Segregation and affirmative action in school choice*

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Abstract

Segregation in schools is prevalent in cities around the world. We analyze the impact of affirmative action policies commonly used in centralized school choice on segregation and efficiency. In a large market model, we show that minority reserves–which guarantee a number of seats to minority students–are an effective tool for reducing segregation in schools. More subtly, minority reserves increase the number of students assigned to their first preferences and improve efficiency. The cost of increasing minority reserves is leaving more students assigned to unattractive schools. The theoretical predictions are confirmed by field evidence from school choice programs in Chile.

1 Introduction

Many school choice programs around the world use centralized procedures to assign students to schools.[1] Based on the Gale-Shapley deferred acceptance algorithm (Gale and Shapley 1962), these procedures result in assignment processes that are considered successful by scholars and policymakers. Yet, our understanding of the impact that affirmative action policies have on segregation and

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[1] The list of places using centralized procedures to match students to schools includes Boston, Chicago, NYC, Paris, Barcelona, and Chile.

different market outcomes is rather limited, despite the fact that segregation in schools is a key societal and economic problem. Indeed, segregation in schools impacts both learning outcomes and social attitudes (Karsten 2010, Rao 2019) and, as policy debates recognize, is in part determined by the algorithm used to admit students.[2] The main goal of this paper is to explore the tradeoffs faced when reducing segregation in school choice programs by using affirmative action policies.

We study the impact of minority reserves on segregation and efficiency in school choice programs. Minority reserves guarantee a given number of seats to minority students and respect each school ranking otherwise (Hafalir, Yenmez, and Yildirim 2013, Ehlers, Hafalir, Yenmez, and Yildirim 2014, Echenique and Yenmez 2015). These reserves are a simple and transparent way to include diversity considerations into centralized school choice programs. We use simulations from the Chilean centralized school choice system and derive theoretical results to understand the impact that minority reserves have on several market outcomes, including segregation, the number of students assigned to their top schools, and efficiency.

Our analysis is motivated by data and simulations from the centralized school choice system used in Chile. The Chilean system reserves 15% of the seats in each school to socially disadvantaged minority students and assigns students to schools using the Gale-Shapley deferred acceptance algorithm (Gale and Shapley 1962). Simulating the system with different minority reserves, we show that reserves can be an important tool to reduce segregation in schools. Surprisingly, minority reserves increase the number of students assigned to their top schools and also improve the overall efficiency of the final assignment. The main cost associated with minority reserves is to leave more students assigned to less attractive schools or unassigned.

The Chilean data shows that minorities apply to high-demand schools with less intensity than regular students. As a result, schools are segregated. Moreover, our theoretical analysis reveals that these differences in application patterns also drive the impact that minority reserves have on several key market outcomes.

We explore a large market model in which a continuum of students apply to a finite number of schools (Abdulkadiroglu, Che, and Yasuda 2015, Azevedo and Leshno 2016). A student is either regular or minority. Schools fall under two tiers, 1 and 2. Each school ranks students randomly. Tier 1 schools are popular and over-demanded, while tier 2 schools are unpopular and under-demanded. Motivated by the different application patterns we uncover in Chilean cities and

[2]For example, segregation in the current New York City centralized school match is an issue of intense debate and some proponents argue that the city should modify the algorithm used to assign students. See the New York Times story at https://www.nytimes.com/2021/03/09/nyregion/nyc-schools-segregation-lawsuit.html. See also the discussion in Alvin Roth's blog at https://marketdesigner.blogspot.com/2019/04/should-nyc-school-choice-diversify.html.

evidence from Europe and the US (Hastings, Kane, and Staiger 2009, Laverde 2020, Oosterbeek, Sóvágó, and van der Klaauw 2021), we assume that regular students apply more intensely to tier 1 schools than minority students. We show that minority reserves favor minorities and decrease segregation, unless too many seats are reserved for minority students.

Our first main result shows that increasing minority reserves leaves more students assigned to their top schools. To understand this result, note that after increasing minority reserves, some regular students are replaced by minorities in popular tier 1 schools. The minority students applying to tier 1 schools are likely to rank the school as her top choice and thus including them will increase the total number of students assigned to their top schools. While increasing minority reserves also reduces the number of seats available to regular students and triggers chains of rejections, we show that, provided the total capacity of overdemanded schools is not too high, the assignment under a higher minority reserve has more students that obtain their top choices.[3]

We also measure the inefficiency of the assignment by the number of students in Pareto improving pairs. Two students form a Pareto improving pair if, by swapping schools, both are better off. We show that increasing minority reserves improves the efficiency of the system by reducing the number of students in Pareto improving pairs.

The theoretical analysis additionally exposes the costs of reducing segregation in schools by using minority reserves. We show that increasing minority reserves leaves more students assigned to less attractive schools. Formally, we prove that as minority reserves increase, the cumulative rank distributions cross and, as a result, cannot be compared in the first order stochastic dominance sense. In particular, increasing the total number of students assigned to unattractive schools is an important consequence of a rise in minority reserves.

Our model and results uncover the impact that changes to minority reserves have on several market outcomes. While our model is stylized and abstracts away from several features, each of our theoretical findings is confirmed by field data from the assignment processes in Chilean cities.

Segregation in schools is pervasive in cities around the world. Centralized school choice programs are often seen as providing equal access to schools. However, recent research shows that systematic differences in the application patterns of different groups may limit the efficacy of centralized school choice programs at reducing social, ethnic, or racial segregation in schools (Laverde 2020, Kutscher, Nath, and Urzua 2020, Son 2020, Oosterbeek, Sóvágó, and van der Klaauw 2021).[4] We build from

[3]The chains of rejections triggered by an increase in minority reserves explain why we need an upper bound on the number of seats to derive these comparative statics results. These effects are discussed in detail after Proposition 2.

[4]Laverde (2020) shows that in some dimensions the outcome of the centralized school choice program in Boston is similar to the outcome generated by an assignment based on proximity between residences and schools. Kutscher, Nath, and Urzua (2020) show that the introduction of the centralized school choice program in Chile has had a limited impact

the premise that centralized school choice alone may not be enough to integrate schools and explore the trade-offs faced when designing minority reserves.

Our analysis has important practical implications. The number of students assigned to their top schools is typically reported by districts implementing centralized school choice platforms (Featherstone 2020). Moreover, authorities in New York and Boston have made algorithmic decisions to maximize the number of students in their top schools (Abdulkadiroğlu, Pathak, and Roth 2009, Abdulkadiroglu, Pathak, Roth, and Sönmez 2006).[5] The number of Pareto improving pairs in the assignment is an inefficiency measure authorities in Amsterdam have also looked at (Ashlagi and Nikzad 2020). Thus our results are of interest to policymakers who may combat segregation in schools, increase the number of families obtaining their top choices, reduce the number of applicants in Pareto improving pairs, but incur the costs of leaving more students unassigned.

Abdulkadiroğlu and Sönmez (2003) apply matching theory to school choice problems. The school choice literature has shown that the design of matching mechanisms involves complex trade-offs (Gale and Shapley 1962, Roth and Sotomayor 1990, Che and Tercieux 2019, Leshno and Lo 2021). Reducing segregation in schools is another important desideratum and our results expose new forces that are important for practical implementations of matching algorithms.

Recent work has explored how different tie-breaking rules impact efficiency in matching markets (Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Arnosti forthcoming, Ashlagi and Nikzad 2020, Allman, Ashlagi, and Nikzad forthcoming). Our results show that the impact of higher reserves on the rank distribution is similar to the impact of switching from multiple to single tie-breaking (Ashlagi and Nikzad 2020, Arnosti forthcoming). The results are unrelated in that our arguments rely on the different application patterns of minorities and regular students.

Our paper contributes to the extensive literature on matching problems with diversity considerations, including work by Abdulkadiroğlu (2005), Kojima (2012), Westkamp (2013), Hafalir, Yenmez, and Yildirim (2013), Ehlers, Hafalir, Yenmez, and Yildirim (2014), Echenique and Yenmez (2015), Kominers and Sönmez (2016), Fragiadakis and Troyan (2017), Dur, Kominers, Pathak, and Sönmez (2018), Nguyen and Vohra (2019), Dur, Pathak, and Sönmez (2020), Aygün and Turhan (2020), Pathak, Sönmez, Ünver, and Yenmez (2020), Sönmez and Yenmez (2022), and Aygun and

on segregation in schools. Oosterbeek, Sóvágó, and van der Klaauw (2021) show that differences in application patterns explain a substantial fraction of segregation in secondary schools in Amsterdam.

[5]As Abdulkadiroğlu, Pathak, and Roth (2009) observe: "The greater number of students obtaining one of their top choices in a similar simulation and in the first year of submitted preference data convinced New York City to employ a single tiebreaker in their assignment system." When discussing the Boston school choice experience, Abdulkadiroglu, Pathak, Roth, and Sönmez (2006) argue that "the ability to tell the public that a high proportion of students receive their top choices may be a reason for the widespread popularity of the Boston mechanism."

Bó (2021). Throughout the paper, we borrow some definitions and concepts from these works. In particular, our formulations of minority reserves and soft bounds follow Hafalir, Yenmez, and Yildirim (2013) and Ehlers, Hafalir, Yenmez, and Yildirim (2014). Following Dur, Kominers, Pathak, and Sönmez (2018), we also contrast minority reserves to set asides (or vertical and horizontal constraints). We contribute to this literature by deriving new theoretical and empirical results on the impact of affirmative action policies on important outcomes neglected by previous research, such as segregation, the rank distributions of assignments, and the number of applicants in Pareto improving pairs.

The rest of the paper is organized as follows. Section 2 describes the Chilean setting and shows some motivating simulations. Section 3 presents our model. Section 4 introduces minority reserves and provides our main comparative statics results. Section 4 also presents variations of our model. Section 5 discusses our findings. Section 6 concludes. The Appendix contains supporting material.

2 Motivation

Chile initiated its centralized school choice system gradually in 2016. Before 2016, the system worked as a decentralized voucher market in which schools selected applicants based on non-transparent criteria. According to policymakers and politicians that pushed for reform, the decentralized nature of the system was the culprit for the high levels of segregation in Chilean schools. However, as noted by Kutscher, Nath, and Urzua (2020) and Honey and Carrasco (2022), the centralized system has had a modest impact on segregation in Chilean schools. Naturally, this leads to the question about the tools and tradeoffs policymakers face when designing algorithms to reduce segregation.

2.1 Centralized school choice in Chilean cities

To get admission to schools, students access a platform and fill a rank order list. The law regulating admission reserves 15% of seats in each school to socially disadvantaged minority students.[6] This reserve policy is an explicit attempt to promote social inclusion in schools. Schools rank students using a variety of criteria, but many of them are relevant for a small fraction of the applicants. Many students cannot be ranked by schools simply using any of the priority criteria.[7] In those

[6]A student is considered minority if her social background impairs her learning process and educational outcomes. See https://sep.mineduc.cl/alumnos-prioritarios-preferente/ for details.

[7]For details on the Chilean system, see Correa, Epstein, Escobar, Rios, Bahamondes, Bonet, Epstein, Aramayo, Castillo, and Cristi (2022).

cases, each school runs a lottery over its whole set of applicants. Students are then assigned to schools by running a Gale-Shapley deferred acceptance algorithm with minority reserves (Gale and Shapley 1962, Hafalir, Yenmez, and Yildirim 2013). The assignment algorithm runs as follows:

Step 1: Each student proposes to her first choice. Each school tentatively assigns seats to its proposers, following the priority and lottery orders, and respecting minority reserves. Remaining proposers are rejected.

Step k: Each student rejected in the previous step proposes to her next best choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats following the priority and lottery orders and the minority reserves. Any remaining proposers are rejected. Go to Step $k+1$.

The algorithm terminates either when there are no new proposals or when all rejected students have exhausted their preference lists.

We focus on the admission process for Pre-Kinder during 2019 in the main urban centers in Chile: Valparaíso, Concepción, Santiago.[8] Table 4 presents summary statistics for each market. Note that in each of the cities, the percentage of minority students far exceeds the current minority reserve of 15%.[9]

Table 1: Valparaiso, Concepcion and Santiago markets

	Valparaíso	Concepción	Santiago
Number of schools	275	250	1,214
Total capacity (seats)	8,754	9,199	56,331
Number of students	6,819	7,523	49,108
Minority students	2,994 (43.91%)	3,233 (42.97%)	18,399 (37.47%)

2.2 Minority reserves and market outcomes

We now present simulation results for Valparaíso, Concepción and Santiago. For each market, we run the algorithm used by the Ministry of Education for different minority reserves.[10] Concretely,

[8]Simulations for smaller Chilean cities are almost identical to the ones reported here for the main urban centers.

[9]Each of our markets include some rural areas in which the supply of schools is limited and therefore naturally families apply to one or two schools. However, virtually all students in our sample live in urban centers.

[10]In particular, our simulation considers all the criteria used by the Ministry of Education to rank students in each school, including special needs and sibling priority.

for each market and for each $f \in \{0, \ldots, 100\}$, we run independent simulations of the algorithm where minority reserves in each school equals $f\%$ of its seats. We assume that variations in the algorithm do not change applications. This assumption is justified since in Chile applicants are allowed to submit rank order lists of arbitrary length and the deferred acceptance algorithm with minority reserves is strategy-proof (Hafalir, Yenmez, and Yildirim 2013).

To measure segregation, we compute the Duncan segregation index (Duncan and Duncan 1955):

$$D = \frac{1}{2} \sum_{c \in C} \left| \frac{\text{Regular students in } c}{\text{Total number regular students}} - \frac{\text{Minority students in } c}{\text{Total number minority students}} \right|$$

The Duncan index measures the fraction of minority students that need to be reassigned so that every school has the same ratio of students of each group. The Duncan index equals 0 under perfect integration, and 1 under perfect segregation.

For each allocation, we compute the Duncan segregation index, the fraction of students assigned to their top schools, and the fraction of students to their top three choices. Figures 1 and 2 illustrate our simulations. The simulations show three important results. First, even when schools' ranks are mostly determined by lotteries, schools are segregated. School choice alone is not enough to eliminate segregation in Chilean schools. Second, segregation is U-shaped. The Duncan index is minimized close to the fraction of minority students in each city.[11] Third, as the minority reserve increases, more students are assigned to their top schools. Yet, increasing reserves also leaves fewer students assigned to their top three schools (and thus more students assigned to schools ranked 4 or worse).

[11] Our theoretical results show that segregation is minimized at a reserve that equals the fraction of minority students in the market. In our simulations, in contrast, segregation is minimized slightly to the right of our theoretical prediction. Two reasons explain this. First, around 2% of seats in each market are reserved for students with special needs. The fraction of seats reserved for minorities is computed excluding the special needs seats and minorities are underrepresented among students with special needs. Second, to compute the Duncan index we are ignoring unassigned students. Minorities are underrepresented among unassigned students.

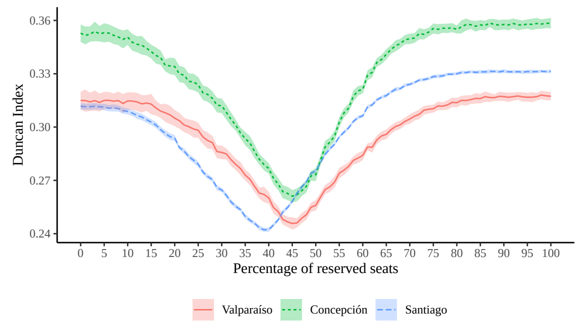
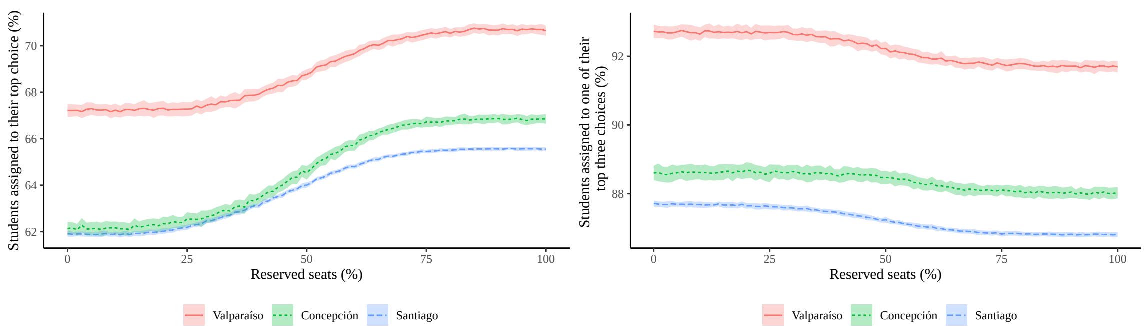


Figure 1: Duncan segregation index.



(a) Percentage of students assigned to top schools.

(b) Percentage of students assigned to one of their top three schools.

Figure 2: Students assigned to their top choices and top three choices.

2.3 Schools and application patterns

In this Subsection, we argue that there are differences in application patterns that can explain the comparative statics results illustrated in Figures 1 and 2.

2.3.1 Popular schools

Schools face different demand levels. We measure the popularity of each school c as the ratio between the number of students for whom school c is their top choice and the capacity of school c. More formally, let $p(c)$ be the number of students that list school c as their top choice and let q_c be the number of seats that school c has. The *popularity* of school c is given by $pop(c) = \frac{p(c)}{q_c}$. Table 2 shows the popularity of schools in our markets. We say that school c is *popular* if $pop(c) \geq 1$. A popular school will fill its seats under different variations of the deferred acceptance algorithm. As shown in Table 2, close to 1/4 of schools in Santiago and Valparaíso are popular schools.

	Valparaíso	Concepción	Santiago
Median	0.53	0.44	0.60
Mean	0.74	0.67	0.80
Third quartile	0.96	0.86	1.03

Table 2: Popularity of schools

In Appendix B.2, we show that all results in this Section (including Figures 3 and 4 below) extend to alternative definitions of popular schools.

2.3.2 Application patterns and market outcomes

For each student s in our database, we define her *application intensity* as the number of popular schools in the top of her application. Formally, letting $(c_1, c_2, \ldots, c_L)$ be the rank order list of student s, we define her application intensity as

$$l(s) = \begin{cases} \max\{i \mid c_1, \ldots, c_{i-1}, c_i \text{ are all popular schools.}\} & \text{if } c_1 \text{ is popular} \\ 0 & \text{if not.} \end{cases}$$

The application intensity $l(s)$ of student s is the number of popular schools student s competes for. Since school $c_{l(s)+1}$ is not popular, if student s is rejected from all her top $l(s)$ schools then she will likely get accepted to school $l(s)+1$ in her list. In particular, whether student s ranks other popular schools below $c_{l(s)+1}$ is likely to be irrelevant for the assignment.

For each group of students $t \in \{r, m\}$ and $l \in \mathbb{N}$, we compute the *empirical distribution function of application intensities* as

$$\hat{\Pi}_t(l) = \frac{|\{s \text{ is in group } t \mid l(s) \leq l\}|}{|\{s \text{ is in group } t\}|}. \tag{2.1}$$

In each city, $\hat{\Pi}_t(l)$ is the fraction of group t students with application intensities weakly less than l. Figure 3 shows the distributions of application intensities. Notably, in each city, $\hat{\Pi}_r$ dominates $\hat{\Pi}_m$ in the first order stochastic dominance sense. Thus minority students apply with lower intensity to popular schools than regular students.[12] We also compute the distributions of application

[12] We are agnostic about why minority students apply less to popular schools. Online Appendix F shows that minority students tend to live farther away from popular schools. We also show that students attending schools with higher popularity tend to perform better in standarized tests.

intensities conditional on $l \geq 1$ as

$$\hat{\Pi}_t(l \mid l \geq 1) = \frac{|\{s \text{ is in group } t \mid 1 \leq l(s) \leq l\}|}{|\{s \text{ is in group } t \text{ and } l(s) \geq 1\}|}.$$

As shown in Figure 4, $\hat{\Pi}_r(\cdot \mid l \geq 1)$ dominates $\hat{\Pi}_m(\cdot \mid l \geq 1)$. Even restricting attention to students that apply first to popular schools, regular students apply more intensely to popular schools.[13]

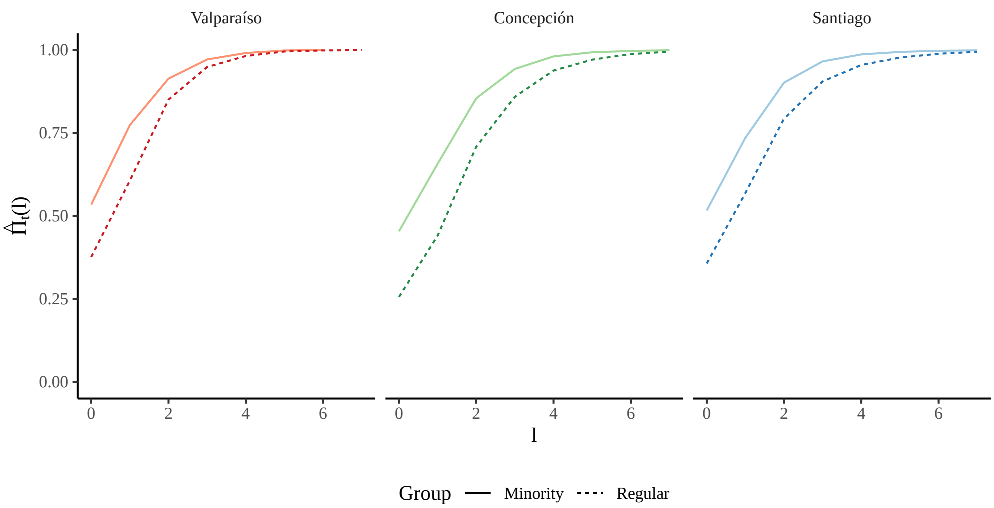


Figure 3: Empirical distributions of application intensities $\hat{\Pi}_t(\cdot)$ for each group t.

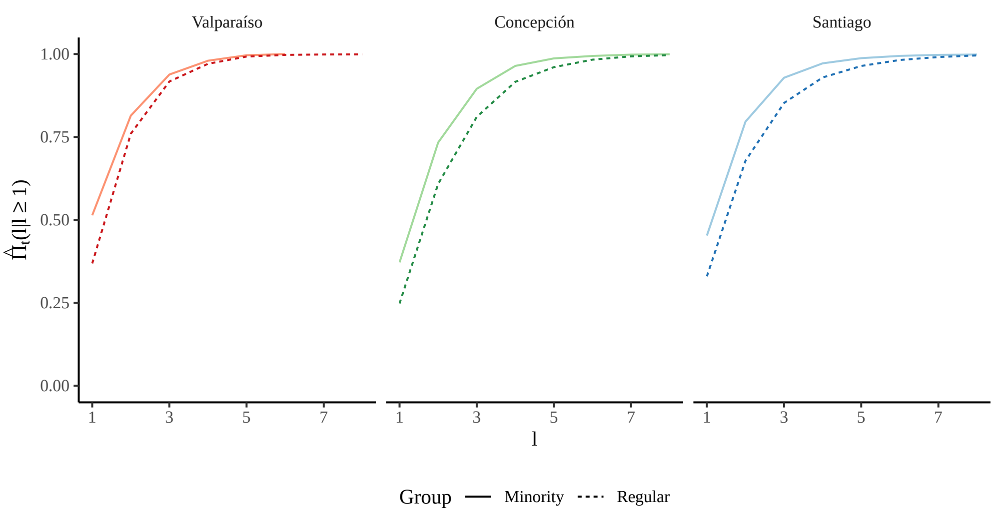


Figure 4: Empirical distributions of application intensities $\hat{\Pi}_t(\cdot \mid l \geq 1)$ conditional on $l \geq 1$.

[13] Appendix D shows that the higher the popularity of a school, the higher the fraction of regular students that demand that school.

The application patterns exposited in Figures 3 and 4 are important to understand segregation in schools and the simulations illustrated in Figures 1 and 2. To see this, consider a popular school c that reserves a relatively low fraction of its seats to minorities. Since minorities are less likely to apply to popular schools, regular students will be overrepresented in school c. Suppose now that school c raises its minority reserve. Thus, some minority students will replace regular students and, up to some reserve level, school c will be less segregated. More subtly, as Figure 4 shows, a minority student that replaces a regular student in c is more likely to rank that school first. Thus, for the new pool of students admitted to c, school c is likely to be the top ranked school.

Sections 3 and 4 provide theoretical results that clarify the discussion in the previous paragraph. One important aspect that is ignored in the discussion above (but carefully explained in our theoretical analysis) is the role that system-wide effects have on the relation between minority reserves and market outcomes. In markets in which the final allocation is the result of the deferred acceptance algorithm, raising reserves causes rejections that trigger new applications and chains of rejections. We will show that these chains of rejections may change the comparative statics results about the impact of minority reserves on efficiency and the number of students assigned to their top schools. We also argue that these effects explain why reserves increase the number of students assigned to less attractive schools (Figure 2).

3 Model

3.1 Environment

We consider a school choice problem with a continuum of students and a finite number of schools (Abdulkadiroglu, Che, and Yasuda 2015, Azevedo and Leshno 2016). There is a measure 1 of regular students (r), and a measure $\beta > 0$ of minority students (m).[14] A student is characterized by $s = (t, x) \in (\{r\} \times [0, 1]) \cup (\{m\} \times [0, \beta])$. The set of all students is denoted S.

The set of schools is $C = \{1, \ldots, n, n+1, \ldots, n+N\}$. Schools $c \in C_1 = \{1, \ldots, n\}$ are tier 1 , while schools $c \in C_2 = \{n+1, \ldots, n+N\}$ are tier 2. Tier i schools have capacity k_i.

For $l \in \{0, 1, \ldots, n\}$, we define the set $Z(l)$ of complete and transitive preferences over schools such that the l-most prefered schools are all tier 1 schools, but the school ranked $l+1$ is tier 2.[15] In particular, $Z(n)$ is the set of all preferences $\succ$ such that for all $c_1 \in C_1$ and all $c_2 \in C_2$, $c_1 \succ c_2$.

[14] Strictly speaking, the total number of minority students could exceed regular students. As will be clear later, all what matters for our results is that minority students are under-represented in over-demanded schools.

[15] In particular, for a preference that belongs to $Z(l)$, with $l \leq n-1$, the school ranked $l+1$ could be tier 1 or tier 2.

For $t \in \{r, m\}$ and $l \in \{0, \dots, n\}$, a fraction $\pi_t(l) \in [0,1]$ of group t students have preferences uniformly distributed over $Z(l)$, with $\sum_{l=0}^{n} \pi(l) = 1$. In particular, a fraction $\pi_t(0)$ of type t students list a tier 2 school as first choice. We denote by $\Pi_t(l)$ the fraction of type t students that list l or less tier 1 schools:

$$\Pi_t(l) = \sum_{l' \leq l} \pi_t(l').$$

We call Π_t the *distribution of application intensities* for type t. The distribution Π_t is the theoretical analog of the empirical distribution $\hat{\Pi}_t$ constructed in (2.1). For both types, the preference profile of a student (t, x) is entirely determined by (t, x).[16]

Tier 1 schools are over demanded, but the total capacity of the market exceeds total demand. We thus assume that $nk_1 \leq (1 - \Pi_r(0)) + \beta(1 - \Pi_m(0))$ and $nk_1 + Nk_2 > 1 + \beta$.

We assume that Π_r first order stochastically dominates Π_m. In other words, for all $l \in \{0, 1, \dots, n\}$,

$$\Pi_r(l) \leq \Pi_m(l).$$

Letting $L_t = \min\{L \mid \Pi_t(L) = 1\}$, we also assume that $L_m < L_r$.[17] This assumption is motivated by Figure 3. It captures the idea that minority students are less likely to apply to popular and high-demand schools (Hastings, Kane, and Staiger 2009). This assumption is relevant in centralized school choice programs in cities in Europe and the US. For example, Laverde (2020) shows that white families are more likely than Black and Hispanic families to rank high-achievement schools in Boston Public Schools choice system. Oosterbeek, Sóvágó, and van der Klaauw (2021) also provide evidence of heterogeneity in school preferences between students from different backgrounds in the Amsterdam centralized system.

An implicit assumption in our model is that all popular schools tend to be more demanded by regular students. In practice, some popular schools are particularly attractive for minority students. However, Appendix D shows a significant correlation between the popularity of a school and the fraction of regular students that demand that school. So, our assumption is a good approximation.[18]

In many school choice systems, schools rank students independently and uniformly. We allow some more generality and assume that ranks are not necessarily uniform. A student $s = (t, x)$ at each school c draws $\omega_c^s \in [0,1]$ independently from the cumulative distribution G_t on $[0,1]$, with

[16] For example, for each $t \in \{r, m\}$, we can divide the interval $[0,1]$ in a finite number of disjoint intervals, each of them corresponding to a preference profile in $\cup_{l=0}^{L_t} Z(l)$.

[17] Since Π_r dominates Π_m, $L_m \leq L_r$.

[18] Moreover, Section 4.4.1 shows that when some popular schools are attractive for minority students (so that the popularity of a school does not correlate with the fraction of regular students that find the school more attractive), our results change radically.

derivative $G_t' = g_t > 0$. The number ω_c^s represents the priority that student s has in school c. A higher number ω_c^s implies that the student has higher priority in school c. We will refer to ω_c^s as the *score* that student s has in school c. We assume that regular students tend to have higher scores than minority students so that G_r dominates G_m in the first order stochastic sense: $G_r(\omega) \leq G_m(\omega)$ for all $\omega \in [0, 1]$. This assumption is natural in college admission in which socially disadvantaged students are likely to perform worse in entrance exams. The assumption that $G_r(\omega) \leq G_m(\omega)$ is also appropriate in school choice programs in which schools rank students according to academic performance, or in school systems in which siblings or children whose parents work in the school have higher priority. Under all these criteria, a minority student is less likely to be highly ranked in a school than a regular student. When $G_r = G_m$ equals the uniform distribution on $[0, 1]$, schools rank all students uniformly as in many school choice programs.

Our two-tier model is natural in school choice applications, in which a tier 1 school tends to be more attractive than a tier 2 school for all students. However, a given minority student is less likely than a regular student to apply to a tier 1 school. When $\beta = 0$ and Π_r is uniformly distributed in $\{1, \ldots, n\}$, our model has only one group and all students in the group prefer all tier 1 schools over any tier 2 school. In this case, our model is analogous to the limit models in Che and Tercieux (2019) and Ashlagi and Nikzad (2020). We extend these models to accomodate different groups of students in the market and distinct application patterns.

3.2 Matchings and cutoffs

A matching is a function $\mu\colon S \cup C \to C \cup 2^S$ such

i. For all $s \in S$, $\mu(s) \in C$;

ii. For all $c \in C_i$, $\mu(c) \subseteq S$ with $|\{s \mid \mu(s) = c\}| \leq k_i$;

iii. For all $c \in C$ and all $s \in S$, $\mu(s) = c$ iff $s \in \mu(c)$.

The first condition says that each student is assigned to a school, the second condition says that each school is assigned to a measure of students that does not exceed its capacity, the third condition says that a student is assigned to a school iff the school is assigned to that student. A matching μ is *stable* if for all $c \in C_i$ and all $s = (t, x) \in S$ with $c \succ_s \mu(s)$, the following two conditions hold: (i) $|\{s \mid \mu(s) = c\}| = k_i$; and (ii) $\omega_c^s < \omega_c^{s'}$ for all $s' = (t', x')$ with $\mu(s') = c$. Intuitively, a matching is stable if there is no pair (s, c) that can block the matching.[19]

[19]This notion defines a stable matching up to a measure 0 set of students. The market outcomes we analyze are not altered by this ambiguity. See Azevedo and Leshno (2016) for discussion.

Following Abdulkadiroglu, Che, and Yasuda (2015) and Azevedo and Leshno (2016), we can characterize a stable matching by means of admission cutoffs $p_c \in [0,1]$, for all $c \in C$. A cutoff p_c determines the lowest lottery number ω_c that a student can have to be admitted to school c. The highest the cutoff p_c, the harder it is to get to school c. Two observations simplify the characterization of cutoffs. First, schools within the same tier are symmetric and therefore $p_c = p_{c'}$ for all $c, c' \in C_i$.[20] Second, in any stable matching a tier two school will have excess capacity and therefore its cutoff will equal 0. We can therefore characterize a stable matching by means of a single cutoff p that clears the market for tier 1 schools:

$$\sum_{l=1}^{L_r} \pi_r(l) \sum_{q=1}^{l} \frac{1}{n} G_r(p)^{q-1}(1 - G_r(p)) + \beta \sum_{l=1}^{L_m} \pi_m(l) \sum_{q=1}^{l} \frac{1}{n} G_m(p)^{q-1}(1 - G_m(p)) = k_1. \tag{3.1}$$

The left hand side in equation (3.1) is the demand for a school $c \in C_1$ when the admission cutoff in all schools is p. The first term on the left hand side of (3.1) is the demand for school c of regular students. For each school $c \in C_1$ and $l \in \{1, \dots, L_r\}$, a fraction $\pi_r(l)$ of regular students will rank l schools. A regular student ranking l tier 1 schools will rank school c in the q-th position with probability $1/n$, for $q \in \{1, \dots, l\}$. A student that ranks school c in the q-th position will demand school c if her scores in schools ranked above c are below the cutoffs (which happens with probability $G_r(p)^{q-1}$) but her score in school c is above the cutoff (which happens with probability $1 - G_r(p)$). The second term on the left of (3.1) is the demand for school c from minority students and is analogously computed.

For each $t \in \{r, m\}$ and function $f \colon \{0, \dots, L_t\}$, we write $\mathbb{E}_t[f(l_t)] = \sum_{l_t=0}^{L_t} \pi_t(l_t) f(l_t)$. A solution $\bar{p} \in [0,1]$ to equation (3.1) also solves:

$$\frac{1}{n}\mathbb{E}_r[1 - (G_r(\bar{p}))^{l_r}] + \frac{\beta}{n}\mathbb{E}_m[1 - (G_m(\bar{p}))^{l_m}] = k_1 \tag{3.2}$$

It is relatively simple to show that equation (3.2) has a unique solution $\bar{p} \in]0,1[$ (for which in general there is no closed form solution). Naturally, $\bar{p}$ increases when the supply of tier 1 schools, nk_1, decreases. The market clearing cutoff also increases when the distribution Π_t increases in the first order stochastic dominance sense.

In the unique stable matching, minority students are underrepresented in tier 1 schools. Indeed, the ratio of minority to regular students in the whole population equals β, while the ratio of minority

[20] If two tier 1 schools had different cutoffs, one of them would have excess demand or excess capacity. If the market clearing condition is satisfied in the school with the highest (resp. lowest) cutoff, then the school with the smallest (resp. highest) cutoff has excess demand (resp. capacity). As a result, the matching would not be stable. This observation also applies to the model with minority reserves studied in Section 4.

to regular students assigned to a tier 1 school is

$$\frac{\beta \mathbb{E}_m[1-(G_m(\bar{p}))^{l_m}]}{\mathbb{E}_r[1-(G_r(\bar{p}))^{l_r}]} < \beta.$$

To see this, note that $\mathbb{E}_m[1-G_m(\bar{p})^{l_m}] \leq \mathbb{E}_m[1-G_r(\bar{p})^{l_m}] < \mathbb{E}_r[1-G_r(\bar{p})^{l_r}]$. The first inequality follows since a minority student tends to have lower scores so $G_m(\bar{p}) \geq G_r(\bar{p})$. The second inequality follows since Π_m is dominated by Π_r and $1-(G_r(\bar{p}))^l$ is increasing in l. These forces combine to result in school segregation.

4 Minority reserves, segregation and efficiency

We now introduce minority reserves and explore their impact on several market outcomes. We also discuss variations of our main model.

4.1 Stable matching under minority reserves

A minority reserve ensures that whenever the number of minority students in a school c is below the reserve, all other minority students must be assigned to schools that they strictly prefer to c. We adapt Hafalir, Yenmez, and Yildirim (2013) to model minority reserves as follows. Let $\rho = (\rho_1, \rho_2)$ be a vector of minority reserves in tier 1 and tier 2 schools. A matching μ is *stable under reserves* ρ if for all $c \in C_i$ and all $s = (t, x) \in S$ with $c \succ_s \mu(s)$, the following three conditions hold:

i. $|\{s \mid \mu(s) = c\}| = k_i$;

ii. if $|\{s' = (t', x') \mid \mu(s') = c, t' = m\}| \geq \rho_i$, then $\omega_c^s < \omega_c^{s'}$ for all $s' = (t', x')$ with $\mu(s') = c$; and

iii. if $|\{s' = (t', x') \mid \mu(s') = c, t' = m\}| < \rho_i$, then $t = r$ and $\omega_c^s < \omega_c^{s''}$ for all $s'' = (r, x'') \in \mu(c)$.

A matching is stable under reserves ρ if whenever a student s would like to move to another school c, that school is filling its seats, it is admitting students having higher priority and exceeding the minority reserves, and if it is not exceeding the minority reserves then s is a regular student having a score below the lowest score of regular students assigned to c. Note that when $\rho \equiv 0$, a matching is stable under reserves ρ iff it is stable.

A matching μ that is stable under reserves ρ always exists. It can be computed by the deferred acceptance algorithm by either properly defining a choice function or by making a copy of each school that targets minority students (Hafalir, Yenmez, and Yildirim 2013). Note that since our model has a continumm of students, the deferred acceptance algorithm need not converge in finite time (Abdulkadiroglu, Che, and Yasuda 2015).

We now characterize the unique stable matching under reserves ρ. First note that if $\rho_1 < \frac{1}{n}\mathbb{E}_m[1 - G_m(\bar{p})^{l_m}]$, then the stable matching characterized by cutoffs $\bar{p}$ is stable under reserves ρ. This simply follows from the observation that the minority reserve ρ_1 is already filled in tier 1 schools and therefore Conditions ii. and iii. in the definition of stability under reserves are equivalent to Condition ii in the definition of stability. Second, note that when $\rho_1 > \min\{\frac{\beta}{n}(1 - \pi_m(0)), k_1\}$, the reserve either is above the number of minority students that demand the school, or exceeds the capacity of the school. We thus define the set where reserves have a nontrivial impact on the final assignment: $R = [\frac{1}{n}\mathbb{E}_m[1 - G_m(\bar{p})^{l_m}], \min\{\frac{\beta}{n}(1 - \pi_m(0)), k_1\}]$.

Take a reserve $\rho_1 \in R$. We can characterize stability under reserves by means of cutoffs p_c^t that depend on the school c and the types $t \in \{r, m\}$ of the applying students. Similar to the analysis in Subsection 3.2, we can restrict attention to cutoffs such that $p_c^t = p_{c'}^t$ for all $c, c' \in C_1$ and $p_c^t \equiv 0$ for all $c \in C_2$ and all t. It is therefore enough to characterize the cutoffs p_m and p_r, with $p_m \leq p_r$, that minority and regular students face in tier 1 schools. First, the market clearing condition can be written as:

$$\frac{1}{n}\mathbb{E}_r[1 - (G_r(p_r))^{l_r}] + \frac{\beta}{n}\mathbb{E}_m[1 - (G_m(p_m))^{l_m}] = k_1. \tag{4.1}$$

This is similar to equation (3.1), but in this market clearing condition different groups face different cutoffs. Second, the minority reserve condition must hold. Since $\rho_1 \geq \frac{1}{n}\mathbb{E}_m[1 - G_m(\bar{p})^{l_m}]$, the reserve must bind and therefore the number of minority students in a tier 1 school equals the reserve:

$$\frac{\beta}{n}\mathbb{E}_m[1 - G_m(p_m)^{l_m}] = \rho_1. \tag{4.2}$$

These two conditions have a unique solution p_m and p_r. Figure 5 illustrates how these cutoffs are determined. Note that increasing ρ_1 moves the minority reserve condition (4.2) to the left in Figure 5. So, after an increase in minority reserves, p_m decreases and p_r increases. Increasing ρ_1 makes the access to tier 1 schools easier for minority students and harder for regular students.[21] We denote by μ_ρ the stable matching under reserves ρ.

The main focus of the paper is the impact of reserves ρ on several market outcomes. Note that since tier 2 schools have excess capacity, ρ_2 is irrelevant for the allocation. We explore the role of reserves by stating several comparative statics results with respect to ρ_1. Section 4.4 explores alternative affirmative action policies.

[21]This means that for each tier 1 school, there are some regular students that would like to be assigned to that school, will be rejected but have higher scores than some minority students that have been accepted in the school.

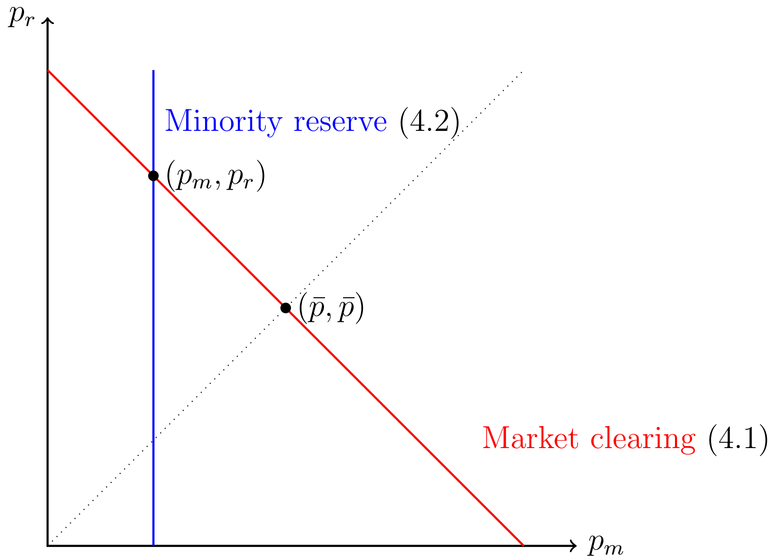


Figure 5: The market clearing condition and the minority reserve condition determine cutoffs p_r and p_m. The cutoff $\bar{p}$ is in the intersection of the market clearing condition and the 45 degree line.

4.2 Segregation

There are several ways to measure segregation in schools, but one of the the most common ones is the Duncan index (Duncan and Duncan 1955). Given a matching μ, the *Duncan index* D_μ is defined by

$$D_\mu = \frac{1}{2} \sum_{c=1}^{n+N} \left| \eta_\mu^r(c) - \frac{\eta_\mu^m(c)}{\beta} \right| \in [0, 1]$$

where $\eta_\mu^t(c)$ is the mass of students of type t assigned to school c in the matching μ. The index equals 0 under perfect integration, where each school is filled by exactly the same number of students of each type. More generally, the Duncan index can be interpreted as the mass of regular students that would need to be moved to different schools so that every school had the same proportions of students of each group.

Given a reserve $\rho \in R$, we denote $D(\rho) = D_{\mu_\rho}$.

Proposition 1. *$D(\rho_1)$ is nonincreasing over $\rho_1 < \frac{\beta}{1+\beta} k_1$ and is non-decreasing over $\rho_1 > \frac{\beta}{1+\beta} k_1$.*

This result shows that reserves have an impact on segregation in schools. The Duncan segregation index is minimized when the fraction of seats reserved to minority students, ρ_1/k_1, equals the share of minority students in the population, $\beta/(1 + \beta)$. Actually, in the proof we show a slightly stronger result: Segregation in each school c, $\left|\eta^r_{\mu_{\rho_1}}(c) - \frac{\eta^m_{\mu_{\rho_1}}(c)}{\beta}\right|$, is non-increasing over $\rho_1 < k_1\frac{\beta}{1+\beta}$ and non-decreasing over $\rho_1 > k_1\frac{\beta}{1+\beta}$. Intuitively, when $\rho_1 < k_1\frac{\beta}{1+\beta}$, minority students are underrepresented in tier 1 schools and overrepresented in tier 2 schools, and increasing ρ_1 moves minority students from tier 2 to tier 1 schools. This stronger property also implies that the index we actually use to measure segregation in our model is rather irrelevant for the Proposition.[22]

4.3 Rank distribution and efficiency

We now explore how ρ_1 impacts the efficiency of the assignment. Obviously, changing ρ_1 does not Pareto improve the assignment for students. We thus evaluate changes to the assignment using two measures. The first measure is the rank distribution of students, which is a function that for each $q \in \{1, \ldots, L_r + 1\}$ yields the fraction of students assigned to one of their q most preferred schools. Our second measure is the number of students that belong to a Pareto improvement pair. As we discuss in the Introduction, both of these measures are important in practical implementations of centralized school choice algorithms.[23] The main results in this Subsection characterize how ρ_1 changes both measures.

Since a type t student ranks at most L_t tier 1 shools and tier 2 schools always have free slots, type t students are assigned to one of their $(L_t + 1)$-most preferred schools. The share of type t students assigned to their q-th preference is

$$f_t(q) = \sum_{l=q}^{L_t} \pi_t(l) G_t(p_t)^{q-1}(1 - G_t(p_t)) + \pi_t(q-1) G_t(p_t)^{q-1}$$

for $q \in \{1, \ldots, L_t\}$. The first term represents all type t students that applying to q or more tier 1 schools get accepted in their q-th preference. The second term represents type t students that applying to $q - 1$ tier 1 schools are assigned to a tier 2 school. Note that for $q = L_t + 1$, $f_t(L_t + 1) = \pi_t(L_t) G_t(p_t)^{L_t}$. The cumulative rank distribution for type t students is thus

$$F_t(q) = \sum_{q' \leq q} f_t(q') = \begin{cases} 1 - G_t(p_t)^q (1 - \Pi_t(q-1)) & \text{if } q \leq L_t, \\ 1 & \text{if } q = L_t + 1. \end{cases}.$$

[22]Proposition 1 and our field evidence also apply to alternative segregation indexes, such as the ones discussed by Hutchens (2004) or Frankel and Volij (2011). See Online Appendix E.

[23]Obviously, both measures are imperfect. For example, it is entirely possible that under some matching very few students belong to Pareto improving pairs, but those improvements are very significant. To analyze this possibility, one would need to make additional assumptions to estimate utility functions. In contrast, our efficiency measures are based only on observable data. The two approaches are complementary.

To understand this formula intuitively, note that the mass of students assigned to schools ranked $q+1, q+2\dots$ is the fractions of students applying to q or more schools (which happens with probability $1-\Pi_t(q-1)$) and rejected in q of them (which happens with probability $G_t(p_t)^q$). Thus, for $q \leq L_t$, $F_t(q) = 1 - G_t(p_t)^q(1-\Pi_t(q-1))$. We will sometimes emphasize the dependence of these distributions on ρ_1 by writing $F_t(q, \rho_1)$.

Lemma 1. *Take $\rho_1 \in R$. Then, $\frac{\partial}{\partial \rho_1} F_m(q, \rho_1) > 0$ for all $q \leq L_m$ and $\frac{\partial}{\partial \rho_1} F_r(q, \rho_1) < 0$ for all $q \leq L_r$.*

This lemma says that increasing ρ_1 reduces (in the first order stochastic dominance sense) the cumulative rank distribution for minority students and increases the rank distribution of regular students. In other words (and not surprisingly), an increase in reserves favors the assignment for minority students but hurts regular students.[24]

Our main results explore the impact of minority reserves on the overall efficiency of the assignment. We thus define the total cumulative rank distribution as

$$F(q, \rho_1) = \frac{1}{1+\beta}\Big(\beta F_m(q, \rho_1) + F_r(q, \rho_1)\Big)$$

which measures the fraction of students assigned to one of their top q schools. Determining the impact of ρ_1 on $F(q, \rho_1)$ is not obvious since Lemma 1 shows that reserves favor minorities but hurt regular students. The following is the first main result in the paper.

Proposition 2. *Suppose $\mathbb{E}_r[l_r \mid l_r \geq 1] > \mathbb{E}_m[l_m \mid l_m \geq 1]$. Define*

$$\bar{K} = \max\Big\{K \in [0,1] \mid \mathbb{E}_r[l_r(1-K)^{l_r} \mid l_r \geq 1] \geq \mathbb{E}_m[l_m \mid l_m \geq 1]\Big\}.$$

Then, $\bar{K} \in]0,1[$ and for all $k_1 < \frac{\bar{K}}{n}$ and all $\rho_1 \in R$:

$$\frac{\partial F}{\partial \rho_1}(1, \rho_1) > 0.$$

Moreover, there exists $\bar{q} \in \{2, \dots, L_m\}$ such that for all $q > \bar{q}$,

$$\frac{\partial F}{\partial \rho_1}(q, \rho_1) < 0.$$

[24]This is related to Hafalir, Yenmez, and Yildirim (2013). Their Theorem 2 shows, in a general matching model, that the introduction of minority reserves favor at least one minority student. They also provide restrictions on preferences such that all minority students are better off when reserves are introduced. Lemma 1 thus complements these results.

Proposition 2 shows conditions under which raising ρ_1 increases the mass of students assigned to their first preferences. But increasing ρ_1 also increases the mass of students assigned to schools that are not highly ranked. The conditions under which these results hold say that (i) the expected reported number of tier 1 schools conditional on that report being at least 1 should be higher for regular types than for minority types, and (ii) the market should be relatively tight in the sense that the total capacity of the popular tier 1 schools, nk_1, is below a threshold $\bar{K}$. This bound is more likely to hold when minority students apply to fewer tier 1 schools (Π_m decreases), or when regular students apply more intensely to tier 1 schools (Π_r increases). Example 1 shows that if the tightness of the market assumption is relaxed, then fewer students are assigned to their top schools when ρ_1 increases.

There are two main forces behind the result that more students are assigned to their most preferred schools when ρ_1 increases. First, by increasing ρ_1, some regular students are replaced by minority students in tier 1 schools. But minority students apply to tier 1 schools with lower intensity: they are more likely to rank first a tier 2 school and, when they do rank first a tier 1 school, they are likely to include more tier 2 schools in the rest of the application. For students that rank first a tier 2 school (those with $l_m = 0$), the reserve ρ_1 does not make any difference. For minority students that actually apply to tier 1 schools (and thus $l_m \geq 1$), the reserve does make a difference. An increase in the reserve will bring some of those students into a tier 1 school, and those students will replace regular students. Thus, the increase in the reserve ρ_1 will replace regular students by minority students for whom the tier 1 school is likely to be very attractive.

The second important force is more subtle and explains why the market should be tight for reserves to increase the total number of students assigned to top choices. Reserves create competition among regular students applying to popular schools. On the one hand, when more seats are reserved for minority students in a tier 1 school c, some regular students are displaced and compete for seats in other schools. This stronger competition for seats in other schools displaces regular students, who in turn may demand seats in school c. On the other hand, when more seats are reserved for minority students in tier 1 schools other than c, displaced regular students also compete for seats in school c. As a result, p_r will be increasing in ρ_1 and decreasing in k_1.

When k_1 is relatively large, there is little competition in tier 1 schools and the majority of regular students are assigned to their top schools. But increasing the reserve ρ_1 activates the two competitive forces mentioned above and, to balance supply and demand in each tier 1 school, p_r increases substantially. This increase in p_r translates into a substantive reduction in the mass of regular students that are assigned to their top school, $\pi_r(0) + (1 - \pi_r(0))(1 - G_r(p_r))$. Thus, increasing the reserve ρ_1 reduces the total number of students assigned to their top choice even when most (even all) minority students that are now accepted in a tier 1 school c rank the school as their top choice. On the other hand, when slots are scarce and k_1 is relatively small, competition is already intense among regular students and increasing the reserves ρ_1 increases competition moderately. This means that the overall number of students assigned to their most preferred schools increases with the reserve.

To see why increasing ρ_1 decreases $F(q)$ for q large enough, note that all minority students are assigned to one of their top $L_m + 1$ schools. As Lemma 1 shows, the cumulative rank distribution for regular students is increasing in ρ_1. As a result, for $q > L_m$, as ρ_1 raises, fewer students are assigned to schools they rank below q.

We now discuss the efficiency impact of minority reserves. Increasing reserves does not Pareto improve the assignment. As Proposition 2 shows, a higher minority reserve does not improve the rank distribution of the assignment either.[25] We thus measure the efficiency of the assignment by the number of students in Pareto improving pairs.

Given a matching μ, students $s = (t, x)$ and $s' = (t', x')$ form a *Pareto improving pair* if $c' = \mu(s') \succ_s c = \mu(s)$ and $c \succ_{s'} c'$. In this case, we say that s is in a Pareto improving pair. Let $P(\rho_1)$ be the total measure of students s that are in a Pareto improving pair. Arguably, $P(\rho_1)$ measures the inefficiency of the matching. The following proposition shows that minority reserves have an unambiguous effect on $P(\rho_1)$.

Proposition 3 (Pareto improvements)**.** *Under the conditions of Proposition 2, $P(\rho_1)$ is decreasing in ρ_1.*

When ρ_1 increases, fewer students are in a Pareto improving pair. Thus, a higher reserve increases the efficiency of the matching. In the proof, we show that a student s can Pareto improve by switching school iff s is assigned to a tier 1 school that is not her top choice. Thus, Proposition 3 follows immediately from Proposition 2. Our characterization also implies that the set of students in a Pareto improving pair coincides with the set of students in Pareto improving cycles.

To conclude this Section, we provide two examples showing that our main comparative statics results do not hold if we relax some of the assumptions in Propositions 2 and 3.

Propositions 2 and 3 apply under the assumption that the market is tight. The following example shows that when the market is slack, minority reserves reduce the number of students assigned to their first preferences.

[25] Featherstone (2020) explores rank efficiency in assignment problems.

Example 1 (Slack markets)**.** *We take our model with* $\beta = 1$, $n = 2$, $N = 1$, $k_2 = 2$, $k_1 = 8/9$. *In this model, all regular students apply to all tier 1 schools, while all minority students apply first to one tier 1 school (and second to the tier 2 school). More precisely,*

$$\pi_r(0) = \pi_r(1) = 0,\ \pi_r(2) = 1 \quad \textit{and} \quad \pi_m(0) = \pi_m(2) = 0,\ \pi_m(1) = 1.$$

Clearly, Π_r *dominates* Π_m *and* $2 = \mathbb{E}_r[l_r \mid l_r \geq 1] > \mathbb{E}_m[l_m \mid l_m \geq 1] = 1$. *Under these conditions,* $\bar{K} = 1 - \sqrt{1/2} \approx 0.29$. *Since* $k_1 > \bar{K}$, *the tightness condition in Proposition 2 does not hold. We will show that less students are assigned their top schools when minority reserves increase.*

Note first that without reserves, $\bar{p} = (-1/2) + \sqrt{1/4 + 2(1 - 8/9)} \approx 0.18$. *Now,* $R \approx [0.4, 0.5]$ *and it is relatively simple to show that for any reserve* $\rho_1 \in R$, *the number of students assigned to their top choice decreases iff* $8/9 - 3/8 > \rho_1$. *Thus, for any* $\rho_1 < 8/9 - 3/8 \approx 0.51$ *with* $\rho_1 \in R$, *increasing the reserve reduces the number of students assigned to their top choice and increases the number of students that can Pareto improve by switching schools.*

We now show that the assumption that the conditional expectation of the application intensities is higher for regular students is key for Propositions 2 and 3.

Example 2 (Expected number of popular schools)**.** *Consider* $\beta = 1$ *and the following distributions of application intensities:*

$$\pi_m(0) = \alpha,\ \pi_m(1) = \cdots = \pi_m(L-2) = 0,\ \pi_m(L-1) = 1 - \alpha$$

and

$$\pi_r(0) = 0,\ \pi_r(1) = \alpha,\ \pi_r(2) = \cdots = \pi_r(L-1) = 0, \pi_r(L) = 1 - \alpha.$$

A fraction α *of minority students ranks first a tier 2 school, while a fraction* $(1-\alpha)$ *lists* $L-1$ *randomly chosen tier 1 schools. Analogously, a fraction* α *of regular students rank first one tier 1 school followed by a tier 2 school, and a fraction* $(1-\alpha)$ *lists* L *tier 1 schools. We assume* $L \leq n$. *Clearly,* Π_r *dominates* Π_m. *When* $\alpha > \frac{1}{L-1}$, $L - 1 = \mathbb{E}_r[l_r \mid l_r \geq 1] < \mathbb{E}_m[l_m \mid l_m \geq 1] = \alpha + (L-1)(1-\alpha)$. *Morever, using equation (A.2) in the Appendix, it is relatively simple to show that* $\frac{\partial F(1)}{\partial \rho_1} < 0$ *when* $\alpha + (1-\alpha)L < (L-1)(1-\rho_1 n)$. *When* $nk_1 < 1 - \frac{(1-\alpha)L+\alpha}{L-1} \in]0,1[$, *it follows that for all* $\rho_1 \in R$, $\frac{\partial F(1)}{\partial \rho_1} < 0$.

4.4 Extensions

We now discuss variations of our model and results. Subsection 4.4.1 explores a model in which preferences are polarized in the sense that minorities and regular students concentrate their applications over different sets of popular schools. In Subsection 4.4.2, we introduce a double reserve policy (Echenique and Yenmez 2015) and argue that, while this policy promotes inclusion with polarized preferences, it has no impact in our main model. Subsection 4.4.3 shows how our results apply when the affirmative action policy is implemented by setting aside seats (Dur, Kominers, Pathak, and Sönmez 2018).

4.4.1 Polarized preferences

In our main model, regular students concentrate their applications on high demand schools, while minority students apply with lower intensity to overdemanded schools. In theory (but not in our field data), segregation could arise because minorities and regular students concentrate their applications on different sets of overdemanded schools. The following example shows that under this type of preferences, our comparative statics results do not hold. Indeed, we show that the number of students assigned to their top school need not increase with reserves. As a result, for minority reserves to improve efficiency, it is not enough that distinct groups have different preferences.

Example 3 (Polarized prefences)**.** *We restrict our main model to $n = 2$, $\beta = 1$, $k_1 \leq 1$, $k_2 = 2$, but now we assume preferences are given by*

$$r : c_1 \succ c_2 \succ c_3 \quad m : c_2 \succ c_1 \succ c_3.$$

Schools rank students uniformly and independently. In this setup, while all students prefer tier 1 schools over the tier 2 school (school c_3), minority students prefer c_2 to c_1 while regular students prefer c_1 to c_2.

When no reserve is imposed, it is relatively simple to find the cutoff $\bar{p} = \sqrt{1 - k_1}$ for each tier 1 school. In the stable matching without reserves, minority students are underrepresented in c_1, and a fraction $F(1) = 1 - \sqrt{1 - k_1}$ of all students are assigned to their top school.

Now, we impose a reserve $\rho_1 \in [\sqrt{1 - k_1}(1 - \sqrt{1 - k_1}), k_1]$. We can characterize the stable matching by solving the market clearing conditions:

$$k_1 - \rho_1 = 1 - p_1^r \quad \rho_1 = p_2(1 - p_1^m) \quad k_1 = 1 - p_2 + p_1^r(1 - p_2)$$

where p_1^r (resp. p_1^m) is the cutoff faced by regular (resp. minority) students in school c_1. Solving the system of equations, we deduce that the fraction of students assigned to their top school is

$$F(1, \rho_1) = \frac{k_1 - \rho_1}{2} + \frac{1}{2}\frac{k_1}{2 - k_1 + \rho_1}.$$

The function $F(1, \rho_1)$ is decreasing in ρ_1.

The example shows that when both groups of students concentrate their applications in different schools, imposing a reserve reduces the number of students assigned to their top schools.[26] There are two forces behind this result. First, after the reserve is imposed in c_1, regular students are replaced by minority students for whom c_1 is not their most preferred school. Second, displaced regular students demand school c_2 and thus $1 - p_2$ decreases. As a result, fewer minority students are assigned to school c_2.

A model with polarized preferences is not a good description of application patterns in Chile. When preferences are polarized as in Example 3, in a stable matching (without reserves) minority students should be overrepresented in many popular schools. While it is true that in the data some popular schools are particularly attractive for minorities, those schools are the exception. Indeed, Appendix D shows that popular schools tend to have a low fraction of minority students.[27]

4.4.2 Double reserves

Another measure that can be used to promote integration in schools is to reserve seats for *both* types of students (Echenique and Yenmez 2015). We consider a double reserve policy such that in each tier 1 school, $\frac{\beta}{1+\beta}k_1$ seats are reserved to minority students and $\frac{1}{1+\beta}$ seats are reserved to regular students.

The double reserve policy will promote integration in Example 3. Indeed, the Duncan index with minority reserves $\frac{\beta}{1+\beta}k_1$ will be higher than the Duncan index under the double reserve policy. Intuitively, a double reserve policy allows regular students to get accepted in the popular school c_2 where minorities are overrepresented.

In contrast, moving from a minority reserve of $\frac{\beta}{1+\beta}k_1$ to a double reserve policy does not change the assignment in our main model. It is relatively simple to show that given a minority reserve $\frac{\beta}{1+\beta}k_1$, in each school the fraction of regular students equals $\frac{1}{1+\beta}$ and thus introducing an additional

[26]Note that this holds for all $k_1 \leq 1$. In particular, it holds even if the market is slack.

[27]Additionally, the comparative statics of a model with polarized preferences does not match the simulations using Chilean data, as shown in Sections 4 and 4.4.2.

reserve for regular students does not give any advantage to them. Importantly, in Appendix C.1, we confirm this theoretical finding using our field data.

4.4.3 Set asides

We have interpreted the affirmative action policy as a minimum guarantee for minority students. As noted by Dur, Kominers, Pathak, and Sönmez (2018), an alternative interpretation of an affirmative action policy is to set aside seats for minority students. Under a set aside policy, a school assigns first the $k_1 - \rho_1$ open seats and reserves the remaining ρ_1 seats for minority students. In this Subsection, we extend our results to this alternative affirmative action implementation.

To characterize a stable matching under set asides, we again consider cutoffs p_r^{SA} and p_m^{SA} that apply to regular and minority students in tier 1 schools under the set aside policy. The market clearing and reserve conditions for a set aside policy are

$$\frac{1}{n}\mathbb{E}_r[1 - G_r(p_r^{SA})^{l_r}] + \frac{\beta}{n}\mathbb{E}_m[1 - G_m(p_m^{SA})^{l_m}] = k_1$$

and

$$\frac{\beta}{n}\mathbb{E}_m[G_m(p_r^{SA})^{l_m} - G_m(p_m^{SA})^{l_m}] = \rho_1. \tag{4.3}$$

Equation (4.3) is the set aside condition. Motivated by Dur, Kominers, Pathak, and Sönmez (2018), the set aside condition says that the number of minority students with scores below the regular cutoff p_r^{SA} and that get admitted to a school should equal the reserve ρ_1. In contrast to minority reserves, under this interpretation of the affirmative action policy, the number of minority students effectively admitted to a tier 1 school exceeds the reserve ρ_1.

We define by $F^{SA}(q, \rho_1)$ as the fraction of students assigned to a school ranked q or below under a set aside affirmative action policy ρ_1. Analogously, we define $P^{AS}(\rho_1)$ as the total measure of students than can Pareto improve in the matching with set aside reserves ρ_1. The following result shows that the main insights from Propositions 2 and 3 extend to the set aside policy.

Proposition 4. *There exists $\bar{k} \in]0, 1[$ such that for all $k_1 < \bar{k}$ and all $\rho_1 < \bar{k}$,*

$$\frac{\partial F^{SA}(1, \rho_1)}{\partial \rho_1} > 0 \text{ and } \frac{\partial P^{SA}(\rho_1)}{\partial \rho_1} < 0.$$

The main intuitions and arguments behind this result are similar to the ones in Subsection 4.3 and are therefore omitted.

As Dur, Kominers, Pathak, and Sönmez (2018) show, in terms of the final assignment of students, the precedence order with which reserves are processed has an impact similar to adjusting reserve sizes. We derive a similar result in our framework. In contrast to the analysis by Dur, Kominers, Pathak, and Sönmez (2018), our focus is on the proportion of students assigned to their top schools and the number of students in Pareto improving pairs.

Proposition 5. *Under the conditions of Proposition 2, $F^{SA}(1, \rho_1) > F(1, \rho_1)$ and $P^{SA}(\rho_1) < P(\rho_1)$.*

Fewer minority students are assigned to tier 1 schools under minority reserves than under set aside. Changing the interpretation of the affirmative action policy from minority reserves to set asides increases the number of students assigned to their top schools and reduces the number of students who can Pareto improve by switching schools. Obviously, compared to the minority reserve policy, the set aside policy may or may not reduce segregation by placing more minority students in tier 1 schools.

Appendix C.2 simulates the set-aside policy and confirms all the theoretical predictions.

5 Discussion

Our results are relevant for policy discussion. For each of our markets, Table 3 reports outcomes with no reserves, reserves equal to 15% (as currently determined by the Law), 75%, 100% and equal to the fraction of minority students in the market. As can be seen, the simulations confirm each of our theoretical results in Section 4. The simulations also confirm the most subtle theoretical prediction from the model, Proposition 2.[28] Table 3 shows that the total number of student assigned to their top school and the total number of students assigned to schools that are not very attractive (ranked fourth or below) move precisely as predicted by our theory.

The simulations thus show that policymakers may integrate schools, increase the proportion of applicants obtaining their top choices, reduce the number of students in Pareto improving pairs, but incur the costs of leaving more students to relatively unattractive schools. In our quantitative exercise, minority reserves can reduce segregation by more than 20% in each of the three cities. At the same time, increasing minority reserves leaves slightly more students assigned to their top schools and reduces the proportion of students in Pareto improving pairs by close to 15%. The main cost of a reduction in segregation is an increase in the number of unassigned students. For

[28]Even though in our theoretical framework all the students are assigned, in the simulations we have computed the fraction of unassigned students as a measure of students whose assignment is unattractive. The fact that more students are unassigned as we increase the reserve is related to the result in Proposition 2 that the fraction of students assigned to schools that are not highly ranked increases with the reserve.

example, Table 3 shows that in Valparaíso, the Duncan index can be reduced from 0.316 to 0.247, while the proportion of unassigned students increases from 9.46% to 9.81% and the proportion of students in Pareto improving pairs reduces from 7.81% to 6.68%. How society weights all these outcomes will determine the optimal minority reserve.[29] More generally, our simulations show that the minority reserve is a significant policy decision that has sizable impact on important market outcomes and its size should be carefully evaluated in practical implementations.

Valparaíso	$f = 0\%$	$f = 15\%$	$f = 44\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.316 (0.005)	0.312 (0.005)	0.247 (0.003)	0.311 (0.003)	0.317 (0.002)
Minority students assigned to their top choice (Lemma 1)	71.73 (0.52)	72.05 (0.4)	78.89 (0.38)	88.69 (0.29)	89.54 (0.25)
Regular students assigned to their top choice (Lemma 1)	63.67 (0.37)	63.45 (0.48)	59.88 (0.35)	56.25 (0.35)	55.96 (0.27)
Students assigned to their top choice (Proposition 2)	67.21 (0.23)	67.23 (0.33)	68.23 (0.25)	70.49 (0.22)	70.7 (0.21)
Students assigned to their fourth choice or worst (Proposition 2)	7.32 (0.21)	7.29 (0.17)	7.6 (0.17)	8.24 (0.16)	8.31 (0.16)
Students unassigned (Proposition 2)	9.46 (0.16)	9.48 (0.19)	9.81 (0.14)	10.56 (0.13)	10.63 (0.13)
Students in Pareto improving pairs (3)	7.81 (0.53)	7.9 (0.51)	6.68 (0.57)	3.66 (0.26)	3.3 (0.33)
Concepción	$f = 0\%$	$f = 15\%$	$f = 43\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.353 (0.004)	0.342 (0.005)	0.264 (0.003)	0.355 (0.003)	0.358 (0.003)
Minority students assigned to their top choice (Lemma 1)	67.77 (0.43)	68.57 (0.44)	76.74 (0.37)	88.01 (0.2)	88.46 (0.22)
Regular students assigned to their top choice (Lemma 1)	58.06 (0.47)	57.4 (0.46)	54.11 (0.39)	50.68 (0.33)	50.5 (0.27)
Students assigned to their top choice (Proposition 2)	62.23 (0.29)	62.2 (0.33)	63.84 (0.26)	66.73 (0.24)	66.81 (0.17)
Students assigned to their fourth choice or worst (Proposition 2)	11.42 (0.21)	11.41 (0.25)	11.46 (0.21)	11.95 (0.15)	11.97 (0.16)
Students unassigned (Proposition 2)	12.65 (0.16)	12.75 (0.13)	13.08 (0.15)	13.61 (0.1)	13.63 (0.1)
Students in Pareto improving pairs (Proposition 3)	12.31 (0.41)	12.34 (0.54)	10.25 (0.44)	5.64 (0.37)	5.35 (0.3)
Santiago	$f = 0\%$	$f = 15\%$	$f = 37\%$	$f = 75\%$	$f = 100\%$
Duncan index (Proposition 1)	0.312 (0.002)	0.303 (0.002)	0.246 (0.001)	0.328 (0.001)	0.331 (0.001)
Minority students assigned to their top choice (Lemma 1)	70.86 (0.19)	71.58 (0.17)	77.81 (0.16)	90.79 (0.08)	91.35 (0.07)
Regular students assigned to their top choice (Lemma 1)	56.48 (0.17)	56.15 (0.19)	54.05 (0.12)	50.29 (0.11)	50.1 (0.1)
Students assigned to their top choice (Proposition 2)	61.87 (0.12)	61.93 (0.13)	62.95 (0.09)	65.46 (0.08)	65.55 (0.07)
Students assigned to their fourth choice or worst (Proposition 2)	12.3 (0.09)	12.32 (0.1)	12.49 (0.09)	13.14 (0.07)	13.19 (0.06)
Students unassigned (Proposition 2)	12.49 (0.06)	12.49 (0.05)	12.67 (0.06)	13.1 (0.05)	13.13 (0.04)
Students in Pareto improving pairs (Proposition 3)	9.5 (0.2)	9.43 (0.19)	8.06 (0.16)	4.52 (0.13)	4.39 (0.12)

Table 3: Average impact of minority reserves on market outcomes. Excluding the Duncan index, all values are percentages. Simulation standard deviations inside parentheses.

[29] The social welfare function $W = W(D, T, P, U)$ is likely to be decreasing in the proportion of students in Pareto improving pairs P, and in the proportion of unassigned students U. The function W will be increasing in the proportion of students assigned to their top schools T. The dependence of W on D will capture how educational outcomes depend on peer effects and how society values integration. For discussion on peer effects and educational outcomes, see Hoxby (2000), Hanushek, Kain, Markman, and Rivkin (2003), Angrist and Lang (2004), and Rao (2019). At least in some nontrivial range, W should be decreasing in D.

6 Conclusions

This paper studies the impact of minority reserves on segregation and efficiency in school choice programs. We show that minority reserves are an important tool to reduce segregation in schools. Minority reserves increase the total number of students assigned to their first preferences and improve efficiency, but more students are unassigned or assigned to unattractive schools. This paper contributes to the market design literature by making explicit the impact that minority reserves have on several market outcomes.

Our model highlights the basic economic forces that determine the impact of minority reserves on the final assignment. In particular, the theoretical analysis shows that the qualitative impact of minority reserves on the matching efficiency is determined by how different the preferences of distinct groups are and how tight the market is. Simulations with data from a large scale implementation complement our theoretical analysis and show the practical relevance of our results.

An important driver of our results is the fact that low income groups apply less to high demand institutions. Similar patterns have been documented in several contexts, such as the school choice programs in Boston and Amsterdam (Laverde 2020, Oosterbeek, Sóvágó, and van der Klaauw 2021) and college admission in the US (Hoxby and Avery 2013). We hope that our findings are relevant when discussing tools to reduce segregation in different environments.

Information policies may also impact segregation in schools by changing the application patterns of minority and regular students. These policies may determine the effectiveness of minority reserves. Reserves will remain a relevant policy instrument inasmuch as minority students apply less intensely to high demand schools. We have ignored any other impacts that changes to minority reserves may have, including migration of regular students to private schools, changes in the application patterns as a result of differences in school compositions, or location decisions (Epple and Romano 2003, Baum-Snow and Lutz 2011, Avery and Pathak 2021, Akbarpour, Kapor, Neilson, van Dijk, and Zimmerman 2022). These are important questions that are left to future research.

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APPENDIX

Appendix A contains proofs. Appendix B provides details about our field data. Appendix C simulates alternative affirmative action policies and confirms the theoretical findings in Section 4.4. Online Appendix D provides further details about our simulations.

A Proofs

Proof of Proposition 1. Note that for $\rho_1 \in R$, each tier 1 school has $k_1 - \rho_1$ regular students and ρ_1 minority students, while a tier 2 school has $(1 - n(k_1 - \rho_1))/N$ regular students and $(\beta - n\rho_1)/N$ minority students.[30] Therefore,

$$D(\rho) = \frac{1}{2}\Big\{ n|\frac{k_1 - \rho_1}{1} - \frac{\rho_1}{\beta}| + N|\frac{(1-(k_1-\rho_1)n)/N}{1} - \frac{(\beta - n\rho_1)/N}{\beta}| \Big\}.$$

The first (resp. second) term inside the bracket captures the summation defining $D(\rho)$ over tier 1 schools (resp. tier 2 schools). Thus, for $\rho_1 \in R$,

$$D(\rho) = \frac{1}{2}\Big\{ n|k_1 - \rho_1(1+\frac{1}{\beta})| + n|k_1 - \rho_1(1+\frac{1}{\beta})| \Big\}.$$

Note that for $\rho_1 \notin R$, $D(\rho)$ is flat. The result follows. $\square$

Proof of Proposition 2. We first note that for $q \leq L_t$, $F_t(q) = 1 - G_t(p_t)^q(1 - \Pi_t(q-1))$. Thus,

$$\begin{aligned}&(1+\beta)\frac{\partial F}{\partial \rho_1}(q, \rho_1)\\ &= -\, qG_r(p_r)^{q-1}g_r(p_r)(1 - M_r(q-1))\frac{\partial p_r}{\partial \rho_1} - \beta q G_m(p_m)^{q-1}g_m(p_m)(1-M_m(q-1))\frac{\partial p_m}{\partial \rho_1}.\end{aligned}$$

Now, recall that $\mathbb{E}_r[1 - G_r(p_r)^{l_r}] = (k_1 - \rho_1)n \quad \beta\mathbb{E}_m[1 - G_m(p_m)^{l_m}] = \rho_1 n$. Taking derivatives

$$\beta g_m(p_m)\frac{\partial p_m}{\partial \rho_1} = \frac{-n}{\mathbb{E}_m[l_m G_m(p_m)^{l_m - 1}]} \quad g_r(p_r)\frac{\partial p_r}{\partial \rho_1} = \frac{n}{\mathbb{E}_r[l_r G_r(p_r)^{l_r-1}]}$$

[30] To see the distribution of students in tier 2 schools, note that $1 - n(k_1 - \rho_1)$ regular students are not assigned to tier 1 schools. Regular students that are not assigned to tier 1 schools demand tier 2 schools uniformly.

We deduce that for $q \le L_m$

$$\frac{\partial F}{\partial \rho_1}(q) < 0 \text{ (resp. } > 0) \quad \text{iff} \quad \frac{G_m(p_m)^{q-1}(1-\Pi_m(q-1))}{\mathbb{E}_m[l_m G_m(p_m)^{l_m-1}]} < \frac{G_r(p_r)^{q-1}(1-\Pi_r(q-1))}{\mathbb{E}_r[l_r G_r(p_r)^{l_r-1}]} \text{ (resp. } >). \tag{A.1}$$

In particular,

$$\frac{\partial F}{\partial \rho_1}(1,\rho_1) > 0 \quad \text{iff} \quad \frac{\mathbb{E}_r[l_r G_r(p_r)^{l_r-1}]}{1-\Pi_r(0)} > \frac{\mathbb{E}_m[l_m G_m(p_m)^{l_m-1}]}{1-\Pi_m(0)}. \tag{A.2}$$

To derive a sufficient condition for the right condition on (A.2), we note that

$$\frac{\mathbb{E}_m[l_m G_m(p_m)^{l_m-1}]}{1-\Pi_m(0)} < \frac{\mathbb{E}_m[l_m]}{1-\Pi_m(0)} = \mathbb{E}_m[l_m \mid l_m \ge 1]. \tag{A.3}$$

Also,

$$G_r(p_r) > \mathbb{E}_r[G_r(p_r)^{l_r}] = 1 - n(k_1-\rho_1) > 1 - nk_1.$$

As a result,

$$\mathbb{E}_r[l_r G_r(p_r)^{l_r-1}] > \mathbb{E}_r[l_r\big(1-nk_1)^{l_r-1}\big)] = \mathbb{E}_r[l_r\big(1-nk_1)^{l_r-1}\big) \mid l_r \ge 1](1-\Pi_r(0)). \tag{A.4}$$

Using (A.2), (A.3) and (A.4), it follows that $\frac{\partial F(1,\rho_1)}{\partial \rho_1} > 0$ when

$$\mathbb{E}_r[l_r\big(1-nk_1)^{l_r-1}\big) \mid l_r \ge 1] \ge \mathbb{E}_m[l_m \mid l_m \ge 1].$$

By assumption, $\mathbb{E}_r[l_r \mid l_r \ge 1] > \mathbb{E}_m[l_m \mid l_m \ge 1]$. As a result, we can define

$$\bar{K} = \max\Big\{K \in [0,1] \mid \mathbb{E}_r[l_r(1-K)^{l_r} \mid l_r \ge 1] \ge \mathbb{E}_m[l_m \mid l_m \ge 1]\Big\} \in]0,1[$$

and for all $nk_1 \le \bar{K}$, $\frac{\partial F}{\partial \rho_1}(1) > 0$ for all ρ_1.

From Lemma 1, for $q \in \{L_m+1, \ldots, L_r\}$,

$$\frac{\partial F}{\partial \rho_1}(q) = \frac{\partial}{\partial \rho_1}\Big(\frac{\beta + F_r(q)}{1+\beta}\Big) < 0.$$

We thus deduce Proposition 2. □

Proof of Proposition 3. Consider any student s who is assigned to a tier 1 school $c = \mu_\rho(s)$ that is not her top choice . Let $\bar{c}$ be the top choice of student s. Consider the (positive measure) set $\bar{S} \subset S$ of all students such that they rank school c first, and school $\bar{c}$ second. Define $\hat{S} \subseteq \bar{S}$ by $\hat{S} = \{s' \in \bar{S},$

$\omega_c^{s'} < p_\rho < \omega_{\bar{c}}^{s'}\}$. By construction, $\hat{S}$ has positive measure. For any $s' \in \hat{S}$, $c \succ_{s'} \mu_\rho(s') = \bar{c}$. As a result, s can Pareto improve by switching school with $s' \in \hat{S}$.

If s is assigned to a tier 1 school that is her top choice, then it is clear that s cannot Pareto improve by switching school.

If s is assigned to a tier 2 school, then s is either assigned to her top choice or s would prefer a tier 1 school. If s is assigned to her top choice, then s cannot Pareo improve by switching school. If s would like to move to some tier 1 school, then all students assigned to that tier 1 school prefer their current school to the tier 2 school s is assigned to. So, s cannot Pareto improve by switching school.

It thus follows that a student s can Pareto improve by switching school iff s is assigned to a tier 1 school that is not her top choice. It thus follows that

$$\begin{aligned} P(\rho_1) &= 1 - F(1, \rho_1) - \frac{\sum_{l_r=1}^{L_r} \pi_r(l) G_r(p_r)^{l_r} + \beta \sum_{l_m=1}^{L_m} \pi_m(l) G_m(p_m)^{l_m}}{1+\beta} \\ &= 1 - F(1, \rho_1) - \frac{(1-\pi_r(0)) + \beta(1-\pi_m) - nk_1}{1+\beta} \end{aligned}$$

which is decreasing in ρ_1 under the conditions of Proposition 2. □

Proof of Proposition 4. The proof is identical to the proof of Proposition 2. □

Proof of Proposition 5. The cutoffs p_m^{SA} and p_r^{SA} are entirely determined by the intersection of the market clearing (4.1) and set aside (4.3) conditions. The set aside condition is to the left of the minority reserve condition (see also Figure 6) and therefore $p_r^{SA} > p_r$ and $p_m^{SA} < p_m$. By increasing ρ_1, the minority reserve condition (4.2) moves to the left. As a result, we can find $\rho_1' > \rho_1$ such that the cutoffs p_m' and p_r' under minority reserves ρ_1' satisfy $p_m^{SA} = p_m'$ and $p_r^{SA} = p_r'$. Proposition 2 implies that for a fixed ρ_1 more students are assigned to their top school under set asides than under minority reserves.

□

B Field data

B.1 Markets

The Chilean centralized system runs nationwide. While any student could apply to any school in the country, virtually all students apply exclusively within their provinces or districts. The system

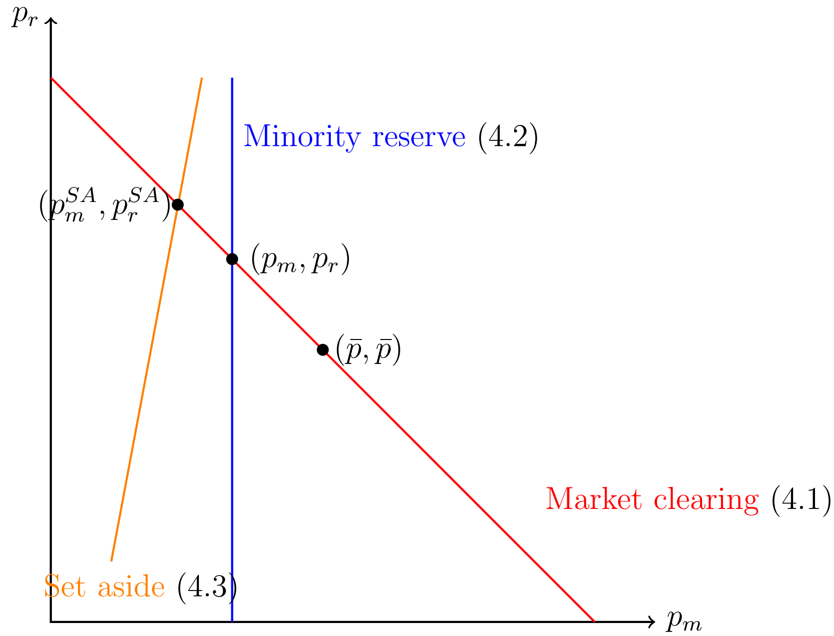


Figure 6: The market clearing condition and the set aside condition determine cutoffs p_r^{SA} and p_m^{SA}. For a given ρ_1, the set aside condition is to the left of the minority reserve condition.

is thus composed of several isolated markets. We show that each of our marketsis indeed isolated and virtually independent from the rest of the markets in the country.

We first define our markets. The Valparaiso market includes each school located in the provincial department of Valparaiso. The Concepcion market includes each school located in the provincial department of Concepcion. The Santiago market includes each school located in the Metropolitan Region of Santiago.[31] So, the boundary of each of our markets follows administrative definitions.

For each market, we consider all students that apply exclusively within the market. Thus a student with a rank order list including some schools in Valparaiso and others outside Valparaiso is excluded from our exercise. This set of students is small as big urban centers heavily concentrate applications. In our database, 99.76% of all nationwide applications listing some school in the Santiago market list exclusively schools in Santiago. The numbers for Valparaiso and Concepcion are 98.85% and 99.66%, respectively. The following table shows the characterization for each market:

[31] As Santiago is the main urban center in Chile the country, the provincial department of Santiago excludes several towns close to Santiago whose students apply to schools in the city. We thus work with the Metropolitan Region of Santiago.

Table 4: Valparaiso, Concepcion and Santiago markets

	Valparaíso	Concepción	Santiago
Number of provincial departments	1	1	7
Number of counties	10	12	52
Number of schools	275	250	1,214
Applicants to the market applying exclusively inside the market	98.85%	99.66%	99.76%

Thus, in practical terms, each of our markets is isolated and independent from all other markets in the country.

B.2 Alternative popularity definitions

We explore an alternative definition of popularity. A school is ν-popular if it is oversubscribed after running the DA algorithm (with no reserves) with probability at least ν (since schools define priorities using random inputs, a school may be oversuscribed for some but not all realizations of the lottery numbers). Clearly, a school c with $pop(c) \geq 1$ will be ν-popular for all $\nu \in [0, 1]$. For concreteness, set $\nu = 0.9$.

We construct the empirical distribution of application intensities for ν-popular schools. As shown below, we obtain the exact same results reported in Section 2.3.2. This shows the robustness of our claim that minority students apply with less intensity to high demand schools.

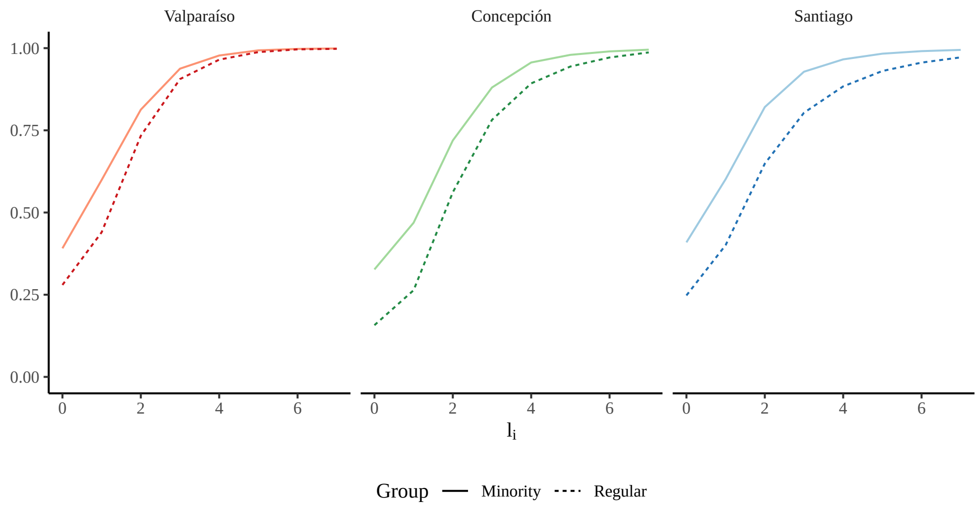


Figure 7: Empirical distributions of application intensities to ν-popular schools for each group t.

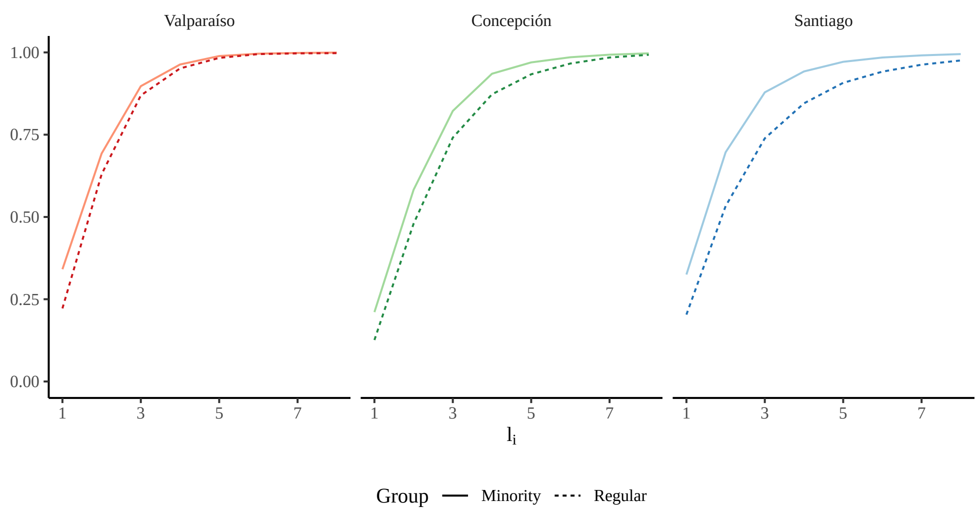


Figure 8: Empirical distributions of application intensities to ν-popular schools, conditional on applying to at least one ν-popular school.

C Other affirmative action policies

To put the design of the minority reserve policy in perspective, we now discuss the impact of other affirmative action policies on market outcomes. We only report results for Santiago (similar results are obtained for Valparaíso and Conception).

C.1 Double reserve policy

We now simulate the double reserve policy (Section 4.4.2). We compare minority reserves to a double reserve policy, where we reserve a fraction of seat equals to the proportion of the group in the market.

	Minority reserve	Double reserve
Santiago		
Duncan index	0.246 (0.001)	0.232 (0.001)
Students assigned to their top choice	62.95 (0.09)	62.89 (0.12)
Students assigned to their fourth choice or worst	12.49 (0.09)	12.37 (0.08)
Students unassigned	12.67 (0.06)	12.66 (0.05)
Students in Pareto improving pairs	8.06 (0.16)	8.06 (0.17)

Table 5: Average impact of single and double reserves on market outcomes. Excluding the Duncan index, all values are percentages. Simulation standard deviations inside parentheses.

Table 5 shows that moving from (single) minority reserves to a double reserve policy has a much smaller impact than moving from no reserve to minority reserve. This is precisely what our theory predicts. Note that the introduction of the ideal point policy reduces the number of students assigned to their top schools, similar to the model with polarized preferences in Example 3.

C.2 Set asides

We also simulated each of the markets using the set aside affirmative action policy. Consistent with Proposition 4, increasing the magnitude of the affirmative action policy has similar impacts under minority reserves and set asides. Tables 3 and 6 also confirm the prediction of Proposition 5 that fixing the reserves ρ, changing the interpretation of the affirmative action policy from minority reserves to set asides increases the number of students assigned to top schools and reduces the number of Pareto improving pairs. Under set asides, segregation is minimized for a reserve below the proportion of minority students in the population.

Santiago	$f = 0\%$	$f = 15\%$	$f = 37\%$	$f = 75\%$	$f = 100\%$
Duncan index	0.312 (0.002)	0.279 (0.001)	0.308 (0.001)	0.331 (0.001)	0.331 (0.001)
Minority students assigned to their top choice	70.91 (0.19)	80.22 (0.18)	88.35 (0.12)	91.33 (0.08)	91.34 (0.06)
Regular students assigned to their top choice	56.5 (0.16)	52.75 (0.14)	50.78 (0.12)	50.12 (0.12)	50.13 (0.1)
Students assigned to their top choice	61.9 (0.11)	63.05 (0.1)	64.85 (0.09)	65.56 (0.08)	65.57 (0.07)
Students assigned to their fourth choice or worst	12.3 (0.08)	12.7 (0.08)	13.04 (0.07)	13.2 (0.08)	13.19 (0.06)
Students unassigned	12.49 (0.05)	12.71 (0.05)	13.01 (0.05)	13.13 (0.04)	13.13 (0.05)
Students in Pareto improving pairs	9.46 (0.2)	8.09 (0.18)	5.53 (0.15)	4.38 (0.13)	4.35 (0.14)

Table 6: Average impact of set asides on market outcomes. Excluding the Duncan index, all values are percentages. Simulation standard deviations inside parentheses.

D Segregation in schools

In the main body of the paper, we have explored how an aggregate segregation index (the Duncan index) changes as minority reserves increase. Figure 9 shows how segregation in each school is determined by its popularity and by the minority reserve. Each school is an observation. As can be seen popular schools tend to have a lower fraction of minority students. The upper graphs are derived without any minority reserve. The upper graphs show that few popular schools have overrepresented minority students. The lower graphs are derived with minority reserves equal to the fraction of minority students in the population.

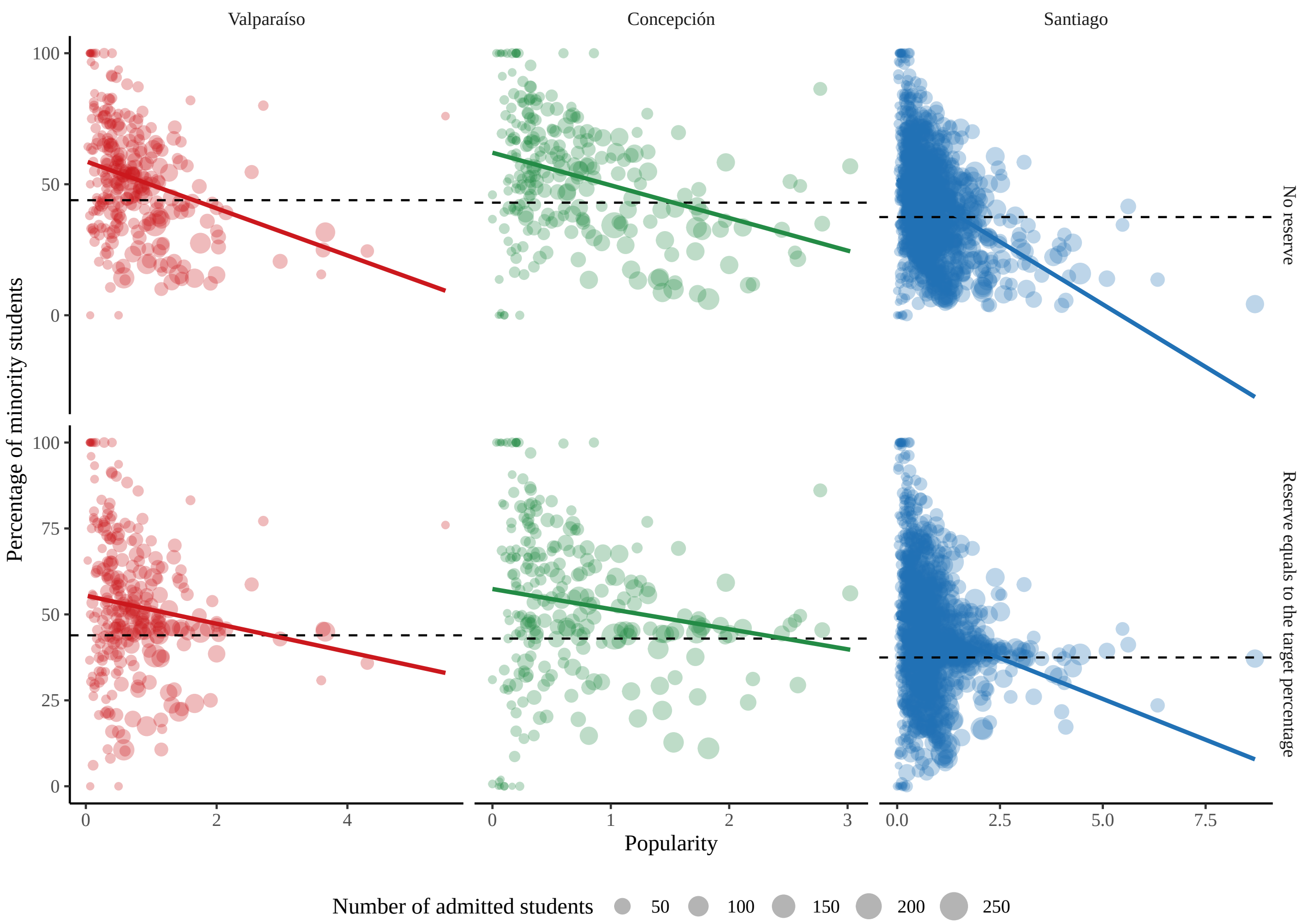


Figure 9: Schools composition. Minority students are under-represented in popular schools.

ONLINE APPENDIX

This Online Appendix contains further supportive results. Online Appendix E provides a version of Proposition 1 for alternative segregation indexes. Online Appendix F details application patterns in schools. Online Appendix G provides some evidence about the impact of the centralized platform in Chile on segregation.

E Other segregation indexes

We adapt Proposition 1 for the Hutchens index (Hutchens 2004):

$$H_\mu = 1 - \sum_{c \in C} \sqrt{\eta^r_\mu(c) \cdot \frac{\eta^m_\mu(c)}{\beta}}$$

Note first that H_μ does not depend on ρ_1 when $\rho_1 \notin [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))^l_m, \min\{\frac{\alpha_m \beta}{n}, k_1\}]$.

Recall that for $\rho_1 \in [\frac{\alpha_m \beta}{n}(1 - G_m(\bar{p}_m))^l_m, \min\{\frac{\alpha_m \beta}{n}, k_1\}]$, each tier 1 school has $k_1 - \rho_1$ regular students and ρ_1 minority ones. Each tier 2 school has $\frac{1-n(k_1-r_1)}{m}$ regular students and $\frac{\beta - nr_1}{m}$ minority ones. Thus, for ρ_1 in this range, the H-index is computed as:

$$H_\mu = 1 - \underbrace{n\sqrt{\frac{\rho_1(k_1 - \rho_1)}{\beta}}}_{H_1} - \underbrace{m\sqrt{\frac{(1 - n(k_1 - \rho_1))(\beta - n\rho_1)}{m^2\beta}}}_{H_2}$$

where the terms H_1 and H_2 correspond to the sum across tier 1 and tier 2 schools respectively.

Taking derivatives we get that:

$$\frac{\partial H_\mu}{\partial \rho_1} = -\frac{n}{2\sqrt{\beta}}\left(\frac{k_1 - 2\rho_1}{\sqrt{\rho_1(k_1 - \rho_1)}} + \frac{\beta - 1 + n(k_1 - 2\rho_1)}{\sqrt{(1 - n(k_1 - \rho_1))(\beta - n\rho_1)}}\right)$$

And also that:

$$\frac{\partial^2 H_\mu}{\partial \rho_1^2} = \frac{n}{4\sqrt{\beta}}\left(\frac{k_1^2}{[\rho_1(k_1 - \rho_1)]^{3/2}} + \frac{n(\beta + 1 - nk_1)^2}{[(1 - n(k_1 - \rho_1))(\beta - n\rho_1)]^{3/2}}\right) > 0$$

So we deduce that H_μ is a strictly convex function. Since $\frac{\partial H_\mu}{\partial \rho_1} = 0$ when $\rho_1 = \frac{\beta}{1+\beta}k_1$, the result follows.

The Atkinson index (Frankel and Volij 2011) can be defined in our setup as:

$$A_\mu = 1 - \left[\sum_{c\in C} \eta_\mu^r(c)^\delta \cdot \left(\frac{\eta_\mu^m(c)}{\beta}\right)^{1-\delta}\right]^{\frac{1}{1-\delta}}$$

Where $\delta \in (0,1)$ is a fixed weight. In the symmetric case in which both types are treated equally in the segregation index, $\delta = \frac{1}{2}$ and thus the Atkinson index is obtained by an increasing transformation of the Hutchens index. The result follows.

F Application patterns, standardized tests, and location

Understanding why minority students apply less to popular schools is beyond the scope of this paper. We observe that distance may be playing a role because minority students tend to live farther away from popular schools. To see this, in each market, we restrict our set of students to those that are market as *properly georeferenced* by the Chilean Ministry of Education[32]. For these set of students, we compute the distance to the closest popular school ($pop(c) > 1$) using the Vincenty (ellipsoid) method provided by the GEOSPHERE package from the R Statistical Software. The resulting distributions are presented below:

	Valparaíso		Concepción		Santiago	
	Regular	Minority	Regular	Minority	Regular	Minority
Sample (number of students)	2494	1833	2907	2029	22508	13089
First quartile	0.47	0.52	0.42	0.48	0.38	0.42
Median	0.81	0.92	0.70	0.82	0.63	0.69
Mean	2.86	1.37	1.13	1.27	0.97	1.19
Third quartile	1.36	1.59	1.16	1.38	1.02	1.07

Table 7: Distance (Km.) to the closest popular school

[32]Students that shared their location when applying on the platform or those whose location held a unique response and was marked as “rooftop” or “range_interpolated” in the “location_type” variable of Google’s Geocoding API.

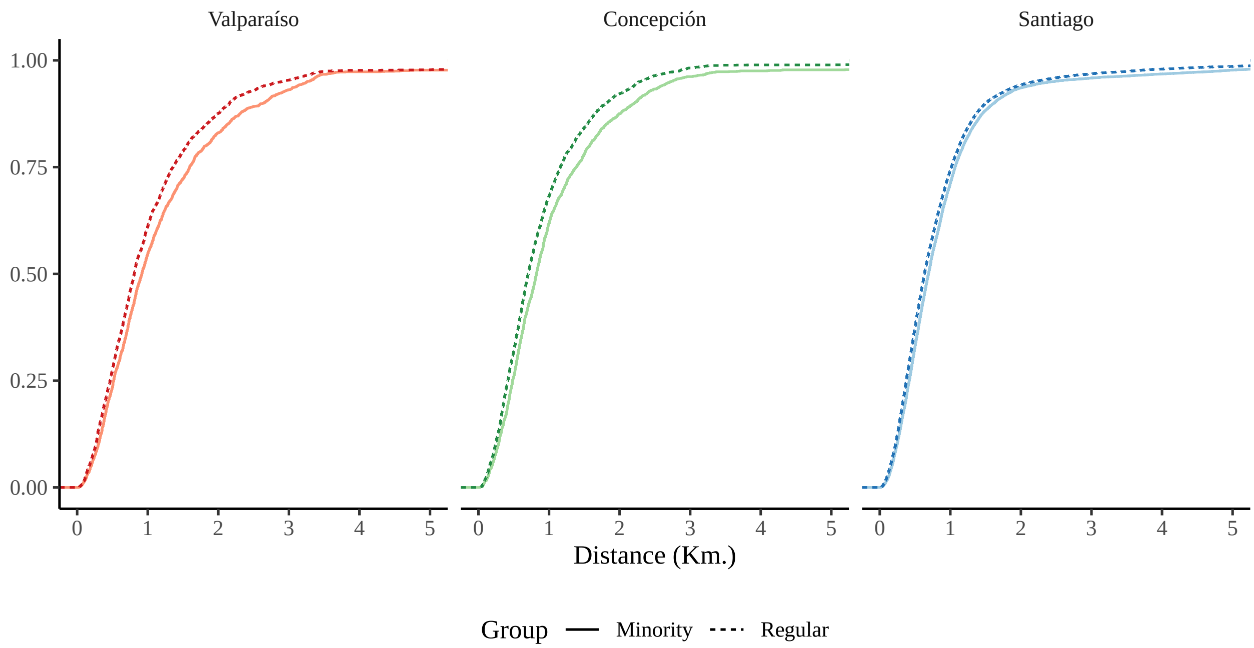


Figure 10: Distance to closest popular school. Minority students live farther away from popular schools than regular students.

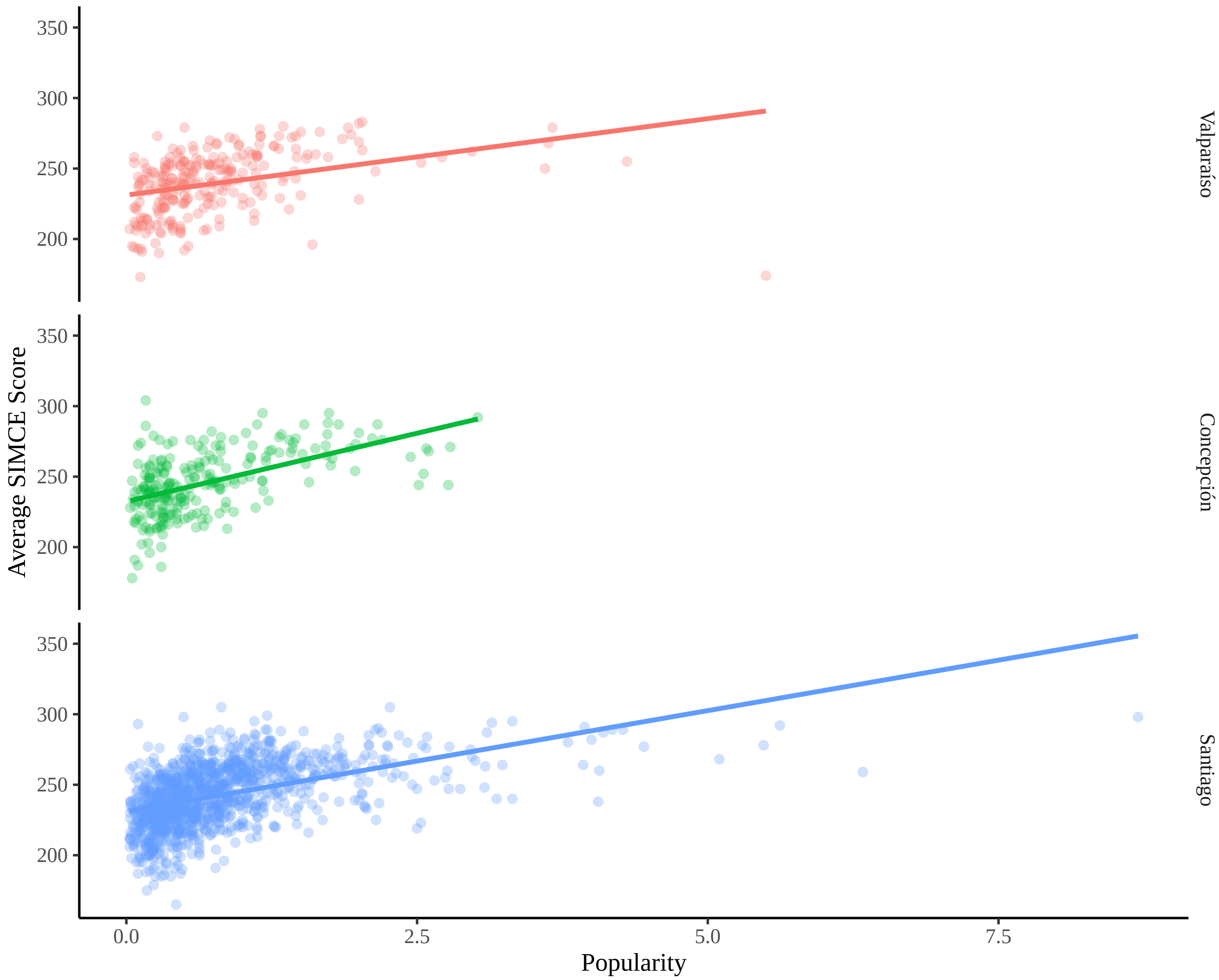
As discussed in the text, popular schools tend to perform better in standardized tests. For each market, we restrict our set of schools to those such that: (1) took part in SIMCE 2015 test[33] (2) reported valid SIMCE scores. This slightly decreases the set of schools we considered (so Table 8 has fewer schools than Table 4). We only use data from the Language test of second degree students in 2015. Popular schools are those such that $pop(c) > 1$.

Table 8: SIMCE scores

	Valparaíso		Concepción		Santiago	
	Not popular	Popular	Not popular	Popular	Not popular	Popular
Sample (number of schools)	200	61	185	53	884	302
First quartile	222	244	224	259	223.75	248.00
Median	239	259	239	269	236	260
Mean	234.84	254.21	239.21	267.47	236.87	259.10
Third quartile	250	269	252	277	251	271

[33]SIMCE is a standardized test taken to all students in the country

Figure 11: Popular schools tend to have higher SIMCE scores



These results show that popular schools have better performance in standardized tests. Obviously, this exercise is just illustrative and we are not claiming any causal effect.

G Centralized platform and its impact on segregation

We now present some evidence about the impact of the centralized system on segregation in Chilean cities. Our main data set is built using 3 sources of information:

- *Student enrollment*, available at:
  http://datos.mineduc.cl/dashboards/19776/descarga-bases-de-datos-de-matricula-por-estudiante/
- *Disadvantaged students*, available at:

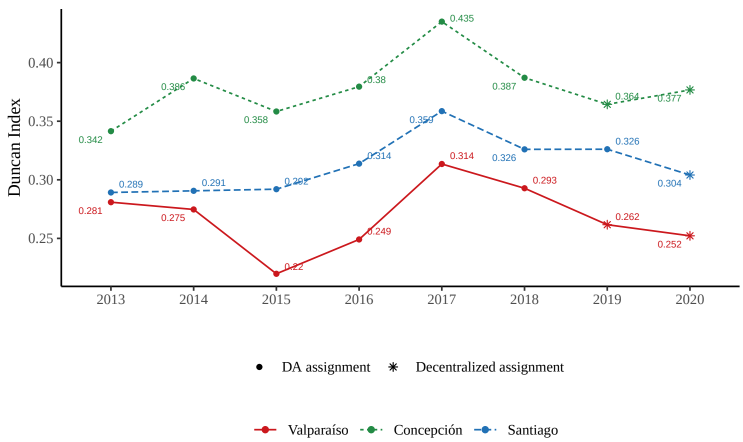
http://datos.mineduc.cl/dashboards/19939/bases-de-datos-alumnos-prioritarios/

- *SAE's supply 2020*, available at:

  http://datos.mineduc.cl/dashboards/20514/descarga-bases-de-datos-de-los-proceso-de-admision-escolar-anos-2016-y-2017/

We consider data from 2013 onward. For each year and each of the 16 regions in Chile, we define a market by selecting every school located in the *main provincial department* of the region. We consider every student enrolled in Pre Kinder in schools that were part of SAE in 2020 (this excludes private schools). We build a dummy variable ($\text{HAS\_SAE}_{i,t}$) if the assignment in region i and year t was centralized using SAE. As discussed in the main body of the paper, SAE was gradually introduced in the country. For each region i and year t, we also compute the fraction of disadvantaged students ($\text{DISAD\_FRAC}_{i,t}$) and the Duncan index ($\text{SEG\_INDEX}_{i,t}$).

The following graph shows the evolution of the Duncan index in Valparaiso, Concepcion, and Valparaiso.



The following are the main regression results. For different specifications, the introduction of SAE results in a relatively modest reductions of the Duncan index. This shows that the impact that minority reserves have on segregation is relatively important.

Table 9: Regression results

	Dependent variable:				
	seg_index				
	OLS				
	(1)	(2)	(3)	(4)	(5)
has_sae1	−0.012 (0.011)	−0.005 (0.012)	−0.012 (0.009)	−0.034 (0.023)	−0.017* (0.010)
disad_frac		0.075 (0.057)	−0.002 (0.083)		
Region fixed effect			Yes		Yes
Year fixed effect				Yes	Yes
Constant	0.313*** (0.006)	0.269*** (0.034)	0.241*** (0.049)	0.292*** (0.013)	0.221*** (0.009)
Observations	122	122	122	122	122
R^2	0.009	0.024	0.717	0.191	0.893
Adjusted R^2	0.001	0.007	0.671	0.134	0.868
Residual Std. Error	0.056 (df = 120)	0.056 (df = 119)	0.032 (df = 104)	0.052 (df = 113)	0.020 (df = 98)
F Statistic	1.112 (df = 1; 120)	1.438 (df = 2; 119)	15.535*** (df = 17; 104)	3.339*** (df = 8; 113)	35.700*** (df = 23; 98)

Note: *p<0.1; **p<0.05; ***p<0.01