Cooperation and Self-Governance in Heterogeneous Communities*

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Abstract

This paper theoretically studies the consequences of heterogeneity on self-governance, cooperation, and trust in large communities. I consider a game model where players belong to a large population and are randomly matched. Players interact with each other infrequently and, when matched, play a prisoners' dilemma. There exists an institution that can convey information on play histories. Players' payoff functions differ, so that some players have a higher tendency towards cooperation. This constitutes the main modeling innovation of this work and makes the model a mixed adverse selection-moral hazard model.

A suitable equilibrium concept is introduced and characterized. Some novel comparative statics results are obtained, showing, in sharp contrast with previous papers, that more heterogeneous societies may sustain more cooperation. Private enforcement mechanisms are explored, showing conditions under which private for profit intermediation leads to Pareto optimal cooperation. We discuss the implications of my results for applied work and show how the disclosure of credit histories impacts the defection rates of credit relations. *JEL* classification numbers: C73, D40, B25

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1 Introduction

This paper studies self-governance through repeated interaction in heterogenous communities. The focus of this work is on transactions among community members, where each member interacts with different partners as time passes by. Applications of this setting abound, including credit relations –where borrowers can borrow from different sources–, and durable goods transactions –where producers sell to different consumers.

Though the impossibility of establishing long term personal relationships is inherent to the economic setting under consideration (which makes the folk theorem, as established by Fudenberg and Maskin (1986), unapplicable), this does not undermine the feasibility of attaining Pareto efficient outcomes. Indeed, when the matched players play a prisoners' dilemma, a consequence of pioneer works by Milgrom et al. (1990), Kandori (1992), and Okuno-Fujiwara and Postlewaite (1995) is that cooperation is enforceable if there is an institution that can credible convey play histories (so that community members can recognize and punish a defector). It has been argued that the role of institutions such as credit bureaus, online feedback systems, clubs, business associations, and even informal worth-of-mouth is precisely to provide such information. So far, this literature has either ignored the presence of heterogeneity in large populations, or been silent on the consequences of heterogeneity in cooperation and welfare.

Recently, economists have given attention to concepts such as social capital and trust, and their determinants as functions of population fundamentals (see Alesina and La Ferrara (2002) and Durlauf and Fafchamps (2005) for a survey). While several conceptual questions remain unanswered,[1] surprisingly economic theorists have devoted little effort to understanding the link between community heterogeneity, cooperation and trust.

Once we acknowledge the presence of population heterogeneity, several questions arise. What are the effects of heterogeneity in cooperation and welfare? What conditions make population heterogeneity more attractive from a social perspective? What mechanism may overcome the adverse selection problem posed by the unobservability of agents' innate cooperation tendencies? Can the community delegate that screening problem?

The present paper offers a theoretical study aimed at answering these and other related questions. Section 2 posits a self-governance model of a community consisting of a continuum of infinitely lived agents. At each period, each agent is randomly matched to some other community member to play a prisoners' dilemma. From a modeling viewpoint, the main innovation of this paper comes from

[1] As Alesina and La Ferrara (2002) point out: "The theory of what determines trust is sketchy at best."

considering agents that enjoy cooperation in different degrees. This cooperation level is called the agent's type and belongs to a compact set of types. Once the match is realized, a player does not know anything about who he is playing with, nor will he be able to recognize his current opponent later on. All what is known to the matched agents (and is common knowledge) is a pair of marks a-la-Kandori. The mark of each player at each round can be either G (standing for good past behavior) or B (standing for bad bad past behavior), and evolves according to the player's play. The evolution of a player's mark creates a link between current and future behavior that may make cooperation enforceable at least for some matches.

Our game model possesses a variety of untractable strategies consistent with sequential equilibrium restrictions. It is therefore necessary to add some more structure to equilibrium play. We first restrict equilibria to be stationary. Crucially, we additionally consider informationally robust equilibria, namely, equilibria that remain so when some additional signal on a player's type is available. We finally impose monotonicity of cooperation, a condition implying that if some agent has a good G mark (henceforth called a cooperative agent), then so does any agent with a higher innate tendency towards cooperation. These restrictions allow us to fully characterize any equilibrium exhibiting some cooperation by the equilibrium lowest cooperative type.

Section 3 fully characterizes the equilibrium set. In particular, it shows that any equilibrium is the fixed point of a nondecreasing function. Roughly speaking, this function maps the *expected* number of cooperative players to the number of players that are *willing* to cooperate given the expected cooperative players. It is monotone because of the following network externality mechanism: The larger the set of agents expected to cooperate, the more the matches in which a player encounters a cooperative player, so the higher the continuation value of a cooperative player, and the larger the set of players indeed willing to cooperate.

In Section 4, we employ standard lattice theory techniques to derive some comparative statics results. It is shown that the Pareto optimal equilibrium can be characterized as the smallest fixed point of a nondecreasing map. Additionally, it is proven that an increase in the discount factor leads to an increase in cooperation. It is also shown that given any equilibrium, an increase in the fraction of types greater than or equal to the lowest cooperative type leads to an increase in cooperation; so that, in particular, a first order stochastic increase in the distribution of types results in an expansion in cooperation. Less obviously, it is shown that a second order stochastic increase of the distribution of types has ambiguous effects on total cooperation and welfare. So, as will be illustrated, in sharp contrast with recent results by Mobius and Szeidl (2007) and Haag and Lagunoff (2007), more heterogenous societies may sustain more cooperation and deliver higher welfare to their members.[2]

The analysis so far assumes that at the beginning of the game marks are assigned as if a central authority could perfectly monitor players' types. Of course, that needs not be the case for players may not be willing to reveal their types. Section 5 shows that a simple way to solve this adverse selection problem is by selling the good marks G at the beginning of the game and then let equilibrium play transpire. Indeed, by setting the right price a social planner can implement the Pareto optimal equilibrium. The mechanism is simple enough as to be implemented in practice and is robust to the existence of a market for names, as studied by Tadelis (2002).

We also explore private intermediation, where the assignment of marks is carried out by a profit-motivated monopolist. Examples of these profit motivated institutions include credit agencies, private clubs, and business associations. We fully characterize the monopoly solution, and show that when the defection payoff is sufficiently high, the monopolist will find optimal to implement the Pareto optimal equilibrium. So, under reasonable circumstances, the community can delegate the intermediation problem to a profit maximizing monopolist, and the monopolist will not see challenged its monopolistic position even in the absence of entry costs; this implies the concentration of markets for clubs and associations memberships.

Section 6 presents some applications. We discuss the empirical literature linking heterogeneity and trust and offer some empirical strategies suggested by our results. We also discuss the role of credit bureaus in credit markets and discuss conditions under which information disclosure of credit histories is welfare enhancing. Here heterogeneity plays a key role: There will be demand for credit histories only if population tendencies towards cooperation are heterogenous. In particular, the disclosure of credit histories plays a role similar to that of intermediaries engaging in punishment activities (Dixit (2003a), Milgrom et al. (1990)).

Several authors have studied cooperation in large communities. In settings where players may play against varying opponents, Milgrom et al. (1990), Okuno-Fujiwara and Postlewaite (1995), and Kandori (1992) show that, by introducing an institution that can credible convey information on past play, cooperation can be sustained even if there are substantial informational asymmetries between opponents. A similar study of information intermediaries is presented by Dixit (2003a). The present work builds on the institutional insights offered by these authors, but studies the consequences of considering payoff-asymmetric agents.[3]

[2]From a modeling perspective, the present work follows more closely the repeated game theories of randomly matched partners than those authors'.

[3]When no institution as the one described above is available, cooperation may still be an equilibrium outcome. Indeed, Kandori (1992) and Ellison (1994) show that cooperation may be enforced by means of contagious defection

A repeated game model with (almost) random matching and *no* information flow is posited by Ghosh and Ray (1996). These authors consider an heterogenous population where the type of a player cannot be identified in advance and, after any match, players can opt to continue the relation. When both agents in the match cooperate, they reveal themselves as cooperative and there are mutual gains at keeping the relation on. While the institutional setting Ghosh and Ray (1996) model is different from mine, their work seems to be the first one giving an explicit role to heterogeneity in models of large community enforcement where agents are randomly matched.[4]

2 The Model

2.1 Games and Assignments

There is a continuum of players alive at each period $t \geq 1$. Denote the set of players by I. At each round t, a matching function $M_t(\cdot)$ is randomly selected so that player $i \in I$ interacts with player $M_t(i)$ at round t.[5]

Each player is characterized by a parameter $\theta \in \mathbb{R}$ which is its private information. From others' perspective, the parameter θ of each player is distributed according to F, a probability distribution with support Θ contained in $\mathbb{R}_+$. At each t, the matched players play a two-person stage game $\Gamma(\theta_i, \theta_j)$, where θ_i and θ_i are the types of the two matched players i and j. We assume that the stage game is a prisoners' dilemma, where each player may either cooperate (C) or defect (D), and where the payoff from cooperation depends on each player's type. The per period payoff matrix is shown in the figure below.

	C	D
C	θ_i, θ_j	$-l$, g
D	g, $-l$	0, 0

Assume that $l \geq 0$ and, to make the game a prisoners' dilemma, assume that for all $\theta \in \Theta$,

pushed by a single defection. It does not seem clear how the contagious equilibrium analysis may overcome the difficulties posed by agents' heterogeneity.

[4] Repeated games models considering heterogeneous agents typically restrict attention to two type models. For example, Kreps et al. (1982), Ghosh and Ray (1996), and Dixit (2003a) consider models where some agents are behavioral (in the sense that they always cooperate or defect) and others are homogenously opportunistically motivated. My model encompasses general forms of agents' heterogeneity, including finite and continuous type set.

[5] A matching function M must satisfy $M(M(i)) = i \neq M(i)$.

$g > \theta \geq 0$. Denote the action set $A = \{C, D\}$ and let $\pi(a_i, a_j, \theta_i)$ be the period payoff function of a type θ_i player when it plays a_i and its rival plays a_j.

Denote a_i^t the action chosen by player i at round t. Then, given a realization of matching functions $(M_t)_{t\geq 1}$, the total payoff of a type θ player is

$$\sum_{t\geq 1} \delta^t \pi_i(a_t^i, a_t^{M_t(i)}, \theta),$$

where $\delta \in]0, 1[$ is the community discount factor.

When two players are matched to play the stage game, they cannot distinguish who they are playing with, nor are they going to be able to identify their current opponent later on. All what the matched players know about each other is a mark which follows a Markov process. The set of marks is $\{G, B\}$, where G stands for "good", and B stands for "bad". There exists a function $\eta\colon \{G, B\} \times \{G, B\} \times A \to \{G, B\}$ such that when a player has a mark m^i, faces an opponent with mark m^j, and plays a^i, his next period mark is given by $\eta(m^i, m^j, a^i)$. Additionally, before the first period of play, players are assigned a mark according to a function $\psi(\theta)$. We call the pair (ψ, η) a *status assignment*. All aspects of the game but the idiosyncratic type of each player are common knowledge.

I assume that η satisfies the following restrictions:

$$\eta(B, \cdot, \cdot) = B \text{ and } \Big(\eta(G, G, a) = G \text{ iff } a = C\Big).$$

The idea behind the first restriction is that punishments are severe in that a mark B is perfectly persistent. The second restriction motivates the G-marked players to cooperate when facing another G-marked player. Many of the results here presented remain valid when alternative restrictions are imposed.

If a type θ player has a mark m_t^i and its rival's mark is $m_t^{M_t(i)}$, then its period t action takes the form $a_t(m_t^i, m_t^{M_t(i)}, \theta)$.[6] Strategies are restricted to be symmetric in that all players of the same type follow the same strategy. Several strategies $a_t(m_t^i, m^{M_t(i)}, \theta)$ satisfying sequential rationality and generating consistent beliefs may lead to a convoluted dynamics that makes the model untractable. This is the reason we impose additional restrictions on equilibrium play.

[6]Standard dynamic programming arguments show that given that rivals follow this kind of strategy, there is always a best response of this type.

2.2 Equilibrium Definition

The tuple (ψ, η, a) is *stationary* if $a_t \equiv a_{t'}$ for all t, t', and, on the play-path, the mark of each player does not change. By this last sentence we mean that if $\psi(\theta) = G$ (resp. $\psi(\theta) = B$) then as the game goes on, the mark of player θ is G (resp. B) as long as the evolution of marks is according to η and all players conform to the strategy a. From here on we simply write $a(m^i, m^j, \theta)$ to denote the stationary period strategy of players.

We say that the strategy is *informationally robust* if, on the play path, $a(m^i, m^j, \theta)$ does not depend on θ. In particular, by observing the marks of each of the matched players it is possible to know what the players will choose. The idea behind this restriction is the following. When matched to other player, a player may know not only the marks given by the status assignment but also some additional information about its partner (e.g., race, clothe brand, educational level, religion). That additional information may be correlated to the idiosyncratic parameter θ of its rival. By imposing robustness we are ruling out the cases in which that additional information is valuable. The presence of some external signals about the partners may provide information about the partners' types but not about how play will unfold.[7]

Beliefs are history-independent. That is, at each period of play players beliefs about the population distribution of marks is

$$\mathbb{P}[\text{a player randomly picked has type } \theta \in \mathcal{O} \text{ and mark } G] = \mathbb{P}[\theta \in \mathcal{O}, \eta(\theta) = G],$$

where $\mathcal{O} \subseteq \Theta$. On-the-equilibrium path this is a consequence of Bayes rule and the stationarity assumption. Off-the-equilibrium path this is justified for, as the game goes on, each player will see at most a finite number of deviations (interpreted as trembles). A finite number of deviations can never modify the distribution in a continuum.

Stationarity, robustness, and sequential rationality still do not restrict equilibrium outcomes enough. To restrict the structure of the problem a little further, consider the set of G marked players

$$P = \{\theta \in \Theta \mid \eta(\theta) = G\}$$

These players are called *cooperative*. The evolution of noncooperative players mark does not depend on actions. So, noncooperative players never cooperate. If a cooperative-cooperative match ended up in defection, then there would be no difference between cooperative and noncooperative players.[8] To

[7] In the Appendix it is shown that informational robustness is intimately linked to stationary.

[8] In this case, cooperative players are not willing to cooperate when faced to a noncooperative player for its continuation reward from doing so is negative.

avoid trivialities, a match of cooperative players is restricted to end up in cooperation. The following lemma shows that a cooperative agent, when faced to a noncooperative agent, will never cooperate.

Lemma 1 *Consider a stationary robust tuple* (ψ, η, a) *such that* a *is sequentially rational. Then, for no type and at no history, the outcome of a match is* (C, D).

If (C, D) were the outcome of a cooperative-noncooperative match, then the value of being noncooperative would be strictly positive. This in turn makes cooperative players willing to defect when faced to a noncooperative partner. A consequence of this result is that we may restrict our attention to assignments η such that $\eta(G, B, a) = C$ if and only if $a = D$.

The following monotonicity restriction is finally imposed: If $\theta \geq \theta'$ and $\theta' \in P$, then $\theta \in P$. That is, if some type is cooperative then so is any type with a higher tendency towards cooperation. The idea behind this restriction is that if we can enforce cooperation of type θ, then enforcing cooperation of type $\theta' \geq \theta$ is not only feasible but Pareto dominates the situation in which θ' is noncooperative. So, there is no reason to exclude θ' from P.

An immediate corollary of the restriction above is that P must be an interval, which is additionally restricted to be closed[9] (eventually empty). An equilibrium with a nonempty set of cooperative players is therefore characterized by a type $\theta^e \in \Theta$ such that

$$\eta(\theta) = \begin{cases} G & \text{if } \theta \geq \theta^e, \\ B & \text{if not.} \end{cases}$$

We define the set

$$\textsc{Equil} = \Big\{ \theta^e \in \Theta \mid \theta^e \text{ characterizes stationary, informationally robust, monotone,} \\ \text{and sequentially rational strategies with } P \neq \emptyset \Big\}.$$

Note that the configuration in which $\eta(\theta) = B$ for all $\theta \in \Theta$ and no player ever cooperates is an equilibrium which however is not in the set Equil. The focus of this paper is on equilibria exhibiting some degree of cooperation. From here on, by equilibrium we mean the cutoff parameter that characterizes a stationary, robust, monotone, and sequentially rational strategy exhibiting some cooperation.

In the Appendix, we analyze a model where neither stationarity nor informational robustness are imposed. It is shown that the long run evolution in that more general model looks exactly as a stationary and informationally robust equilibrium.

[9]This restriction is a normalization which will prove useful.

2.3 Observations

Several remarks are in order. In the model, the community problem is twofold. On the one hand, there is a moral hazard problem in that actions cannot be enforced externally. The second problem, which at this moment is absent, is an adverse selection problem. So far we have assumed that by means of some mechanism it is possible to monitor the type θ of each player and assign the marks according to $\eta(\theta)$ at the outset of play. Of course, when asked its type, a player would pretend to have a high type. In the next section, we will investigate in detail a price mechanism through which the adverse selection problem can be solved.

The analysis assumes the presence of some degree of institutional development. Though actions cannot be enforced, there must be an outsider able to observe, keep record of, and credible convey transpired societal play. Our preferred way to solve the adverse selection problem consists of allowing transactions between game agent (buyers) and an outsider (seller). Later on, we will analyze a profit-maximizing monopoly that can play the seller role. The model therefore seems suited to a somehow structured community.

A crucial consequence of the robustness assumption is that any equilibrium remains so when all histories become public. In this case, there is no need for the marks.[10] Different applications of the model will restrict the publicity of information in different ways. I have preferred to work in a setting with partially public histories (where the mark is the only proxy for histories) to expand and highlight the informational richness of the analysis.

At a more technical level, there are well known measurability problems when working with random matching models. While the law of large numbers had been extensively employed in random matching economic models, these difficulties had been overcome only recently. See Duffie and Sun (2006).

[10]This result contrasts with the models of cooperation with asymmetrically informed agents living during a finite number of periods. In those models, the crux to keep cooperation is the ignorance of one agent about the motives of the other to cooperate.This is the case in the finitely repeated prisoners' dilemma studied by Kreps et al. (1982). In that model, cooperation can be enforced for a finite number of period as long as the rationality of one of the parts is not common knowledge.

3 Characterization and Existence of Equilibrium

Consider any $\theta^e \in \textsc{Equil}$. Then, for all $\theta \geq \theta^e$, $\eta(\theta) = G$ and player θ is willing to cooperate when so does his rival. The total expected continuation payoff of player θ is given by

$$v(\theta, \theta^e) = \frac{\theta(1 - F(\theta^e -))}{1 - \delta},$$

where $F(x-) = \lim_{y \nearrow x} F(y)$ is the probability of encountering a type greater than or equal to x. Since type θ is cooperative, it must be the case that

$$\theta + \delta v(\theta, \theta^e) \geq g. \tag{3.1}$$

The left hand side in this equation is the payoff that player θ obtains by cooperating, which is the current payoff θ plus the total discounted payoff when its next period mark is G. The right hand side in (3.1) is player θ's payoff when it defects. In this case, it will get g in the current period plus the payoff when its mark is B (in which case no match in which the player is involved will be cooperative). In particular, there exists $x \geq 0$ such that

$$\theta^e + \delta v(\theta^e, \theta^e) = g + x. \tag{3.2}$$

Define

$$T^x(\theta^e) = \min\{\theta \in \Theta \mid \theta + \delta v(\theta, \theta^e) \geq g + x\}$$

Since Θ is compact and θ^e satisfies equation (3.2), $T^x(\theta^e)$ is well defined and $T^x(\theta^e) = \theta^e$.

Inspired by the above analysis, we define the closed set[11]

$$\Theta^x = \{\theta \in \Theta \mid \bar{\theta}\Big(1 + \frac{\delta}{1 - \delta}(1 - F(\theta-))\Big) \geq g + x\},$$

with $\bar{\theta} = \max\{\theta \in \Theta\}$ and $x \in [0, \bar{x}]$, where $\bar{x} = \frac{\bar{\theta}}{1-\delta} - g$ is assumed positive. Define the map $T^x \colon \Theta \mapsto \Theta$ by

$$T^x(\theta^e) = \begin{cases} \min\{\theta \in \Theta \mid \theta + \delta v(\theta, \theta^e) \geq g + x\} & \text{if } \theta^e \leq \bar{\theta}^x, \\ \bar{\theta} & \text{if not,} \end{cases}$$

where $\bar{\theta}^x = \max\{\theta \in \Theta^x\}$. Note that $\Theta^x = \Theta \cap [0, \bar{\theta}^x]$ and $\bar{\theta}^x$ is non increasing. Importantly, given $T^x|_{\Theta^x}$, T^x defined on Θ is its smallest nondecreasing extension.

$T^x(\theta^e)$ is the smaller type willing to cooperate even if the gain from deviating were $g + x$ provided all types greater than or equal to θ^e are being cooperative. The parameter x represents the degree at which the incentive constraint of the lowest type is satisfied.

[11] To see that the set is closed, note that $(1 - F(x-))$ is upper semi-continuous in $x \in \mathbb{R}$.

The following lemma will prove useful.

Lemma 2 *Suppose that $\bar{x} \geq 0$. For all $x \in [0, \bar{x}]$, Θ^x is nonempty and $T^x \colon \Theta \to \Theta$ is a well defined nondecreasing function.*

Intuitively, the monotonicity property stated in the lemma holds because when more agents are expected to cooperate (lower θ^e), an agent will be more willing to be cooperative for it will be matched more frequently to cooperative agents, which in turn increases the number of agents that indeed want to cooperate (lower $T^x(\theta^e)$). This complementarity property (leading to the monotonicity of T^x) will be useful when deriving some comparative statics results.[12]

For each $x \in [0, \bar{x}]$, consider the set of fixed points

$$B^x = \{\theta^e \in \Theta \mid T^x(\theta^e) = \theta^e\}.$$

This set is trivially nonempty as a consequence of Tarski fixed point theorem. Moreover, if $\Theta^x \neq \Theta$, $\bar{\theta} = T^x(\bar{\theta})$.

The following theorem is the main result of this subsection. It characterizes the set Equil and provides conditions for its non emptiness.

Theorem 3 *Characterization:*

$$\begin{aligned} \textsc{Equil} = & \bigcup_{x \in [0,\bar{x}]} B^x \cap \Theta^x \\ = & \{\theta \in \Theta \mid \theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta-))\Big) \geq g\}. \end{aligned}$$

Existence: Define

$$\bar{g}(\delta) = \max\{\theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta-))\Big) \mid \theta \in \Theta\} \tag{3.3}$$

The following condition is necessary and sufficient for the existence of a cooperative equilibrium:

$$\bar{g}(\delta) \geq g.$$

Under this condition, $\bar{x} \geq 0$

[12] The logic behind this property is in line with Coleman's insight on the importance of network closure. In the present model, and consistent with Coleman's view, a dense social network –understood as a high fraction of individuals expected to belong to the set of cooperative players– makes cooperation more attractive. In contrast to Chwe (2000), where efficient outcomes are attained in dense networks by creating common knowledge, in this model mutually beneficial cooperation is enhanced through collective punishment. See Sobel (2002) for discussion.

The first part of the theorem characterizes the equilibrium set as the union of all fixed points of the nondecreasing map T^x that additionally belong to the set Θ^x. This restriction will in general be binding for Θ^x should not be expected to coincide with Θ. It is additionally shown that the equilibrium set will be equal to a set of types satisfying a simple inequality. Both characterizations will be employed in the paper.

The condition for existence –which is assumed in the sequel to avoid trivialities– resembles the standard existence inequality in personal enforcement repeated games with homogenous agents. In that case, considering $\bar{\theta}$ as the only type, cooperation can be enforced if and only if

$$\bar{\theta} + \frac{\delta}{1-\delta}\bar{\theta} \geq g,$$

which is precisely the condition given in the theorem for the homogenous population case.

In the Appendix, we study in more detail the equilibrium set. It is shown that $\textsc{Equil}$ is closed, generically continuous, and may or may not be convex. While at the moment we can dispense with those results, the reader may want to go over that analysis before proceeding.

Example 4 Suppose that F is a uniform distribution on the interval $[0,1]$, $\delta = .9$ and $g = 2$ so that an equilibrium exists and can be characterized by employing Theorem 3. For all $x \in [0,8]$, we have that

$$\Theta^x = [0, \frac{8}{9} - \frac{x}{9}]$$

and for $\theta^e \in \Theta^x$,

$$T^x(\theta^e) = \frac{2+x}{10-9\theta^e}.$$

For all $x \in [0, 7/9]$,

$$B^x \cap \Theta^x = \left\{ \frac{10+\sqrt{28-36x}}{18}, \frac{10-\sqrt{28-36x}}{18} \right\},$$

and for $x > \frac{7}{9}$, the intersection is empty. As a consequence,

$$\textsc{Equil} = [\frac{5-\sqrt{7}}{9}, \frac{5+\sqrt{7}}{9}].$$

Figure 1 illustrates the fixed point characterization.

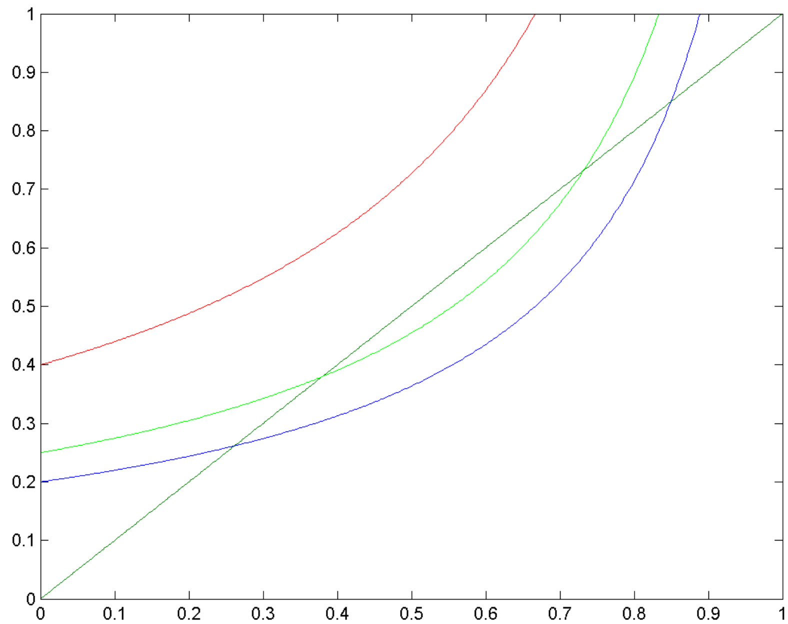

Figure 1: The blue line plots T^0, the green line plots $T^{1/2}$, and the red line plots T^2. As already shown, for $x > \frac{7}{9}$, T^x has only a trivial fixed point $\bar{\theta} = 1$ which does not belong to Θ^x.

4 Pareto Optimality and Comparative Statics

4.1 Results

Let θ^e and $\hat{\theta}^e$ belong to EQUIL and suppose that $\theta^e < \hat{\theta}^e$. Then θ^e Pareto-dominates $\hat{\theta}^e$. To see why, note that types $\theta \geq \hat{\theta}^e$ cooperates in more encounters under θ^e than under $\hat{\theta}^e$. Types $\theta^e \leq \theta < \hat{\theta}^e$ never cooperate under $\hat{\theta}^e$ but cooperate in some matches under θ^e. Types $\theta < \theta^e$ never cooperate under either equilibrium. It therefore seems natural to define

$$\theta^c(F, \delta) = \arg\min\{\theta^e \mid \theta^e \in \text{EQUIL}(F, \delta)\}, \tag{4.1}$$

where we highlight the equilibrium outcomes dependance on the fundamentals F and δ (when clear from context, we omit the dependance). The equilibrium $\theta^c = \theta^c(F, \delta)$ is the Pareto optimal equilibrium for the game defined by F, δ.

The following result provides a simple condition under which equilibrium set comparisons can be made.[13]

[13] Echeñique and Sabarwal (2003) presents results guaranteeing that any old equilibrium is larger than every new

Proposition 5 *Let θ^e be a fixed point of the map $T^x(\cdot \mid F, \delta)$, where $x \in [0, \bar{x}]$ and where we explicit the dependance of the map T^x (defined on subsection 3) on F and δ. Consider an alternative model characterized by a discount factor δ' and a distribution of types G. Suppose that F and G have the same support Θ. If for some x'*

$$T^{x'}(\theta^e \mid G, \delta') \leq \theta^e, \tag{4.2}$$

then, there exists a fixed point $\hat{\theta}^e$ of $T^{x'}(\cdot \mid G, \delta')$ such that $\hat{\theta}^e \leq \theta^e$. Moreover, the last inequality can be taken strict if so is the inequality in (4.2).

While this result can be seen as a corollary to Theorem 6 in Milgrom and Roberts (1990), here we provide a simple and illustrative proof. Consider the restriction of $T^x(\cdot \mid G, \delta')$ to $A = \{\theta \in \Theta \mid \theta \leq \theta^e\}$. Since $T^{x'}(\cdot \mid G, \delta')$ is nondecreasing and by virtue of (4.2), $T^{x'}(A \mid G, \delta') \subseteq A$. Tarski fixed point theorem implies the existence of a fixed point $\hat{\theta}^e$ of $T^{x'}(\cdot \mid G, \delta')$ in A. This completes the argument.[14]

The following two corollaries are immediate consequences to the proposition above.

Corollary 6

$$\theta^c = \min\{\theta \in \Theta \mid T^0(\theta) = \theta\}$$

Corollary 7 *Let $\theta^c(F, \delta)$ be the solution of (4.1) and suppose that*

$$T^0(\theta^c(F, \delta) \mid G, \delta') \leq \theta^c(F, \delta).$$

Then

$$\theta^c(G, \delta') \leq \theta^c(F, \delta)$$

If the first inequality is strict, then so is the second one.

The interest of the first corollary is in computing the Pareto optimal equilibrium. It proves that we only need to search for the Pareto optimal equilibrium among the set of fixed point of the map T^0, and there is no need to check whether that fixed point belongs to Θ^0. The result suggests a simple way to compute θ^c: Set $x_0 = \underline{\theta}$ and $x_n = T^0(x_{n-1})$. It follows that for all n, $x_n \in \Theta^0$ and $x_n \nearrow \theta^c$.[15]

The second corollary permits to do some comparative statics for the Pareto optimal equilibrium. Of course, these results will be useful only when there are circumstances under which the Pareto

equilibrium. While imposing more stringent assumptions, those results can be applied in our setting too.

[14] The same argument shows that when the inequality in (4.2) is reversed, it is possible to ensure the existence of $\hat{\theta}^e = T^{x'}(\hat{\theta}^e \mid G, \delta') > \theta^e$. This result is not so interesting in our framework for whenever $\Theta^x \neq \Theta$, $\bar{\theta} = T^x(\bar{\theta})$.

[15] The algorithm will converge even if there is no equilibrium, provided $\bar{x} \geq 0$.

optimal equilibrium is a good prediction of equilibrium play. This will be discussed in the next section.

Finally, the following result summarizes this subsection main findings.

Theorem 8 *Suppose that F and G have the same support Θ and $\delta, \delta' \in]0,1[$.*

(a) $\theta^c(F,\delta) \geq \theta^c(G,\delta')$, provided either of the followings holds:

(i) $\delta' \geq \delta$, and $G \succeq F$ in the first order stochastic dominance order;

(ii) $\delta' \geq \delta$, and $(1 - F(\theta^c(F,\delta)-)) \leq (1 - G(\theta^c(F,\delta)-))$.

(b) $\theta^c(F,\delta) \leq \theta^c(G,\delta')$, provided the following holds:

(iii) $\delta \geq \delta'$, and $(1 - F(\theta-)) \geq (1 - G(\theta-))$ for all $\theta \leq \theta^c(F,\delta)$.

The consequences of condition (i) in part (a) are to be expected. If agents discount less the future or obtain more utility from cooperation, then it is possible to support more cooperation. Less obvious is the condition stated in (ii). It says that if the fraction of agents having type grater than or equal to $\theta^c(F,\delta)$ increases, then the new Pareto optimal equilibrium exhibits at least as much cooperation as the original one. Part (b) states a partial converse to part (a) (ii). As a result, as will be illustrated soon, a second order stochastic increase of the distribution of types may have an ambiguous impact on cooperation.

Theorem 8 provides cooperative statics results for the set P. Yet, it says nothing about whether the proportion of cooperative players increases or decreases when the distributions are modified. The following corollary provides such results.

Corollary 9 *Suppose that F and G have the same support Θ and $\delta, \delta' \in]0,1[$.*

(a) Under either (i) or (ii) above, the proportion of cooperative players is bigger under G than under F; in other words,

$$\big(1 - G(\theta^c(G,\delta')-)\big) \geq \big(1 - F(\theta^c(F,\delta)-)\big).$$

(b) Under (iii) above, the proportion of cooperative players is bigger under F than under G; in other words,

$$\big(1 - G(\theta^c(G,\delta')-)\big) \leq \big(1 - F(\theta^c(F,\delta)-)\big).$$

4.2 Examples

The following example illustrates some of our comparative statics results.

Example 10 (Heterogeneity: Second order stochastic dominance) Consider a four type model $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ where all types are equally likely: $F(\theta_i) = i/4$. The discount factor is $\delta = 0.9$ and $g = 1$. The model is constructed so that the only equilibrium is $\theta^c = \theta_4$. Note that the necessary and sufficient conditions for $\textsc{Equil} = \{\theta_4\}$ are

$$\theta_4\Big(1 + \frac{\delta}{1-\delta}\frac{1}{4}\Big) \geq g$$

and, for $i = 1, 2, 3$,

$$\theta_i\Big(1 + \frac{\delta}{1-\delta}\frac{5-i}{4}\Big) < g.$$

These conditions can be equivalently stated as

$$\theta_4 \geq \frac{4}{13},\ \theta_3 < \frac{2}{11},\ \theta_2 < \frac{4}{31},\ \theta_1 < \frac{1}{10}.$$

While not necessary, restrict attention to the case $\theta_4 = \frac{1}{3}$, $\theta_3 = \frac{1}{6}$, $\theta_2 = \frac{1}{8}$, $\theta_1 = \frac{1}{12}$.

Define $\alpha = \frac{1}{6}$ so that $\theta_2 = \alpha\theta_4 + (1-\alpha)\theta_1$. For $\beta > 0$, consider the following mean preserving spread of F:

$$\mathbb{P}^\beta(\theta = \theta_i) = \begin{cases} \frac{1}{4} + \beta(1-\alpha) & \text{if } i = 1, \\ \frac{1}{4} - \beta & \text{if } i = 2, \\ \frac{1}{4} & \text{if } i = 3, \\ \frac{1}{4} + \beta\alpha & \text{if } i = 4, \end{cases}$$

where $\beta < 1/4$.[16]

It is possible to find β such that the best equilibrium under F^β is θ^3. To see that, note that

$$\theta_3\Big(1 + \frac{\delta}{1-\delta}(1 - F^\beta(\theta_3-))\Big) \geq g$$

if and only if $\beta \geq \frac{\frac{2}{11} - \theta_3}{9\alpha} = \frac{1}{99}$. Moreover, it is relatively simple to see that this is the Pareto optimal equilibrium. This proves that a second order stochastic decrease may lead to an increase in cooperation and total welfare. From here on, set $\bar{\beta} = \frac{1}{99}$.

[16] We can interpret probability P^β as follows. Draw θ according to F. If $\theta = \theta_2$, set $\widetilde{\theta} = \theta_2$ with probability $(1-4\beta)$, $\widetilde{\theta} = \theta_1$ with probability $4\beta(1-\alpha)$, and $\widetilde{\theta} = \theta_4$ with probability $4\beta\alpha$. If $\theta \neq \theta_2$, then set $\widetilde{\theta} = \theta$. $\mathbb{P}^\beta$ is the distribution of $\widetilde{\theta}$.

Now, define $\gamma = \frac{1}{3}$ so that $\theta_3 = \gamma\theta_4 + (1-\gamma)\theta_1$. Consider a mean preserving spread of $F^{\bar{\beta}}$:

$$P^{\lambda}(\theta_i) = \begin{cases} \frac{1}{4} + \bar{\beta}(1-\alpha) + \lambda(1-\gamma) & \text{if } i = 1, \\ \frac{1}{4} - \bar{\beta} & \text{if } i = 2, \\ \frac{1}{4} - \lambda & \text{if } i = 3, \\ \frac{1}{4} + \bar{\beta}\alpha + \lambda\gamma & \text{if } i = 4, \end{cases}$$

where $\lambda < \frac{1}{4}$. Now, I argue that we can pick $\lambda < \frac{1}{4}$ so that the only equilibrium under the distribution P^{λ} is θ_4. It is sufficient to impose the conditions.

$$1 - F^{\lambda}(\theta_3-) = \frac{1}{2} + \bar{\beta}\alpha - \lambda(1-\gamma) < \frac{1}{2}.$$

This condition holds for $\lambda > \frac{\bar{\beta}\alpha}{1-\gamma} > \frac{1}{99}$. This proves that a second order stochastic decrease may lead to a decrease in cooperation.

In this example, a second order stochastic decrease in the distribution of types (which holds by setting $\bar{\beta} = \frac{1}{99}$) implies that more agents have types above the cooperation threshold θ_4 and so, the new equilibrium exhibits more cooperation (Theorem 8 (ii)). Once the distribution $F^{\bar{\beta}}$ is fixed by setting $\bar{\beta} = \frac{1}{99}$, a new second order stochastic decrease was implemented. This new distribution puts more weight on the set $\{\theta_1, \theta_2\}$ so that those types as well as type θ_3 players become less willing to cooperate. As a consequence (Theorem 8 (iii)), the new equilibrium exhibits less cooperation.

This analysis contrasts with that by Haag and Lagunoff (2007). These authors posit a dynamic game model where a set of agents play repeatedly a stage game against the *same opponents* so that long term cooperation is sustained through personal enforcement. Haag and Lagunoff (2007) show that a second order increase in the distribution of types unambiguously leads to an increase in average cooperation (a definition which is not related to welfare in that model). The difference arises in the varying opponent aspect of my model. In this model, an increase in the fraction of cooperative players may push some formerly noncooperative players to cooperate for the value of cooperation increases. This increase in the fraction of cooperative players is consistent with a second order stochastic decrease in the distribution of types. In the Haag-Lagunoff model, players play against the same opponent always which makes the above discussed effect absent.[17]

In the next example, we draw a more detailed parallel between my result and Haag and Lagunoff (2007)'s. This example also exhibits the no-monotonicity property of our previous example.

[17]Mobius and Szeidl (2007) propose a social network model, where aggregate trust (understanding trust as the amount of money that one agent can lend from another) is decreasing in heterogeneity.

Example 11 Consider first an homogenous model, where the only type in population is $\theta = \theta^f$. The necessary and sufficient condition for cooperation is

$$\theta^f \geq g(1-\delta).$$

We can also consider the following personal enforcement model. There are two agents, who play a prisoners' dilemma repeatedly. So the only difference between the personal enforcement model and the main model discussed in the text is that in the former each of the two agents interact with the same partner. In the personal enforcement model, if types are homogenous $\theta = \theta^f$, cooperation takes place if and only if

$$\theta \geq g(1-\delta).$$

Therefore, the random matching model and a continuum of personal enforcement models are undistinguishable from a behavioral perspective. This is so because given homogeneity, in each round a player in the random matching model is facing an opponent which is identical in all respects to the ones previously faced and all relevant information is public. From here on, we set $\theta^f = g(1-\delta)$.

Now, add heterogeneity to the models. To do that, consider two types $\underline{\theta} = \theta^f - \alpha$ and $\bar{\theta} = \theta^f + \alpha$, where $\alpha \in]0, \theta^f]$. In the personal enforcement model, suppose that each of the two players has a different type. For all $\alpha > 0$, no cooperation can take place. Indeed, the lower type player will never find optimal to cooperate for $\underline{\theta} = \theta^f - \alpha < g(1-\delta)$, and as a consequence, the hight type will never cooperate.

In the random matching model, suppose that half of the population is of type $\theta^f - \alpha$ and the other half is of type $\theta^f + \alpha$. We may think of α as parameterizing the distribution of types, so that an increase in α is equivalent to a second order stochastic decrease in the distribution of types. In the random matching model, low type players will never cooperate. The high type players may or may not cooperate depending on α. To see that, note that a high type player will cooperate when matched to a high type opponent if and only if

$$\bar{\theta}\Big(1 + \frac{\delta}{1-\delta}\frac{1}{2}\Big) \geq g.$$

This condition holds whenever

$$\alpha \geq \frac{\delta(1-\delta)}{2-\delta} g.$$

Since $\theta^f > \frac{\delta(1-\delta)}{2-\delta} g$, this condition can be met.

A more general personal enforcement model is studied by Haag and Lagunoff (2007). Two are the main differences between the personal enforcement model here studied and the one studied by them. First, Haag and Lagunoff (2007) allow players to play and monitor mixed strategies. So, in principle, the low type players could cooperate with some positive probability. Second, in the work by Haag and Lagunoff (2007), there is heterogeneity in players discount factors but not in the gains from cooperation. Neither of these differences seems relevant to either their results or our purposes.

5 Adverse Selection and Pricing Strategies

So far our analysis has been not fully comprehensive in that we have not described the way in which players will be sorted out by the function ψ at the onset of the game. We may be interested, for example, in implementing the Pareto optimal equilibrium θ^c. One imaginable mechanism is to ask the players about their types. A player gets a mark G if and only if it claims to have a type $\theta \geq \theta^c$. Of course, under this arrangement players have an incentive to misreport and claim to have a high type. It seems therefore important to look at mechanisms that may solve this problem in a simple way.

We propose a simple mechanism where, at the beginning of the game, marks G are sold at a price $p \in \mathbb{R}$ (to be defined). That is, instead of assigning marks according to ψ, a seller offers an unlimited amount of marks G at a price p. Those agents who do not buy a mark G get a mark B. Once the marks are assigned, each agent's mark evolves according to the assignment η previously described. We can interpret the price p as a club membership fee. (We go over this interpretation in more detail later on.) We consider two cases; in the first one, the seller is a social planner, while in the second one the seller is a monopolist.

5.1 Social Planner Pricing

Suppose that we can implement the Pareto optimal equilibrium by setting a price p. Consider the equilibrium cutoff type θ^c and all types above θ^c. At the beginning of the game, a type $\theta \geq \theta^c$ has total expected payoff equal to

$$\pi^c(\theta) = \frac{\theta(1 - F(\theta^c -))}{1 - \delta}$$

if it is assigned a G mark, and 0 if its mark is B. So, a type $\theta \geq \theta^c$ will be willing to buy a mark G if and only if

$$\pi^c(\theta^c) \geq p,$$

Now, a type $\theta < \theta^c$ has a slightly different payoff for if it gets a mark G, then it will defect always and, in particular, whenever its rival is a good type. So, its payoff is

$$\pi^c(\theta, G) = \frac{(1 - F(\theta^c -))g}{1 - \delta F(\theta^c -)}.$$

So, for a type $\theta < \theta^c$ not to buy a mark G, this quantity has to be less than or equal to p.

The following proposition proves that it is possible to set a price so that the proposed assignment implements the Pareto optimal equilibrium.

Proposition 12 *The set of prices over which a seller can implement the optimal equilibrium is nonempty.*

This result provides a simple mechanism through which a social planer may implement the optimal equilibrium θ^c. Note that this price mechanism, which can also be implemented by charging a membership fee each period, is robust to several variations. It is robust to a secondary club membership market, where club members may sell their membership to nonmembers. This secondary membership market resembles the market for names, as studied by Tadelis (2002). So, the sorting mechanism is robust to a market for names. Moreover, in the continuous distribution case, incentives are strict for all but a negligible population fraction. Indeed, only the cutoff type is indifferent between buying and not buying a membership. In particular, had we solved its indifference in a different way, it would not have changed the incentives of the rest of the agents. These are also features of the price mechanisms discussed below.

The price mechanism here studied may be seen as providing a theory of network formation. The coalition can be interpreted as a club and the agent in charge of the club (that is, in charge of selling the memberships and providing the marks as play histories unfold) may be seen as an information intermediary. The club membership fee is p and an agent stays in the club so long as he complies with the club's norms.The "law merchants" in medieval France (Milgrom et al. (1990)) and Sicilian mafia in southern Italy (Dixit (2003a)) may be seen as chief examples of institutions providing the information intermediation activities here studied.[18]

As noted by Milgrom et al. (1990) and Dixit (2003a), some information intermediary institutions also engaged in enforcement activities by directly punishing defectors. Formally speaking, in our

[18]More generally, several other organizations may be seen as providing information dissemination after charging a membership fee. While not their main service, clubs aimed at resolving the problem of provision of public goods (Buchanan (1965), Ellickson et al. (1999)) do facilitate the spread of information. In the absence of altruistic motives, charity and rotary clubs may also be seen as providing such a flow of information.

model such punishment implies a decrease in g which, as a consequence of our previous analysis, leads to a decrease in θ^c and, in turn, to an increase in aggregate cooperation. We will show later on in the paper how the presence of heterogeneity among community members allows the information intermediary to engage in punishing activities in a much more subtle and indirect way.

5.2 Monopoly Intermediation

As noted by Dixit (2003a), in the absence of a legal system or a social planner able to perform the information intermediation activities previously explored, private for-profit intermediation may be an equilibrium outcome. In this subsection I study how a monopolist will price club's memberships. I will abstract from extortion and double-crossing practices by simply focusing on the quantity-price tradeoff faced by a monopolist (who can credible commit to convey play histories) when setting the membership fee.[19]

Consider first any equilibrium $\theta^e \in \textsc{Equil}$. The same argument we employed to ensure the implementability of θ^c allows us to show that θ^e can be implemented by setting a price $p \in \mathbb{R}_+$. Moreover, the monopoly will optimally set the highest of those prices. It is not hard to see that such highest price is given by

$$p(\theta^e) = \theta^e \frac{1 - F(\theta^e-)}{1 - \delta}$$

so that the monopoly extracts all the rent of the cutoff type θ^e. The monopoly problem is therefore defined as

$$\max\{p(\theta^e)(1 - F(\theta^e-)) \mid \theta^e \in \textsc{Equil}\},$$

a problem which has always a solution.[20] In a departure from the standard monopoly problem, the tradeoff faced by our monopolist is not clear-cut. To see that, note that an increase in θ^e implies a decrease in total quantity $(1 - F(\theta-))$. But this decrease in the total memberships may not lead to an increase in the price $p(\theta^e)$. Indeed, even when the type of the marginal type θ^e increases, its

[19]The for-profit intermediary may extort agents with a clean history (by threatening to assert they cheated in the past) or double-cross some agents (by allowing some players to cheat while keeping their history clean). Dixit (2003a) shows how these moral hazard problems on the intermediary side restrict the intermediary pricing problem. Those constraints could be considered in this model too.

[20] In particular, we are restricting the monopoly to implement outcomes that are consistent with our equilibrium notion. This is somehow similar to the mechanism design literature methodology, where the modeler restricts the mechanism designer to implement mechanism that possess an equilibrium. In our model, the monopolist (mechanism designer) set prices (mechanisms) such that the game (mechanism) played after the price (mechanism) is set has an equilibrium.

willingness to pay may decrease as a result of the decrease in the number of cooperative encounters. So, $p(\theta^e)$ may be decreasing in θ^e.

To see how the mechanism above may work, suppose that for some $x \in [0, \bar{x}]$, θ^e and $\theta^{e\prime}$ are two cooperative equilibria such that $\theta^e, \theta^{e\prime} \in B^x \cup \Theta^x$. For simplicity, suppose that F is continuous. So, both θ^e and $\theta^{e\prime}$ solve the equation

$$\theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta))\Big) = g + x.$$

This equation can be equivalently written as

$$\theta + \delta p(\theta) = g + x.$$

An increase in θ implies a decrease in $p(\theta)$. This means that by rising the marginal type the monopolist is not only selling less marks but also at a lower price per unit. The monopolist will therefore prefer $\min\{\theta^e, \theta^{e\prime}\}$ over $\max\{\theta^e, \theta^{e\prime}\}$.

The above discussion seems promising in terms of the incentives of the monopolist to implement the optimal equilibrium θ^c. It happens, however, that the ability of the monopolist to pick the slackness of the incentive constraint may make profitable to implement some other equilibrium.

From here on, assume that F is continuously differentiable with support $[a, b]$, where $0 \leq a < b$. Denote its derivative by $f(\theta)$, and assume that $f(\theta) > 0$ for all $\theta \in [a, b]$. Define the function

$$\Phi(\theta) = 2\theta - \frac{1 - F(\theta)}{f(\theta)}.$$

The following assumption is key in our analysis.

Monotonicity: $\Phi(\theta)$ is increasing on $[a, b]$.

This assumption is less demanding than the monotone likelihood ratio condition usually employed in mechanism design; see for example Myerson (1981). In this sense, we do not regard the monotonicity assumption as particularly stringent.

We define

$$\underline{g}(\delta) = \theta^*\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^*))\Big),$$

where

$$\theta^* = \min\{\theta \in [a, b] \mid \Phi(\theta) = 0\}.$$

It is easy to see that $\underline{g}(\delta) < \bar{g}(\delta)$, where $\bar{g}(\delta)$ is defined by Equation (3.3), and both of these functions are nondecreasing in δ.

Proposition 13 *Under Monotonicity, the following assertions hold:*

(i) If $g \in [\underline{g}(\delta), \bar{g}(\delta)]$, then the monopoly problem has $\theta^m = \theta^c$ as solution.

(ii) If $g \in]b, \underline{g}(\delta)[$, then the monopoly problem has solution $\theta^m > \theta^c$.

(iii) Defining θ^1 as the only solution to

$$\theta^1 = \frac{1 - F(\theta^1)}{f(\theta^1)},$$

it follows that

$$\lim_{\delta \to 1} \frac{\bar{g}(\delta) - \max\{b, \underline{g}(\delta)\}}{\bar{g}(\delta) - b} = 1 - \frac{\theta^*(1 - F(\theta^*))}{\theta^1(1 - F(\theta^1))}. \tag{5.1}$$

The first two statements characterize the monopoly problem solutions. The logic behind these results is the following. When g is sufficiently big, less players are willing to cooperate, and therefore the equilibrium set EQUIL is small. It is then proven that when EQUIL is sufficiently small, the whole equilibrium set belongs to the decreasing portion of the monopolist objective function. So, the best the monopolist can do is to set $\theta^m = \theta^c$.

To interpret Equation (5.1), note that $\bar{g}(\delta) - b$ is the Lebesgue measure of those g for which all the period games are prisoners' dilemma having an equilibrium exists. On the other hand $\bar{g}(\delta) - \max\{b, \underline{g}(\delta)\}$ is the Lebesgue measure of the set of g under which the model has an equilibrium and the monopoly problem has the Pareto optimal equilibrium as its solution. Therefore, viewing g as randomly draw from a uniform distribution, the quotient in the right hand side of (5.1) can be interpreted as the probability of getting a model under which the Pareto optimal problem can be solved by giving to a monopolist the right to sell the marks, conditional on the model having an equilibrium. The formula gives us the asymptotic value of this probability, showing in particular that it will be strictly less than 1.

The monopoly could achieve the same payoff by charging a per period membership fee. When the monopolist acquires a club with an efficient level of cooperation, Proposition 13 shows that the monopolist is willing to exclude some members from the club. Of course, a straightforward way to do it is by rising the membership fee. However, for the monopolist this may be hard to implement if club's members have some influence on the monopolist's decisions and may not be willing to accept the exclusion of some members. Indeed, rising the membership fee and excluding some agents can only harm club's members. A subtler way to do it is by misreporting play histories, so that when the membership fee is raised, no member drops out of the club.[21]

[21]Some evidence suggests that the fraction of errors in credit histories in the US is substantial (Hunt (2002)). The

When the monopolist implements the Pareto optimal equilibrium, its monopoly position cannot be challenged. When it does not, depending on the strength of the competitive forces and the way in which agents coordinate, competition may lead to implement the Pareto optimal outcome through a single club. We leave for future research the exploration of these possibilities.

Purely informational private intermediation is studied by Lizzeri (1999) and Biglaiser (1993). Those authors deal with the problem of quality-certification in an adverse selection static setting. The certification process in the present model is much simpler, in part because players's payoffs do not depend on their partners' types but only on their actions.

The following example illustrates the monopoly pricing schemes in our model.

Example 14 Suppose that $F(\theta) = \theta$. So the monotonicity assumption is satisfied. Moreover $\theta^* = \frac{1}{3}$ and

$$\underline{g}(\delta) = \frac{1}{3} + \frac{\delta}{1-\delta}\frac{2}{9}.$$

The threshold $\bar{g}(\delta)$ can be analytically derived. Indeed,

$$\bar{g}(\theta) = \begin{cases} \frac{1}{4\delta(1-\delta)} & \text{if } \delta \geq \frac{1}{2}, \\ 1 & \text{if } \delta < \frac{1}{2}. \end{cases}$$

There exists a cooperative equilibrium only if $\delta \geq \frac{1}{2}$. The following table shows the values of g for which there exists a cooperative equilibrium and the monopoly finds optimal to implement the Pareto optimal equilibrium.

δ	$\underline{g}(\delta)$	$\bar{g}(\delta)$	$\frac{\bar{g}(\delta)-\max\{1,\underline{g}(\delta)\}}{\bar{g}(\delta)-1}$
0.6	0.66	1.04	1
0.75	1	1.33	1
0.8	1.22	1.56	0.60
0.9	2.33	2.77	0.25
0.99	22.33	25.25	0.12

Given that we assume that the prisoners' dilemma is actually a dilemma for all matches, we assume that $g > 1$. The table shows that for all $\delta \leq 3/4$, provided an equilibrium exists, the monopoly will

mechanism here explored offers an alternative explanation to the conventional views: By misreporting, credit bureaus are shrinking the credit demand but at the same time this pushes away smaller creditors. After rising the membership fees, no creditor is expelled. It may be possible that credit bureaus act in behave of large and efficient creditors, and that misreporting credit histories allow large creditors to exploit a less competitive position.

always end up picking θ^c. For $\delta = 0.9$, the monopoly will pick θ^c only if $g \geq 2.33$, quite a stringent condition given that for equilibrium existence we require $g \leq 2.77$. When g fails to be in the region $[\underline{g}(\delta), \bar{g}(\delta)]$, the monopoly will unambiguously set $\theta^m = \frac{1}{3}$.

This example suggests that the higher the agents' patience, the less likely the monopolist will implement the Pareto optimal equilibrium. Intuitively, this should be so because the higher the discount factor, the larger the equilibrium set, and so the more the alternatives the monopolist have to improve upon θ^c. However, I have not been able to provide a general statement of this result.

6 Discussion and Applications

6.1 Trust, Social Capital, and Community Heterogeneity

Recently, economists have given considerable attention to concepts such as social capital and trust, and their impact on output and growth. While many conceptual questions need to be responded –among others, what exactly we mean by social capital – the analysis here presented contributes to this discussion.

An empirical work by Alesina and La Ferrara (2002) shows that in more racially and economically heterogenous communities, people are less likely to trust others. While how people interpret 'trust' when asked whether they trust others is an open question, community agents in our model would interpret it as whether they are expected to meet someone willing to cooperate during a round encounter. In this sense, the probability of meeting a cooperative type may be seen as a good measure of trust in our model.

Our theoretical results identifies the impact of heterogeneity on trust and cooperation. In particular, it shows that more heterogenous communities need not exhibit lower levels of trust. This is so because more heterogeneous societies may have a higher fraction of people with high tendencies towards cooperation, an aspect that facilitates cooperation.

It seems important to mention that in our model, heterogeneity is non observable (payoff structure), while in most of empirical studies heterogeneity is observable (race, income). This is an explanation for why our results might differ from the empirical results by Alesina and La Ferrara (2002): It is possible that more racially diverse communities have lesser payoff relevant asymmetries.

Durlauf and Fafchamps (2005) provide an exhaustive survey of the literature on social capital and economic performance. They show that the relationship between social capital and aggregate output is by no means empirically established. Simple OLS regressions fail to capture this relation because social capital is likely an endogenous variable.[22] The present work suggests that a way to settle this issue is to look at different measures of heterogeneity,[23] and exploit the mechanisms here studied as an answer for why we observe differences in social capital and output.

6.2 Credit Markets and Credit Bureaus: Information Disclosure

Two somehow disconnected literatures have emerged to model credit bureaus and their importance on solving the intrinsical information asymmetry in credit markets. On the one hand, Klein (1992) and Kandori (1992) focus on the moral hazard aspect involving credit transactions in that borrowers may renege on their payments. On the other hand, Pagano and Jappelli (1993) stress the importance of credit bureaus as a means to solve the adverse selection in that each consumer's repayment probability is his private information. I will argue below that both problems seem important in practice.

The fact that financial market outsiders are willing to spend resources to obtain the financial market histories of prospective partners suggests that some form of adverse selection is present. Indeed, if it were otherwise (or in other words, if there were no payoff relevant difference between defector and cooperative agents), even if agents were to use different strategies in the new relation, outsiders (such as landlords and employers) could renegotiate and prompt financial market defectors to use the strategies followed by non defectors in the new relation.[24] This new contract is optimal for both the outsider and the defector because the contract would be voluntarily signed by the outsider and a non defector. Therefore, an outsider wishes to contract both a defector and a non defector and, as a consequence, is not willing to spend resources to know the financial situation of a prospective partner. This contradicts the evidence.[25]

By restricting the model so that some agents repay with some exogenous probability, it is being assumed that the legal system works so that, at least for some population members, actions can be enforced ex-post (Pagano and Jappelli (1993)). In other words, this line of modeling works under the assumption that there is a working legal system with some capability to enforce contracts. The alternative enforcement mechanism is self-governance and is the focus of this paper. After all, even in

[22]Some attempts to run IV regressions have been made. As discussed by Durlauf and Fafchamps (2005), the IV variables up to now considered do not seem exogenous.

[23]Of course, what forms of observable heterogeneity are important for our model is an empirical question.

[24]The argument assumes that the cooperative strategies are feasible for a defector. This need not be so. For example, financial market defectors may be credit constrained and that may impair them to honor housing market contracts. This example does not seem so relevant for the labor relation.

[25]The presence of adverse selection in credit relations has also been suggested by Ausubel (1999).

countries with a well functioning legal system, economic agents need to consider that if they defect, they not only will be legally sanctioned but also will be credit constrained and eventually expelled from the financial system.

A natural question to ask is whether the disclosure of information is a Pareto improvement upon a nondisclosure policy. For the question to be under consideration, it is necessary to have outsiders interested in such each agent's type. We assume that those outsiders exist and are interested in establishing relationship only with agents having a sufficiently high type θ. This is consistent with the practice among financial market outsiders above discussed. It turns out that disclosing information decreases the defection payoff g (pretty much in the same way as studied by Milgrom et al. (1990), and Dixit (2003a)) because outsiders will be less willing to establish relationships with B marked agents. In other words, by disclosing information about play histories, the information intermediary is punishing defectors *de facto*.

We assume that $\Theta = [0, 1]$ and F has no atom. Suppose that there is a secondary market (e.g. the labor market), where agents in our model may participate.[26] In the secondary market, there is a continuum of outsiders (prospective employers) who, by trading with a type θ agent at a round t, get a payoff $V(\theta)$, where V is nondecreasing. To make the problem interesting, we assume that for an outsider it is not always optimal to trade so that $\mathbb{P}[V(\theta) < 0] > 0$. If an agent trades, then he gets a payoff $W \geq 0$, so for an agent trade is always beneficial. We assume that, at each t, trade in the secondary market takes place right after the randomly matched agents play the prisoners' dilemmas. Two designs are possible: To disclose agents' marks or to not disclose those marks.

In the disclosure model where outsiders only trade with G marked agents (a situation we deem as a good description of reality), an equilibrium in the main game is a fixed point of the map

$$T^{x,W}(\theta^e) = \min\{\theta \mid (\theta + W)\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^e))\Big) \geq g + x\},$$

so defined whenever the feasible set is nonempty. An outsider will indeed be willing to hire only G marked players if $\mathbb{E}[V(\theta)|\theta \geq \theta^e] \geq 0$ and $\mathbb{E}[V(\theta)|\theta \leq \theta^e] \leq 0$. So, the Pareto optimal equilibrium in this class is characterized by

$$\theta^c_W = \min\{\theta^e \mid \theta^e = T^{0,W}(\theta^e),\ \mathbb{E}[V(\theta)|\theta \geq \theta^e] \geq 0,\ \mathbb{E}[V(\theta)|\theta \leq \theta^e] \leq 0\}.$$

Now, in the model with no disclosure, the equilibrium is $\theta^c = \min\{\theta \mid \theta = T^0(\theta)\}$ and is assumed to be strictly positive. We assume that in the no disclosure model, secondary market transactions take place, or in other words, $\mathbb{E}[V(\theta)] \geq 0$. Disclosure will increase cooperation if and only if $\theta^c_W \leq \theta^c$.

[26] We stick to the random matching model previously presented. While the moral hazard problem in credit relations may be better modeled as a one-sided prisoner's dilemma, the results we develop can be easily adapted to that setting.

Proposition 15 *(i) If* $\mathbb{E}[V(\theta)|\theta \leq \theta^c] \leq 0$*, then for all* $W > 0$*,* $\theta^c_W < \theta^c$*.*

(ii) If $\mathbb{E}[V(\theta)|\theta \leq \theta^c] > 0$*, then there exists* $\bar{W} > 0$ *such that for all* $W \leq \bar{W}$*,* $\theta^c_W = \theta^c$ *and employers ignore agents' marks. For* $W > \bar{W}$*, then* $\theta^c_W \leq \theta^c$*.*

The argument behind this proposition is the following. Suppose that information is not being disclosed so that the equilibrium of the model is characterized by θ^c. Now, suppose that agents' marks are disclosed. In (i), outsiders find profitable only to hire agents having a G mark (or a type $\theta \geq \theta^c$). So, the original equilibrium is also an equilibrium of the disclosure model and consequently the disclosure model cannot have a Pareto dominated equilibrium. In (ii), however, once marks are disclosed, outsiders will still find optimal to hire B marked agents. When W is small, even if outsiders were not to trade with defectors, that would only slightly decrease θ^c so that, by continuity, outsiders must be willing to trade with everyone. When W small, therefore, it is not possible to improve upon the no disclosure situation. For W sufficiently large, however, outsiders are indeed willing to trade only with G marked players and so $\theta^c_W < \theta^c$.

The mechanism above somehow resembles the multimarket interaction model studied by Bernheim and Whinston (1990). The main difference is that in my model agents punishing defectors in the secondary market may not be part of the primary game model. Agents in the secondary market punish defectors not because of some relational force but because defecting is a bad signal on the defector agents' intrinsic cooperation tendency.

The result shows that information disclosure cannot decrease cooperation. However, the result does not say anything about whether efficiency in the secondary market is improved. By assuming that the outsider's payoff function takes the form $V(\theta) = u(\theta) - W$, a transaction involving a type θ agent is efficient if and only if $u(\theta) \geq 0$. So, if for all $\theta \in [0,1]$, $u(\theta) \geq 0$, then marks disclosure cannot improve efficiency in the secondary market.[27] If, on the other hand, with positive probability $u(\theta) < 0$, then information disclosure may improve information in the secondary market.[28]

[27] Two are the key assumptions that open the possibility for efficiency reduction in the secondary market. First, we are assuming that $\mathbb{E}[V(\theta)] > 0$, so that in the no disclosure case outsiders are willing to trade with game agents. Second, we are assuming that the price W is exogenous, and in particular, is not affected by the quality of the expected agent. These two assumptions are made for tractability and to clarify the role of information disclosure on cooperation in the *main game*.

[28] Information disclosure in adverse selection environments has ambiguous effects. See Levin (2001) for examples and results.

7 Concluding Remarks

I have argued that heterogeneity is important when studying game models of large communities. Based on this observation, this paper offers a repeated game model of a large community with heterogeneous agents. The existence and characterization of equilibrium strategies have been studied and novel comparative statics results have been presented. Contrary to previous results, it has been shown that more heterogeneous communities may sustain more cooperation. Conditions under which the intermediation problem can be efficiently delegated to a for-profit monopoly who will not see challenged its monopolistic position have been derived.

Many economic situation can be thought of as quasi random matching models. In those models, there are two large populations and, at each round, a population member is randomly matched to some member in the other population. For example, firms (the first group) produce high or low quality goods and consumers (the second group) demand goods but may renege on payment. As discussed by Dixit (2003a), in practice cooperation among firms and consumers may be sustained by spreading information about the quality of the goods produced by each firm (e.g. media and online feedback systems) and by creating networks that allow firms to exchange information (e.g. credit bureaus and business associations). Our methods can be exploited to analyze this class of models too.

Several research questions remain open. While considerable progress has been made describing the institutional environments that facilitate cooperation among community members, agents' incentives to provide such information have not been fully understood. Why do people participate in online feedback systems and not just free-ride from others' reports? It seems reasonable to think that at some degree people enjoy writing a negative report after being cheated, but much more (empirical and experimental) evidence needs to be analyzed before reaching a conclusive answer.

Extensions of the main framework are also possible. It is clear that the studied equilibrium strategies do not carry over to the finite population game. To see why, note that after a defection the total number of cooperative players change. As a consequence, the incentives off-the-equilibrium path are altered and so are therefore the on-the-equilibrium path conditions. By employing the empirical distribution of types (that is, assuming that a central authority may observe the type of each player and assign marks accordingly), one could use finite but sufficiently large punishments after a defection. This would require to expand the mark set, unless we consider 1-period punishment. The finite punishment equilibrium is sequential for, after a finite number of periods, cooperation is restored and the long run distribution of play does not depend on current period play.[29]

[29]This argument is set forth by Kandori (1992). One needs to guarantee not only cooperation among cooperative

8 Appendix

8.1 More General Equilibrium Notions

8.1.1 Nonstationary Equilibrium

Consider the model introduced in Subsection 2.1, but now let us work with a weaker equilibrium notion. We drop the stationarity and informational robustness restrictions and stick to the monotonicity and closeness restrictions we worked with in the main text. We assume that F is continuous.[30]

Let P_t be the set of G marked players at the outset of period t. We restrict our attention to equilibria such that, on the play path, P_t is closed and if $\theta \in P_t$ is cooperating at round t when faced to a partner in P_t, then so is any player in P_t with a type $\theta' \geq \theta$.

Consider ψ such that $\psi(\theta) = G$ if and only if $\theta \geq \theta_0$, where $\theta_0 \in \mathbb{R}$ is fixed. Define $P_1 = [\theta_0, \infty[\cap \Theta$. Given the sequence of matching functions $(M_t)_{t\geq 1}$, for all $t \geq 1$

$$P_{t+1} = \Big(P_t \cap \big\{i \in I \mid \theta_i \in [\theta_{t+1}, \infty[\big\}\Big) \cup N_t$$

where θ_t is the smallest type willing to cooperate in round t given that its partner has a mark G and $N_t \subseteq P_t \setminus [\theta_t, \infty[$ is the set of players who would have defected had their partners had a G mark but they keep their good records because their partners had a B mark. It is evident that $P_{t+1} \subseteq P_t$, and $(\theta_t)_{t\geq 1}$ is a nondecreasing sequence which converges, say, to $\theta^* \in \Theta$. The random matching assumption implies that, almost sure in the sequence $(M_t)_t$, $\mathbb{P}[i \in N_t] \to 0$ for with probability 1 an agent encounters a player willing to cooperate at some t. This implies that in the long run, players' strategies become increasingly robust and, almost sure, $\mathbb{P}[P_{t+1}] = (1 - F(\theta_t)) + \mathbb{P}[N_t] \to (1 - F(\theta^*))$. Since equilibrium strategies are asymptotically informationally robust, θ^* must satisfy the conditions characterizing a stationary informationally robust equilibrium.

8.1.2 Nonmonotonic Equilibrium

We consider equilibria where the set of cooperative agents P may not be a closed interval but any closed subset of Θ. For each closed set $P \subseteq \Theta$, define the set valued map

$$T^x(P^e) = \{P \subseteq \Theta \mid \text{ For all } \theta \in P, \theta + \frac{\delta}{1-\delta}\mathbb{P}(P^e) \geq g + x\},$$

players but also the off-the-equilibrium restriction that a cooperative defector is willing to incur the cost of becoming cooperative. See Theorem 2 in Kandori (1992).

[30]It is also assumed that the matching functions $(M_t)_{t\geq 1}$ are not observed. In the stationary model analyzed in the main text, whether or not agents observe the matching functions does not change the analysis.

whenever the right hand side is nonempty. Consider the equivalence relation $\mathcal{R}$: $A, B \subset \Theta$, $A\mathcal{R}B$ if and only if $\mathbb{P}[A] = \mathbb{P}[B]$. Define $\mathcal{C}$ as the set of all equivalence classes of $\mathcal{R}$, and endow it with the partial order $A \geq B$ if and only if $\mathbb{P}[A] \geq \mathbb{P}[B]$. So defined, $T^x \colon \mathcal{C} \to 2^{\mathcal{C}}$ is nondecreasing, and so following the main text analysis it is possible to characterize the equilibrium set.

8.2 The Structure of the Equilibrium Set

8.2.1 Preliminary Results

Proposition 16 *The equilibrium set* EQUIL *is closed in* $\mathbb{R}$.

Proof: It is immediate from the characterization given in Theorem 3 and the lower semi-continuity of $F(\theta-)$. □

In the rest of this subsubsection, we assume that F is continuous differentiable and its derivative $f(\theta)$ is (strictly) positive on $[a, b]$

Proposition 17 *Under Monotonicity,* EQUIL *is convex.*

Proof: Immediate from Theorem 3.□

By means of an example, we show that the equilibrium set may not be convex.

Example 18 Suppose that $\Theta = [0, 1]$ and

$$F(\theta) = \begin{cases} 0 & \text{if } \theta < 1/3, \\ 6(\theta - 1/3) & \text{if } \theta \in [1/3, 1/2[, \\ 1/2 & \text{if } \theta \in [1/2, 3/4[, \\ 2(\theta - 3/4) + 1/2; & \text{if } \theta \in [3/4, 1], \end{cases}$$

with $\delta = 0.9$, $g = 3$. By using Theorem 3, it is easy to see that the equilibrium set is

$$\text{EQUIL} = [0.3, 0.37] \cup [0.54, 0.86]$$

The equilibrium set is depicted in Figure 2. The solid line shows the function

$$\theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta))\Big),$$

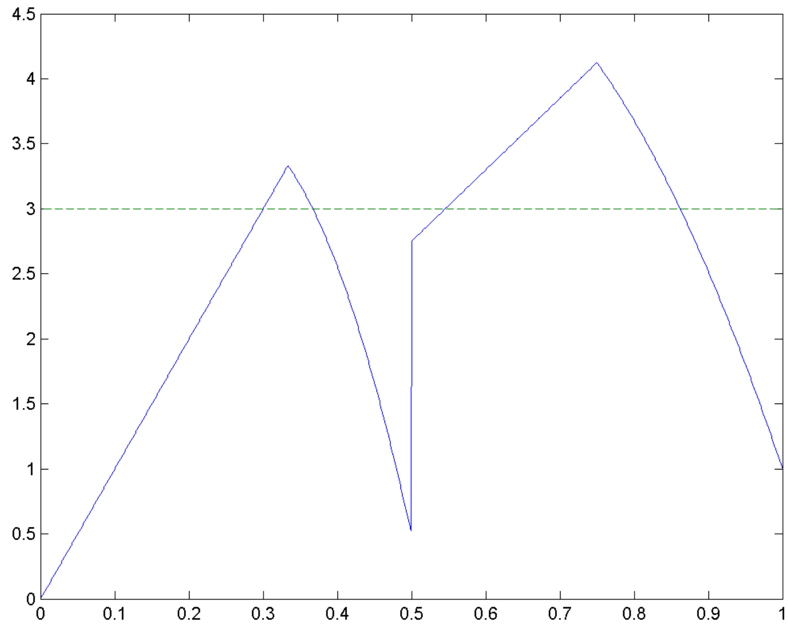

Figure 2: The equilibrium set may not be convex.

while the dashed line is at the level $g = 3$.

It should be clear that the nonconvexity could also be obtained with a differentiable distribution function whose density is strictly positive on.

8.2.2 Equilibrium Set Cardinality

Now, we investigate generic properties concerning the cardinality of the equilibrium set.

To do that, parameterize each model according to its discount factor $\delta \in]0, 1[$ (so, we fix g, F, and l). As shown in the text, a necessary and sufficient condition for equilibrium existence is given by Equation (**??**). So, consider the closed set of models for which an equilibrium exists:

$$\Delta = \{\delta \in]0, 1[| \; \bar{g}(\delta) \geq g\}.$$

It is easy to see that Δ must be an interval. Endow Δ with the Lebesgue measure.

Theorem 19 *Suppose that F is continuous. Then, generically in Δ, the set of equilibria is a continuum.*

It is useful to state the following lemma.

Lemma 20 *Suppose that $\bar{g}(\delta) > g$. Then, the set of equilibria is a continuum.*

Proof: The function $\theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta))\Big)$ is continuous in θ. Therefore, there exists an interval $[\theta_1, \theta_2]$, where $\theta_1 < \theta_2$, such that for all $\theta \in [\theta_1, \theta_2]$, $\theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta))\Big) > g$. So, $[\theta_1, \theta_2] \subseteq \textsc{Equil}$ and so $\textsc{Equil}$ must be a continuum.□

Proof of Theorem 19: It follows by noting that $\bar{g}(\delta)$ is continuous. Indeed, $\theta\Big(1+\frac{\delta}{1-\delta}(1-F(\theta))\Big)$ is continuous in (θ, δ) and so we can apply the maximum theorem. Moreover, $\bar{g}(\cdot)$ is strictly increasing on Δ. The result follows.□

8.3 Additional Related Results

8.3.1 More General Heterogeneity

It is possible that agents not only differ in the gains that obtain from cooperation, but, more generally, in their whole payoff profiles. To study this possibility, suppose that the payoff matrix in the stage game takes the form:

	C		D	
C	$c(\theta_1)$	$c(\theta_2)$	$-l(\theta_1)$	$g(\theta_2)$
D	$g(\theta_1)$	$-l(\theta_2)$	0	0

where $c(\cdot), g(\cdot), l(\cdot)$ are functions of the type. Each player's discount factor is considered as a function its type $\delta(\theta) \in [0, 1[$. The following prisoners' dilemma restrictions are imposed: For all $\theta \in \Theta$, $g(\theta) > c(\theta) \geq 0$ and $l(\theta) \geq 0$. Additionally, we assume that higher type players are more cooperative than lower type players. Formally,

Increasing cooperation attitudes (ICA): The functions $c(\theta), c(\theta) - g(\theta)$, and $\delta(\theta)$ are nondecreasing in θ.

We restrict our attention to stationary robust and monotone equilibria, which can be shown to be characterized by a cutoff point θ^e satisfying

$$c(\theta^e)\Big(1 + \frac{\delta(\theta^e)}{1-\delta(\theta^e)}(1 - F(\theta^e-))\Big) \geq g(\theta^e).$$

Define the map

$$\bar{T}^x(\theta^e) = \min\{\theta \in \Theta \mid c(\theta)\Big(1 + \frac{\delta(\theta)}{1-\delta(\theta)}(1 - F(\theta^e-))\Big) \geq g(\theta) + x\}$$

whenever the set over which the minimum is taken is nonempty. The map T^x is nondecreasing, and any equilibrium is a fixed point of $\bar{T}^x$ for some x. The characterization and comparative statics results can be proven following the arguments of Section **??**.

The decentralization result also holds in this more general model: By setting the right price, a social planer can implement the Pareto optimal equilibrium. The presence of other forms of heterogeneity may make more likely for the monopolist to implement the optimal equilibrium. Two particular cases are considered.

Case 1: The only source of heterogeneity is g. In other words, $c(\theta) = \bar{c}$ and $\delta(\theta) = \bar{\delta}$ are constant functions of θ. Then, by implementing $\theta^e \in \text{EQUIL}$, the monopolist obtains

$$\frac{\bar{c}}{1-\bar{\delta}}(1 - F(\theta^e-))^2$$

an expression which is decreasing in θ^e. Consequently the monopolist optimally sets the Pareto optimal equilibrium: $\theta^m = \theta^c$.

Case 2: The only source of heterogeneity is δ. In other words, $c(\theta) = \bar{c}$ and $g(\theta) = \bar{g}$ are constant functions of θ. By implementing $\theta^e \in \text{EQUIL}$, the monopolist gets

$$\frac{\bar{c}}{1-\delta(\theta)}(1 - F(\theta^e))^2.$$

Just for simplicity, suppose that $\delta(\theta) = \theta$. The objective function is decreasing in θ whenever $R(\theta) = 2(1-\theta) - \frac{1-F(\theta)}{f(\theta)} \geq 0$. Since $R(1) = 0$, the last condition will be met whenever $R(\theta)$ is decreasing (a condition that holds, for example, when F is uniform).

8.3.2 Expanding the Mark Set

Consider the model introduced in Section 2 but now expand the mark set to $M = \{m_1, \dots, m_M\}$. Assume that each player's mark evolves according to $\eta\colon M \times M \times A \to M$. At the outset of the game, marks are assigned according to $\psi(\theta) \in M$. The following result shows that there are no benefits from expanding the mark set.

Proposition 21 *Let θ^c be the Pareto optimal equilibrium defined by Equation (4.1). Then, no stationary informationally robust equilibrium of the model with mark set M can Pareto dominate θ^c.*

This result, whose logic is quite simple, shows us that adding more marks cannot lead to a Pareto improvement upon the Pareto optimal equilibrium. The following example shows that it is possible that expanding the mark set may lead to an increase in total welfare.

Example 22 Consider a two-type model $\Theta = \{\underline{\theta}, \bar{\theta}\}$ where both types are equally likely. Suppose that

$$0 < \underline{\theta} < (1-\delta)g$$

so that types $\underline{\theta}$ are not willing to cooperate always. We also suppose that $\bar{\theta}$ is sufficiently large so that those types cooperate when matched. In other terms,

$$(1-\delta)\bar{\theta} + \delta\frac{1}{2}\bar{\theta} \geq (1-\delta)g$$

Consequently, there is a single equilibrium $\theta^c = \bar{\theta}$ and the expected payoffs of types $\underline{\theta}$ and $\bar{\theta}$ are 0 and $\frac{1}{2}\bar{\theta}$ respectively.

Consider now a three mark model, where the mark set is $\{\bar{G}, \underline{G}, B\}$. We construct an equilibrium so that that when a type $\underline{\theta}$ is matched to a type $\bar{\theta}$, then the former defects and the latter cooperates; when a type $\underline{\theta}$ is matched to some other type $\underline{\theta}$, they cooperate. Marks are assigned so that $\bar{G}$ (resp. $\underline{G}$) is the mark for type $\bar{\theta}$ (resp. $\underline{\theta}$) that conforms to the equilibrium play, and B is the mark of any player off-the-equilibrium path.

The continuation values are

$$v(\bar{\theta}) = \frac{1}{2(1-\delta)}(\bar{\theta} - l)$$

and

$$v(\underline{\theta}) = \frac{1}{2(1-\delta)}(\underline{\theta} + g).$$

This profile is enforceable whenever $-l + \delta v(\bar{\theta}) \geq 0$ and $\underline{\theta} + \delta v(\underline{\theta}) > g$. These conditions can be equivalently stated as

$$g \leq \frac{2-\delta}{1-\delta}\underline{\theta},\ l \leq \frac{\delta}{2-\delta}\bar{\theta}$$

The expected payoffs of players $\underline{\theta}$ and $\bar{\theta}$ are $\frac{\underline{\theta}+g}{2}$ and $\frac{\bar{\theta}-l}{2}$, respectively.

Our proposed profile cannot Pareto dominates the Pareto optimal equilibrium of the two type model. However, if $l < \underline{\theta} + g - \bar{\theta}$, it produces higher total welfare. To see that all these inequalities may be satisfied, set $g = 2$, $\bar{\theta} = 1$, $\underline{\theta} = 1/2$, $\delta = 3/4$ and $l \leq 3/5$.

8.4 Proofs

Proof of Lemma 1. Let p be the probability mass of P. The only interesting case is $p > 0$. If (C, D) is the outcome of a cooperative-noncooperative match, then the stationary equilibrium restrictions imply that

$$-l + \delta v_G(\theta) \geq 0 + \delta v_B,$$

where $v_G(\theta) = \frac{p\theta - l(1-p)}{1-\delta}$ is the total expected payoff when type θ is cooperative and and $v_B = \frac{pg}{1-\delta}$ is what a player would obtain if marked as noncooperative. Note that in this case, it must be the case that $\eta(G, B, C) = G$ for otherwise either the stationarity restriction or the incentive restriction would be violated. In other words,

$$-l\big((1-\delta) + \delta(1-p)\big) \geq \delta p(g - \theta)$$

which is a contradiction for $\theta < g$ and $l \geq 0$. □

Proof of Lemma 2. Note that for $x \in [0, \bar{x}]$, $\underline{\theta} \in \Theta^x$. Indeed, $1 - F(\underline{\theta}-) = \mathbb{P}[\theta \geq \underline{\theta}] = 1$ so

$$\bar{\theta}\Big(1 + \frac{\delta}{1-\delta}(1 - F(\underline{\theta}-))\Big) = \frac{\bar{\theta}}{1-\delta} \geq g + x.$$

So, Θ^x is nonempty. Moreover, for $\theta^e \in \Theta^x$,

$$\bar{\theta}\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^e-))\Big) \geq g + x,$$

so that the optimization problem defining T^x is nonempty. Additionally, note that the function $\theta + \delta v(\theta, \theta^e)$ is continuous in θ. Therefore $T^x(\theta^e) = \min\{\theta \in \Theta \mid \theta + \delta v(\theta, \theta^e) \geq g + x\} \in \Theta$, and $T^x(\theta^e)$ is well defined.

Now, let us prove that T^x is nondecreasing. The restriction of T^x to Θ^x, $T^x|_{\Theta^x}$, is nondecreasing. Indeed, by increasing θ^e the feasible set in the minimization problem defining $T^x(\theta^e)$ (weakly) shrinks. So, the increase of θ^e leads to a weak increase in $T^x(\theta^e)$. It is further clear that for any $\theta^e > \bar{\theta}^x$, $T^x(\theta^e) = \bar{\theta} \geq T^x(\bar{\theta}^x)$. This completes the proof. □

Proof of Theorem 3. To see the characterization part, note that any equilibrium θ^e is a fixed point of T^x, for some x, and additionally belongs to Θ^x for

$$\bar{\theta}\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^e-))\Big) \geq \theta^e\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^e-))\Big) = g + x.$$

To see the converse inclusion note that for any $\theta^e \in B^x \cap \Theta^x$, it must be the case that

$$\theta^e\Big(1+\frac{\delta}{1-\delta}(1-F(\theta^e-))\Big) = g+x.$$

This implies that all types greater than or equal to θ^e are willing to cooperate when so does θ^e.

Let us now prove the second characterization. Suppose that $\theta^e \in \textsc{Equil}$. Then from the first characterization, it must exist a fixed point θ^e of T^x, for some x, which in turn satisfies

$$\theta^e\Big(1+\frac{\delta}{1-\delta}(1-F(\theta^e-))\Big) \geq g.$$

Now, suppose that there is a point satisfying the condition above. Then, for some $x \geq 0$

$$\theta^e\Big(1+\frac{\delta}{1-\delta}(1-F(\theta^e-))\Big) \geq g+x.$$

It readily follows that $T^x(\theta^e) = \theta^e$ and $\theta^e \in \Theta^x$. The result follows.

The existence part is immediate.□

Proof of Corollary 6. Suppose that $\theta^e \in B^x \cap \Theta^x$ for $x \geq 0$. Note that $T^0(\theta) \leq T^x(\theta)$ for all θ. Then, from Proposition 5 there must be a fixed point $\theta \leq \theta^e$ of the map T^0. But $\Theta^x \leq \Theta^0$ (in the strong set order) and $\Theta^x \subseteq \Theta^0$. So $\theta \in \Theta^0$. We have proven that for any equilibrium θ^e we can find a fixed point θ of T^0 (which happens to be equilibrium) and such that $\theta \leq \theta^e$. Therefore,

$$\min\{\theta^e \in \Theta \mid \theta^e \in \textsc{Equil}\} \geq \min\{\theta \in \Theta \mid T^0(\theta) = \theta\},$$

which completes the proof.□

Proof of Corollary 7. It is immediate from Proposition 5. □

Proof of Proposition 12. We need to prove that

$$\frac{\theta^c(1-F(\theta^c-))}{1-\delta} - \frac{(1-F(\theta^c-))g}{1-\delta F(\theta^c-)} \geq 0$$

A little of algebra shows that the condition above is less stringent than

$$\theta^c\Big(1+\frac{\delta}{1-\delta}(1-F(\theta^c-))\Big) \geq g,$$

a condition that is met. Note that this condition is also sufficient for, by fixing any price as the one described above, it is easy to see that the incentive constraints are satisfied.□

Proof of Proposition 13: The monopoly problem can be written as

$$\max\{\theta^e(1-F(\theta^e))^2 \mid \varphi(\theta^e) \geq g\}$$

where

$$\varphi(\theta^e) = \theta^e\Big(1 + \frac{\delta}{1-\delta}(1-F(\theta^e))\Big).$$

Note that

$$\varphi'(\theta^e) = 1 - \frac{\delta}{1-\delta} f(\theta^e)(\Phi(\theta^e) - \theta^e)$$

is non-increasing. Moreover, the objective function of the monopoly problem has derivative

$$\frac{1}{\theta^e} - \frac{2f(\theta^e)}{1-F(\theta^e)}$$

which is negative if and only if $\Phi(\theta^e) = 2\theta^e - \frac{1-F(\theta^e)}{f(\theta^e)}$ is positive. Since $\Phi(\theta^e)$ is nondecreasing as a consequence of Monotonicity, θ^c is the monopoly problem solution if and only if

$$\Phi(\theta^c) \geq 0.$$

Define $\hat{\theta}$ as the biggest solution to

$$\max\{\varphi(\theta) \mid \theta \in [0,1]\}.$$

Let us prove that $\theta^* \leq \hat{\theta}$. To do that, it is enough to prove that the function

$$(\alpha, \theta) \to \alpha \log(\theta\Big(1 + \frac{\delta}{1-\delta}(1-F(\theta))\Big)) + (1-\alpha)\log(\theta(1-F(\theta))^2)$$

is supermodular. But the derivative of this function with respect to α is

$$\log(1 + \frac{\delta}{1-\delta}(1-F(\theta))) - 2\log((1-F(\theta))),$$

so the result will obtain if the derivative of this function with respect to $F(\theta)$ is positive. But this derivative equals

$$\frac{2}{1-F(\theta)} - \frac{\frac{\delta}{1-\delta}}{1+\frac{\delta}{1-\delta}(1-F(\theta))}$$

which is positive whenever

$$2 + \frac{\delta}{1-\delta}(1-F(\theta)) \geq 0.$$

It follows that for any $g \in [\underline{g}(\delta), \bar{g}(\delta)]$, the monopoly solution is $\theta^m = \theta^c$. If not, $\theta^c < \theta^m$.

Let us now prove the result

$$\lim_{\delta\to 1} \frac{\bar{g}(\delta) - \underline{g}(\delta)}{\bar{g}(\delta) - 1} = 1 - \frac{\theta^*(1-F(\theta^*))}{\theta^1(1-F(\theta^1))},$$

where we employ the fact that $\lim_{\delta\to 1} \underline{g}(\delta) = \infty$. Consider θ^δ as the only solution to

$$\max_{\theta\in[a,b]} \theta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta))\Big)$$

This problem is concave and so θ^δ is the only solution to the necessary and sufficient first order condition

$$\frac{1-\delta}{\delta} = \Phi(\theta^\delta).$$

It is therefore clear that any converging sequence θ^{δ_n} must converge to θ^1, where $\Phi(\delta^1) = 0$. This proves that

$$\lim_{\delta\to 1} \theta^\delta = \theta^1.$$

Therefore

$$\begin{aligned}
\frac{\bar{g}(\delta) - \underline{g}(\delta)}{\bar{g}(\delta) - 1} &= \frac{\theta^\delta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^\delta))\Big) - \theta^*\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^*))\Big)}{\theta^\delta\Big(1 + \frac{\delta}{1-\delta}(1 - F(\theta^\delta))\Big) - 1} \\
&= \frac{\theta^\delta - \theta^* + \frac{\delta}{1-\delta}(\theta^\delta(1 - F(\theta^\delta)) - \theta^*(1 - F(\theta^*)))}{\theta^\delta - 1 + \frac{\delta}{1-\delta}\theta^\delta(1 - F(\theta^\delta))} \\
&\to \frac{\theta^1(1 - F(\theta^1) - \theta^*(1 - F(\theta^*)))}{\theta^1(1 - F(\theta^1))}.
\end{aligned}$$

This completes the proof. □

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