

# Business Cycles and Lending Standards\*

Nicolás Figueroa and Oksana Leukhina<sup>†</sup>

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Aggregate productivity falls in recessions and rises in expansions. Several empirical studies suggest that the systematic behavior of lending standards, with laxer (tighter) standards applied during expansions (recessions), is responsible for reverting trends in aggregate productivity. We build a dynamic model that rationalizes these findings. Higher liquidity brought about by high levels of capital increases the early loan payments necessary for screening out unproductive entrepreneurs. Because the early payments hurt productive entrepreneurs by restricting their investment, competition among lenders results in contracts with no screening. Low productivity entrepreneurs enter production, causing a recession. The opposite happens for low capital levels.

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<sup>†</sup>*Nicolás Figueroa:* CEA-DII, Universidad de Chile, Santiago, Chile, nicolasf@dii.uchile.cl; *Oksana Leukhina (CORRESPONDING AUTHOR):* Department of Economics, 305 Savery Hall, Box 353330, University of Washington, Seattle, WA 98195; e-mail: oml@uw.edu; tel: (919) 259-4428, fax: (206) 685-7477.

# 1. Introduction

Aggregate productivity falls in recessions and rises in expansions. Reasons behind this pattern are still not well understood. We are motivated by several empirical studies that suggested that the systematic behavior of lending standards<sup>1</sup> over the business cycle that may be responsible for reversing trends in aggregate productivity. Laxer standards during economic booms allow for unproductive firms to be funded, reducing aggregate productivity through the producer composition effect. On the contrary, tight credit standards during economic downturns tend to exclude bad projects, thus sowing the seeds of an economic recovery. Asea and Blomberg (1998), using a panel data set of two million commercial and industrial loans, document that laxer lending standards occur during expansions and tighter standards occur during recessions, and that such behavior considerably influences the dynamics of aggregate fluctuations. Lown and Morgan (2006) use a survey of loan officers to document similar systematic behavior of lending standards. Moreover, they point out that loan standards are more important than loan rates in explaining variation of business loans and output in the time series.

Consistent with the hypothesis that less productive producers enter the market in expansions, the aggregate data reveals that delinquency rates and loan charge-off rates<sup>2</sup> begin rising while the economy is still expanding, peaking in recessions (Figure 1a).<sup>3</sup> Moreover, the difference between the prime loan rate and the riskless rate, reported in Figure 1b, which can be thought of as reflecting the risk of lending, also begins rising before the economy enters a recession.

Due to the widespread practice of financing production through banks, the seeming imperfections arising from credit standards that are either too lax or too tight, have generated much concern among the policy makers.<sup>4</sup> Alan Greenspan, speaking at the Chicago Bank Structure Conference in 2001, alarmingly stated that “the worst loans are made at the top of the business cycle...”

We build a model with information asymmetry in credit markets that can endogenously generate substandard lending standards in expansions, allowing for the less productive entrepreneurs to be funded and thus reverting trends in aggregate productivity through the extensive margin. The opposite happens in recessions. Our model, which features competitive credit markets and fully rational agents, delivers an endogenous screening cost that rises with economic fundamentals. Note that to produce a deterioration (tightening) of lending terms during expansions (recessions), our model does not rely on non-fully rational expectations on the part of loan officers, as in Berger and Udell (2004). A version of our model with

<sup>1</sup>Lending standards are contract terms other than the interest rate that are used to screen borrowers, for example, loan size or credit line limit, time to loan maturity or first payment towards the loan balance, or borrowers’ balance sheet variables.

<sup>2</sup>Charge-offs, which are the value of loans removed from the books and charged against loss reserves, are measured net of recoveries as a percentage of average loans and annualized. Delinquent loans are those past due thirty days or more and still accruing interest as well as those in nonaccrual status. They are measured as a percentage of end-of-period loans. Source: The Board of Governors, downloaded from <http://www.federalreserve.gov/releases/chargeoff/default.htm>. GDP and GDP deflator data are taken from the Bureau of Economic Analysis.

<sup>3</sup>To measure real business cycles, we first apply the Hodrick-Prescott filter to the log of the series, we then report deviations from the trend, scaled by 25 for better visualization.

<sup>4</sup>Non-corporate business, representing roughly 1/3 of the total U.S. business net worth, relies entirely on bank loans. Although corporate business has considerably reduced its dependence on bank financing, it still holds over 25% of its debt in bank loans and mortgages and, as recently as in the mid-1970s, held around 45% of its debt in these instruments. The aggregate balanced sheet data for corporate and non-corporate business is taken from the Board of Governors website.

an endogenous supply of loanable funds, studied in Section VI, is qualitatively consistent with much of the empirical evidence generating procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle, default rates lagging after the business cycle, and finally, the possibility of slow recoveries and abrupt and severe recessions.<sup>5</sup>

We now explain the channel through which the state of the economy influences the lending standards in our model. There are two privately known types of entrepreneurs that differ in their productivity. They produce capital in two stages, for both of which they must seek external financing. Banks screen entrepreneurs by requiring that upon completion of the first stage of production, they make an early payment towards the loan balance, which we interpret as a stylized version of the lending standards.<sup>6</sup> This payment can always be set high enough so that the unproductive type cannot afford it. Making this payment, however, is costly as it lowers entrepreneurs' reinvestment into the second stage of production. Entrepreneurs who obtain financing for both stages of production can default after the second stage of production, in which case they abscond with a fraction of their output. Along the equilibrium paths analyzed in this paper, both entrepreneurs seek financing whenever the size of the early payment allows it, but only the productive type repays.

We draw on Hellwig's (1987) result that equilibrium contracts are selected as the best contracts for the good type subject to the lenders' zero profit condition. Changes in lending standards arise from an endogenously evolving cost of effective screening. Two forces deliver pooling contracts at the top of the cycle (for high capital levels) and separating contracts, with unproductive entrepreneurs unfinanced, at the bottom. First, at peaks, when all entrepreneurs enjoy higher liquidity, the early payment required to accomplish screening out the unproductive types is high, and hence good entrepreneurs must be hurt more to ensure viable separation. Second, pooling contracts that allows bad entrepreneurs to enter are less costly at peaks, as the bank recovers more money after default, and hence a lower interest rate is needed to ensure bank participation. Hence, at peaks, good entrepreneurs are then more willing to accept a pooling contract with no screening, which allows for higher reinvestment, even though asks for a higher than risk-free interest rate as a cross-subsidy.

The main insight captured by our model is that better economic fundamentals may raise effective screening costs:<sup>7</sup> a given signal about an entrepreneur's productivity (a particular amount met as early payment) is informative at the trough, but not informative at the peak. Consequently, screening out the bad projects becomes more costly at the peak and separating contracts (with strict lending terms) are beaten by pooling contracts with no screening at all. Low productivity entrepreneurs are financed along with high productivity types, default rates rise, and the composition effect induces a recession. This

<sup>5</sup>Although quantitative analysis is outside of the scope of this paper, one quick check of our model's potential quantitative relevance is to see how much of aggregate productivity variation over time transpires through the extensive margin (new firm entry/exit). In fact, Lee and Mukoyama (2008) document that in the manufacturing sector the extensive margin is slightly more important than the intensive margin for job creation/destruction over the cycle. Job creation by startups in booms is greater than net job creation of preexisting firms. Likewise, in downturns, job destruction by exiting firms is greater than net job destruction by continuing firms.

<sup>6</sup>In the empirical work of Asea and Blomberg (2004), how early loan payments are due is included in the definition of a lending standard.

<sup>7</sup>This link, modelled here, could be further reinforced if, for example, in expansions, observable entrepreneurs' revenues became less informative of their type as everyone's revenues increased, or if the pool of potential applicants became biased in favor of low productivity entrepreneurs.

situation, in turn, eventually generates conditions – a low enough level of capital and liquidity – conducive to the emergence of separating contracts that exclude low productivity entrepreneurs from production, thereby increasing aggregate productivity in the capital good sector, and leading to an economic recovery. Endogenous cycles may arise. Thus, our model endogenizes aggregate productivity through the producer composition effect, which in turn, depends on the endogenously evolving lending standards.

One important stylized fact about recessions that we emphasize is that default rates begin rising already before the economy enters into a recession. This fact cannot be captured through a standard RBC-type model, in which a low aggregate productivity shock is the cause of both a productivity drop and project failures, and thus default rates rise strictly after a bad shock. We have in mind a model with heterogeneity in entrepreneurs’ productivity and a common productivity shock. Banks finance entrepreneurs based on expected returns to their project. Financed projects are always ex-ante good, but may become ex-post bad investments due to the low aggregate shock. Instead, in our model, ex-ante bad projects can be financed. Financing of ex-ante bad projects, i.e. the emergence of substandard lending, is probably best exemplified with the dot-com mania of the late nineties, during which projects with no discernible sources of revenue were heavily financed. These were not good projects going bust due to an exogenous shock. If anything, the late nineties saw a strong technology improvement in the computer and internet industries. A more likely explanation is that ex-ante bad projects were financed because screening was particularly costly. The recent subprime mortgage lending also exemplifies poor screening standards (zero money down, low credit scores) arising in equilibrium. A straight-forward extension of our model to include 3 types of entrepreneurs (example 2), which identifies periods of expansions and recessions more clearly, delivers the empirically observed pattern of comovement of default rates and productivity.

Thus, we develop a new insight into why lending standards vary with the state of the economy and may contribute to turning expansions into recessions and vice versa. The rest of the paper is organized as follows. Section II overviews related literature. In Section III, we introduce the general model, derive static equilibrium contracts for given prices and define the dynamic equilibrium. In Section IV, we study a fully dynamic economy with externalities in the production sector (which simplifies the analysis by making the price of capital constant) and show the existence of equilibrium paths along which the model economy exhibits cyclical behavior. The general model with no externalities is analyzed in Section V, where we find cyclical behavior and also the possibility of indeterminacy of equilibrium on some range of state variables (due to the existence of more than one self-fulfilling belief). Finally, in Section VI, we endogenize the amount of funds available to finance entrepreneurs’ projects. We conclude in Section VII.

## 2. Related Literature

A number of theoretical models illustrate potentially important interactions between informational frictions in credit markets<sup>8</sup> and economic fluctuations. For the purpose of our discussion, we focus on two strands of related work that pursue the idea that credit markets influence the course of the business cycle. One strand argues that credit market imperfections amplify exogenous shocks and make them more

<sup>8</sup>The early works on informational frictions in credit markets are Stiglitz and Weiss (1981), Bester (1985) and De Meza and Webb (1987).

persistent. The other strand argues that credit market imperfections can be responsible for a reversion in output.

A well-known example of an amplification mechanism is Bernanke and Gertler (1989),<sup>9</sup> where the borrowers' balance sheets amplify exogenous external shocks in a model of costly state verification. Economic upturns improve the borrowers' net worth, which lowers agency costs of financing investment, increases investment and hence amplifies the upturn, while the opposite happens in the presence of a downturn. Another example is Kiyotaki and Moore (1997), which assumes that loan payments cannot be enforced, and hence only collateralized debt arises in equilibrium. A temporary shock that reduces a credit constrained firm's net worth reduces this firm's ability to obtain new loans and therefore its investment, thus propagating the effect of a temporary shock. In Rampini (2004), entrepreneurial activity which consists in the undertaking of risky, but in expected terms productive, projects increases at peaks. This is due to agents' higher willingness to take risks when the economic situation is booming, and the smaller need for bearing such a risk in those times, due to slacker incentive constraints. Other studies focusing on the amplification of exogenous shocks arising from informational frictions include Williamson (1987), Greenwald and Stiglitz (1993) and Bernanke, Gertler and Gilchrist (1999).

Our paper falls into the second strand of literature, which argues that credit market imperfections can also be responsible for a reversion in output. The stark difference between the existing papers in this strand and our work is that the reversion mechanism in our model relies on the endogenously relaxed lending standards during expansions and tightening of lending standards in recessions. This empirical fact has been widely documented, and it has been suggested to affect the course of aggregate productivity through its influence on the composition of producers. Our model is the first one to rationalize the emergence of substandard lending (tight lending) during expansions (recessions), entry (exclusion) of the low productivity producers and hence a reversal of aggregate productivity.

The existing papers in the second strand include Suarez and Sussman (1997), Azariadis and Smith (1998), Reichlin and Siconolfi (2003) and Martin (2006). Suarez and Sussman (1997) generate a reversion mechanism that works through the effect of equilibrium prices on liquidity constraints. The model is a dynamic extension of the Stiglitz-Weiss (1982) model of lending under moral hazard to an overlapping generations model with three generations. During booms, old entrepreneurs sell high quantities and, as a consequence, prices are low implying that young entrepreneurs must finance a higher fraction of output externally. Because external financing generates excessive risk-taking, booms are followed by high project failure rates. Although it delivers an endogenous reversion mechanism, the main channel through which this mechanism works – higher reliance on external financing at peaks – appears to be at odds with the data (see Levy and Henessey 2007).

Reichlin and Siconolfi (2003) generalize the Rothschild and Stiglitz (1976) adverse selection problem by including moral hazard. Both safe and risky projects can be implemented. They show that endogenous cycles may arise: when loanable funds are high, equilibrium contracts are such that a large fraction of entrepreneurs chooses to engage into risky production, high setup costs decrease output and wages sending the economy into a recession. As in our model, the reversion mechanism is driven by the composition

<sup>9</sup>Carlstrom and Fuerst (1997) perform the quantitative analysis of the mechanism in Bernanke and Gertler (1989).

effect: risky projects (that are worse than safe projects in expected terms) are chosen at the top of the cycle causing a recession. In fact, the opposite assumption, i.e. that risky projects are better in expected terms, is made in Rampini (2004), and the informational frictions deliver an amplification mechanism. Another related paper that generates a reversion mechanism is Martin (2006). In that model, at the top of the cycle, separating contracts with strict lending terms arise, generating low investment. The low investment then causes the downturn in the economy. Hence, it is the countercyclicality of investment that is at the root of the reversion mechanism. The opposite is true in our model: lending standards are lax and investment is high at the top of the cycle, both of which are consistent with empirical evidence. It is the entry of low productivity entrepreneurs at the top that drives the economy down. Hence, we generate reversals in aggregate productivity trends due to the composition effect of producers, and despite the procyclicality of investment. In Azariadis and Smith (1998), the presence of adverse selection leads to the indeterminacy of equilibrium. Savers moving their funds in and out of the banking system, as a result of their anticipation of the real interest rate, may lead to endogenous cycles.

Similar in spirit to our work is Dell’Ariccia and Marquez (2006), motivated by the deterioration of lending standards observed in emerging economies prior to financial crises. The paper considers an exogenous change in the information structure in credit markets. This change, which is associated with financial liberalization, leads to a deterioration of lending standards and loan portfolios for the bank, making the economy more susceptible to low aggregate shocks. In our work, we endogenize the link between current economic state and screening costs.

### 3. The Model

#### 3.1. Environment

Consider a model economy where time is discrete and indexed by  $t = 0, 1, 2, \dots$ . It is populated with overlapping generations of entrepreneurs who live for two periods and there exist two types of goods: consumption and capital. When young, entrepreneurs are endowed with 1 unit of time and an ability to implement projects that produce capital. We assume entrepreneurs do not suffer disutility from labor and enjoy utility from consuming both when young and old, according to homothetic preferences represented by  $u(c^y, c^o)$ . A savings technology is available to them at the risk free rate  $R_f$ .

The consumption good is produced by overlapping generations of competitively behaved firms that live for two periods. A representative firm born in period  $t - 1$  purchases capital from the young entrepreneurs in period  $t - 1$  at price  $\rho_t$  and uses it in production along with labor in period  $t$ , according to technology  $F(K_t, L_t, t) = A_t K_t^\beta L_t^{1-\beta}$ . Capital fully depreciates upon use. We assume that firms are born with nothing but can borrow at the risk free rate  $R_f$  in order to make capital purchases.<sup>10, 11</sup> Since both the decision about capital  $K_t$  and labor demanded  $L_t$  must be simultaneously made in period  $t - 1$  (when capital is purchased), firms must then form beliefs about the next period wage  $w_t$ . These beliefs must be

<sup>10</sup>Taking price  $\rho_t$  and beliefs  $w_t$  as given, a firm born in period  $t - 1$  solves  $\max_{K_t, L_t} A_t K_t^\beta L_t^{1-\beta} - R_f \rho_t K_t - w_t L_t$ .

<sup>11</sup>We also assume that the firm born in period  $-1$  is endowed with  $K_0$  and a debt in the amount of its value  $R_f F_1(K_0, 1, 0) K_0$ .

consistent with the actual wage in the next period, which must in turn clear the labor market given the capital  $K_t$  bought by the consumption good sector in the previous period.

Capital goods are produced by entrepreneurs. Each generation consists of measure  $\mu$  of type  $G$  and measure  $1-\mu$  of type  $B$ . Types differ in their productivity and are private information. Each entrepreneur can implement a project within a single period, but in two stages. A fixed investment  $M$  is required at each stage of production, the second stage allowing additional investment. Project  $i$  transforms investment into capital goods at the rate of  $f_i/M$  in the first stage and  $g_i/M$  in the second stage. We assume that type  $G$  is more productive at each stage.

**Assumption 1**  $f_G > f_B$  and  $g_G > g_B$ .

There is a competitive banking sector that loans investment funds to the young entrepreneurs. Each period, banks are endowed with  $2M\mu$  loanable units of the consumption good, exactly the fixed cost amount of implementing projects of all type  $G$  entrepreneurs. This particular amount is assumed for analytical simplicity.<sup>12</sup> We relax this assumption and endogenize the supply of funds in Section VI. A risk-free savings technology is available to the bank at rate  $R_f$ .

We consider contracts signed in the beginning of the entrepreneurs' young period. If an entrepreneur enters into a contract  $(\delta, R)$ , he receives an amount  $M$  in the beginning of the first stage and another amount  $M$  in the beginning of the second stage, conditional on meeting a partial payment  $\delta$  towards the loan balance. At the end of the period, the remaining loan repayment is  $2MR - \delta$ . With the early payment  $\delta$ , we aim to capture the banks' need to use a signalling device that is costly in order to gather "hard evidence" about an entrepreneurs' type.<sup>13</sup> We also assume that whenever financing is obtained, the entrepreneur must implement the project, which is justified by the availability of a monitoring technology.

Upon completion of the second stage, an entrepreneur can avoid repayment, in which case he retains a fraction  $\alpha$  of wealth is kept by the entrepreneur, while a fraction  $1 - \alpha$  is seized by the bank. We interpret the magnitude of  $\alpha$  as determined by entrepreneurs' ability to profit from managing a firm and defaulting on the loan later, such as compensation through managerial salary and perquisites, hiding of income, fraud and protection provided by bankruptcy institutions. We assume that default can take place only at the end of the second stage. This captures the idea that it is more difficult to hide income during initial stages of production and is made only for analytical simplicity.<sup>14</sup>

Alternatively, we could assume that banks offer a credit line to entrepreneurs and contracts specify the credit limit and the interest rate on the borrowed funds. In the appendix, we prove that identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with credit limit  $2M - \delta$  and interest rate  $\frac{2MR - \delta}{2M - \delta}$ .

<sup>12</sup>With both types applying for loans, some crowding out of type  $G$  will take place as long as the amount of funds is less than  $2M$ , which is needed for the results derived under the present setup. However, our results also extend to a more general setup with any amount of loanable funds as long as financing of type  $B$  entrepreneurs replaces investment in some safe capital production technology with returns higher than those generated by  $B$ .

<sup>13</sup>Note that for this repayment to reveal the type, it must be costly. If not, for example, if the banks compensated the entrepreneur for  $\delta$  with a larger future loan, or if the banks simply asked to show  $\delta$ , the bad entrepreneurs would be induced to obtain short-term liquidity to meet the conditions and consequently obtain the large loan, which would enable them to repay the short-term lender and still have enough funds to start production in the next period.

<sup>14</sup>This essentially means that in case of default at the end of the first stage, the bank seizes the entire wealth of the defaulting entrepreneur.

It is instructive to examine the cash flows for an entrepreneur who enters into a contract  $(\delta_t, R_t)$  at time  $t$ , reinvests everything into the second stage of production, and does not default. At time  $t$ , a young entrepreneur obtains  $M$  units of the consumption good from the bank in the beginning of the first stage of production and invests it into his project. At the end of the first stage he receives  $\rho_{t+1}f_i$  in payment for his capital and  $w_t$  as labor income from supplying 1 unit of time to the the consumption good sector. He pays  $\delta_t$  towards the loan balance upon completion of the first stage of production. He receives  $M$  units of the consumption good from the bank and invests into the second stage, along with his own income net of the loan payment,  $w_t + \rho_{t+1}f_i - \delta_t$ , which transforms into capital at the rate of  $\frac{g_i}{M}$ . At the end of the period, the entrepreneur sells his capital  $g_i + (w_t + \rho_{t+1}f_i - \delta_t)\frac{g_i}{M}$  at price  $\rho_{t+1}$  and pays the remaining loan balance  $2MR_t - \delta_t$  to the bank. The net worth at the end of the period,  $\rho_{t+1} [g_i + (w_t + \rho_{t+1}f_i - \delta_t)\frac{g_i}{M}] - (2MR_t - \delta_t)$ , is then optimally allocated between consumption at time  $t$  and  $t + 1$ .

Two stages of production and the assumption that the consumption good sector delivers the payment for capital goods produced in the first stage before the second stage of production starts are necessary to allow for the possibility of screening. In our case, the screening tool is the requirement of a partial payment towards the loan balance before production is completed. Thus, we capture the idea that the lenders always have the ability to screen out bad entrepreneurs, because type  $G$  can always afford a higher payment than type  $B$ . However, with liquidity (as determined by  $w_t$  or  $\rho_{t+1}$ ) evolving endogenously through time, the cost of effective screening will also evolve endogenously through time, determining whether or not the screening tool is used in equilibrium at a particular point in time.

The previous discussion points out that our theory allocates an important role to the presence of liquidity positively correlated with the state of the economy. Although we chose to model this liquidity as arising due from wage income, a number of other modeling choices would suffice (e.g. income from land holdings).

### 3.2. Entrepreneurs' Behavior

For a given contract  $(\delta_t, R_t)$  and prices  $w_t$  and  $\rho_{t+1}$ , a time  $t$  young entrepreneur of type  $i$ , who reinvests all the proceedings from the first stage (net of the early payment  $\delta_t$ ), chooses among the options summarized below. Since we focus on static choices, we drop the time subscripts for the rest of the section.

(O1) Do not enter into the contract. End of period net worth is  $W_{O1} = w$ .

(O2) Enter into the contract, meet the payment  $\delta$ , default. End of period net worth is  $W_{O2} = \alpha\rho [g_i + (w + \rho f_i - \delta)\frac{g_i}{M}]$ .

(O3) Enter into the contract, meet the payment  $\delta$ , repay. End of period net worth is  $W_{O3} = \rho [g_i + (w + \rho f_i - \delta)\frac{g_i}{M}] - (2MR - \delta)$ .

Hence, an entrepreneur solves

$$\max_{O1, O2, O3} \{W_{O1}, W_{O2}, W_{O3}\}.$$

Note that since default is not an option before the second stage, entrepreneur  $i$  will always choose O1 if  $\delta < w + \rho f_i$ , i.e., if he cannot afford the early payment.

We denote the maximum early payment affordable by type  $B$  entrepreneurs by

$$\tilde{\delta}(w, \rho) := w + \rho f_B. \quad (1)$$

Note that  $\tilde{\delta}(w, \rho)$  increases in both  $w$  and  $\rho$ , which raise labor and capital income respectively.

Both adverse selection (due to the existence of private information about types) and moral hazard (due to the possibility of not repaying the loan) arise in our model of competitive lending. We are, however, interested in equilibrium paths along which (i) type  $B$  entrepreneurs seek financing whenever the early payment is affordable ( $\delta < \tilde{\delta}(w, \rho)$ ) and in case of obtaining it, default on the loan; (ii) type  $G$  entrepreneurs seek financing whenever the early payment is affordable ( $\delta < w + \rho f_G$ ) and repay. Focusing on a particular behavior of entrepreneurs, i.e. a particular resolution of the moral hazard, allows us to use the results from Hellwig (1987) which apply to competitive markets with adverse selection and determine equilibrium contracts for a given state of the economy.

In short, we will seek restrictions on the parameter space that induce behavior (i) and (ii) along any equilibrium path. Clearly, the trade-offs that entrepreneurs face depend on the endogenously determined prices, which in turn depend on the state of the economy. In Assumption 8 (in the appendix), we state sufficient conditions on prices  $w$  and  $\rho$  that guarantee the existence of a non-empty set of contract menus that are incentive-compatible, induce entrepreneurs to behave according to (i) and (ii), and yield nonnegative profits. When we analyze the dynamic behavior of the economy, we will derive parametric conditions and restrictions on the initial conditions such that Lemma 1 applies to the entire equilibrium path.

**Lemma 1** *Suppose that Assumptions 1 and 8 (in the appendix) hold. For any contract menu  $((R_B, \delta_B), (R_G, \delta_G))$  that induces self-selection, generates nonnegative profits and satisfies  $R_i \geq R_f$  the following are true:*

- *Both types, if financed, reinvest all the cash flows from the first stage (after the early payment  $\delta$ ) into second stage production.*
- *If  $\delta_B < \tilde{\delta}(w, \rho)$  (i.e., the contract is affordable to type  $B$ ), type  $B$  chooses to enter and default.*
- *Type  $G$  chooses to enter and repay.*

*Moreover, such menu of contracts exists. In particular,*

- *For any  $\delta_B < \tilde{\delta}(w, \rho)$ , the pooling menu  $((\delta_B, \hat{R}), (\delta_B, \hat{R}))$  generates zero profits for some uniquely determined  $\hat{R} > R_f$ .*
- *The separating menu that offers  $(\tilde{\delta}(w, \rho), R_f)$  to type  $G$  and non-participation to type  $B$  generates zero profits and induces self-selection.*

**Proof.** See the appendix. ■

The previous lemma asserts that in the set of feasible contract menus, type  $B$  enters and defaults. It also shows that it is always possible to find feasible contracts, either by excluding type  $B$  (by setting a

high enough  $\delta_B$ ) or inducing type  $G$  to accept a contract under which he repays and cross-subsidizes type  $B$ .

In what follows, we will show that in equilibrium, banks will offer exactly one of the two types of contracts characterized above: either pooling contracts or contracts where type  $B$  is excluded.

### 3.3. Equilibrium contracts

Next we characterize equilibrium contracts that emerge for given prices  $w_t$  and  $\rho_{t+1}$ . As is known from Rothschild and Stiglitz (1976), equilibrium does not always exist in a context of competitive markets with adverse selection. In short, if separating contracts are preferred by the good type, equilibrium exists. If, however, pooling contracts are preferred by the good type and therefore Pareto superior, competition imposes a zero-profit condition which induces cross-subsidization, and this implies that a bank offering such a contract is exposed to cream-skimming from a competitor targeting the good clients. Hellwig (1987) resolved this dilemma by incorporating a third stage into the game, so that after banks have offered contracts and agents have applied, banks have the possibility to reject their applicants, based on all the contracts offered by competing firms. Using the concept of sequential equilibrium, Hellwig proved that the pooling equilibrium that maximizes the utility of the good type subject to the zero-profit condition can be sustained.

When considering feasible contract menus as defined in Lemma 1, we are focusing on a particular resolution of the moral hazard. Therefore, Hellwig's results that apply to competitive markets with adverse selection apply to our setting, which allows us to follow the standard practice of finding equilibrium contracts  $((\delta_G, R_G), (\delta_B, R_B))$ , whether pooling or separating, as maximizers of the utility of type  $G$  subject to the zero-profit condition.

The next lemma states that financing of both types cannot occur under different contracts. Intuitively, if both types receive financing through different contracts, then each one of these contracts must yield zero profit to be robust to cream-skimming. Since type  $B$  never repays, the only way for type  $B$  to be financed is through a pooling contract.

**Lemma 2** *If both types receive financing, then  $(\delta_B, R_B) = (\delta_G, R_G)$ .*

**Proof.** See the appendix. ■

We can thus restrict our attention to feasible menus that specify identical contracts  $(\delta, R)$  for both types, whether or not they exclude type  $B$ . If a bank chooses to exclude type  $B$  entrepreneurs, it can do so by offering a single contract  $(\tilde{\delta}(w, \rho), R_f)$  which induces type  $G$  to repay and yields zero profits.

Moreover, by Lemma 1, type  $B$  enters and defaults for any  $\delta < \tilde{\delta}(w, \rho)$ , but type  $G$  can be induced to cross-subsidize type  $B$  with a pooling contract that yields zero profits. Under this contract, both types enter, type  $G$  repays in full and type  $B$  defaults. Because types are unobservable a proportion  $\mu$  of entrepreneurs of type  $G$  and  $(1 - \mu)$  of type  $B$  are financed. The funds obtained as loan repayment consist of full repayment from type  $G$  entrepreneurs and  $\delta_B$  collected as early payment together with fraction  $1 - \alpha$  of output seized from type  $B$ . Hence, the interest rate in this contract is a solution to

$$\mu 2MR + (1 - \mu) \left[ \delta + (1 - \alpha)\rho \left( g_B + (w + \rho f_B - \delta) \frac{g_B}{M} \right) \right] = 2MR_f, \quad (2)$$

where the right hand side represents the opportunity cost of a loan of size  $2M$ .

The zero profit condition that must be satisfied in competitive markets, implies an interest rate  $R$  for each contract such that  $\delta \leq \tilde{\delta}(w, \rho)$  that can be summarized by

$$R(w, \rho, \delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} [\delta + (1-\alpha)\rho(g_B + (w + \rho f_B - \delta)\frac{g_B}{M})], & \text{if } \delta < \tilde{\delta}(w, \rho) \\ R_f, & \text{if } \delta = \tilde{\delta}(w, \rho) \end{cases}, \quad (3)$$

where we solved (2) for  $R$ . Note that  $R(w, \rho, \delta)$  is greater than  $R_f$  by Lemma 1 and decreases in both  $\rho$  and  $w$  in the interval  $[0, \tilde{\delta}(w, \rho))$ . Intuitively, as income increases, the amount recovered from type  $B$  increases, and hence a lower interest rate is needed to ensure zero profit for the lender. Moreover,  $R(w, \rho, \delta)$  decreases in  $\delta$ : an increase in  $\delta$  (which lowers reinvestment for type  $B$  but implies a higher interim payment for the bank) actually raises the total collection from type  $B$  and thus requires a lower cross-subsidy from type  $G$  under a pooling contract.

Under any contract, for given prices  $(w, \rho)$ , the net worth obtained by type  $G$  is given by  $\rho(g_G + (w + \rho f_G - \delta)\frac{g_G}{M}) - (2MR - \delta)$ . Therefore, the equilibrium contract offered by banks is a solution to

$$\begin{aligned} \max_{(\delta, R)} \quad & \rho \left( g_G + (w + \rho f_G - \delta) \frac{g_G}{M} \right) - (2MR - \delta) \\ \text{s.t} \quad & \\ & R = R(w, \rho, \delta), \end{aligned}$$

where  $R(w, \rho, \delta)$  is given by equation (3).

This problem, which yields the equilibrium contract, is summarized by the feasible set (zero-profit condition) and type  $G$  indifference curves, depicted in Figure 2. To find its, we note that indifference curves describing his trade-off between  $R$  and  $\delta$ , given current prices, have the slope

$$ICS := -\frac{\rho \frac{g_G}{M} - 1}{2M} < 0, \quad (4)$$

which is negative by condition (a) of Assumption 8. Moreover, indifference curves become steeper as  $\rho$  rises. This last fact is intuitive since, as capital sells for a higher price, the cost of foregone investment associated with  $\delta$  rises, and type  $G$  requires a greater reduction in  $R$  to compensate for a given rise in  $\delta$ . The solution depends on the relative size of  $ICS$  and the slope of the line connecting the following two points on the lender's zero-profit curve: the pooling contract  $(\delta, R) = (0, R(w, \rho, 0))$  and the separating contract  $(\tilde{\delta}(w, \rho), R_f)$ . We call this slope  $IPS$ , the first two letters referring to the iso-profit,

$$IPS := -\frac{R(w, \rho, 0) - R_f}{\tilde{\delta}(w, \rho)} < 0. \quad (5)$$

It is easy to see from Figure 2 that the solution to the (linear) banking problem is that a pooling contract  $(0, R(w, \rho, 0))$  is offered if  $ICS < IPS$ , and a separating contract  $(\tilde{\delta}(w, \rho), R_f)$  is offered otherwise.

Note that all of type  $G$  entrepreneurs (measure  $\mu$ ) and none of type  $B$  entrepreneurs are financed under the separating contract. Hence, total capital production amounts to the production of type  $G$

entrepreneurs:

$$k_s(\rho) = \mu \left( f_G + g_G + (\rho f_G + w - \tilde{\delta}(w, \rho)) \frac{g_G}{M} \right) = \mu \left( f_G + g_G + (\rho (f_G - f_B)) \frac{g_G}{M} \right). \quad (6)$$

Under the pooling contract, a measure  $\mu^2$  of type  $G$  entrepreneurs and a measure  $\mu(1 - \mu)$  of type  $B$  entrepreneurs obtain financing and enter production. Since the production of type  $i$  under the contract that involves no early payment is given by  $f_i + g_i + (\rho f_i + w) \frac{g_i}{M}$ , the total capital production is given by

$$k_p(w, \rho) = \mu^2 \left( f_G + g_G + (\rho f_G + w) \frac{g_G}{M} \right) + \mu(1 - \mu) \left( f_B + g_B + (\rho f_B + w) \frac{g_B}{M} \right). \quad (7)$$

The following proposition summarizes these results.

**Proposition 1** *If  $\frac{1 - \rho \frac{g_G}{M}}{2M} \geq -\frac{R(w, \rho, 0) - R_f}{\tilde{\delta}(w, \rho)}$ , then a separating contract  $(\tilde{\delta}(w, \rho), R_f)$  is offered, and total capital production is given by (6). Otherwise, a pooling contract  $(0, R(w, \rho, 0))$  is offered, and total capital production is given by (7).*

**Proof.** *Follows from the above discussion. ■*

### 3.4. Dynamic Equilibrium

Up to this point, we derived equilibrium contracts and capital production for given prices. As usual prices are determined endogenously, but in this case the consumption good firms producing in period  $t + 1$  must also form beliefs in period  $t$  about wages  $w_{t+1}$ , which are not announced in period  $t$  but influence the decision about the demand for capital  $k_{t+1}$ . These beliefs must be consistent with the actual wages, which in turn must clear the labor market in period  $t + 1$ , given the total capital purchased by the consumption good sector  $k_{t+1}$ .

An expectation about future wages can then be seen as an expectation about the future level of capital  $k_{t+1}$ , which must be consistent with the actual decisions of entrepreneurs and firms in period  $t$ . This is the convention we use throughout the paper.

**Definition 1** *A dynamic equilibrium for given  $k_0$  is given by sequences of prices  $\{w_t^*, \rho_t^*\}_{t=0}^\infty$ , capital levels  $\{k_{t+1}^*\}_{t=0}^\infty$ , beliefs  $\{k_t^*\}_{t=0}^\infty$  and contracts  $\{R_t^*, \delta_t^*\}_{t=0}^\infty$  such that the following holds:*

- *Expectations are consistent with equilibrium outcomes (rational):  $k_t^* = k_{t+1}^*$ ,*
- *Prices are given by  $w_t^* = A_t(1 - \beta)k_t^{*\beta}$  and  $\rho_{t+1}^* = A_{t+1}\beta(k_t^*)^{\beta-1}/R_f$ , and satisfy Assumption 8.*
- *Contracts and capital production for given prices are determined according to Proposition 1, i.e.,*

$$(\delta_t^*, R_t^*) = \begin{cases} (\tilde{\delta}(w_t^*, \rho_{t+1}^*), R_f) & \text{if } \frac{1 - \rho_{t+1}^* \frac{g_G}{M}}{2M} \geq -\frac{R(w_t^*, \rho_{t+1}^*, 0) - R_f}{\tilde{\delta}(w_t^*, \rho_{t+1}^*)} \\ (0, R(w_t^*, \rho_{t+1}^*, 0)) & \text{otherwise} \end{cases},$$

where  $\tilde{\delta}(w, \rho)$  is defined in (1) and  $R(w_t^*, \rho_t^*, 0)$  is defined in (3);

$$k_{t+1}^* = \begin{cases} k_s(\rho_{t+1}^*) & \text{if } \frac{1 - \rho_{t+1}^* \frac{g_G}{M}}{2M} \geq -\frac{R(w_t^*, \rho_{t+1}^*, 0) - R_f}{\tilde{\delta}(w_t^*, \rho_{t+1}^*)} \\ k_p(w_t^*, \rho_{t+1}^*) & \text{otherwise} \end{cases},$$

where  $k_s(\rho)$  and  $k_p(w, \rho)$  are given by (6) and (7).

## 4. A Benchmark Case

We now turn to study the existence of equilibrium in a simple case, where we assume the presence of a productivity externality due to the size of the economy. This makes the price of capital constant, while the wage  $w_t$  varies linearly with the capital level  $k_t$ . The reversion mechanism, which is the focus of this paper, is easy to observe in this context. In this case, liquidity changes arise only due to changes in labor income (and not changes in revenues from capital sales), and therefore expectations about future capital level do not play a role. Higher labor income raises the level of the early payment needed for effective screening. In addition, with higher labor income, default by type  $B$  entrepreneurs is less costly for the banks, implying that a lower interest rate is charged in case of pooling. These two forces combined make the pooling contract emerge for a high enough level of capital, the composition of producers subsequently dampening the future capital level. A similar intuition explains why a separating contract appears for a low enough level of capital.

### 4.1. Prices and Equilibrium Contracts

We now consider a positive externality on the production of consumption goods, by assuming  $A_t = K_t^{1-\beta}$ . We then have

$$R_f \rho_t = \beta \text{ and } w_t = (1 - \beta) k_t. \quad (8)$$

Since the rental price of capital is constant,  $k_t$  is the only state variable and expectations do not play any role. We drop the time subscript for this subsection. To keep the notation simple, from now on we also set  $R_f = 1$ .

Drawing on the earlier derivation of equilibrium contracts for given prices, we first obtain equilibrium contracts as a function of the state variable  $k$ . The minimum early payment that accomplishes separation becomes  $\tilde{\delta}(k) = (1 - \beta)k + \beta f_B$  and the maximization problem that determines the equilibrium contracts simplifies to

$$\begin{aligned} \max_{(\delta, R)} \quad & \rho \left( g_G + ((1 - \beta)k + \beta f_G - \delta) \frac{g_G}{M} \right) - (2MR - \delta) \\ \text{s.t.} \quad & R = R(k, \delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} [\delta + (1 - \alpha)\beta (g_B + ((1 - \beta)k + \beta f_B - \delta) \frac{g_B}{M})], & \text{if } \delta < \tilde{\delta}(k) \\ R_f, & \text{if } \delta = \tilde{\delta}(k) \end{cases} \end{aligned}$$

As explained in the previous section, the solution to this problem is given by comparing the slope of the indifference curves (ICS) and the slope that represents the trade-off between  $R$  and  $\delta$  in order to guarantee bank's participation (IPS). Expressions (4) and (5) now become

$$ICS = -\frac{\beta \frac{g_G}{M} - 1}{2M} \text{ and } IPS = -\frac{R(k, 0) - R_f}{\tilde{\delta}(k)}.$$

Proposition 1, which characterized contracts offered in equilibrium, can be now expressed in terms of

$k$ . If  $\frac{1-\beta\frac{g_G}{M}}{2M} \geq -\frac{R(k,0)-R_f}{\tilde{\delta}(k)}$ , then a separating contract  $(\tilde{\delta}(k), R_f)$  is offered. Otherwise, a pooling contract  $(0, R(k, 0))$  is offered.

Figure 3 illustrates the equilibrium contract determination for two different levels of capital, low and high, and clarifies the main intuition for the link between the state of the economy and stringency of lending standards. Note that  $ICS$  is independent of  $k$ , while  $IPS$  increases in  $k$  for two reasons, thus leading to the selection of separating contracts for low levels of  $k$  and pooling contracts for high levels of  $k$ . Intuitively, labor income increases in the state of the economy, which improves the ability of low types to meet the early payment, and thus raises  $\tilde{\delta}(k)$ , the payment that keeps out type  $B$ . Selecting a separating contract is less attractive for type  $G$  when  $k$  is relatively high as it means having relatively little reinvestment into second stage production. In other words, when times are good and liquidity is high for everyone, it is more costly for productive types to reveal their type. In addition,  $R(k, 0)$  decreases with higher income, because a higher collection from defaulting entrepreneurs allows for a lower interest rate to guarantee zero profits for the banks. Selecting a pooling contract thus becomes more attractive for type  $G$  when  $k$  is relatively high, because the cross-subsidy it entails is relatively low.

Note that there is a threshold  $\bar{k}$ , which divides the state space into regions of pooling and separating contracts.

**Lemma 3** Define  $\bar{k} = \left( \frac{(1-\mu)(2MR_f - g_B(1-\alpha)\beta)}{\mu(\beta\frac{g_G}{M}-1) + \beta\frac{g_B}{M}(1-\mu)(1-\alpha)} - \beta f_B \right) / (1-\beta)$ . For  $k > \bar{k}$ , a pooling contract is selected, while for  $k \leq \bar{k}$ , a separating contract is selected.

**Proof.** Follows immediately from  $\bar{k}$  being the unique solution to  $\frac{\beta\frac{g_G}{M}-1}{2M} = \frac{R(k,0)-R_f}{\tilde{\delta}(k)}$ . ■

## 4.2. Dynamic Equilibrium

We now focus on the model dynamics. While we already explained how we propose to rationalize the link between the state of the economy and the stringency of lending standards observed in the data, we now explain how our model can deliver the productivity reversion mechanism. Screening out the bad projects becomes more costly in expansions, which leads to the selection of pooling contracts with no screening at all. Low productivity entrepreneurs enter production along with productive types, the composition effect setting off a recession. This situation, in turn, eventually generates conditions – a low enough level of capital and liquidity – conducive to the emergence of separating contracts that exclude low productivity entrepreneurs from production, thereby increasing aggregate productivity in the capital good sector, and leading to an economic recovery.

We seek the parametric restrictions that give rise to equilibrium paths along which prices satisfy Assumption 8 and we will show existence of cyclical equilibria, which demonstrate the reversion mechanism in the most stark manner.

Total production, characterized in Proposition 1 for given prices, can be determined as a function of  $k$  alone:

$$\begin{aligned} k_s &= \mu \left( f_G + g_G + \beta (f_G - f_B) \frac{g_G}{M} \right), \\ k_p(k) &= \mu^2 \left( f_G + g_G + ((1-\beta)k + \beta f_G) \frac{g_G}{M} \right) + (1-\mu) \mu \left( f_G + g_B + ((1-\beta)k + \beta f_B) \frac{g_B}{M} \right). \end{aligned}$$

Note that  $k_s$  is independent of capital because an additional unit of capital translates into  $1 - \beta$  additional units of labor income, which are paid to the bank before the second stage takes place in order to keep separation viable, and therefore they are not reinvested. However,  $\frac{dk_p(k)}{dk} > 0$ , because a pooling contract involves no early payment and therefore every additional unit of labor income is reinvested, augmenting current capital production. Invoking Lemma 3 we can then derive the transition function for capital as

$$k_{t+1} = \begin{cases} k_s & \text{if } k_t \leq \bar{k} \\ k_p(k_t) & \text{otherwise} \end{cases} . \quad (9)$$

How does  $k_s$  relate to  $k_p(\bar{k})$ ? On the one hand, under separation, all of the productive entrepreneurs (entire measure  $\mu$ ) engage in production. Type  $B$  entrepreneurs do not apply for financing and do not crowd out type  $G$  from getting financed. On the other hand, to make separation possible, the income obtained in the first stage cannot be reinvested, so each type  $G$  entrepreneur produces less. In this paper, we focus on the set of parameters for which the composition effect dominates the per agent production effect at  $\bar{k}$ . We therefore make the following assumption.

**Assumption 2**  $k_p(\bar{k}) < k_s$ .

This implies that as type  $B$  gets pooled into the mix, capital output goes down due to the crowding out of type  $G$ ,<sup>15</sup> despite the fact that more resources are invested in production.

Also, we make an assumption to ensure there is no perpetual growth in this economy. We make sure that for pooling contracts, an extra unit of capital, which translates into an extra  $1 - \beta$  units of input into the second stage of production, results in less than one unit of additional capital.

**Assumption 3**  $\frac{dk_p(k)}{dk} = (1 - \beta) \mu \left( \mu \frac{q_G}{M} + (1 - \mu) \frac{q_B}{M} \right) < 1$ .

In the following proposition we find restrictions on the set of parameters and initial condition  $k_0$  that ensure existence of the certain type of dynamic equilibrium, in particular, the one that exhibits cyclical behavior. Note that we need to ensure that Assumption 8 holds for all admissible  $k$ .

**Proposition 2** *Suppose Assumptions 1, 2, 3 hold and that  $k_p(\bar{k}) < \bar{k} < k_s$ . Consider  $k_{min} := k_p(\bar{k})$  and  $k_{max} := k_s$ , and suppose that Assumption 8 holds for  $\rho = \beta$  and  $w = w(k_{max})$  in (a), (b), (d), (f) and  $w = w(k_{min})$  in (c). Then for any  $k_0 \in [k_{min}, k_{max}]$  there exists a dynamic equilibrium with the capital stock (and output) exhibiting cycles, not necessarily trivial (see Figure 4a).*

**Proof.** See the appendix. ■

For cyclical equilibria,<sup>16</sup> the length of the cycle can be easily calculated from the primitives.

<sup>15</sup>Crowding out of type  $G$  caused by financing of type  $B$  entrepreneurs occurs because there is a limited supply of funds ( $2M\mu$ ). This, however, is not essential for our reversion mechanism. What would suffice is the existence of any technology, which is more productive than type  $B$  technology and financing of type  $B$  causing crowding out of funds away from that technology. [[For exposition purposes, however, we focus on the possibility of crowding out of funds from the most productive entrepreneurs.]]

<sup>16</sup>Note that it is also possible to derive parametric restrictions and bounds on  $k_0$  that guarantee that  $\bar{k} < k_p(\bar{k}) < k_s$  and existence of equilibrium such that the capital stock converges to  $k_{ss} = k_p(\bar{k})$  (Figure 5). Finally, another possible equilibrium behavior is for the capital stock to converge to  $k_s$ . Such behavior would arise for parametric restrictions and bounds on  $k_0$  that guarantee existence of equilibrium and ensure that  $k_p(\bar{k}) < k_s < \bar{k}$ .

**Corollary 1** Consider an economy satisfying  $k_p(\bar{k}) < \bar{k} < k_s$ . If  $n$  is the smallest number such that  $k_p^{(n-1)}(k_s) > \bar{k}$  but  $k_p^n(k_s) < \bar{k}$ , then an economy starting at  $k_0 = k_s$  exhibits cycles of length  $n + 1$ . In each cycle, the capital level declines for the first  $n - 1$  periods and goes up to  $k_s$  in the last period.

**Proof.** See the appendix. ■

We give a numerical example satisfying the constraint of Proposition 2 which generates a non-trivial cycle.

*Example 1.* Parameters are  $\alpha = 0.42$ ,  $\beta = 0.9$ ,  $\mu = 0.64$ ,  $f_G = 2$ ,  $g_G = 2.3$ ,  $f_B = 0$ ,  $g_B = 1.55$ ,  $R_f = M = 1$ . With these parameters,  $\bar{k} = 4.39$ ,  $k_s = 5.4$ ,  $k_p(k_s) = 4.52$ . Because these parameters imply  $k_p(k_s) > \bar{k}$ , we obtain a 4-period cycles (5.4, 4.52, 4.4, 4.39, 5.4, 4.52...).

In our model, an economic expansion leads to the endogenous relaxation of lending standards, which allows for financing of less productive entrepreneurs and consequently negatively impacts future productivity through changing the composition of entrepreneurs. In order to make the comovement of macroeconomic variables more clear, in particular, to clarify that our mechanism can generate default rates that rise before the onset of a recession and peak afterwards, we extended our model in a straight-forward way to include three types of entrepreneurs:  $G$ ,  $M$  and  $B$ . Type  $M$  is more productive than type  $B$  in both stages, but it still chooses to default in equilibrium.

The parametric example given below delivers a three-period cycle characterized by the following dynamics. For low  $k$ , only type  $G$  is financed and hence default rates are zero. The economy grows due to the strong composition effect, and despite the low reinvestment rates of type  $G$  producers. Next period, lending standards are relaxed and allow the less productive type  $M$  to be financed along with type  $G$ . The economy, however, continues to grow as the increase in reinvestment due to laxer lending standards dominates the negative composition effect. Default rates are now positive as the type  $M$  producers choose to default. Next period the economy has reached its peak. The worst type  $B$  is financed along with  $G$  and  $M$ , which leads to the large drop in productivity due to the strong composition effect. Default rates peak after the onset of the recession as both type  $B$  and  $M$  producers choose to default.

*Example 2.* (Figure 4b<sup>17</sup>) Parameters are  $\mu_G = 0.45$ ,  $\mu_M = 0.2$ ,  $\mu_B = 0.35$ ,  $f_G = 2.1$ ,  $f_M = 0.725$ ,  $f_B = 0$ ,  $g_G = g_M = g_B = 2.15$ ,  $\alpha_G = 0.38$ ,  $\alpha_M = 0.83$ ,  $\alpha_B = 0.38$ ,  $\beta = 0.81$ ,  $R_f = M = 1$ . With these parameters, we obtain a 3-period cycle described above (2.87, 2.99, 3.036, 2.87, 2.99, 3.036, ...).

## 5. The General Case

We now consider the case with no externalities, that is  $A_t := A$ . The rental price of capital goods is no longer a constant; it depends on the belief of the the consumption good sector about the capital stock in the next period, which we denote by  $k'$ . Prices are given by  $\rho(k') = A\beta k'^{\beta-1}$  and  $w(k) = A(1 - \beta)k^\beta$ .

Although this introduces complications in the analysis of the model's dynamics, the reversion of the aggregate productivity arises according to the same intuition as discussed before. However, a new interesting phenomenon, an indeterminacy region, appears. For certain levels of capital  $k$ , two possible

<sup>17</sup>In the figure, we mark midpoints of time periods. Recall that default rates occur in the second stage.

forecasts about future capital  $k'$  are self-fulfilling. If every entrepreneur believes that a separating contract will arise yielding high levels of capital holdings by the consumption good sector tomorrow, then currently produced capital will trade at a low price  $\rho(k')$ . This low price, in turn, will induce a separating contract, thus making the belief consistent with the equilibrium outcome. In the same way, if they believe that a pooling contract and hence lower levels of future capital holdings will arise, capital will trade at a high price. This price, in turn, will induce a pooling contract, making the belief consistent.

### 5.1. Prices, Equilibrium Contracts and Consistent Beliefs

As we discussed, the consumption good sector's belief regarding future capital holdings influences the behavior of entrepreneurs and equilibrium outcomes through its effect on the rental price of capital. We refer to  $(k, k')$  as the state of the economy, which determines equilibrium contracts and total production. Moreover, when looking for a dynamic equilibrium we require beliefs  $k'$  to be consistent with actual production.

We can write the minimum level of the early payment that ensures separation  $\tilde{\delta}(w, \rho)$  and the interest rate  $R(w, \rho, 0)$  given in (1) and (3) as functions of  $(k, k')$ ,

$$\tilde{\delta}(k, k') = w(k) + \rho(k')f_B, \quad (10)$$

$$R(k, k', 0) = \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} \left[ (1-\alpha)\rho(k') \left( g_B + (w(k) + \rho(k')f_B) \frac{g_B}{M} \right) \right]. \quad (11)$$

As before, separating contracts emerge if and only if

$$\frac{1 - \rho(k') \frac{g_G}{M}}{2M} \leq - \frac{R(k, k', 0) - R_f}{\tilde{\delta}(k, k')}. \quad (12)$$

Note that as the belief  $k'$  increases, the price of capital  $\rho(k')$  decreases, generating two important effects. First, an increase in the early payment  $\delta$  now hurts type  $G$  less, as the benefits from extra reinvestment (due to the sale of extra units of capital) decrease, and therefore type  $G$ 's indifference curves (with slope  $\frac{1-\rho(k') \frac{g_G}{M}}{2M}$ ) flatten in the  $R - \delta$  plane. Second, the critical slope  $IPS$  ( $-\frac{R(k, k', 0) - R_f}{\tilde{\delta}(k, k')}$ ) steepens, because the minimum early payment that ensures separation declines, and the interest rate charged in case of pooling rises. As we can see, an increase in the belief  $k'$  has the same qualitative effect as a decline in  $k$  does, although for different reasons.

For a fixed  $k$ , there exists a cutoff level of beliefs, defined as the unique solution to (12) rewritten as equality, and denoted by  $\tilde{k}'(k)$ , which characterizes the equilibrium contracts. For beliefs  $k'$  below (above) this cutoff level, pooling (separating) contracts are selected. Hence, capital production is given by

$$\kappa(k, k') = \begin{cases} \kappa_p(k, k') & \text{if } k' < \tilde{k}'(k), \\ \kappa_s(k') & \text{if } k' \geq \tilde{k}'(k) \end{cases} \quad (13)$$

where

$$\kappa_p(k, k') = \mu^2[f_G + g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] + (1 - \mu)\mu[f_B + g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}], \quad (14)$$

$$\kappa_s(k') = \mu[f_G + g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}]. \quad (15)$$

Figure 5 illustrates  $\kappa(k, k')$  as a function of beliefs  $k'$  (for a fixed  $k$ ). It consists of two segments: for low beliefs  $k'$ , pooling arises, and for high beliefs  $k'$ , separation arises. Both segments are downward sloping, as higher  $k'$  reduces reinvestment and hence total capital production. Moreover, for a given  $k'$ ,  $\kappa_p(k, k')$  increases in  $k$  (due to higher reinvestments) and  $\kappa_s(k')$  is independent of  $k$  (as the extra income is used for repayment). Finally, note that as  $k$  increases, a higher belief  $k'$  is needed to induce the selection of a separating contract. In fact, as  $k$  goes up the critical slope *IPS* flattens (as in Section 4), so it takes a higher belief  $k'$  (which implies a lower price  $\rho(k')$  and therefore a steeper *IPS* and flatter indifference curves) to restore indifference between the two contracts for type  $G$  entrepreneurs. These facts are summarized in the following lemma.

**Lemma 4** *The following are true*

$$\frac{\partial \kappa_p(k, k')}{\partial k} \geq 0, \frac{\partial \kappa_p(k, k')}{\partial k'} \leq 0, \frac{d\kappa_s(k')}{dk'} \leq 0, \frac{d\tilde{k}'(k)}{dk} \geq 0.$$

**Proof.** See the appendix. ■

We require that in accordance with the rational expectations hypothesis, equilibrium beliefs  $k'$  are consistent with equilibrium outcomes, i.e.,

$$k' = \kappa(k, k'). \quad (16)$$

The right hand side gives the future level of capital as a function of current capital  $k$  and beliefs about future capital  $k'$ . These beliefs, i.e. the left hand side, must equal the actual level of capital in the next period. We define the consistent beliefs correspondence  $k'(k)$  as the set of solutions to equation (16). We now turn our attention to ensuring that this correspondence is non-empty, which is needed for equilibrium existence.

Note that because  $\kappa_p(k, \cdot)$  and  $\kappa_s(\cdot)$  are single-valued, the consistent belief correspondence can attain at most two values: a belief associated with pooling, and a belief associated with separation. We define these two candidate values for consistent beliefs below.

**Definition 2** *Denote by  $k_s$  the solution to  $\kappa_s(k') = k'$ , and by  $k_p(k)$  the solution (for fixed  $k$ ) to  $\kappa_p(k, k') = k'$  (both exist since  $\kappa_s(\cdot)$  and  $\kappa_p(k, \cdot)$  are decreasing).*

In order for the belief  $k_s$  to be consistent, it must actually induce separation, that is, we must have  $k_s \geq \tilde{k}'(k)$ . Note that if this inequality is satisfied for some  $k$ , it is also satisfied for all smaller  $k$ , because  $\tilde{k}'(\cdot)$  is increasing. The maximum level of  $k$  satisfying this inequality is given by the solution to  $\kappa_s(\tilde{k}'(k)) = \tilde{k}'(k)$ , which we denote by  $k_{hi}$  (See Figure 6).<sup>18</sup> Hence, the belief  $k_s$  is consistent for all

<sup>18</sup>If the solution does not exist, let  $k_{hi} = 0$ .

$k \in [0, k_{hi}]$ .

In order for the belief  $k_p(k)$  to be consistent, it must induce pooling, that is, we must have  $k_p(k) \leq \tilde{k}'(k)$ . We define the minimum level of capital for which  $k_p(k)$  is consistent as  $k_{li} = \inf\{k | k_p(k) \leq \tilde{k}'(k)\}$  (See Figure 7.). If this set is empty,  $k_{li} = +\infty$ .

For an equilibrium to exist, a consistent belief must exist for every capital level  $k$  in the equilibrium path. We already showed that for  $k \leq k_{hi}$ , there is at least one consistent belief ( $k_s$ ). We will now seek the restrictions needed to guarantee the existence of a consistent belief for  $k > k_{hi}$ . We do this in two steps. First, we guarantee that  $k_{li} < k_{hi}$ . Second, we guarantee that  $k_p(k)$  is consistent for all  $k > k_{li}$ .

The next assumption guarantees that  $k_p(k)$  is consistent for  $k = k_{hi}$  and therefore that  $k_{li} \leq k_{hi}$  (see Lemma 5 and Figure 6).

**Assumption 4**  $\tilde{k}'(k_{hi}) > \kappa_p(k_{hi}, \tilde{k}'(k_{hi})) > 0$ .

**Lemma 5** *If Assumption 4 is satisfied, then  $k_{hi} \in \{k | k_p(k) \leq \tilde{k}'(k)\}$  and therefore  $k_{li} \leq k_{hi}$ .*

**Proof.** By Assumption 4,  $\kappa_p(k_{hi}, \tilde{k}'(k_{hi})) < \tilde{k}'(k_{hi})$ . Since  $\kappa_p(k_{hi}, 0^+) > 0$ , the continuity of  $\kappa_p(k_{hi}, \cdot)$  implies the existence of  $k_p(k_{hi}) \in (0, \tilde{k}'(k_{hi}))$ . We then conclude that  $k_{hi} \in \{k | k_p(k) \leq \tilde{k}'(k)\}$ . Recalling the definition of  $k_{li}$ , we have  $k_{li} \leq k_{hi}$ . ■

The next assumption, made parametric in Proposition 3, ensures that  $k_p(k)$  is consistent for  $k \geq k_{li}$  (See Lemma 6).

**Assumption 5**  $\frac{d}{dk}[k_p(k) - \tilde{k}'(k)] \leq 0$  for  $k \geq k_{li}$ .

This assumption is more restrictive than necessary to ensure existence of equilibrium, but it makes the form of the consistent belief correspondence more tractable.

**Lemma 6** *If Assumptions 4 and 5 are satisfied for all  $k \geq k_{li}$ , there exists a consistent pooling belief.*

**Proof.** The belief about pooling is consistent as long as  $k_p(k) - \tilde{k}'(k) \leq 0$ . By Lemma 5,  $k_{li}$  exists. Moreover, this inequality is satisfied for  $k_{li}$  by definition of  $k_{li}$ . Assumption 5 then guarantees the result. ■

Lemma 4 allows us to deduce the shape of the consistent belief correspondence  $k'(k)$ :

$$k'(k) = \begin{cases} k_s & \text{if } k < k_{li} \\ \{k_s, k_p(k)\} & \text{if } k \in [k_{li}, k_{hi}] \\ k_p(k) & \text{if } k > k_{hi} \end{cases}, \quad (17)$$

For low levels of capital, a belief associated with separation is the unique consistent belief. For intermediate levels of capital, a belief associated with separation and a belief associated with pooling are both consistent. We refer to this range of capital levels as the indeterminacy region. Finally, for high levels of capital, a belief associated with pooling is the unique consistent belief (See Figures 8-10.).

The transition function for capital is then

$$k_{t+1} = \begin{cases} k_s & \text{if } k_t < k_{li} \text{ or } (k_t \in [k_{li}, k_{hi}] \text{ and } k'_t = k_s) \\ k_p(k_t) & \text{if } k_t > k_{hi} \text{ or } (k_t \in [k_{li}, k_{hi}] \text{ and } k'_t = k_p(k_t)) \end{cases}. \quad (18)$$

## 5.2. Dynamic Equilibrium

The consistent belief correspondence and the transition function, derived above, allow us to characterize the dynamic behavior of the capital stock, aggregate productivity and total output in the economy. The next assumption, made parametric in Proposition 3, corresponds to Assumption 3 of Section IV, and is sufficient to guarantee that there is no perpetual growth.

**Assumption 6**  $\frac{\partial \kappa_p(k, k')}{\partial k} < 1$ .

Figure 11 illustrates one possible transition correspondence. Assumption 6 together with Lemma 4 imply that  $\frac{dk_p(k)}{dk} \in [0, 1]$ .<sup>19</sup> Assumption 4 implies  $k_p(k_{hi}) < k_s$ .

In the following two propositions, we show the existence of cyclical equilibria with different characteristics. In Proposition 3, cycles always exist, but the selection of beliefs in the indeterminacy region dictates their length and amplitude. In Proposition 4, the selection of beliefs dictates whether the equilibrium exhibits cycles or convergence to a steady state.

**Proposition 3** *Suppose that Assumptions 1 and 4 are satisfied,  $k_p(k_{li}) \leq k_{li} \leq k_s$  and  $k_p(k_{hi}) \leq k_{hi} \leq k_s$ . Moreover, suppose that Assumptions 8, 5, 6 (in the appendix) are satisfied along the equilibrium path.<sup>20</sup> Then for any  $k_0 \in [k_{min}, k_{max}]$  defined in (17), there exists a dynamic equilibrium with the capital stock (and output) exhibiting perpetual cycles regardless of the selection from the correspondence  $k'(\cdot)$ . However, if the selection is “optimistic” (i.e.  $k'(k) = k_s$  for  $k \in [k_{li}, k_{hi}]$ ), the cycles are shorter and smaller in amplitude.*

**Proof.** See the appendix. ■

**Proposition 4** *Suppose the hypothesis of Proposition 3 holds, except that  $k_p(k_{li}) \leq k_{li} \leq k_s$  and  $k_{hi} \geq k_s$ . Then, for any  $k_0 \in [k_{min}, k_{max}]$ , equilibrium exists, and its characteristics depend on the selection from the correspondence  $k'(\cdot)$ . If the selection is optimistic, there are no cycles, both  $k$  and  $k'$  converging to  $k_s$ . However, if the consumption good sector is pessimistic, the economy exhibits perpetual cycles.*

**Proof.** Analogous to the proof of Proposition 3. ■

We now give a numerical example, which exhibits the cyclical behavior presented in Proposition 3.

*Example 2.* Consider parameters  $A = 2.66, \alpha = 0.82, \beta = 0.89, \mu = 0.33, f_G = 2, g_G = 2, f_B = 0, g_B = 1.6, R_f = 1, M = 1$ . With these parameters,  $k_s = 4.06, k_{li} = 2.316, k_{hi} = 2.63, k_p(k_s) = 2.327$ . Because these parameters imply  $k_p(k_s) > k_{li}$ , we obtain a three-period cycle ( $k = 4.06, 2.301, 2.114, 4.06\dots$ ) as long as the selection of beliefs in the indeterminacy region is pessimistic. The dynamics for this case are illustrated in Figure 12. If the selection of beliefs for  $k$  in the indeterminacy region is optimistic, then the two-period cycle emerges ( $k = 4.06, 2.301, 4.06\dots$ ).

<sup>19</sup>Indeed, implicitly differentiating  $\kappa_p(k, k_p(k)) = k_p(k)$  gives  $\frac{dk_p(k)}{dk} = \frac{d\kappa_p(k, k')}{dk} / \left(1 - \frac{d\kappa_p(k, k')}{dk'}\right) \in [0, 1]$ .

<sup>20</sup>Parametric conditions guaranteeing this can be found in the restatement of this proposition in the appendix.

## 6. A Model with Endogenous Savings

In our previous setup, entrepreneurs saved a part of their end of period net worth at the risk-free rate in international markets. Their net worth depended on the current level of capital in the economy, but we assumed that the amount of loanable funds available for financing the young entrepreneurs of each generation was fixed at the level of  $2M\mu$ . In this section, we extend the analysis of the case presented in Section IV by endogenizing the supply of loanable funds. In particular, we require that loans to the young entrepreneurs are financed with the old entrepreneurs' savings. To keep the analysis tractable, we keep the risk-free interest rate fixed as in the case of a small open economy.

Endogenizing the supply of loanable funds not only confirms the possibility of cyclical economic behavior, but it also gives rise to the possibility of sudden drops and slow recoveries, which have been widely documented in the literature. This extended model also generates predictions that are qualitatively consistent with much of the empirical evidence: procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle and default rates lagging after the business cycle.

The intuition for slow expansions and sudden recessions is as follows. For the same reasons as those discussed in Section IV, for low enough levels of capital, separating contracts emerge and only type  $G$  entrepreneurs enter production. However, if the supply of funds is also low, only a small fraction of type  $G$  entrepreneurs is financed. Aggregate productivity and capital production increase from one period to the next, and so do the savings. As savings increase, a higher fraction of type  $G$  entrepreneurs is financed and the recovery continues until capital reaches a high enough level that induces the selection of pooling contracts. As pooling emerges, however, high levels of funds, which increase individual production, may not be enough to offset the decline in productivity generated by the mix of entrepreneurs engaged in production. Hence, in contrast to the previous setup, endogenous supply of funds can give rise to cyclical dynamic equilibria exhibiting slow expansions and sudden recessions. Even though we focus on the case of slow recoveries and sudden drops, which we find particularly interesting, endogenizing the evolution of loanable funds in general allows for a richer cyclical dynamics.

In what follows, we derive the dynamical system describing the evolution of the two state variables (capital and savings). We then analyze their behavior using the phase diagram and illustrate the possibility of cyclical dynamic equilibria exhibiting sudden drops and slow recoveries.

### 6.1. Transition Functions

The funds used to finance the young generation are given by the savings  $S_t$  of the old generation. If funds are not sufficient to finance all applicants, crowding out occurs, with the unfinanced entrepreneurs staying out of capital production and obtaining only  $w(k)$  as labor income. If funds are in excess of applicants' demand, every applicant gets financed. The excess funds are saved in international markets at the risk-free rate  $R_f$ .

It follows from the discussion in Section IV that whether pooling or separating contracts emerge depends on  $k$  (according to Lemma 3) and not on  $S$ . The amount of savings  $S$ , however, is important as it affects the measure of entrepreneurs financed and influences capital and loanable funds in the next

period. In case of a pooling contract, the available funds are used to finance both type  $G$  and type  $B$  entrepreneurs, the total demand for funds is  $2M$ . In case of a separating contract, only type  $G$  is financed, and the total demand for funds equals to  $2M\mu$ . Capital stock in period  $t + 1$ , given the current state variables  $k_t, S_t$ , is then

$$k_{t+1}(S_t, k_t) = \begin{cases} \frac{S_t}{2M} [\mu k_p^G(k_t) + (1 - \mu)k_p^B(k_t)] & \text{if } k_t > \bar{k} \text{ and } S_t \leq 2M \\ \mu k_p^G(k_t) + (1 - \mu)k_p^B(k_t) & \text{if } k_t > \bar{k} \text{ and } S_t > 2M \\ \frac{S_t}{2M} k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t \leq 2M\mu \\ \mu k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t > 2M\mu \end{cases}, \quad (19)$$

where

$$\begin{aligned} k_s^G &= f_G + g_G + \beta(f_G - f_B) \frac{g_G}{M}, \\ k_p^i(k_t) &= f_i + g_i + ((1 - \beta)k_t + \beta f_i) \frac{g_i}{M}, i = G, B, \end{aligned}$$

denote individual capital production of a type  $G$  entrepreneur under separation and of each type under pooling respectively.

Homotheticity of preferences and constancy of  $R_f$  imply that entrepreneurs always save a constant fraction ( $:= \xi$ ) of their end of period net worth. Note that the unfinanced entrepreneurs, whether they are type  $B$  due to exclusion under separation or both types due to crowding out under pooling, save fraction  $\xi$  of the wage income  $(1 - \beta)k$ . We obtain the supply of funds tomorrow as a function of the current state variables:

$$S_{t+1}(S_t, k_t) = \begin{cases} \xi \left[ \frac{S_t}{2M} (\mu W_p^G(k_t) + (1 - \mu)W_p^B(k_t)) + (1 - \frac{S_t}{2M}) (1 - \beta)k_t \right] & \text{if } k_t > \bar{k} \text{ and } S_t \leq 2M \\ \xi [\mu W_p^G(k_t) + (1 - \mu)W_p^B(k_t)] & \text{if } k_t > \bar{k} \text{ and } S_t > 2M \\ \xi \left[ \frac{S_t}{2M} W_s^G + (1 - \frac{S_t}{2M}) (1 - \beta)k_t \right] & \text{if } k_t \leq \bar{k} \text{ and } S_t \leq 2M\mu \\ \xi [\mu W_s^G + (1 - \mu) (1 - \beta)k_t] & \text{if } k_t \leq \bar{k} \text{ and } S_t > 2M\mu \end{cases}, \quad (20)$$

where

$$\begin{aligned} W_s^G &= \beta [g_G + \beta(f_G - f_B) \frac{g_G}{M}] - 2MR_f, \\ W_p^G(k_t) &= \beta \left[ g_G + ((1 - \beta)k_t + \beta f_G) \frac{g_G}{M} \right] - \frac{2MR_f}{\mu} + \frac{1 - \mu}{\mu} (1 - \alpha) \beta \left[ g_B + ((1 - \beta)k_t + \beta f_B) \frac{g_B}{M} \right], \\ W_p^B(k_t) &= \alpha \beta \left[ g_B + ((1 - \beta)k_t + \beta f_B) \frac{g_B}{M} \right] \end{aligned}$$

represent the net worth of type  $G$  entrepreneur under separation, type  $G$  entrepreneur under pooling, and type  $B$  entrepreneur under pooling, respectively.

## 6.2. The Phase Diagram

As usual for the analysis of such a dynamical system, we now divide the state space  $(S_t, k_t)$  into regions where it is possible to sign the changes  $k_{t+1} - k_t$  and  $S_{t+1} - S_t$ . Because only  $k$  matters for whether

separating or pooling contracts emerge, the horizontal line given by  $k = \bar{k}$  splits the state space  $(S_t, k_t)$  into the region of separating contracts (the region below the line) and the region of pooling contracts (the region above the line). Our analysis is carried out separately for these two regions.

First, consider the region of separation ( $k_t \leq \bar{k}$ ). From (19) and (20), we derive that  $k_{t+1} = k_t$  iff

$$k_t = \begin{cases} \frac{S_t}{2M} k_s^G & \text{if } S_t \leq 2M\mu \\ \mu k_s^G & \text{if } S_t > 2M\mu \end{cases}, \quad (21)$$

and  $S_{t+1} = S_t$  iff

$$k_t = \begin{cases} \frac{(2M/\xi - W_G^s)S_t}{(1-\beta)(2M-S_t)} & \text{if } S_t \leq 2M\mu \\ \frac{S_t/\xi - \mu W_G^s}{(1-\beta)(1-\mu)} & \text{if } S_t > 2M\mu \end{cases}. \quad (22)$$

Note that (21) is an upward sloping linear curve for  $S_t \leq 2M\mu$ , at which point it connects to a horizontal line. Moreover,  $k_{t+1} - k_t < 0$  for points above and to the left of this curve, and  $k_{t+1} - k_t > 0$  otherwise. The expression in (22) is a hyperbola in the range  $S_t \leq 2M\mu$ , connecting at  $S_t = 2M\mu$  to an upward sloping line. Moreover,  $S_{t+1} - S_t > 0$  above the resulting curve and  $S_{t+1} - S_t < 0$  below it.

Second, we consider the region of pooling ( $k_t > \bar{k}$ ). From (19) and (20), we obtain that  $k_{t+1} = k_t$  iff

$$k_t = \begin{cases} \frac{A'S_t}{2M-B'S_t} & \text{if } S_t \leq 2M \\ \frac{A'}{1-B'} & \text{if } S_t > 2M \end{cases}, \quad (23)$$

where  $A' = \mu(f_G + g_G + \beta f_G \frac{g_G}{M}) + (1-\mu)(f_B + g_B + \beta f_B \frac{g_B}{M})$  and  $B' = \mu(1-\beta)\frac{g_G}{M} + (1-\mu)(1-\beta)\frac{g_B}{M}$ , and  $S_{t+1} = S_t$  iff

$$k_t = \begin{cases} \frac{(2M/\xi - A)S_t}{BS_t + (2M - S_t)(1-\beta)} & \text{if } S_t \leq 2M \\ \frac{S_t/\xi - A}{B} & \text{if } S_t > 2M \end{cases}, \quad (24)$$

where  $A = \beta \left[ \mu \left( g_G + \frac{\beta f_G g_G}{M} \right) + (1-\mu) \left( g_B + \frac{\beta f_B g_B}{M} \right) \right] - 2MR_f$  and  $B = \beta(1-\beta) \left( \mu \frac{g_G}{M} + (1-\mu) \frac{g_B}{M} \right)$ .

Assuming that  $B' < 1$  to avoid perpetual growth,<sup>21</sup> we have that (23) is a positive hyperbola connecting to a horizontal line at  $S_t = 2M$ . In this case,  $k_{t+1} - k_t < 0$  to the left and above of the curve and  $k_{t+1} - k_t > 0$  below and to the right of the curve. The curve given by (24) in the range  $S_t \leq 2M$  connects to a positively sloping line at  $S_t = 2M$ . We have  $S_{t+1} - S_t > 0$  above and  $S_{t+1} - S_t < 0$  below the resulting curve.

Note that the definition of the dynamic equilibrium (Definition 1) must be modified for the extended environment. In particular, the equilibrium path of  $\{S_t^*\}_{t=0}^\infty$  must be specified. While the prices and contracts are determined as previously,  $k_t^*$  and  $S_t^*$  must now evolve according to (19) and (20). Note that prices must still satisfy Assumption 8. Our goal is to illustrate an example of a cyclical dynamic equilibrium that exhibits long expansions and short but severe recessions.

By setting the restrictions below, we essentially focus on a particular configuration of the phase diagram, one that allows for the possibility of equilibrium paths that exhibit slow expansions and sudden drops.

<sup>21</sup>This is slightly stronger than Assumption 3, as it ensures no perpetual growth with all entrepreneurs entering production under pooling, not only a fraction  $\mu$  of them. If  $B' > 1$ , then the curve is negative in the range  $S_t \in (2M/B', 2M)$ , connecting to a zero-sloping line at  $S_t = 2M$ . In that case,  $k_{t+1} - k_t < 0$  to the left of the convex curve in range  $S_t \in (0, 2M/B')$  and  $k_{t+1} - k_t > 0$  for all the points to its right in the first quadrant, thus allowing for perpetual growth in that region.

**Assumption 7** Suppose that  $\mu k_s^G > \bar{k}$  and  $B' < 1$ . In that case, the solution to (21) at  $k = \bar{k}$  is given by  $S_1 = \frac{2M\bar{k}}{k_s^G}$ , and the solution to (23) at  $k = \bar{k}$  is given by  $S_3 = \frac{2M\bar{k}}{A'+B'\bar{k}}$ . Denote by  $S_2$  and  $S_4$  the respective solutions to (22) and (24) at  $k = \bar{k}$ . Suppose that  $S_4 < S_1 < \min\{S_2, S_3\}$ .

Assuming that  $S_1 < S_3$  is equivalent to the parametric restriction we made in Assumption 2. Assuming  $S_4 < S_1$  ensures that the equilibrium path does not converge to a steady state with separating contracts, while  $S_1 < S_2$  assumes away the convergence to a steady state with pooling contracts.

We find the parametric example satisfying Assumption 7 that gives rise to the cyclical dynamic equilibrium exhibiting sudden drops and slow recoveries.

*Example 3.* Suppose that  $\xi = 0.75$ ,  $\alpha = 0.44$ ,  $\beta = 0.975$ ,  $\mu = 0.26$ ,  $f_G = 2.41$ ,  $g_G = 1.83$ ,  $f_B = 0.77$ ,  $g_B = 1.825$ ,  $R_f = 1$ ,  $M = 1$ . There exists a cyclical dynamic equilibrium for  $(S_0, k_0) = (0.4, 1.2)$  exhibiting 11-period cycles with slow recoveries and sudden drops ( $s_t = 0.35, 0.362, 0.374, 0.385, 0.397, 0.41, 0.422, 0.435, 0.448, 0.46, \dots 0.35\dots$  and  $k_t = 1.197, 1.26, 1.29, 1.34, 1.38, 1.42, 1.46, 1.51, 1.55, 1.6003, \dots 1.197\dots$ ). Capital and supply of funds rise slowly and drop suddenly. The drop occurs immediately upon  $k$  exceeding  $\bar{k}$ . (See Figures 13 a and b.)

In the example given above, capital and supply of funds are low at the trough. Separating contracts emerges, but as the demand for funds ( $= \mu 2M$ ) exceeds the supply, even type G entrepreneurs are rationed. Aggregate productivity, capital and savings rise. With more loanable funds available more type G entrepreneurs enter production and the expansion continues. Because the expansion is very slow, the peak level of capital is just slightly above  $\bar{k} = 1.6$ . At the peak, capital is high enough to warrant pooling. However, the demand for funds ( $= 2M$ ) still exceeds the supply, and there is crowding out of type G entrepreneurs by type B entrepreneurs, thus setting off a recession.

Because we construct the proposed equilibrium path using the appropriate contract determination and the appropriate evolution of capital and savings, to prove that the resulting paths comprise the dynamic equilibrium, it remains to check that the implied prices satisfy the conditions of Assumption 8. These can be easily checked using the equilibrium price definitions and the provided values of  $\{k_t^*\}$ .

In the example economy, capital, savings, investment, loanable funds, output, cash flows and net worth move together. These variables are procyclical. Moreover, because of procyclicality of cash flows (labor income and revenues from capital sales after the first stage), the reliance on banks for financing of second stage production (and hence overall production) declines during the expansions and rises at troughs. Finally, the default rates are highest following the peak. Hence, our extended model generates predictions that are qualitatively consistent with much of the empirical evidence.

## 7. Conclusions

Recent empirical work has suggested that the systematic behavior of lending standards over the business cycle, with laxer standards applied during expansions and tighter standards applied during recessions is responsible for the reversion of trends in aggregate productivity. In this paper, we build a model that can rationalize the emergence of substandard lending (tight lending) during expansions (recessions), entry (exclusion) of the low productivity producers and hence a rise in default rates and reversal of aggregate productivity.

We model fully rational entrepreneurs of privately known types which must seek financing from a competitive lending sector. Screening is done through an early payment on the loan. The equilibrium contract is found as the best for the productive entrepreneurs subject to zero lenders' profits.

As the (endogenously evolving) productivity rises, liquidity is high for all types of producers, allowing even the unproductive type to meet the early payments on the loan. The early payment required to accomplish screening out the unproductive types thus rises. Because the early payment hurts productive entrepreneurs by lowering their investments, a separating contract menu becomes less attractive for them. Moreover, a pooling menu (with zero early payments) become more attractive, as it calls for a lower cross-subsidy in periods with high liquidity. Hence, low productivity entrepreneurs enter production along with productive types, the composition effect setting off a recession. The opposite happens in periods with low productivity. Thus, the aggregate productivity evolves due to the composition of types of projects being financed and implemented.

Our model delivers the empirical observation that default rates begin rising prior to the onset of recessions. When the supply of loanable funds is endogenized, we find the possibility of much richer cyclical dynamics and predictions that are qualitatively consistent with much of the empirical evidence: procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle, and finally, the possibility of slow recoveries and abrupt and severe recessions.

Future research can benefit from embedding our productivity reversion mechanism within the general equilibrium model with infinitely lived agents and assessing its importance quantitatively.

## 8. Appendix

### Credit Line as the Alternative Form of Contracts

Identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with the credit limit  $2M - \delta$  and interest rate  $\frac{2MR - \delta}{2M - \delta}$ .

In the first stage the entrepreneur borrows  $M$  and starts production (he does not borrow more since it would not be productive to do so). In the second stage he taps the credit line for the remaining  $M - \delta$ , which could be complemented with his own savings, so his disposable income is  $M - \delta + \rho f_i + w$ , exactly as in the paper. Finally, he would have to repay  $2MR - \delta$  in the last stage, so the effective interest rate is  $\frac{2MR - \delta}{2M - \delta}$ .

Note that in this case the only requirement is that the bank can monitor that production is started in the second stage, which in the case of a bad entrepreneur requires he puts at least  $\delta$  extra towards the project.

#### Assumption for Lemma 1

**Assumption 8** Suppose the following conditions hold for prices  $w, \rho$ :

- a).  $\frac{\rho g_B}{M} > 1$ .
- b).  $w + \rho f_B + (1 - \alpha)\rho g_B \leq 2MR_f$ .
- c). If

$$\mu\left((1 - \alpha)\frac{\rho g_G}{M} - 1\right) + (1 - \mu)\left((1 - \alpha)\frac{\rho g_B}{M} - 1\right) \leq 0, \quad (25)$$

then

$$2MR_f \leq (1 - \alpha)\rho\left[\mu\left[g_G + (w + \rho f_G)\frac{g_G}{M}\right] + (1 - \mu)\left[g_B + (w + \rho f_B)\frac{g_B}{M}\right]\right]. \quad (26)$$

If (25) fails, then

$$2MR_f \leq \mu\left[(1 - \alpha)\rho\left[g_G + \rho(f_G - f_B)\frac{g_G}{M}\right] + \rho f_B + w\right] + (1 - \mu)\left[\rho f_B + w + (1 - \alpha)\rho g_B\right]. \quad (27)$$

- d).  $\alpha\rho g_B \geq w$ .
- e).  $1 > (1 - \alpha)\frac{\rho g_B}{M}$ .
- f).  $(1 - \alpha)\rho g_B + w + \rho f_B < 2MR_f$ .

#### Proof of Lemma 1

► By Assumption 1, it is enough to prove that type  $B$  is willing to reinvest. One extra unit of consumption good reinvested into the second stage of production yields  $g_B/M$  units of capital, generating  $\rho g_B/M$  units of the consumption good. Therefore, for reinvestment to be profitable, we must have  $\rho g_B/M > 1$ , which is true by condition (a).

► We now show that type  $B$  enters and defaults whenever offered a contract with  $\delta_B < \tilde{\delta}(w, \rho)$  and  $R_B \geq R_f$ . First, note that repaying is dominated by defaulting whenever

$$\alpha\rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] \geq \rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] - (2MR_B - \delta_B).$$

Noting that the most restrictive case is for  $R_B = R_f$  and  $\delta_B = \tilde{\delta}(w, \rho)$  (this last fact is due to condition (e)), the above inequality is implied by

$$\begin{aligned} \alpha\rho[g_B + (w + \rho f_B - \tilde{\delta}(w, \rho))\frac{g_B}{M}] &\geq \rho[g_B + (w + \rho f_B - \tilde{\delta}(w, \rho))\frac{g_B}{M}] - (2MR_f - \tilde{\delta}(w, \rho)), \text{ i.e.,} \\ \alpha\rho g_B &\geq \rho g_B - 2MR_f + w + \rho f_B, \text{ i.e.,} \\ w + \rho f_B + (1 - \alpha)\rho g_B &\leq 2MR_f, \end{aligned}$$

which is true by condition (b).

Second, note that type  $B$  prefers to enter (and default) rather than reject the contract whenever for any  $\delta_B < \rho f_B + w$ ,

$$\alpha\rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] \geq w,$$

which is implied by condition (d).

► Fix an arbitrary  $\delta_B < \tilde{\delta}(w, \rho)$ . We want to show that there exists an  $\hat{R}$  such that a pooling menu  $((\delta_B, \hat{R}), (\delta_B, \hat{R}))$  generates nonnegative profits. Note that any  $R > R_f$  would induce type  $B$  to enter and default (as shown above). Consider an  $\hat{R}$  constructed to yield exactly a zero profit to the bank under the assumption that type  $G$  enters and repays:<sup>22</sup>

$$\hat{R} = R(w, \rho, \delta_B) = \frac{R_f}{\mu} - \frac{(1 - \mu)}{2M\mu} \left[ \delta_B + (1 - \alpha)\rho \left( g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M} \right) \right]. \quad (28)$$

Next we will show that  $\hat{R} > R_f$ , i.e. that the contract  $(\delta_B, \hat{R})$  requires cross-subsidization from type  $G$ . Note that this would be implied by

$$(1 - \alpha)\rho \left[ g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M} \right] + \delta_B < 2MR_f, \quad (29)$$

i.e. if the bank collected less from defaulting type  $B$  than the opportunity cost of funds. The derivative of the left hand side with respect to  $\delta$  is given by  $1 - (1 - \alpha)\rho g_B/M$ , which is positive by condition (e). Hence, if (29) holds for  $\delta_B = \tilde{\delta}(w, \rho)$ , which is guaranteed by condition (f), it holds for all  $\delta_B < \tilde{\delta}(w, \rho)$ . So, we have  $\hat{R} = R(w, \rho, \delta_B) > R_f$ .

In order to prove that  $((\delta_B, R(w, \rho, \delta_B)), (\delta_B, R(w, \rho, \delta_B)))$  is in  $F$ , it remains to show that type  $G$  indeed enters and repays.

<sup>22</sup>Under such pooling contract  $(R, \delta_B)$ , both types enter, type  $G$  repaying in full while type  $B$  defaulting. Because types are unobservable and only  $2M\mu$  can be loaned out, a measure  $\mu^2$  of entrepreneurs of type  $G$  and a measure  $(1 - \mu)\mu$  of type  $B$  are financed. Hence, the zero profit condition is given by  $\mu^2 2MR + (1 - \mu)\mu \left[ \delta_B + (1 - \alpha)\rho \left( g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M} \right) \right] = \mu 2MR_f$ , which implicitly defines  $R(w, \rho, \delta_B)$  given in (28).

First, note that type  $G$  prefers repaying to defaulting whenever

$$\begin{aligned}
\alpha\rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] &\leq \rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] - (2MR(w, \rho, \delta_B) - \delta_B), \text{ i.e.,} \\
(1 - \alpha)\rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] &\geq 2MR(w, \rho, \delta_B) - \delta_B, \text{ i.e.,} \\
(1 - \alpha)\rho[g_G + (w + \rho f_G)\frac{g_G}{M}] &\geq (1 - \alpha)\rho\delta_B\frac{g_G}{M} + 2MR(w, \rho, \delta_B) - \delta_B.
\end{aligned} \tag{30}$$

Because the derivative of the right hand side with respect to  $\delta_B$  is given by

$$\mu((1 - \alpha)\frac{\rho g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho g_B}{M} - 1),$$

there are two possible cases.

*Case 1:* Suppose  $\mu((1 - \alpha)\frac{\rho g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho g_B}{M} - 1) \leq 0$ . Then inequality (30) is most difficult to satisfy for  $\delta = 0$  and the corresponding  $R(w, \rho, 0)$  defined above, in which case the inequality simplifies to

$$2MR_f \leq (1 - \alpha)\rho \left[ \mu[g_G + (w + \rho f_G)\frac{g_G}{M}] + (1 - \mu)[g_B + (w + \rho f_B)\frac{g_B}{M}] \right].$$

This is implied by (26).

*Case 2:* Suppose  $\mu((1 - \alpha)\frac{\rho g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho g_B}{M} - 1) \geq 0$ . Then inequality (30) is most difficult to satisfy for  $\delta \rightarrow \tilde{\delta}$  and the corresponding  $R(w, \rho, \delta)$ , in which case the inequality simplifies to

$$2MR_f \leq \mu \left[ (1 - \alpha)\rho[g_G + \beta(f_G - f_B)\frac{g_G}{M}] + \rho f_B + w \right] + (1 - \mu) [\rho f_B + w + (1 - \alpha)\beta g_B].$$

This is implied by (27). Hence, condition (c) implies that  $G$  prefers to repay in full if he enters the pooling contract constructed here.

Next, we show that type  $G$  enters this contract. As already shown, type  $G$  prefers repaying to defaulting under the pooling contract. Since his utility of default is larger than type  $B$ 's utility of default (due to Assumption 1), and since type  $B$  prefers entering and defaulting rather than working for  $w$ , we conclude without extra assumptions that type  $G$  enters the pooling contract (and repays).

► We show that  $((\tilde{\delta}(w, \rho), R_f), (\tilde{\delta}(w, \rho), R_f))$  is in  $F$ . Type  $B$  cannot afford the contract and rejects it. From the previous part, and by continuity, we know that type  $G$  decides to enter and repay when offered a contract  $(\tilde{\delta}(w, \rho), R(w, \rho, \tilde{\delta}(w, \rho)))$ . Replacing the interest rate by  $R_f < R(w, \rho, \tilde{\delta}(w, \rho))$  makes the repayment option more attractive without changing the default or non-entry options, hence type  $G$  will enter and repay in full when offered  $(\tilde{\delta}(w, \rho), R_f)$ .

## Proof of Lemma 2

From Lemma 1 and the zero profit condition, we know that if  $\delta_B < \tilde{\delta}(w, \rho)$ , the only possible menus are in the set  $\{((\delta_B, R_B), (\delta_G, R_G)) | \delta_G \geq \delta_B, R_B \geq \hat{R}\}$ . WLOG, we can consider  $R_B = \hat{R}$ , since the interest rate for type  $B$  entrepreneurs is irrelevant for both the bank and the entrepreneurs themselves. Moreover, any menu  $((\delta_B, \hat{R}), (\delta_G, \hat{R}))$  with  $\delta_G > \delta_B$  is strictly dominated, for type  $G$  entrepreneurs, by  $((\delta_B, \hat{R}), (\delta_B, \hat{R}))$ , therefore it is never offered in equilibrium.

## Proof of Proposition 2

Consider  $\{k_t^*\}_{t \geq 0}$  defined inductively as  $k_0^* = k_0$ , and for a given  $k_t^*$ , define  $k_{t+1}^*$  as in (9). Let  $k_{min}^* := \inf_t k_t^*$  and  $k_{max}^* := \sup_t k_t^*$ . Define a sequence of beliefs  $\{k_t'^*\}_{t \geq 0}$  by setting  $k_t'^* = k_{t+1}^* \forall t$ . Define price sequences by  $w_t^* = (1 - \beta)k_t^*$  and  $\rho_t^* = \beta$ . Given these prices, define  $(R_t^*, \delta_t^*)$  as in Proposition 1. It remains to check that conditions of Assumption 8 are satisfied for  $\rho_t^*$  and  $w_t^*$  for all  $t$ . Noting that  $w(k)$  is increasing in  $k$ , the hypothesis guarantees exactly this as long as  $k_{min}^* \geq k_p(\bar{k})$  and  $k_{max}^* \leq k_s$ . We prove this by showing inductively that the entire sequence  $\{k_t^*\}_{t \geq 0}$  lies within  $[k_p(\bar{k}), k_s]$ .

We have  $k_p(\bar{k}) \leq k_0^* \leq k_s$  by hypothesis. Suppose that  $k_p(\bar{k}) \leq k_t^* \leq k_s$ . Then  $k_{t+1}^* \geq \min\{k_s, k_p(k_t^*)\} \geq \min\{k_s, k_p(\bar{k})\} \geq k_p(\bar{k})$ , where the second inequality is due to  $\frac{dk_p(k)}{dk} > 0$  and the last one is due to the hypothesis that  $k_p(\bar{k}) \leq k_s$ . Also,  $k_{t+1}^* \leq \max\{k_s, k_p(k_t^*)\} \leq \max\{k_s, k_t^*\} \leq k_s$  where the second inequality is implied by Assumption 3 and hypothesis  $k_p(\bar{k}) \leq k_t^*$ , while the last inequality is due to hypothesis that  $k_t^* \leq k_s$ .

Now, to see the existence of cycles, consider  $k_0 < \bar{k}$  (the other case is analogous). Then  $k_1 = k_s > \bar{k}$ . It remains to show that  $\exists N$  such that  $k_p^N(k_1) < \bar{k}$ . Since  $k_p(k)$  is a linear function, we can write  $k_p(k) = a + bk$ , with  $b < 1$  by Assumption 3. This, together with Assumption 2, implies that  $k_1 - k_p(k) > 0$ . Then  $k_n - k_{n+1} = b^n [(1 - b)k_1 - a] = b^n [k_1 - k_p(k_1)] \rightarrow 0^+$ . Since by hypothesis  $\bar{k} - k_p(\bar{k}) > 0$ ,  $\exists N$  such that  $k_p^N(k_1) < \bar{k}$ . Then  $k_{N+1} = k_s$  and the cycle repeats itself.

## Proof of Corollary 1

We know that  $k_0 = k_s > \bar{k}$  and therefore  $k_1 = k_p(k_s)$ . For all  $m \leq n$  we have then  $k_m = k_p^{(m)}(k_s)$  (since by hypothesis  $k_p^{(m-1)}(k_s) > \bar{k}$ ). Since, also by hypothesis,  $k_m > k_s$ , we have that  $k_{m+1} = k_s$  and the result follows.

## Proof of Lemma 4

The first four results are obvious from straightforward differentiation. We obtain  $\frac{d\tilde{k}'(k)}{dk} \geq 0$  by differentiating equation (12) implicitly:

$$-\frac{\rho'(k') \frac{q_G}{M} \frac{d\tilde{k}'_k}{dk}}{2M} = - \left[ \frac{\left( \frac{dR(k, k', 0)}{dk'} \frac{d\tilde{k}'(k)}{dk} + \frac{dR(k, k', 0)}{dk} \right) \delta_{k, k'} - R(k, k', 0) \left( w'(k) + \rho'(k') f_B \frac{d\tilde{k}'(k)}{dk} \right)}{\delta_{k, k'}^2} \right],$$

$$\frac{w'(k) R(k, k', 0)}{\delta_{k, k'}^2} - \frac{\frac{dR(k, k', 0)}{dk}}{\delta_{k, k'}} = \frac{d\tilde{k}'_k}{dk} \left[ -\frac{\rho'(k') \frac{q_G}{M}}{2M} + \frac{\frac{dR(k, k', 0)}{dk'}}{\delta_{k, k'}} - \frac{\rho'(k') f_B R(k, k', 0)}{\delta_{k, k'}^2} \right].$$

Because  $\rho'(k') < 0$ ,  $\frac{dR(k, k', 0)}{dk'} > 0$  and  $\frac{dR(k, k', 0)}{dk} < 0$ , the result follows directly.

## Proof of Proposition 3

We first make parametric assumptions that guarantee that along the cycle, the assumptions hold. For that, consider  $k_{min} := k'_{min} := k_p(k_{li})$  and  $k_{max} := k'_{max} = k_s$  and suppose that Assumption 8 holds for  $\rho = \rho(k'_{max})$  in (a),  $\rho = \rho(k'_{min})$  and  $w = w(k_{max})$  in (b),  $\rho = \rho(k_p(k_{max}))$  and  $w = w(k_{max})$

in (d),  $\rho = \rho(k'_{min})$  in (e) and (f). For (c), assume that if (25) is satisfied for  $\rho = \rho(k'_{min})$ , then (26) holds for  $\rho = \rho(k_p(k_{max}))$  and  $w = w(k_{min})$ . If (25) fails for  $\rho = \rho(k'_{max})$ , then (27) holds for  $\rho = \rho(k_p(k_{max}))$  and  $w = w(k_{min})$ . If neither is true, then both (26) and (27) hold for  $\rho = \rho(k_p(k_{max}))$  and  $w = w(k_{min})$ . Finally, suppose that  $\max_{[k_{li}, k_{max}]} \frac{d}{dk} [k_p(k) - \tilde{k}'(k)] \leq 0$  (for Assumption 5 to hold) and  $[\mu^2 \frac{gG}{M} + (1 - \mu)\mu \frac{gB}{M}] w'(k_{max}) < 1$  (for Assumption 6 to hold).

Now, consider  $\{k_t^*\}_{t \geq 0}$  defined inductively as  $k_0^* = k_0$ ,  $k_{t+1}^* \in k'(k_t^*)$  according to (17). Let  $k_{min}^* := \inf_t k_t^*$  and  $k_{max}^* := \sup_t k_t^*$  and define a sequence of beliefs by setting  $k_t'^* = k_{t+1}^* \forall t$ . Define price sequences by  $w_t^* = A(1 - \beta)k_t^{*\beta}$  and  $\rho_t^* = A\beta k_{t+1}^{*\beta-1}$ . Given these prices, define  $(P_t^*, \delta_t^*)$  as in Proposition 1.

It remains to check that conditions of Assumption 8 are satisfied for  $\rho_t^*$  and  $w_t^*$  for all  $t$ . Noting that  $w(k)$  is increasing in  $k$  and  $\rho(k')$  is decreasing in  $k'$ , the hypothesis established at the beginning of the proof guarantees exactly this as long as  $k_{min}^* \geq k_p(k_{li})$  and  $k_{max}^* \leq k_s$ . We prove this by showing inductively that the entire sequence  $\{k_t^*\}_{t \geq 0}$  lies within  $[k_p(k_{li}), k_s]$ .

It is true that  $k_p(k_{li}) \leq k_0^* \leq k_s$  by hypothesis. Suppose that  $k_p(k_{li}) \leq k_t^* \leq k_s$ . Then  $k_{t+1}^* \geq \min\{k_s, k_p(k_t^*)\} \geq \min\{k_s, k_p(k_{li})\} = k_p(k_{li})$ , where the second inequality is from  $\frac{dk_p(k)}{dk} > 0$  (Lemma 3) and the equality is due to the hypothesis that  $k_p(k_{li}) \leq k_s$ . Moreover,  $k_{t+1}^* \leq \max\{k_s, k_p(k_t^*)\} \leq \max\{k_s, k_t^*\} \leq k_s$ , where the second inequality is due to  $\frac{dk_p(k)}{dk} \leq 1$  (Lemma 3 and Assumption 7) and hypothesis  $k_p(k_{li}) \leq k_{li}$  and the last inequality is due to the hypothesis.

The existence of cycles is analogous to the proof of Proposition 2. To see that the length of cycles depends on the selection from the beliefs correspondence, consider two sequences  $\{k_t^*\}$ , and  $\{k_t^{**}\}$ , with a pessimistic and an optimistic selection of beliefs respectively. Suppose that for some  $t$ ,  $k_{t-1}^*, k_{t-1}^{**} \leq k_{li}$  (the other cases are identical). Then  $k_t^* = k_t^{**} = k_s$ , and  $k_{t+m^*+1}^* = k_s$ , where  $m^* = \min\{r : k_p^{(r)}(k_s) < k_{li}\}$ , and  $k_{t+m^{**}+1}^{**} = k_s$ , where  $m^{**} = \min\{r : k_p^{(r)}(k_s) < k_{hi}\}$ . Since  $k_{li} < k_{hi}$  and the function  $k_p^{(\cdot)}(k_s)$  is decreasing, we have that  $m^* \geq m^{**}$ . Finally, since  $k_p(\cdot)$  is increasing and below the 45 degree line, the cycles in the sequence  $\{k_t^*\}$  are smaller in amplitude than their counterparts in the sequence  $\{k_t^{**}\}$ .

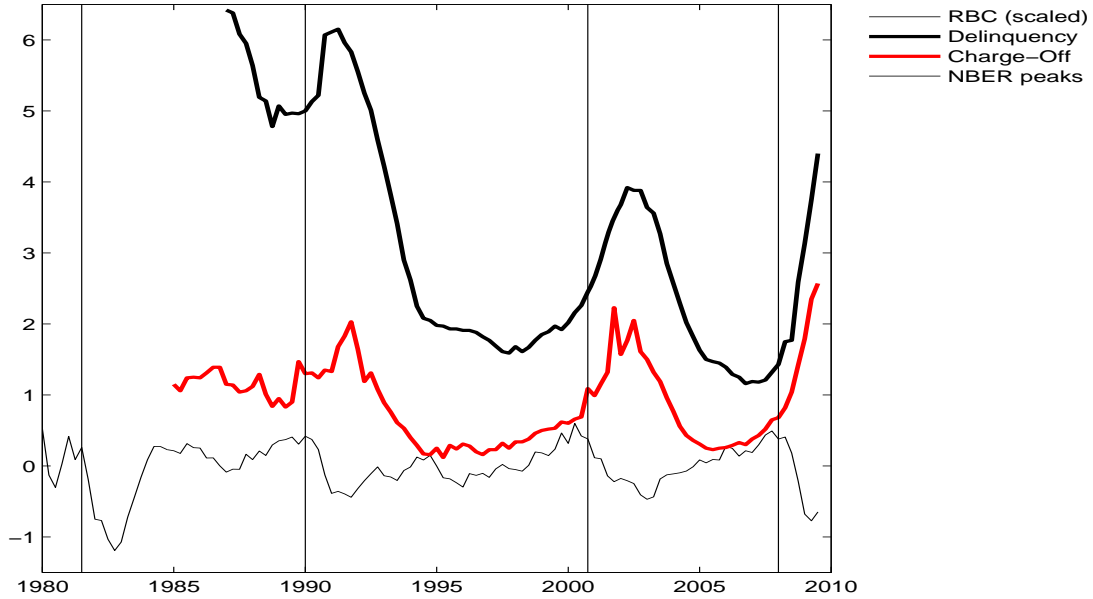
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FIGURE 1A.

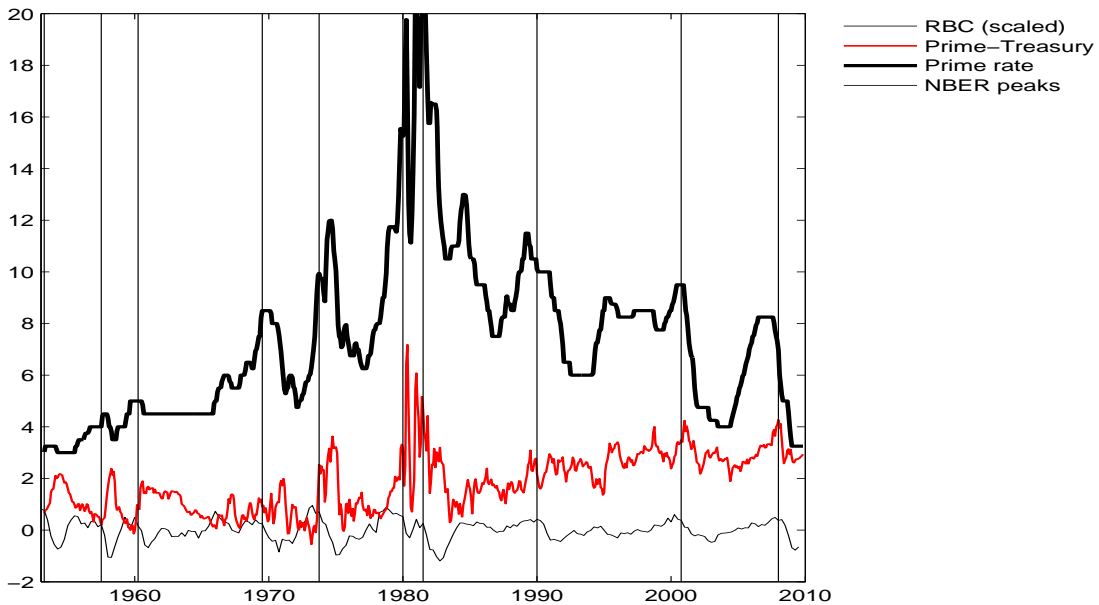
Real Business Cycles, Delinquency and Charge-Off Rates on C&I Loans.



Charge-offs, which are the value of loans removed from the books and charged against loss reserves, are measured net of recoveries as a percentage of average loans and annualized. Delinquency and default rates begin rising prior to the business cycle peak, i.e. during expansions, and peak in recessions. These rates bottom out following a business cycle trough.

FIGURE 1B.

Real Business Cycles, Prime Loan Rate, and Prime - Riskless Rate.



Prime Rate (rate charged to the most credible business borrowers) and Prime- $R_f$  rise prior to the onsets of recessions.

FIGURE 2.

Equilibrium Contract Determination (for given  $w$  and  $\rho$ ).

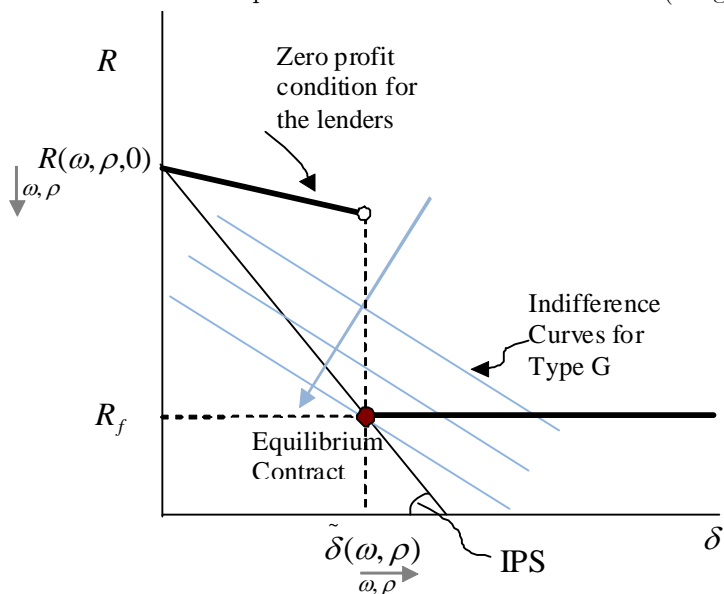


FIGURE 3.

Simplified Model. Equilibrium Contract Determination, 2 states,  $k_H > k_L$ .

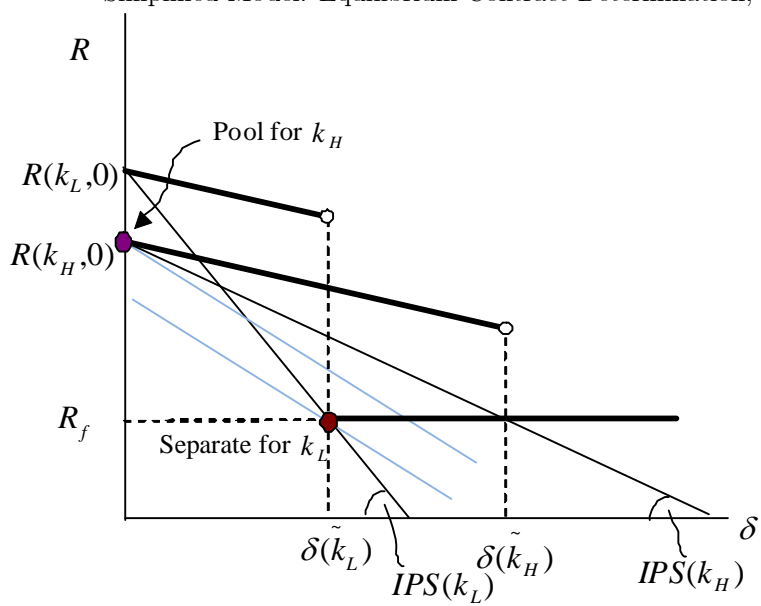


FIGURE 4A.

Transition Function  $k_{t+1}(k_t)$  in the Benchmark Case (Example 1)

The case of endogenous cycles:  $k_P(\bar{k}) < \bar{k} < k_S$ .

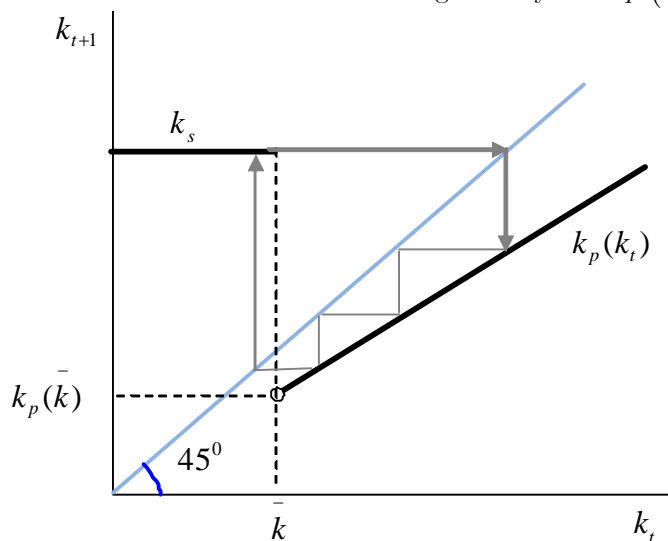


Figure 4B.

Transition Function  $k_{t+1}(k_t)$  and Cycles in the Benchmark Economy with 3 types (Example 2)

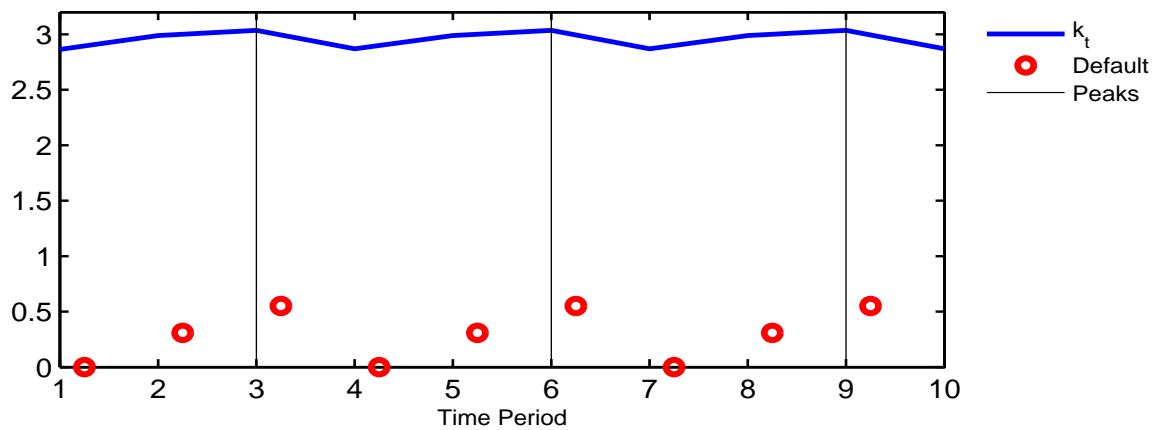
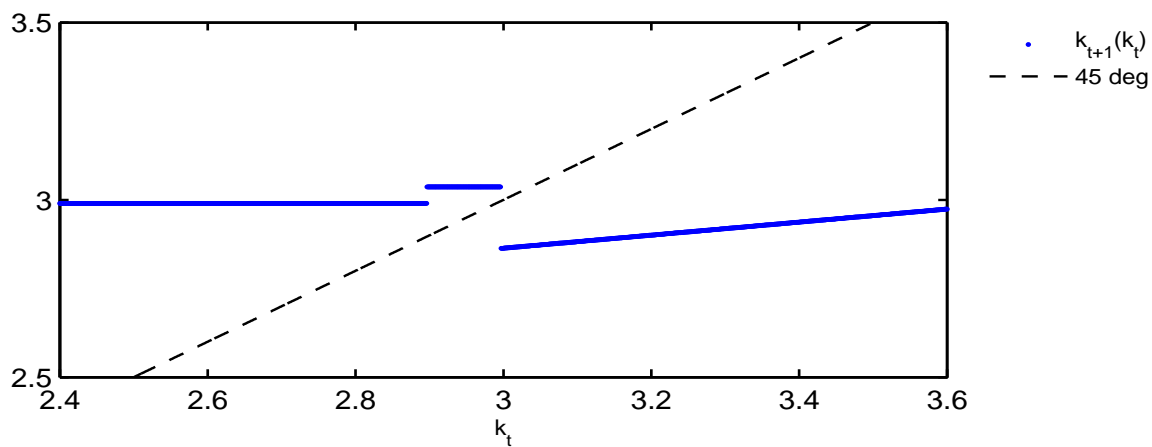


FIGURE 5.

Model with No Externalities. Illustration of  $\kappa(k, k')$ , for a fixed  $k$ ,

$$\tilde{k}'(k) \text{ is defined as a solution to } \frac{1-\rho(k')\frac{gG}{M}}{2M} = -\frac{R(k, k', 0) - R_f}{\delta(k, k')}.$$

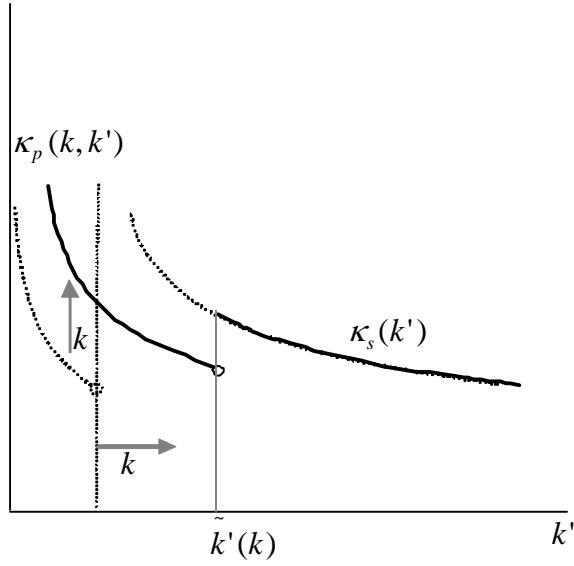


FIGURE 6.

Model with No Externalities. Illustration of  $k_{hi}$  and Assumption 4:  $\tilde{k}'(k_{hi}) > \kappa_p(k_{hi}, \tilde{k}'(k_{hi}))$ .

$k_{hi}$  is the maximum capital that allows for separation, it is given by the solution to  $\kappa_s(\tilde{k}'(k)) = \tilde{k}'(k)$ .

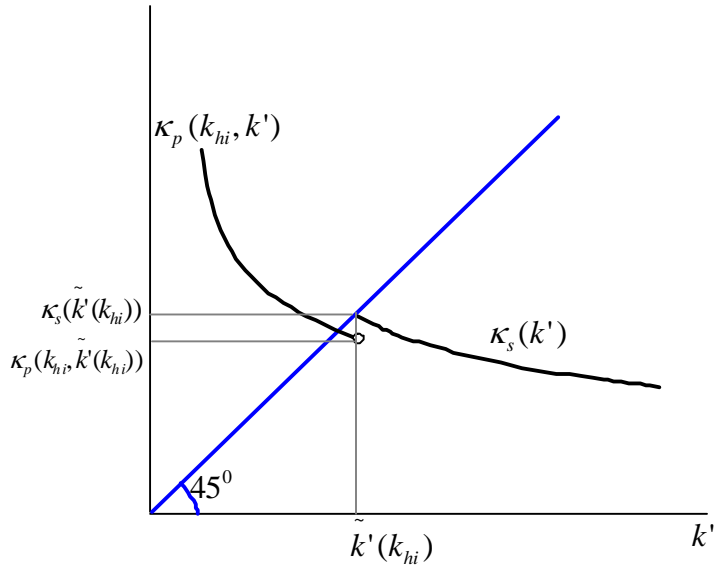


FIGURE 7.

Illustration of  $k_{li}$ .

$k_{li}$  is the minimum capital that allows for pooling. If it exists, it solves  $\kappa_p(k, \tilde{k}'(k)) = \tilde{k}(k)$ .

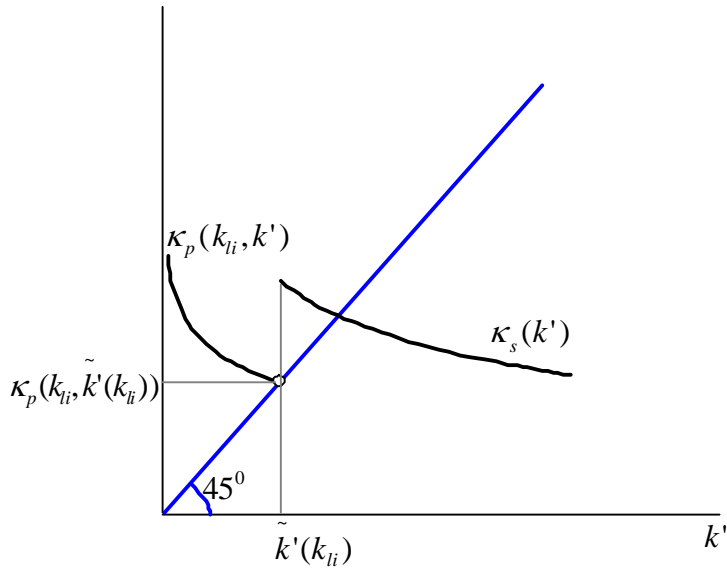


FIGURE 8.

Model with No Externalities. Determination of  $k'(k)$ , for some capital level  $k < k_{li}$ .

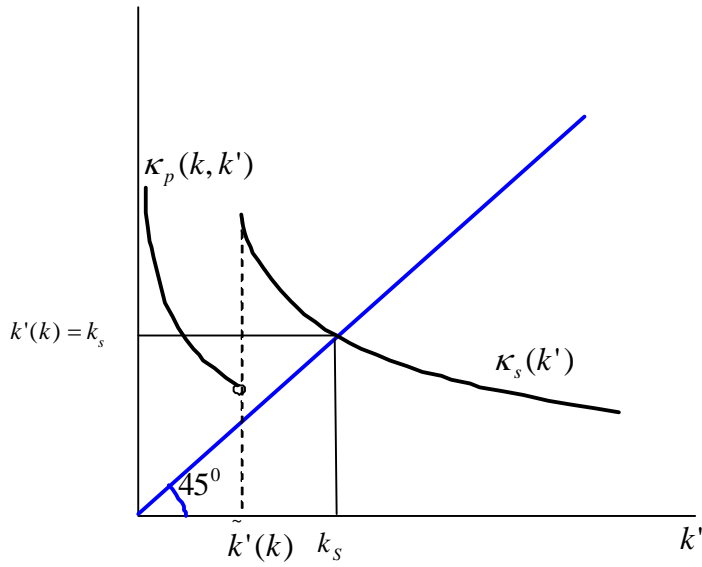


FIGURE 9.

Model with No Externalities. Determination of  $k'(k)$ , for some capital level  $k \in [k_{li}, k_{hi}]$ .

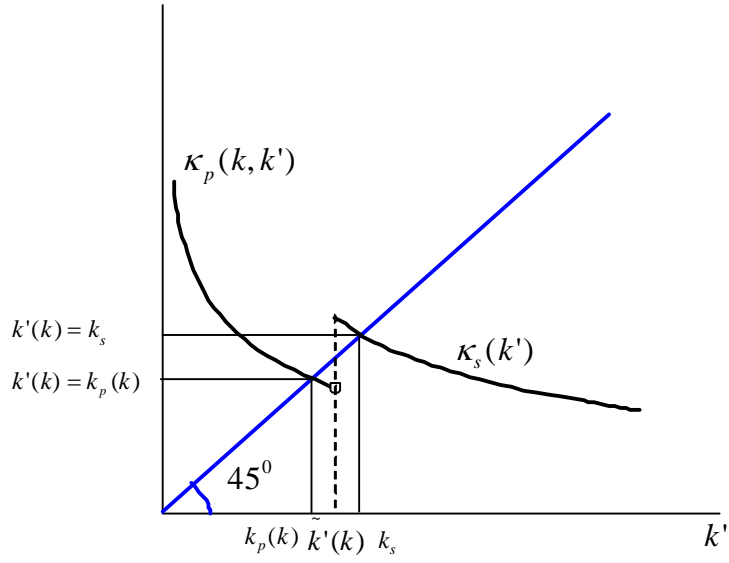


FIGURE 10.

Model with No Externalities. Determination of  $k'(k)$ , for some capital level  $k > k_{hi}$ .

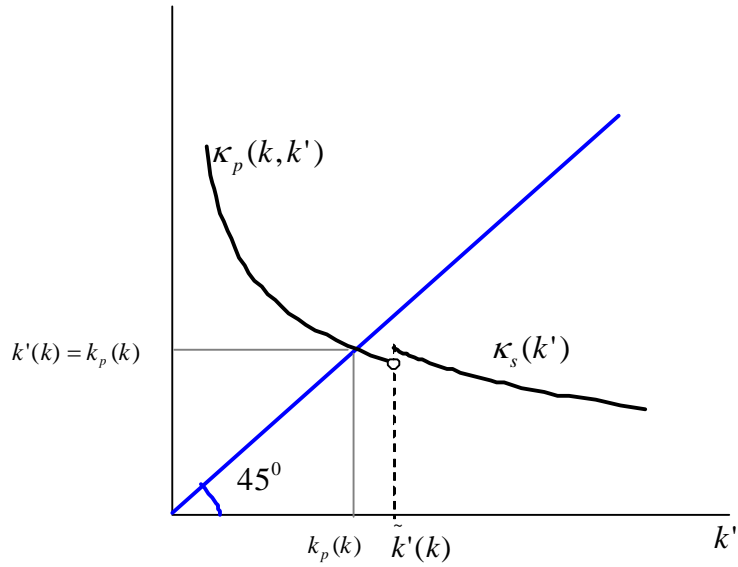


FIGURE 11.

Model with No Externalities. Consistent Belief Correspondence  $k'(k)$ .

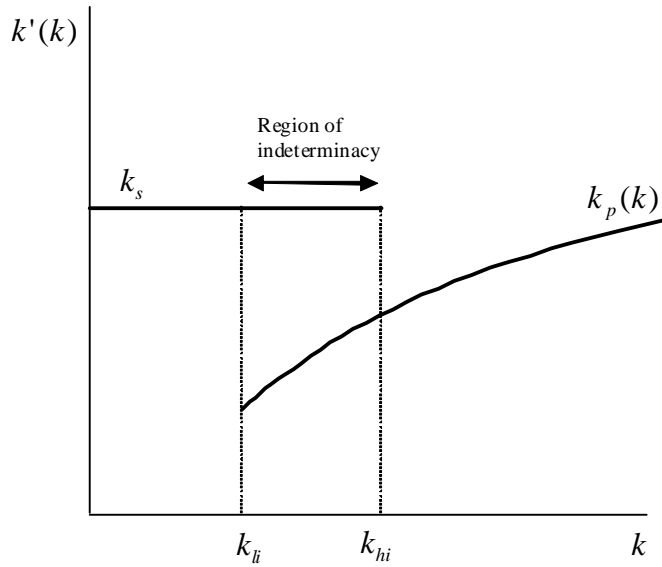


FIGURE 12.

Example 2. Cycles for the Model with no Externalities. The Case of the Pessimistic Belief Selection.

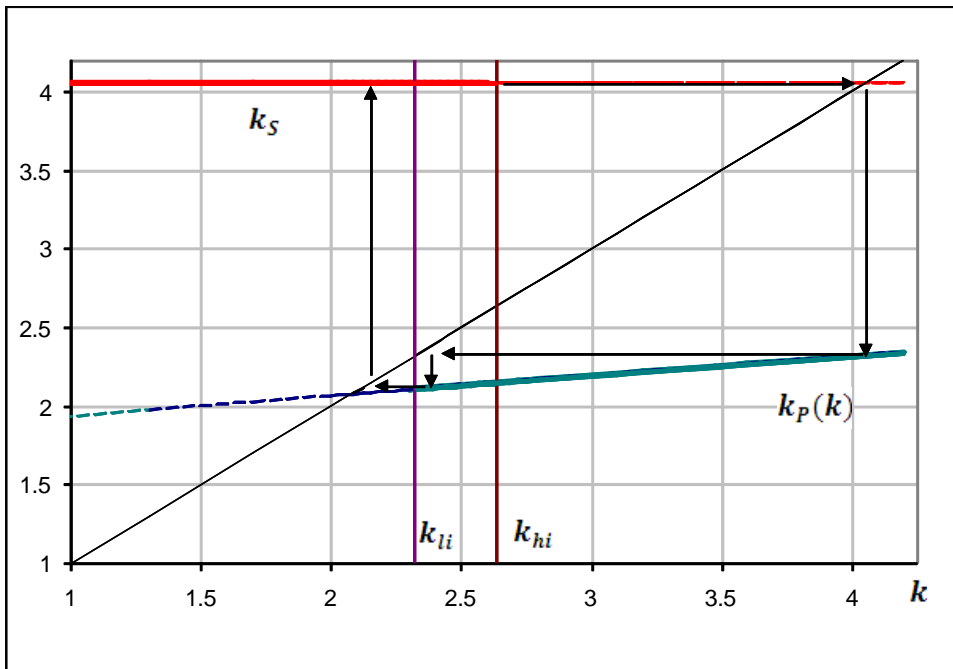
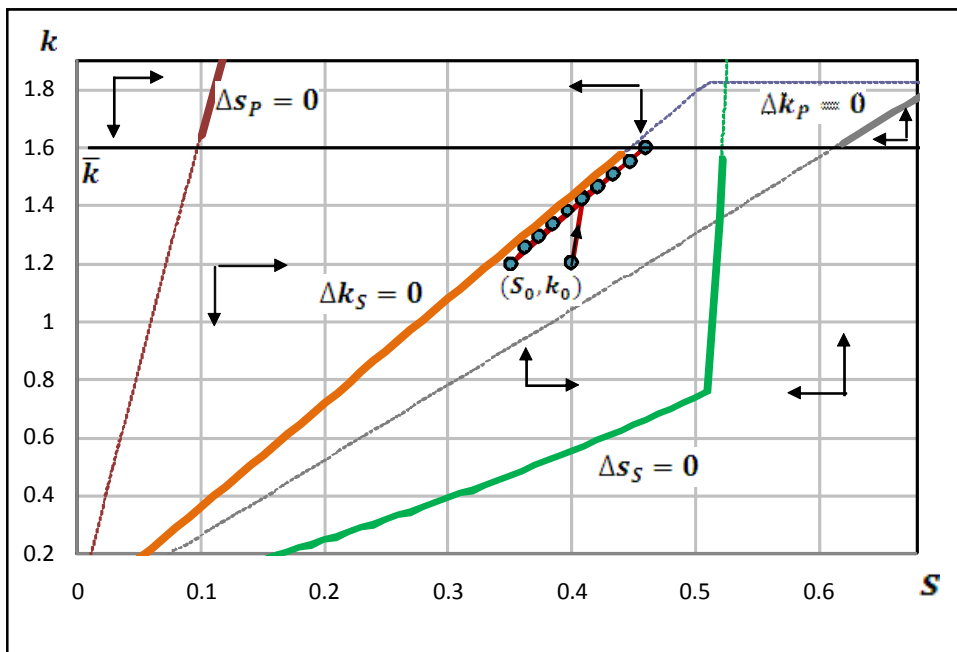


FIGURE 13

Example 3. Cycles for the Model with the Endogenous Supply of Funds (Section VI).

13A. The Phase Diagram.



13B. Capital and the supply of funds dynamics.

