Chapter 1

FUNDAMENTAL LIMITS OF NETWORKED SENSING

The Flow Optimization Framework

Bhaskar Krishnamachari
Department of Electrical Engineering-Systems
University of Southern California
Los Angeles, CA, USA 90089
bkrishna@usc.edu

Fernando Ordóñez
Department of Industrial Systems Engineering
University of Southern California
Los Angeles, CA, USA 90089
fordon@usc.edu

Abstract We describe a useful theoretical approach — the flow optimization framework — that can be used to identify the fundamental performance limits on information routing in energy-limited wireless sensor networks. We discuss the relevant recent literature, and present both linear constant-rate and non-linear adaptive rate models that optimize the tradeoff between the total information extracted (Bits) and the total energy used (Joules) for a given sensor network scenario. We also illustrate the utility of this approach through examples, and indicate possible extensions.

Keywords: Wireless Sensor Networks, Optimization, Network Flows, Fundamental Limits

Introduction

Because of the unique characteristics of wireless sensor networks (severe energy constraints, unattended operation, many-to-one flows, data-centric operation), information in these systems presents novel design challenges. Several protocols have been proposed for querying, routing and in these sensor networks, that have been primarily validated via simulations and limited experimentation. These include cluster-based and chain-based data gathering techniques [1, 2], attribute-based routing [3, 4], indexed storage and retrieval [5], database-style querying [6, 7], and active query techniques [8, 9]. Given the severe resource constraints, application-specificity, and need for robust performance in sensor networks, it is clearly crucial to complement these ongoing routing protocol development efforts by the concurrent development of a strong theoretical understanding.

There are significant challenges inherent in developing such a theory — traditional tools such as queuing theory [10] can be used to analyze throughput and delay issues, but these are of secondary importance in energy-limited sensor networks. Network information theoretic approaches involving exact analysis have traditionally resulted in limited progress [11], but more recently there has been an attempt to provide useful results by focusing on asymptotic behavior. For example, Gupta and Kumar [12] and Xie and Kumar [13] have analyzed the asymptotic capacity of multihop wireless networks; work that has been extended to consider mobility [14] as well as directional antennae [15]. The capacity of wireless sensor networks has been addressed by taking into account spatial correlations in data from nearby nodes [16, 17]. Other theoretical efforts in the area of sensor networks have been focused on understanding the complexity and optimality of data-aggregation [18, 19], and first-order mathematical modeling of specific querying and routing protocols such as ACQUIRE [9] and Directed Diffusion [20].

In this chapter, we describe a useful theoretical approach — the framework — that can be used to identify the fundamental performance limits on information routing for a specified network. In particular, we focus on a data-gathering application, consisting of n nodes with given locations and finite remaining energies, and explore how information should be routed in the network to maximize the total information extractable from the network. We first survey the recent literature on variants of flow optimization models that have been proposed for wireless networks by several authors in recent years. We then present two models for the flow optimization approach – the first is an optimization model with non-linear convex constraints permitting rate adaptation through transmit power control; the second is a simpler linear optimization model in which the rate is kept fixed on all links. We then illustrate the usefulness of such models through some numerical examples, showing how the information extracted varies with available energy, how the two models (linear and non-linear) compare, and how reception costs impact optimal routing behavior.

The key feature of flow optimization-based modeling of sensor networks is that it is a computational framework. Thus, it yields performance bounds not in the form of closed-form expressions, but rather numerically, for specified scenarios. As we show, this framework is well suited to explore the impact of different design variables such as node location, energy distribution, rate-adaptation etc. The framework is also useful as a benchmark for comparing the performance of implementable protocols on a test-suite of scenarios. The basic optimization models we present are for single-sink data gathering scenarios, and incorporate energy constraints and costs for transmission, reception and sensing, channel capacity constraints, as well as information constraints including fairness. These models assume a (TDMA/FDMA like) scheduled medium access scheme with no interference. However, as we shall discuss, these models can be extended in principle to incorporate soft interference, data aggregation, richer energy models, and to some extent even mobility.

1. Flow Optimization in Wireless Networks

Network flow optimization, which forms the foundation of the approach we describe in this chapter, is an established area of Operations Research and is described in detail in the book by Ahuja, Magnanti and Orlin [21]. In the simplest max-flow problem, a graph $G = (V, E)$ is given with a specified source node s and a sink node t. The edges of the graph have capacities C_{ij} and the objective is to determine the flows on the edges f_{ij} with capacity and flow conservation constraints that maximize the total flow from s to t. A generalization of this is the multi-commodity flow problem where the goal is to maximize the sending of several different commodities (each possibly having different sources and sinks) over a network with restricted capacity. In recent years, flow optimization has been applied by several researchers to the analysis of multi-hop wireless networks.

Toumpis and Goldsmith analyze capacity regions for general-purpose multi-hop wireless networks [22, 23]. Using a linear-programming optimization based formulation (that is equivalent to a network flow problem), the authors study the characteristics of the maximum information throughput that can be obtained in a wireless network with arbitrary topology. Non-linear constraints are considered in the optimization models for wireless networks discussed in [24, 25]. In these works the authors consider jointly optimizing the routing as well as rate-adaptive power control and bandwidth allocation. They also treat the constraints imposed by interference in their models. It should be noted however that all of these models do not consider energy constraints that are important in sensor networks.

Chang *et al.* use the flow optimization formulation to maximize the lifetime of an ad hoc network in [26]. They propose a class of flow augmentation and flow redirection algorithms that balance the energy consumption rates across nodes based on the remaining battery power of these nodes. Optimization models have also been used to study maximum lifetime conditions for sensor networks. Bhardwaj and Chandrakasan [27] develop upper bounds on the based on optimum role assignments to sensors (e.g. whether they should act as routers or aggregators). Kalpakis *et al.* examine the MLDA (Maximum Lifetime Data Aggregation) problem and the MLDR (Maximum Lifetime Data Routing) problem in [28], again formulating it using network flows. They use the solution obtained by solving the LP to construct an optimal data gathering schedule. Ordóñez and Krishnamachari have developed non-linear models (permitting rate-adaptation) for maximizing information extraction in wireless sensor networks subject to energy and fairness constraints [29, 30].

Recent research has also looked into obtaining implementable algorithms (based on flow optimization) that provide near-optimal performance. Garg and Konemann propose and present an excellent discussion of fast approximation techniques for solving the multi-commodity flow problem [31]. Chang, et al apply the Garg-Konemann algorithm to the problem of maximizing the network lifetime of an ad hoc network in [32]. Sadagopan and Krishnamachari [33] extend the Garg-Konemann to provide approximation algorithms for a maximum information extraction problem in sensor networks, and also present faster, implementable, heuristics that are also shown to be near-optimal.

We will now describe both non-linear and linear flow optimization models for wireless sensor networks, and present some illustrations of the utility of this framework.

2. Optimization Models for Wireless Sensor Networks

We now present flow optimization models that investigate the trade-off between maximum information extraction and minimum energy requirements for a given network topology. We begin by looking at non-linear models with , in which the flow rates on each link can be adapted by varying transmit powers. We then examine simpler linear models in which the rates are kept constant. In both models, we assume that there are n source nodes, and a sink numbered $n+1$, located with pairwise distances d_{ij} in a given area. In the basic models it is assumed that there is an overall energy budget of E_{tot} (Joules) to distribute among the sensor nodes (this could be easily modified to a per-node energy budget of E_{tot}/n if needed). The transmit power on link (i,j) is P_{ij} (Joules/sec), while β and C are the sensing and reception energy costs (Joules/bit). It is assumed that the relation between the flow rate f_{ij} and transmission power P_{ij} on a link is given by Shannon's capacity:

$$f_{ij} \le \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) . \tag{1.1}$$

This expression assumes that the decay factor of the medium is d_{ij}^{-2}, the communication channel has a noise power of η, and that all transmissions are scheduled (e.g. via TDMA/FDMA) such that they are non-interfering.

Non-linear Adaptive Rate Models

In this model the link rates f_{ij} and transmission powers P_{ij} are design variables, and T is total time duration of (in seconds) communication on each link. Thus $\sum_{j=1}^{n} f_{j,n+1}T$ represents the total number of bits extracted by the sink from the network. The objective of our first model is therefore to find the coordinated operation of all nodes by setting transmission powers and flow rates in order to maximize the amount of information that reaches the sink:

$$\begin{aligned}
\max \quad & \sum_{j=1}^{n} f_{jn+1}T \\
\text{s.t.} \quad & \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \ge 0 \\
& \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \le \alpha_i \sum_{j=1}^{n} f_{jn+1} \\
& \sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1})T + \sum_{i=1}^{n}\sum_{j=1}^{n} (Cf_{ij} + P_{ij})T \le E_{\text{tot}} \\
& f_{ij} \le \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) \\
& f_{ij} \ge 0, P_{ij} \ge 0
\end{aligned} \tag{1.2}$$

Since we do not consider aggregation in this simple model, the first constraint ensures that outgoing flow from a node is never less than the incoming flow. The second constraint imposes the fairness requirement that source i may not contribute more than a fraction α_i of the total flow to the sink. The third constraint imposes a network-wide energy constraint on the weighted costs of transmissions, receptions and sensing. Note that we do not explicitly constrain the communication between any pairs of nodes here, since we assume that we can make use of an ideal non-interfering schedule of transmissions; however it would be trivial to incorporate an additional constraint that limits the maximum transmit power of all nodes to minimize interference.

We simplify this problem using the arc-incidence matrix N, which for a network with $n+1$ nodes and m arcs, is a $n+1$ by m matrix with coefficients equal to 0, 1 or -1. The matrix is defined by

$$N_{i(k,l)} = \begin{cases} 1 & \text{if } i = k \\ -1 & \text{if } i = l \\ 0 & \text{otherwise} . \end{cases}$$

With this notation we can show that the problem is equivalent to:

$$\begin{aligned} \max \quad & \sum_{j=1}^{n} f_{jn+1} T \\ \text{s.t.} \quad & 0 \le Nf \le \alpha \sum_{j=1}^{n} f_{jn+1} \\ & \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} T + \eta d_{ij}^2 \left(e^{f_{ij}} - 1 \right) T = E_{\text{tot}} \\ & f \ge 0 \ , \end{aligned} \tag{1.3}$$

where we define $\kappa_j = C$ if $j = 1 : n$ and $\kappa_{n+1} = \beta$. A related dual problem to the problem above, which minimizes the energy to obtain a certain amount of information b_{out} (in bits), is

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \sum_{j=1}^{n+1} \left(\kappa_j f_{ij} + \eta d_{ij}^2 (e^{f_{ij}} - 1) \right) \\ \text{s.t.} \quad & 0 \le Nf \le \alpha \textstyle\sum_{j=1}^{n} f_{jn+1} \\ & \sum_{j=1}^{n} f_{jn+1} T = b_{\text{out}} \\ & f \ge 0 \ . \end{aligned} \tag{1.4}$$

We can show that the optimal solutions for Problems (1.3) and (1.4) are in fact related, a relationship that illustrated with the following example. In Figure 1.1 we plot both the maximal information extracted as a function of the energy bound and minimum energy needed as a function of the information bound. The experiments that originated these results considered the same WSN with all nodes placed in a straight line, the sink node at one end, 10 sensor nodes uniformly distributed from a distance 1 to 10 of the sink, and the following values for other problem parameters: $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, $T = 1$, and $\alpha_i = 0.20$ for all i. The minimum information bound was varied from $b_{\text{out}} = 1$, to $b_{\text{out}} = 20$ when solving Problem (1.4), and the maximum energy bound was varied from $E_{\text{tot}} = 0.01$ to $E_{\text{tot}} = 0.2$ when solving Problem (1.3).

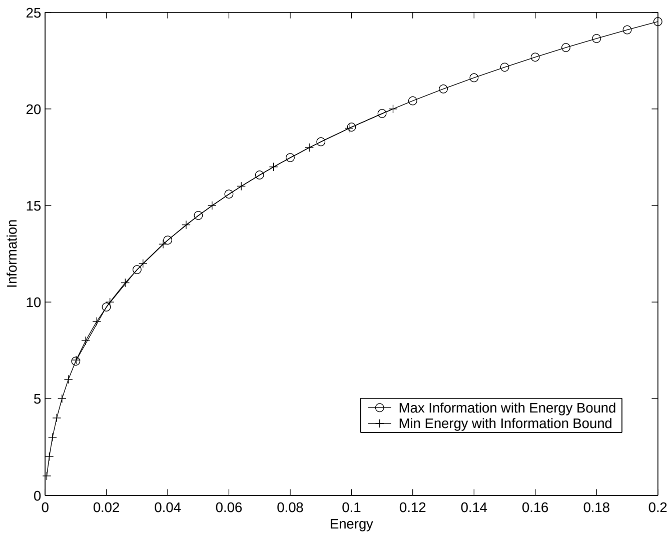

Figure 1.1. Optimal energy versus information

Linear Constant Rate Model

An alternative, computationally simpler, linear flow optimization model is obtained if we do not permit rate adaptation and assume that there is a fixed transmission rate $f_{ij} = R$ (bits/sec) for each link in the network. The transmission powers are therefore also fixed and given by $P_{ij} = \eta d_{ij}(e^R - 1)$ (in J/sec). In this model our decision variables are how many bits to send from i to j, b_{ij}. Given that the transmission rate is fixed the time taken for transmission on a given link is therefore variable and depends on the number of bits sent (recall that this was a constant T for all links in the previous model).

The corresponding problem to Problem (1.3) in which the goal is to maximize the bits extracted for a given total energy budget is

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{n} b_{jn+1} \\
\text{s.t.} \quad & 0 \leq Nb \leq \alpha \sum_{j=1}^{n} b_{jn+1} \\
& \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j b_{ij} + \frac{\eta d_{ij}^2}{R} \left(e^R - 1\right) b_{ij} = E_{\text{tot}} \\
& b \geq 0 \ .
\end{aligned} \tag{1.5}
$$

The corresponding problem to Problem (1.4) in which we minimize the energy usage subject to a given information requirement b_{out} is

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j b_{ij} + \frac{\eta d_{ij}^2}{R} \left(e^R - 1\right) b_{ij} \\
\text{s.t.} \quad & 0 \leq Nb \leq \alpha b_{\text{out}} \\
& \sum_{j=1}^{n} b_{jn+1} = b_{\text{out}} \\
& b \geq 0 \ .
\end{aligned} \tag{1.6}
$$

We will now undertake a comparison of the adaptive and constant rate models. The latter model, involving a linear program is computationally more tractable, but as one may expect, we find that the loss of a degree of freedom (rate adaptation) results in inefficiency.

3. A Comparison of the Non-linear and Linear models

We consider Problems (1.3) and (1.5). In other words we will compare how much information b_{out} (bits) can be extracted from a given sensor network according to each model, when we are given a limited budget of overall energy E_{tot} (Joules). We will compare the total information that can be extracted with the same budget of energy for each model. For the comparisons we will tune the model parameter T for the non-linear adaptive rate model, and the parameter R for the linear constant rate model. These parameters in effect tune the throughput (bits/second) with which information is extracted from the network in each model.

Simple example

As an illustration, we first consider a simple problem with two source nodes in line with the sink; one of the nodes provides all the information (i.e. $\alpha_1 = 0$ and $\alpha_2 = 1$). We assume that all the information is originating from the node furthest from the sink. In this example we compare the performance of the non-linear and linear models by studying which outputs more data for a given amount of energy E_{tot}. We assume that the sink is at $(0,0)$ and the nodes at $(1,0)$ and $(2,0)$, and that we have a sensing cost β (in J/bit), reception cost C (in J/bit) and a noise power η. It is straightforward to show that for the non-linear model the optimal flow rate f for this example satisfies

$$\begin{aligned} CfT + \beta fT + 2\eta\left(e^f - 1\right)T &= E_{\text{tot}} \\ \Rightarrow (C+\beta)f + 2\eta\left(e^f - 1\right) &= \frac{E_{\text{tot}}}{T} \end{aligned} \tag{1.7}$$

and the optimal number of bits b for the linear model in turn satisfies the relation

$$\begin{aligned} Cb + \beta b + 2\eta\left(e^R - 1\right)\frac{b}{R} &= E_{\text{tot}} \\ b &= \frac{E_{\text{tot}}R}{2\eta\left(e^R - 1\right) + R(C+\beta)} \end{aligned} \tag{1.8}$$

Note that the optimal solution in both cases depend on a tunable parameter T for the non-linear model and R for the linear model. We study now how the optimal solutions vary with these parameters.

We considered additional problem parameters as $E_{\text{tot}} = 1$, $\eta = 0.01$, $\beta = 0.001$, and $C = 0.001$. We compute the total amount of bits that can be extracted from the linear and non-linear models (b and $f * T$ respectively) for different values of their respective tunable parameters (R and T respectively). In Figure 1.2 we plot the total bits that are extracted versus the flow rate in doing so for both models:

Analytically it is easy to show that $\lim_{R\to 0} b(R) = \frac{E_{\text{tot}}}{2\eta + C + \beta}$, and $\lim_{R\to\infty} b(R) = 0$. The function $b(R)$ is maximized for the rate that satisfies $e^R(1-R) = 1$, which occurs when R is zero, i.e. as small as possible.

As there is no explicit analytical expression of the optimal solution for the optimal flow rate f, we can only obtain approximate limits for the total flow extracted at T tends to 0 and ∞. As $T \to \infty$ the expression $(C+\beta)f + 2\eta(e^f - 1) \to 0$, therefore $(C+\beta)f + 2\eta(e^f - 1) = (C + \beta)f + 2\eta\sum_{k=1}^{\infty}\frac{f^k}{k!} \approx (C+\beta+2\eta)f$, and thus $\lim_{T\to\infty} Tf(T) \approx \frac{E_{\text{tot}}}{2\eta + C + \beta}$.

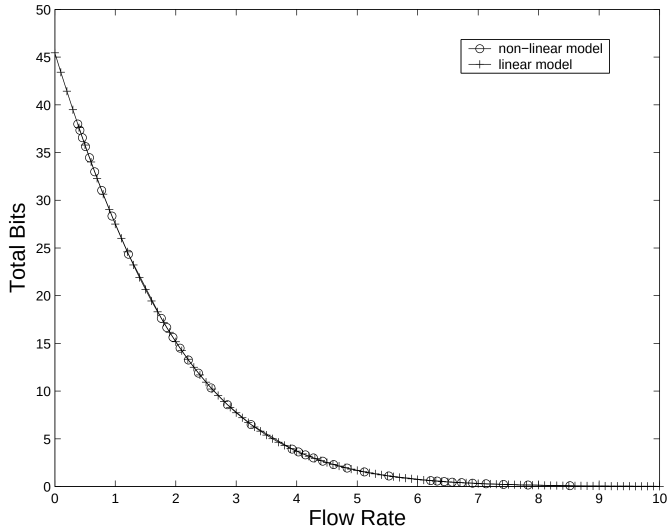

Figure 1.2. Total bits sent to sink versus flow rate for linear and non-linear models, simple example

Likewise as $T \to 0$, then $(C+\beta)f + 2\eta(e^f - 1) \to \infty$ and thus $(C+\beta)f + 2\eta(e^f - 1) \approx 2\eta e^f$, which implies that $Tf(T) \approx T\log(\frac{E_{\text{tot}}}{2\eta T}) \to 0$. Thus for the non-linear model the total information extracted is maximized for a large T. For both the linear and non-linear models, these observations are consistent with figure 1.2 which shows that the maximum total information is extracted when the overall throughput is kept as low as possible (i.e. to the left of the curve).

General case

We consider a square grid scenario, with a 3×3 uniform square grid of sensor nodes in a $[0, 10]^2$ square sending information to a sink located outside the square, located at $(-3, 5)$, other problem parameters were set as $\alpha_i = 0.15$ for all nodes, $\beta = 0.00001$, $\eta = 0.0001$, $E_{\text{tot}} = 10$, and cost $C = 0.001$.

In figure 1.3 we compare the total bits extracted for each the linear and non-linear models respectively as a function of the total rate to the sink. Note that the non-linear model outputs much more information for the same level of energy, at all rate levels. This shows that the computational tractability of the constant rate linear models comes at the expense of some inefficiency. Rate adaptation can provide significant additional

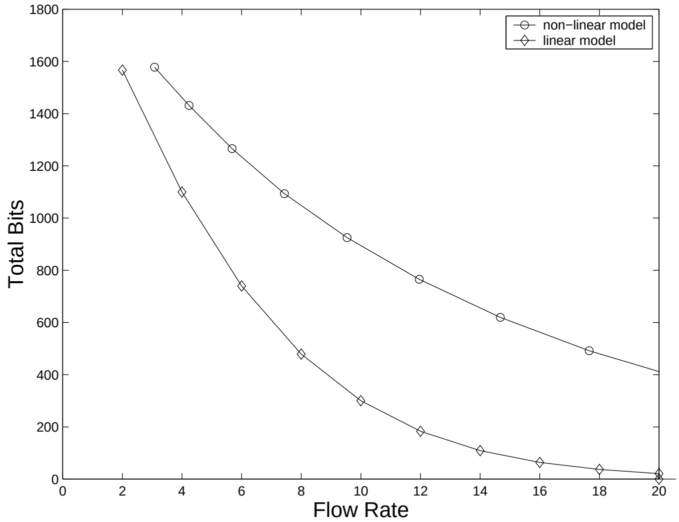

Figure 1.3. Total bits sent to sink versus flow rate for linear and non-linear models, square example

information for the same budget (it is nearly an order of magnitude higher in this scenario).

4. Multi-hop Behavior

Here we investigate how varying the reception cost C affects the hopping behavior of the optimal solution. In this section we are simply considering the non-linear model. Clearly a very high reception cost will make preferable to send the information directly to the sink, while a very inexpensive reception will allow for hopping in the optimal solution. An approach is to use an optimization model to see for which values of C the optimal routing hops and for which it does not. We first consider a very simple example that is also amenable to an analytical solution and then see how the insights can be generalized.

Simple example

To provide an initial solution to this question we consider a very simplified problem consisting of only two sensor nodes, one of which provides all the information (that is $\alpha_1 = 0$ and $\alpha_2 = 1$). The question is to try to predict when node 2 will prefer to send the information directly to the sink and when it will prefer to route it through node 1. In order to avoid a trivial solution we place node 1 closer to the sink than node two, in fact for simplicity we place it exactly mid-way between node 2 and the sink. Assume also that we have $b_{\text{out}} = 1$, that we have a sensing cost of β, and noise parameter of η. Clearly the decision of whether to route information or not will be affected for different reception costs C. We note that for small values of C node 2 will find more attractive to route its information through node 1. In this case node two has the alternative to route part of the information and send the rest directly to the sink. For high values of C the network will decide that it's too expensive to route information through node 1 and node 2 will send everything directly. Here we investigate for what values of C will the network decide to hop and for which to send the information directly.

With the use of optimization models we solve for the optimal routing behavior given different values of the reception cost C. In Figure 1.4 we plot the total value of flow that is sent directly to the sink from node 2 for different reception cost values. This computational example additionally has the following parameters values $\beta = 0.00001$ and $\eta = 0.1$. We note

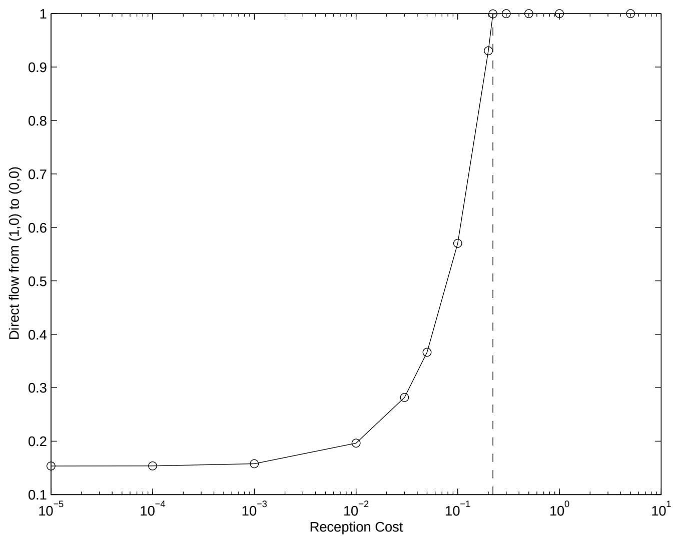

Figure 1.4. Flow to the sink without hopping as a function of the reception cost C

that for reception costs higher than a critical value (plotted as a dashed vertical line) node 2 sends all the information directly to the sink. We also note that it is never optimal to hop all the information, as for any reception cost there is some fraction of the information being sent directly. We finally note that there is a dramatic change in the type of the routing solution as C varies from 10^{-2} to 0.3.

Due to the simplicity of this example, we can analyze the results further. We are simply comparing the solution in which we route all the information directly at a cost

$$h_s = \beta + \eta(e - 1)$$

with the case in which we send f_1 from node 2 to node 1 and then to the sink, and $f_2 = 1 - f_1$ directly form node 2 to the sink, at a cost

$$\begin{aligned} h_c(f_1) &= \beta + \frac{1}{4}\eta(e^{f_1} - 1) + Cf_1 + \frac{1}{4}\eta(e^{f_1} - 1) + \eta(e^{1-f_1} - 1) \\ &= \beta + Cf_1 + \frac{1}{2}\eta(e^{f_1} - 1) + \eta(e^{1-f_1} - 1) \ . \end{aligned}$$

The amount of information that will be routed will be the minimizer of function $h_c(f_1)$ on the domain $[0, 1]$.

We need to determine for what values of C will $h_c(f_1) < h_s$ for some $f_1 \in (0, 1]$, which means that it is more convenient to route f_1 than to send everything directly. Equivalently we will determine for what value of C, the function $H_C(f_1) = h_c(f_1) - h_s \geq 0$ for all $f_1 \in [0, 1]$, these are the values of C that will make it more convenient to send directly rather than route the information. It is easy to show that $H_C(f_1)$ is a convex function and $H_C(0) = 0$, therefore to guarantee that $H_C(f_1) \geq 0$ for all $f_1 \geq 0$ it is sufficient to show that $H'_C(f_1) \geq 0$. This last condition reduces to $C \geq \eta(e - \frac{1}{2})$. This critical value for the reception cost is $C = 0.221828$ for the problem parameters of this example. We plot this value a vertical line in Figure 1.4. Note that the the optimal solution routes all the information from sources directly to the sink precisely at that critical value.

General case

To illustrate the general case we considered a 3×3 uniform square grid of sensor nodes sending information to a sink located outside the square. The example considers 9 nodes uniformly distributed in a $[0, 10]^2$ square with the sink located at $(-3, 5)$, other problem parameters were set as $\alpha_i = 0.15$ for all nodes, $\beta = 0.00001$, $\eta = 0.0001$, and $b_{\text{out}} = 10$. The reception cost C was varied between 10^{-5} and 1.

To analyze the result for a general WSN we also construct a function $H_C(f)$ that quantifies the difference between a hopping solution and a non-hopping solution. Let $\hat{f}$ be the optimal routing of information that sends all information directly. That is it solves Problem (1.4) with the additional constraint that $\sum_{i,j=1}^{n} f_{ij} = 0$. Then the minimal energy needed to send directly to the sink a given amount of information b_{out}

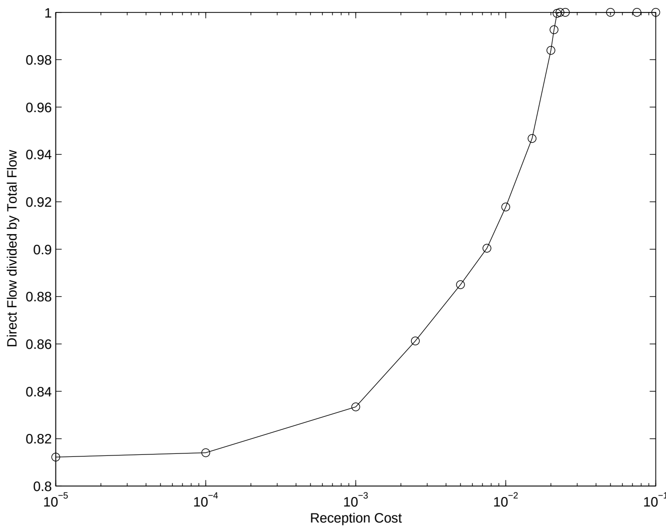

Figure 1.5. Fraction of total flow on the network that goes directly to the sink as a function of the reception cost C

is

$$h_s = \beta b_{\text{out}} + \sum_{i=1}^{n} \eta d_{in+1}^2 (e^{\hat{f}_{in+1}} - 1) \ .$$

We define $H_C(f)$ for any feasible flow f to be the difference between the energy consumed to send b_{out} to the sink by routing through f and h_s, that is

$$\begin{aligned} H_C(f) &= \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} + \eta d_{ij}^2 \left(e^{f_{ij}} - 1 \right) - h_s \\ &= \sum_{i,j=1}^{n} C f_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n+1} \eta d_{ij}^2 \left(e^{f_{ij}} - e^{\hat{f}_{ij}} \right) \ . \end{aligned}$$

The condition that the reception cost must satisfy, in order to indicate that it forces direct routing as the optimal solution, is that for every feasible solution f we have $H_C(f) \geq 0$. It is easy to see that if $\tilde{f}$ is a routing solution that sends all the information directly to the sink, then $H_C(\tilde{f}) \geq 0$, and $H_C(\hat{f}) = 0$. Therefore C induces a no-hopping optimal routing solution if $\nabla H_C(\hat{f})^t (f - \hat{f}) \geq 0$ for all feasible solutions

f. Taking derivatives we have that

$$\begin{aligned}
\nabla H_C(\hat{f})^t(f-\hat{f}) &= \sum_{i,j=1}^{n}(C+\eta d_{ij}^2)f_{ij} + \sum_{i=1}^{n}\eta d_{in+1}^2 e^{\hat{f}_{in+1}}\left(f_{in+1}-\hat{f}_{in+1}\right) \\
&= C\sum_{i,j=1}^{n} f_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n+1}\eta d_{ij}^2 e^{\hat{f}_{ij}}\left(f_{ij}-\hat{f}_{ij}\right)\ .
\end{aligned}$$

If f is a non-hopping feasible flow then we can show that $\nabla H_C(\hat{f})^t(f-\hat{f}) \geq 0$, from the KKT conditions of $\hat{f}$. If f is a solution that does some hopping, that is $\sum_{i,j=1}^{n} f_{ij} > 0$, then the condition that C has to satisfy to force a non-hopping solution is

$$C \geq \frac{\sum_{i=1}^{n}\sum_{j=1}^{n+1}\eta d_{ij}^2 e^{\hat{f}_{ij}}\left(\hat{f}_{ij}-f_{ij}\right)}{\sum_{i,j=1}^{n} f_{ij}}\ .$$

This condition implies that the threshold value for the reception cost C^*, above which it is preferable to send all the information directly, is the solution to the following maximization problem over an open domain:

$$\begin{array}{rll}
C^* = & \max_{f,\kappa} & \frac{1}{\kappa}\sum_{i=1}^{n}\sum_{j=1}^{n+1}\eta d_{ij}^2 e^{\hat{f}_{ij}}\left(\hat{f}_{ij}-f_{ij}\right) \\
& \text{s.t.} & 0 \leq Nf \leq \alpha\sum_{j=1}^{n} f_{jn+1} \\
& & \sum_{i,j=1}^{n} f_{ij} = \kappa \\
& & f \geq 0 \\
& & \kappa > 0
\end{array}$$

This example illustrates the possible use of the flow optimization models to examine qualitative issues such the use of multi-path routing versus direct transmission to a base station.

5. Conclusions

In this chapter, we have described the flow optimization framework for analyzing the fundamental limits of sensor network performance. We reviewed some of the relevant recent literature on the subject, and illustrated the framework by presenting several models including both non-linear adaptive rate models as well as linear constant rate models. We illustrated the utility of this framework by studying the energy-information tradeoffs in sensor network, investigating the gains that are possible through rate adaptation, and exploring the conditions under which multi-hop routes are present in the optimal solution. Another important use of the flow optimization framework is that the numerical fundamental performance limits that it provides can be used as a benchmark for measuring the performance of practical implementations.

There are a number of directions in which the flow optimization models we have presented can be extended. The models above assume conservation of flows at each node. If nodes can aggregate data, this constraint must be changed. Data aggregation constraints can be best modelled by incorporating multi-commodity flows. The models above assume that the network has scheduled communications on all links. For a CDMA-like environment, interference poses a non-convex constraint. There are some techniques that can be used to handle such constraints approximately [25]. Models can also consider the possibility of mobile nodes, in which locations and therefore inter-node distances can be varied as a design parameter at the expense of some energy for motion. However, this may also introduce a non-convex constraint. Another area of open research is in exploiting the structure of these optimization problems to develop constructive, implementable algorithms that obtain near-optimal performance in practice.

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