# *Adjustment of the WACC with Subsidized Debt in the Presence of Corporate Taxes: the N-Period Case*


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***ABSTRACT***

In the Weighted Average Cost of Capital (WACC) applied to the free cash flow (FCF), we assume that the cost of debt is the market, unsubsidized rate. With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain a loan at a rate that is below the market rate. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would be unequal. The benefit of lower tax savings are offset by the benefit of the subsidy. These two benefits have to be introduced explicitly.

In this paper we present the adjustments to the WACC with subsidized debt and taxes and the cost of leveraged equity for multiple periods. We demonstrate the analysis for both the WACC applied to the FCF and the WACC applied to the capital cash flow (CCF). We use the calculation of the Adjusted Present Value, APV, to consider both, the tax savings and the subsidy. We show how all the methods match.



KEY WORDS

Adjusted Present Value, APV, weighted average cost of capital, discounted cash flow, DCF equity value, cost of equity, WACC, subsidized debt with taxes, valuation of cash flows, project evaluation, project appraisal, firm valuation, cost of capital, cash flows, free cash flow, capital cash flow.

JEL CLASSIFICATION

D61, G30, G31, G32, H43


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## INTRODUCTION

In the Weighted Average Cost of Capital (WACC) applied to the free cash flow (FCF), we assume that the cost of debt is the market, unsubsidized rate. With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain a loan at a rate that is below the market rate. In a previous work we showed how to adjust the WACC in the presence of a subsidy and no taxes. There we showed that plugging the lower cost of debt into the WACC formula is not the correct approach to measuring the value creation due to the subsidy. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would be unequal. The benefit of lower tax savings TS, are offset by the benefit of the subsidy. These two benefits have to be introduced explicitly.

In this paper we present the adjustments to the WACC and the cost of leveraged equity for multiple periods with subsidized debt and taxes. We demonstrate the analysis for both the WACC applied to the FCF and the WACC applied to the capital cash flow (CCF). We use the calculation of the Adjusted Present Value, APV, to consider both, the TS and the subsidy. We show how all the methods match.

The issue of the effect of subsidy in interest rate on the WACC is not widely dealt in the literature. Ross et al, 1999 mention the effect on value and propose to use the APV method and Damodaran 1996 suggests including the value of the subsidy in the cash flow. Dailami and Klein, 1997, say "investors ask for government support in the form of grants, preferential tax treatment, debt or equity contributions, or guarantees" and that "Guarantees themselves do not appear to affect the cost of capital, which is determined by the risks of the project, not the financing structure." On the other hand, Krishnaswami and Subramaniam, 2000 and Fratantoni and Niculescu, 2005 discuss the effect of subsidy in interest in the acquisitions of households. Most literature studies the subsidy from the government due to the tax savings that arise from the corporate taxes. These references suggest that the real effect of subsidy in debt is not well incorporated in the cost of capital.In this paper we do properly incorporate the subsidy effect in the cost of capital. More, we show that when improperly done, a lower cost of debt might destroy value instead of create value.

This paper is organized as follows: In Section One we present the expressions for the cost of capital in the presence of subsidy and corporate taxes for multiple periods and illustrate it with an example. In Section Two we conclude. In an Appendix we show the derivation of the formulation used in Section One.

## SECTION ONE

In summary we have

Table 1a. Summary of formulae for different discount rates

| Cash Flow | Discount rate |
|---|---|
| CFE | $Ke = Ku + \frac{D}{E}\left(Ku - Kd^{Sub}\right) + V_0^{L\,Sub}\frac{\lambda - Ku}{E} + V_0^{TS}\frac{\psi - Ku}{E}$ |
| CCF | $WACC^{CCF} = Ku + \frac{V_0^{TS}}{V_0^{L}}\left(\psi - Ku\right) + \frac{V_0^{L\,Sub}}{V_0^{L}}\left(\lambda - Ku\right)$ |
| FCF | $WACC^{FCF} = Ku + \frac{V_0^{TS}}{V_0^{L}}\left(\psi - Ku\right) + \frac{V_0^{L\,Sub}}{V_0^{L}}\left(\lambda - Ku\right) - \frac{TS}{V_0^{L}} - \frac{V_0^{L\,Sub}\left(1+\lambda\right)}{V_0^{L}}$ |

Where Ke is the cost of leveraged equity and Ku is the cost of unleveraged equity, D is the market value of debt, E is the market value of equity, $V^{L}$ is the leveraged value, let $V^{Un}$ is the unleveraged value, $V^{TS}$ is the value of the TS, $V^{LSub}$ is the value of the interest subsidy, $\lambda$ is the appropriate discount rate for the interest subsidy and $\psi$ is the discount rate for the tax savings, TS.

From this summary, we can obtain simpler formulations depending on the assumptions regarding the discount rate for TS and subsidy. For instance, if we assume that $\psi$ and $\lambda$ are equal to Ku, then the formulae for the different costs are

Table 1b. Formulae assuming $\lambda = \psi = Ku$

| Cash Flow | Discount rate |
|---|---|
| CFE | $Ke=Ku+(Ku-Kd^{Sub})D/E$ |
| CCF | $WACC^{CCF}=Ku$ |
| FCF | $WACC^{FCF}=Ku-TS/V^L_0-Sub/V^L_0$ |

The formula for Ke resembles the typical formulation of Ke when $\psi$ is Ku, except that Kd is replaced by $Kd^{Sub}$. For the CCF we have $WACC^{CCF}$ equal to Ku; this is what is expected when we use the CCF and assume Ku as the discount rate for TS. Finally, for discounting the FCF we have $WACC^{FCF}$ equal to $Ku - TS/V^L_0 - Sub/V^L_0$ and this resembles the adjusted WACC. (See Tham and Velez-Pareja 2004).

We illustrate these ideas with a three period numerical example. The values of the various parameters are shown below. We present the input variables and the final tables after solving the circularity[2].

The input variables are shown in Table 2.

[2] See Velez-Pareja and Tham 2001, Tham and Velez-Pareja, 2004 and Velez-Pareja and Tham, 2005

Table 2. Input variables for single period example

| | |
|---|---|
| Tax rate | 20.0% |
| Cost of unleveraged equity, Ku | 15.0% |
| Debt, $D_0$ | 842.67 |
| Market cost of debt, $Kd^{NS}$ | 10.0% |
| FCF, constant | 1,230.2 |
| Subsidy on Kd | 2.0% |
| Discount rate for Subsidy, $\lambda$ | 10.0% |
| Discount rate for TS, $\psi$ | 8.0% |

Next we calculate the CFD with $Kd^{Sub}$, the TS, the subsidy and the CFE. These values will be needed to calculate Ke and WACC for FCF and CFE.

Table 3a. $Kd^{Sub}$, CFD, TS, Subsidy CFE, $V^{TS}_0$ and $V^{LSub}_0$

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Kdsub | | 8.0% | 8.0% | 8.0% |
| Value of debt | 842.669 | 842.669 | 842.669 | |
| CFD | | 67.4 | 67.4 | 910.1 |
| TS | | 13.5 | 13.5 | 13.48271 |
| Subsidy | | 16.9 | 16.9 | 16.9 |
| FCF | | 1,230.2 | 1,230.2 | 1,230.2 |
| CFE = FCF + TS + Sub - CFD | | 1,193.2 | 1,193.2 | 350.5 |
| $V^{TS}_0$ | 34.7463 | 24.0432 | 12.4840 | |
| $V^{LSub}_0$ | 41.9119 | 29.2497 | 15.3213 | |

Now we can calculate the value of Ke for every year and we calculate the market value of equity.

Table 3b Leverage D% at market value, Ke and leveraged value of equity

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| D% | 29.2% | 41.0% | 76.8% | |
| D%/E% | 0.412 | 0.696 | 3.306 | |
| Ke | | 17.6658% | 19.6126% | 37.4975% |
| Leveraged equity value | 2,042.8866 | 1,210.6236 | 254.9033 | |
| Leveraged value = Equity+debt | 2,885.5560 | 2,053.2929 | 1,097.5727 | |

For instance, for year 1, in the previous table we apply equation

$$Ke=Ku+(D/E)(Ku-Kd^{Sub})+V^{LSub}{}_0(\lambda-Ku)/E+V^{TS}{}_0(\psi-Ku)/E$$

$$15\%+0.412\times(15\%-8\%)+41.912\times(10\%-15\%)/2{,}042.887+34.746\times(8\%-15\%)/2{,}042.887=17.666\%$$

(allow for rounding errors if the reader tries to replicate this calculation).

Table 4 FCF, $WACC^{FCF}$ and leveraged value

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| FCF | | 1,230.2 | 1,230.2 | 1,230.2 |
| $WACC^{FCF}$ | | 13.8% | 13.4% | 12.1% |
| PV of FCF @ WACC | 2,885.5560 | 2,053.2929 | 1,097.5727 | |

In the case of $WACC^{FCF}$ we have for year 1,

$$Ku+(V^{TS}{}_0/V^L{}_0)(\psi-Ku)+V^{LSub}{}_0/V^L{}_0(\lambda-Ku))-TS/V^L{}_0-Sub/V^L{}_0$$

$$15\%+(34.7463/2{,}885.5560)\times(8\%-15\%)+(41.9119/2{,}885.5560)\times(10\%-15\%)-13.5/2{,}885.5560-16.9/2{,}885.5560=13.8\%$$

Table 5 Unleveraged values, values of TS and subsidy and APV

| Year | 0 | 1 | 2 |
|---|---|---|---|
| Unleveraged value | 2,808.8979 | 2,000.0000 | 1,069.7674 |
| $V^{TS}{}_0$ | 34.7463 | 24.0432 | 12.4840 |
| $V^{LSub}{}_0$ | 41.9119 | 29.2497 | 15.3213 |
| Leveraged value APV | 2,885.5560 | 2,053.2929 | 1,097.5727 |

The figures from this table are taken from previous tables except the unleveraged value that is calculated as the present value of the FCF at Ku.

Table 6 Capital Cash Flow, CCF, $WACC^{CCF}$ and leveraged value

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| WACC for CCF | | 14.84% | 14.85% | 14.85% |
| CCF=FCF+TS+Sub | | 1,260.6 | 1,260.6 | 1,260.6 |
| CCF=CFD+CFE | | 1,260.6 | 1,260.6 | 1,260.6 |
| PV(CCF) | 2,885.5560 | 2,053.2929 | 1,097.5727 | |

The CCF is derived from data from table 2. The $WACC^{CCF}$ is derived using the next equation. For year 1 we have.

$Ku+(V^{TS}{}_0/V^L{}_0)(\psi-Ku)+V^{LSub}{}_0/V^L{}_0(\lambda-Ku)$

$15\%+(34.7463/2{,}885.5560)\times(8\%-15\%)+(41.9119/2{,}885.5560)\times(10\%-15\%)=14.84\%$

Now we calculate the leveraged value assuming what is the current practice: to include the $Kd^{Sub}$ in the traditional formula for WACC for the FCF. First we calculate the leveraged value without subsidy. This is what is shown in the next table.

Table 7. Calculation of value using $Kd^{NS}$ and FCF

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Market cost of debt, $Kd^{NS}$ | | 10.0% | 10.0% | 10.0% |
| Debt (% of leveraged value) | 29.6% | 41.6% | 77.7% | |
| Debt-equity ratio | 0.420 | 0.711 | 3.486 | |
| Ke | | 17.10% | 18.56% | 32.43% |
| WACC | | 14.4% | 14.2% | 13.4% |
| FCF | | 1,230.2 | 1,230.2 | 1,230.2 |
| Leveraged value | 2,847.4 | 2,027.4 | 1,084.4 | |

Now we calculate the value using the traditional WACC for the FCF and including $Kd^{Sub}$ as the cost of debt.

Table 8. Calculation of value using $Kd^{Sub}$ and FCF

| Year | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Debt (% of leveraged value) | 29.7% | 41.7% | 77.9% | |
| Debt-equity ratio | 0.422 | 0.715 | 3.528 | |
| Ke | | 17.95% | 20.00% | 39.70% |
| WACC | | 14.5% | 14.3% | 13.8% |
| FCF | | 1,230.2 | 1,230.2 | 1,230.2 |
| Leveraged value | 2,839.7 | 2,021.9 | 1,081.5 | |

Observe that the leveraged value has been reduced compared when we use the traditional WACC and include the $Kd^{Sub}$. A lower cost of debt destroys value! This is counter evident. This occurs because we have lost part of the value generated by the TS and because the Ke calculation absorbs the reduction of the cost of debt. This means that the subsidy has to be explicitly included in the analysis.

In the next table we present a summary of the different calculations for values:

Table 9. Different values with different methods

| Method | Leveraged value | Equity value |
|---|---|---|
| No subsidy | 2,847.4 | 1,997.03 |
| With subsidy using KdSub in the WACC | 2,839.7 | 2,004.73 |
| With subsidy using new formulation for WACC | 2,885.5560 | 2,042.8866 |

In the numerical example, we assume that the appropriate discount rate for the interest subsidy λ is the market rate of interest. However, we could also use the subsidized rate $Kd^{Sub}$ or the Ku. For completeness, in the next table we show the consistent results for the two other values for λ, namely $Kd^{Sub}$ and Ku.

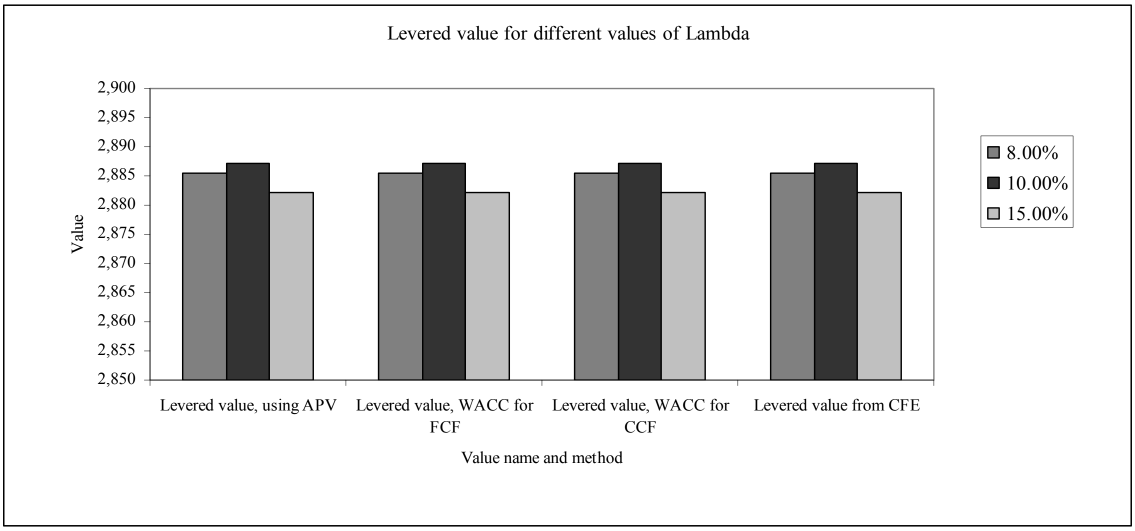
It might be argued that the differences in this example are irrelevant. However, we think that it is not a matter of precision; it is a matter of correctness that can be reached without extra cost. More, it is usual to assume that differences are assigned to rounding errors or that the magnitude is negligible or that practical approaches are more important than theoretical and precise ones. However, while errors could cancel out, sometimes errors cumulate. See for instance Vélez Pareja 2004 and 2005.

Table 10. Results for different values of λ

| | Unsubsidized | Subsidized | Ku |
|---|---|---|---|
| λ | 10.00% | 8.00% | 15.00% |
| Equity, unsubsidized debt | 2,004.71 | 2,004.71 | 2,004.71 |
| Equity, subsidized debt | 2,042.89 | 2,044.41 | 2,039.45 |
| Value, using APV, WACC for FCF, CFE with Ke and WACC for CCF | 2,885.56 | 2,887.08 | 2,882.12 |

In the next figure we show the same results in graphical form

Figure 1. Values for different levels of λ the discount rate of the subsidy




CONCLUSION

In this paper, we show the adjustments that have to be made to the WACC in the presence of a subsidized loan and taxes. It is interesting to observe that when obtaining a subsidy in the cost of debt, using that lower cost in the WACC is not the correct approach to measure the increase in value due to the subsidy. The adjustments to the WACC and the explicit introduction of the subsidy in the analysis, give the proper result.

We found that the discount rate for the subsidy affects the value of the firm. As expected, when λ the discount rate of the subsidy is Ku, the value is lower, however, the use of Kd as discount rate for the subsidy does not result in a lower value, instead, it is the highest value.

As can be noticed there is consistency between all the values calculated with different methods. This consistency is attained using the proper formulation of Ke the cost of levered equity and WACC, the weighted average cost of capital and solving the circularity relationship that arises when we calculate value and cost of capital. These findings and the procedure can be found in Vélez-Pareja, Ignacio and Joseph Tham, 2000 and 2005 and Tham and Vélez-Pareja, 2004.

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## APPENDIX

In this appendix we derive the proper formulations for Ke and WACC.

First we derive the cost of leveraged equity, Ke. Let $V^L$ be the leveraged value, let $V^{Un}$ be the unleveraged value, let $V^{TS}$ the value of the TS, let T the corporate tax rate and let $V^{LSub}$ be the value of the interest subsidy. Then, with respect to the end of year 0, the leveraged value equals the sum of the unleveraged value, plus the value of the TS and the value of the interest subsidy.

$$V^L{}_0 = V^{Un}{}_0 + V^{TS}{}_0 + V^{LSub}{}_0 \tag{1}$$

Using the APV approach, it would be very easy to estimate the value of the subsidized debt. Let $Kd^{NS}$ be the cost of the non subsidized debt, and let $Kd^{Sub}$ be the cost of the subsidized debt. The value of the debt at the end of year 0 is $D_0$.

Let $L^{Sub}{}_1$ be the interest subsidy at the end of year 1 and $TS_1$ be the TS at the end of year 1. Then the interest subsidy equals the value of the debt times the difference between the two interest rates adjusted for taxes and the TS are the cost of unsubsidized debt times the debt, $D_0$ and times the tax rate, T.

$$L^{Sub}{}_1=D_0(Kd^{NS}-Kd^{Sub}) \tag{2}$$

and

$$TS_1=Kd^{Sub}\times T\times D_0 \tag{3}$$

The expression for the value of the interest subsidy is as follows, where $\lambda$ is the appropriate discount rate for the interest subsidy.

$$V^{LSub}{}_0=L^{Sub}/(1+\lambda)=D_0(Kd^{NS}-Kd^{Sub})/(1+\lambda) \tag{4}$$

The expression for the value of the TS is as follows, where $\psi$ is the appropriate discount rate for the TS.

$$V^{TS}_0=Kd^{Sub}\times T\times D_0/(1+\psi)=D_0\times T\times Kd^{Sub}/(1+\psi) \quad (5)$$

Where $\psi$ is the discount rate for the tax savings, TS

DERIVATION OF KE

Let $CCF_1$ be the capital cash flow at the end of year 1 with financing. At the end of year 1, the capital cash flow equals the sum of the FCF, plus the TS and the interest subsidy.

Then,

$$CCF_1=FCF_1+L^{Sub}_1+TS_1 \quad (6)$$

Also, at the end of year 1, the capital cash flow equals the sum of the cash flow to equity (CFE) and the cash flow to debt (with the subsidized interest rate).

$$CCF_1=CFE_1+CFD_1 \quad (7)$$

Putting these two equations together, we obtain,

$$CCF_1 = CFE_1 + CFD_1 = FCF_1 + L^{Sub}_1 + TS_1 \quad (8)$$

The corresponding value relationship is as follows.

$$V^L_0 = E_0 + D_0 = V^{Un}_0 + V^{LSub}_0 + V^{TS}_0 \quad (9)$$

Substituting the appropriate value expressions for each of the cash flow items in equation 8, we obtain,

$$E_0\times(1+Ke)+D_0\times(1+Kd^{Sub}) =V^{Un}_0\times(1+Ku)+V^{LSub}_0\times(1+\lambda)+V^{TS}_0(1+\psi) \quad (10)$$

where Ke is the cost of leveraged equity and Ku is the cost of unleveraged equity.

Applying equation 9 to equation 10, we obtain,

$$E_0\times Ke+D_0\times Kd^{Sub} = V^{Un}_0\times Ku + V^{LSub}_0\times\lambda + V^{TS}_0\times\psi \quad (10.1)$$

$$E_0\times Ke+D_0\times Kd^{Sub}=(E_0+D_0-V^{LSub}_0-V^{TS}_0)\times Ku+V^{LSub}_0\times\lambda+V^{TS}_0\times\psi \quad (10.2)$$

Rearranging, we obtain,

$$E_0 \times Ke = E_0 \times Ku + D_0 \times (Ku - Kd^{Sub}) + V^{LSub}{}_0 \times (\lambda - Ku) + V^{TS}{}_0 \times (\psi - Ku) \quad (11)$$

Substituting equation 4 and 5 into equation 11, we obtain the expression for the Ke.

$$E_0 \times Ke = E_0 \times Ku + D_0(Ku - Kd^{Sub}) + D_0(Kd^{NS} - Kd^{Sub})(\lambda - Ku)/(1+\lambda)$$

$$+[D_0 \times T \times Kd^{Sub}/(1+\psi)](\psi - Ku) \quad (12.1)$$

$$Ke = Ku + (Ku - Kd^{Sub})D_0/E_0 + (Kd^{NS} - Kd^{Sub})[(\lambda - Ku)/(1+\lambda)]D_0/E_0$$

$$+[T \times Kd^{NS}/(1+\psi)](\psi - Ku)D_0/E_0 \quad (12.2)$$

But

From (4) $V^{LSub}{}_0 = (Kd^{NS} - Kd^{Sub})/(1+\lambda)$ and

From (5) $V^{TS}{}_0 = Kd^{Sub} \times T \times D_0/(1+\psi) = D_0 \times T \times Kd^{Sub}/(1+\psi)$

then

$$Ke = Ku + (D/E)(Ku - Kd^{Sub}) + V^{LSub}{}_0(\lambda - Ku)/E + V^{TS}{}_0(\psi - Ku)/E \quad (12.3)$$

If we assume that the appropriate discount rate for the interest subsidy and for the TS is equal to the cost of unleveraged equity, then the third and fourth terms in equation 12.2 are zero.

### Derivation of $WACC^{CCF}$

We now derive the WACC for the capital cash flow, CCF. From (8) we can write the following

$$V^L{}_0 \times (1 + WACC^{CCF}) = CCF_1 = FCF_1 + LSub_1 + TS_1 \quad (13)$$

and

$$V^L{}_0 \times (1 + WACC^{CCF}) = CCF_1 = V^{Un}{}_0(1+Ku) + V^{LSub}{}_0 \times (1+\lambda) + V^{TS}{}_0 \times (1+\psi) \quad (14a)$$

As per (9) then

$$V^L{}_0 \times WACC^{CCF} = V^{Un}{}_0 \times Ku + V^{LSub}{}_0 \times \lambda + V^{TS}{}_0 \times \psi \quad (14b)$$

and

$$V^{L}_{0}\times WACC^{CCF} = (V^{L}_{0}-V^{LSub}_{0}-V^{TS}_{0})\times Ku + V^{LSub}_{0}\times\lambda + V^{TS}_{0}\times\psi \quad (14c)$$

Rearranging terms

$$V^{L}_{0}\times WACC^{CCF} = V^{L}_{0}\times Ku + V^{LSub}_{0}\times(\lambda-Ku) + V^{TS}_{0}\times(\psi-Ku) \quad (14d)$$

Dividing by $V^{L}_{0}$

$$WACC^{CCF} = Ku + (V^{TS}_{0}/V^{L}_{0})(\ \psi-Ku) + (V^{LSub}_{0}/V^{L}_{0})(\lambda-Ku) \quad (14e)$$

DERIVATION OF WACC$^{FCF}$

Now we derive the WACC to be applied to the FCF. As before, from (8) we can write the following

$$V^{L}_{0}\times(1+WACC^{FCF}) + LSub_1 + TS_1 = FCF_1 = CFE_1+CFD_1 = CCF \quad (15a)$$

$$V^{L}_{0}\times(1+WACC^{FCF}) + V^{LSub}_{0}\times(1+\lambda) + V^{TS}_{0}\times(1+\psi) = V^{L}_{0}\times(1+WACC^{CCF}) \quad (15b)$$

Replacing the expression for WACC$^{CCF}$ we have

$$V^{L}_{0}\times(1+WACC^{FCF}) + V^{LSub}_{0}\times(1+\lambda) + V^{TS}_{0}\times(1+\psi)$$
$$= V^{L}_{0}\times(1+Ku + (V^{TS}_{0}/V^{L}_{0})(\psi-Ku) + V^{TS}_{0}/V^{L}_{0}(\lambda-Ku)) \quad (15c)$$

But

$V^{LSub}_{0}\times(1+\lambda)$=Sub and
$V^{TS}_{0}\times(1+\psi)$=TS

Then

$$V^{L}_{0}\times(WACC^{FCF}) + Sub + TS$$
$$= V^{L}_{0}\times(Ku + (V^{TS}_{0}/V^{L}_{0})(\psi-Ku) + V^{LSub}_{0}/V^{L}_{0}(\lambda-Ku)) \quad (15d)$$

Dividing by $V^{L}_{0}$

$$WACC^{FCF} + Sub/V^{L}_{0} + TS/V^{L}_{0}$$
$$= Ku + (V^{TS}_{0}/V^{L}_{0})(\psi-Ku) + V^{LSub}_{0}/V^{L}_{0}(\lambda-Ku)) \quad (15e)$$

Rearranging terms

$$WACC^{FCF} = Ku + (V^{TS}_{0}/V^{L}_{0})(\psi-Ku) + V^{LSub}_{0}/V^{L}_{0}(\lambda-Ku)) - TS/V^{L}_{0} - Sub/V^{L}_{0} \quad (15f)$$