From imperfection to nonidealness

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Beyond perfection, the field to be explored is too wide. In this context, several ways to classify imperfect graphs or, equivalently, $0, 1$ (clique-node) imperfect matrices can be found in the literature.

From the polyhedral point of view, ideal matrices are to set covering problems what perfect matrices are to set packing problems. In fact, if a matrix is ideal, its linear relaxation is an integer polyhedron and its blocker and minors are also ideal. However, idealness seems to be more difficult or, at least, not as well studied as imperfection. No characterization in terms of forbidden minors is known. Even worse, the packing property, which would correspond to the combinatorial definition of perfection, is only satisfied by a proper family of ideal matrices. Nevertheless, when dealing with polyhedral aspects, several key results for perfect graphs (matrices) seem to be naturally transferable. In this talk we focus on some of these results, concerning the different ways of classifying imperfection and nonidealness.

Most polyhedral classifications of imperfection deal with the question of how far the clique relaxation is from the stable set polytope. Symmetrically, nonideal $0, 1$ matrices can be classified according to the distance between the set covering polyhedron and its linear relaxation.

A first way to measure this distance is through the performance of sequential tightening procedures. Working with Balas, Ceria and Cornuéjols's procedure, every graph is as imperfect as its complement and every $0, 1$ matrix is as nonideal as its blocker. The same result can be obtained defining a *nonidealness ratio*, by symmetry with the imperfection ratio defined by Gerke and McDiarmid.

Imperfection has also been classified according to the facet defining inequalities of the stable set polytope. The same idea can be used to classify nonidealness. In fact, *near ideal* matrices give polyhedral characterizations of minimally nonideal (mni) matrices as near perfect graphs, defined by Shepherd, do for minimally imperfect graphs. Moreover, considering the blockers of near ideal matrices, only a few and very easy conditions must be checked in order to identify a mni matrix. Web's clique-node matrices are circulant matrices, and many results on imperfect webs can be completely translated in terms of nonideal circulant matrices. In particular, Wagler proved that antiwebs are rank perfect graphs and it can also be proved that the blockers of circulant matrices are *rank ideal*.

Even though it is not possible to completely translate properties from perfection to idealness, the polyhedral approach still gives a promising field for further research.

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