Closed formulae for revenue-maximizing mechanisms in 2-D sequencing mechanism design

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Revenue maximizing mechanism design

Selling product (goods/services) under incomplete information.

- Combinatorial optimization problem
- Agents 'own' parameters
- May misrepresent
- Mechanism = set of rules:
- Input: strategies of the agents
- Output: feasible solution + payments

Example

Single item auction

Myerson optimal single item auctions

Selling a single item to a group of agents [Meyerson, 1981].

- Agents: private information on valuation
- Priors on the private information
- Mechanism outcome: allocation + payments

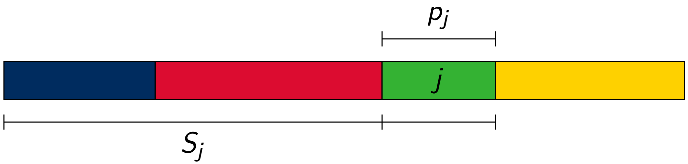
Optimal mechanism:

- Strategies: revealing information
- Truth telling w.l.o.g.
- 'Nice' properties

Focus of this talk

Properties of 1-D, 1.5-D and 2-D revenue optimal mechanisms for sequencing.

Sequencing jobs on a single processor



- Job: unit waiting cost, w_j; processing requirement, p_j
- Jobs must be scheduled
- Payments, π_j, reimburse jobs for waiting cost ($= w_j S_j$)
- Minimize total payment

All data known:

- $\sum \pi_j = \sum w_j S_j$
- Priorities according to w_j / p_j (Smith's Rule [Smith 1956])

Mechanism design problem

- Type $t_j = (w_j, p_j) \in T_j$ is private to agent j (owns job j)
- Probability distribution $\varphi_j : T_j \to (0, 1]$ public knowledge
- Agents may lie to maximize utility, $u_j = \pi_j - w_j S_j$
- Mechanism = schedule + payments
- Optimal mechanism, minimizing total payment

Mechanism design: example

- Three jobs
- $p_j = 1$ for all j
- $w_1 = 5$, $w_2 = 2$ and $w_3 = 3$ or $w_3 = 1$

σ_1: $w_1 = 5$ | $w_3 = 3$ | $w_2 = 2$ $\quad \pi_2 = 4, \pi_3 = 3$

σ_2: $w_1 = 5$ | $w_2 = 2$ | $w_3 = 1$ $\quad \pi_2 = 2, \pi_3 = 2$

Mechanism design: example

- Three jobs
- $p_j = 1$ for all j
- $w_1 = 5$, $w_2 = 2$ and $w_3 = 3$ or $w_3 = 1$

σ_1:	$w_1 = 5$	$w_3 = 3$	$w_2 = 2$	$\pi_2 = 4, \pi_3 = 3$
σ_2:	$w_1 = 5$	$w_2 = 2$	$w_3 = 1$	$\pi_2 = 2, \pi_3 = 2$

- $\pi_3(\sigma_2) - S_3(\sigma_2) < \pi_3(\sigma_1) - S_3(\sigma_1)$: Job 3 prefers σ_1

Mechanism design: example

- Three jobs
- $p_j = 1$ for all j
- $w_1 = 5$, $w_2 = 2$ and $w_3 = 3$ or $w_3 = 1$

σ_1: $w_1 = 5$ | $w_3 = 3$ | $w_2 = 2$ $\qquad \pi_2 = 4, \pi_3 = 3$

σ_2: $w_1 = 5$ | $w_2 = 2$ | $w_3 = 1$ $\qquad \pi_2 = 2, \pi_3 = 4$

- $\pi_3(\sigma_2) - S_3(\sigma_2) < \pi_3(\sigma_1) - S_3(\sigma_1)$: Job 3 prefers σ_1
- Increasing $\pi_3(\sigma_2)$ reduces total payment

Model

- Agents with jobs: types $t_j = (w_j, p_j) \in T_j$; (partly) private
- Mechanism strategies: report type $t'_j \in T_j$
- Mechanism output: machine sequence (ES) + payments
- Truthful mechanisms
- Payments: individual rational (IR) & incentive compatible (BNIC)

$$\text{(IR)} \qquad \pi_j(t_j) - w_j(t_j)ES_j(t_j) \geq 0$$

$$\text{(BNIC)} \qquad \pi_j(t_j) - w_j(t_j)ES_j(t_j) \geq \pi_j(t'_j) - w_j(t_j)ES_j(t'_j)$$

Overview

Open Problem [Heydenreich et al. 2008]

"Identify (closed formulae for) optimal 2-D mechanisms."

Model	Comments	Solution method
0-D	Optimization problem	Priorities: w_j/p_j
1-D	Only w_j private	Priorities: $\overline{w}_j/p_j$
1.5-D	Reported $p_j \geq$ true p_j	LP-compactification
2-D		Priorities: $\overline{w}_j/\mathbb{E}(p_j|w_j)$

Lemma

Priorities result in 'nice' properties

1-Dimensional

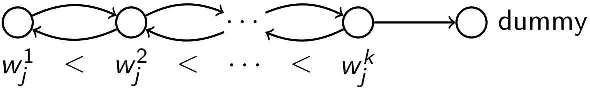
- Agents with jobs: p_j known, w_j private
- Strategies: report w_j'
- Mechanism output: sequences (ES) + payments
- Truthful mechanisms: Bayes-Nash incentive compatible payments
- [Heydenreich et al., WINE 2008; Duives et al. 2015]

Type graph

Given output sequences (ES), construct a type graph for each agent:

- Complete di-graph
- Node for each type + dummy
- Length of arc (w_j, w_j'): gain by reporting type w_j' if really w_j

$$l(w_j, w_j') = w_j(ES_j(w_j') - ES_j(w_j))$$



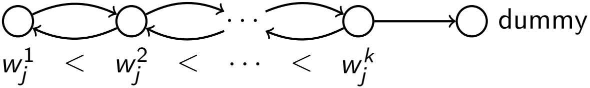
Lemma

Bayes-Nash implementable ⇔ no negative cycles ⇔ monotonicity.

Lemma

Given ES, the minimal BNIC payment for agent j reporting w_j is $-Dist(w_j, dummy)$.

Optimal 1-D mechanism



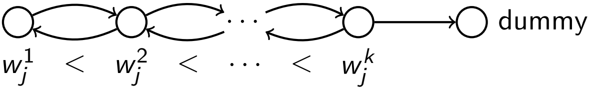
Lemma

Shortest path from w_j^i to the dummy traverses $(w_j^i, \ldots, w_j^k, dummy)$.

Lemma

$Dist(w_j^i, dummy) = -w_j^i ES_j(w_j) + \sum_{h>i} ES_j(w_j^h)(w_j^{h-1} - w_j^h)$.

Optimal 1-D mechanism



Lemma

Optimal mechanism minimizes

$$\sum_j \sum_i ES_j(w_j^i) \left(\varphi_j(w_j^i) w_j^i + (w_j^{i-1} - w_j^i) \sum_{h<i} \varphi_j(w_j^h) \right)$$

$$= \sum_{(w_1,\ldots,w_n)} \prod_j \varphi_j(w_j) \sum_j \overline{w}_j ES_j(w_j) \; ,$$

where $\overline{w}_j^i = w_j^i + (w_j^{i-1} - w_j^i) \frac{\sum_{h<i} \varphi_j(w_j^h)}{\varphi_j(w_j^i)}$

Optimal 1-D mechanism

$$\min \sum_{(w_1,\ldots,w_n)} \prod_j \varphi_j(w_j) \sum_j \overline{w}_j ES_j(w_j)$$

Many sequencing optimization problems $\rightarrow$ priority: $\overline{w}_j / p_j$.

Corollary

Optimal mechanism can be implemented as dominant strategies.

Corollary

Optimal mechanism is deterministic.

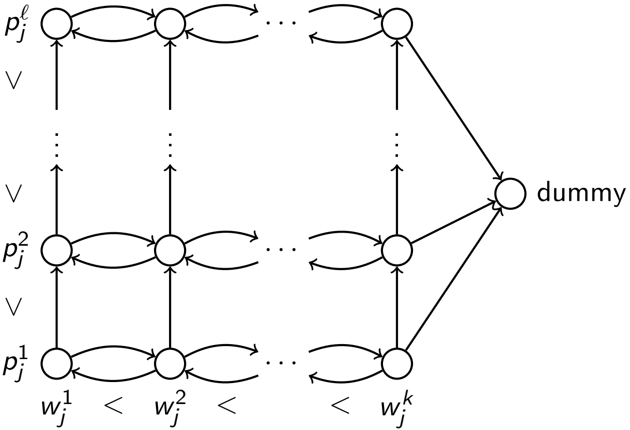
Corollary

Optimal mechanism is IIA.

1.5-Dimensional

- Agents with jobs: $t_j = (w_j, p_j)$ private
- Strategies: report t'_j with $p_j(t'_j) \geq p_j$
- Mechanism output: sequences (ES) + payments
- Truthful mechanisms: Bayes-Nash incentive compatible payments
- [H. & Uetz, IPCO 2013]

Type graph



Lemma

No 'dominating' shortest path.

Optimal 1.5-D mechanism

Theorem (H. & Uetz, IPCO 2013)

Polynomial size LP formulation for (BNIC) 1.5-D problem.

Results in randomized outcome, i.e. a lottery over sequences for each vector of types.

Lemma

Optimal randomized mechanism $>$ optimal deterministic mechanism.

Lemma

Optimal determinist mechanism $>$ optimal deterministic IIA mechanism.

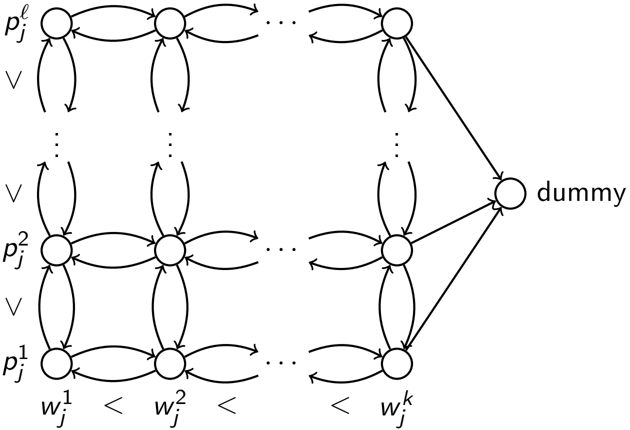
Corollary

Optimal mechanism does not have priorities.

2-Dimensional

- Agents with jobs: $t_j = (w_j, p_j)$ private
- Strategies: report any t'_j
- Mechanism output: sequences (ES) + payments
- Truthful mechanisms: Bayes-Nash incentive compatible payments

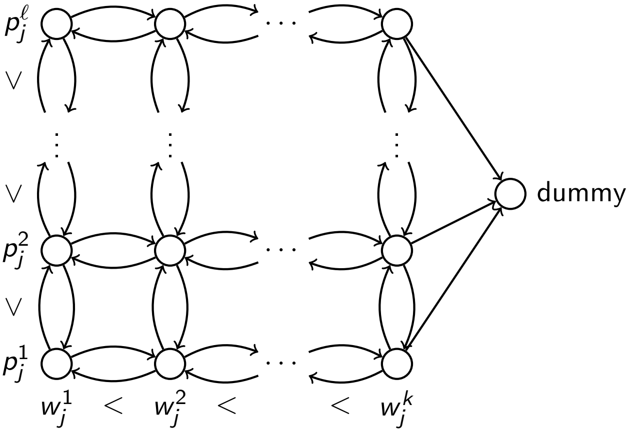
Type graph



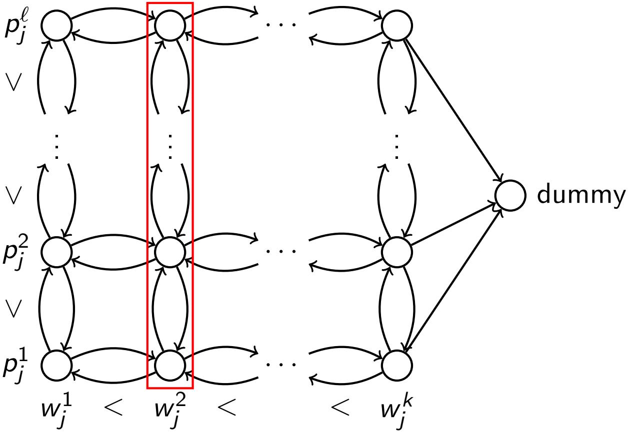
Lemma

$ES_j(w_j, p_j) = ES_j(w_j, p_j')$ *for all* j, w_j, p_j, p_j'.

Proof

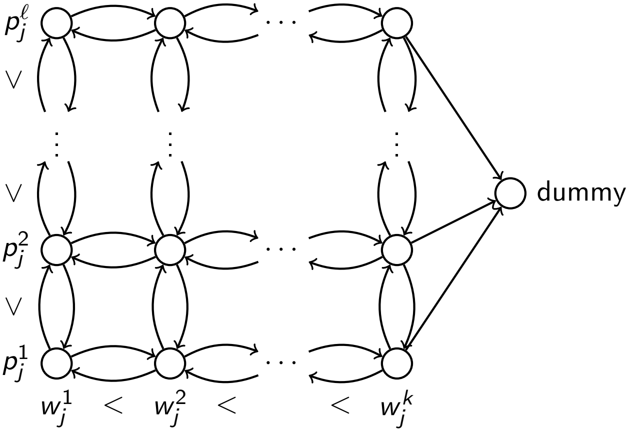


Proof



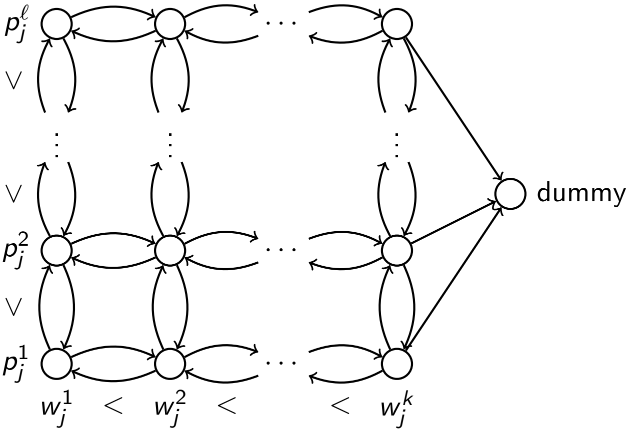
Equal utility for all types with equal w_j.

Proof



Monotonicity:
$w_j \geq w_j' \Leftrightarrow ES_j(w_j, p_j) \leq ES_j(w_j', p_j') \quad \forall w_j, w_j', p_j, p_j'.$

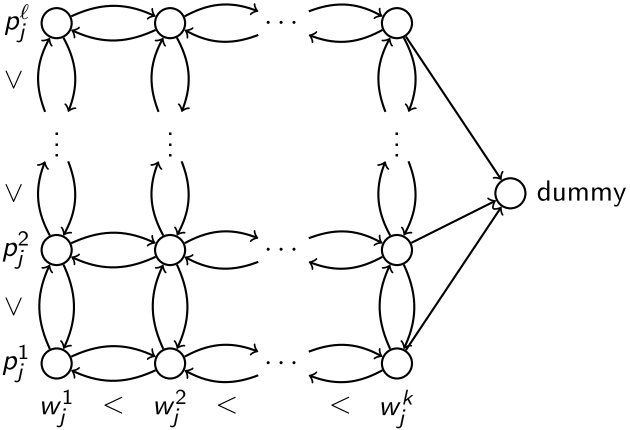
Proof



For all choices of $p_j^i, \ldots, p_j^h$:

$$\pi_j(w_j^i, p_j^i) \geq w_j^k Es_j(w_j^k, p_j^k) + \sum_{h=i}^{k-1} w_j^h \left(Es_j(w_j^h, p_j^h) - Es_j(w_j^{h+1}, p_j^{h+1}) \right) \quad .$$

Proof



$ES_j(w_j, p_j) = ES_j(w_j, p_j')$ for all j, w_j, p_j, p_j'. $\square$

Optimal 2-D mechanism

- Reduction to 1-D case with (conditional) stochastic processing requirement
- Solved by priorities: $\overline{w}_j/\mathbb{E}(p_j|w_j)$ [Rothkopf, 1966]
- Dominant strategy implementation
- IIA

Summary

- 2-D sequencing mechanism design reduces to 1-D case
- Priority sequencing rule
- 1.5-D optimal mechanism has no priority sequencing rule

Open problem:

- 2-D mechanism as an approximately optimal 1.5-D mechanism?