Trust in Cohesive Communities*

Felipe Balmaceda
Diego Portales University†

Juan F. Escobar
University of Chile‡

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Abstract

This paper studies which social networks maximize trust and welfare when agreements are implicitly enforced. We study a repeated trust game in which trading opportunities arise exogenously and a social network determines the information each player has. We show that cohesive communities, modeled as social networks of complete components, emerge as the optimal community design. Cohesive communities generate some degree of common knowledge of transpired play that allows players to coordinate their punishments and, as a result, yield relatively high equilibrium payoffs. We also show that when news swiftly travel through the network, Pareto efficient networks are minimally connected: the removal of any link isolates some community members. Our results then clarify a sociological debate on the merits of different social structures to foster cooperation and trust.

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†Economics Department, Diego Portales University, Avda. Santa Clara 797, Santiago, Chile, E-mail: `felipe.balmace@udp.cl`

‡Center for Applied Economics, Department of Industrial Engineering, University of Chile, Republica 701, Santiago, Chile, E-mail: `jescobar@dii.uchile.cl`

1 Introduction

The use of implicit mechanisms of misconduct deterrence has been widely recognized and documented by economists (Milgrom, North, and Weingast 1990), political scientists (Ostrom 1990, Fearon and Laitin 1996), sociologists (Coleman 1990, Raub and Weesie 1990), and legal scholars (Bernstein 1992). Crucial to their use is the way in which trading partners get informed about mischievous actions. As illustrated by Greif (2006) study of contract enforcement between medieval Maghribi traders, a close-knit community can quickly disseminate information about its members' behaviors and, as a result, can align its members' incentives by employing implicit, community-based sanctions that punish behaviors deemed unacceptable or opportunistic.[1] Several authors have stressed the importance the community structure has as a determinant of information flows and the success of trust-based transactions (Coleman 1990, Greif 1993, Rauch 2001, Burt 2001, Dixit 2006). However, our understanding of the basic economic forces that make some community structures more attractive than others is somewhat limited. This paper uncovers some of those forces in a repeated game model of imperfect information on networks.

We study a repeated trust game played by N investors and one agent. At each round $t \geq 1$ one out of the N investors is randomly and uniformly selected to play a trust game with the agent. More specifically, the investor decides whether or not to participate and, if he participates, he also picks an action or investment level; then the agent chooses whether to cooperate (or share the investment's return with the agent) or to defect (or appropriate the return in full). The equilibrium of the stage game is inefficient as the agent will defect after an investment is already made and, anticipating this behavior, the investor will not participate. The agent's temptation to defect may be curtailed by the existence of community sanctions governed by a social network of investors G. We assume that if the agent misbehaves when facing investor i, then i and all his direct connections in G become aware of that, and may act upon by changing their play in the continuation game. We focus on perfect Bayesian equilibria that sustain cooperation on the path of play.

Our results characterize optimal networks. Theorem 1 establishes the Pareto optimality of any social network of equally sized complete components (i.e., networks in which all players have the same number of connections and if a player is connected to two other players, then all three players are connected). Namely, a social network of equally sized complete components G^* yields a higher

[1] Other cases abound. In the automobile industry, for example, firms usually outsource large amounts of work and suppliers are routinely called upon to make specific investments. Hold-up problems are overcome by the threat of future business losses. As McMillan (1995) documents, one of the keys to deter opportunistic behavior in vertical relationships is the existence of cohesive business associations, such as Japanese keiretsus or Korean chaebols, that facilitate information exchange about parties' previous performances. McMillan (1995) observes that "the institutionalization of links among firms that is provided by the keiretsu system arguably serve as ... an information-provision device." He also notes that "by providing a mechanism for keeping track of any opportunistic behavior ..., the keiretsu provides a disincentive to such behavior."

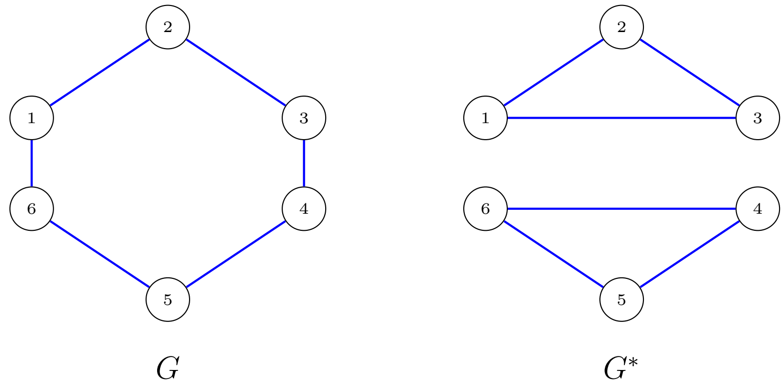


Figure 1: Illustration of Theorem 1. Even when each investor has the same number of connections in each network, network G^* yields higher payoffs than all equilibria of network G.

expected payoff to each community member than any other network G in which each investor has at most the same number of connections as in G^*. Hence, even when each investor has the same number of connections in G^* and G, equilibrium payoffs sustained by appropriately designed trigger strategies in G^* Pareto dominate any equilibrium in G. As illustrated in Figure 1, this implies that when links are scarce and the number of connections per player is exogenously given, the formation of a fully cohesive social network is really the best that all game players can hope for.

The mechanism behind this result can be understood as follows. Consider networks G and G^* in Figure 1 and note that in both networks each investor is connected to two other players. Take investors 1 and 5 in network G. Since investor 5 is not directly connected to 1, a defection against 1 is observed by 2 and 6 but not by 5. A defection against 1 should be punished by investors 2 and 6 in order to increase the relative value of on-path cooperation. The agent can defect against 1 and then against investor 5, but investor 3 will still be willing to trade. Network G therefore allows the agent to defect twice (first against 1 and then against 3) by incurring a rather small loss in continuation value. In contrast, in network G^*, play within each component is common knowledge and punishments can be immediately implemented leaving no room for further off-path defections (subsequent defections against 1 and 5 leave the agent with no investor willing to participate in G^*). Consequently, the network of complete components G^* results in higher equilibrium payoffs.

In Section 4 we study a variation of our model and assume that a defection against investor i is immediately transmitted to all investors that are direct and indirectly connected to i. Compared to the baseline model in Section 2, this alternative formulation can be seen as a polar assumption on the role of the social network –extremely slow information flows in the baseline model, very fast dissemination of news in Section 4. Theorem 3 shows that Pareto efficient networks are minimally connected: in an optimal network, any two investors that are indirectly connected, must be connected by a single path. Intuitively, with fast information flows, a minimally connected network maximizes the number of players that become aware of a defection and therefore more severe punishments are available.

Taken together, our theorems illustrate how the role of the social network as an information transmission device determines the architecture of optimal networks. Cohesion is important when a social network does not immediately create common knowledge of play within a component (Theorem 1). In contrast, when such common knowledge is granted, sparse (or loose) social structures emerge so that each player can bridge different sources of information (Theorem 3). The mechanisms here identified seem new to the literature and are likely to emerge in more general repeated game models of imperfect information transmission in networks.

Our results clarify two different sociological views on the merits of alternative social architectures. Coleman (1990) contends that network closure (or cohesion) fosters the creation of trust by facilitating the implementation of sanctions. In contrast, Burt (1992) argues that loose social networks allow players to bridge different sources of information and therefore result in higher payoffs. Theorems 1 and 3 nail down the assumptions on the information transmission technology under which each of the these views is more appropriate, and nail down the economic mechanisms at work.

We complement Theorems 1 and 3 by exploring social networks that maximize the sum of players' equilibrium payoffs given an exogenous number of links. In this problem, reassigning links effectively translates into utility transfers between different investors. This implies that the characterization of welfare-maximizing networks will depend, in part, on the convexity properties of the map from connections to utility levels. Theorem 2 characterizes optimal networks for the baseline model in Section 2. In line with Theorem 1, Theorem 2 shows conditions under which welfare-maximizing networks have complete components. Perhaps surprisingly, we also show that for some specifications the maximization of total welfare can be attained only when connections are unevenly distributed, giving raise to star networks. Finally, Theorem 4 applies to the model of rapid information transmission, nailing down the sizes of the different minimally connected components of welfare-maximizing networks.

Related Literature Cohesive networks play an important role in collective action games (Chwe 1999, Chwe 2000, Morris 2000). While some degree of common knowledge is also important to attain efficient coordination in those static games, our insights emphasize off-path coordination as a vehicle to attain high equilibrium payoffs.

Important antecedents for this paper come from work on community enforcement by Kandori (1992), Ellison (1994), Harrington (1995), and Okuno-Fujiwara and Postlewaite (1995). These authors study repeated games in which players interact infrequently, but the sources of information transmission are not captured by a social network. There is also a more recent and growing literature studying repeated games on networks, with a particular focus on the social network as a determinant of transaction opportunities among players. Some authors emphasize how heterogeneity in payoffs may make attractive the use of third party sanctions (Bendor and Mookherjee 1990), favoring the formation of cohesive networks (Haag and Lagunoff 2006, Lippert and Spagnolo 2011).[2] When players only observe their own interactions, a social network of trading opportunities has also a key role spreading information about distant interactions. In particular, as Mihm, Toth, and Lang (2009), Lippert and Spagnolo (2011) and Ali and Miller (2013) illustrate, cohesive networks quicken information dissemination punishments.[3] Jackson, Rodriguez-Barraquer, and Tan (2012) study a repeated favor exchange model and show that equilibrium networks satisfying renegotiation-proofness and contagion-freeness (which they term "robustness to social contagion") must consist of cliques joined in a tree-like form. In their perfect information game, off-path behaviors take a very different form and therefore the forces behind our model and theirs are different.[4]

Ahn and Suominen (2001) study a model similar to ours, in which the network of information transmission is drawn at the beginning of each round. They do not study the problem of optimal network design, nor do they explore how networks of complete components minimize the costs of contagion. Wolitzky (2013) studies a repeated public provision game in which the social network determines the monitoring technology. Wolitzky (2013) does not focus on the optimality of networks of complete components, so our research question is different. As he does, we use some lattice theory techniques to prove our results, but in our model once a player defects whether or not defections keep occurring depends on the actions being implemented on the path of play. Since such off-path behavior is arbitrary in our model, on-path actions are neither complements nor substitutes in general networks.[5] To prove our optimality results, we obtain estimates of continuation values and derive relaxed incentive constraints.

Organization Section 2 presents the baseline model. Section 3 states and discusses the optimality of networks of complete components. Section 4 presents and studies the model of rapid information transmission. Section 5 discusses our main results. Section 6 presents some conclusions and extensions. All proofs are placed in the Appendix.

[2]Other contributions to this topic include Raub and Weesie (1990), Bloch, Genicot, and Ray (2008), Karlan, Mobius, Rosenblat, and Szeidl (2009), Fainmesser (2012) and Nava and Piccione (2013).

[3]The precise meaning of cohesiveness in some of these papers is rather different from ours. Mihm, Toth, and Lang (2009) show that enforcing cooperation requires triadic closure (Coleman 1990), while Lippert and Spagnolo (2011) highlight the existence of cycles to spread punishments and attain cooperation. The main result in Ali and Miller (2013) shows that in networks of complete components, a contagion-equilibrium Pareto dominates all stationary equilibria of networks having incomplete components.

[4]Double-defections do not arise in Jackson, Rodriguez-Barraquer, and Tan (2012) model. As we show in Proposition 1, incentive constraints deterring double-defections are key in networks of incomplete components.

[5]It is easy to construct examples of our model in which a player's own maximal on-path action increases in the neighbors' actions and decreases as a function of the actions of more distant players.

2 Set Up

2.1 The Environment

At the beginning of each round $t \geq 1$, an investor i^t is randomly and uniformly selected from the set $\{1, \ldots, N\}$. Investor i^t and an agent (hereinafter, player 0) may produce some surplus in round t by playing a trust game as in Kreps (1996). First, the investor decides whether to participate or not (P or NP). If he chooses not to participate, then per-period payoffs equal 0 and the game moves on to round $t+1$. If investor i^t participates, he chooses an action $a \in \mathbb{R}_+$. This action can be thought of as the investor choosing how much to invest or trust the agent. After observing the investor's decision, the agent decides whether to cooperate (C) or to defect (D). Figure 2 illustrates the game when investor i^t participates.

Investors who are not selected get a per-period payoff equal to 0. Given a stream of per-period payoffs $(u_i^t)_{t\geq 1}$ for player $i \in \{0, \ldots, N\}$, his utility function equals $\sum_{t\geq 1} \delta^{t-1} u_i^t$, where $\delta \in]0, 1[$ is the common discount factor.

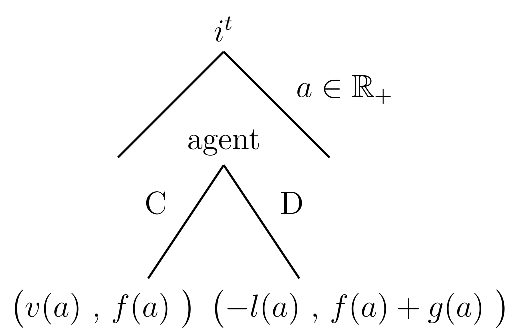


Figure 2: The game between investor i^t and the agent when the investor participates.

The following restrictions on payoffs are maintained throughout the paper.

Condition 1.

1. *All functions* f, v, l, g *are continuous on* $\mathbb{R}_+$ *and strictly positive on* $\mathbb{R}_{++}$.
2. g *is increasing,* f *is nondecreasing and* v *is increasing.*
3. $g(0) \leq \frac{\delta}{1-\delta}\frac{1}{N} f(0)$ *and if the equality holds, then* $g'(0) < \frac{\delta}{1-\delta}\frac{1}{N} f'(0)$.[6] *There exists* $\bar{b} > 0$ *such that* $g(a) > \frac{\delta}{1-\delta} f(a)$ *for all* $a \geq \bar{b}$.

[6] Here it is also assumed that the derivatives exist at 0.

Restriction 1 says that for any investment level, the agent's static best response is to defect. As a result, the investor decides not to participate and both players' payoffs equal 0. This outcome is Pareto dominated by the outcome in which the investor participates and makes positive investment and the agent cooperates. Restriction 2 implies that when the agent cooperates, both players benefit from a higher investment. Yet, the higher the investment, the larger the agent's temptation to defect. The third restriction is mainly technical and allows us to bound equilibrium investments in the repeated game. It ensures that the game has a nontrivial equilibrium in which players participate and make strictly positive investments.

There is a social network of investors that determines the monitoring technology. More formally, a social network of investors is a symmetric matrix $G \in \{0,1\}^{n\times n}$ such that $G_{ij} = 1$ if and only if i and j are linked.[7] We also write $ij \in G$ whenever $G_{ij} = 1$. We assume that $G_{ii} = 0$. Denote by $N(i,G) = \{j \mid ij \in G\}$ the set of i's neighbors in G and define the closed neighborhood of i as $\bar{N}(i,G) = N(i,G) \cup \{i\}$.

The monitoring technology is as follows. Let $i^t \in \{1,\ldots,N\}$ be the investor chosen at round t. If i^t plays NP, then the signal investors receive is empty. If i^t chooses P, then all investors $j \in N(i^t)$ become aware of that and observe whether the agent cooperated or defected. More formally, if investor i^t participates and the agent played $x^t \in \{C,D\}$, then player $j \in N(i^t)$ receives a signal $s_j^t = (i^t, x^t)$, while if $j \notin \bar{N}(i^t)$ then j receives a signal $s_j^t = \emptyset$. If i^t did not participate, then all players $j \neq i$ receive signal $s_j^t = \emptyset$. Player i^t perfectly observes play during round t. Players receive signals only about current interactions; in particular, we assume that information does not travel any further. A history h_i^t for investor i at the beginning of round t will consist of all the signals $(s_i^1,\ldots,s_i^{t-1})$ he has received during past play. We assume the agent has perfect information and therefore his information sets are all singletons.

These assumptions capture the idea that the network determines the transactions each investor can observe. This modeling choice is consistent with the view that information is hard to be transmitted throughout the network. One could also interpret these assumptions as saying that the victim (and only the victim) of a mischievous action let all his connections know that the agent is a miscreant. In on-line markets, for example, one is called upon to rate the seller only after being involved in some transaction. We postpone further discussion to Sections 4, 5, and 6.

Our model can be interpreted broadly and captures key aspects of several repeated interactions. The agent can be seen as a firm that may or may not hold-up the specific investments made by suppliers (investors), as in Williamson (1979). The social network represents all the business ties among the different suppliers (Greif 1993, McMillan 1995, Uzzi 1996). The game can also be seen as a model in which consumers (investors) decide whether or not to buy experience goods from a

[7] By considering a symmetric matrix G, we assume that the network is undirected

monopolist (agent) who may be tempted to sell low-quality goods. The social network represents all sources of information on the monopolist performance (such as online feedback systems, word-of-mouth) as in Dellarocas (2003). The basic strategic structure of our repeated trust game also appears in models of relational contracting with multiple employees (investors), in which the employer may renege payments to the employees.

We introduce some network terminology (for details consult Jackson 2008). We consider the set of investors that are at distance 2 of i, $N_2(i,G)$, formally defined as $N_2(i,G) = \{j \notin N(i,G) \mid jk \in G \text{ for some } k \in N(i,G)\}$. The component of investor i, $C(i,G)$, will include i and the set of all j such that there exists $i_0, \ldots, i_K$ with $i_n i_{n+1} \in G$ with $i_0 = i$ and $i_K = j$. It follows that if $j \in C(i,G)$, then $C(j,G) = C(i,G)$. Sometimes, we will also use $C(i,G)$ to denote the links among the different investors in $C(i,G)$. Each network G has a finite number of components that partition the set of investors. If $C(i,G) = \{i\}$, we will say that the component is trivial. A network G has *complete components* if for any $j \in C(i,G)$, $ij \in G$; a network G *has some incomplete component* if there exists i,j such that $j \in C(i,j)$ but $ij \notin G$. We will say that G is *κ-regular*, for some $\kappa \geq 1$, if for all i such that $|N(i,G)| \geq 1$, we have $|N(i,G)| = \kappa$. A network G is *minimally connected* if for any link $ij \in G$, $G \setminus \{ij\}$ has more components than G. When there is no risk of confusion, we will omit the dependence of these sets on G (and simply write $N(i)$ instead of $N(i,G)$, for example).

2.2 Strategies That Sustain Cooperation

A pure strategy for investor i is a family of functions $\sigma_i = (\sigma_i^t)_{t\geq 1}$ such that each σ_i^t maps private histories $h_i^t = (s_i^1, \ldots, s_i^{t-1})$ to stage game actions in $\{NP\} \cup (P \times \mathbb{R}_+)$.[8] A strategy σ_0 for the agent maps game histories, including the decision of investor i^t to participate in period t, to $\{C, D\}$. For any strategy profile $\sigma = (\sigma_0, \sigma_1, \ldots, \sigma_N)$, let $\mathbb{P}_\sigma$ be the probability measure induced over the set of histories by σ and let $\tilde{H} = \tilde{H}_\sigma$ be the set of on-path histories, or histories that have positive probability according to $\mathbb{P}_\sigma$.

We focus on equilibrium strategies in which the investors trust the agent and the agent cooperates. We say that a strategy profile σ *sustains cooperation* if σ is a perfect Bayesian equilibrium of the game and on the path of play, in all encounters, the selected investor participates and the agent cooperates. Let $\Sigma(G)$ be the set of all strategies σ that sustain cooperation. For any $\sigma \in \Sigma(G)$ and given any history of selected investors $(i^1, \ldots, i^t)$, we can define $\alpha_\sigma(i^1, \ldots, i^t) \in \mathbb{R}_+$ as the investment that player i^t chooses in round t on the path of play.

[8] While we are restricting attention to pure strategies, Theorems 1, 2, 3 and 4 can be extended to allow investors to use behavior strategies.

3 Characterization of Optimal Networks

In this section, we derive some upper bounds on equilibrium actions and then characterize the architecture of Pareto-dominant and welfare-maximizing social networks.

3.1 Bounding Equilibrium Investments

Our first observation is that investments along an equilibrium path can be bounded above. Indeed, take any network G and $\sigma \in \Sigma(G)$ such that α_σ is stationary, that is, $\alpha_\sigma(i^1, \ldots, i^t) = \alpha_\sigma(i^t)$ for all $i^1, \ldots, i^t$. Since $\sigma \in \Sigma(G)$, the following incentive compatibility restriction must hold

$$g(\alpha_\sigma(i)) \leq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j \in \bar{N}(i,G)} f(\alpha_\sigma(j)) \tag{3.1}$$

for all $i = 1, \ldots, N$. This inequality is derived after noting that, since it is in the agent's interest to cooperate in each encounter, defecting against i and cooperating in subsequent encounters with players outside $\bar{N}(i,G)$ cannot be optimal. Investors $j \in \bar{N}(i,G)$ are aware of the defection and can inflict a punishment yielding period payoffs to the agent greater than or equal to 0.[9] After such defection, investors $j \notin \bar{N}(i,G)$ are still willing to participate and it is feasible for the agent to keep cooperating in those encounters. The left-hand side of (3.1) is the short-term gain from a defection against i, whereas the right-hand side is an upper bound for the losses in continuation value from the same defection.

Equation (3.1) defines a system of inequalities, where the unknowns are actions $a = (a_i)_{i=1}^N \in \mathbb{R}_+^N$. Using lattice theory tools (Topkis 1998), such system of inequalities has a largest solution, $\bar{a}^G$, so that $\alpha_\sigma(i) \leq \bar{a}_i^G$ for all $i = 1, \ldots, N$. Moreover, at $\bar{a}^G$, all inequalities must actually bind. While we have assumed that α_σ is stationary, the following result shows such restriction is not really needed.[10]

Lemma 1. *For any G, the following hold.*

[9] It will be 0 only if investors $j \in \bar{N}(i,G)$ use trigger-strategies. Investors do not need not to use trigger strategies though.

[10] The way in which this upper bound is derived is similar to arguments appearing in the proof of Theorem 1 in Wolitzky (2013). Wolitsky obtains a system of incentive constraints and proves such system has a largest solution. Our argument here is different because we derive a system of relaxed incentive constraints that has a largest solution. More importantly, for our purposes the key question is whether such largest solution can actually be implemented as an equilibrium outcome –a question answered in Propositions 1 and Proposition 3. We then build on these preliminary results to derive the architecture of optimal networks.

a. The system of inequalities

$$g(a_i) \leq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in\bar{N}(i,G)} f(a_j),\ i = 1,\dots,N$$

has a largest solution $\bar{a}^G \gg 0$. At $\bar{a}^G$, all inequalities above bind.

b. For all $\sigma \in \Sigma(G)$ and all $(i^1,\dots,i^t) \in \{1,\dots,N\}^t$

$$\alpha_\sigma(i^1,\dots,i^t) \leq \bar{a}^G_{i^t}.$$

Having established upper bounds on equilibrium actions, we now turn to the problem of whether this bound is tight.

Definition 1. *Let G be a network and $a \in \mathbb{R}^N_{++}$. We say that the equilibrium $\sigma \in \Sigma(G)$* implements *a if*

$$\alpha_\sigma(i^1,\dots,i^t) = a_{i^t},\ \textit{for all } (i^1,\dots,i^t) \in \{1,\dots,N\}^t.$$

The incentive constraints exploited to obtain the upper bound $\bar{a}^G$ need not be sufficient for equilibrium since optimal behavior following a defection by the agent could, for example, involve defections when facing some of the remaining investors. As a result, it is entirely possible that for some networks, no equilibrium can actually attain $\bar{a}^G$ on the path of play. The following result, which is the main contribution of this subsection, provides necessary and sufficient conditions for implementability of $\bar{a}^G$.

Proposition 1. *Let G be a network. The following are equivalent:*

a. G is a network of complete components.

b. There exists $\sigma \in \Sigma(G)$ that implements $\bar{a}^G$.

Proposition 1 shows that the action profile $\bar{a}^G$ can be implemented as an equilibrium outcome if and only if G is a network of complete components. When G is a network of complete components, it is relatively easy to construct σ to implement $\bar{a}^G$ along the path of play. Indeed, trigger strategies suffice: on the path of play, investor i participates and makes investment decision $\bar{a}^G_i$; if the agent ever defects against $j \in \bar{N}(i,G)$, investor i refuses trade in all subsequent interactions. The agent cooperates against any investor who decides to participate and who has not observed any defection. Since G is a network of complete components and (3.1) holds, $\sigma \in \Sigma(G)$.

The converse is more subtle and reveals some important constraints arising from the private monitoring problem in networks with incomplete components. Take, for example, network G in

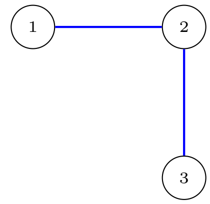


Figure 3: A network where investors 1 and 3 are not connected.

Figure 3. Suppose that there exists $\sigma \in \Sigma(G)$ such that in all encounters $\alpha_\sigma(i) = \bar{a}_i^G$, $i = 1, 2, 3$. Since $\bar{a}^G$ is the largest solution to the system of relaxed incentive constraints, the strategy profile σ must prescribe investor 2 to punish whenever a defection against either 1 or 3 is observed. We argue that the strategy σ is not sequentially rational for the agent. Indeed, following an off-path defection against 1, both players 1 and 2 must refuse trade in all subsequent rounds. In the continuation game right after a defection against 1, player 3 has less leverage because he is uninformed and player 2 is already punishing the defection against 1. It is therefore in the interest of the agent to defect once more when facing 3. In other words, after a defection against 1, the agent becomes contagious, defects when facing 3, and gets higher payoffs than if he would have cooperated against 3. This implies that the agent actually prefers defecting when facing investor 1. Intuitively, if the upper bound $\bar{a}^G$ is being implemented, the agent can defect twice and incur a relatively small loss in continuation value. The strategy profile σ cannot be sequentially rational and there must be a positive probability event where the selected player i chooses an action strictly less than $\bar{a}_i^G$. This shows that the upper bound $\bar{a}^G$ cannot be implemented in the network of incomplete components G.

3.2 The Design of Optimal Networks

We now present our main results characterizing optimal networks (Theorems 1 and 2). We start by showing the Pareto efficiency of networks of complete components.

Given G and a strategy profile σ, let $u_i(\sigma, G)$ be the expected sum of discounted payoffs for player i when play evolves according to σ and the social network is G. We will say that G^* *Pareto dominates* G if there exists $\sigma^* \in \Sigma(G^*)$ such that for all $\sigma \in \Sigma(G)$, $u_i(\sigma^*, G^*) \geq u_i(\sigma, G)$ for all $i = 0, 1, \ldots, N$ with at least some strict inequality.

Theorem 1. *Let G^* be a κ-regular network of complete components. Let G be any other network having some incomplete component such that $|N(i,G)| \leq |N(i,G^*)|$ for all $i = 1, \ldots, N$. Then, G^* Pareto dominates G.*

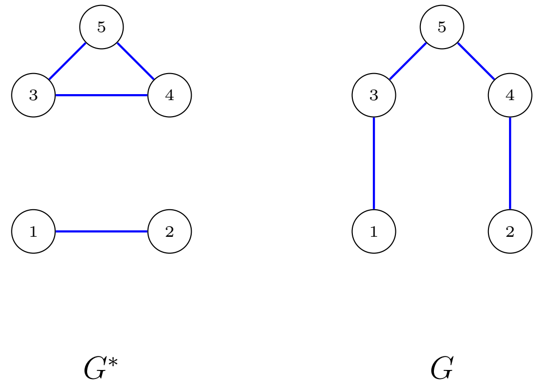


Figure 4: The number of connections each player has is the same in both networks.

By showing that all game players prefer a network of equally sized complete components over any other network in which no investor has more connections, this theorem provides a concrete answer to the question of what makes fully cohesive communities attractive. In a social network of complete components, there is common knowledge of play within a given component and therefore connected investors can coordinate their punishment leaving no room for off-path gaming. In contrast, as discussed previously, in an incomplete component the agent can defect twice by incurring a rather small loss in continuation values.

Theorem 1 is proven by noticing that, for all $i = 1, \ldots, N$, $\bar{a}_i^G \leq \bar{a}_i^{G^*}$. Lemma 1 and Proposition 1 show that no equilibrium in network G can implement $\bar{a}^G$, whereas some equilibrium $\sigma^* \in \Sigma(G^*)$ implements $\bar{a}^{G^*}$. The result follows since payoffs are increasing in actions.

As the following example illustrates, the assumption that G^* is regular cannot be dispensed with.

Example 1. *Consider networks G^* and G in Figure 4. Each investor has the same number of connections in each network. Network G^* is a network of complete components that is not regular, while network G has a single component which is incomplete. Assuming that both f and g are strictly increasing, we will show that there exists an equilibrium in network G that results in higher payoffs for player 1 than* all *equilibria in network G^*.*

In network G^*, the Pareto-dominant equilibrium is characterized by the largest solution (a_1^*, a_2^*) to the equation

$$\max\{g(a_1), g(a_2)\} \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_2)\Big). \tag{3.2}$$

Therefore, no equilibrium can implement investments higher than a_1^* for investor 1 in network G^*.

To construct an equilibrium for network G, take $\bar{a} = (\bar{a}_i)_{i=1}^5$ as the largest solution to

$$g(a_1) \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_3)\Big) \tag{3.3}$$

$$g(a_3) \leq \frac{\delta}{1-\delta}\frac{1}{5}\Big(f(a_1) + f(a_3) + f(a_5)\Big) \tag{3.4}$$

$$g(a_i) \leq \frac{\delta}{5} f(a_i) \quad \text{for } i = 2, 4, 5 \tag{3.5}$$

We can construct an equilibrium that implements $\bar{a}$ in network G as follows. Along the path of play, investor i participates and chooses $\bar{a}_i$. If investor 1 or investor 3 observes a defection against either of them, both refuse trade in all subsequent rounds. Investor 3 ignores any defection against 5. Player 5 does not participate in subsequent trading opportunities if he observes a defection against 3. A defection against either 2, 4, or 5 is punished only by the victim by refusing trade only in the next round (which happens with probability 1/5) and then resuming play as no defection had occurred. The agent cooperates when facing any investor who, according to his strategy, should participate and invest (including off-path histories). This strategy profile is sequentially rational for all players. For the investors, optimality follows by construction. By construction, following any defection, it is in the agent's interest to keep cooperating against investors still willing to cooperate.

We finally observe that $\bar{a}_1 > a_1^*$. To see this, just fix $\bar{a}^5$ and consider the system (3.3 - 3.4) in (a_1, a_3). Since f and g are strictly increasing and $f(\bar{a}^5) > 0$, $(\bar{a}_1, \bar{a}_3) \gg (a_1^*, a_2^*)$.

In this example, investor 1 can enjoy a higher equilibrium payoff in network G because in this network he is connected to investor 3, who is also connected to 5. In contrast, in network G^* investor 1 is connected to investor 2 who is not connected to other players. A simple way to rule out the network effects illustrated in the example is by ensuring that $\bar{a}_i^G$ is determined by the number of connections i has in G. The next assumption is crucial to achieve that.

Assumption 1. *f attains its maximum.*

This assumption says that the payoff accruing to the agent when cooperating does not grow beyond an upper bound. Assumption 1 allows us to characterize $\bar{a}^G$ in simpler terms. Indeed, we can define

$$\bar{\delta} = \min\left\{\delta \mid \frac{\delta}{1-\delta}\frac{1}{N}\max f \geq g\Big(\min\big(\arg\max f\big)\Big)\right\}$$

which belongs to $]0, 1[$.[11]

[11] When f is constant, the restriction $\delta > \bar{\delta}$ is equivalent to the restriction imposed in Condition 1, $g(0) \leq \frac{\delta}{1-\delta}\frac{1}{N}f(0)$. A constant f can be interpreted as saying that regardless of the investment made in a given round, the way in which the investor and the agent divide surplus leaves the investor with a constant amount that, for example, covers its (constant) effort cost plus a premium.

Lemma 2. *Under Assumption 1, for $\delta > \bar{\delta}$,*

$$\bar{a}_i^G = g^{-1}\left(\frac{\delta}{1-\delta}\frac{\max f}{N}|\bar{N}(i,G)|\right),$$

for all $i = 1, \ldots, N$.

This Lemma allows us to simplify the design problem by focusing on the network effects that arise purely due to information frictions. Intuitively, when $\delta > \bar{\delta}$, the i-th component of the upper bound $\bar{a}^G$ is large enough so that it lies in the flat portion of f and therefore, as a function of G, it is entirely determined by the number of connections investor i has, $|\bar{N}(i,G)|$.

The following optimality result applies to networks that are not necessarily regular. It follows by combining Lemma 2 with the arguments in Theorem 1.

Proposition 2. *Under Assumption 1, for any $\delta > \bar{\delta}$, any network of complete components G^* and any network G having some incomplete component with $|N(i,G)| \leq |N(i,G^*)|$ for all $i = 1, \ldots, N$, G^* Pareto dominates G.*

We are also interested in characterizing networks that maximize total welfare. We will say that G^* *gives strictly more total welfare* than G if there exists $\sigma^* \in \Sigma(G^*)$ such that for all $\sigma \in \Sigma(G)$, $\sum_{i=0}^N u_i(\sigma^*, G^*) > \sum_{i=0}^N u_i(\sigma, G)$. We will also write

$$U(x) = v \circ g^{-1}\left((x+1)\frac{\delta}{1-\delta}\frac{1}{N}\max f\right),$$

for $x \in \mathbb{N}$. Under the conditions of Lemma 2, $U(x)$ can be interpreted as the period payoff an investor obtains when trading given a number x of connections when the upper bound on equilibrium actions is implemented.

Theorem 2. *Under Assumption 1, let $\delta > \bar{\delta}$, G^* and G be networks such that $\sum_{i=1}^N |N(i,G)| \leq \sum_{i=1}^N |N(i,G^*)|$. The following hold:*

i. If U is linear and G^* is a network of complete component but G is not, then G^* gives strictly more total welfare than G.

ii. If U is strictly concave and G^* is a network of equally sized complete components but G is not, then G^* gives strictly more total welfare than G.

iii. If U is such that

$$U(x+1) - U(x) \geq \max\left\{x(U(2) - U(0)), 2(U(x) - U(x-1))\right\}, \ 2 \leq x \leq N-2 \tag{3.6}$$

and G^* is a star but G is not, then G^* gives strictly more total welfare than G.[12]

Theorem 2 characterizes welfare maximizing networks given a number of links to be freely allocated among the different investors. The main determinants of the architecture of optimal networks are the convexity properties of U. These restrictions are more easily understood if the payoff to the investor when the agent cooperates is linear in the investment, $v(a) = a$ for all a, and therefore $U \equiv g^{-1}$. In this case, the architecture of welfare maximizing networks is determined by the relationship between investments, a, and defection gains, $g(a)$. When g is convex, an uneven assignment of links implies that the equilibrium actions of highly connected investors only slightly decrease when a link is reassigned to a barely connected investor, who gains substantial leverage by the addition of the new link. This is a force towards equally sized components, that must be completed to avoid the implementation losses described in Proposition 1. This explain part (ii). Part (i) follows a similar logic, but the linearity of g implies that the incremental gains and losses when links are reassigned do not depend on how connected the different players are. To attain optimality, therefore, it is enough to avoid the implementation losses of networks with incomplete components. Finally, in part (iii), the restriction implies that $U = g^{-1}$ is sufficiently convex. In this case, the star G^* is attractive as it attains an extreme form of heterogeneity in connections and the central investor's investment can take high actions. However, as the star is a network of incomplete components, some implementation losses must be incurred –Proposition 1 shows that $\bar{a}^{G^*}$ cannot be implemented in the star. The proof shows that in the star G^*, the implementation losses are small compared to the gains from unevenness and, therefore, total welfare is maximal.

Finally, we can also quantify the implementation losses in networks of incomplete components. Recall that a function $h\colon \mathbb{R} \to \mathbb{R}$ is Lipschitz-continuous with constant $L_h \geq 0$ in $X \subseteq \mathbb{R}$ if for all $x, y \in X$, $|h(x) - h(y)| \leq L_h|x - y|$.

Proposition 3. *Assume that f and g are Lipschitz-continuous with constants L_f and L_g in $[0, \bar{b}]$ (where $\bar{b}$ was introduced in Condition 1). Let G be a network and $\sigma \in \Sigma(G)$ that implements $a \in \mathbb{R}^N_{++}$. Then, for all $i = 1, \ldots, N$,*

$$\begin{aligned} &\frac{\delta}{1-\delta}\frac{1}{N}\sum_{k\in N(i)} f(a_k)|N(k)\setminus N(i)| \qquad (3.7)\\ &\leq \frac{N(1-\delta)+\delta|N_2(i)|}{\delta}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{k\in\bar{N}(i)} L_f|a_k - \bar{a}^G_k| + L_g|a_i - \bar{a}^G_i|\Big)\\ &\quad + \sum_{k\in N_2(i)}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{l\in\bar{N}(k)} L_f|a_l - \bar{a}^G_l| + L_g|a_k - \bar{a}^G_k|\Big) \end{aligned}$$

[12] The convexity restrictions are easy to check. For example, if $v \circ g^{-1}(x) = \exp(x)$, (3.6) holds provided $\frac{\delta}{1-\delta}\frac{\max f}{N} \geq \ln(2)$.

This result proves that if an action profile $a \in \mathbb{R}^N_{++}$ can be implemented in network G, where some investor i is such that a_k is close to $\bar{a}^G_k$ for all k within distance 3 of i, then the neighborhood $\bar{N}(i)$ must be sufficiently cohesive: the weighted sum of links that "leave" the neighborhood must be sufficiently small. As in Proposition 1, the basic force here arises due to the imperfect monitoring problem in networks of incomplete components. Intuitively, if some equilibrium in network G is implementing a profile close to $\bar{a}^G$ on some extended neighborhood of i, then the deterrence of double defections (first against i and then against investors within distance 2 of i) implies that investors $k \in N_2(i)$ cannot loose too much leverage following the first defection. As a result, investors $k \in N_2(i)$ cannot have many connections in common with i.

Proposition 3 suggests that investors can attain relatively high equilibrium actions only when their direct connections are not connected to more distant players. This implies a notion of cohesiveness that contrasts with the standard clustering coefficients that measure the cohesiveness of a player's neighborhood according to the (normalized) number of links among the player's connections (see Chapter 2.2.3 in Jackson 2008). The following example illustrates this difference.

Example 2. *We consider a simple example assuming $N = 5$, $f(a) = 1$ and $g(a) = a$. See Figure 5.*

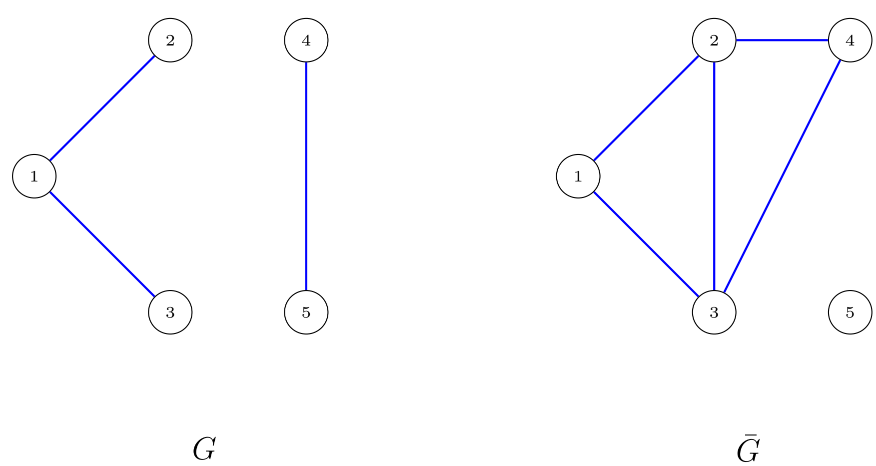


Figure 5: Investors 1 and 4 are in a more favorable position in network G.

In network G, investor 1 is connected to investors 2 and 3. Since investors 2 and 3 do not have additional connections, we can deem investor 1's neighborhood as cohesive. Indeed, Proposition 3 imposes no bound over the equilibrium actions that investor 1 can play. Moreover, $\bar{a}^G_1 = \frac{\delta}{1-\delta}\frac{3}{5}$ can be implemented by an equilibrium in which investor 1 plays $\bar{a}^G_1$ on the play-path, whereas 2 and 3 play $\frac{\delta}{1-\delta}\frac{1}{5}$. A defection against 1 triggers no further participation by 1, 2, and 3; a defection against 2 (resp. 3) triggers no further participation by 2 (resp. 3), investor 1 keeps participating choosing

$\frac{\delta}{1-\delta}\frac{2}{5}$, *investor 1 refuses trade only after a second defection. It is clear that it is in the interest of the agent to cooperate when facing either 1, 2, or, 3. Since investors 4 and 5 belong to a different component (which is complete), we can construct a strategy profile sustaining cooperation in which investor 1 invests* $\frac{\delta}{1-\delta}\frac{3}{5}$ *and investor 4 invests* $\frac{\delta}{1-\delta}\frac{2}{5}$.

Take now network $\bar{G}$. *At first glance, it could seem that investors 1 and 4 are in a more advantageous position (compared to G) as in* $\bar{G}$ *investor 4 has more direct connections and, moreover, both investors' neighborhoods are fully clustered. We show that this is not the case. Take* $a \in \mathbb{R}^5_{++}$ *that can be implemented by some* $\sigma \in \Sigma(G)$ *and apply Proposition 3 to deduce that*

$$\frac{\delta}{1-\delta}\frac{2}{5} \leq \frac{5(1-\delta)+\delta}{\delta}\left|\frac{\delta}{1-\delta}\frac{3}{5} - a_1\right| + \left|\frac{\delta}{1-\delta}\frac{3}{5} - a_4\right|.$$

This inequality immediately shows that if $a_1 = \frac{\delta}{1-\delta}\frac{3}{5}$ *then* $a_4 \leq \frac{\delta}{1-\delta}\frac{1}{5}$ *(*$< \frac{\delta}{1-\delta}\frac{2}{5}$*).*

This example illustrates how clustering measures are inappropriate to capture the cohesiveness of a player's neighborhood in our model. Network $\bar{G}$ is unattractive for investors 1 and 4 as both investors have neighbors that are connected to players outside their own neighborhood.

4 Social Networks of Rapid Information Transmission

We explore a variation of our main model in which information is rapidly transmitted through the network. More specifically, the payoffs and the matching technology are as in Section 2, but the network G plays a different role.

Let $i^t \in \{1, \dots, N\}$ be the investor chosen at round t. If i^t plays NP, then the signal all investors receive is empty. If i^t chooses P, then all investors in the same component as i become aware of that and observe whether the agent cooperated or defected. More formally, if investor i^t participates and the agent played $x^t \in \{C, D\}$, then player $j \in C(i^t)$ receives a signal $s^t_j = (i^t, x^t)$, while if $j \notin C(i^t)$ then j receives a signal $s^t_j = \emptyset$. If i^t did not participate, then all players $j \neq i$ receive signal $s^t_j = \emptyset$. Player i^t perfectly observes play during round t. A history h^t_i for investor i at the beginning of round t will consist of all the signals $(s^1_i, \dots, s^{t-1}_i)$ he has received during past play.

These assumptions capture the idea that once a player becomes informed, he can rapidly transmit his information to all his direct connections. Those direct connections can also transmit the new information to their own connections almost immediately, and so on. The important thing to keep in mind is that once an interaction involving i occurs, all players in i's component become aware of the result of the transaction before the next investor is determined.

We adapt to this setting the definition of strategies and strategies that sustain cooperation in the obvious way. We also define $u_i(\sigma, G)$ as the expected sum of discounted payoffs given the network G and a strategy profile σ. The definitions of Pareto dominance and total welfare are as in Section 3. In particular, we say that network G maximizes total welfare if there exists some equilibrium $\sigma \in \Sigma(G)$ such that for all network G' with $\sum_{i=1}^{N}|N(i,G')| \leq \sum_{i=1}^{N}|N(i,G)|$ and all $\sigma' \in \Sigma(G')$, $\sum_{i=0}^{N} u_i(\sigma, G) \geq \sum_{i=0}^{N} u_i(\sigma', G')$.

We observe that analyzing optimal equilibria in this model is relatively simple. As in Proposition 1, for any network G, the system of incentive compatibility constraints

$$g(a_i) \leq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in C(i)} f(a_j), \quad i = 1, \ldots, N$$

has a largest solution $\hat{a}^G \in \mathbb{R}^N_{++}$. Moreover, $\hat{a}^G$ can be implemented as an equilibrium of the game for all G by using trigger strategies and no other equilibrium can implement higher actions.

Theorem 3. *Let G be a network which is not minimally connected having two or more components. Then, G is Pareto-dominated by a network G^* having the same number of links.*

This result shows that Pareto-efficient networks must be minimally connected. Intuitively, in a network which is not minimally connected, we can give an alternative use to one of the links in order to expand the number of players that become aware of any defection. In the new network more players can punish a defection and therefore on-path actions are higher.

To characterize welfare-maximizing networks, we note that for each investor i, $\hat{a}_i^G$ depends on G only through the size of the component i belongs to, $|C(i,G)|$. We therefore write $\hat{a}_i^G = \hat{a}_i(|C(i,G)|)$. Up to a constant, the equilibrium payoff player i obtains in component C is

$$\hat{U}(|C|) = v\big(\hat{a}_i(|C|)\big).$$

We also define the total welfare members of component C obtain as $\Phi(|C|) = |C|\hat{U}\big(|C|\big)$.

Theorem 4. *Let G^* be a network that maximizes total welfare, with $\frac{1}{2}\sum_{i=1}^{N}|N(i,G^*)| \leq N-1$. Then, G^* is minimally connected. Moreover, the following hold:*

i. If Φ is strictly convex, G^* has a single non-trivial component.

ii. If Φ is strictly concave, and G^* has two components of n_1 and n_2 members respectively, with $n_1 > n_2$, then $n_1 = n_2 + 1$.

The restriction $\frac{1}{2}\sum_{i=1}^{N}|N(i,G^*)| \leq N-1$ implies that the problem is nontrivial; if it did not hold welfare maximizing networks would be networks formed by N or more links and having a single component. When Φ is strictly convex, as in Theorem 2, forming a single nontrivial component is attractive as it makes the allocation of (direct and indirect) contacts extremely heterogeneous. This implies that in order to maximize welfare, a network must have a single nontrivial components. In contrast, when Φ is strictly concave, the maximization of total welfare requires that all components are of a similar size and therefore their sizes differ in at most one player.[13]

5 Closure versus Structural Holes

There are two sociological perspectives on the benefits of different social architectures for welfare (Burt 2001, Sobel 2002). On the on hand, the closure view (Coleman 1990) emphasizes that in networks with high closure and cohesion, the enforcement of cooperative behaviors is more effective. In Coleman's (1990) argument, cohesion facilitates the implementation of sanctions and therefore leads to higher welfare. Granovetter (1973), in contrast, advances a view in which information can be transmitted from any player to all his connections.[14] This leads to the structural hole argument, as presented by Burt (1992), in which the existence of players linking otherwise disconnected groups is beneficial as those players can bridge non-redundant sources of information.[15]

As our analysis makes clear, this seemingly conflicting views can be reconciled after observing that they apply to different models of information flows on networks. The closure view is consistent with the model presented in Section 2, in which once a player becomes aware of a defection against one of his direct connections, he cannot pass that information to other players. As shown in Section 3, the key game theoretical reason that makes cohesion desirable is that harsher punishments are available when, within a component, players have common knowledge of play. Burt's (1992) perspective is supported by the model of Section 4, in which players can transmit any available information to all their direct and indirect contacts. In this context, scattered networks are optimal as they maximize the number of players that become aware of a defection and thus result in more severe punishments.

These observations are similar to those by Chwe (1999) and Sobel (2002). In particular, Sobel (2002) observes that "widely scattered weak links are better for obtaining information, while strong

[13]Under Assumption 1 and $\delta > \bar{\delta}$, $\hat{a}_i(|C|) = g^{-1}(\frac{\delta}{1-\delta}\frac{1}{N}\max f|C|)$ and therefore $\Phi(|C|) = |C|v \circ g^{-1}(\frac{\delta}{1-\delta}\frac{1}{N}\max f|C|)$. Determining the convexity properties of Φ is therefore immediate.

[14]In Granovetter's (1973) analysis, there is a distinction between weak and strong links. Weak links exist between acquaintances, while strong links exists between friends. Granovetter argues that networks of weak links are scattered. See alo Granovetter (1995).

[15]Burt (1992) also argues that players bridging different groups gain relatively high rents. Our results are silent about this mechanism.

and dense links are better for collective action." Our results not only formalize this intuition in a full-fledged dynamic model of imperfect information in networks, but also clarify the role that off-path coordination has as a vehicle to attain higher equilibrium payoffs.

In closing, we observe that our results provide a new angle to the discussion on structural holes versus closure. Theorems 2 and 4 reveal that the topology of welfare-maximizing networks is also determined by how direct and indirect connections map into utility levels. We show that such map is determined by the the details of each transaction. In particular, for some payoff specifications, a star can be optimal in the model of Section 2 because it creates an extreme form of heterogeneity in the assignment of links. A similar force determines the size of minimally connected networks in the model of rapid information flows. We are not aware of any paper arguing about the importance of heterogeneity in connections for the optimality different network structures in models of trust-based transactions.

6 Concluding Remarks

This paper studies the emergence of cooperation and trust in a repeated game model in which information flows are governed by the community architecture. When players only observe the interactions of their direct connections, networks of complete components allow players to coordinate their play, reduce the scope for off-path gaming, and therefore result in high equilibrium payoffs. In contrast, social networks in which some components are not complete leave room for off-path behaviors that are detrimental for incentives and should be avoided. We explore an important variation of the model in which information is transmitted to both directed and indirectedly connected investors, and show that optimal community design favors the formation of loose social networks. Our theorems yield testable implications of our repeated game model on networks.

Our analysis can accommodate some variations. For example, investments need not be continuous (Appendix D). It would be interesting to explore a model in which news travel through the networks at intermediate speeds, perhaps with noise. In such model, information percolates the network, and the agent must selectively decide whether or not to defect following a chain of defections. We think that the intuitions offered by this paper should be present in that richer model, but more formal analysis is needed. In the Appendix, we study a model in which news travel one step at each time, and show that networks of complete components are optimal when we restrict attention to robust equilibria that are immune to mistakes. Details are provided in Appendix D.

Appendix

This Appendix consists of four parts. Appendix A presents proofs for Section 3.1. Appendix B presents proofs for Section 3.2. Appendix C presents proofs for Section 4. Finally, some variations of the main model are presented in Section D.

A Proofs for Section 3.1

Proof of Lemma 1. The proof is organized as follows. In Claim 1, we derive necessary conditions for implementability and then, in Claim 2, we show that those necessary conditions have a larger solution.

Claim 1. *Let $\sigma \in \Sigma(G)$, $(i^1, \ldots, i^T) \in \{1, \ldots, N\}^T$ be any selection of randomly chosen investors, and a^T be the action chosen by i^T on the path of play. Then,*

$$g(a^T) \leq \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \in \bar{N}(i^T)} \frac{1}{N} \mathbb{E}_{\sigma}\big[f(a^t_{i^t}) \mid i^1, \ldots, i^T,\ i^t = j\big]. \tag{A.1}$$

To prove this Claim, denote by $V(C)$ (resp. $V(D)$) the continuation value accruing to the agent by cooperating (resp. defecting) against investor i^T at the on path history selecting $(i^1, \ldots, i^T)$. Then,

$$g(a^T) \leq V(C) - V(D).$$

Now, since σ sustains cooperation

$$V(C) = \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_{\sigma}[f(a^t_{i^t}) \mid i^1, \ldots, i^T] = \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j=1}^{N} \frac{1}{N} \mathbb{E}_{\sigma}[f(a^t_{i^t}) \mid i^1, \ldots, i^T, i^t = j].$$

Now, after a defection against i^T, it is still feasible for the agent to cooperate when facing players $i \notin \bar{N}(i^T)$. Thus, following a defection against i^T, the agent can secure a total discounted payoff

$$V(D) \geq \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j \notin \bar{N}(i^T)} \frac{1}{N} \mathbb{E}_{\sigma}[f(a^t_{i^t}) \mid i^1, \ldots, i^T, i^t = j].$$

It follows that

$$
\begin{aligned}
g(a^T) &\le \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1,\dots,i^T] - \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j\notin \bar N(i^T)} \frac{1}{N}\mathbb{E}_\sigma[f(a^t_{i^t}) \mid i^1,\dots,i^T, i^t=j] \\
&= \sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j\in \bar N(i)} \frac{1}{N}\mathbb{E}_\sigma\big[f(a^t_{i^t}) \mid i^1,\dots,i^T,\ i^t=j\big].
\end{aligned}
$$

This completes the proof of the claim.

Consider now the set

$$
\mathcal{A} = \Big\{\alpha \mid \alpha\colon \bigcup_{t=1}^{\infty}\{1,\dots,N\}^t \to \mathbb{R}_+\Big\}.
$$

For any $\sigma \in \Sigma(G)$, we can generate the same distribution over on path actions by using a particular element $\bar\alpha = \bar\alpha_\sigma \in \mathcal{A}$. Inspired by this observation and the incentive constraint (A.1), we define $\mathcal{T}^G\colon \mathcal{A}\to\mathcal{A}$ by

$$
\Big(\mathcal{T}^G(\alpha)\Big)(i^1,\dots,i^T) = g^{-1}\Big(\sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j\in \bar N(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1,\dots,i^T,i^{T+1},\dots,i^t)) \mid i^1,\dots,i^T,\ i^t=j\big]\Big).
$$

We observe that this quantity is well defined as

$$
\begin{aligned}
\Big(\sum_{t=T+1}^{\infty} \delta^{t-T} \sum_{j\in \bar N(i^T)} \frac{1}{N}\mathbb{E}\big[f(\alpha(i^1,\dots,i^T,i^{T+1},\dots,i^t)) \mid i^1,\dots,i^T,\ i^t=j\big]\Big) &\in \frac{\delta}{1-\delta}[\frac{1}{N}\min f, \max f] \\
&\subseteq \text{range}(g).
\end{aligned}
$$

Claim 2. *For any network G, there exists $\bar a^G \in \mathbb{R}^N_+$ which is the largest solution to the equation $a = \Phi(a,G)$, with*

$$
\Phi_i(a,G) = g^{-1}\Big(\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in \bar N(i,G)} f(a_j)\Big).
$$

Moreover, any solution $\alpha\in\mathcal{A}$ to the system $\alpha \le \mathcal{T}^G(\alpha)$ satisfies

$$
\alpha(i^1,\dots,i^T) \le \bar a^G_{i^T}
$$

for all $(i^t)_{t=1}^T \in \{1,\dots,N\}^T$ and for all T.

To prove this claim, take $\bar A = \max_{a\ge 0} g^{-1}(\frac{\delta}{1-\delta}f(a))$ and note that for any α, $\mathcal{T}^G(\alpha)(i^1,\dots,i^T) \le \bar A$ for all $(i^1,\dots,i^T)\in\{1,\dots,N\}^T$ and all T. Define $\mathcal{A}^* = \mathcal{A}\cap\{\alpha \mid \alpha(i^1,\dots,i^T)\le \bar A\}$. In order to find solutions to the system $\alpha \le \mathcal{T}^G(\alpha)$, it is enough to restrict the domain and range of $\mathcal{T}^G$

to $\mathcal{A}^*$ and, abusing notation, we write $\mathcal{T}^G \colon \mathcal{A}^* \to \mathcal{A}^*$. Let $\alpha^* \in \mathcal{A}^*$ be the largest element in $\mathcal{A}^*$ defined as $\alpha^*(i^1, \ldots, i^T) = \bar{A}$ for all $(i^1, \ldots i^T)$. Since $\mathcal{T}^G$ is increasing, Tarski's fixed point theorem implies the existence of a largest fixed point $\bar{\alpha} \in \mathcal{A}$, which can be computed as the limit point of the sequence $((\mathcal{T}^G)^n(\alpha^*))_{n\geq 1}$. Observing that $(\mathcal{T}^G)^n(\alpha^*)(i^1, \ldots, i^T)$ is only a function of i^T (and not of the vector $(i^1, \ldots, i^{T-1})$), we can actually represent the sequence $((\mathcal{T}^G)^n(\alpha^*)) \subseteq \mathcal{A}^*$ as a sequence of vectors $(\alpha^n)_{n\geq 1} \subseteq \mathbb{R}^N$ with $(\mathcal{T}^G)^n(\alpha^*)(i^1, \ldots, i^T) = \alpha^n_{i^T}$. Denote $\bar{a}^G = \lim_{n\to\infty} \alpha^n$. By construction, the sequence (α^n) satisfies $\alpha^n = \Phi(\alpha^{n-1}, G)$ with α^1 an upper bound for the set of fixed points of $\Phi(\cdot, G)$. As a result, $\bar{a}^G$ is the largest fixed point of the increasing function $\Phi(\cdot, G)$. Since $\bar{a}^G$ actually represents the largest fixed point of $\mathcal{T}^G$, it readily follows that for any $\alpha \leq \mathcal{T}^G(\alpha)$, $\alpha(i^1, \ldots, i^T) \leq \bar{a}^G_{i^T}$ for all $(i^1, \ldots, i^T)$.

□

Proof of Proposition 1. Proving $a \Rightarrow b$ is immediate. We prove the converse.

Let $\sigma \in \Sigma(G)$. Observe that if $\bar{a}^G_i$ is played on-path by player i, for all i, then a defection against some i must entail a loss in continuation values equal to

$$V(C) - V(D) = \frac{\delta}{1-\delta}\frac{1}{N} \sum_{j \in \bar{N}(i,G)} f(\bar{a}^G_j).$$

Since $V(C) = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j=1}^N f(\bar{a}^G_j)$, we deduce that the continuation value after a defection against i equals

$$V(D) = \frac{\delta}{1-\delta}\frac{1}{N} \sum_{j \notin \bar{N}(i,G)} f(\bar{a}^G_j).$$

Now, decompose the continuation value $V(D)$ following the defection against i as payoffs accruing from encounters with investors $j \in \bar{N}(i,G)$, $\bar{v}$, and payoffs accruing from encounters with investors $j \notin \bar{N}(i,G)$, $\hat{v}$. By definition $V(D) = \bar{v} + \hat{v}$. Since agents' payoffs are nonnegative, $\bar{v} \geq 0$. Moreover, following a defection against i, it is feasible for the agent to cooperate in all remaining encounters against $j \notin \bar{N}(i,G)$ and therefore $\hat{v} \geq \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin \bar{N}(i,G)} f(\bar{a}^G_j)$. It follows that $\bar{v} = 0$ and $\hat{v} = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\notin \bar{N}(i,G)} f(\bar{a}^G_j)$. Moreover, following a defection against i, continuation play must be such that (i) all investors $j \in \bar{N}(i,G)$ do not participate, and (ii) there is no contagion: investors $j \notin \bar{N}(i,G)$ keep investing according to $\bar{a}^G_j$ and the agent cooperates in all those encounters.

Now, assume that G is not of complete components and let i and k be within distance 2. Consider a defection against i in $t = 1$ and consider the incentives the agent faces if he meets k in $t = 2$. From the previous paragraph, the path of play in encounters $j \notin \bar{N}(i)$ should involve cooperation. Following an argument similar to that in the proof of Lemma 1, we derive the incentive constraint for cooperation when facing k after a defection against i

$$g(\bar{a}_k^G) \leq \frac{\delta}{1-\delta}\frac{1}{N}\Big(\sum_{j\notin \bar{N}(i)} f(\bar{a}_j^G) - \sum_{j\notin \bar{N}(i)\cup\bar{N}(k)} f(\bar{a}_j^G)\Big)$$

Since i and k are within distance 2, it follows that $\sum_{j\notin \bar{N}(i)} f(\bar{a}_j^G) - \sum_{j\notin \bar{N}(i)\cup\bar{N}(k)} f(\bar{a}_j^G) < \sum_{j\in \bar{N}(k)} f(\bar{a}_j^G)$ and therefore $\bar{a}_k^G < \Phi_k(\bar{a}^G)$, yielding a contradiction and concluding the proof. □

B Proofs for Section 3.2

Proof of Theorem 1. We first argue that for G and G^* be as in the statement of Theorem 1, $\bar{a}^G \leq \bar{a}^{G^*}$. To see this, just note that $\bar{a}^G$ and $\bar{a}^{G^*}$ can be computed by iterative applications of $\Phi(\cdot, G)$ and $\Phi(\cdot, G^*)$. Starting both iterative procedures from a common upper bound $\bar{A}$, denote by $\bar{a}^n$ and $\bar{a^*}^n$ the corresponding sequences. It follows that in each round of iteration, $\bar{a}^n \leq \bar{a^*}^n$. The result follows by passing to the limit.

To finally prove Theorem 1, let $\sigma \in \Sigma(G)$, and note that $\alpha_\sigma(i^1, \ldots, i^T) \leq \bar{a}_{i^T}^G$ from Lemma 1, where the inequality is strict for some history due to Proposition 1. Noting that $\bar{a}^{G^*}$ can actually be implemented using trigger strategies in G^* and $\bar{a}^G \leq \bar{a}^{G^*}$, the result follows since v is strictly increasing and f is nondecreasing. □

Proof of Lemma 2. Define $\tilde{a}$ as $\tilde{a}_i = g^{-1}\big(\frac{\delta}{1-\delta}\frac{|\bar{N}(i,G)|}{N}\bar{f}\big)$, where $\bar{f} = \max f$, and note that $\bar{a}_i^G \leq \tilde{a}_i$ for all i. Since $\delta > \bar{\delta}$, $\tilde{a}_i \geq \min(\arg\max f)$ for all i, and therefore $\Phi_i(\tilde{a}) = g^{-1}(\frac{\delta}{1-\delta}\frac{|\bar{N}(i,G)|}{N}\bar{f}) = \tilde{a}_i$. It follows that for $\delta > \bar{\delta}$, $\tilde{a}$ is a fixed point of Φ_i. Since $\bar{a}^G$ is the largest fixed point, $\tilde{a}_i \leq \bar{a}_i^G$ for all i and we conclude that $\tilde{a} = \bar{a}^G$. □

Proof of Theorem 2. Let $\rho = \sum_{i=1}^N |N(i, G^*)|$ and define $\bar{f} = \max f$. From Lemma 1, for a social network G and $\sigma \in \Sigma(G)$

$$\alpha_\sigma(i^1, \ldots, i^t) \leq g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i^t, G)}{N}\bar{f}\big) \quad \forall\ (i^1, \ldots, i^t)$$

with at least some inequality strict if G has some incomplete component. Taking expectations and summing

$$\sum_{i=1}^N u_i(\sigma, G) \leq \sum_{i=1}^N v \circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i^t, G)}{N}\bar{f}\big)$$

with strict inequality for G having some incomplete component.

Take G^* having complete components (as in parts (i) and (ii)). From Proposition 1, there exists $\sigma^* \in \Sigma(G^*)$ such that

$$\sum_{i=1}^{N} u_i(\sigma^*, G^*) = \sum_{i=1}^{N} v \circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i, G^*)}{N}\bar{f}\big).$$

Part (i) follows immediately. To prove (ii), consider the relaxed problem

$$\max_{(x_1,\ldots,x_N)} \sum_{i=1}^{N} v \circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{x_i+1}{N}\bar{f}\big).$$

subject to

$$x_i \in \{1, 2, \ldots, N\},\ \sum_{i=1}^{N} x_i \leq \rho.$$

This problem can be thought of as the problem of assigning a total of ρ pennies between N persons with the purpose of maximizing average utility. Since $v \circ g^{-1}$ is strictly concave and all components of G^* are of the same size $\gamma \geq 2$, this problem has a single solution $x_i^* = \gamma - 1$ for all i. Noting that $(x_i)_{i=1}^N$ and $(x_i^*)_{i=1}^N$ defined as $x_i = |N(i, G)|$ and $x_i^* = |N(i, G^*)| = 2\rho/N$ are, respectively, feasible and optimal for the maximization above, it follows that

$$\sum_{i=1}^{N} v \circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i, G)}{N}\bar{f}\big) < \sum_{i=1}^{N} v \circ g^{-1}\big(\frac{\delta}{1-\delta}\frac{\bar{N}(i, G^*)}{N}\bar{f}\big).$$

It remains to prove part (iii). Observe first that in the star G^*, we can construct a strategy profile σ^* that yields average total welfare equal to $U(E) + (N-1)U(0)$, where $E = \rho/2$ is the number of available links. Take investor 1 as the center of the star and investors $2, \ldots E+1$ as the leaves of the star. On the path of play, investor 1 invests $g^{-1}(\frac{\delta}{1-\delta}\frac{E+1}{N}\bar{f})$, whereas all other investors take action $g^{-1}(\frac{\delta}{1-\delta}\frac{1}{N}\bar{f})$. A defection against player 1 implies all other star members refuse trade in subsequent rounds (and after any second deviation). A defection against any $i \neq 1$ implies only player i refuse trade in subsequent rounds, while player 1 keeps participating and playing $g^{-1}(\frac{\delta}{1-\delta}\frac{E}{N}\bar{f})$ (more generally, after a number n of defections against leaves, 1 chooses $g^{-1}(\frac{\delta}{1-\delta}\frac{E+1-n}{N}\bar{f})$). The agent cooperates on the play path and, after a defection, keeps cooperating when facing any investor who according to his strategy should participate (defecting otherwise). This strategy profile is a sequential equilibrium that sustains cooperation and results in total average welfare equal to $U(E) + (N-1)U(0)$.

Now, consider the maximization problem

$$\bar{W} = \max_G \sum_{i=1}^{N} U(|N(i,G)|) \tag{B.1}$$

subject to

$$\sum_{i=1}^{N} |N(i,G)| \leq \rho, \text{ and } G \text{ is not a star.}$$

Problem (B.1) yields an upper bound for total equilibrium welfare over all networks G different from a star and having less than 2ρ links. Let $\bar{G}$ be optimal for (B.1) and let $i^* \in \arg\max_i |N(i,\bar{G})|$.

We first note that if there exists $ij \in \bar{G}$, with $i \neq i^* \neq j$, such that the new network formed from $\bar{G}$ by deleting ij and using that link to connect some isolated player k to i^*, then $\bar{G}$ cannot be optimal. Indeed, the net gain from forming the new network would be

$$U(|N(i^*,\bar{G})|+1) - U(|N(i^*,\bar{G})|) + U(|N(k,\bar{G})|+1) - U(|N(k,\bar{G})|)$$
$$+ U(|N(i,\bar{G})|-1) - U(|N(i,\bar{G})|) + U(|N(j,\bar{G})|-1) - U(|N(j,\bar{G})|)$$

Define $\Delta(x) = U(x) - U(x-1)$ and rewrite the inequality above as

$$\Delta(|N(i^*,\bar{G})|+1) + \Delta(1) \geq \Delta(|N(i,\bar{G})|) + \Delta(|N(j,\bar{G})|).$$

As Δ is increasing, the inequality holds as $\Delta(x+1) + \Delta(1) \geq 2\Delta(x)$. It therefore follows that the new gain would be positive. It follows that if $\bar{G}$ has two or more components, then only two of them are nontrivial, one of the nontrivial components is a star formed by $E-1$ links, and the other nontrivial component is a line formed by two nodes. Such network yields total welfare which is less than the welfare from the star. Indeed,

$$U(E-1) + (E-1)U(1) + 2U(1) + (N-E-2)U(0) \leq U(E) + (N-1)U(0)$$

if and only if $U(E) - U(E-1) \geq (E-1)(U(1) - U(0))$ and the inequality above holds true. It is therefore enough to consider the case where $\bar{G}$ has a single nontrivial component.

Define $I = \{i \mid |N(i,\bar{G})| \geq 2\}$. Using the construction above, it follows that $|I| \leq 3$. Consider first the case $|I| = 3$ and let $I = \{i^*, j_1, j_2\}$. We claim that $N(j_n,\bar{G}) = \{i^*, j_{3-n}\}$ as otherwise, and following the construction above, deleting $j_n l$, with $l \notin I$, to form $i^* m$, with $N(m,\bar{G}) = \emptyset$, would be worthwhile. If $|I| = 3$, the value of (B.1) equals

$$\bar{W} = U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0).$$

Now, assume $|I| = 2$ and let $I = \{i^*, j\}$. Observe that $|N(j, \bar{G})| = 2$ as otherwise $|N(j, \bar{G})| \geq 3$ and we could remove one of the links connecting j and use that link to connect i^* to a new player. If $j \in N(i^*, \bar{G})$, then it is relatively easy to see that the objective function at $\bar{G}$ is less than or equal to $U(E-1)+(E-3)U(1)+2U(2)+(N-E+1)U(0)$ and therefore $j \notin N(i^*, \bar{G})$. Take $l, m \in N(j, \bar{G})$ and note that deleting jm and connecting j to i^*, deleting lj and connecting l so that in the new network the distance between l and i^* equals 2, one can increase the objective. We deduce that $|I| = 3$ and no equilibrium in a network which is not a star can give total payoffs above

$$\bar{W} = U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0).$$

By assumption $\Delta(E) \geq (E-2)(U(1) - U(0)) + \Delta(2) + U(2)$ and thus

$$U(E) + (N-1)U(0) \geq U(E-1) + (E-3)U(1) + 2U(2) + (N-E+1)U(0)$$

proving the result. □

Proof of Proposition 3. Conditional on facing investor i in t, the expected discounted total equilibrium payoff for the agent is

$$f(a_i) + \frac{\delta}{1-\delta}\frac{1}{N}\sum_{k=1}^{N} f(a_k).$$

Alternatively, the agent could defect against i in t, defect the first round after meeting $k \in N_2(i)$, and otherwise keep cooperating. Such deviation strategy yields payoffs which are bounded above by

$$f(a_i) + g(a_i) + \delta V$$

where V is implicitly defined by

$$V = \frac{1}{N}\sum_{k\in N_2(i)}\Big(f(a_k) + g(a_k) + \frac{\delta}{1-\delta}\frac{1}{N}\sum_{l\notin \bar{N}(i)\cup\bar{N}(k)} f(a_l)\Big) + \frac{1}{N}\sum_{k\notin \bar{N}(i)\cup N_2(i)} f(a_k) + \frac{N-|N_2(i)|}{N}\delta V.$$

Arrange terms to deduce that

$$\delta V = \frac{\delta}{1-\delta}\frac{1}{N}\sum_{k\notin \bar{N}(i)} f(a_k) + \frac{\delta}{N(1-\delta)+\delta|N_2(i)|}\sum_{k\in N_2(i)}\Big(g(a_k) - \frac{\delta}{1-\delta}\frac{1}{N}\sum_{l\in \bar{N}(i)} f(a_l)\Big)$$
$$+ \frac{\delta}{N(1-\delta)+\delta|N_2(i)|}\sum_{k\in N_2(i)}\frac{\delta}{1-\delta}\frac{1}{N}\sum_{l\in \bar{N}(i)\cap\bar{N}(k)} f(a_l).$$

Since the deviation strategy cannot result in payoffs that are higher than the equilibrium payoffs

$$f(a_i) + \frac{\delta}{1-\delta}\frac{1}{N}\sum_{k=1}^{N} f(a_k) \geq f(a_i) + g(a_i) + \delta V$$

and arranging terms

$$\sum_{k\in N_2(i)} \frac{\delta}{1-\delta}\frac{1}{N} \sum_{l\in \bar{N}(i)\cap \bar{N}(k)} f(a_l) \leq \sum_{k\in N_2(i)} \Big(\frac{\delta}{1-\delta}\frac{1}{N} \sum_{l\in \bar{N}(k)} f(a_l) - g(a_k)\Big) + \frac{N(1-\delta)+\delta|N_2(i)|}{\delta}\Big(\frac{1}{N}\frac{\delta}{1-\delta}\sum_{k\in \bar{N}(i)} f(a_k) - g(a_i)\Big)$$

The result follows by noting that for all k, $\frac{\delta}{1-\delta}\frac{1}{N}\sum_{j\in \bar{N}(k)} f(\bar{a}_j^G) - g(\bar{a}_k^G) = 0$ (Lemma 1) and using the fact that the functions on the right of the inequality above are Lipschitz continuous. □

C Proofs for Section 4

Proof of Theorem 3. For any G, if i and j belong to the same component C, $\hat{a}_i^G = \hat{a}_j^G$, which is purely determined by the size of the component $|C|$. Now, for G as in the statement of the Theorem, take the component which is not minimally connected, C_1, and link $ij \in C_1$ such that $C \setminus \{ij\}$ is connected. Take a second component C_2 and any $k \in C_2$, and use the link ij to form a new connection ik. We therefore obtain a new network G^* such that, for all i, $\hat{a}_i^{G^*} \geq \hat{a}_i^G$ for all i, with strict inequality for $i \in C_1 \cup C_2$. It follows that G^* Pareto dominates G. □

Proof of Theorem 4. Since $\frac{1}{2}\sum_{i=1}^{N}|N(i,G^*)| \leq N-1$, G^* has two or more components and from Theorem 3, G^* is minimally connected. The characterization of part i follows by noting that if G^* has two non-trivial components of sizes n_1 and n_2, each component must be minimally connected and we can form two components of size $n_1 + n_2 - 1$ and 1. This new network yields strictly more total welfare. A similar argument shows part ii. □

D Some Extensions and Variations

In this Section, we explore how our results can be extended to accommodate alternative modeling assumptions.

Networks of Information Transmission and Contagion-Freeness

We consider the same set up described in Section 2, but modify the way in which monitoring occurs. Given a social network G, monitoring is such that the signals investors receive are recursively determined as follows. More specifically, let $h_i^0 = \emptyset$ bet the history i has at the beginning of $t = 1$ (before play). Let h_i^{t-1} be the history player i has at the beginning of t. Let $i^t \in \{1, \ldots, N\}$ be the investor chosen at round t. If i^t chooses P, then all investors $j \in N(i^t)$ become aware of that and observe whether the agent cooperated or defected. More formally, if investor i^t participates and the agent played $x^t \in \{C, D\}$, then player $j \in N(i^t)$ receives a signal $s_j^t = (i^t, x^t)$, while if $j \notin \bar{N}(i^t)$ then j receives a signal $s_j^t = \emptyset$. If i^t did not participate, then all players $j \neq i$ receive signal $s_j^t = \emptyset$. Player i^t perfectly observes play during round t. The history player $i = 1 \ldots, N$ has at the beginning of $t + 1$ is then defined as

$$h_i^t = (h_i^{t-1},\ s_i^t,\ h_{N(i,G)}^{t-1})$$

where $h_{N(i,G)}^{t-1}$ denotes the concatenation of the histories that players $j \in N(i, G)$ had at the beginning of round t (so that any information that $j \in N(i, G)$ had at the beginning of t becomes available for i at the beginning of $t + 1$).

In this model, information is spread through the networks as play transpires. While in Section 2 the network determines the transactions a player observes, in this model the network is used to transmit information through the links: whenever investor i becomes aware of some defection against k in some previous round, that information becomes immediately and automatically available to $j \in N(i, G)$ at the beginning of the next round.

The definition of strategy σ_i for player i is similar to that presented for the main model. We also define the set $\bar{\Sigma}(G)$ of strategy profiles $\sigma = (\sigma_i)_{i=0}^N$ that sustain cooperation on the path of play.

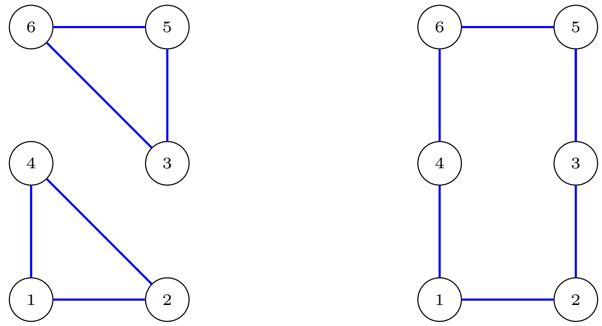
As the following example illustrates, Theorem 1 does not apply to this environment.

Example 3. *Suppose that $N = 6$, $g(a) = a$ and $f(a) = 1$. Consider the regular networks G^* and G in Figure 6. The optimal equilibrium in the network of complete components G^* has investors using trigger strategies and choosing actions*

$$a^* = \frac{\delta}{1-\delta}\frac{1}{2} \tag{D.1}$$

on the path of play.

In network G, we can construct an equilibrium such that on the path of play investors choose $a > 0$ and whenever information about a defection arrives, the investor refuses trade in all ensuing



(a) Network G^* (b) Network G

Figure 6: When news travel through the network, G^* is Pareto dominated by G.

rounds. The incentive compatibility restriction for the agent is

$$\frac{1}{1-\delta} \geq (1+a) + \delta\frac{1}{2}(1+a) + \delta^2\frac{1}{2}\frac{1}{6}(1+a).$$

The left side can be understood by noting that, after a defection at t (which results in period payoffs $(1+a)$), it is in the interest of the agent to defect against any of the uninformed investors in $t+1$ (and obtain discounted payoffs $\delta(1+a)$). There is an extra defection opportunity for the agent in $t+2$ if at $t+1$ he faced the informed investors and, at $t+1$, faces the still uninformed investor (this explains the last term). Solving for the investment $\bar{a}$ at which the incentive compatibility constraint binds

$$a = \frac{\frac{\delta}{1-\delta} - \frac{\delta}{2} - \frac{\delta^2}{12}}{1 + \frac{\delta}{2} + \frac{\delta^2}{12}}. \tag{D.2}$$

It is relatively simple to see that $a > a^*$. Therefore, G Pareto dominates G^*.

In this example, the Pareto-dominant network G sustains high levels of cooperation because any defections is punished by the whole communities, whereas in the network of complete components G^* punishments are kept local. On the other hand, the equilibrium constructed for network G seems utterly fragile as any mistake (say there is some tiny probability that signals are imperfectly observed) causes a complete community breakdown. As Kandori (1992) forcefully argues, when evaluating the merits of a social norm one should also consider its robustness to mistakes. An arrangement in which small trembles have incommensurate impact on continuation play seems inappropriate. Our focus therefore will be on equilibria that are immune to mistakes.

We present two preliminary definitions. Let h' and $\bar{h}'$ be two histories of the same length T. We say that h' and $\bar{h}'$ *coincidentally select players* if for all $\tau \in \{1, \ldots, T\}$ and all $i \in \{1, \ldots, N\}$, $i^\tau = i$

in h' if and only if $i^\tau = i$ in $\bar{h}$. We will say that h' and $\bar{h}'$ *have equivalent outcomes at* i, if for all $\tau \in \{1, \ldots, T\}$, all a and all $x, y \in \{C, D\}$, $(i^\tau, a^\tau, x^\tau) = (i, a, x)$ in h' iff $(i^\tau, a^\tau, x^\tau) = (i, a, x)$ in $\bar{h}'$.

A strategy profile $\sigma \in \Sigma$ is *contagion-free* if for all $h \in \tilde{H}_\sigma$, with $h = (i^1, a^1, C, \ldots, i^T, a^T, C)$ and $\bar{h} = (i^1, a^1, C, \ldots, i^{T-1}, a^{T-1}, C, i^T, a^T, D)$, and all continuation histories $h' \succsim_\sigma h$ and $\bar{h}' \succsim_\sigma \bar{h}$ of the same length, if h' and $\bar{h}'$ coincidentally select investors, then h' and $\bar{h}'$ have equivalent outcomes at $i \notin \bar{N}(i^T)$. Let $\bar{\Sigma}^{cf} = \bar{\Sigma}^{cf}(G)$ be the set of all contagion-free strategy profiles.

Contagion-free strategies restrict off-path play following a defection to remain unchanged in matches involving investors not instantaneously aware of the defection. This normative prescription is motivated by the observation that (unmodeled) mistakes may occur, but those mistakes should have a moderate impact on the whole community.[16] While after a single defection play among unaware players remain unchanged, a second defection may unchain a sequence of defections and cause a complete community breakdown. Alternative robustness requirements could be explored too. For example, one may actually want to impose equilibrium robustness to a larger number of mistakes. In particular, while players could make an arbitrary number of mistakes, contagion-free equilibria are robust only to a single mistake. As our Proposition 4 shows, optimal networks are robust to an arbitrary number of mistakes.

In general, the contagion-free restriction has costs in terms of equilibrium payoffs. Examples of contagion-free equilibria include trigger strategies in networks of complete components, as well as one-period memory equilibrium strategies (as in Barlo, Carmona, and Sabourian 2009) in which following a defection, neighbors aware of it punish for a single round, while more distant players ignore the defection. We revisit Example 3 to construct a simple contagion-free equilibrium.

Example 4. *Consider network G in Example 3. We construct a contagion-free symmetric equilibrium sustaining investment $a > 0$ (to be determined) on the path of play. A defection against any investor will be punished by all his direct connections, whereas other players will keep investing a (even after receiving the news of the defection). Even when some players ignore first defections, those players refuse trade in all subsequent rounds after being aware of a second defection.*

To write the restrictions ensuring it is in the agent's interest to cooperate, note first that the on-path incentive constraints take the form

$$\frac{1}{1-\delta} \geq (1+a) + \frac{1}{2}\frac{\delta}{1-\delta}.$$

Consider now a history following a defection against 1. If the agent faces investor 5, then the incentive constraint coincides with the one above as a defection against 5 yields net period gains

[16] In a contagion-free equilibrium, a player's incentives remain unchanged if he believes there was a single defection. Yet, this is different from belief-free equilibria (Ely and Välimäki 2002, Ely, Hörner, and Olszewski 2005) as in a contagion-free equilibrium if a player believes two or more defections occurred his behavior could be modified.

a and continuation losses equal to $\frac{\delta}{1-\delta}\frac{1}{2}$. Instead, if the agent defects when facing 3, he will have one more opportunity to defect against 6 in the subsequent round (investor 6 will be aware of the first defection against 6, but it will take him one more round to know the defection against 3). The incentive constraint for off-path cooperation is therefore

$$1+\frac{1}{2}\frac{\delta}{1-\delta}\geq 1+a+\frac{1}{6}\delta(1+a)$$

Solving for the maximal investment $\bar{a}$ that satisfies both incentive constraints, we find

$$\bar{a}=\frac{\frac{1}{2}\frac{\delta}{1-\delta}-\frac{\delta}{6}}{1+\frac{\delta}{6}}.$$

In network G, on-path investments are lower when we restrict attention to contagion-free equilibria:

$$\bar{a}<a$$

where a was defined in (D.2). Intuitively, high on-path investments make the network fragile. Ensuring a defection against 1 does not infect investors $\{3,5,6\}$ implies investor 3 cannot count with 2 to punish a defection against 3. As a result, players on path investments must be relatively low in a contagion-free equilibrium of network G. In contrast, none of these "second-defections" are relevant for the network of complete components G^*. Not surprisingly, then, higher investments can be implemented in a contagion-free manner in network G^*: $\bar{a}<a^*$. We will show this result extends more generally.

We will say that G^* *Pareto-dominates G in contagion-free strategies* if there exists $\sigma^*\in\bar{\Sigma}^{cf}(G^*)$ such that for all $\sigma\in\bar{\Sigma}^{cf}(G)$, $u_i(\sigma^*,G^*)\geq u_i(\sigma,G)$ for all $i=1,\ldots,N$, with at least some strict inequality.

Proposition 4. *Let G^* be a κ-regular network of complete components. Let G be any other network having some incomplete component such that $|N(i,G)|\leq|N(i,G^*)|$ for all $i=1,\ldots,N$. Then, G^* Pareto dominates G in contagion-free strategies.*

The proof of this result is similar to the proof of Theorem 1. Details are available upon request.

On The Continuous Actions Assumption

We now explore a model in which the scale of the project cannot be chosen by the investor. The only decision that the investor i^t makes is whether or not to participate. If the investor participates, the agent decides whether to cooperate or to defect. Figure 7 illustrates the stage game. The

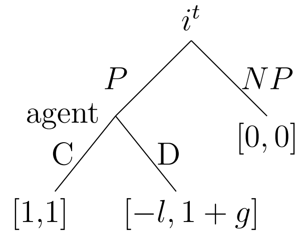


Figure 7: The game between investor i^t and the agent. We assume that $l, g > 0$.

monitoring technology is also determined by a social network G of investors such that if i^t decides to participate in round t, then all of i^t's neighbors become aware of that and observe the behavior of the agent in period t. No further information is transmitted.

We import the definition of strategies that sustain cooperation from Section 2. We say that a strategy profile σ *sustains cooperation* if it is a perfect Bayesian equilibrium and on the path of play all investors participate and the agent cooperates.

Proposition 5. *Let σ be any strategy profile sustaining cooperation in network G. Let G^* be any network of complete component such that $N(i,G) \leq N(i,G^*)$ for all i. Then, there exists a strategy profile σ^* sustaining cooperation in network G^*.*

This result shows that if a strategy profile can sustain cooperation in a network having an incomplete component, then it is possible to construct strategies –indeed trigger strategies– that sustain cooperation in a network of complete components. As in Theorem 1, Proposition 5 shows that forming a network of complete components is the best that game players can hope for. The logic behind both results is the same: in networks of complete components there is less room for off-path gaming. Yet, Proposition 5 is weaker as it is still possible that, given an exogenous number of links, networks G and G^* result in the same equilibrium payoffs.[17]

Proof of Proposition 5. Let a strategy profile σ sustains cooperation in network G. Following a defection, a feasible (but not necessarily optimal) strategy for the agent is to cooperate in remaining encounters and therefore the incentive condition for cooperation on the path of play implies $g \leq \frac{\delta}{1-\delta}\frac{\bar{N}(i,G)}{N}$ for all i. Since network G^* is denser than network G, $g \leq \frac{\delta}{1-\delta}\frac{\bar{N}(i,G^*)}{N}$ for all i. This condition is necessary and sufficient for trigger strategies to sustain cooperation in network G^*. □

[17] This model is studied in Balmaceda and Escobar (2011), where conditions are shown under which networks of complete components are (strictly) optimal.

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