partial I^w}(c,I^{w*}) - \Psi_w'(I^{w*}) \\
&\geq \int_C [1-F(J_0^{-1}(c),I^{l,no})]\frac{\partial G}{\partial I^w}(c,I^{w*}) - \Psi_w'(I^{w*})
\end{aligned}$$

thanks to the distributional upgrade. Since this last function is concave in I^w it must be that $I^{w,no} \leq I^{w*}$. That the inequality is strict follows from observing that $F(J_0^{-1}(c),I^{l,no}) > F(J_0^{-1}(c),0)$ iff $I^{l,no} > 0$.

□

Proof of Proposition 34: Let $BR_w^o(\cdot)$ and $BR_l^o(\cdot)$ be the best response functions of the winner and the loser, respectively, in the fictitious game when investment is observable. Analogously let $BR_w^{no}(\cdot|0)$ and $BR_l^{no}(\cdot|0)$ be the corresponding best response functions in the non-observable case (in this setting the buyer chooses a mechanism indexed by zero investment). Recall that all this best response functions are decreasing under the assumptions stated in the proposition. Since $[1-F(J_I^{-1}(c),I)] > [1-F(J_0^{-1}(c),I)]$ for $I > 0$, we have that $BR_w^{no}(\cdot|0) < BR_w^o(\cdot)$. Finally, we have that

$$BR_l^{no}(I^w|0) \in arg\max_{I^l \geq 0} \int_{\underline{c}}^{J_0^{-1}(\bar{c})} [1-G(J_0(c),I^w)]F(c,I^l)dc - \Psi_l(I^l) \tag{65}$$

$$= arg\max_{I^l \geq 0} \int_C [1-G(c,I^w)]F(J_0^{-1}(c),I^l)\frac{d}{dc}(J_0^{-1}(c))dc - \Psi_l(I^l) \tag{66}$$

Assumption (B) ensures that

$$\int_C [1-G(c,I^w)]\frac{\partial F}{\partial I}(J_0^{-1}(c),I^l)\frac{d}{dc}(J_0^{-1}(c))dc - \Psi_l'(I^l) > \int_C [1-G(c,I^w)]\frac{\partial}{\partial I}(F(J_{I^l}^{-1}(c),I^l))dc - \Psi_l'(I^l) \quad (67)$$

which implies that $BR_l^{no}(\cdot|0) < BR_l^{o}(\cdot)$ using the concavity of $I \mapsto F(J_0^{-1}(c),I)$ and the convexity of $\Psi_l(\cdot)$. Since all these best response functions are decreasing, we can conclude that $I^{l*} \leq I^{l,no}$ and $I^{w,no} \leq I^{w*}$.

If condition (B) does not hold, we can no longer ensure that $BR_l^{no}(\cdot|0) < BR_l^{o}(\cdot)$, hence, given that the best response functions are decreasing, we cannot say anything about the investment levels.

$\square$