Online Cost-Sharing Mechanism Design for Demand-Responsive Transport Systems

Masabumi Furuhata, Kenny Daniel, Sven Koenig, *Senior Member, IEEE,*
Fernando Ordóñez, Maged Dessouky, Marc-Etienne Brunet, Liron Cohen, and Xiaoqing Wang

***Abstract*—Demand-responsive transport (DRT) systems provide flexible transportservices for passengers who request door-to-door rides in shared-ride mode without fixed routes and schedules. DRT systems face interesting coordination challenges. For example, one has to design cost-sharing mechanisms for offering fare quotes to potential passengers so that all passengers are treated fairly. The main issue is how the operating costs of the DRT system should be shared among the passengers (given that different passengers cause different amounts of inconvenience to the other passengers), taking into account that DRT systems should provide fare quotes instantaneously without knowing future ride request submissions. We determine properties of cost-sharing mechanisms that make DRT systems attractive to both the transport providers and passengers, namely online fairness, immediate response, individual rationality, budget balance and ex-post incentive compatibility. We propose a novel cost-sharing mechanism, called Proportional Online Cost Sharing (POCS), that provides passengers with upper bounds on their fares immediately after their ride request submissions despite missing knowledge of future ride request submissions, allowing them to accept their fare quotes or drop out. We examine how POCS satisfies these properties in theory and computational experiments.**

***Index Terms*—Demand-Responsive Transport Systems, Cost Sharing, Online-Mechanism Design**

I. Introduction

Demand-responsive transport (DRT) systems provide flexible transport services where individual passengers request door-to-door rides by specifying their desired start and end locations. Multiple shuttles (or vans or small busses) service these ride requests in shared-ride mode without fixed routes and schedules. DRT services are more flexible and convenient for passengers than buses since they do not operate on fixed routes and schedules, yet are cheaper than taxis due to the higher utilization of transport capacity. In the United States, DRT services are commonly used to service the transport needs of disabled and elderly citizens and have experienced rapid growth [1], [2]. Furthermore, the National Transit Summaries and Trends report for 2008 states that the average operating cost per passenger trip is \$30.0 for DRT systems but only \$3.3 for buses, the average operating cost per passenger mile is \$3.4 for DRT systems but only \$0.8 for buses, and the revenues from fares cover less than 10% of the operating costs of DRT systems. Hence, it is important to identify opportunities for reductions in cost and improvements in efficiency [1], [3], especially if one wants to expand DRT services to provide a transport option for urban populations in general.

Two important research issues in the context of DRT systems are how to determine the routes and schedules of the shuttles (including how to assign passengers to shuttles) in the presence of conflicting objectives, such as maximizing the number of serviced passengers, minimizing the operating cost or minimizing the passenger inconvenience, and how much to charge the passengers. The first (optimization) and second (cost-sharing) problems are highly interrelated since the routes and schedules of the shuttles determine the operating cost that needs to be shared. Conversely, the cost-sharing mechanism imposes constraints on the routes and schedules that need to be optimized, for example, because the fares of the passengers should not exceed their fare quotes. The optimization problem has received considerable attention in the literature and is often solved as a pickup and delivery problem [1], [4]–[9]. The cost-sharing problem, on the other hand, has largely been neglected in the literature, which might be due to shuttles being highly subsidized and most passengers thus enjoying transport services at affordable fares, typically determined by flat rates within service zones that do not cover the operating cost. Without significant subsidies, the fares would substantially increase and the passengers would then be more concerned about how the operating cost is shared among them in a fair manner. This article presents a cost-sharing mechanism for transport systems where the fares are not heavily subsidized by exploring the ideal assumption that they are not subsidized at all.

How passengers should share the operating cost in an online setting, where knowledge of future ride request submissions is missing, is a non-trivial problem for the following reasons: First, passengers do not submit their ride requests at the same time but should be given incentives to submit them as early as possible to allow the DRT systems more time to find routing

M. Furuhata is with DWANGO Corporation Limited, Kabukiza Tower, 4-12-15 Ginza, Chuo-ku, Tokyo, Japan 104-0061, e-mail: k439bk439b@gmail.com. K. Daniel is with Algorithmia Incorporated, San Francisco, California 94107, United States of America, e-mail: kenny@algorithmia.io. S. Koenig and L. Cohen are with the Computer Science Department, University of Southern California, 941 Bloom Walk, Los Angeles, California 90089-0781, United States of America, e-mails: skoenig@usc.edu and lironcoh@usc.edu. Fernando Ordóñez is with the Industrial Engineering Department, Universidad de Chile, Republica 701, Santiago, Chile, e-mail: fordon@dii.uchile.cl. Maged Dessouky and Xiaoqing Wang are with the Daniel J. Epstein Department of Industrial and Systems Engineering, University of Southern California, 3715 McClintock Avenue, Los Angeles, California 90089-0193, United States of America, e-mails: maged@usc.edu and xiaoqinw@usc.edu. Marc-Etienne Brunet is with the Department of Electrical and Computer Engineering, McGill University, 3630 University Street, Montreal, Quebec H3A 0C6, Canada, e-mail: marc-etienne.brunet@mail.mcgill.ca. M. Furuhata, K. Daniel, Fernando Ordóñez and Marc-Etienne Brunet all were with the University of Southern California in the past.

solutions that can offer subsequent passengers lower fares due to synergies with the early ride requests, which might allow them to service more passengers. Second, passengers have different start and end locations and thus cause different amounts of inconvenience to the other passengers, which should be reflected in the fares. Finally, passengers should be quoted fares immediately after their ride request submissions because, this way, passengers have no uncertainty about they cost of service while the DRT systems reduce their uncertainty about passengers dropping out. This requires DRT systems to make instantaneous and irreversible decisions despite having no knowledge of future ride request submissions [10].

In this article, we define the online cost-sharing problem for DRT systems and describe typical cost-sharing mechanisms, focusing on proportional and incremental cost sharing and some of their shortcomings in an online setting. We then determine properties of cost-sharing mechanisms for DRT systems that we believe are attractive to both the shuttles and the passengers, namely online fairness, immediate response, individual rationality, budget balance and ex-post incentive compatibility. We propose a novel cost-sharing mechanism, called Proportional Online Cost Sharing (POCS), and show that it satisfies all five properties, provided that the routes and schedules of the shuttles satisfy some restrictions, for example, minimize the operating cost. However, DRT systems need to determine the routes and schedules of the shuttles after every ride request submission and minimizing the operating cost takes time, which would prevent them from operating in real-time. We therefore also present experimental results to evaluate DRT systems where the routes and schedules of the shuttles minimize the operating cost only approximately.

II. Online Cost Sharing

In this section, we define the online cost-sharing problem for demand responsive transport (DRT) systems, provide an example, discuss existing cost-sharing mechanisms and some of their shortcomings, and finally derive a list of desirable properties for online cost-sharing mechanisms for DRT systems.

A. Problem Definition

DRT systems provide flexible transport services where individual passengers request door-to-door rides. Multiple shuttles service these ride requests without fixed routes and schedules. Passengers share shuttles. For example, after a passenger has been picked up and before it is dropped off, other passengers can be picked up and dropped off, resulting in a longer ride for the passenger. Passengers need to pay a share of the operating cost. Passengers submit their ride requests one after the other by specifying their desired start and end locations. The submit time of a passenger is the time when it submits its ride request. In case the passenger decides to delay its ride request submission, we distinguish its truthful submit time, which is its earliest possible submit time, from its actual, perhaps delayed, submit time. We assume, for simplicity, that all passengers submit their ride requests before the shuttles start to service the passengers. We also assume, without loss of generality, that exactly one passenger submits its ride request at each time $k = 1, \ldots, t$, namely that passenger $\pi(k)$ submits its ride request at time k under submit order π, where a submit order is a function that maps submit times to passengers.

Definition 1. *For all times k and all submit orders π with $1 \leq k$, the alpha value $\alpha_{\pi(k)}$ of passenger $\pi(k)$ quantifies the demand of its ride request, that is, how much of the transport resources it requests. We assume that it is positive and independent of the submit time of the passenger.*

These assumptions are, for example, satisfied for the shortest point-to-point travel distance from the start location to the end location of a passenger, which is the quantity that we use in this article as its alpha value.

Definition 2. *For all times t and all submit orders π with $1 \leq t$, the total cost $totalcost^t_\pi$ at time t under submit order π is the operating cost required to service passengers $\pi(1), \ldots, \pi(t)$. We define $totalcost^0_\pi := 0$ and assume that 1) the total cost is non-decreasing over time, that is, for all times t and t' and all submit orders π with $t \leq t'$, $totalcost^t_\pi \leq totalcost^{t'}_\pi$; and 2) the total cost at time t is independent of the submit order of passengers $\pi(1), \ldots, \pi(t)$, that is, for all times t and all submit orders π and π' with $1 \leq t$ and $\{\pi(1), \ldots, \pi(t)\} = \{\pi'(1), \ldots, \pi'(t)\}$, $totalcost^t_\pi = totalcost^t_{\pi'}$.*

These assumptions are, for example, satisfied for the minimal operating cost, which is the quantity that we use in this article for the total cost. The minimal operating cost includes the cost for deadhead miles for the shuttles to provide the transport services. These assumptions can also be satisfied by other types of costs. For instance, the part of the minimal operating cost incurred by shuttles when passengers are on board also satisfies them. The DRT system can accommodate advanced features, such as operating times and capacities of the shuttles and time constraints of the passengers, as long as it can calculate total costs that satisfy the assumptions. The assumptions are typically not satisfied if passengers can submit their ride requests after the shuttles have started to service passengers since the shuttle locations influence the total cost. We initially assume for simplicity (in the theoretical part of this article) that the DRT system can easily calculate the total cost at any given time. This is not always true since the problem could be NP-hard and thus time-consuming, as is the case for the minimal operating cost. However, the DRT system needs to calculate the minimal operating cost after every ride request submission, which would prevent it from operating in real-time. We therefore also include an experimental part to evaluate DRT systems that minimize the operating cost only approximately.

Definition 3. *For all times k and all submit orders π with $1 \leq k$, the marginal cost $mc_{\pi(k)}$ of passenger $\pi(k)$ under submit order π is the increase in total cost due to its ride request submission, that is, $mc_{\pi(k)} := totalcost^k_\pi - totalcost^{k-1}_\pi$.*

Definition 4. *For all times k and t and all submit orders π with $1 \leq k \leq t$, the shared cost $cost^t_{\pi(k)}$ of passenger $\pi(k)$ at time t under submit order π is its share of the total cost at time t.*

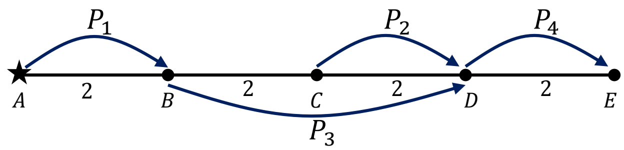


Fig. 1. DRT Example 1

TABLE I
DRT VALUES

		$k=1$ $\pi(k)=P_1$	$k=2$ $\pi(k)=P_2$	$k=3$ $\pi(k)=P_3$	$k=4$ $\pi(k)=P_4$
Alpha Value:	$\alpha_{\pi(k)}$	2	2	4	2
Total Cost:	$totalcost^k_\pi$	40	120	120	160
Marginal Cost:	$mc_{\pi(k)}$	40	80	0	40

The DRT system provides a (myopic) fare quote to a passenger immediately after its ride request submission. The fare quoted to passenger $\pi(k)$ immediately after its ride request submission at time k is $cost^k_{\pi(k)}$. (A fare quote of infinity means that the passenger cannot be serviced.)

Definition 5. *For all times k and all submit orders π with $1 \leq k$, the fare limit $w_{\pi(k)}$ of passenger $\pi(k)$ is the maximum amount that it is willing to pay for its requested ride.*

Passenger $\pi(k)$ drops out and is not serviced if its fare limit $w_{\pi(k)}$ is lower than its fare quote, that is, $w_{\pi(k)} < cost^k_{\pi(k)}$. In this case, the DRT system simply pretends that the passenger never submitted its ride request, which explains why we assume, without loss of generality, that all passengers accept their fare quotes. When the passenger accepts its fare quote and is serviced, its fare is $cost^t_{\pi(k)}$ (which is not guaranteed to equal its fare quote) if the shuttles start to service passengers after passenger $\pi(t)$ submitted its ride request.

B. Demand-Responsive Transport Example

We use the DRT example in Figure 1 to illustrate typical cost-sharing mechanisms. There is one shuttle that can transport up to four passengers and starts at the star. The shuttle incurs an operating cost of 10 for each unit of distance traveled and needs to return to its initial location. There are four passengers with submit order $\pi(1) = P_1$, $\pi(2) = P_2$, $\pi(3) = P_3$ and $\pi(4) = P_4$. For example, Passenger P_3 requests a ride from location B to location D, as shown in Figure 1. All passengers accept all fare quotes. Table I shows the alpha value of each passenger, the total cost after the ride request submission of each passenger and the marginal cost of each passenger. For example, the alpha value of Passenger P_3 is the shortest point-to-point travel distance from its start location B to its end location D. Thus, $\alpha_{\pi(3)} = 4$. The total cost at time 3, after the ride request submission of Passenger P_3, is 10 times the minimal travel distance of the shuttle required to service Passengers P_1, P_2 and P_3 and return to its initial location. Thus, $totalcost^3_\pi = 120$ since the shuttle has to drive from location A (to pick up Passenger P_1) via location B (to drop off Passenger P_1 and pick up Passenger P_3) and location C (to pick up Passenger P_2) to location D (to drop off Passengers P_2 and P_3) and to return to its initial location A. The marginal cost of Passenger P_3 is the increase in total cost due to its ride request submission. Thus, $mc_{\pi(3)} = totalcost^3_\pi - totalcost^2_\pi = 120 - 120 = 0$ since the total cost remains 120.

TABLE II
SHARED COSTS UNDER PROPORTIONAL COST SHARING: $cost^t_{\pi(k)}$

	$k=1$ $\pi(k)=P_1$	$k=2$ $\pi(k)=P_2$	$k=3$ $\pi(k)=P_3$	$k=4$ $\pi(k)=P_4$
$t=1$	40			
$t=2$	60	60		
$t=3$	30	30	60	
$t=4$	32	32	64	32

C. Typical Cost-Sharing Mechanisms

Online cost-sharing mechanisms determine the shared costs in an online setting, where knowledge of future ride request submissions is missing. We present typical cost-sharing mechanisms and some of their shortcomings in an online setting, using the DRT example in Section II-B.

1) Proportional Cost Sharing: One commonly used cost-sharing mechanism is *proportional cost sharing* [11], [12], where the total cost is distributed among all passengers proportionally to their alpha values, which reflects that passengers with higher demands should contribute more toward the total cost. Consequently, for all times k and t and all submit orders π with $1 \leq k \leq t$, the shared cost of passenger $\pi(k)$ at time t under submit order π is

$$cost^t_{\pi(k)} := totalcost^t_\pi \frac{\alpha_{\pi(k)}}{\sum_{j=1}^t \alpha_{\pi(j)}}.$$

Instead of distributing the total (operating) cost among all passengers, one could also distribute the operating cost of each shuttle among all passengers serviced by that shuttle, which results in identical properties for the DRT example in Section II-B since there is only one shuttle in the DRT example.

Table II shows the shared costs for the DRT example. For example, the total cost at time 3 is 120. It is distributed among all passengers who submitted their ride requests by time 3, namely Passengers P_1, P_2 and P_3, proportionally to their alpha values, namely 2, 2 and 4, respectively. Consequently, the shared cost of Passenger P_3 at time 3 and thus the fare quoted to Passenger P_3 after its ride request submission is $cost^3_{\pi(3)} = 60$. Similarly, the total cost at time 4 is 160. It is distributed among all passengers who submitted their ride requests by time 4, namely Passengers P_1, P_2, P_3 and P_4, proportionally to their alpha values, namely 2, 2, 4 and 2, respectively. Consequently, the shared cost of Passenger P_3 at time 4 and thus its fare is $cost^4_{\pi(3)} = 64$, implying that its fare is higher than its fare quote at time 3. This is undesirable because Passenger P_3 might accept the fare quote but not the higher fare, meaning that it has to drop out shortly before receiving its ride and then needs to search for a last-minute alternative to using the DRT system, which might be pricy and is not guaranteed to exist. Thus, we suggest that a fare quote should be an upper bound on the fare (*immediate-response property*). We also suggest that the upper bound should be reasonably low since passengers might otherwise look for alternatives to using the DRT system, commit to one and then drop out unnecessarily. Obtaining reasonably low upper bounds can be difficult since the DRT system has no knowledge of future ride request submissions.

2) Incremental Cost Sharing: Another commonly used cost-sharing mechanism is *incremental cost sharing* [13], where the shared cost of each passenger is its marginal cost, which is the increase in total cost due to its ride request submission. Consequently, for all times k and t and all submit orders π with $1 \leq k \leq t$, the shared cost of passenger $\pi(k)$ at time t under submit order π is

$$cost^t_{\pi(k)} := mc_{\pi(k)}.$$

Table III (left) shows the shared costs for the DRT example in Section II-B. For example, the marginal cost of Passenger P_3 is 0. Consequently, the shared cost of Passenger P_3 from its ride request submission at time 3 on is 0, and thus both its fare quote and fare are 0 as well. In general, incremental cost sharing satisfies the immediate-response property since the marginal costs are independent of time. The fares of Passengers P_1, P_2 P_3 and P_4 are 40, 80, 0 and 40, respectively. Thus, Passenger P_3 is a free rider, which is undesirable especially in the context of the DRT example since Passenger P_3 has the highest demand, which should be reflected in the fares. Proportional cost sharing does not suffer from this problem.

Because the demands of different passengers (quantified by their alpha values) are not necessarily identical, their fares should be divided by their alpha values to compare them. The fares per alpha value of Passengers P_1 and P_3 are 20 and 0, respectively. Thus, the fare per alpha value of Passenger P_1 is larger than that of Passenger P_3 even though Passenger P_1 submits its ride request before Passenger P_3. An indirect way of providing incentives for passengers to submit their ride requests truthfully (that is, as early as possible) is to ensure that the fares per alpha value of passengers are never higher than those of passengers who submit their ride requests after them (*online-fairness property*). Incremental cost sharing does not satisfy this property, as shown above.

Table III (right) shows the shared costs for the DRT example in Section II-B if Passenger P_1 delays its ride request submission and the passengers submit their ride requests in order P_2, P_1, P_3 and P_4. Now, the shared cost of Passenger P_1 from its ride request submission at time 2 on is 0, and thus both its fare quote and fare are 0 as well. Thus, Passenger P_1 can reduce its fare from 40 to 0 by strategically delaying its ride request submission. A direct way of providing incentives for passengers to submit their ride requests truthfully is to ensure that no passenger can decrease its fare by delaying its ride request submission (*incentive-compatibility property*). Incremental cost sharing does not satisfy this property, as shown above.

3) Other Cost-Sharing Mechanisms: There has been a considerable amount of research on designing cost-sharing mechanisms in the fields of cooperative game theory and multi-agent systems, mostly focusing on an offline setting, see [14]–[16] for related work. Two cost-sharing mechanisms have received a fair amount of attention in the mechanism design literature in addition to proportional and incremental cost sharing, namely the *value mechanism* [17] and the *serial mechanism* [18]. However, these mechanisms for the offline setting encounter similar problems as proportional cost sharing in the online setting. Online cost-sharing mechanisms have been studied in [10] but without considering fairness.

Online fair division problems for cake cutting [19] and resource allocation [20] deal with agents that submit their requests one after the other, just like the online cost-sharing problem for DRT systems, but the available amount of cake and resources of those problems are independent of the requests of the agents, while the total cost of online cost-sharing problems for DRT systems depends on the ride requests that have been submitted.

D. Desirable Properties

None of the cost-sharing mechanisms discussed so far are well-suited for the DRT problem. Based on their shortcomings, we derive a list of desirable properties for online cost-sharing mechanisms. Our primary objective is to design an online cost-sharing mechanism that provides incentives for passengers to submit their ride requests truthfully while satisfying basic properties of cost-sharing mechanisms in general, such as fairness and budget balance.

- **Online Fairness:** The shared costs per alpha value of passengers are never higher than those of passengers who submit their ride requests after them, that is, for all times k_1, k_2 and t and all submit orders π with $1 \leq k_1 \leq k_2 \leq t$,
$$\frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \leq \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}}.$$
- **Immediate Response:** Passengers are provided immediately after their ride request submissions with (ideally low) upper bounds on their shared costs at any future time, that is, for all times k, t_1 and t_2 and all submit orders π with $1 \leq k \leq t_1 \leq t_2$,
$$cost^{t_1}_{\pi(k)} \geq cost^{t_2}_{\pi(k)}.$$
- **Individual Rationality:** The shared costs of passengers who accepted their fare quotes never exceed their fare limits at any future time, that is, for all times k and t and all submit orders π with $1 \leq k \leq t$,
$$cost^t_{\pi(k)} \leq w_{\pi(k)}.$$
- **Budget Balance:** The total cost equals the sum of the shared costs of all passengers, that is, for all times t and all submit orders π with $1 \leq t$,
$$\sum_{j=1}^{t} cost^t_{\pi(j)} = totalcost^t_{\pi}.$$

TABLE III
SHARED COSTS UNDER INCREMENTAL COST SHARING: $cost^t_{\pi(k)}$

	Truthful Submission				Delayed Submission			
	$k=1$	$k=2$	$k=3$	$k=4$	$k=1$	$k=2$	$k=3$	$k=4$
	$\pi(k)=P_1$	$\pi(k)=P_2$	$\pi(k)=P_3$	$\pi(k)=P_4$	$\pi(k)=P_2$	$\pi(k)=P_1$	$\pi(k)=P_3$	$\pi(k)=P_4$
$t=1$	40				120			
$t=2$	40	80			120	0		
$t=3$	40	80	0		120	0	0	
$t=4$	40	80	0	40	120	0	0	40

- **Ex-Post Incentive Compatibility:**[1] The best strategy of every passenger is to submit its ride request truthfully, provided that all other passengers do not change their submit times and whether they accept or decline their fare quotes, because it then cannot decrease its shared cost by delaying its ride request submission, that is, for all times k_1, k_2 and t and all submit orders π and π' with $1 \leq k_1 < k_2 \leq t$ and

$$\pi'(k) = \begin{cases} \pi(k+1) & \text{if } k_1 \leq k < k_2 \\ \pi(k_1) & \text{if } k = k_2 \\ \pi(k) & \text{otherwise,} \end{cases}$$

$$cost^t_{\pi(k_1)} \leq cost^t_{\pi'(k_2)}.$$

In other words, consider any time t, any submit order π and any passenger $\pi(k_1)$. Now assume that the passenger delays its ride request submission and now submits the k_2th instead of the k_1th ride request, with everything else being equal. Then, its shared cost $cost^t_{\pi'(k_2)}$ at time t under the new submit order π' should not be lower than its shared cost $cost^t_{\pi(k_1)}$ at time t under the previous submit order π.

The online fairness and ex-post incentive-compatibility properties are similar but one does not imply the other. Basically, they provide incentives for passengers to submit their ride requests truthfully. Thus, the DRT systems have more time to prepare and might also be able to offer subsequent passengers lower fares due to synergies with the early ride requests, which might allow them to service more passengers. The online-fairness property is also meant to ensure that passengers consider the fares to be fair. The immediate-response property enables DRT systems to provide fare quotes, in form of upper bounds on the fares, to passengers immediately after their ride request submissions despite missing knowledge of future ride request submissions. Thus, passengers have no uncertainty about whether they can be serviced or how high their fares are at most, while the DRT systems reduce their uncertainty about passengers dropping out. Yet, they still retain some flexibility to optimize the routes and schedules of the shuttles after future ride request submissions. The budget-balance property guarantees that the sum of the fares of all passengers always equals the total cost. Thus, no profit is made and no subsidies are required.

We stated sufficient rather than necessary conditions for the properties. For example, the immediate-response property could be weakened to state that passengers are provided immediately after their ride request submissions with (ideally low) upper bounds on their shared costs after the ride request submission of the last passenger since this implies that their fare quotes are upper bounds on their fares. Similarly, the budget-balance property could be weakened to state that the total cost equals the sum of the shared costs of all passengers after the ride request submission of the last passenger. Requiring the properties to be satisfied at any time rather than only after the ride request submission of the last passenger simplifies the development of the online cost-sharing mechanism since it does not know in advance which ride request submission is the last one.

III. Proportional Online Cost Sharing

Several online cost-sharing mechanisms might satisfy the properties listed in Section II-D. In this section, we describe a novel online cost-sharing mechanism, called Proportional Online Cost Sharing (POCS), which satisfies the properties and is thus a first step toward addressing some of the problems raised by the missing knowledge of future ride request submissions. The idea behind POCS is the following: POCS partitions passengers into coalitions, where coalitions contain all passengers who submit their ride requests within given time intervals (rather than, for example, all passengers serviced by the same shuttle). Initially, each passenger forms its own coalition when it submits its ride request. However, passengers can choose to form coalitions with passengers who submit their ride requests directly after them to decrease their shared costs per alpha value, which implies the online fairness, immediate response, and ex-post incentive-compatibility properties. For example, the immediate-response property is satisfied because passengers add other passengers to their coalitions only when this decreases their shared costs per alpha value and thus also their shared costs (since the alpha values are positive). We

[1] We would like the ex-post incentive-compatibility property ideally to state that the best strategy of every passenger is to submit its ride request truthfully because it cannot decrease its shared cost by delaying its ride request submission. However, we impose two conditions in this article that we hope to be able to relax in the future. The first condition is that all other passengers do not change their submit times, which, for example, rules out collusion of several passengers. In general, the literature on online-mechanism design [21] distinguishes two types of incentive compatibility, namely *dominant-strategy incentive compatibility* and *ex-post incentive compatibility*. Dominant-strategy incentive compatibility does not require the first condition, while ex-post incentive compatibility does. Dominant-strategy incentive compatibility is difficult to achieve in an online setting [21], which is why we impose the first condition in this article. The second condition is that the other passengers do not change whether they accept their fare quotes or drop out, even though, for example, the delayed ride request submission of a passenger could cause the fare quotes of subsequent passengers to increase, which might make them drop out. The submit orders with and without the delayed ride request submission are then difficult to relate, which is why we impose the second condition in this article.

prove in Section IV that POCS indeed satisfies all properties listed in Section II-D.

A new passenger accepting its fare quote can thus decrease the fares of earlier passengers, perhaps at the expense of detours that increase their ride time. However, their ride times cannot increase by arbitrary amounts, for the following reason: The operating cost and thus also the total cost typically increase as the ride times of passengers increase. In turn, the fare quote of the new passenger increases. If it increases up to the point where it is larger than the fare limit of the new passenger, then the new passenger declines its fare quote and the fares of all earlier passengers thus do not increase at all. POCS can also accommodate ride time limits of passengers directly as long as it can calculate the minimal operating costs (or other total costs that satisfy the assumptions of Definition 2) under these constraints sufficiently fast. However, it is future work to develop a cost-sharing mechanism that minimizes a joint objective of cost and time.

A. Calculation of Shared Costs

We first define the coalition cost per alpha value to be able to describe formally how POCS calculates the shared costs.

Definition 6. *For all times k_1, k_2 and t and all submit orders π with $k_1 \leq k_2 \leq t$, the coalition cost per alpha value of passengers $\pi(k_1), \ldots, \pi(k_2)$ at time t under submit order π is*

$$ccpa_{\pi(k_1,k_2)} := \frac{\sum_{j=k_1}^{k_2} mc_{\pi(j)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}}.$$

We now describe how POCS calculates the shared costs.

Definition 7. *For all times k and t and all submit orders π with $k \leq t$, the shared cost of passenger $\pi(k)$ at time t under submit order π is*

$$cost^t_{\pi(k)} := \alpha_{\pi(k)} \min_{k \leq j \leq t} \max_{1 \leq i \leq j} ccpa_{\pi(i,j)}.$$

B. Relationship to Other Cost-Sharing Mechanisms

We first define coalitions to be able to explain why POCS is a combination of proportional and incremental cost sharing.

Definition 8. *For all times k_1, k_2 and t and all submit orders π with $k_1 \leq k_2 \leq t$, a coalition (k_1, k_2) at time t is a group of passengers $\pi(k_1), \ldots, \pi(k_2)$ with*

$$\frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} = \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}$$

for all times k with $k_1 \leq k \leq k_2$ and

$$\frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} \neq \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}$$

for all times k with ($k = k_1 - 1$ or $k = k_2 + 1$) and $1 \leq k \leq t$.

The following lemma, whose proof is provided in Section IV, helps to understand the similarities between POCS and other cost-sharing mechanisms. It states that the shared costs per alpha value of all passengers in any coalition are always identical and equal to the coalition cost per alpha value of the coalition.

Lemma. 2 *The shared cost per alpha value of any passenger in any coalition at any time equals the coalition cost per alpha value of the coalition, that is, for all times k_1, k, k_2 and t and all submit orders π with $1 \leq k_1 \leq k \leq k_2 \leq t$ such that (k_1, k_2) is a coalition at time t,*

$$\frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} = ccpa_{\pi(k_1,k_2)}.$$

Lemma 2 implies that POCS is a combination of proportional and incremental cost sharing, for the following reasons:

- The sum of the marginal costs of all passengers in any coalition ("the total cost of all passengers in the coalition") at time t is distributed among all passengers in the coalition proportionally to their alpha values since, for all times k_1, k, k_2 and t and all submit orders π with $k_1 \leq k \leq k_2 \leq t$ such that (k_1, k_2) is a coalition at time t,

$$\begin{aligned} cost^t_{\pi(k)} &\overset{Lem.2}{=} \alpha_{\pi(k)} ccpa_{\pi(k_1,k_2)} \\ &\overset{Def.6}{=} \alpha_{\pi(k)} \frac{\sum_{j=k_1}^{k_2} mc_{\pi(j)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}} \\ &= \left(\sum_{j=k_1}^{k_2} mc_{\pi(j)} \right) \frac{\alpha_{\pi(k)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}}, \end{aligned}$$

  which is similar to proportional cost sharing, where the total cost (of all passengers) is distributed among all passengers proportionally to their alpha values.
- The sum of the shared costs of all passengers in any coalition ("the shared cost of the coalition") at time t equals the sum of the marginal costs of all passengers in the coalition ("the marginal cost of the coalition") at the same time since, for all times k_1, k_2 and t and all submit orders π with $k_1 \leq k_2 \leq t$ such that (k_1, k_2) is a coalition at time t,

$$\begin{aligned} \sum_{j=k_1}^{k_2} cost^t_{\pi(j)} &\overset{Lem.2}{=} ccpa_{\pi(k_1,k_2)} \sum_{j=k_1}^{k_2} \alpha_{\pi(j)} \\ &\overset{Def.6}{=} \frac{\sum_{j=k_1}^{k_2} mc_{\pi(j)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}} \sum_{j=k_1}^{k_2} \alpha_{\pi(j)} \\ &= \sum_{j=k_1}^{k_2} mc_{\pi(j)}, \end{aligned}$$

  which is similar to incremental cost sharing. where the shared cost of a passenger is its marginal cost. It also implies the budget-balance property since summing over all passengers in all coalitions is identical to summing over all passengers and the sum of the marginal costs of all passengers equals the total cost.

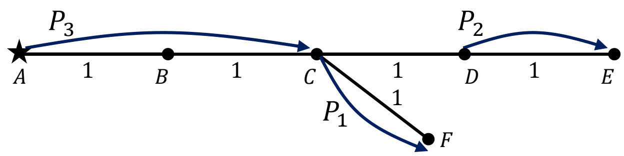


Fig. 2. DRT Example 2

TABLE V
SHARED COSTS UNDER POCS: $cost^t_{\pi(k)}$

	$k=1$ $\pi(k)=P_1$	$k=2$ $\pi(k)=P_2$	$k=3$ $\pi(k)=P_3$	$k=4$ $\pi(k)=P_4$
$t=1$	40			
$t=2$	40	80		
$t=3$	30	30	60	
$t=4$	30	30	60	40

C. Illustration

Table IV shows the coalition costs per alpha value for the DRT example in Section II-B. For example, the coalition cost per alpha value of Passengers P_1, P_2 and P_3 (at all times) is $ccpa_{\pi(1,3)} = \frac{\sum_{j=1}^{3} mc_{\pi(j)}}{\sum_{j=1}^{3} \alpha_{\pi(j)}} = \frac{40+80+0}{2+2+4} = 15$. The coalition costs per alpha value are used to calculate the shared costs, shown in Table V. The shared costs, in turn, are used to calculate the shared costs per alpha value, shown in Table VI, by dividing the shared costs by the alpha values, shown in Table I. For example, at time 4, Passengers P_1, P_2 and P_3 form a coalition (since their shared costs per alpha value are equal), and Passenger P_4 forms a coalition by itself. The sum of the marginal costs of the three passengers in the first coalition ("the total cost of all passengers in the coalition") is 120 and is distributed among all passengers in the coalition proportionally to their alpha values, namely 2, 2 and 4, respectively. Consequently, the shared cost of Passenger P_3 at time 4 and thus its fare is $cost^4_{\pi(3)} = 60$. Table VI shows that the shared costs per alpha value in each row are monotonically non-decreasing from left to right, corresponding to the online-fairness property. Table V shows that the shared costs in each column are monotonically non-increasing from top to bottom (and consequently Table VI shows that the shared costs per alpha value have the same property), corresponding to the immediate-response property. Table V also shows that the sum of the shared costs in each row equals the total cost at the corresponding time, corresponding to the budget-balance property.

D. Ex-Post Incentive Compatibility

We use the DRT example in Figure 2 to illustrate that POCS does not satisfy the ex-post incentive-compatibility property if the second condition (namely that the other passengers do not change whether they accept their fare quotes or drop out) is removed. There is one shuttle that can transport up to four passengers and starts at the star. The shuttle incurs an operating cost of 10 for each unit of distance traveled and needs to return to its initial location. There are three passengers. Passengers P_1 and P_3 accept all fare quotes, while Passenger P_2 accepts all fare quotes up to 60. Assume that the passengers submit their ride requests in order P_1, P_2 and P_3. First, Passenger P_1 submits its ride request, receives a fare quote of 60 and accepts it. Second, Passenger P_2 submits its ride request, receives a fare quote of 50 and accepts it. Third, Passenger P_3 submits its ride request, receives a fare quote of 50 and accepts it. In the end, Passengers P_1, P_2 and P_3 are serviced with fares of 25, 25 and 50, respectively. Now assume that Passenger P_1 delays its ride request submission, and the passengers submit their ride requests in order P_2, P_3 and P_1. First, Passenger P_2 submits its ride request, receives a fare quote of 80 and drops out since the fare quote exceeds its fare limit of 60. Second, Passenger P_3 submits its ride request, receives a fare quote of 40 and accepts it. Third, Passenger P_1 submits its ride request, receives a fare quote of 20 and accepts it. In the end, Passengers P_1 and P_3 are serviced with fares of 20 and 40, respectively. Thus, Passenger P_1 managed to decrease both its fare quote and fare by delaying its ride request submission since this caused Passenger P_2 to drop out.

TABLE VI
SHARED COSTS PER ALPHA VALUE UNDER POCS: $cost^t_{\pi(k)}/\alpha_{\pi(k)}$

	$k=1$ $\pi(k)=P_1$	$k=2$ $\pi(k)=P_2$	$k=3$ $\pi(k)=P_3$	$k=4$ $\pi(k)=P_4$
$t=1$	20			
$t=2$	20	40		
$t=3$	15	15	15	
$t=4$	15	15	15	20

IV. ANALYSIS OF PROPERTIES

In this section, we prove that POCS satisfies all properties listed in Section II-D, making use of the following corollary to Definition 7.

Corollary 1. *For all times k and t and all submit orders π with $1 \leq k \leq t$,*

$$\frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} = \min_{k \leq j \leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}}.$$

Proof. Consider any times k and t and any submit order π with $1 \leq k \leq t$. Then,

$$\begin{aligned} \frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} &\overset{Def.7}{=} \min_{k \leq j \leq t} \max_{1 \leq i \leq j} ccpa_{\pi(i,j)} \\ &= \min_{k \leq j \leq t} \min_{j \leq j' \leq j} \max_{1 \leq i \leq j'} ccpa_{\pi(i,j')} \\ &\overset{Def.7}{=} \min_{k \leq j \leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}}, \end{aligned}$$

which proves the corollary. □

A. Online Fairness

In this section, we prove that POCS satisfies the online-fairness property, namely that the shared costs per alpha value of passengers are never higher than those of passengers who submit their ride requests after them.

TABLE IV
COALITION COSTS PER ALPHA VALUE UNDER POCS: $ccpa_{\pi(k_1,k_2)}$

		$k_2 = 1$ $\pi(k_2) = P_1$	$k_2 = 2$ $\pi(k_2) = P_2$	$k_2 = 3$ $\pi(k_2) = P_3$	$k_2 = 4$ $\pi(k_2) = P_4$
$k_1 = 1$	$\pi(k_1) = P_1$	20	30	15	16
$k_1 = 2$	$\pi(k_1) = P_2$		40	13 1/3	15
$k_1 = 3$	$\pi(k_1) = P_3$			0	6 2/3
$k_1 = 4$	$\pi(k_1) = P_4$				20

Theorem 1. *POCS satisfies the online-fairness property, that is, for all times k_1, k_2 and t and all submit orders π with $1 \leq k_1 \leq k_2 \leq t$,*

$$\frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \leq \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}}.$$

Proof. Consider any times k_1, k_2 and t and any submit order π with $1 \leq k_1 \leq k_2 \leq t$. Then,

$$\frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \overset{Cor.1}{=} \min_{k_1 \leq j \leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{k_1 \leq k_2}{\leq} \min_{k_2 \leq j \leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{Cor.1}{=} \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}},$$

which proves the theorem. □

B. Immediate Response

In this section, we prove that POCS satisfies the immediate-response property, namely that passengers are provided immediately after their ride request submissions with upper bounds on their shared costs at any future time because they are provided with their shared costs immediately after their ride request submissions and their shared costs are monotonically non-increasing over time.

Theorem 2. *POCS satisfies the immediate-response property, that is, for all times k, t_1 and t_2 and all submit orders π with $1 \leq k \leq t_1 \leq t_2$,*

$$cost^{t_1}_{\pi(k)} \geq cost^{t_2}_{\pi(k)}.$$

Proof. Consider any times k, t_1 and t_2 and any submit order π with $1 \leq k \leq t_1 \leq t_2$. Then,

$$\begin{aligned} cost^{t_1}_{\pi(k)} &\overset{Cor.1}{=} \alpha_{\pi(k)} \min_{k \leq j \leq t_1} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \\ &\overset{t_1 \leq t_2}{\geq} \alpha_{\pi(k)} \min_{k \leq j \leq t_2} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \\ &\overset{Cor.1}{=} cost^{t_2}_{\pi(k)}, \end{aligned}$$

which proves the theorem. □

C. Individual Rationality

In this section, we prove that POCS satisfies the individual-rationality property, namely that the shared costs of passengers who accept their fare quotes never exceed their fare limits at any future time.

Theorem 3. *POCS satisfies the individual-rationality property, that is, for all times k and t and all submit orders π with $1 \leq k \leq t$,*

$$cost^t_{\pi(k)} \leq w_{\pi(k)}.$$

Proof. Consider any times k and t and any submit order π with $1 \leq k \leq t$. It holds that

$$cost^k_{\pi(k)} \leq w_{\pi(k)} \tag{1}$$

since passenger $\pi(k)$ accepted its fare quote $cost^k_{\pi(k)}$ at time k, which implies that its fare limit $w_{\pi(k)}$ is no lower than its fare quote. Then,

$$cost^t_{\pi(k)} \overset{Thm.2}{\leq} cost^k_{\pi(k)} \overset{Eq.1}{\leq} w_{\pi(k)},$$

which proves the theorem. □

D. Budget Balance

In this section, we prove that POCS satisfies the budget-balance property, namely that the total cost equals the sum of the shared costs of all passengers. We first prove some properties of coalitions and then that the sum of the marginal costs of all passengers in any coalition ("the total cost of all passengers in the coalition") equals the sum of the shared costs of all passengers in it.

Lemma 1. *The shared cost of the last passenger in any coalition at any time equals its shared cost immediately after its ride request submission, that is, for all times k_1, k_2 and t and all submit orders π with $1 \leq k_1 \leq k_2 \leq t$ such that (k_1, k_2) is a coalition at time t,*

$$cost^t_{\pi(k_2)} = cost^{k_2}_{\pi(k_2)}.$$

Proof. Consider any times k_1, k_2 and t and any submit order π with $1 \leq k_1 \leq k_2 \leq t$ such that (k_1, k_2) is a coalition at time t. At time $t = k_2$, the lemma trivially holds. At time $t > k_2$,

$$\frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \overset{Th.1}{<} \frac{cost^t_{\pi(k_2+1)}}{\alpha_{\pi(k_2+1)}} \tag{2}$$

since passengers $\pi(k_2)$ and $\pi(k_2+1)$ are in different coalitions at time t and thus do not have the same shared cost per alpha value at time t according to Definition 8. Thus,[2]

$$\min_{k_2\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{Cor.1}{=} \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \overset{Eq.2}{<} \frac{cost^t_{\pi(k_2+1)}}{\alpha_{\pi(k_2+1)}} \overset{Cor.1}{=} \min_{k_2+1\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \quad (3)$$

$$cost^t_{\pi(k_2)} \overset{Cor.1}{=} \alpha_{\pi(k_2)}\min_{k_2\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{Eq.3*}{=} \alpha_{\pi(k_2)}\frac{cost^{k_2}_{\pi(k_2)}}{\alpha_{\pi(k_2)}} = cost^{k_2}_{\pi(k_2)},$$

which proves the lemma. □

Lemma 2. *The shared cost per alpha value of any passenger in any coalition at any time equals the coalition cost per alpha value of the coalition, that is, for all times k_1, k, k_2 and t and all submit orders π with $1 \le k_1 \le k \le k_2 \le t$ such that (k_1, k_2) is a coalition at time t,*

$$\frac{cost^t_{\pi(k)}}{\alpha_{\pi(k)}} = ccpa_{\pi(k_1,k_2)}.$$

Proof. Consider any times k_1, k, k_2 and t and any submit order π with $1 \le k_1 \le k \le k_2 \le t$ such that (k_1, k_2) is a coalition at time t. We prove the lemma for $k = k_2$. It then also holds for all k with $k_1 \le k \le k_2$ since all passengers in the same coalition at time t have the same shared cost per alpha value at time t according to Definition 8. We prove the lemma for $k = k_2$ by contradiction by assuming that

$$\frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \ne ccpa_{\pi(k_1,k_2)}. \quad (4)$$

Then,

$$ccpa_{\pi(k_1,k_2)} \overset{Eq.4}{\ne} \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \overset{Lem.1}{=} \frac{cost^{k_2}_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \overset{Def.7}{=} \max_{1\le j\le k_2} ccpa_{\pi(j,k_2)} \quad (5)$$

$$ccpa_{\pi(k_1,k_2)} \overset{Eq.5,1\le k_1\le k_2}{<} \max_{1\le j\le k_2} ccpa_{\pi(j,k_2)}. \quad (6)$$

Thus, there exists a time k' with $1 \le k' < k_2$ such that

$$ccpa_{\pi(k_1,k_2)} \overset{Eq.6}{<} \max_{1\le j\le k_2} ccpa_{\pi(j,k_2)} = ccpa_{\pi(k',k_2)}. \quad (7)$$

Assume without loss of generality that k' is the earliest such time. Then, for all times k'' with $1 \le k'' < k'$,

$$ccpa_{\pi(k'',k_2)} < ccpa_{\pi(k',k_2)}. \quad (8)$$

We distinguish the following cases to prove that such a k' does not exist. The cases are exhaustive since $1 \le k' < k_2$ and $1 \le k_1 \le k_2$:

- **Case $1 \le k' < k_1$:**
  Let $A := \sum_{j=k'}^{k_1-1} mc_{\pi(j)}$, $B := \sum_{j=k_1}^{k_2} mc_{\pi(j)}$, $C := \sum_{j=k'}^{k_1-1} \alpha_{\pi(j)}$ and $D := \sum_{j=k_1}^{k_2} \alpha_{\pi(j)}$. Then,

$$\tfrac{B}{D} \overset{Def.6}{=} ccpa_{\pi(k_1,k_2)} \overset{Eqs.7}{<} ccpa_{\pi(k',k_2)} \overset{Def.6}{=} \tfrac{A+B}{C+D} \quad (9)$$

$$BC \overset{Eq.9,C>0,D>0}{<} AD \quad (10)$$

$$ccpa_{\pi(k',k_2)} \overset{Def.6}{=} \tfrac{A+B}{C+D} \overset{Eq.10}{<} \tfrac{A}{C} \overset{Def.6}{=} ccpa_{\pi(k',k_1-1)}. \quad (11)$$

  Separately,

$$\frac{cost^t_{\pi(k_1-1)}}{\alpha_{\pi(k_1-1)}} \overset{Th.1}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \quad (12)$$

  since passengers $\pi(k_1 - 1)$ and $\pi(k_1)$ are in different coalitions at time t and thus do not have the same shared cost per alpha value at time t according to Definition 8. Thus,

$$\min_{k_1-1\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{Cor.1}{=} \frac{cost^t_{\pi(k_1-1)}}{\alpha_{\pi(k_1-1)}} \overset{Eq.12}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \overset{Cor.1}{=} \min_{k_1\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \quad (13)$$

  and[3]

$$\begin{aligned}\frac{cost^{k_1-1}_{\pi(k_1-1)}}{\alpha_{\pi(k_1-1)}} &\overset{Eq.13*}{<} \min_{k_1\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{k_1\le k_2}{\le} \min_{k_2\le j\le t}\frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \overset{Cor.1}{=} \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \overset{Lem.1}{=} \frac{cost^{k_2}_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \\ &\overset{Def.7}{=} \max_{1\le j\le k_2} ccpa_{\pi(j,k_2)} \overset{Eq.7}{=} ccpa_{\pi(k',k_2)} \overset{Eq.11}{<} ccpa_{\pi(k',k_1-1)} \\ &\overset{1\le k'\le k_1-1}{\le} \max_{1\le j\le k_1-1} ccpa_{\pi(j,k_1-1)} \overset{Def.7}{=} \frac{cost^{k_1-1}_{\pi(k_1-1)}}{\alpha_{\pi(k_1-1)}},\end{aligned}$$

  which is a contradiction since a value cannot be lower than itself.
- **Case $k' = k_1$:**
  It holds that

$$ccpa_{\pi(k',k_2)} \overset{k'=k_1}{=} ccpa_{\pi(k_1,k_2)} \overset{Eq.6}{<} \max_{1\le j\le k_2} ccpa_{\pi(j,k_2)} \overset{Eq.7}{=} ccpa_{\pi(k',k_2)},$$

[2] At the position marked with an asterisk, we use that $\min_{k_2\le j\le t} x_j < \min_{k_2+1\le j\le t} x_j$ implies $\min_{k_2\le j\le t} x_j = x_{k_2}$.

[3] At the position marked with an asterisk, we use that $\min_{k_1-1\le j\le t} x_j < \min_{k_1\le j\le t} x_j$ implies $x_{k_1-1} < \min_{k_1\le j\le t} x_j$.

which is a contradiction since a value cannot be lower than itself.

- **Case** $k_1 < k' < k_2$**:**
Let $A := \sum_{j=k''}^{k'-1} mc_{\pi(j)}$, $B := \sum_{j=k'}^{k_2} mc_{\pi(j)}$, $C := \sum_{j=k''}^{k'-1} \alpha_{\pi(j)}$ and $D := \sum_{j=k'}^{k_2} \alpha_{\pi(j)}$. Then, for all times k'' with $1 \le k'' < k'$,

$$\frac{A+B}{C+D} \overset{Def.6}{=} ccpa_{\pi(k'',k_2)} \overset{Eq.8}{<} ccpa_{\pi(k',k_2)} \overset{Def.6}{=} \frac{B}{D} \quad (14)$$

$$AD \overset{Eq.14, C>0, D>0}{<} BC \quad (15)$$

$$ccpa_{\pi(k'',k'-1)} \overset{Def.6}{=} \frac{A}{C} \overset{Eq.15}{<} \frac{B}{D} \overset{Def.6}{=} ccpa_{\pi(k',k_2)}. \quad (16)$$

Thus,[4]

$$\begin{aligned} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} &\overset{Cor.1}{=} \min_{k_1 \le j \le t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}} \\ &\overset{k_1 \le k'-1 < t}{\le} \frac{cost^{k'-1}_{\pi(k'-1)}}{\alpha_{\pi(k'-1)}} \\ &\overset{Def.7}{=} \max_{1 \le j \le k'-1} ccpa_{\pi(j,k'-1)} \\ &\overset{Eq.16*}{<} ccpa_{\pi(k',k_2)} \\ &\overset{Eq.7}{=} \max_{1 \le j \le k_2} ccpa_{\pi}(j,k_2) \\ &\overset{Def.7}{=} \frac{cost^{k_2}_{\pi(k_2)}}{\alpha_{\pi(k_2)}} \\ &\overset{Lem.1}{=} \frac{cost^t_{\pi(k_2)}}{\alpha_{\pi(k_2)}}, \end{aligned}$$

which is a contradiction since passengers $\pi(k_1)$ and $\pi(k_2)$ are in the same coalition at time t and thus have the same shared cost per alpha value at time t according to Definition 8.

□

Theorem 4. *POCS satisfies the budget-balance property, that is, for all times t and all submit orders π with $1 \le t$,*

$$\sum_{j=1}^{t} cost^t_{\pi(j)} = totalcost^t_\pi.$$

Proof. Consider any times k_1, k_2 and t and any submit order π with $1 \le k_1 \le k_2 \le t$ such that (k_1, k_2) is a coalition at time t. Then,

$$\begin{aligned} \sum_{j=k_1}^{k_2} cost^t_{\pi(j)} &\overset{Lem.2}{=} ccpa_{\pi(k_1,k_2)} \sum_{j=k_1}^{k_2} \alpha_{\pi(j)} \\ &\overset{Def.6}{=} \frac{\sum_{j=k_1}^{k_2} mc_{\pi(j)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}} \sum_{j=k_1}^{k_2} \alpha_{\pi(j)} \\ &= \sum_{j=k_1}^{k_2} mc_{\pi(j)}. \end{aligned} \quad (17)$$

Summing over all passengers in all coalitions is identical to summing over all passengers. Thus,

$$\begin{aligned} \sum_{j=1}^{t} cost^t_{\pi(j)} &\overset{Eq.17}{=} \sum_{j=1}^{t} mc_{\pi(j)} \\ &\overset{Def.3}{=} \sum_{j=1}^{t} \left(totalcost^t_\pi - totalcost^{t-1}_\pi\right) \\ &= totalcost^t_\pi, \end{aligned}$$

which proves the theorem. □

E. Ex-Post Incentive Compatibility

In this section, we prove that POCS satisfies the ex-post incentive-compatibility property, namely that the best strategy for every passenger is to submit its ride request truthfully, provided that all other passengers do not change their submit times and whether they accept or decline their fare quotes, because the passenger then cannot decrease its shared cost by delaying its ride request submission.

Theorem 5. *POCS satisfies the ex-post incentive-compatibility property, that is, for all times k_1, k_2 and t and all submit orders π and π' with $1 \le k_1 < k_2 \le t$ and*

$$\pi'(k) = \begin{cases} \pi(k+1) & \text{if } k_1 \le k < k_2 \\ \pi(k_1) & \text{if } k = k_2 \\ \pi(k) & \textit{otherwise,} \end{cases} \quad (18)$$

$$cost^t_{\pi(k_1)} \le cost^t_{\pi'(k_2)}.$$

Proof. Consider any times k_1, k_2 and t and any submit orders π and π' with $1 \le k_1 < k_2 \le t$ and $\pi'(k)$ as given in Equation 18. π is the submit order where passenger $\pi(k_1)$ submits its ride request truthfully and the passengers submit their ride requests in order $\pi(1), \pi(2), \ldots, \pi(k_1-1), \pi(k_1), \pi(k_1+1), \ldots, \pi(k_2-1), \pi(k_2), \pi(k_2+1), \ldots, \pi(t)$, while π' is the submit order where passenger $\pi(k_1)$ delays its ride request submission and the passengers submit their ride requests in order $\pi(1), \pi(2), \ldots, \pi(k_1-1), \pi(k_1+1), \ldots, \pi(k_2-1), \pi(k_2), \pi(k_1), \pi(k_2+1), \ldots, \pi(t)$. We prove the theorem by contradiction by assuming that

$$cost^t_{\pi'(k_2)} < cost^t_{\pi(k_1)}. \quad (19)$$

Then,

$$\frac{cost^t_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}} \overset{Eq.19}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}, \quad (20)$$

since the alpha values are independent of the submit order according to Definition 1 and thus $\alpha_{\pi'(k_2)} = \alpha_{\pi(k_1)}$. Assume without loss of generality that t is the earliest such time. Thus, if $k_2 < t$,

$$\frac{cost^{t-1}_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}} \ge \frac{cost^{t-1}_{\pi(k_1)}}{\alpha_{\pi(k_1)}}. \quad (21)$$

[4]At the position marked with an asterisk, we use that, for all k'' with $1 \le k'' < k'$, $x_{k''} < y$ implies $\max_{1 \le k'' < k'} x_{k''} < y$.

Separately,

$$\sum_{j=1}^{t} cost^t_{\pi'(j)} \overset{Th.4}{=} totalcost^t_{\pi'} = totalcost^t_{\pi} \overset{Th.4}{=} \sum_{j=1}^{t} cost^t_{\pi(j)} \quad (22)$$

due to the budget-balance property since the total cost at time t is independent of the submit order according to Definition 2. Equations 21 and 22 together imply that there exists a passenger $\pi(m)$ with $1 \leq m \leq t$ and $m \neq k_1$ such that the shared cost of passenger $\pi(m)$ at time t is higher under submit order π' than submit order π because the shared cost of passenger $\pi(k_1)$ at time t is lower under submit order π' than submit order π (*existence property*). We distinguish the following cases to prove that such a passenger does not exist due to the online-fairness and immediate-response properties. The cases are exhaustive since $1 \leq m \leq t$, $m \neq k_1$ and $1 \leq k_1 < k_2 \leq t$:

- **Case** $1 \leq m < k_1$**:**
  In this case, $\pi'(m) \overset{Eq.18}{=} \pi(m)$. We distinguish the following subcases. They are exhaustive since

$$\frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}} \overset{Th.1}{\leq} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}.$$

  – **Sub-Case** $\frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}} < \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}$**:**
  It holds that

$$\begin{aligned}\frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}} &\overset{Cor.1}{=} \min_{m\leq j\leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}}\\ &= \min\left(\min_{m\leq j\leq k_1-1} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}}, \min_{k_1\leq j\leq t} \frac{cost^j_{\pi(j)}}{\alpha_{\pi(j)}}\right)\\ &\overset{Cor.1}{=} \min\left(\frac{cost^{k_1-1}_{\pi(m)}}{\alpha_{\pi(m)}}, \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}\right). \end{aligned} \quad (23)$$

  Thus,[5]

$$\frac{cost^{k-1}_{\pi(m)}}{\alpha_{\pi(m)}} \overset{Eq.23*,subcase-assumption}{=} \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}}. \quad (24)$$

  Put together,

$$\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} \overset{Th.2}{\leq} \frac{cost^{k_1-1}_{\pi'(m)}}{\alpha_{\pi'(m)}} \overset{\forall_{1\leq k<k_1}\pi(k)=\pi'(k)}{=} \frac{cost^{k_1-1}_{\pi(m)}}{\alpha_{\pi(m)}} \overset{Eq.24}{=} \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}},$$

  which contradicts the existence property, namely that there exists a passenger $\pi(m) = \pi'(m)$ with

$$\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} > \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}}.$$

  – **Sub-Case** $\frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}} = \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}$**:**
  It holds that

$$\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} \overset{Th.1}{\leq} \frac{cost^t_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}} \overset{Eq.20}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \overset{subcase-assumption}{=} \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}},$$

  which contradicts the existence property, namely that there exists a passenger $\pi(m) = \pi'(m)$ with

$$\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} > \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}}.$$

- **Case** $k_1 < m \leq k_2$**:**
  In this case, $\pi'(m-1) \overset{Eq.18}{=} \pi(m)$. It holds that

$$\frac{cost^t_{\pi'(m-1)}}{\alpha_{\pi'(m-1)}} \overset{Th.1}{\leq} \frac{cost^t_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}} \overset{Eq.20}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}} \overset{Th.1}{\leq} \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}},$$

  which contradicts the existence property, namely that there exists a passenger $\pi(m) = \pi'(m-1)$ with

$$\frac{cost^t_{\pi'(m-1)}}{\alpha_{\pi'(m-1)}} > \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}}.$$

- **Case** $k_2 < m \leq t$**:**
  In this case, $\pi'(m) \overset{Eq.18}{=} \pi(m)$. It holds that

$$\begin{aligned}\min_{k_2\leq j\leq t} \frac{cost^j_{\pi'(j)}}{\alpha_{\pi'(j)}} &\overset{Cor.1}{=} \frac{cost^t_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}}\\ &\overset{Eq.20}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}\\ &\overset{Th.2}{\leq} \frac{cost^{t-1}_{\pi(k_1)}}{\alpha_{\pi(k_1)}}\\ &\overset{Eq.21}{\leq} \frac{cost^{t-1}_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}}\\ &\overset{Cor.1}{=} \min_{k_2\leq j\leq t-1} \frac{cost^j_{\pi'(j)}}{\alpha_{\pi'(j)}} \end{aligned} \quad (25)$$

  and thus[6]

$$\frac{cost^t_{\pi'(t)}}{\alpha_{\pi'(t)}} \overset{Eq.25*}{<} \min_{k_2\leq j\leq t-1} \frac{cost^j_{\pi'(j)}}{\alpha_{\pi'(j)}}. \quad (26)$$

[5]At the position marked with an asterisk, we use that $x < z$ and $x = \min(y, z)$ implies $x = y$.

[6]At the position marked with an asterisk, we use that $\min_{k_2\leq j\leq t} x_j < \min_{k_2\leq j\leq t-1} x_j$ implies $x_t < \min_{k_2\leq j\leq t-1} x_j$.

Therefore,[7]

$$\begin{aligned}\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} &\overset{Th.1}{\leq} \frac{cost^t_{\pi'(t)}}{\alpha_{\pi'(t)}}\\ &\overset{Eq.26*}{=} \min\left(\min_{k_2\leq j\leq t-1}\frac{cost^j_{\pi'(j)}}{\alpha_{\pi'(j)}}, \frac{cost^t_{\pi'(t)}}{\alpha_{\pi'(t)}}\right)\\ &= \min_{k_2\leq j\leq t}\frac{cost^j_{\pi'(j)}}{\alpha_{\pi'(j)}}\\ &\overset{Cor.1}{=} \frac{cost^t_{\pi'(k_2)}}{\alpha_{\pi'(k_2)}}\\ &\overset{Eq.20}{<} \frac{cost^t_{\pi(k_1)}}{\alpha_{\pi(k_1)}}\\ &\overset{Th.1}{\leq} \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}},\end{aligned}$$

which contradicts the existence property, namely that there exists a passenger $\pi(m) = \pi'(m)$ with

$$\frac{cost^t_{\pi'(m)}}{\alpha_{\pi'(m)}} > \frac{cost^t_{\pi(m)}}{\alpha_{\pi(m)}}.$$

□

V. Experimental Analysis

We have proved that POCS satisfies five properties that make DRT systems more attractive to both the transport providers and the passengers, provided that our assumptions are satisfied. For example, Definition 2 assumes that the total cost satisfies two properties that hold for the minimal operating cost, which is therefore the quantity that we have used so far for the total cost. Calculating the minimal operating cost is typically an NP-hard problem and thus time-consuming. However, DRT systems need to calculate the minimal operating cost after each ride request submission, which would prevent them from operating in real-time. We thus present an experimental study with a transport simulation where the DRT system uses a heuristic to compute a low operating cost that is not guaranteed to be minimal [22]. In this case, the assumption in Definition 2 that the total cost is independent of the submit order (which implies that the decisions of passengers to accept their fare quotes or drop out and thus also their fare quotes themselves are independent of the submit order) is not satisfied. This assumption is used (only) to prove that POCS satisfies the ex-post incentive-compatibility property, namely in Equation 22 of the proof of Theorem 5. We thus investigate whether the best strategy of every passenger remains to submit its ride request truthfully, for example because the likelihood of transport capacity still being available tends to decrease over time.

A. Transport Simulator

Our transport simulator first generates a given number of shuttles and passengers. Each shuttle is characterized by its capacity, start location, end location, operating time window and operating cost for each unit of distance traveled. Each passenger is characterized by its truthful submit time, start location, end location, pick-up time window, drop-off time window and fare limit. The settings of our simulator are slightly more general than what we have used in the DRT examples because operating time windows of shuttles and pick-up and drop-off time windows of passengers are taken into account. The transport simulator then simulates each passenger. Once a passenger is assigned to a shuttle, it is never re-assigned to a different shuttle, which makes it possible to calculate the marginal cost of a passenger as the lowest operating cost increase of adding the passenger to any shuttle, but is also a reason why the total cost (which equals the sum of the operating costs of all shuttles) is not guaranteed to be equal to the minimal operating cost or to be independent of the submit order of the passengers. When a new passenger submits its ride request, the transport simulator requests from each shuttle the operating cost increase from adding the passenger to all passengers previously assigned to it, selects a shuttle with the lowest operating cost increase and then uses POCS to calculate a fare quote for the passenger under the assumption that the passenger is assigned to the selected shuttle. If the fare limit of the passenger is lower than this fare quote, then the passenger drops out and the transport simulator does not service it. Otherwise, the passenger accepts the fare quote, and the transport simulator adds it to all passengers previously assigned to the selected shuttle and then updates the shared costs of all passengers assigned to the shuttles.

Each shuttle has to calculate its route, schedule and operating cost increase (or, equivalently, operating cost) when adding a new passenger to all passengers previously assigned to it. The shuttle maintains an itinerary for all passengers assigned to it - in the form of a sequence of locations, namely its start location, its end location and the start and end locations of all passengers assigned to it. It calculates its travel distance as the shortest travel distance needed to visit all locations in the order given in its itinerary, and it calculates its operating cost as the product of its travel distance and its operating cost for each unit of distance traveled. Determining an itinerary for the new passenger and all passengers previously assigned to it that minimizes its operating cost is time-consuming. The shuttle therefore uses a non-optimal scheduling method [23], [24], which is another reason why the total cost is not guaranteed to equal the minimal operating cost and not guaranteed to be independent of the submit order. In the construction phase of the scheduling method, the shuttle uses a cheapest-insertion method to construct a (feasible) itinerary by inserting the start and end locations of the new passenger into the cached itinerary for the passengers previously assigned to it. In the subsequent improvement phase of the scheduling method, the shuttle uses tabu search [25]–[28], a form of hill climbing, to improve the itinerary from the construction phase.

B. Experiment 1

In Experiment 1, we evaluate the probability that passengers accept their fare quotes and, in case they do, how their fares

[7]At the position marked with an asterisk, we use that $x < y$ implies $x = \min(y, x)$.

depend on their submit times. We perform 10,000 simulations with the transport simulator in a grid city of size 11×11 (that is, with 121 locations) and report average results. There are 25 shuttles that can each transport up to 10 passengers and operate the same hours from dawn (time 101) to dusk (time 1440). These experimental conditions allow us to focus on the effect that POCS has on the shared costs of passengers. We assume that passengers submit their ride requests before dawn (the departure time of the shuttles) since the total cost is not independent of the submit order otherwise. We also assume that shuttles have sufficient time to service all passengers before dusk. The shuttles start at a depot in the center of the city. Each shuttle incurs an operating cost of 1 for each unit of distance traveled and needs to return to its initial location at dusk. There are 100 passengers who submit their ride requests truthfully one at a time (that is, their submit times range from time 1 to time 100). Thus, the passengers submit their ride requests from time 0 to time 100 and are serviced by the shuttles from time 101 to time 1440. The start location of 20 percent of the passengers is the depot. The start locations of the other passengers and the end locations of all passengers are randomly selected from all locations with uniform probability. The pick-up and drop-off time windows are identical for each passenger but might be different from passenger to passenger. Their lower bounds are dawn, and the differences between their upper and lower bounds are randomly selected from being 2.5 to 3.0 times higher than their alpha values (that is, the shortest point-to-point travel distances from their start locations to their end locations). Thus, passengers do not have tight schedules, resulting in low fare quotes. The fare limits of passengers are randomly selected from being 1.5 to 3.0 times higher than their alpha values. Thus, passengers have high fare limits. For both of these reasons, the fare quotes often do not exceed the fare limits. Many passengers therefore accept their fare quotes and are serviced.

Figure 3 shows the probability that passengers accept their fare quotes ("Matched Probabilities of Passengers") as a function of their submit times k, that is, the percentage of simulations with $cost^k_{\pi(k)} \leq w_{\pi(k)}$. The probability that passengers accept their fare quotes is around 75 percent. It decreases as their submit times increase (since their fare quotes tend to increase as their submit times increase) but only very slowly. Figure 3 also shows the fares per alpha value of all passengers who accepted their fare quotes ("Normalized Shared Costs") as a function of their submit times k, that is, $cost^{100}_{\pi(k)}$ averaged over all simulations with $cost^k_{\pi(k)} \leq w_{\pi(k)}$. The fares per alpha value of passengers increase as their submit times increase (as suggested by the online fairness property) but only very slowly. The only exception is the sharp increase for submit times close to 100 since passengers who submit their ride requests then can no longer share their costs with a high number of passengers who submit their ride requests after them. Thus, Experiment 1 demonstrates that passengers have an incentive to submit their ride requests truthfully since their fare quotes and fares tend to increase as their submit times increase. Thus, it is more likely that they accept their fare quotes and are serviced for low fares if they submit their ride requests as early as possible.

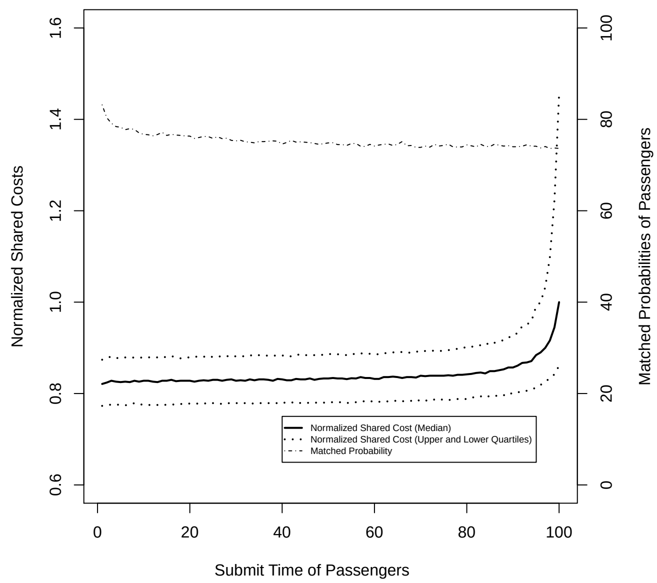


Fig. 3. Results of Experiment 1

C. Experiment 2

The definition of ex-post incentive compatibility states that the best strategy of every passenger is to submit its ride request truthfully, provided that all other passengers do not change their submit times and whether they accept or decline their fare quotes. However, these two assumptions are not guaranteed to be satisfied in practice. We have already shown in Section III-D that POCS does not satisfy the ex-post incentive-compatibility property if the second condition is removed. In Experiment 2, we therefore evaluate how likely it is that passengers can decrease their fares by delaying their ride request submissions if the second condition is removed. Experiment 2 is similar to Experiment 1, except that we distinguish four scenarios with different flexibilities of the shuttles and the passengers and use experimental parameters that decrease the scale of the experiment since each simulation is now more time-consuming. We perform 1,000 simulations with the transport simulator in a grid city of size 5×5 and report average results. Each simulation consists of at most 45 runs in addition to a run where Passengers $P_1 \dots P_{10}$ submit their ride requests truthfully in order $P_1 \dots P_{10}$ (truthful case), namely runs where all passengers submit their ride requests truthfully except that Passenger P_i delays its ride request submission and submits its ride request only immediately after Passenger P_j (delayed case) for all i and j with $1 \leq i < j \leq 10$ where Passenger P_i accepts its fare quote when all passengers submit their ride requests truthfully. There are either 2 or 10 shuttles (for two scenarios) that can each transport up to 3 passengers, operate the same hours from dawn to dusk and start at a depot in the center of the city. Each shuttle incurs an operating cost of 1 for each unit of distance traveled and needs to return to its initial location at dusk. There are 10 passengers who submit their ride requests one at a time (that is, their submit times range from time 1 to time 10) before the shuttles start to service them. The start and end locations of all passengers are randomly selected from all locations with uniform probability. The pick-up and drop-off time windows are identical for each passenger but might be different from passenger to passenger. Their lower bounds are dawn, and the differences between their upper and lower bounds are either 3.0 or 4.0 times (for two scenarios) higher than their alpha values. The fare limits of passengers are 3.0 times higher than their alpha values.

Table VII shows, for each scenario, the number of runs, (in the top row) the probabilities that passengers who delay their ride request submissions improve their situations (since their fares decrease), do not change their situations (since their fares remain unchanged) or worsen their situations (since either their fare quotes increase sufficiently for them to drop out or - in case they do not drop out - their fares increase) and (in the bottom row) the medians of their shared costs per alpha value (with their standard deviations in parentheses) both for the truthful cases (left) and delayed cases (right). Experiment 2 demonstrates that passengers have an incentive to submit their ride requests truthfully since, in all scenarios, the probability that passengers who delay their ride request submissions improve their situations is lower than 20 percent while the probability that they worsen their situations is higher than 50 percent. Also, it turns out that the shared cost per alpha value, averaged over all passengers, decreases for each scenario. (The standard deviations, averaged over all passengers, are similar in Experiments 1 and 2.) Experiment 2 does not measure one advantage of passengers who delay their ride request submissions, namely the situations where passengers originally dropped out since their fare quotes exceeded their fare limits and by delaying their ride request submissions decrease their fare quotes so much that they no longer drop out. Also, Experiment 2 assumes that passengers delay their ride request submissions randomly (rather than strategically) due to missing knowledge of future ride request submissions. The probability that the situations for passengers who delay their ride request submissions worsen is zero if passengers are able to delay their ride request submissions strategically since they can always decide to submit their ride requests truthfully instead, in which case their situations do not change. We thus expect the probability that their situations improve to increase.

We now analyze Scenario 2 in Table VII in more depth. We select 50 simulations randomly where Passenger P_1 accepts its fare quote when all passengers submit their ride requests truthfully and, for each simulation, consider the 9 runs where all passengers submit their ride requests truthfully except that passenger P_1 delays its ride request submission and submits its ride request only immediately after Passenger P_j for all j with $1 < j \leq 10$. The situation of Passenger P_1 improves in 70 of the resulting 450 runs. The reason for three of these runs is identical to the one given in Section III-D, namely that other passengers drop out when Passenger P_1 delays its ride request submission, which reduces its fare. The reason for the remaining 67 runs is that the operating cost is not independent of the submit order, both because passengers are never re-assigned to different shuttles and because the scheduling method is non-optimal, in particular because its construction phase does not find feasible itineraries even when they exist (in which case the improvement phase is ineffective). We therefore expect that smarter scheduling methods are able to reduce the number of cases where passengers improve their situations by delaying their ride request submissions.

VI. Conclusions

In this article, we determined properties of cost-sharing mechanisms that we believe make DRT systems attractive to both the shuttles and the passengers, namely online fairness, immediate response, individual rationality, budget balance and ex-post incentive compatibility. We then proposed a novel cost-sharing mechanism, called Proportional Online Cost Sharing (POCS), that provides passengers with upper bounds on their fares immediately after their ride request submissions despite missing knowledge of future ride request submissions, allowing them to accept their fare quotes or drop out. Thus, passengers have no uncertainty about whether they can be serviced or how high their fares are at most, while DRT systems reduce their uncertainty about passengers dropping out. Yet, they still retain some flexibility to optimize the routes and schedules of the shuttles after future ride request submissions. The sum of the fares of all passengers always equals the operating cost. Thus, no profit is made and no subsidies are required. POCS provides incentives for passengers to submit their ride requests truthfully (that is, as early as possible) since the fares of passengers per mile of requested travel are never higher than those of passengers who submit their ride requests after them. Thus, the DRT systems have more time to find routing solutions that can offer subsequent passengers lower fares due to synergies with the early ride requests, which might allow them to service more passengers. Another incentive for passengers to submit their ride requests truthfully is that the likelihood of transport capacity being available tends to decrease over time, which alleviates the issue that some of the properties of POCS depend on assumptions that are only approximately satisfied in practice.

Overall, POCS is a first step towards addressing some of the problems raised by the missing knowledge of future ride request submissions, which differentiates our research from previous research [29]–[32]. However, some issues remain to be addressed by more advanced online cost-sharing mechanisms, including integrating more complex models of passengers, shuttles and transport environments. Our current simplifying assumptions include, for example, that the availability of the shuttles does not change unexpectedly, that all passengers submit their ride requests before the shuttles start to service passengers, that fares depend only on the ride requests and no other considerations (for example, that DRT systems do not face competition), that all passengers evaluate their trips uniformly according to the criteria quantified by the alpha values (for example, that all passengers consider travel time to be equally important), that DRT systems provide fare quotes to passengers without predicting future ride request submissions (for example, that DRT systems service hard-to-accommodate passengers even though these passengers increase the shared costs of subsequent passengers and might make subsequent

TABLE VII
RESULTS OF EXPERIMENT 2

Scenario	Number of Shuttles	Time Window	Number of Runs	Situation Improves	No Change	Situation Worsens	
						Not Dropping Out	Dropping Out
1	2	3.0	33,116	11%	32%	24%	33%
				1.38 (0.24) → 1.20 (0.19)	1.29 (0.25) → 1.29 (0.25)	1.23 (0.22) → 1.50 (0.34)	1.38 (0.30) → -(-)
2	2	4.0	37,047	15%	31%	39%	15%
				1.31 (0.23) → 1.16 (0.19)	1.20 (0.24) → 1.20 (0.24)	1.14 (0.21) → 1.43 (0.36)	1.32 (0.29) → -(-)
3	10	3.0	36,975	16%	31%	51%	2%
				1.49 (0.24) → 1.33 (0.22)	1.37 (0.30) → 1.37 (0.30)	1.32 (0.27) → 1.71 (0.45)	1.46 (0.28) → -(-)
4	10	4.0	37,911	17%	29%	51%	3%
				1.32 (0.22) → 1.17 (0.19)	1.21 (0.25) → 1.21 (0.25)	1.18 (0.21) → 1.56 (0.46)	1.32 (0.28) → -(-)

passengers drop out), that passengers try to decrease their fares only by delaying their ride request submissions (rather than, for example, by colluding with other passengers or submitting fake ride requests under false names) and that passengers honor their commitments (for example, that passengers do not change their ride requests, cancel them, show up late or do not show up at all).

Finally, it is future work to apply POCS to problems similar to demand-responsive transport, such as taxi sharing [33] and ridesharing [34]. An application of POCS to taxi sharing is straightforward since the main difference between taxi sharing and demand-responsive transport is how the operating cost is calculated. An application of POCS to ridesharing is more complicated since it is more likely for transport providers in the context of ridesharing than demand-responsive transport (that is, drivers and shuttles, respectively) to start offering their services at arbitrary points in time during the course of a day. Also, transport providers are more likely to consider some trips unacceptable to them in the context of ridesharing than demand-responsive transport. In general, fairness is a concept that has remained understudied in the context of all these applications.

Acknowledgment

The article is based upon research supported by METRANS grant 09-19 via the California Department of Transportation and follow-up contract DTFH61-10-C-00030 from the Federal Highway Administration under the Broad Agency Announcement of Exploratory Advanced Research (EAR).

References

[1] L. Quadrifoglio, M. Dessouky, and F. Ordóñez, "A simulation study of demand responsive transit system design," *Transportation Research Part A: Policy and Practice*, vol. 42, no. 4, pp. 718–737, 2008.

[2] R. Cervero, *Paratransit in America: Redefining Mass Transportation*. Praeger Publishers, 1997.

[3] E. Ellis and B. McCollom, *Guidebook for Rural Demand-Response Transportation: Measuring, Assessing, and Improving Performance*. Transportation Research Board of the National Academies, 2009.

[4] I. Ioachim, J. Desrosiers, Y. Dumas, M. Solomon, and D. Villeneuve, "A request clustering algorithm for door-to-door handicapped transportation," *Transportation Science*, vol. 29, no. 1, pp. 63–78, 1995.

[5] M. Savelsbergh and M. Sol, "The general pickup and delivery problem," *Transportation Science*, vol. 29, no. 1, pp. 17–29, 1995.

[6] P. Toth and D. Vigo, "Heuristic algorithms for the handicapped persons transportation problem," *Transportation Science*, vol. 31, no. 1, pp. 60–71, 1997.

[7] M. Diana and M. Dessouky, "A new regret insertion heuristic for solving large-scale dial-a-ride problems with time windows," *Transportation Research Part B*, vol. 38, no. 6, pp. 539–557, 2004.

[8] J. Cordeau and G. Laporte, "The dial-a-ride problem: Models and algorithms," *Annals of Operations Research*, vol. 153, no. 1, pp. 29–46, 2007.

[9] G. Berbeglia, J. Cordeau, and G. Laporte, "Dynamic pickup and delivery problems," *European Journal of Operational Research*, vol. 202, no. 1, pp. 8–15, 2010.

[10] J. Brenner and G. Schäfer, "Online cooperative cost sharing," in *Algorithms and Complexity*, T. Calamoneri and J. Diaz, Eds. Springer, 2010, vol. 6078, pp. 252–263.

[11] Y. Wang and D. Zhu, "Ordinal proportional cost sharing," *Journal of Mathematical Economics*, vol. 37, no. 3, pp. 215–230, 2002.

[12] Y. Sprumont, "Ordinal cost sharing," *Journal of Economic Theory*, vol. 81, no. 1, pp. 126–162, 1998.

[13] H. Moulin, "Incremental cost sharing: Characterization by coalition strategy-proofness," *Social Choice and Welfare*, vol. 16, no. 2, pp. 279–320, 1999.

[14] ——, "Axiomatic cost and surplus sharing," in *Handbook of Social Choice and Welfare*, K. Arrow, A. Sen, and K. Suzumura, Eds. Elsevier, 2002, vol. 1, ch. 6, pp. 289–357.

[15] K. Jain and M. Mahdian, "Cost sharing," in *Algorithmic Game Theory*, N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani, Eds. Cambridge University Press, 2007, ch. 15, pp. 385–410.

[16] Y. Shoham and K. Leyton-Brown, *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2009.

[17] M. Shubik, "Incentives, decentralized control, the assignment of joint costs and internal pricing," *Management Science*, vol. 8, no. 3, pp. 325–343, 1962.

[18] H. Moulin and S. Shenker, "Serial cost sharing," *Econometrica*, vol. 60, no. 5, pp. 1009–1037, 1992.

[19] T. Walsh, "Online cake cutting," in *Algorithmic Decision Theory*, R. Brafman, F. Roberts, and A. Tsoukiàs, Eds. Springer, 2011, vol. 6992 of Lecture Notes in Computer Science, pp. 292–305.

[20] I. Kash, A. Procaccia, and N. Shah, "No agent left behind: Dynamic fair division of multiple resources," in *Proceedings of the International Joint Conference on Autonomous Agents and Multi-Agent Systems*, 2013, pp. 351–358.

[21] D. Parkes, "Online mechanisms," in *Algorithmic Game Theory*, N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani, Eds. Cambridge University Press, 2007, ch. 16, pp. 411–439.

[22] H. Moulin, "Cost sharing in networks: Some open questions," *International Game Theory Review*, vol. 15, no. 2, pp. 1–10, 2013.

[23] Z. Xiang, C. Chu, and H. Chen, "A fast heuristic for solving a large-scale static dial-a-ride problem under complex constraints," *European Journal of Operational Research*, vol. 174, no. 2, pp. 1117–1139, 2006.

[24] W. Nanry and J. Barnes, "Solving the pickup and delivery problem with time windows using reactive tabu search," *Transportation Research Part B*, vol. 34, no. 2, pp. 107–121, 2000.

[25] F. Glover, E. Taillard, and D. de Werra, "A user's guide to tabu search," *Annals of Operations Research*, vol. 41, no. 1, pp. 1–28, 1993.

[26] C. Reeves, *Modern Heuristic Techniques for Combinatorial Problems*. Blackwell, 1993.

[27] M. Gendreau, "An introduction to tabu search," in *Handbook of Metaheuristics*, F. Glover and G. Kochenberger, Eds. Springer, 2003, vol. 57, pp. 37–54.

[28] M. Gendreau and J. Potvin, "Tabu search," in *Search Methodologies*, E. Burke and G. Kendall, Eds. Springer, 2005, pp. 165–186.

[29] M. Frisk, M. Göthe-Lundgren, K. Jörnsten, and M. Rönnqvist, "Cost allocation in collaborative forest transportation," *European Journal of Operational Research*, vol. 205, no. 2, pp. 448–458, 2010.

[30] F. Tohmé and T. Sandholm, "Coalition formation processes with belief

revision among bounded-rational self-interested agents," *Journal of Logic and Computation*, vol. 9, no. 6, pp. 793–815, 1999.

[31] J. Lemaire, "An application of game theory: Cost allocation," *Astin Bulletin*, vol. 14, no. 1, pp. 61–81, 1984.

[32] R. Anderson and A. Claus, "Cost allocation in transportation systems," *Southern Economic Journal*, pp. 793–803, 1976.

[33] P. d'Orey, R. Fernandes, and M. Ferreira, "Empirical evaluation of a dynamic and distributed taxi-sharing system," in *International IEEE Conference on Intelligent Transportation Systems*, 2012, pp. 140–146.

[34] M. Furuhata, M. Dessouky, F. Ordóñez, M. Brunet, X. Wang, and S. Koenig, "Ridesharing: the state-of-the-art and future directions," *Transportation Research Part B*, vol. 57, pp. 28–46, 2013.

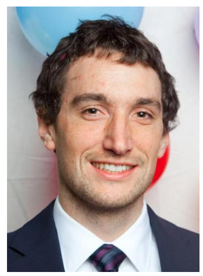
Masabumi Furuhata Masabumi Furuhata was a postdoctoral research associate at the University of Southern California and is now a section manager at DWANGO Corporation Limited. He received B.S. and M.S. degrees in Management Science from the University of Tsukuba and a Ph.D. degree in Computer Science from the University of Western Sydney and the University of Toulouse 1 Capitole. He was awarded a Japan Society for Artificial Intelligence Incentive Award. He was also awarded the first prize in the Trading Agent Competition in Mechanism Design (CAT competition) as a team member of Jackaroo. He is interested in mechanism design, multi-agent systems and data mining for transportation, robotics, finance and supply chain management.

Kenny Daniel Kenny Daniel was a graduate student at the University of Southern California and is now with Algorithmia Incorporated, which he cofounded. He received a B.S. degrees in Computer Science and Mathematics from Carnegie Mellon University and an M.S. degree from the University of Southern California.

Sven Koenig Sven Koenig is a professor in Computer Science at the University of Southern California. He received M.S. degrees from the University of California at Berkeley and Carnegie Mellon University in Computer Science and a Ph.D. degree in Computer Science from Carnegie Mellon University. He is a fellow of the Association for the Advancement of Artificial Intelligence and a distinguished speaker of the Association for Computing Machinery.

Fernando Ordóñez Fernando Ordóñez is an associate professor in Industrial Engineering at the Universidad de Chile and has an adjunct research appointment in Industrial and Systems Engineering at the University of Southern California. He received a B.S. degree in Engineering Sciences and a Mathematical Civil Engineer degree from the University of Chile (Chile) and a Ph.D. degree in Operations Research from Massachusetts Institute of Technology.

Maged M. Dessouky Maged M. Dessouky is a professor in Industrial and Systems Engineering at the University of Southern California and the director of the Epstein Institute. He received B.S. and M.S. degrees from Purdue University and a Ph.D. in Industrial Engineering from the University of California, Berkeley. He is area/associate editor of the IEEE Transactions on Intelligent Transportation Systems and the IIE Transactions and Computers and Industrial Engineering, on the editorial board of Transportation Research Part B: Methodological and Transportation Research Part E: Logistics and Transportation Review, and previously served as area editor of the ACM Transactions of Modeling and Computer Simulation. He is a fellow of IIE and was awarded the 2007 Transportation Science and Logistics Best Paper Prize.



Marc-Etienne Brunet Marc-Etienne Brunet was a visiting student at the University of Southern California. He received a B.Eng. (honours) degree in Electrical Engineering from McGill University (Canada) and is currently pursuing a graduate diploma in Social Innovation at the University of Waterloo (Canada), sponsored by the McConnell Family Foundation. He is also co-founder and president of Glencross Brunet Incorporated. He was awarded the Ernest Brown Gold Medal for Highest Ability for graduating at the top of his engineering class and the McGill Emerald Key for a significant and long-lasting impact on the sustainability of McGill's campuses.

Liron Cohen Liron Cohen is a Ph.D. candidate in Computer Science at the University of Southern California. He received a B.S. degree in Computer Engineering and an M.S. degree in Computer Science from the Hebrew University of Jerusalem (Israel). He was awarded Best Lecturer Awards from the Jerusalem College of Engineering (Israel) and the Final Engineering Best Project Award from Hebrew University of Jerusalem.

Xiaoqing Wang Xiaoqing Wang is a Ph.D. candidate in Industrial and Systems Engineering at the University of Southern California. She received a B.S. degree in Industrial Engineering from Zhejiang University (China) and M.S. degrees in Computer Science and Industrial and Systems Engineering from the University of Southern California.