Scheduling the South American Qualifiers to the 2018 FIFA Wold Cup by Integer Programming

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Abstract

Every four years, the 10 national teams members of the South American Football Confederation (CONMEBOL) compete for one of the slots available to South America in the final phase of the FIFA World Cup. The qualifying phase consists of a double round robin tournament, played in 9 match-days, with every team playing twice on each match-day. The overall tournament is spread throughout 2 years, so match-days are months apart from each other. After using a same mirrored schedule for about twenty years, and persistent complaints from its members, CONMEBOL decided to change the schedule for the 2018 World Cup. Supported by one of CONMEBOL's members, we used integer programming to construct schedules that overcome the main drawbacks of the previous approach. After exploring many design criteria, we proposed a candidate schedule based on a *French scheme*. The main feature of the proposed schedule is that every team plays once at home and once away on each match-day, stepping away from traditional symmetric (mirrored) schemes. This proposal was unanimously approved by CONMEBOL members and its currently being implemented.

Keywords: OR in practice; Sports scheduling; Integer programming; Football.

1   Introduction

Organized every 4 years by the Fédération Internationale de Football Association (FIFA), the FIFA World Cup (WC) is the most important football competition in the world. Attracting dozens of sponsors and intense media coverage, the final phase of the WC receives more international attention than any other single-sport event: some 3.4 million spectators filled the stadiums for the 2014 World Cup finals matches in Brazil and the television coverage reached 3.2 billion people around the world (FIFA, 2014a,b).

The WC begins with a qualifying stage in which each of FIFA's six continental federations selects their representatives for the WC's final stage: out of FIFA's 209 member associations, only 32 teams qualifies for this final stage. In the qualifying phase, the South American Football Confederation (CONMEBOL) selects its representatives based on the results of a double round robin tournament that takes place during nine FIFA match-days. A FIFA match-day is a one-week period during which private clubs must allow players to represent their countries' teams. Because of the scarcity of these match-days, CONMEBOL schedules two games for each country on each match-day. While match-days are usually scheduled months apart from each other, games within a match-day are only 4-5 days apart. Hereafter, we refer to a match-day as a *double round*, to distinguish it from a single *round*, where each team plays a single game (thus, a double round consists of two single rounds).

Starting with the 1998 WC and until the 2014 WC, CONMEBOL used a *mirrored* schedule for the qualifying tournament: that is, a team would face every rival once during the first 9 rounds (in some order), and then again during the last 9 rounds, following the same order, but reversing the home-away status. Moreover,the exact same fixture was used between the 2002 WC and 2010 WC (i.e. teams played rivals on the same order in all these tournaments).

The situation above elicited many complaints from some of CONMEBOL members throughout the years, as it was perceived that such a fixture unjustly benefited some teams. This, either because some teams started the tournament playing against traditionally weak teams, or because other teams played stronger teams late on the tournament, often once the latter teams had already secured a place on the final phase of the WC, thus not always featuring their best players in their starting line-ups. More importantly, due to the structure of the schedule, some teams went for almost a year without receiving a rival at home. Understandably, most complaints came from teams who repeatedly did not qualify for the final phase of the WC.

Considering the situation depicted above, for the 2018 WC CONMEBOL decided to change

the tournament's fixture, and that whatever schedule ended up being adopted, teams would have to be assigned to it by a random draw. To maintain fairness in the process, all members could submit candidate fixture-templates (that is, a tournament's fixture prior to assigning teams to positions on the fixture) which would be voted by all CONMEBOL members.

In partnership with Chile's National Professional Football Association (ANFP), we used integer programming (IP) to develop a series of generic schedules among which Chile's proposal was selected. There is a relatively long tradition of using IP to scheduling sporting tournaments. Its use has been particularly popular in Football; some examples are tournaments in Holland (Schreuder, 1992), Austria and Germany (Bartsch et al., 2006), Chile (Durán et al., 2007, 2012), Denmark (Rasmussen, 2008), Belgium (Goossens and Spieksma, 2009), Norway (Flatberg et al., 2009), Honduras (Fiallos et al., 2010), Brazil (Ribeiro and Urrutia, 2012) and Ecuador (Recalde et al., 2013). We refer the reader to Rasmussen and Trick (2008), Wright (2009), Kendall et al. (2010), and Ribeiro (2012) for reviews of the literature on the use of IP in sport scheduling.

In our work, candidate proposals were designed so as to balance several sporting fairness considerations. For example, our focus was on eliminating the occurrence of *double round breaks* in which a team plays either at home or away both games on a same double round. Unfortunately, such a feature can not be achieved in a mirrored fixture, as will be shown in this paper. Thus, our schedules were designed to eliminate double round breaks while maintaining certain *symmetry*, so as to *resemble* a mirrored scheme. Some of these designs were inspired by symmetric schemes adopted in European Leagues (Goossens and Spieksma, 2012).

The ANFP's proposal followed a *French* symmetry scheme, was voted unanimously by CONMEBOL's members, and was adopted to schedule the 2018 WC qualifying tournament, which at the moment of this writing is currently underway.

The remainder of the paper is organized as follows. Section 2 provides background on the WC qualification tournament. Section 3 describes the basic principles and symmetric schemes for scheduling, and provides IP formulations. Section 4 summarizes our results and analyzes the schedule adopted by CONMEBOL. Finally, Section 5 presents our conclusions.

2   Background

2.1   South American Qualifier Tournament

FIFA's South American federation, CONMEBOL, is the organization overseeing the WC qualifying phase for its ten nation members: Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador,

Paraguay, Peru, Uruguay and Venezuela.

CONMEBOL is arguably one of the strongest among FIFA's federations: its members have won 9 out of the 20 WC championships disputed between 1939 and 2014 (consider that FIFA has 209 nation members, more than the United Nations). Because of this, and despite having the lowest number of members among FIFA's continental federations, four of its teams qualify directly to the WC's final phase, while a fifth one competes on a wild-card two-games tie for an additional place on the final phase.

CONMEBOL selects the teams that advance to the final phase of the WC based on the results of a double round robin tournament in which each team plays all other rival teams twice; once at home and one away. This tournament is played along 18 *rounds*, each consisting of 5 games and in which each team plays exactly once. The entire tournament consists of 90 matches: after each match, 3 points are granted to the winner team, and none to the loser; one point is granted to each team in case of a tie. At the end of the tournament, the 4 teams with the most cumulated points advance to the final phase of the WC (various tie-breaking rules apply), and the fifth team plays a two-games tie against a team from other federation for an additional place.

Many players from several teams members of CONMEBOL play in the best football clubs in the world, which are always reluctant to grant permission to said players to represent their countries in international games. For this reason, FIFA publishes in advance a calendar of *match-days*, which are one-week periods during which private clubs must allow players to represent their national teams. These dates are carefully picked so as to not interfere with international club-level competitions and national club leagues, and because of this, their number is rather scarce. As a result, consecutive match-days are usually months apart from each other.

The South American qualifying tournament takes places during 9 of FIFA's match-days, which are spread along a period of 2 to 3 years. During any particular match-day, two rounds of the qualifying tournament are played only 4-5 days apart. We refer to a match-day as a *double round*, to the first (second) round on it as the *even* (*odd*) round, in reference to their index among the 18 total rounds in the tournament. The practice of playing two games during a single match-day has been promoted by FIFA not only for CONMEBOL but for other confederations as well.

2.2 Qualifier Tournament Fixture 1998-2014

Starting with the 1998 WC and until the 2014 WC, CONMEBOL used a *mirrored* fixture for the qualifying tournament: that is, a team would face every rival once during the first 9 rounds (in some order), and then again during the last 9 rounds, following the same order, but reversing the home-away status. Moreover,the exact same fixture was used between the 2002 WC and 2010 WC (i.e. teams played rivals on the same order in all these tournaments); see Table 1. To the best of our knowledge, there is no official documentation detailing the criteria driving the construction of this fixture. The lack of transparency in this regard has left place for speculation. For example, Gilardi (2013) suggests that the fixture was designed so as to favor a particular team that did not qualify to the 1998 WC. (The author lists a series of factors that arguably favors a particular country, which would be behind the construction of the fixture and that, paradoxically, subsequently failed to qualify to WCs to date.)

Table 1: The 2002-2014 schedule of the South American Qualifier Tournament.

Team	1	2	3	4	5	6	7	8	9	10	...	18
ARG	CHI	@VEN	BOL	@COL	ECU	@BRA	PAR	@PER	URU	@CHI		@URU
BOL	@URU	COL	@ARG	@VEN	CHI	PAR	@ECU	@BRA	PER	URU		@PER
BRA	@COL	ECU	@PER	URU	@PAR	ARG	@CHI	BOL	@VEN	COL		VEN
CHI	@ARG	PER	@URU	PAR	@BOL	@VEN	BRA	COL	@ECU	ARG		ECU
COL	BRA	@BOL	VEN	ARG	@PER	@ECU	URU	@CHI	PAR	@BRA		@PAR
ECU	VEN	@BRA	@PAR	PER	@ARG	COL	BOL	@URU	CHI	@VEN		@CHI
PAR	@PER	URU	ECU	@CHI	BRA	@BOL	@ARG	VEN	@COL	PER		COL
PER	PAR	@CHI	BRA	@ECU	COL	@URU	VEN	ARG	@BOL	@PAR		BOL
URU	BOL	@PAR	CHI	@BRA	VEN	PER	@COL	ECU	@ARG	@BOL		ARG
VEN	@ECU	ARG	@COL	BOL	@URU	CHI	@PER	@PAR	BRA	ECU		@BRA

The 2002-2014 fixture has some positive features. For example, no team plays more than two consecutive home or away games; also, each team plays once at home and once away in the first and last double rounds. This latter feature is perceived as contributing to the fairness of the tournament. Also, no team plays consecutively against Argentina and Brazil, which are considered the two strongest South American teams.

Unfortunately, the 2002-2014 fixture also posess many undesirable features. Traditionally, the literature in sports scheduling has paid special attention to the number of *breaks* in a fixture. A break occurs when a team plays two games consecutively either at home or away (see Section 4 in Rasmussen and Trick (2008)). Given the temporal separation of double rounds, in the context of the qualifying tournament, it stands to reason that the focus ought to be on double-round breaks; that is, double rounds in which both games are played either at home or away. In the 2002-2014 fixture, there are 18 double-round breaks. This is largely a feature of the mirrored

structure of the fixture: it can be shown that the minimum number of such breaks for a mirrored tournament is 16.

Compounding such a problem, double-round breaks in the 2002-2014 fixture are unevenly distributed among teams. Table 2 provides summary statistics on the number of home (B_h), away (B_a) and total (B) double-round breaks per team. We see, for example, that Bolivia has the most (4) double-round breaks while Argentina and Brazil have none.

Table 2: Double round breaks in the 2002-2014 South American Qualifier Tournament.

Team	B_h	B_a	B	H-A	A-H
ARG	0	0	0	9	0
BOL	2	2	4	2	3
BRA	0	0	0	0	9
CHI	1	1	2	1	6
COL	1	1	2	6	1
ECU	1	1	2	4	3
PAR	1	1	2	3	4
PER	1	1	2	6	1
URU	1	1	2	4	3
VEN	1	1	2	1	6

In terms of sport fairness, away double-round breaks are undesirable as they imply not having a home-field advantage for two consecutive games (the significance of home-field advantage in football has been largely documented in the literature; see Pollard (2006)). Moreover, in the context of the South American qualifying tournament, they imply that teams might not play at home in almost a year. For example, in the qualifiers for the 2010 World Cup, Uruguay did not play at home for more than 6 months (from September 10, 2010 to March 28, 2011) due to an away break on double round 5. Given all the local excitement and the economic benefits that matches at home entail (ticket/advertisement sales, increase in retails sales, etc), away double-round breaks are also unwelcomed by fans, team officials and the media.

Away double-round breaks might also involve long travel sequences to foreign countries within just a few days, with negative effects on players' stamina. For example, within double round 2, Bolivia has an away break against Argentina and Venezuela. This trip involves 7,340 km (slightly more than the distance from London to Mumbai), and is the fifth longest of the side's 72 possible travel sequences. By contrast, Peru's only double round away break is against Bolivia and Paraguay in double round 5, the second shortest of the Peruvian side's 72 possible trips.

Furthermore, Table 2 also shows an unbalanced distribution of home-away (H-A) and away-

home (A-H) sequences in double rounds (for example, Argentina had 9 sequences H-A while Brazil did not have any). For logistical reasons, national teams generally prefer the H-A sequence. This allows more time to prepare for the initial game of a double round, and allows players playing abroad to return to their clubs immediately after the second game on the double round, thus reducing travel times (players typically travel to their respective countries before the first game of a double round, irrespective of whether such a game is played at home or away).

2.3 Controversy and the CONMEBOL decision

Aside from the structural deficiencies of the 2002-2014 fixture, its repeated use throughout the years gave rise to other problems that might have not appear had a random draw been involved prior to each qualifying tournament. For example, the fixture had Uruguay playing at home against Argentina in the final round: while the former team is typically still trying to secure a spot among the five top teams, Argentina, one of the historically strongest teams, has usually already secured a place on the final stage of the WC. As neighboring countries, ties between them are close, and accusations of less than competitive play in key matches have been voiced more than once. This was the case in the 2002, 2006 and 2014 tournaments, when Uruguay secured one of the 5 top places in the last round. In some occasions, understandably, Argentina played most of the bench on those games. By contrast, in the 2010 tournament, Argentina had not secured a place in the WC by the final round, and they went to win the last match against Uruguay.

The above is just one of many examples of controversies arising from the repeated use of an unbalanced (in terms of sporting fairness) fixture. Whether or not the suspicions of football fans and the media are well-founded, CONMEBOL took notice of the many complaints received throughout the years. In January 2015, CONMEBOL members agreed that a new methodology was to be used for scheduling the 2018 qualifier tournament. In this new methodology, teams would be assigned to a fixture template (one where teams are denoted only by numbers) by means of a random draw (where teams are assigned to numbers on the template). It was also agreed that the fixture template would be selected among proposals brought up by the ten CONMEBOL members. By 2015, our research team had been working together with Chile's ANFP for more than ten years, period in which IP techniques have been applied to schedule Chilean professional soccer leagues (Durán et al., 2007, 2012). Together with ANFP we developed a series of fixture templates, among which ANFP's proposal was selected.

3 Integer programming approach

Encouraged by our results in scheduling the Chilean professional leagues, we approach the scheduling of the WC qualifiers using IP. As mentioned in Section 1, there is ample evidence of the use of IP in sports scheduling. The double-round feature of the setting, however, is almost absent in the literature, to the best of our knowledge. Goossens and Spieksma (2011) generalize the concept of breaks by considering pairs of arbitrary rounds instead of pairs of all consecutive rounds (a double round in our problem is equivalent to choosing such arbitrary rounds as all the pairs formed by an odd round and its subsequent even round). However, unlike ours, their work focuses on minimizing generalized breaks in a single round robin tournament without additional constraints. (See also Buiteveld et al. (2014))

3.1 Base IP Formulation

Let $I = \{1, \ldots, n\}$ denote the set of teams ($n = 10$), and I_s the set of *strong* teams ($I_s \subset I$). Also, let $K = \{1, \ldots, 18\}$ the set of rounds, and K_{odd} the set of first rounds on a double round. For $i, j \in I$ and $k \in K$, define the binary variable $x_{i,j,k}$ as equal to 1 if team i plays at home against team j in round k, and zero otherwise.

Double round robin constraints. In a double round robin tournament, all teams must play twice against any other team. Here, we maintain traditional features of the tournament: every team faces every other team once on the first half of the tournament and once on the second half, where exactly one of these is a home game and the other an away game. These constraints are formulated below.

$$\sum_{k \in K: k \leq n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j$$

$$\sum_{k \in K: k > n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j$$

$$\sum_{k \in K} x_{i,j,k} = 1 \quad \forall i \in I, j \in I : i \neq j$$

Compactness. We also require the schedule to be compact, that is, all teams play one match on each round.

$$\sum_{i \in I: i \neq j} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall j \in I, k \in K$$

Top team constraints. We prevent any team from playing strong teams in consecutive rounds.

$$\sum_{j \in I_S} (x_{i,j,k} + x_{j,i,k} + x_{i,j,k+1} + x_{j,i,k+1}) \leq 1 \quad \forall\, i \in I \setminus I_S, k \in K : k < |K|$$

Balance constraints. We balance the distribution of $H - A$ sequences in double rounds among teams: given that there are $n - 1$ double rounds, we impose that each team has between $n/2 - 1$ and $n/2$ $H - A$ sequences in double rounds. These constraints are encoded with the help of auxiliary variables $y_{i,k}$ which take value 1 if team i has a $H - A$ sequence on the double round that start at round k, and zero otherwise. These variables must satisfy the following constraints:

$$n/2 - 1 \leq \sum_{k \in K_{odd}} y_{i,k} \leq n/2, \qquad \forall i \in I,$$

$$\sum_{j \in I : i \neq j} (x_{i,j,k} + x_{j,i,k+1}) \leq 1 + y_{i,k}, \qquad \forall i \in I, k \in K_{odd},$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{i,j,k}, \qquad \forall i \in I, k \in K_{odd},$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1}, \qquad \forall i \in I, k \in K_{odd}.$$

Objective function. We aim at minimizing the total number of away breaks within double rounds across all teams. For that, we define auxiliary variables $w_{i,k}$ which take value 1 team i has an away break in the double round that start at round k, and zero otherwise. These variables must satisfy the following constraints:

$$\sum_{j \in I : i \neq j} (x_{j,i,k} + x_{j,i,k+1}) \leq 1 + w_{i,k}, \qquad \forall i \in I, k \in K_{odd},$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k}, \qquad \forall i \in I, k \in K_{odd},$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1}, \qquad \forall i \in I, k \in K_{odd}.$$

Note also that because if a team has an away break on a double round, another team must have a home break on the same double round (de Werra, 1988), thus minimizing the total number of breaks within double rounds is captured by the following objective function:

$$\min z = \sum_{i \in I} \sum_{k \in K_{odd}} w_{i,k}.$$

3.2 Symmetric schemes

Unlike the mirrored structure of previous WC qualifier tournaments, the base formulation above does not impose any type of symmetry between the first 9 rounds and the last 9. It is important to note that *symmetry* is present in most tournament formats: Goossens and Spieksma (2012) surveyed the schedules of 25 European football leagues, using data of the season 2008-2009, and found that 20 of them used some form of symmetric scheme, with the mirrored one being the most frequently used (15 out of 25 leagues use it).

Anticipating reluctance from CONMEBOL to adopt any scheme that did not incorporate any form of symmetry, we studied several forms of symmetry, and prepare proposals accordingly.

Mirrored scheme. The mirroring condition can be incorporated in the base IP formulation adding the constraints

$$x_{i,j,k} = x_{j,i,k+n-1}, \quad \forall\, i,j \in I : i \neq j,\ k \in K : 1 \leq k \leq n-1.$$

Unfortunately, imposing only logical constrains in addition to the mirrored condition above results on at least 16 double round breaks (which we aim to minimize). Adding the H-A balance constraints results in infeasibility. Because of these drawbacks, we explored other forms of symmetry.

French scheme. In this scheme, the first and last rounds are identical, but with the home conditions inverted; the same holds for rounds k and $k+n-2$, $k = 2, \ldots, n-1$. That is,

$$x_{i,j,1} = x_{j,i,2n-2}, \quad x_{i,j,k} = x_{j,i,k+n-2}, \quad \forall\, i,j \in I : i \neq j,\ 2 \leq k \leq n-1.$$

English scheme. In this scheme, rounds $n/2 - 1$ and $n/2$ are identical, but with the home conditions reversed; the same holds for rounds k and $k+n$,$k = 2, \ldots, n-2$. That is

$$x_{i,j,n-1} = x_{j,i,n}, \quad x_{i,j,k} = x_{j,i,k+n}, \quad \forall\, i,j \in I : i \neq j,\ 2 \leq k \leq n-2.$$

Inverted scheme. In this scheme, round k and $2n - k - 1$ are identical, but with the home conditions reversed, for $k = 1, \ldots, n-1$. That is

$$x_{i,j,k} = x_{j,i,2n-1-k}, \quad \forall\, i,j \in I : i \neq j,\ 1 \leq k \leq n-1.$$

Back-to-back scheme. In this scheme, round k and $k+1$ are identical, but with the home conditions reversed, for $k \in K_{odd}$. That is

$$x_{i,j,k} = x_{j,i,k+1}, \quad \forall i, j \in I : i \neq j, \; k \in K_{odd}.$$

Min-Max separation scheme. In this scheme, consecutive games between any two teams are separated by at least c rounds and at most by d consecutive. That is,

$$\sum_{\bar{k} \in K : k \leq \bar{k} \leq k+c} (x_{i,j,\bar{k}} + x_{j,i,\bar{k}}) \leq 1, \quad \forall i, j \in I : i \neq j, \; k \leq |K| - c,$$
$$\sum_{\bar{k} \in K : (k-d) \wedge 1 \leq \bar{k} \leq (k+d) \vee 2(n-1) : \bar{k} \neq k} x_{i,j,\bar{k}} \geq x_{j,i,k}, \quad \forall i, j \in I : i \neq j, \; k \in K.$$

Although not as popular as the mirrored scheme, some of the schemes above have been used in the professional football leagues of various countries: the French scheme has been used in Czech Republic, France, Luxembourg and Russia, while the English and the inverted schemes have been used in Austria and Switzerland, respectively (Goossens and Spieksma, 2012). In South America, the inverted scheme has been used for the prestigious *Copa Libertadores* (international clubs) tournament.

We solved the base formulation in combination with the various symmetry-related constraints and obtained a number of template schedules. As some constraints are incompatible with some symmetric schemes, we relaxed the former when necessary. For example, for the back-to-back scheme, we relaxed the constraint that every team faces every other team once on the first half of the tournament and once on the second half, which is clearly incompatible with the scheme. In each setting, we obtained schedules that eliminated double round breaks. For the case of the min-max separation scheme, we use $c = 7$ and $d = 11$, as these were the closest possible values to the infeasible mirrored case ($c = 9$ and $d = 9$) that resulted in the elimination of double round breaks. Among all schemes, the French one allowed us to satisfy all constraints and, more important, was well accepted by ANFP officials, as we detail next.

4 Results and implementation in practice

Among the templates presented in the previous section, ANFP officials selected the one following the French scheme. The ANFP proposed this schedule at a meeting of CONMEBOL's members in April 2015. Although other countries also presented proposals, ours was unanimously selected

by all members for the 2018 WC qualifiers.

In July 2015, the random draw that assigned teams to the template fixture took place in Saint Petersburg at an event involving the 209 FIFA-affiliated national associations, and that was broadcasted worldwide. Former *Best World Cup Award* players Ronaldo and Forlán drew balls from different pots to define the South American qualifiers schedule. Some pots contained the names of the participating teams while the others contained the numbers indicating the position each team would take in the schedule we constructed. The outcome of the draw resulted in a schedule presented in Table 3.

Table 3: Rounds 1 to 10 of 2018 fixture of the South American Qualifier tournament.

Team	1	2	3	4	5	6	7	8	9	10	...	18
ARG	ECU	@PAR	BRA	@COL	@CHI	BOL	URU	@VEN	@PER	PAR		@ECU
BOL	URU	@ECU	VEN	@PAR	COL	@ARG	PER	@CHI	@BRA	ECU		@URU
BRA	@CHI	VEN	@ARG	PER	URU	@PAR	@ECU	COL	BOL	@VEN		CHI
CHI	BRA	@PER	COL	@URU	ARG	@VEN	@PAR	BOL	@ECU	PER		@BRA
COL	PER	@URU	@CHI	ARG	@BOL	ECU	VEN	@BRA	@PAR	URU		@PER
ECU	@ARG	BOL	URU	@VEN	PAR	@COL	BRA	@PER	CHI	@BOL		ARG
PAR	@VEN	ARG	@PER	BOL	@ECU	BRA	CHI	@URU	COL	@ARG		VEN
PER	@COL	CHI	PAR	@BRA	VEN	@URU	@BOL	ECU	ARG	@CHI		COL
URU	@BOL	COL	@ECU	CHI	@BRA	PER	@ARG	PAR	VEN	@COL		BOL
VEN	PAR	@BRA	@BOL	ECU	@PER	CHI	@COL	ARG	@URU	BRA		@PAR

By construction, the definitive schedule departs from a mirrored scheme only in that the last round of matches in the second half is the reverse of the first round in the first half, this while ensuring that all teams play once at home and once away in each double round. Also, it greatly improves the balance of H-A and A-H sequences by giving all teams at least 4 and at most 5 of these sequences. These features improves significantly upon the previous schedule, as shown in Table 4. First, the total number of breaks within double rounds was reduced from 18 to 0. Also, because teams have no double round breaks, it improves sport fairness significantly. Finally, the schedule balances the H-A sequences within double rounds, especially for Brazil, Chile and Venezuela, which instead of having 0 or 1 of such sequences, as in the 2002-2014 fixture, now have 4 or 5 of these sequences. As in the 2002-2014 schedule, in the new schedule no team plays consecutively against Argentina and Brazil. In the draw, numbers associated with these two teams (that formed set I_s in our formulation) where placed on a different pot, and then assigned randomly using a separate pot containing their names.

Table 4: Comparison between the schedules of the 2018 and 2002-2014 South American qualifiers tournaments.

	Schedule 2002-2014					Schedule 2018				
Team	B_h	B_a	B	H-A	A-H	B_h	B_a	B	H-A	A-H
ARG	0	0	0	9	0	0	0	0	5	4
BOL	2	2	4	2	3	0	0	0	5	4
BRA	0	0	0	0	9	0	0	0	4	5
CHI	1	1	2	1	6	0	0	0	5	4
COL	1	1	2	6	1	0	0	0	5	4
ECU	1	1	2	4	3	0	0	0	4	5
PAR	1	1	2	3	4	0	0	0	4	5
PER	1	1	2	6	1	0	0	0	4	5
URU	1	1	2	4	3	0	0	0	4	5
VEN	1	1	2	1	6	0	0	0	5	4
Total	9	9	18	36	36	0	0	0	45	45

5 Conclusions

In this paper, we have shown our use of IP to the design of a new schedule for the South American qualification phase of the FIFA World Cup football championship. Our work contributes to the theory and practice of sports scheduling by presenting what is, to the best of our knowledge, the first real-world application of OR to soccer scheduling at the *international* level. The proposed schedule replaces a fixture used by CONMEBOL for the last two decades. Resistance to change, a barrier to the adoption of technology, is specially strong in the culture of an international organization such as CONMEBOL, which must align the interests of its ten members, each with their own cultural identity and points of view.

Our proposal introduced a series of modifications that, from a rather objective perspective, improved upon the traditional schedule, and was selected unanimously by CONMEBOL members. The proposed schedule, with teams assigned as in Table 3, is currently being used in the qualification tournament for the 2018 WC. The tournament started in October 2015 and will conclude in October 2017.

While having less teams than typical football tournaments scheduled using IP techniques, the scheduling problem for the South American qualifiers has features that present interesting technical challenges. One of these is the temporal spacing of the games and the structure derived from the double rounds. Our focus on minimizing double round breaks resulted in a fixture that has all teams playing at least once at home on every double round. This, in sharp contrast to mirrored schedules where teams can go without playing home for several months. With the double round feature also adopted by other confederations, such as UEFA and AFC, solutions

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addressing this characteristic are all the more necessary.

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