

# School Choice in Chile

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Centralized school admission mechanisms are an attractive way for improving social welfare and fairness in large educational systems. In this paper we report the design and implementation of the newly established school choice mechanism in Chile, where over 274,000 students applied to more than 6,400 schools. The Chilean system presents unprecedented design challenges that make it unique. On the one hand, it is a simultaneous nationwide system, making it one of the largest school admission problems worldwide. On the other hand, the system runs at all school levels, from Pre-K to 12th grade, raising at least two issues of outmost importance; namely, the system needs to guarantee their current seat to students applying for a school change, and the system has to favor the assignment of siblings to the same school. As in other systems around the world, we develop a model based on the celebrated Deferred Acceptance algorithm. The algorithm deals not only with the aforementioned issues, but also with further practical features such as soft-bounds and overlapping types. In this context we analyze new stability definitions, present the results of its implementation and conduct simulations showing the benefits of the innovations of the implemented system.

## 1 INTRODUCTION

According to the Duncan Index of segregation, Chilean schools are extremely socially segregated [34]. Several authors have shown that the costs of school segregation are important and include low social cohesion and mobility, and lack of equal opportunities [14, 25, 35–37]. While the drivers of school segregation in Chile include societal aspects well beyond the school system, social movements and politicians were provably right at blaming features of the admission system.

The School Inclusion Law marks a breaking point in the organization and functioning of the school admission system. The Law, promulgated in 2015, changed the old admission system drastically by (i) eliminating co-payments in publicly subsidized schools, (ii) forbidding publicly subsidized schools from selecting their students based on social, religious, economic, or academic criteria, and (iii) defining priorities that must be used to assign students to schools.

In this paper we report the results of an ongoing collaboration with the Chilean Ministry of Education (MINEDUC) addressing the practical challenges of implementing the School Inclusion Law. To this end, we designed and implemented a centralized system that (i) provides information about schools—seats available, mission, values, educational project, among others—to help parents and students in building their preferences; (ii) collects families' preferences through an online platform, reducing the time and cost that visiting each school implied in the past; and (iii) assigns students to schools using a transparent, fair and efficient procedure.

At the core of the system is the assignment algorithm, which adapts the celebrated Deferred Acceptance (DA) algorithm—introduced in the seminal paper by Gale and Shapley [21]—to incorporate all the elements required by law and by MINEDUC. In particular, the system considers a set of priority groups—students with siblings in the school, students with parents that work in the school and former students of the school—that are served in strict order of priority. The system also includes quotas for students (i) from disadvantageous environments, (ii) with special educational needs and disabilities, and (iii) with high academic performance. Within each priority group, ties are broken randomly.

One of the distinctive features of the new school admission system in Chile is its broadness; it runs nationwide and throughout all school levels. These two aspects bring new challenges for the design and implementation of a school choice system. Being nationwide, the school admission system changes the way in which families are assigned to schools, and this makes the transmission of information essential to the implementation. The key to these challenges is gradualism. The system was first implemented in 2016 in the least populous of the sixteen regions in Chile. This allowed to

gain practical experience to improve the system as more regions were subsequently added. The system will be fully in place by 2020. From a computational perspective, our implementation of the Deferred Acceptance algorithm relies in an adaptation of the approach of directed graphs proposed by Baïou and Balinski [12], which is efficient and allows us to find an assignment in a few seconds. The admission system runs throughout all educational levels from the highest (12th grade) to the lowest (Pre-K). This makes patent the need to secure their current enrollment to students who want to change to another school. Additionally, some families may have two or more of their children simultaneously applying to schools and may naturally want their children to attend the same school. Two features of our implementation favor the assignment of siblings to the same school. First, the tie-breaking lotteries are run over families (rather than over students). Second, families can express their willingness to have their children assigned to the same school by filling a *block application*. A block application ensures that once the oldest child is assigned to some school, the application of the younger is modified to put that school as their most preferred one.

In summary, our design presents at least three innovative features:

- It allows each student currently enrolled in a given school to apply to a different school while guaranteeing him/her a seat in his/her current school.
- It runs tie-breaking lotteries over families, significantly increasing the fraction of siblings that end up assigned to the same school.
- It adds a block application heuristic that improves the chances of having siblings assigned to the same school.

The results reported in this paper consider the current state of the system, which includes all regions, except the Metropolitan area of Santiago, reaching 274,990 students and 6,421 schools in the main round. In the current admission process—for students who will start their academic year in March, 2019—students applied to 3.4 schools on average, and 59.2% of students were assigned to their top preference. Moreover, 82.5% of students were assigned to one of the schools in their application list, 8.6% were assigned by secured enrollment to their current schools, and only 8.9% resulted unassigned. In addition, there were 10,301 blocks involving 21,424 students and 65.3% of these were successful, i.e. siblings got assigned to the same school, while 3% were partially successful, i.e. only a subset of siblings got assigned together.<sup>1</sup> We also provide simulations evaluating different elements of our design.

Designing, implementing and improving the Chilean school choice system has resulted in many lessons that could be useful for other practitioners designing large-scale clearinghouses. From a practical standpoint, having a continuous communication and collaboration with policy-makers is essential, as many aspects evolve over time and must be incorporated in the design. In addition, fragmenting the implementation in a given number of steps allowed us to gain experience, solve unexpected problems and continuously improve the system. As centralized procedures to assign students to schools are becoming the norm in many countries, we expect that the lessons and solutions offered in this work are deemed useful in other implementations.

The remainder of the paper is organized as follows. In Section 2 we describe the school choice problem in Chile, with the main features requested by law and by MINEDUC. In Section 3 we discuss how this paper relates to several strands of the literature. In Section 4 we present our model and describe its implementation. In Section 5 we present the results, focusing on the admission process of 2018. In addition, we evaluate the effects of (i) quotas for disadvantaged students and (ii) block applications via simulations. Finally, in Section 6 we conclude and provide directions for future work.

<sup>1</sup>This 3% corresponds to 307 partially successful blocks. However, only 750 blocks were of size 3 or more, therefore this represents 41% of the possibly successful blocks.

## 2 THE PROBLEM IN CHILE

The Chilean school choice system considers fourteen levels, ranging from pre-kindergarten to 12th grade. Among these there are five entry levels: pre-kindergarten, kindergarten, 1st, 7th and 9th grade, which are the levels where a school can start. Depending on their type of funding, schools can be classified in three types: (1) private, for those schools that are independent and privately funded; (2) voucher, where families make co-payments to complement state subsidies; and (3) public, for those schools that are fully funded and operated by local governments.<sup>2</sup> Voucher and public schools, which are the focus of this paper, account for more than 90.3% of the total number of students in primary and secondary education [30].

Before the introduction of the School Inclusion Law, schools ran their admission processes independently, often selecting their students based on arbitrary rules, such as interviews with the students and their parents, results of unofficial admission exams, past academic records, among many others. Since the admission processes were not coordinated, in many cases parents were forced to strategically decide whether to accept an offer or to reject it and wait until other schools released their admission offers. Moreover, many schools used “first-come first-served” rules to prioritize students, resulting in many parents waiting in long overnight queues to secure a seat for their children.<sup>3</sup> Overall, the freedom of schools to choose their students and the existence of voucher schools are considered among the main reasons that explain the polarization and segregation of the Chilean school system [33].

To address these problems, the School Inclusion Law forbids any sort of discrimination in the admission processes of schools that receive (partial or full) government funding, and mandates schools to use a centralized system that collects families’ and students’ preferences and returns a fair allocation. In this system, students and families can access a platform where they can collect information—number of open seats, number of students per classroom and level, educational project, rules and values, co-payments required, among others—to build their preferences, and later they can use it to apply to as many schools as they want by submitting a strict order of preferences. The system collects all these applications and runs a mechanism that aims to assign each student to their top preference provided that there are enough seats available. More specifically, if the number of applicants is less than the number of open seats, the law requires that all students applying to that school are admitted, unless they can be allocated in a school they prefer. On the other hand, for schools that are over-demanded the law defines a set of priority groups that are used to order students. In particular, there are three priority groups, which are processed in strict order of priority:

- (1) *Siblings*. For students that have a sibling already enrolled at the school.
- (2) *Working parent*. For students that have a parent working at the school.
- (3) *Former students*. For students that were enrolled at the school in the past and were not expelled from it.

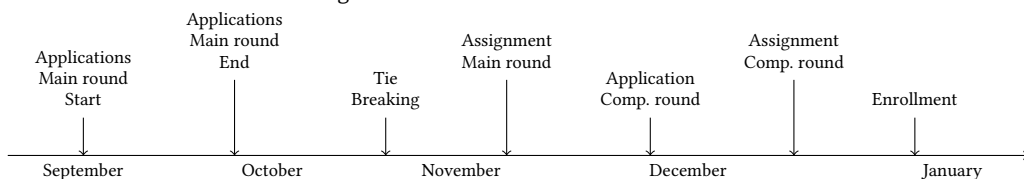
Within each priority group students are randomly ordered and each school uses a different random tie-breaker. In addition to these priorities, the law specifies three different types of quotas:

- (1) *Special needs*. This quota serves students with disabilities. It reserves at most two seats per classroom per school and it is processed before any other priority group or quota. The quota only applies to schools who have a validated special program.

<sup>2</sup> The requirements for a school to be partially funded are relatively minimal. The most relevant ones are: (1) have a school building that is certified and allowed to be used as a school, (2) have at least 15% of disadvantaged students if there is enough demand, (3) have a set of rules of procedure that define the relation between students and teachers and (4) have all salaries paid at the moment of applying to government funds.

<sup>3</sup> Some recent news articles illustrating the problem are [1], [16], [32].

Fig. 1. Timeline of the Admission Process



- (2) *High-achieving*. This quota applies to students with high academic performance. It is processed right after the special needs quota, and considers between 30% to 85% of the total number of seats depending on the school. Only a subset of pre-selected schools can implement this quota in 7th and 9th grade, taking an exam to the applicants to rank them.
- (3) *Disadvantaged*. This quota prioritizes the most vulnerable students (bottom third in terms of income).<sup>4</sup> In each level of all schools 15% of seats are reserved to disadvantaged students, and this group of students must be processed right after the first priority group, i.e. students with siblings.

The example in Appendix A.1 illustrates how quotas and priorities are processed.

There are two additional features that are relevant in the design of the system. First, students that are currently enrolled at a school and apply in the system to change their allocation have a *secured enrollment* in their current school, i.e. in case of not being assigned to a school of their preference they can keep their current assignment. Second, families having two or more children that participate in the centralized system can choose to *apply as a block*, which means that they prioritize having their children assigned to the same school over a better school in their reported preferences.

The timing of the admission process is summarized in Figure 1. After collecting information, families submit their application between September and October. After all applications are received, the centralized mechanism generates the lotteries that will be used to order students in over-demanded schools and executes the main round of the process. For those students that result unassigned or did not apply in the main round, there is a complementary process where students submit a new application list that includes schools with available seats. Finally, students that result unassigned in the complementary round are assigned to the closest school with available seats that does not charge a co-payment. We refer to this as *distance assignment*. In case that there are no schools with seats available within 17km, students remain unassigned and MINEDUC gives them a solution. The process ends in late December with the enrollment.

### 3 LITERATURE

This paper is related to four strands of the literature: school choice, affirmative action, assignment of families and tie-breaking.

*School Choice*. In the last two decades, and starting from the theoretical formalization of the school choice problem by Abdulkadiroğlu and Sönmez [8], there have been reforms to the school choice systems of many places around the world. The first major reform was introduced in New York, where a variation of the Deferred Acceptance (DA) algorithm with restricted lists was implemented [5]. In 2005, the Boston Public School system decided to switch from the so called *Boston Mechanism* (BM), also known as *Immediate Acceptance* (IA) mechanism, to DA in order to address the strategic incentives introduced by the former algorithm [7]. Despite its lack of strategy-proofness, BM has

<sup>4</sup>According to the Social Registry of Homes

been revisited in the last few years since it better captures cardinal preferences and therefore can lead to higher social welfare [4]. Since then, other systems such as Barcelona [15], Amsterdam [22], New Orleans [3], among others, have implemented centralized school choice systems using some variant of DA, BM or *top-trading cycles* (TTC). This paper contributes to this literature by adding a new case study with some additional features that have not been explored in previous literature, such as the admission of siblings in different levels and the secured enrollment problem. In addition, this is one of the first papers that describes the implementation of a system at a country level.

*Priorities and Affirmative Action.* Many school choice systems include affirmative action policies to promote diversity in the classrooms. Abdulkadiroğlu [2] explores DA under type-specific quotas, finding that the student-proposing DA is strategy proof for students if schools' preferences satisfy responsiveness. Kojima [27] studies the implementation of majority quotas and shows that this may actually hurt minority students. Consequently, Hafalir et al. [23] propose the use of minority reserves to overcome this problem, showing that DA with minority reserves Pareto dominates the one with majority quotas. Ehlers et al. [20] extend the previous model to account for multiple disjoint types, and propose extensions of DA to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima [26], who study problems with distributional constraints motivated by the Japanese Medical Residency. Dur et al. [18] analyze the Boston's school system and Dur et al. [19] analyze impact of policies using Chicago's system data. However, these models consider disjoint types of students. Kurata et al. [29] study the overlapping types problem and show that, even in the soft-bound minority quotas scenario, a stable matching might not exist. As a solution, they propose a model in which stability is recovered by counting each student towards the seat type he was assigned to. Our contribution to this literature is by implementing a new case study and analyzing the effect of different minority quotas in the implementation. Moreover, we show that quotas depend on students truly being a minority and that there can also be some auto-segregation of minorities.

*Assignment of families.* When designing a school choice mechanism, families are in general considered. We will analyze two aspects related to families in our system (i) block application for two or more siblings who apply to the same schools and (ii) the tie-breaking rule used. The block application can be thought both as the existence of coalitions in the system, or as a many-to-many matching between families and schools. A similar structure can be found in the labor market of medical residents, where couples want to work in the same city. Studying that market, Roth [31] shows that one cannot guarantee the existence of a matching without justified envy when couples have arbitrary preferences over pairs of hospitals. Kojima et al. [28] show that a stable matching exists if the number of couples is relatively small and preference lists are sufficiently short relative to the size of the market. Another positive result is presented by Ashlagi et al. [10], who introduce a new algorithm that finds a stable matching with high probability (in large matching markets) and where truth-telling becomes an approximate equilibrium for the induced game.

*Tie-breaking.* A common approach to deal with ties in school choice is to use random tie-breaking rules, such as *single tie-breaking* (STB) —all schools use the same ordering for breaking ties— and *multiple tie-breaking* (MTB) —each school uses its own random order. Abdulkadiroğlu et al. [6] are the first to empirically compare these tie-breaking rules, and they find that there is no stochastic dominance in NYC. A similar pattern is found in Amsterdam, as described by De Haan et al. [17]. These findings are in line with the theoretical results by Ashlagi et al. [11], who find that when there is low competition there is no stochastic dominance between random assignments. However, they also show that when there is a shortage of seats STB almost dominates MTB, and also leads to a lower variance in students' rankings. Moreover, Arnosti [9] shows that STB can lead to more

matches when preferences are short and random. We contribute to this literature by introducing and studying the effect of block applications and breaking ties between families instead of students.

#### 4 MODEL AND IMPLEMENTATION

The Chilean school choice problem can be formalized as follows. Let  $K$  be the set of all levels, including pre-kindergarten and kindergarten in preschool, 1st to 8th grade in primary school and 9th to 12th grade in secondary school. Each school offers some or all of these levels, and students can only apply to schools that offer their level. For simplicity, we will first focus on a fixed level to define the basic setting, and later (in Section 4.3) we will introduce how levels interact with each other through the block application.

At a fixed level, let  $S = \{s_1, \dots, s_n\}$  be the set of students and  $C = \{c_1, c_2, \dots, c_m\}$  be the set of schools. Every school  $c$  has a capacity  $q_c \in \mathbb{N}$  that accounts for the number of available seats. Students have a strict preference profile  $\succ_S = (\succ_{s_1}, \dots, \succ_{s_n})$  over schools, where  $c \succ_s c'$  means that student  $s$  strictly prefers school  $c$  to  $c'$ . Students who rank a subset of schools implicitly declare that they prefer to be unassigned than to be assigned to a school that is not in their preference list.

Schools, on the other hand, have a weak priority profile  $\succeq_C = (\succeq_{c_1}, \dots, \succeq_{c_m})$  over students, where  $s \succeq_c s'$  means that at school  $c$ , student  $s$  has a higher or equal priority ranking than student  $s'$ . Students within a priority group are randomly ordered, creating a strict priority profile  $\succ_C = (\succ_{c_1}, \dots, \succ_{c_m})$  over students.

A **matching**  $\mu$  is a function from the set  $C \cup S$  to the subsets of  $C \cup S$  such that:

- (i)  $\mu(s) \in C$  or  $\mu(s) = \emptyset$  for every student  $s \in S$ .
- (ii)  $\mu(c) \subseteq S$  and  $|\mu(c)| \leq q_c$  for every school  $c \in C$ .
- (iii)  $c \in \mu(s)$  if and only if  $s \in \mu(c)$  for every student  $s \in S$  and school  $c \in C$ .

This definition formalizes that a feasible matching cannot assign more students to a school than its capacity, and it cannot assign a student to more than one school.

In what follows, we extend this model to address the particular features of the Chilean problem, namely: (1) secured enrollment, for students already enrolled at a school; (2) quotas for students of different types; (3) block applications; and (4) the tie breaking rule.

##### 4.1 Secured enrollment

The system allows students of any level to apply to schools of their choice, provided that those schools offer their level. In particular, some students may want to change to another school they prefer, but are better off at their current school than remaining unassigned or assigned to other schools they prefer less. The School Inclusion Law gives students the right to keep a seat at their current school in case they do not get a better assignment.

To address this requirement, we add to each student that participates in the system and seeks to change to another school a preference over the current school at the bottom of their preference list. In addition, we consider the secured enrollment of a student as a priority criterion with a higher priority than any other priority group. With this extra criterion, the complete list of priority groups is given by:

$$\text{Secured Enrollment} \succ_c \text{Siblings} \succ_c \text{Working Parent} \succ_c \text{Former Students} \quad \forall c \in C. \quad (1)$$

##### 4.2 Quotas

In order to promote diversity within schools, the School Inclusion Law contemplates the implementation of affirmative action policies for financially disadvantaged students as well as children with special needs. Furthermore, a limited number of schools are allowed to reserve seats for students with high-achieving records.

Let  $T = \{\textit{Special needs}, \textit{High-achieving}, \textit{Disadvantaged}, \textit{Regular}\}$  be the set of all possible types that students may belong to. In general, for each student, these types are school-dependent<sup>5</sup> and may overlap. Each student belongs to at least one type, being *Regular* the default (i.e., *Regular* encodes the absence of type). We define a mapping  $\tau : S \times C \rightarrow 2^T$  that maps students to their types on each school.

A function  $p : C \times T \rightarrow \mathbb{N}$  defines type-specific quotas for each school, where  $p_{ct}$  represents a soft lower bound for school  $c$ , i.e. a flexible limit that regulates school  $c$ 's priorities dynamically, giving higher priority to students of type  $t$  up to filling  $p_{ct}$  seats. Furthermore, we assume that in every school the quotas can be met without violating its capacity, i.e.  $\sum_{t \in T} p_{ct} \leq q_c$  holds for every school  $c \in C$ .

As shown by Kurata et al. [29], when student types can overlap, the general concepts of stability for a matching with soft lower bounds proposed in literature [20, 23] are insufficient to guarantee the existence of a stable matching. To overcome this difficulty, they propose a new model based on the framework of matching with contracts due to Hatfield and Milgrom [24]. In this new model, schools provide distinct reserved seats for each student type, and assignments are interpreted as contracts that explicitly state that a student is assigned to a particular reserved seat at a school, in contrast to previous models where a student accounts for seats of multiple types.

We adhere to Kurata's setting by extending our model as follows. Every student  $s$  has now strict preferences  $\succ_s$  over contracts of the form  $(c, t) \in C \times T$  and every school  $c$  has now a weak priority profile  $\succeq_c$  over contracts of the form  $(s, t) \in S \times T$ . Then, a **matching**  $\mu$  is a function from  $(S \cup C) \times T$  to the subsets of  $(S \cup C) \times T$  such that:

- (i)  $\mu(s) \in C \times T$  or  $\mu(s) = \emptyset$  for every student  $s \in S$ .
- (ii)  $\mu(c) \subseteq S \times T$  and  $|\mu(c)| \leq q_c$  for every school  $c \in C$ .
- (iii)  $\mu(s) = (c, t)$  if and only if  $(s, t) \in \mu(c)$  for every student  $s \in S$ , school  $c \in C$  and type  $t \in T$ .

In other words, a student  $s$  is either unassigned or assigned to a seat of type  $t$  in school  $c$ ,  $\mu(c)$  is the set of students assigned at school  $c$ , each one to a type-specific seat, and student  $s$  is assigned to a seat of type  $t$  in school  $c$  if and only if school  $c$ 's assignment contains  $s$  assigned to a seat of type  $t$ . Note that this definition does not require that type  $t$  students must be matched to seats of type  $t$ .

Let  $\mu_t(c) := \{s \in S : \mu(s) \in \{c\} \times T \text{ and } t \in \tau(s, c)\}$  be the set of students of type  $t$  assigned to school  $c$ . Two well-known and desirable properties of a matching studied in the literature of school choice problems are to be fair or justified envy-free and non-wasteful. In our setting, a student  $s$  has **justified envy** towards a student  $s'$  with assignment  $\mu(s') = (c', t')$  in matching  $\mu$  if there exists a type  $t \in T$  such that:

- (i)  $(c', t) \succ_s \mu(s)$ ,
- (ii)  $(s, t) \succ_{c'} (s', t')$ ,
- (iii) and either  $t' = t$  or  $|\mu_{t'}(c')| > q_{c't'}$ .

That is,  $s$  has justified envy towards  $s'$  assigned to school  $c'$  in a seat of type  $t'$  if either  $s$  prefers  $(c', t')$  to his assignment and is preferred by the school on that seat, or for some type  $t \neq t'$ ,  $s$  prefers  $(c', t)$  to his assignment, school  $c'$  has exceeded the quota of type  $t'$  students and prefers  $s$  in a seat of type  $t$  to  $s'$  in a seat of type  $t'$ .

A student  $s$  **claims an empty seat** of a school  $c$  in matching  $\mu$  if there exists  $t \in T$  such that  $(c, t) \succ_s \mu(s)$  and one of the following conditions hold:

- (i)  $|\mu(c)| < q_c$  or
- (ii)  $\mu(s) = (c, t')$  for some type  $t' \in T$ ,  $(s, t) \succ_c (s, t')$  and  $|\mu_{t'}(c)| > q_{ct'}$ .

<sup>5</sup>As an exception, the disadvantaged type is school-independent.

A student  $s$  **claims an empty seat by type** in school  $c$  in matching  $\mu$  if there exists  $t \in T$  such that  $(c, t) \succ_s \mu(s)$  and the following condition holds:

$$(iii) |\mu_t(c)| < q_{ct}.$$

Namely,  $s$  claims an empty seat of type  $t$  at  $c$  if  $s$  prefers that contract to her assignment and either  $c$  has empty seats, or  $s$  is assigned to  $c$  in a seat that exceeded its quota and the school prefers having  $s$  in a seat of type  $t$ , or the quota of type  $t$  students has not been met at  $c$ .

When no student claims an empty seat or claims an empty seat by type at any school, we say the matching is **non-wasteful**. Finally, a matching is **stable** if it is non-wasteful and eliminates justified envy for all students. This notion of stability matches the one proposed by Kurata et al. [29], even when we assume some students might remain unassigned.

In Table 1 we describe the schools' weak preference profiles over contracts  $(s, t) \in S \times T$  for each fixed type  $t \in T$ . At every school  $c$ , students currently enrolled at the school that are participating in the system have the highest priority in all types of seats. Then, for the special needs and high-achieving seats, students of the corresponding type are given the second highest priority, and the rest of the students are given priorities according to (1). As required by law, students that have siblings currently enrolled at the school have higher priority than disadvantaged students, even in seats reserved for that type.

Table 1. Weak priorities by type-specific seats. Lower numbers indicate higher priority.

| Priority | $\succsim_{c, \text{special needs}}$ | $\succsim_{c, \text{high-achieving}}$ | $\succsim_{c, \text{disadvantaged}}$ | $\succsim_{c, \text{regular}}$ |
|----------|--------------------------------------|---------------------------------------|--------------------------------------|--------------------------------|
| 1        | Secured enrollment                   | Secured enrollment                    | Secured enrollment                   | Secured enrollment             |
| 2        | Special needs                        | High-achieving                        | Siblings                             | Siblings                       |
| 3        | Siblings                             | Siblings                              | Disadvantaged                        | Working parent                 |
| 4        | Working parent                       | Working parent                        | Working parent                       | Former students                |
| 5        | Former students                      | Former students                       | Former students                      | No priority                    |
| 6        | No priority                          | No priority                           | No priority                          |                                |

Priorities of schools over contracts  $(s, t) \in S \times T$  for each fixed student and preferences of students over contracts  $(c, t) \in C \times T$  also need to be defined to fully state our model, even though neither schools nor students have real preferences over the type of seat defined by the contract, i.e. schools rank students for each type of seat and students rank schools. The way to break ties is not straightforward: as shown by Dur et al. [19], different tie-breaking rules might favor some type of students. To reduce over-representation of quotas, we break ties in a way such that students and schools favor assignments of students to seats of one of their corresponding types.

### 4.3 Block application

Families that have two or more children participating in the system are given the option to prioritize assigning them to the same school of their preference list to the detriment of better schools reported in each individual preference list. This is called *block application*.

We implement block applications by defining an equivalence relation on  $S$  that captures the sibling relationship among students that are participating in the system. Thereby, the blocks correspond to the equivalence classes of size two or more induced by this relation. We only consider blocks that have at least one school in common in their members' preference lists.

It is important to emphasize the different roles of siblings in the system. On the one hand, there is a sibling's priority that gives special priority to students applying to schools where they have



siblings already enrolled at. On the other hand, there is the block application, which addresses siblings (either in the same or in a different level) that are all participating in the admission process.

Similar to the matching with couples problem, a stable matching might not exist in the school admission problem with block applications. To explain why, we must first define stability in this context. Consider the simplest version of the school choice problem: without blocks, types or quotas. A stable matching can be described as a matching  $\mu$  where there is no pair of student  $s$  and school  $c$  such that  $c \succ_s \mu(s)$  and  $|\{s' \in \mu(c) : s' \succ_c s\}| \leq q_c - 1$ . Now, a school  $c$  in level  $k$  has capacity  $q_c^k$  and a block  $A$  has preferences over the possible assignments of its members. In particular, we consider that a family  $A$  has one strict order of preferences  $\succ_A$  over a set of acceptable schools, and that they prefer the most assignments where  $A \subseteq \mu(c)$  for some  $c$  and the rest of comparisons are given by the Pareto partial ordering induced by  $\succ_A$ . Then, we say that a matching  $\mu$  is stable if (i) there are no triplets of student, school and level  $s, c, k$  where  $s$  is not matched to the same school as the rest of the block  $A \ni s$ , such that  $c \succ_s \mu(s)$  and  $|\{s' \in \mu(c, k) : s' \succ_c s\}| \leq q_c^k - 1$ ; and (ii) there are no pairs of block and school  $A, c$ , such that  $|\{s' \in \mu(c, k) : s' \succ_c A\}| \leq q_c^k - |A_k|$  for all  $k \in K$  and either  $c \succ_A \mu(A)$  if  $|\mu(A)| = 1$  or  $c$  is just acceptable for  $A$  if  $|\mu(A)| > 1$ .

**PROPOSITION 4.1.** *If there are block applications, a stable matching might not exist. This is true even in the case where the blocks have at most two siblings, the preferences of schools are over families, and students of the same block have the same preferences.*

In Figure 2 we show a diagram that represents an instance with two levels, four families  $A, B, C, D$  represented as rows, and four schools  $c_1, c_2, c_3, c_4$  represented as columns, where no stable matching exists. Each of the two bigger squares represent the levels and the small squares are applications to acceptable schools. Each school has one seat at each level and the arrows point towards more preferred applications. Families  $A$  and  $C$  have only one kid and families  $B$  and  $D$  have two, one in each level. Note that each family is the most preferred for some school, so no student can result unassigned and siblings must be together in any stable matching. To show that there are no stable matchings consider the two cases for family  $B$ . Suppose we have a stable matching where  $B$  is in  $c_1$  (in both levels). Then  $C$  will take their favourite seat in  $c_2$  in the right level, then  $D$  can go to their favourite school  $c_3$  (in both levels), so  $A$  takes their favourite seat in  $c_4$ . But now no one with higher priority than  $B$  is matched to  $c_2$ , so  $B$  should be there, contradicting our initial assumption. Suppose on the other hand that  $B$  is matched to  $c_2$ . Then  $C$  can only be in  $c_3$ ,  $D$  can only be in  $c_4$ , but now we do not have space for  $A$ , contradicting our claim that everyone would be assigned in a stable matching. A similar instance exists when siblings are applying to the same level, as it is shown in Appendix B.1.

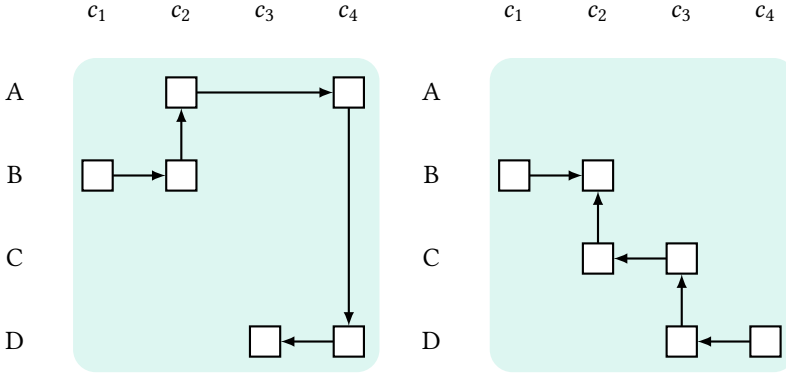
In our setting, and due to this result, we implement a heuristic to find a feasible solution to the block application problem. First, we obtain an assignment in the highest level. Then we use this assignment to update preferences of students in lower levels dragging their sibling's assigned school to the top of their preference list. We repeat this process subsequently from the highest to the lowest levels, and use the outcome of previous levels to update the preference list of students of the current level that participate in block applications.

#### 4.4 Tie breaking rule

In order to run the matching algorithm, ties between students need to be broken. By law, schools are allowed to have their own lotteries, so a multiple tie-breaking rule must be used.

As explained before, the system seeks to give siblings a higher chance of being assigned together. In order to do so, a second feature we implement is a tie-breaking rule by family, where each family can account for one or more students. More specifically, all families— independent of the number of students, and considering all levels—that apply to a given school are first (randomly) ordered to

Fig. 2. Instance in two levels where no stable matching exist because of the block preferences. Each of the two big squares represents a level.



break ties. Then, for those families having more than one student, a second random order is used to break ties among siblings. This random order over students is used to break ties in each level of the school. In case that a student has a priority criterion or belongs to a quota, the school updates priorities in each seat type and each level independently.

To illustrate the idea behind the family lottery and how this can help to get more siblings assigned to the same school consider the following example.

*Example 4.2.* Consider two schools,  $c$  and  $c'$ , that have a single seat in levels  $k_1$  and  $k_2$ , and suppose that level  $k_1$  is processed first. In addition, suppose that there are two families,  $f$  and  $f'$ , with one child applying to each level, i.e.  $f = \{s_1, s_2\}$  and  $f' = \{s'_1, s'_2\}$  ( $s_1$  and  $s'_1$  apply to  $k_1$ ). Finally, we assume that all students prefer  $c$  over  $c'$ .

Using our approach, families are first ordered according to a random order. As a result, we may get that  $f \succ_c f'$ . Then, students within each family are randomly ordered, which may result in the order  $s_1 \succ_c s_2 \succ_c s'_1 \succ_c s'_2$ . As a result, students  $s_1$  and  $s_2$  are assigned to school  $c$  and students  $s'_1$  and  $s'_2$  are assigned to school  $c'$ .

To illustrate how this helps families, suppose that a lottery by student is used instead. One possible result is to obtain  $s_1 \succ_c s'_1 \succ_c s'_2 \succ_c s_2$ , and therefore  $s_1$  is assigned to school  $c$  in level  $k_1$ , but his sibling  $s_2$  gets assigned to  $c'$ . Similarly, students  $s'_1$  and  $s'_2$  end up assigned to schools  $c'$  and  $c$  respectively, and thus they are also assigned to different schools.

#### 4.5 Implementation

Finally, we present the algorithm that determines the matching of students to schools:

For each level  $k \in K$ , from the highest (12th grade) down to the lowest (Pre-K) do:

- (1) Update preferences of students in block application
- (2) Create the weak priority list for each school, and subsequently randomly break the ties between and within families.
- (3) Create the strict priority list for each type of seats at each school.
- (4) Create student preferences over seats in each school.
- (5) Run the student-optimal *Deferred Acceptance* algorithm for overlapping types on all students and schools that belong to level  $k$ .

Table 2. Evolution of the System - Main Round

|                           | 2016  | 2017   | 2018    |
|---------------------------|-------|--------|---------|
| Regions                   | 1     | 5      | 15      |
| Schools                   | 63    | 2,174  | 6,421   |
| Students                  | 3,436 | 76,821 | 274,990 |
| % assigned 1st preference | 58.0  | 56.2   | 59.2    |
| % assigned any preference | 86.4  | 83.0   | 82.5    |
| % unassigned              | 9.0   | 8.7    | 8.9     |

Our implementation of the Deferred Acceptance algorithm is based in the approach of directed graphs first proposed by Balinski and Ratier [13] for one-to-one stable matchings and later extended by Baiou and Balinski [12] for many-to-one matchings. At each level, we encode the instance as a graph in which every node represents a student applying to a type-specific seat in a school, and every directed edge connects either two preferences or two priorities, from the least preferred (or prioritized) one to the most. Then, the algorithm eliminates nodes that do not belong to any stable matching, until no further eliminations are possible. Finally, in order to find the student-optimal matching, we pick the top preference of each student that has at least one preference remaining in the graph. This algorithm allows us to solve all levels of the nationwide instance of the school admission problem in just a few seconds.

## 5 RESULTS

In this section we report part of the implementation results. We start by describing how the system evolved from 2016 to 2018. Then, we focus on the current admission process and report the results of the main and complementary rounds in Sections 5.1 and 5.2 respectively. In Section 5.3 we analyze the effect of the quota for disadvantaged students. Finally, in Section 5.4 we study the impact of the block application.

In Table 2 we summarize how the admission system evolved. In 2016 we considered only the entry levels of the Magallanes region, located in the extreme south of the country. In 2017, the system was extended to all levels in Magallanes, and to four more regions considering only their entry levels. For the 2018 process all the levels of the aforementioned regions were added, and all the remaining regions (except for the Metropolitan area) were included at their entry levels. By 2020, the plan is to implement the system in the entire country and considering all levels, i.e. from Pre-K to 12th grade. As the table shows, most of the relevant performance metrics of the main round—fraction of students assigned to their top choice and unassigned—have remained stable over time.

### 5.1 Main Round

In 2018, 274,990 students and 6,421 schools—divided in 32,198 sections, i.e. school-level pairs—participated in the system, with a total of 522,859 available seats (average of 16.2 seats per section). In Table 3 we classify students based on (1) their gender, (2) whether they have any type of priority in the schools they applied to, and (3) whether they were eligible for any quota in the schools of their choice.<sup>6</sup> Notice that the percentage of disadvantaged students exceeds 50% of the total number of applicants. As the quota for this group is only 15%, an interesting design question is whether

<sup>6</sup>Priorities and quotas may depend on the school (e.g. being the child of a school employee is only valid for that school), and are not mutually exclusive.

Table 3. Characterization of Applicants - Main Round

|          |                     | # applicants | % of total applicants |
|----------|---------------------|--------------|-----------------------|
| Gender   | Female              | 134,973      | 49.1%                 |
|          | Male                | 140,016      | 50.9%                 |
| Priority | Siblings            | 66,743       | 24.3%                 |
|          | Working parent      | 3,700        | 1.3%                  |
|          | Former students     | 9,165        | 3.3%                  |
| Quota    | Special needs       | 1,631        | 0.6%                  |
|          | Academic excellence | 6,534        | 2.4%                  |
|          | Disadvantaged       | 150,287      | 54.7%                 |

having a quota has any impact when the targeted population is relatively large. We analyze this in Section 5.3.

Analyzing the submitted preferences we observe that students apply on average to 3.4 schools. Among these applications, 73.1% are to public schools and 26.9% to voucher schools, although only 11% of the total seats available are of the latter type. Out of the 485,905 applications made by disadvantaged students, 22.0% were made to voucher schools, which is significantly less than the general population. These differences are not surprising considering that disadvantaged students have less resources, and therefore their willingness to pay is probably lower.

In Figure 3 we present the distribution of assignments by preference. We observe that 59.2% and 12.8% of the applicants are assigned to their first and second preference respectively. In addition, 8.6% are assigned to their current school via secured enrollment, and 8.9% are left unassigned<sup>7</sup>. Results of the assignment by type of level and by the priority criteria of students are presented in Appendix C.1 and C.2 respectively.

Figure 4 shows the fraction of students that (1) are assigned to one of their preferences, (2) are assigned to their current school by secured enrollment, and (3) result unassigned, conditional on the number of reported preferences. We observe that when the number of declared preferences increases so does the probability of being assigned, but the average preference of assignment also increases. Moreover, we find that students who result unassigned apply on average to fewer schools (3.36, with std. dev. 1.49) than those who result assigned (3.42, with std. dev. 1.83). Applicants assigned by secured enrollment usually submit even less preferences (3.05, with std. 1.49), which is expected as they have a secured option.

## 5.2 Complementary Round

A total of 46,698 students participated in the complementary round including unassigned students from the main round and new applicants. In Table 4 we characterize these students based on their gender, priority type and eligibility for disadvantaged quota—the other quotas are not considered in the complementary round. In general we observe that there are no significant differences relative to the main round.

In Figure 5 we present the distribution of preference of assignment in the complementary round. We observe that the results are not as good as in the main round, as 47% are assigned to their top choice, 28% are assigned by distance, and 3.6% resulted unassigned. This result can be explained by

<sup>7</sup>Recall that these students—the unassigned— have the chance to participate in the complementary process, whose results are described in Section 5.2.

Fig. 3. Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment and students that were not assigned - Main Round

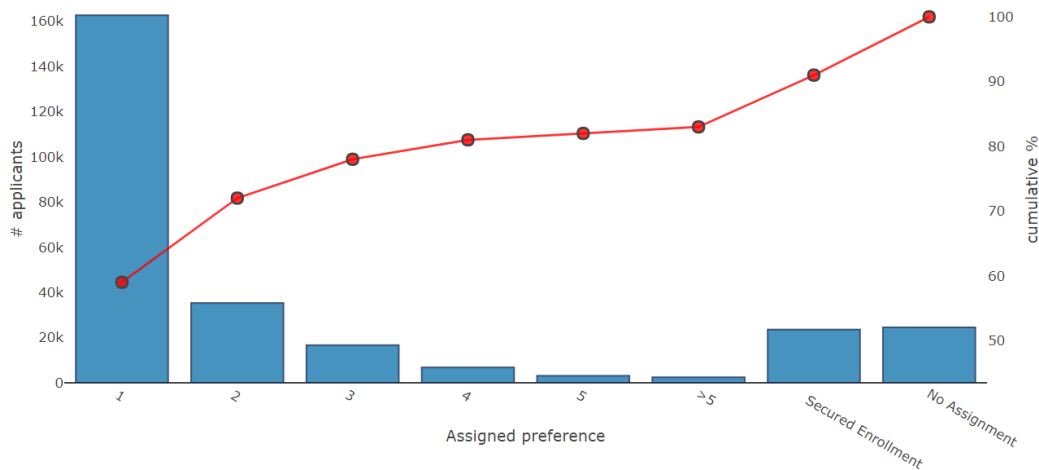
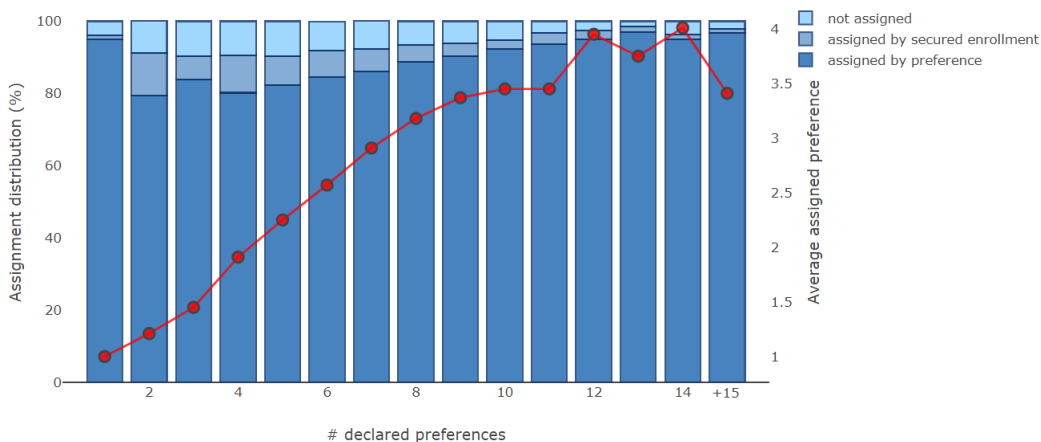


Fig. 4. Assignment distribution and average rank distribution by number of declared preferences - Main Round



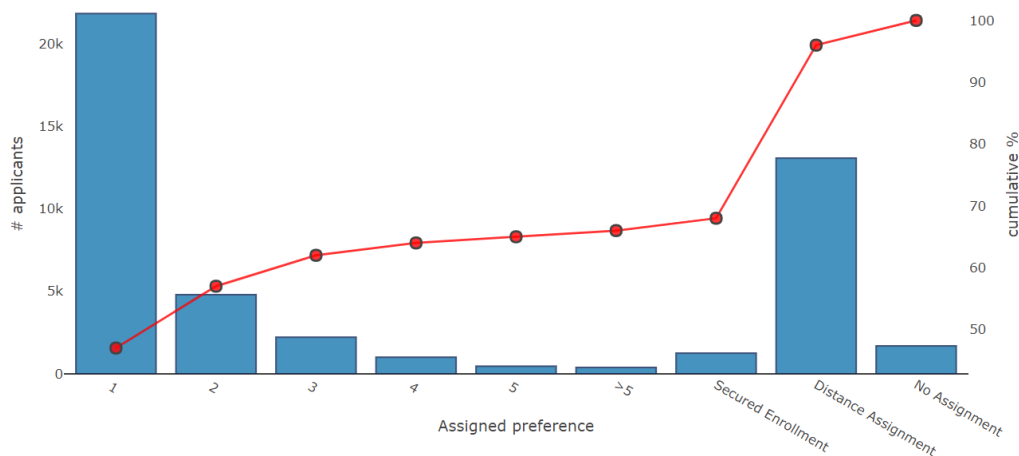
the number of submitted preferences, as students that get assigned apply to 3.49 (std. 1.83) schools on average, compared to 2.57 (std. 0.98) for students with secured enrollment, 3.28 (std. 1.47) for students assigned by distance, and 2.79 (std. 2.08) for unassigned students.

Recall that those students who are not assigned to any of their preferences in the complementary round may be allocated to the nearest public school with remaining open seats within 17km, i.e. distance assignment. Indeed, 13,064 students were assigned by distance, and the average distance for these students was 2.17km, compared to 2.19km for those students who were assigned to one of their preferences in the complementary process and 3.35km for those assigned by secured

Table 4. Characterization of Applicants - Complementary Round

|          |                 | # applicants | % of total applicants |
|----------|-----------------|--------------|-----------------------|
| Gender   | Female          | 23,063       | 49.4%                 |
|          | Male            | 23,635       | 50.6%                 |
| Priority | Siblings        | 5,443        | 11.7%                 |
|          | Working parent  | 328          | 0.7%                  |
|          | Former students | 2,441        | 5.2%                  |
| Quota    | Disadvantaged   | 23,414       | 50.1%                 |

Fig. 5. Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment, distance and students that were not assigned - Complementary Round



enrollment. Finally, only 1,691 students—0.6% of the total number of applicants considering both rounds—resulted unassigned and were manually allocated by MINEDUC.

### 5.3 Quotas

An important design question is whether quotas for disadvantaged students should be considered, especially when these students are the majority (54.7%) of those who participate in the system and when random numbers are used to break ties. In Table 5 we compare how well these students perform compared to those who are not eligible for this quota.<sup>8</sup> As expected, students eligible for the quota perform better than those who are not, with more disadvantaged students being assigned to their top choice and less of them resulting unassigned.

In addition to helping students from disadvantageous environments, a second goal of the quota is to reduce segregation and achieve a higher level of socioeconomic heterogeneity in all the

<sup>8</sup>The results for the other quotas, i.e. for students with special needs and for high-achieving students, are reported in the Appendix C.3.

Table 5. Results for disadvantaged and non-disadvantaged students - Main Round

|                      | Disadvantaged | Non disadvantaged |
|----------------------|---------------|-------------------|
| Number of applicants | 150,287       | 124,703           |
| 1st preference       | 66.0%         | 51.0%             |
| Other preference     | 21.1%         | 26.1%             |
| Secured enrollment   | 7.3%          | 10.1%             |
| Not assigned         | 5.7%          | 12.8%             |
| Average rank         | 1.53          | 1.82              |
| Average applications | 3.02          | 3.37              |

Table 6. Results with and without socioeconomic quota - Simulations

|                                | With quota    |             | Without quota |             |
|--------------------------------|---------------|-------------|---------------|-------------|
|                                | Disadvantaged | Non-disadv. | Disadvantaged | Non-disadv. |
| 1st preference                 | 66.1%         | 50.1%       | 65.8%         | 51.3%       |
| Other preference               | 21.1%         | 26.1%       | 21.0%         | 26.1%       |
| Secured enrollment             | 7.2%          | 10.0%       | 7.4%          | 10.0%       |
| Not assigned                   | 5.6%          | 12.8%       | 5.8%          | 12.6%       |
| Heterogeneity of schools       |               |             |               |             |
|                                | Mean          | Std. Dev.   | Mean          | Std. Dev.   |
| Fraction of disadv. classmates | 0.5661        | 0.2173      | 0.5650        | 0.2198      |

classrooms. An analysis on the heterogeneity of schools in pre-kindergarten can be found in Appendix C.4.

To complement this analysis, we conducted a simulation study to better understand the effect of having this quota. In particular, we performed 20,000 simulations, where half of them were done considering the quota of 15% and the other half were done eliminating this quota. In the latter case, seats were assigned regularly by priority groups.

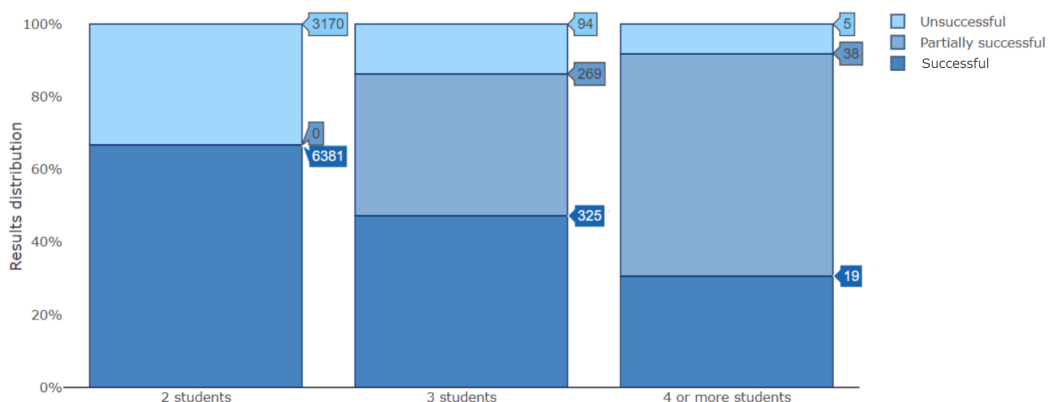
In Table 6 we show the results of the simulations. As expected, disadvantaged students perform better when there is a quota, but the differences are not significant compared to the case with no quota. The fact that there is almost no change in performance when removing the quota could be due to auto-segregation, i.e. disadvantaged students apply to different schools than non disadvantaged students. Table 6 also shows the mean and standard deviation of the fraction of disadvantaged classmates that each student has.<sup>9</sup> We observe that the results are practically the same with and without quota, and therefore we conclude that having the quota has no major impact in the heterogeneity of schools.

#### 5.4 Block Application

Another distinctive feature of the Chilean school choice problem is the block application, which aims to increase the number of siblings that get assigned to the same school. In 2018, a total of

<sup>9</sup>Refer to the Appendix C.5 for detail in the computation.

Fig. 6. Number of successful and partially successful blocks by size - Main Round



21,424 students were part of 10,301 blocks in the main round, with 2,869 (27.9%) formed by students that belong to the same level, and 7,432 (72.1%) having at least two students of different levels. For concreteness, we focus on the results for the main round.

We say that a block application is successful if all of its students are assigned to the same school. Similarly, for blocks that include three or more students, we say that a block is partially successful if at least two of its students, but not all of them, are assigned to the same school. Overall we observe that 6,725 (65.3%) blocks were successful, while 307 (3%) were partially successful. Figure 6 shows the success of block applications by size. As expected, larger blocks are less likely to be successful, as they require more students being allocated to the same school.<sup>10</sup>

As described in Section 4.3, to implement block applications we consider lotteries by families and update students' preferences. However, there may be other approaches that could lead to more successful blocks. To explore the benefits of our approach, we conduct a simulation study to compare our current approach with three other alternatives: (i) lotteries by student in each school and updating preferences (ii) lotteries by families without updating preferences, and (iii) lotteries by student without updating preferences. Notice that the latter is the standard approach used in other school choice settings.

In Table 7 we report the results of 10,000 simulations for each approach. We observe that the fraction of successful blocks is larger when both components—lottery per family and updating preferences—are implemented. In addition, comparing the results of the current mechanism with alternatives (i) and (ii) we observe that updating preferences seems to be more relevant than having a lottery per family, as the former explains 9% of successful blocks, while the latter only explains 4%. These results suggest that updating preferences should be prioritized to improve the chances of siblings of getting assigned together.

## 6 CONCLUSIONS

Centralized procedures to assign students to schools are becoming the norm in many countries. In this paper we describe the design and implementation of the new school choice system in Chile, which expands previous applications in three main areas.

<sup>10</sup>Refer to Appendix C.6 for results of block application depending on the number of common schools a block declares.



Table 7. Block application - Simulations

| Approach | Successful blocks | Partially successful blocks |
|----------|-------------------|-----------------------------|
| Current  | 6,747 (66%)       | 303 (3%)                    |
| (i)      | 6,342 (62%)       | 309 (3%)                    |
| (ii)     | 5,906 (57%)       | 344 (3%)                    |
| (iii)    | 5,446 (53%)       | 351 (3%)                    |

First, we introduce, analyze and evaluate the impact of features of our design intended to favor siblings in getting assigned to the same school. Concretely, we propose the use of two lotteries, one to order families and the other to break ties among siblings. In addition, our design allows families to update students preferences to prioritize them getting assigned with their siblings. Our results show that these features improve the fraction of siblings assigned in the same school by 13% compared to the standard approach of breaking ties at the student level. Second, we propose a multi-level mechanism that allows students to have a secured enrollment in their current school. This feature of the system eliminates the risk of ending up unassigned when trying to move to a new school. Finally, we implement a mechanism with multiple quotas and priority groups. We show via simulations that having quotas for disadvantaged students is not very effective when the majority of students are eligible and the quota is relatively small. Hence, other approaches should be considered if the goal is to really benefit this group.

The experience of implementing a large-scale nationwide system stresses the importance of having a continuous collaboration with policy makers, and the need of implementing changes in small steps. Having a gradual implementation not only allows to learn from the experience and continuously improve the system, but also gives time to the general public—and final users of the system—to get information, learn and understand the benefits of the new system. Overall, we will continue working to improve the system, increasing its efficiency and fairness to give equal opportunities to all students, regardless of their background.

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## A THE PROBLEM IN CHILE

### A.1 Quota processing

*Example A.1.* Consider an over-demanded school with a certified special program for students with special needs and that is pre-selected to have high-achieving quota. In this case, seats are first filled with special needs students until the available quota is completed or the number of applicants with special needs is exhausted. Then, seats are filled with high-achieving students up to the quota, or until the number of applicants for the quota is exhausted, in the strict order of the results of the entrance exams. Next, if there are seats left, students with siblings priority are processed in order based on the random tie-breaker. Then, if there are seats left, these are filled with disadvantaged students until either reaching 15% of the total seats, there are no seats left, or the number of disadvantaged applicants is exhausted. Afterwards, students in the second and third priority group are processed respecting their priorities and random orders, and if there are seats available. Finally, all remaining students –those with no priority and that were not accepted in a quota– are processed according to their random order until all seats are filled or there are no more applicants. In the cases that there are no applicants for a certain quota, the seats are filled by students in the priority groups and the remaining students (those with no priority and no quota), ensuring there is no demand for empty seats after the assignment.

## B MODEL AND IMPLEMENTATION

### B.1 Block application stability

Regarding stability with block application, in Figure 7 we show an instance with one level, three families and three schools, where no stable matching exist. Each row represents a family and each column represents a school, and in parentheses are the size of the family and the number of available seats. The arcs represent the preferences or priorities, pointing from the less preferred to the most preferred. In this case, each family is the most preferred for one school, so in any stable matching all students are matched. This also means that the two kids of family  $B$  must be assigned together. It is easy to check that none of the  $2^3$  possible combinations is a non-wasteful matching without justified envy.

Fig. 7. Example of instance where no stable matching exists. Each row is a family and each column is a school. In parenthesis is the size of the family or the number of available seats

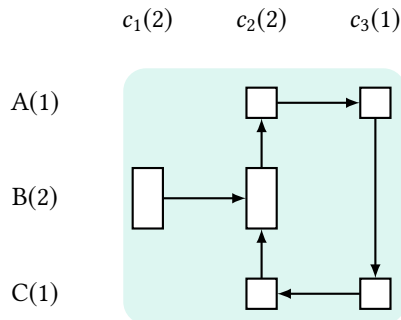
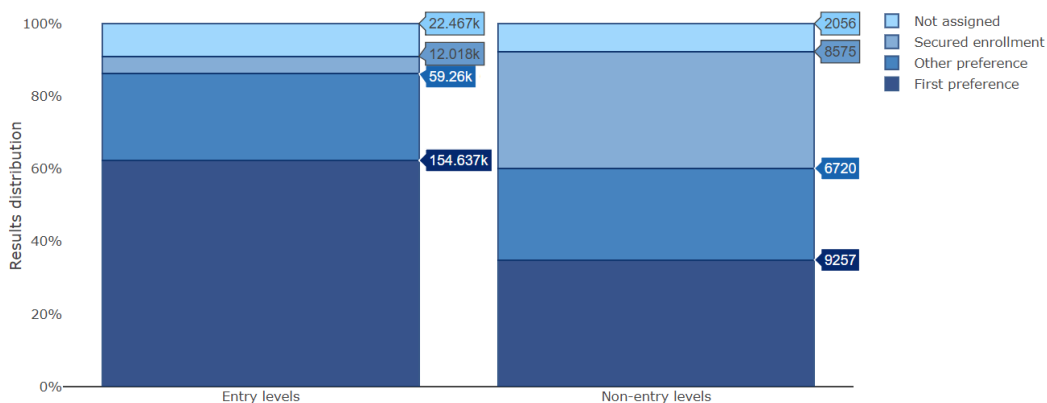


Table 8. Composition by level - Main Round

| Level            | # applicants | # schools | # available seats |
|------------------|--------------|-----------|-------------------|
| Pre-kindergarten | 84,626       | 3,714     | 107,584           |
| Kindergarten     | 26,323       | 3,915     | 51,863            |
| 1st grade        | 38,999       | 5,957     | 88,677            |
| 2nd grade        | 4,242        | 1,914     | 15,483            |
| 3rd grade        | 3,839        | 1,880     | 14,082            |
| 4th grade        | 3,476        | 1,879     | 13,838            |
| 5th grade        | 3,362        | 1,868     | 13,401            |
| 6th grade        | 2,958        | 1,867     | 13,435            |
| 7th grade        | 19,997       | 4,082     | 58,658            |
| 8th grade        | 3,124        | 1,209     | 14,355            |
| 9th grade        | 78,437       | 1,710     | 102,442           |
| 10th grade       | 2,945        | 494       | 9,812             |
| 11th grade       | 2,266        | 861       | 9,697             |
| 12th grade       | 396          | 848       | 9,532             |

Fig. 8. Percentage of assigned students by level type - Main Round



## C RESULTS

### C.1 Main Round by level

In Table 8 we classify applications based on level. We observe that 90% of all applicants and 78% of available seats correspond to entry levels (pre-kindergarten, kindergarten, 1st grade, 7th grade and 9th grade). In addition, we observe that the number of seats offered exceeds the total number of applicants in each level. Thus, unassigned students are a result of congestion and not a consequence of lack of supply.

In Figure 8 we show the assignment results for entry levels and non-entry levels. We observe that the former perform significantly better, having 27% more students assigned in their top choice and 27% less assigned to their secured enrollment. This happens because of the availability of seats in each type of level, where in non-entry levels there are few seats in demanded schools.

Table 9. Results by priority group - Main Round

|                          | Siblings | Working parent | Former students | No priority |
|--------------------------|----------|----------------|-----------------|-------------|
| Number of applicants     | 66,743   | 3,700          | 9,165           | 196,676     |
| 1st preference           | 86.7%    | 81.1%          | 60.6%           | 49.9%       |
| Other preference         | 9.5%     | 14.8%          | 27.2%           | 28.0%       |
| Secured enrollment       | 2.4%     | 2.6%           | 7.8%            | 10.6%       |
| Not assigned             | 1.3%     | 1.5%           | 4.4%            | 11.5%       |
| Average rank             | 1.19     | 1.29           | 1.71            | 1.83        |
| Average sections applied | 2.82     | 2.89           | 3.85            | 3.59        |

Table 10. Total seats declared by schools - Main Round

| Quota          | # seats | % of total |
|----------------|---------|------------|
| Special needs  | 15,324  | 2.9%       |
| Disadvantaged  | 43,336  | 8.3%       |
| High-achieving | 2,591   | 0.5%       |
| Regular        | 462,927 | 88.3%      |

## C.2 Results by priority group

We compare the assignment results of students depending on the priority groups they belong to including students that do not have any priority as a group. Recall that a student may belong to more than one group depending on the schools she is applying to i.e. a student might have siblings priority in one school, former students' priority in another and be a regular student in a third school, but a student cannot belong to more than one priority group in the same school.

In Table 9 we show the number of applicants and the results by priority group. Students with siblings priority outperform other students, which could be because they are processed even before the disadvantaged quota.

## C.3 Results by quota

Recall that students can belong to three quotas: (1) special needs, (2) high-achieving, and (3) disadvantaged, and that a student can belong to more than one quota even in the same school. Students are indifferent to being assigned by any quota or non of them and schools only declare their total available seats and the mechanism calculates seats for the different types of quotas that are allowed by the system. In Table 10 we show the distribution of the 524,178 declared seats for the 2018 process.

Figure 9 shows the results for students belonging to different quotas. It is clear to see that students belonging to special needs and disadvantaged quotas outperform students belonging to high-achieving quota or students that do not belong to any quota in both percentage of students assigned to their first choice and percentage of unassigned students. The low performance of the high-achieving quota compared to the other two quotas could be explained by the fact that high-achieving students apply to a subset of very over-demanded schools.

Fig. 9. Results by quota - Main Round

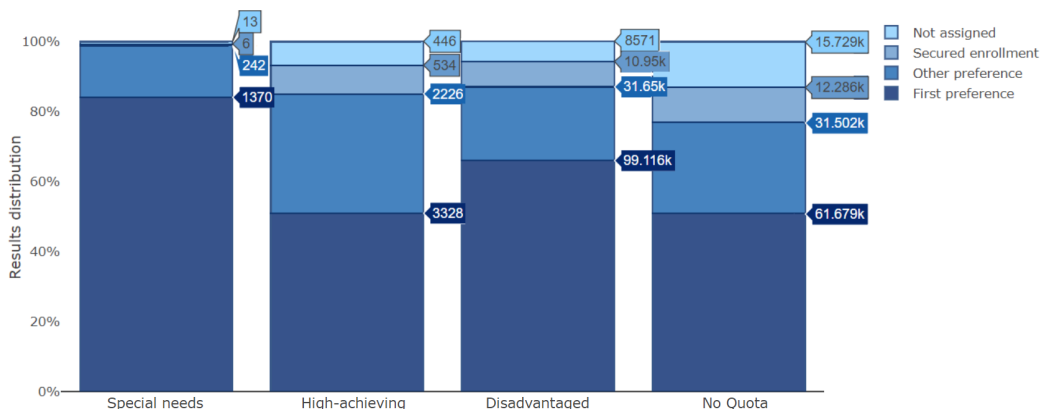
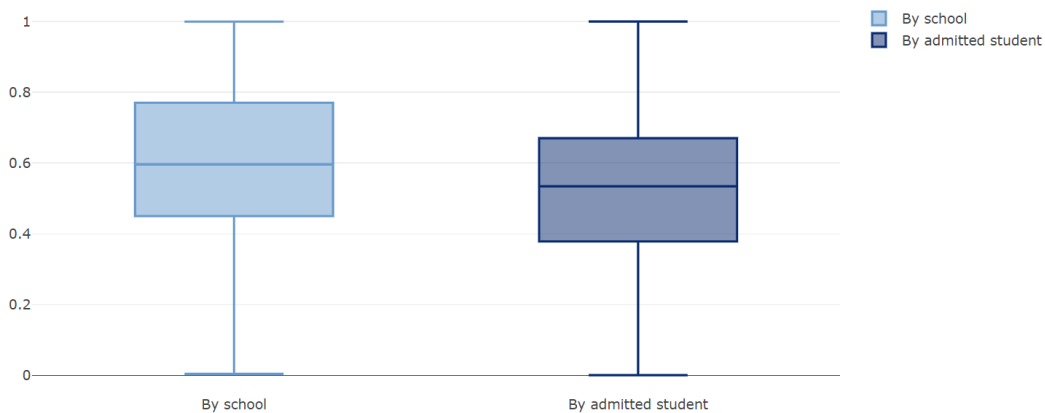


Fig. 10. Distribution of the percentage of disadvantaged students in Pre-K - Main Round



### C.4 Heterogeneity in pre-kindergarten

To analyze segregation in schools, in Figure 10 we show the distribution of the fraction of disadvantaged students assigned in pre-kindergarten (the lowest level). We focus on this level because there are no students with secured enrollment.

From this figure we observe that most schools have a significant fraction of disadvantaged students, ranging from 45% to 77%. In addition, if we analyze this at the student level, we observe that 75% of students are assigned to a school that has between 38% and 67% of disadvantaged students. These results suggest that the disadvantaged quota has a positive effect in making heterogeneous sections.

### C.5 Heterogeneity simulations

From the point of view of schools, MINEDUC seeks to have balanced and heterogeneous schools with respect to the socioeconomic composition. In this sense, we analyze in both scenarios (with and without the quota) the balance of disadvantaged students in schools. For this aim, we consider the following measure of heterogeneity: among all the students of the first round that get an assignment, we pick a student uniformly at random and count the fraction of disadvantaged students that are assigned to her course. This defines a random variable that depends both on the lottery used for the tie-breaking rule and on the selected student.

Let  $S_{\text{dis}}$  be the set of disadvantaged students that participate in the first round. Given an assignment  $\mu$ , let  $S(\mu) := \{s \in S : \mu(s) \neq \emptyset\}$  be the set of all students that get an assignment in  $\mu$ , and similarly, let  $S_{\text{dis}}(\mu) := S(\mu) \cap S_{\text{dis}}$  be the set of all the disadvantaged students that get an assignment in  $\mu$ .

For a fixed lottery, let  $\mu^{\text{lottery}}$  be the assignment obtained from its induced tie-breaking and  $s$  be a student chosen at random among all the students that get an assignment in  $\mu^{\text{lottery}}$ . For a school  $c \in C$  with  $\mu(c) \neq \emptyset$ , let  $f^{\text{lottery}}(c) := \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|}{|\mu^{\text{lottery}}(c)|}$  be the fraction of disadvantaged students assigned to  $c$  in  $\mu$ . Then, our random variable can be expressed as  $f^{\text{lottery}}(\mu^{\text{lottery}}(s))$ . Its conditional expectation given the lottery turns out to be the ratio of all the disadvantaged students assigned in  $\mu^{\text{lottery}}$  to all the students assigned in  $\mu^{\text{lottery}}$ , since

$$\begin{aligned} \mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \mid \text{lottery}] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} f^{\text{lottery}}(\mu^{\text{lottery}}(t)) \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} f^{\text{lottery}}(c) |\mu^{\text{lottery}}(c)| \\ &= \frac{|S_{\text{dis}}(\mu^{\text{lottery}})|}{|S(\mu^{\text{lottery}})|}. \end{aligned}$$

Its second moment, on the other hand, is given by

$$\begin{aligned} \mathbb{E} \left[ \left( f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \right)^2 \mid \text{lottery} \right] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} \left( f^{\text{lottery}}(\mu^{\text{lottery}}(t)) \right)^2 \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} \left( f^{\text{lottery}}(c) \right)^2 |\mu^{\text{lottery}}(c)| \\ &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C: \mu^{\text{lottery}}(c) \neq \emptyset} \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|^2}{|\mu^{\text{lottery}}(c)|}. \end{aligned}$$

We estimate  $\mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$  by computing the average of  $\mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \mid \text{lottery}]$  over the results of the 10,000 simulations. Similarly, we estimate  $\text{Var} [f^{\text{lottery}}(\mu^{\text{lottery}}(s))] = \mathbb{E} \left[ \left( f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \right)^2 \right] - \mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s))]^2$  by averaging  $\mathbb{E} \left[ \left( f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \right)^2 \mid \text{lottery} \right]$  over the 10,000 simulations and then subtracting the square of the estimator for  $\mathbb{E} [f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$ . Finally, we calculate the standard deviation as the square root of the variance.

### C.6 Block application results

We measure the success of block applications of size two as a function of the number of schools their students declare in common in their preference lists. Table 11 shows, as expected, that the

Table 11. Results of block applications for blocks of size 2 by number of schools in common

| # schools in common | Main round |              | Complementary round |              |
|---------------------|------------|--------------|---------------------|--------------|
|                     | # blocks   | % of success | # blocks            | % of success |
| 1                   | 1,291      | 38.5%        | 497                 | 67.7%        |
| 2                   | 3,216      | 69.8%        | 2,245               | 76.6%        |
| 3                   | 2,441      | 71.7%        | 1,750               | 77.8%        |
| $\geq 4$            | 2,603      | 72.6%        | 1,889               | 80.5%        |
| Total               | 9,551      |              | 1,421               |              |

success rate increases with the number of common preferences, having its greatest increment when the number of common schools grows from one to two. Furthermore, in both rounds, blocks of size two of the same level were more successful in percentage than those of different levels. Indeed, the main round has 2,832 blocks of size two of the same level and 6,719 of different levels, with success rates of 77.8% and 62.2% respectively. For the complementary round, there are 362 such blocks of the same level, with a success rate of 82.3%, and 1,059 of different levels, with a success rate of 70.7%.