# NEGATIVE LIQUIDITY PREMIA AND THE SHAPE OF THE TERM STRUCTURE OF INTEREST RATES 

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#### Abstract

From the early 1980's until the late 1990's the term structure of interest rates in Chile was usually downward sloping, particularly for long maturities. Although this fact was common knowledge, no one attempted to explain formally the reason why of this phenomenon. We postulate that the explanation is behind liquidity premium of the term structure of interest rates. Based upon a parsimonious theoretical model, we show that the sign of liquidity premium depends on both expected return and risk.


For our sample period 1983-1999, liquidity premium was negative about 50 percent of the time, and when positive it was very small. This implies that investors were willing to hold long-term assets even though their return was relatively lower. This appeared to be a consequence of the indexation of the Chilean economy, which reduced risk of long-term bonds as their return was linked to past inflation.

The existence of a negative liquidity premium would explain why the term structure of interest rates in Chile was downward sloping for long maturities over our sample period. Data of spreads of Central Bank indexed bonds show that these were usually negative over January 1994-December 1998. However, since 2000 onwards, spreads have switched sign due to an expansionary monetary policy. As a consequence, the term structure has become upward sloping for short maturities, and rather flat for longer maturities. At the same time, a declining inflation rate has made inflation-linked bonds less attractive than before.

JEL classification: E4, G1
Keywords: liquidity premium, $\mathrm{ARCH}-\mathrm{M}$ models.

[^0]
## I Introduction

There has been a long debate in the field of finance about the compensation riskaverse agents should get in order to be willing to hold assets whose future returns are uncertain. For instance, several articles have found that the traditional expectations hypothesis ${ }^{2}$ is inconsistent with the levels of interest rates observed for different maturities (e.g. Shiller, 1979, 1981; Campbell and Schoenholtz, 1983, Mankiw and Summers, 1984). In particular, there is evidence that long-term interest rates exhibit volatility that cannot be explained by this theory. Moreover, some researchers have concluded that predictors implicit in the future interest rates-derivable from the term structure-are both inconsistent and biased. This implies that the one-period excess return, which is unpredictable ex ante, can be forecasted from information currently available in the market.

Such findings have been interpreted as evidence of some sort of bounded rationality or time-dependent liquidity ${ }^{3}$ and risk $^{4}$ premiums of the interest rates. Attempts by Campbell and Schoenholtz (1983) and by Mankiw and Summers (1984) to model bounded rationality have been unsuccessful. Therefore, the assumption of a time-varying risk premium or a time-varying liquidity premium has been generally adopted in later work related to the U.S. term structure of interest rates (e.g., Engle, Lilien and Robins (1987); Adams and Moghaddam (1991)), to inter-temporal models of asset pricing (e.g., Evans (1994), Flannery, Hameed and Harges (1997)), and to volatility of stock returns (e.g., Hyytinen (1999)). In the context of the study of the term structure of interest rates, such an approach has made it possible to explain changes in the slope of the yield curve through fluctuations in the risk and liquidity premiums. An application to the relationship between exchangerate volatility and risk-premium is analyzed by Soto and Valdes (1999) for a sample of 16 countries with different exchange rate systems.

Engle, Lilien and Robins (1987) postulate that the risk premium is due to unanticipated movements in the interest rates, and that this can be quantified by the conditional variance of the one-period excess return of long-term financial instruments. The authors introduce the so-called ARCH-M model, an extension of the ARCH model ${ }^{5}$. The ARCH-M model allows for changes in the conditional variance of the excess return to affect directly the expected return of a portfolio. Engle et al.'s methodology was later applied by Adams and Moghaddam (1991) to the risk premium implicit in the excess returns of municipal bonds over Treasury bills.

Unlike developed economies, one thing that characterized the shape of the term structure of interest rates in Chile over the 1980's and 1990's was its downward slope for long maturities. This phenomenon was previously reported by Lefort and Walker (2000), and Fernandez (2001). However, the economic forces behind such phenomenon have yet to

[^1]be studied. One possible explanation is the existence of a negative liquidity premium. In this article we investigate that possibility resorting to the ARCH-M family.

We also analyze the impact of the curvature of the term structure, expected inflation, expected depreciation of the nominal exchange rate, and of economic activity on changes of the liquidity premium over time. We are particularly interested in studying whether liquidity premium fluctuates with economic agents' perception of the current uncertainty in financial markets. The issue is particularly relevant to pension funds and insurance companies portfolio decisions. Zurita (1999) has argued that Chilean pension funds (AFPs) follow an investment strategy biased towards short-term assets. If a positive liquidity premium existed, then, on average, such a strategy would lead to overpriced pensions. Our estimation results show that, on the contrary, liquidity premium has been predominantly negative in Chile for the time period 1983-1999.

This article is organized as follows. Section II describes the theoretical model upon which our econometric estimation is based. Section III presents and discusses the results for the Chilean economy obtained from data of commercial bank deposits and indexed bonds issued by the Central Bank of Chile during 1983-1999. Finally, Section IV presents a summary of our main findings.

## II Model

Our model extends Engle et al (1987)'s work. Let us consider a two-period world with a short-maturity asset and a long-maturity asset, both of which are risky. The shortmaturity asset (the numeraire) has a price of 1 , one period until maturity, and provides a rate of return $\mathrm{q}_{1}$ with expectation and variance $\theta_{1}$ and $\phi_{1}$, respectively. The long-maturity asset has a price of p , two periods until maturity, and provides a return of $\mathrm{q}_{2}$-measured in terms of the short-maturity asset-with expectation and variance, $\theta_{2}$ and $\phi_{2}$, respectively. The covariance between the two returns is $\eta$.

The excess return on each dollar invested on the long-maturity asset, measured in units of the numeraire, is given by:

$$
\begin{equation*}
\mathrm{y}=\frac{\mathrm{q}_{2}}{\mathrm{p}}-\mathrm{q}_{1} . \tag{1}
\end{equation*}
$$

This implies that the expectation and the variance of $y$ are, respectively, given by:

$$
\begin{equation*}
\mathrm{E}(\mathrm{y}) \equiv \mu=\frac{\theta_{2}}{\mathrm{p}}-\theta_{1} \quad \operatorname{Var}(\mathrm{y}) \equiv \sigma^{2}=\frac{\phi_{2}}{\mathrm{p}^{2}}+\phi_{1}-\frac{2}{\mathrm{p}} \eta . \tag{2}
\end{equation*}
$$

Agents hold $x$ units of the short-maturity asset and $s$ units of the long-maturity asset. The former is held only in period 1, whereas the latter is held in both periods. For simplicity, we assume that an agent hold $s / 2$ units of the long maturity asset each period.

Under the assumption of normally distributed returns and absolute constant risk aversion, expected utility at the end of period 1 is given by:

$$
\begin{align*}
E(U)= & 2 E\left(q_{1} x+\frac{1}{2} q_{2} s+\frac{q_{2} s}{2(1+\rho)}\right)-b \operatorname{Var}\left(q_{1} x+\frac{1}{2} q_{2} s+\frac{q_{2} s}{2(1+\rho)}\right)  \tag{3}\\
& \equiv 2 E\left(q_{1} x+q_{2} \tilde{s}\right)-b \operatorname{Var}\left(q_{1} x+q_{2} \tilde{s}\right),
\end{align*}
$$

where $\rho$ is the agent's discount factor, and $\tilde{s} \equiv \frac{s}{2}\left(1+\frac{1}{1+\rho}\right)=\frac{s}{2}\left(\frac{2+\rho}{1+\rho}\right)$.
That is, only the expectation and variance of the return on the portfolio enter the expected utility function. The maximization of (3) subject to the budget constraint (measured in terms of the numeraire) $\mathrm{W}=\mathrm{p} \tilde{\mathrm{s}}+\mathrm{x}$, and condition (2) leads to:

$$
\tilde{\mathrm{s}}^{*}=\frac{\theta_{2}-\mathrm{p} \theta_{1}+\mathrm{bW}\left(\phi_{1} \mathrm{p}+\eta\right)}{\mathrm{b}\left(\mathrm{p}^{2} \phi_{1}+\phi_{2}-2 \mathrm{p} \eta\right)}
$$

and consequently,

$$
\begin{align*}
\mathrm{p} \tilde{\mathrm{~s}}^{*}= & \frac{\mathrm{p}\left(\theta_{2}-\mathrm{p} \theta_{1}+\mathrm{bW}\left(\phi_{1} \mathrm{p}+\eta\right)\right)}{\mathrm{b}\left(\mathrm{p}^{2} \phi_{1}+\phi_{2}-2 \mathrm{p} \eta\right)}=\frac{\left(\theta_{2}-\mathrm{p} \theta_{1}\right) / \mathrm{p}+\mathrm{bW}\left(\phi_{1}+\eta / \mathrm{p}\right)}{\mathrm{b}\left(\phi_{1}+\frac{\phi_{2}}{\mathrm{p}^{2}}-\frac{2 \eta}{\mathrm{p}}\right)}=\frac{\mu+\mathrm{bW}\left(\phi_{1}+\eta / \mathrm{p}\right)}{\mathrm{b} \sigma^{2}} \\
& =\frac{\mu+\mathrm{bW}\left(\phi_{1}+\eta / \mathrm{p}\right)}{\mathrm{b} \sigma^{2}} . \tag{4}
\end{align*}
$$

From equation (2), it follows that $\mathrm{p}=\frac{\theta_{2}}{\mu+\theta_{1}}$. Therefore:

$$
\frac{\theta_{2} \tilde{\mathrm{~s}}^{*}}{\mu+\theta_{1}}=\frac{\mu+\mathrm{bW}\left(\phi_{1}+\eta / \mathrm{p}\right)}{\mathrm{b} \sigma^{2}}
$$

This in turn implies that the mean excess-return on the long-maturity asset is given by:

$$
\begin{equation*}
\mu=\frac{-\left(\theta_{1}+b W\left(\phi_{1}+\eta / p\right)\right)+\sqrt{\left(\theta_{1}-b W\left(\phi_{1}+\eta / p\right)\right)^{2}+4 b \tilde{s}^{*} \sigma^{2} \theta_{2}}}{2} . \tag{5}
\end{equation*}
$$

The sign of $\mu$ depends upon both the magnitude and sign of the parameters of the model. A special case is when the short-maturity asset is offered at a sure rate ( $\phi_{1}=\eta=0$ ), as in Engle et al. In that scenario, the mean excess-return is always positive. However, as discussed below, the Chilean data is characterized for long periods of negative excess-
returns of long-maturity over short-maturity assets. Therefore, it seems more realistic to allow for the possibility of a negative mean excess-return.

Table 1 shows simulations for different values of the relevant parameters. Scenario 1 describes a situation where the long-maturity asset offers a lower rate of return than the short-maturity asset, but it has lower risk. In this case, 51.82 percent of the total wealth is allocated to the long-maturity asset, and the mean excess return is slightly negative $(-0.83$ percent). In Scenario 2, the short-maturity asset becomes riskier than in Scenario 1. As a result, a bigger share is invested on the long-maturity asset ( 54.2 per cent), and the mean excess return is unambiguously negative ( -6.1 percent). Finally, in scenario 3 the situation reverts and the long-maturity asset is riskier than the short-maturity asset, but it offers a higher rate of return. In this case, a lower share is invested on the long-maturity asset (48.3 per cent), and the mean excess return is positive ( 6.4 per cent). From these simulations it is clear that both the sign and the magnitude of the mean excess return on the long-maturity asset are highly sensitive to the value of the volatility and expected return of the short and long-maturity assets.

## [Table 1 about here]

How does the liquidity premium connect with the excess return on the long-maturity asset? In order to address this point, let us consider the one-period holding return on the long-maturity asset. For simplicity let $\mathrm{p}=1$, and $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ be realizations of the short and long rates, respectively:

$$
\frac{\left(1+\mathrm{q}_{2}\right)^{2}}{1+\mathrm{E}\left(\mathrm{r}_{2}\right)}-1=\frac{\left(1+\mathrm{q}_{1}\right)\left(1+\mathrm{f}_{2}\right)}{1+\mathrm{E}\left(\mathrm{r}_{2}\right)}-1
$$

where $E\left({ }_{1} r_{2}\right)$ is the expected short rate between $t=1$ and $t=2$, and $f_{2}$ is the forward rate. Then the one-period excess return is given by:

$$
\begin{equation*}
\frac{\left(1+\mathrm{q}_{1}\right)\left(1+\mathrm{f}_{2}\right)}{1+\mathrm{E}\left(\mathrm{r}_{2}\right)}-\left(1+\mathrm{q}_{1}\right)=\frac{\left(1+\mathrm{q}_{1}\right)}{1+\mathrm{E}\left(\mathrm{r}_{2}\right)}\left(\mathrm{f}_{2}-\mathrm{E}\left({ }_{1} \mathrm{r}_{2}\right)\right) \tag{6}
\end{equation*}
$$

where $f_{2}-E\left({ }_{1} r_{2}\right)$ is the liquidity premium. Equation (6) implies that the one-period excess return is proportional to the liquidity premium. Moreover, the sign of the excess return is given by that of the liquidity premium. In particular, a negative excess return would be indicative of a negative liquidity premium.

Now, from equation (5), we see that if the variance of the excess return, $\sigma^{2}$, is zero, the excess return is also zero. On the other hand, if $\sigma^{2}$ is large as compared with the other parameters in equation (5), then the excess return will be proportional to the standard deviation, $\sigma$. Therefore, if $\theta_{1}, s^{*}$, and $\theta_{2}$ do not vary much over time when compared with $\sigma^{2}$, the mean excess return and its variance will move in the same direction, but not proportionally. This implies that our specification can be statistically formulated as an ARCH-M model:

$$
\begin{equation*}
\mathrm{y}_{\mathrm{t}}=\mu_{\mathrm{t}}+\varepsilon_{\mathrm{t}} \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{E}\left(\varepsilon_{\mathrm{t}}\right)=0 \quad \operatorname{Var}\left(\varepsilon_{\mathrm{t}} \mid \Pi_{\mathrm{t}}\right)=\mathrm{h}_{\mathrm{t}}^{2} \tag{8}
\end{equation*}
$$

$\Pi_{\mathrm{t}} \equiv$ set of information available at time t , and

$$
\begin{equation*}
\mu_{\mathrm{t}}=\beta+\delta \mathrm{h}_{\mathrm{t}} \quad \mathrm{~h}_{\mathrm{t}}^{2}=\alpha_{0}+\alpha_{1} \sum_{\mathrm{i}=1}^{\mathrm{p}} \omega_{\mathrm{i}} \varepsilon_{\mathrm{t}-\mathrm{i}}^{2} \tag{9}
\end{equation*}
$$

The variable $y_{t}$ represents the excess return on the long-maturity asset, $\mu_{t}$ is the expected value of $y_{t}$ conditional on $\Pi_{t}$, and the error term $\varepsilon_{t}$ is the difference between the ex-ante and ex-post excess return. Expression (9) assumes that the mean of the excess return is a linear function of the conditional standard deviation of the excess return, $\mathrm{h}_{\mathrm{t}}$, and that changes in variance are reflected less than proportionally in the mean.

The model stated in (7) through (9) can be generalized to allow for the presence of exogenous regressors:

$$
\begin{equation*}
\mu_{\mathrm{t}}=\boldsymbol{\beta}^{\prime} \mathbf{x}_{\mathrm{t}}+\delta \mathrm{h}_{\mathrm{t}} \tag{10}
\end{equation*}
$$

so that:

$$
\begin{equation*}
\mathrm{E}\left(\mathrm{y}_{\mathrm{t}} \mid \mathbf{x}_{\mathrm{t}}, \Pi_{\mathrm{t}}\right)=\boldsymbol{\beta}^{\prime} \mathbf{x}_{\mathrm{t}}+\delta \mathrm{h}_{\mathrm{t}} \quad \operatorname{Var}\left(\mathrm{y}_{\mathrm{t}} \mid \mathbf{x}_{\mathrm{t}}, \Pi_{\mathrm{t}}\right)=\mathrm{h}_{\mathrm{t}}^{2} \tag{11}
\end{equation*}
$$

Such a formulation makes it possible to model the excess return not only as a function of financial innovations, but also as a function of observable economic indicators. For example, given that the Central Bank of Chile adjusts the so-called 'stance-of-monetary policy rate' (tasa de instancia monetaria) according to past fluctuations of the monthly indicator of economic activity ('Indice Mensual de Actividad Económica’, IMACEC), one would expect the excess return to be a function of the IMACEC.

In order to take account of the fact that the conditional variance of the excess return responds asymmetrically to decreases and increases in the excess return, we will consider alternative functional forms, such as the Threshold ARCH (T-ARCH):

$$
\begin{equation*}
\mathrm{h}_{\mathrm{t}}^{2}=\alpha_{0}+\alpha_{1} \sum_{\mathrm{i}=1}^{\mathrm{p}} \omega_{\mathrm{i}} \varepsilon_{\mathrm{t}-\mathrm{i}}^{2}+\gamma \mathrm{d}_{\mathrm{t}-1} \varepsilon_{\mathrm{t}-1}^{2}, \tag{12}
\end{equation*}
$$

where $d_{t-1}=1$ if $\varepsilon_{t-1}$ is negative-the ex-post excess return at $t-1$ is lower than expected-('bad news'), and $\mathrm{d}_{\mathrm{t}-1}=0$ otherwise. Therefore, in this model, good news and bad news have differential effects on the conditional variance. Good news has an impact of $\alpha_{0}$, while bad news has an impact of $\alpha_{0}+\gamma$. If $\gamma>0$, the leverage effect exists, while if $\gamma \neq 0$,
the news impact is asymmetric. A good revision of this family of non-linear models is in Engle and Ng (1993).

## III Estimation Results

In this section we present an application for commercial banks deposits, and bonds issued by the Central Bank of Chile. The sample comprises data of deposits from January 1983 to April 1999, whereas for bonds issued by the Central Bank the sample comprises the time period February 1993-December 1998. ${ }^{6}$

In order to give a flavor of how the financial market works in Chile, it is important to mention that, due to indexation, most transactions with maturities over two years carried out by commercial banks are indexed according to past inflation. Specifically, longer-term deposits and loans are denominated in what is known as the 'U.F' (Unidad de Fomento). The U.F is an accounting measure, whose daily variation depends on the previous month inflation rate.

As of the sample period, the Central Bank of Chile issued bonds with maturities ranging from 42 days to 20 years. Non-indexed bonds were issued for maturities of 42,90 , 180 , and 360 days, whereas indexed bonds were issued for maturities of $8,10,12$, and 20 years. No bonds with maturities between 360 days and 8 years were issued by the Central Bank. However, in secondary markets government bonds with maturities ranging from 5 to 15 years were traded. ${ }^{7}$ These are known as Bonos de Reconomiento (literary, Bonds of Validation), which were issued when the old 'pay-as-you-go' pension scheme changed to the current scheme of individual capitalization accounts in 1980-known as the AFP system (Administradoras de Fondos de Pensiones). Table 2 presents some figures of the Chilean economy for the sample period. As we see, inflation has declined since the mid1990's and the growth rate has been relatively high.
[Table 2 about here]

### 3.1 Commercial Banks Instruments

In this section we present an application for commercial banks deposits. The sample period covers January 1983 through April 1999. In order to carry out the estimation it was necessary to make some simplifying assumptions. Monthly data available for deposits, both non-indexed and indexed, are aggregate into two categories: operations whose maturity range from 30 to 89 days, and operations whose maturity range from 90 to 365 days (source: Central Bank of Chile). There is no public information that makes it possible to learn the temporal composition of those deposits within categories. Therefore, in order to compute the excess return of 90-365 days operations over 30-89 days operations, we considered the former as 90 -day operations and the latter as 1 -month operations. This

[^2]assumption seems a reasonable approximation because most $90-365$ day deposits correspond with 90-day deposits, and most 30-89 deposits are actually 30 -day deposits. We later analyze disaggregate data of deposits to check the robustness of our results.

Therefore, the excess holding yield of 90-365 day deposits over 30-89 day deposits is given by:

$$
\begin{equation*}
y_{t}=\frac{\left(1+R_{t}\right)^{3}}{\left(1+r_{t+1}\right)\left(1+r_{t+2}\right)}-\left(1+r_{t}\right) \tag{13}
\end{equation*}
$$

Equation (13) shows the excess return of a 90-day deposit over the strategy of rolling over a 30-day deposit. Specifically, $r_{t}$ represents the monthly return of a 30-day deposit, whereas $R_{t}$ indicates the monthly return of a 90-day deposit. As indicated previously, our proxy for $r_{t}$ is the rate paid on $30-89$ day operations, while $R_{t}$ is approximated by the rate paid on 90-365 day operations.

Notice that equation (13) can be written in terms of forward rates:

$$
\mathrm{y}_{\mathrm{t}}=\frac{\left(1+\mathrm{r}_{\mathrm{t}}\right)}{\left(1+\mathrm{r}_{\mathrm{t}+1}\right)\left(1+\mathrm{r}_{\mathrm{t}+2}\right)}\left\{\left(1+\mathrm{f}_{\mathrm{t}+2}\right)\left(1+\mathrm{f}_{\mathrm{t}+3}\right)-\left(1+\mathrm{r}_{\mathrm{t}+1}\right)\left(1+\mathrm{r}_{\mathrm{t}+2}\right)\right\}
$$

where $\tilde{f} \equiv \sqrt[2]{\left(1+f_{t+2}\right)\left(1+f_{t+3}\right)}$ and $\tilde{r} \equiv \sqrt[2]{\left(1+r_{t+1}\right)\left(1+r_{t+2}\right)}$ can be interpreted as geometric averages of forward and short rates. Such an interpretation makes it possible to relate the excess return in equation (13) to the liquidity premium.

Figure 1 shows our estimation of the excess return calculated according to formula (13), and the spread of the interest rates paid on $90-365$ day and $30-89$ day deposits (quarterly basis). The figures are monthly, and correspond to the time period January 1983April 1999. In order to construct the excess-return series, some observations had to be omitted from the estimation, so that the actual sample comprises January 1983-November 1998. Both series are stationary at the 1 percent level according to a Phillips-Perron unitroot test. (The statistic takes on the value of -10.51 for the excess return, and -12.49 for the spread series. The 1-percent critical value is -3.47 , for a Barlett kernel smoothing parameter equal to 4).
[Figure 1 Excess Return and Interest Rate Spread of Non-Indexed Bank Deposits]
The mean of the excess return for our sample period is approximately zero, but the series was highly volatile over our sample period. For example, the maximum nominal excess return of a balanced portfolio based on the strategy of lending at the 90-365 day rate and borrowing at the $30-89$ day rate was 61.7 percent per quarter ( 250 percent per year). Meanwhile the minimum excess return of such a portfolio was -13.25 percent per quarter ( -53 percent per year). Table 3 shows descriptive statistics of the excess return for different value intervals. As we see, about 50 percent of all observations takes on negative values, illustrating our point of the existence of a negative liquidity premium.
[Table 3 about here]
In turn the sample mean of the spread of the interest rates paid on the 90-365 day and 30-89 day deposits was -0.17 percent points in an annual basis, while the sample maximum and minimum were 30 and -19 percent points per year, respectively. The nonparametric estimate of the density of the excess return is both highly leptokurtic and skewed to the right, as shown in Figure 2 (a). The non-parametric estimate of the density of the spread by contrast presents both lower asymmetry and kurtosis, but still higher than those of a normal distribution, as Figure 2 (b) shows.
[Figures 2 (a), (b)]

Table 4 presents the TARCH-M model that allows for the presence of economic variables to explain the evolution of the excess return over time. The variables chosen are the 12 -month percent change in the IMACEC, the spread of 90-365 day and 30-89 day deposits (term premium), the expected depreciation of the nominal exchange rate (Chilean pesos per U.S. dollar)-measured from the uncovered parity of interest rates, and expected inflation-measured by the difference between the interest rates paid on 90-356 day nonindexed deposits and 90-365 inflation-linked deposits.
[Table 4 about here]
The 12-month percent variation in the IMACEC has a negative impact on the excess return. In particular, a 100 -basis point (bsp) increase in the annual growth of IMACEC reduces the excess return in 38 bsp per year (i.e., 0.38 percent points). Intuitively, booms in economic activity lead, on average, to a lower spread of interest rates, possibly in response to a tighter monetary policy of the Central Bank. In addition, we see that the excess return is positively correlated with the spread of interest rates, and that an increase in expected depreciation of the nominal exchange rate leads to a higher excess return, as medium-term investors have to be compensated for a more depreciated peso. (In particular, a 100-bps increase in monthly depreciation would require a compensation of 3.07 bps per month).

An increase in expected inflation affects volatility positively, but in an almost negligible fashion. Finally, the coefficient on $\varepsilon_{t-1} d_{t-1}$ is both positive and highly significant, suggesting the existence of a leverage effect in the volatility of the excess return. ${ }^{8}$ Figure 3 shows two estimates of $\mathrm{h}_{\mathrm{t}}$ : the one obtained from the estimation reported in Table 4, and the so-called naive estimate. The latter is calculated as the absolute monthly change in the excess return.
[Figure 3 about here]

[^3]Figure 4 shows the forecast of the excess return $\pm$ two standard deviations. As we see, the episodes of highest volatility took place in the mid-1980's, and at the beginning of the 1990's. More precisely, the peak of the volatility of the excess return is observed during the economic contraction of 1990 and 1991. (This can also be seen in Figure 3). This suggests that periods of more economic uncertainty translate into a greater volatility of the excess return.

## [Figure 4 about here]

As pointed out above, the spread of non-indexed interest rates has a positive and statistically significant impact on the excess return. This implies that information contained in the current term structure of interest rates would make it possible to predict the excess return more accurately. Mankiw and Summers (1984) point that such a phenomenon would imply a violation of the expectations hypothesis. In its purest form, this states that all financial assets have the same expected return for a given holding period, $\tau$. That is to say,

$$
\begin{equation*}
\mathrm{E}_{\mathrm{t}}\left(\mathrm{r}_{\mathrm{L} ; \mathrm{t}, \mathrm{t}+\tau}-\mathrm{r}_{\mathrm{C} ; \mathrm{t}, \mathrm{t}+\tau}\right)=0 \tag{14}
\end{equation*}
$$

where L and C stand for a long and short-holding period, respectively.
Engle et al also find that the spread of interest rates is positively correlated with the excess return, but only at the 10 percent level. The authors point out that it is not surprising to find some residual effect on the spread, because the expected value of the spread is approximately proportional to the current excess return. Given that the spread shows some persistence, it contains information about the next-period excess return. ${ }^{9}$ If there is additional information to predict the excess return, which is not contained in the past innovations, then one would expect that past observations of the spread would explain to some extent fluctuations in the excess return. We tested an alternative specification in which we allowed for the conditional variance of the excess return to depend on the spread. We concluded that the direct effect of the spread on the excess return continues to be statistically significant, but the spread does not help to better predict the variance of the excess return.

Given the availability of daily data for non-indexed and inflation-linked deposits, we analyze the behavior of the excess return at a disaggregate level. Figure 5 shows the daily excess return computed for indexed 360-day deposits for the sample period December 1992-November 1999. The investment strategy considered was holding a 360-day deposit as opposed to the alternative of rolling over a 180-day deposit. As we see, the excess return is always positive approximately between December 1992 and March, 1995, but it becomes negative during all 1997 and the beginning of 1998. This again suggests an anti-cyclical behavior of the excess return, given that the GDP growth was relatively high until 1999. That is, prior to the aftermath of the 1997 Asian crisis.

[^4]Figure 6 shows the excess return of a 60-day non-indexed deposit over a 30-day non-indexed deposit. The portfolio strategy considered was to compare the interest rate paid on a 60-day deposit vis-à-vis that obtained by rolling over a 30-day deposit. As we see, the return of such a strategy is negative for most of the sample period.
[Figures 5 and 6 about here]
Table 5(a) shows descriptive statistics for the 360-day excess return. This was negative for about 32 percent of the observations, and below 2 percent per quarter for about 96 percent of the sample. Meanwhile the maximum was 3.8 percent per quarter. In turn Table 5(b) shows that the 60-day excess return for non-indexed bank deposits was negative for 76 percent of the sample, and the maximum excess return was only 1.15 percent per quarter.

## [Table 5 about here]

In order to estimate ARCH-M type models for the daily data, we had to compute daily series of expected inflation and expected depreciation of the nominal exchange rate. The latter was compute from the 1 -month LIBOR (in U.S. dollar) and the domestic 1month interest rates (source: Bloomberg). The daily series of expected inflation was constructed as follows. We had daily data on non-indexed interest rates paid on 30 and 60day deposits. However, no data is publicly available on interest rates paid on 90-day nonindexed deposits. Therefore, we estimated the 90 -day non-indexed rate by considering a strategy of rolling a 60 day-deposit over a 30 -day period at the current 30-day rate:

$$
\begin{equation*}
\hat{\mathrm{r}}_{90-\text { day }}=\frac{1}{3}\left(\left(1+\frac{\mathrm{r}_{60-\text { day }}}{12}\right)^{2}\left(1+\frac{\mathrm{r}_{30-\text { day }}}{12}\right)-1\right) \tag{15}
\end{equation*}
$$

where $\mathrm{r}_{60 \text {-day }}$ and $\mathrm{r}_{30 \text {-day }}$ are annualized rates, and $\hat{\mathrm{r}}_{90-\text { day }}$ is the estimated 90 -day non-indexed rate in a monthly basis. Expected inflation was approximated by the difference between $\hat{\mathrm{r}}_{90-\text { day }}$ and the interest rate paid on 90-day inflation-linked deposits, for which daily data is available (source: Bloomberg). The result of our estimation for the sample period, December 1992-November 1999, is depicted in Figures 7 a and b.

## [Figures 7a and $b$ about here]

It is worth noticing that our estimate of expected inflation, within any given month, is relatively stable. This makes sense because we would not expect sharp changes of expected inflation overnight, particularly in an economy where one-digit annual inflation has been the rule since 1994. Also, it is noticeable that expected inflation has gone down over time as realized inflation has decline, and so has its volatility.

Table 6 shows our estimation for inflation-linked deposits. As we see, the interest rate spread helps explain the variation observed in the excess return. All estimates, except for expected inflation, are statistically significant, and we again observe evidence in favor
of a time-varying liquidity premium. The dummy variable labeled as 'adjustment of the monetary-stance policy rate' takes account of the sharp increase experienced by the interest rate controlled by the Central Bank of Chile in mid-September, 1998. Indeed, the monetarystance policy rate was raised from 8.5 percent to 14 percent (annualized, 'real' rate), to then be lowered to 12 percent in mid-October. The rate was again reduced at the beginning of November to reach 10 percent. At the end of December 1998 the rate was 7.8 per cent.
[Table 6 about here]
As shown in the variance equation of Table 6, these episodes of sharp fluctuations of the monetary-stance policy rate led to a higher volatility of the excess return (although the effect is quite small). Figure 8 also illustrates how the sharp increase in the monetarystance policy rate transmitted to the 90 -day Central Bank indexed bond rate.
[Figure 8 about here]
Finally, Table 7 shows our estimation for the 60-day deposit excess return. We again find evidence about a time-dependent liquidity premium. As suggested earlier by the aggregate data, depreciation of the nominal exchange rate is both highly significant and positively correlated with the excess return. In turn the spread has a statistically significant but almost negligible impact on volatility. ${ }^{10}$
[Table 7 about here]

### 3.2 Long-Maturity Financial Instruments: Central Bank of Chile Indexed Bonds

As explained earlier, due to the indexation of the Chilean economy, most financial instruments yields are linked to past inflation. Indexed bonds are denominated in the Unidad de Fomento (U.F.), whose value changes daily according to the previous month percent variation of the CPI (daily basis). In this section, we focus on indexed bonds issued by the Central Bank of Chile. The maturity of its debt ranges from 90 days to 20 years. Over our sample period, indexed bonds were issued for 90 days, $8,10,12$, and 20 years. ${ }^{11}$

In order to analyze the liquidity premium of long-term bonds, we concentrated on a 3-month horizon investment strategy involving a zero-coupon 90-day bond (Pagaré Reajustable del Banco Central de Chile, PRBC), and 10 and 20-year bonds that pay biannual coupons (Pagaré Reajustable con Cupones, PRC). For example, using biannual compounding, the quarterly excess return of a 20 -year bond that pays a biannual coupon over a 90 -day zero-coupon bond-where both bonds have a face value equal to 1 -is given by:

[^5]\[

$$
\begin{equation*}
y_{t}=\left(\frac{1}{2} \sum_{i=1}^{40} \frac{R_{t}}{\left(1+\frac{R_{t+1}}{2}\right)^{i}}+\frac{1}{\left(1+\frac{R_{t+1}}{2}\right)^{40}}-1\right)-\left(\frac{\left(1+\frac{r_{t}}{2}\right)^{\frac{1}{2}}}{\left(1+\frac{r_{t+1}}{2}\right)^{\frac{1}{3}}}-1\right) \tag{16}
\end{equation*}
$$

\]

where $R_{t}$ and $r_{t}$ are the annualized rates of the 20 -year and 90 -day bonds at time $t$, respectively.

An approximation to the above formula is:

$$
\begin{equation*}
\mathrm{y}_{\mathrm{t}}=\ln \left(1+\frac{\mathrm{R}_{\mathrm{n}, \mathrm{t}}}{2}\right)-\ln \left(1+\frac{\mathrm{r}_{\mathrm{t}}}{2}\right)-(\mathrm{n}-1)\left[\ln \left(1+\frac{\mathrm{R}_{\mathrm{n}-1, \mathrm{t}+1}}{2}\right)-\ln \left(1+\frac{\mathrm{R}_{\mathrm{n}, \mathrm{t}}}{2}\right)\right] . \tag{17}
\end{equation*}
$$

Formula (17) gives the natural logarithm of the one-period excess return for a zerocoupon bond with maturity of $n$ periods ( $n=40$ semesters, for a 20 -year PRC). When the time horizon is short enough, such that there are no coupon payments, (17) yields a very good approximation of (16).

Figure 9 shows the excess returns of a 20 -year PRC over a PRBC, and of a 10 -year PRC over a PRBC for the sample period February 1993-December 1998. The sample mean of the 20 -year 'real' excess return is 0.8 percent per year, while the sample mean of the spread of the 20 -year and 90 -day bond rates is -0.72 basis points per year. For a 10 -year PRC and a PRBC these figures are 0.4 percent per year and -0.54 basis points per year, respectively.
[Figure 9 about here]
The data suggest, therefore, that on average there is a positive liquidity premium in the term structure of indexed bonds, but that this is relatively small for the sample period. Table 8 sheds more light on this point. For instance, the excess return on 10-year indexed bonds and 20-year indexed bonds were negative for 49 percent and 52 percent of the sample, respectively. This shows that, despite the fact that excess returns were on average positive, half the sample shows evidence of a negative liquidity premium.

## [Table 8 about here]

Tables 9 and 10 show our estimation results for the 10 -year and 20 -year PRC, respectively. In this case, the regressors included in the ARCH-models are the 12-month percent change in the IMACEC, interest rate spreads, expected depreciation of the nominal exchange rate, and a proxy for country risk. The latter is measured as the difference between the U.S. denominated 90-365 day deposit rate and the 180-day LIBOR rate.
[Tables 9 and 10
about here]
We find in both cases that there is evidence of a time-varying liquidity premium, being this larger for 20-year PRC (that is, for the longer-maturity bond). The impact of 12month percent variations in the IMACEC on the excess return is statistically insignificant
for both maturities, which contrasts with our evidence for bank deposits. This may be due to the fact that bank deposits involve shorter maturities, for which fluctuations in the monthly indicator of economic activity might be more relevant. It is interesting to see that the spread is negatively correlated with the excess return of both bonds, in particular for the 20 -year PRC. This phenomenon has been previously reported for Chile by Lefort and Walker (2000). The authors argue that the interest rate spread is positive when the return on the long-maturity bond is low relative to its long-run mean. Therefore, the excess return on long-maturity bonds would be lower as the spread increases.

The expected depreciation of the nominal exchange rate is only statistically significant in the 10 -year PRC equation, while our proxy for country risk does not play any role to explain fluctuations of the excess return in either case. We also quantified the effect of changes in the spread of interest rates on the volatility of the excess return. In both cases, an increase in the spread of interest rates leads to higher volatility of the excess return, although the overall impact is relatively small.

In summary, our estimation results for bank deposits and long-maturity bonds show the excess return on long-maturity assets, and therefore liquidity premium, is not only timevarying but that it may also depend on the curvature of the term structure, expected inflation, expected depreciation of the nominal exchange rate, and on economic activity, contradicting the expectations hypothesis. In addition, we find that, on average, liquidity premium is close to zero, but about half of the observations show negative excess returns for long term bonds. This implies that there are investors who hold long-maturity assets even though their return is relatively lower than that on shorter-maturity assets. This may be a consequence of indexation, which reduces the risk of long-term bonds as their return is linked to past inflation.

The existence of negative liquidity premia would explain why the term structure of interest rates in Chile has been downward-sloping for long maturities over our sample period. Figure 10 depicts weekly data of the spreads of Central Bank indexed bonds (PRCs), whose maturity ranges from 8 to 20 years, for the time period January 1994December 2000 (source: Bloomberg). It is noticeable that spreads have been usually negative for this time period. (Look, for example, at the spread between 20-year and 14year bonds). However, since 2000 onwards, spreads have become positive due to an expansionary monetary policy. Indeed, Figure 11 shows how the Central Bank's monetary policy rate has sharply declined since 1999 onwards.
[Figures 10 and 11 about here]
Figures 12 a) and b) show monthly estimates of the term structure of interest rates for our sample period, and for January 1999-July 2001. As we see, panel (a) suggests the existence of a downward sloping curve. By contrast the term structure depicted in panel b) exhibits an upward sloping curve for shorter maturities, and it becomes flatter for longer maturities. It is important to mention that, despite an expansionary monetary policy, annual inflation has been fairly low over the last two years. Indeed, annual inflation reached 3.8 and 3.6 percent in 2000 and 2001, respectively. As inflation goes down, long maturity inflation-indexed bonds become less attractive than otherwise. Therefore, this might be
another reason why the term structure is no longer downward sloping for long maturity bonds.

## IV Conclusions

This article has looked at the determinants of liquidity premium of the term structure of interest rates. Based upon a parsimonious theoretical model, we show that liquidity premium is not necessarily positive. This point is illustrated empirically with Chilean data on bank deposits and long-term bonds for the sample period 1983-1999. Our estimation results show that liquidity premium is not only time-varying but that it may also be a function of the curvature of the term structure, expected inflation, expected depreciation of the nominal exchange rate, and of economic activity. These findings contradict the expectations hypothesis, which states that forward interest rates are an unbiased estimate of the expected value of future interest rates.

In addition, for our sample period, we find that liquidity premium is negative for half the sample, and close to zero on average. This fact might a consequence of indexation, which reduces the risk of long-term bonds as their return is linked to past inflation. In particular, the existence of negative liquidity premia would explain why the term structure of interest rates in Chile was downward sloping for long maturities over our sample period. Data of spreads of Central Bank indexed bonds show that these have been usually negative over January 1994-December 1998. However, since 2000 onwards, spreads have switched sign due to an expansionary monetary policy. As a consequence, the term structure has become upward sloping for short maturities, and rather flat for longer maturities.

Previous research for Chile has concluded that liquidity premia have been positive over time. Therefore, if pension funds (AFPs) tend to hold portfolios whose assets have a much shorter duration than that of their liabilities, future pensions would be overpriced. Our study contradicts this view given that liquidity premiums may be actually negative.

## References

Adams, R. and M. Moghaddam (1991), "The Risk Premia in Municipal Bonds: An Application of the ARCH-M Model". Journal of Macroeconomics 13(4), 725-31.

Campbell J. and K. Schoenoltz (1983), "Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates". Brookings Papers on Economic Activity, 173-217.

Engle R., D. Lilien and R. Robins (1987), "Estimating Time-Varying Risk Premia in the Term Structure: the ARCH-M Model" en ARCH Selected Readings, R. Engle editors., pages: 24-41. Advanced Texts in Econometrics. Oxford University Press, 1995.

Engle R. and V. Ng (1993), "Measuring and Testing the Impact of News on Volatility". The Journal of Finance 58 (5), 1749-78.

Evans, M. (1994), "Expected Returns, Time-Varying Risk, and Risk Premia". The Journal of Finance 49(2), 655-79.

Fernandez, V. (2001), "A Non-parametric Approach to Model the Term Structure of Interest Rates: The Case of Chile". The International Review of Financial Analysis 10(2), special issue on the Latin American Financial Markets, 99-122.

Flannery, M., A. Hameed and R. Harjes (1997), "Asset Pricing, Time-Varying Risk Premia and Interest Rate Risk". Journal of Banking and Finance 21(3), 315-35.

Hyytinen, A. (1999), "Stock Return Volatility on Scandinavian Stock Markets and the Banking Industry. Evidence from the Years of Financial Liberalization and Banking Crisis". Bank of Finland Discussion Papers 19/99.

Lefort F. and E. Walker (2000), "Characterization of the Term Structure of Real Interest Rates in Chile". Economia Chilena 3(2), 31-52.

Mankiw, G. and L. Summers (1984), "Do Long-Term Interest Rates Overreact to ShortTerm Interest Rates?". Brookings Papers on Economic Activity, 173-217.

Schiller, R. (1979), "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure". The Journal of Political Economy 87, 1190-219.
___ (1981), "Alternative Tests of Rational Expectations Models: The Case of the Term Structure". The Journal of Econometrics 16, 71-87.

Soto C., and R. Valdes (1999), "Exchange Rate Volatility and Risk-Premium". Working Paper No. 46, Central Bank of Chile.

Zurita F. (1999), "Are Pension Funds Myopic?" en Revista de Administración y Economía 39, Catholic University of Chile, Spring, 13-15.

## TABLES

Table 1 Excess Return on the Long-maturity Asset

## Scenario 1

| $\theta_{1}$ | $\theta_{2}$ | b | W | $\phi_{1}$ | $\phi_{2}$ | $\eta$ | p |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.075 | 0.06 | 1 | 100 | 0.009 | 0.008 | 0.0001 | 1.1 |  |
|  |  | $\tilde{\mathrm{~s}}$ | $\sigma^{2}$ | $\mu$ |  |  |  |  |
|  | 51.821 | 0.0154 <br> Scenario 2 | -0.0083 |  |  |  |  |  |
| $\theta_{1}$ | $\theta_{2}$ | b | W | $\phi_{1}$ | $\phi_{2}$ | $\eta$ | p |  |
| 0.075 | 0.06 | 1 | 100 | 0.01 | 0.008 | 0.0001 | 1.1 |  |
|  |  | $\tilde{\mathrm{~s}}$ | $\sigma^{2}$ | $\mu$ |  |  |  |  |
|  |  | 54.200 | 0.0164 | -0.061 |  |  |  |  |
| $\theta_{1}$ | $\theta_{2}$ | b | Scenario 3 | W | $\phi_{1}$ | $\phi_{2}$ | $\eta$ | p |
| 0.08 | 0.1 | 1 | 100 | 0.008 | 0.009 | 0.0001 | 1.1 |  |
|  |  | $\tilde{\mathrm{~s}}$ | $\sigma^{2}$ | $\mu$ |  |  |  |  |
|  |  | 48.320 | 0.0152 | 0.064 |  |  |  |  |

Table 2 Annual Percent Variations of the CPI and the IMACEC

| Year | CPI (1) | IMACEC (2) |
| :---: | :---: | :---: |
| 1983 | 19.0 | -1.7 |
| 1984 | 19.1 | 6.1 |
| 1985 | 21.7 | 1.8 |
| 1986 | 14.3 | 5.5 |
| 1987 | 17.6 | 6.6 |
| 1988 | 10.5 | 7.3 |
| 1989 | 17.6 | 10.6 |
| 1990 | 22.4 | 3.7 |
| 1991 | 15.4 | 8.1 |
| 1992 | 10.5 | 12.3 |
| 1993 | 10.1 | 7.0 |
| 1994 | 7.4 | 5.7 |
| 1995 | 6.8 | 10.7 |
| 1996 | 5.5 | 7.4 |
| 1997 | 5.0 | 7.1 |
| 1998 | 3.9 | 4.2 |
| Sample Average 1983-1998 | $\mathbf{1 2 . 9}$ | $\mathbf{6 . 4}$ |

Notes: (1) Annual average of monthly rates (in annual terms) (2) Annual average of 12-month percent variations observed every year. Source: Central Bank of Chile.

Table 3 Descriptive Statistics of the Excess Return of 90-365 Day Non-indexed Bank Deposits
Sample: 1983:01 1998:11

| Excess <br> return | Mean | Max | Min. | Std. Dev. | Skewness | Kurtosis | Frequency <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[-0.2,0)$ | -0.040 | -0.001 | -0.200 | 0.0342 | -1.567 | 6.979 | 50.3 |
| $[0,0.2)$ | 0.035 | 0.168 | 0.000 | 0.0349 | 1.889 | 6.961 | 49.2 |
| $[0.4,0.6)$ | 0.508 | 0.508 | 0.508 | -- | -- | -- | 0.5 |
| All | $-9.95 \mathrm{E}-05$ | 0.508 | -0.200 | 0.063 | 2.722 | 24.322 | 100 |

Table 4 TARCH-M Model for the Excess Return of 90-365 Day Non-indexed Bank Deposits
Dependent Variable: Excess Return; Sample Period: 1983:01 1998:11

| Regressor | Coefficient | Standard Error | Asymptotic t-Statistic | Probability |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{h}_{\mathrm{t}}$ | 0.342 | 0.135 | 2.529 | 0.011 |
| Constant $^{2}$ | -0.006 | 0.005 | -1.2513 | 0.211 |
| Percent Change in IMACEC $^{\text {Spread }}{ }^{(1)}$ | $-9.40 \mathrm{E}-04$ | $3.44 \mathrm{E}-04$ | -2.732 | 0.006 |
| Expected Depreciation $^{(2)}$ | 0.030 | 0.002 | 12.657 | 0.000 |


|  | Conditional Variance Equation $\left(\mathrm{h}_{\mathrm{t}}^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Constant | $-1.38 \mathrm{E}-04$ | $1.74 \mathrm{E}-05$ | -7.966 | 0.000 |
| $\varepsilon_{\mathrm{t}-1}^{2}$ | -0.079 | 0.022 | -3.533 | 0.000 |
| $\varepsilon_{\mathrm{t}-1 \mathrm{~d}_{\mathrm{t}-1}{ }^{(3)}}$ | 0.602 | 0.156 | 3.857 | 0.000 |
| $\mathrm{~h}_{\mathrm{t}-1}^{2}$ | 0.636 | 0.0512 | 12.248 | 0.000 |
| Expected Inflation ${ }^{(4)}$ | $4.18 \mathrm{E}-05$ | $4.03 \mathrm{E}-08$ | 1038.123 | 0.000 |
| Log likelihood | 374.3590 | Akaike info criterion |  | -3.815 |
| Engle ARCH test (3 lags) | $1.89(\mathrm{p}$ value=0.59) | Schwarz criterion | -3.645 |  |

Notes: (1) The spread is measured by the difference of the interest rates paid on non-indexed 90-365 day deposits and non-indexed 30-89-day deposits. (2) Expected depreciation of the nominal exchange rate is measured from the uncovered parity of interest rates. (3) $d_{t-1}=1$ if $\varepsilon_{t-1}<0$, and zero otherwise. (4) Expected inflation is measured by the difference between $90-365$ day non-indexed and $90-365$ day inflation-linked deposits. The equation is estimated by the method of quasi maximum likelihood, and the standard errors correspond with the Bollerslev-Wooldrige robust standard errors.

Table 5 Descriptive Statistics of Excess Returns computed with Daily Data
(a) 360 Day Inflation-Linked Bank Deposits

Sample period: December 1992-November 1999 (Daily Data)

| Excess return | Mean | Max | Min. | Std. Dev. | Skewness | Kurtosis | Frequency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[-0.04,-0.02)$ | -0.026 | -0.020 | -0.038 | 0.005 | -0.653 | 2.324 | $2.16 \%$ |
| $[-0.02,0)$ | -0.005 | 0.000 | --0.019 | 0.004 | -0.753 | 3.457 | $30.89 \%$ |
| $[0,0.02)$ | 0.004 | 0.020 | 0.000 | 0.004 | 1.586 | 5.804 | $65.25 \%$ |
| $[0.02,0.04)$ | 0.030 | 0.038 | 0.020 | 0.006 | -0.493 | 1.911 | $1.70 \%$ |
| All | 0.001 | 0.038 | -0.038 | 0.008 | -0.173 | 8.181 | $2.16 \%$ |

(b) Excess Return of 60-Day Non-indexed Bank Deposits

| Sample period: December 1992-November 1999 (Daily Data) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Excess return | Mean | Max | Min. | Std. Dev. | Skewness | Kurtosis | Frequency. |
| $[-0.005,0)$ | $-3.43 \mathrm{E}-04$ | $-1.00 \mathrm{E}-04$ | $-1.80 \mathrm{E}-03$ | $2.49 \mathrm{E}-04$ | -1.752 | 7.383 | $76.00 \%$ |
| $[0,0.005)$ | $2.32 \mathrm{E}-04$ | $3.00 \mathrm{E}-03$ | 0.000 | $4.47 \mathrm{E}-04$ | 3.816 | 20.043 | $23.88 \%$ |
| $[0.01,0.015)$ | $1.15 \mathrm{E}-02$ | $1.15 \mathrm{E}-02$ | $1.15 \mathrm{E}-02$ | 0.000 | -- | -- | $0.12 \%$ |
| All | $-1.91 \mathrm{E}-04$ | $1.15 \mathrm{E}-02$ | $-1.80 \mathrm{E}-03$ | $5.64 \mathrm{E}-04$ | 11.269 | 224.511 | $100 \%$ |

Table 6 ARCH-M Model for the Excess Return of 360 Day Inflation-Linked Bank Deposits

| Dependent Variable: Excess Return; Sample period: December 1992-November 1999 (Daily Data) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Regressor | Coefficient | Standard Error | Asymptotic t-Statistic | Probability |
| $\mathrm{h}_{\mathrm{t}}$ | 0.517 | 0.049 | 10.376 | 0.000 |
| Constant | 0.002 | $5.73 \mathrm{E}-05$ | 30.979 | 0.000 |
| Spread $^{(1)}$ | 0.007 | $3.12 \mathrm{E}-04$ | 21.507 | 0.000 |


|  | Conditional Variance Equation $\left(\mathrm{h}_{\mathrm{t}}^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Constant | $5.96 \mathrm{E}-07$ | $1.11 \mathrm{E}-07$ | 5.347 | 0.000 |
| $\varepsilon_{\mathrm{t}-1}^{2}$ | 0.316 | 0.0386 | 8.179 | 0.000 |
| $\mathrm{~h}_{\mathrm{t}-1}^{2}$ | 0.237 | 0.078 | 3.024 | 0.003 |
| Adjustment of Monetary Policy Rate ${ }^{(2)}$ | $6.76 \mathrm{E}-05$ | $1.19 \mathrm{E}-05$ | 5.665 | 0.000 |
| Expected Inflation ${ }^{(3)}$ | $-1.41 \mathrm{E}-05$ | $9.99 \mathrm{E}-06$ | -1.412 | 0.158 |
| Log likelihood | 6421.999 | Akaike info criterion |  | -8.412 |
|  |  | Schwarz criterion |  | -8.384 |

Notes: (1) The spread is measured by the difference of the interest rates paid on inflation-linked 360 day deposits and inflation-linked 180-day deposits. (2) The variable 'Adjustment of monetary policy rate' is a dummy that takes on the value of 1 between August and December 1998. (3) Expected inflation is calculated using daily data of non-indexed and inflation-linked interest rates deposits. The equation is estimated by the method of quasi maximum likelihood, and the standard errors correspond with the Bollerslev-Wooldrige robust standard errors.

Table 7 ARCH-M Model for the Excess Return of 60-Day Non-indexed Bank Deposits

| Dependent Variable: Excess Return; Sample period: December |  |  |  | 1992-November 1999 (Daily Data) |
| :---: | :---: | :---: | :---: | :---: |
| Regressor | Coefficient | Standard Error | Asymptotic t-Statistic | Probability |
| $\mathrm{h}_{\mathrm{t}}$ | 0.259 | 0.145 | 1.777 | 0.076 |
| Constant | $-3.56 \mathrm{E}-04$ | $3.22 \mathrm{E}-05$ | -11.055 | 0.000 |
| Expected Depreciation ${ }^{(2)}$ | 0.003 | 0.001 | 2.948 | 0.003 |
|  |  | Conditional Variance Equation $\left(\mathrm{h}_{\mathrm{t}}^{2}\right)$ |  |  |
| Constant $^{2}$ | $2.00 \mathrm{E}-08$ | $1.64 \mathrm{E}-08$ | 1.219 | 0.222 |
| $\varepsilon_{\mathrm{t}-1}^{2}$ | 0.150 | 0.048 | 3.148 | 0.002 |
| $\mathrm{~h}_{\mathrm{t}-1}^{2}$ | 0.599 | 0.156 | 3.845 | 0.000 |
| Spread $^{2}$ | $3.67 \mathrm{E}-06$ | $2.23 \mathrm{E}-07$ | 16.418 | 0.000 |
| Log likelihood | 11437.64 | Akaike info criterion |  | -13.616 |
|  |  | Schwarz criterion | -13.593 |  |

Notes (1) The spread is measured by the difference of the interest rates paid on 60-day non-indexed deposits and non-indexed 30 deposits (2) Expected depreciation of the nominal exchange rate is measured from the uncovered parity of interest rates. The equation is estimated by the method of quasi maximum likelihood, and the standard errors correspond with the Bollerslev-Wooldrige robust standard errors.

Table 8 Descriptive statistics of the Excess Returns of Central Bank Bonds
(a) 10-year PRC

|  | Sample: 1993:02 1998:12 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Excess <br> return | Mean | Max | Min. | Std. Dev. | Skewness | Kurtosis | Frequency. |
| $[-0.2,-0.1)$ | -0.129 | -0.129 | -0.129 | -- | -- | -- | $1.41 \%$ |
| $[-0.1,0)$ | -0.020 | 0.000 | -0.056 | 0.015 | -0.688 | 2.483 | $47.89 \%$ |
| $[0,0.1)$ | 0.021 | 0.079 | 0.000 | 0.022 | 1.089 | 2.903 | $49.30 \%$ |
| $[0.1,0.2)$ | 0.125 | 0.125 | 0.125 | -- | -- | - | $1.41 \%$ |
| All | 0.001 | 0.125 | --0.129 | 0.035 | 0.205 | 6.317 | $100 \%$ |

(b) 20-year PRC

| Excess <br> return | Mean | Max | Min. | Std. Dev. | Skewness | Kurtosis | Frequency. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[-0.3,-0.2)$ | -0.211 | -0.211 | -0.211 | -- | - | -- | $1.41 \%$ |
| $[-0.1,0)$ | -0.036 | 0.000 | -0.097 | 0.027 | -0.661 | 2.522 | $50.70 \%$ |
| $[0,0.1)$ | 0.032 | 0.098 | 0.000 | 0.028 | 0.833 | 2.597 | $40.85 \%$ |
| $[0.1,0.2)$ | 0.127 | 0.157 | 0.107 | 0.021 | 0.687 | 1.588 | $5.63 \%$ |
| $[0.2,0.3)$ | 0.231 | 0.231 | 0.231 | -- | -- | -- | $1.41 \%$ |
| All | 0.002 | 0.231 | -0.211 | 0.064 | 0.454 | 5.594 | $100 \%$ |

Table 9 ARCH-M model for the Excess Return of 10-Year Central Bank Bonds (PRC)
Dependent Variable: Excess Return; Sample Period: 1993:02 1998:12

| Regressor | Coefficient | Standard Error | Asymptotic t-Statistic | Probability |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{h}_{\mathrm{t}}$ | 0.749 | 0.258 | 2.905 | 0.004 |
| Constant | -0.024 | 0.008 | -3.198 | 0.001 |
| Percent Change in IMACEC | -0.050 | 0.063 | -0.807 | 0.419 |
| Spread ${ }^{(1)}$ | -0.044 | 0.017 | -2.625 | 0.009 |
| Country Risk ${ }^{(2)}$ | $7.81 \mathrm{E}-04$ | 0.002 | 0.375 | 0.708 |
| Expected Depreciation ${ }^{(3)}$ | $3.52 \mathrm{E}-04$ | $1.28 \mathrm{E}-04$ | 2.755 | 0.006 |
| Conditional Variance Equation ( $\mathrm{h}_{\mathrm{t}}^{2}$ ) |  |  |  |  |
| Constant | $5.07 \mathrm{E}-05$ | $2.32 \mathrm{E}-05$ | 2.187 | 0.029 |
| $\varepsilon_{t-1}^{2}$ | 0.741 | 0.172 | 4.310 | 0.000 |
| $\varepsilon_{\mathrm{t}-2}^{2}$ | -0.531 | 0.183 | -2.891 | 0.004 |
| $\mathrm{h}_{\mathrm{t}-1}^{2}$ | 0.777 | 0.106 | 7.325 | 0.000 |
| Spread | $2.04 \mathrm{E}-04$ | $9.12 \mathrm{E}-05$ | 2.236 | 0.025 |
| Log likelihood | 180.233 | Akaik | info criterion | -4.767 |
| Engle ARCH test (3 lags) | 5.316 (p-value=0.150) | Schw | arz criterion | -4.417 |

Notes (1) The spread is measured by the difference of the interest rates paid on inflation-linked 10 -year Central Bank bonds (PRC) and inflation-linked 90 -day Central Bank bonds. (2) Country risk is measured as the difference between the U.S. dollar denominated 90-365 day deposit rate and the 180-day LIBOR rate. (3) Expected depreciation of the nominal exchange rate is measured from the uncovered parity of interest rates. The equation is estimated by the method of quasi maximum likelihood. The errors of the equation are not leptokurtic, according to a Jarque-Bera test, so the standard errors were not computed by BollerslevWooldrige's method.

Table 10 ARCH-M model for the Excess Return of 20-Year Central Bank Bonds (PRC)

| Dependent Variable: Excess Return; Sample period: 1993:02 1998:12 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Regressor | Coefficient | Standard Error | Asymptotic t-Statistic | Probability |
| $\mathrm{h}_{\mathrm{t}}$ | 0.813 | 0.175 | 4.649 | 0.000 |
| Constant | -0.054 | 0.015 | -3.504 | 0.001 |
| Percent Change in IMACEC | -0.088 | 0.211 | -0.419 | 0.675 |
| Spread ${ }^{(1)}$ | -0.082 | 0.024 | -3.487 | 0.001 |
| Country Risk ${ }^{(2)}$ | $4.82 \mathrm{E}-05$ | 0.002 | 0.019 | 0.984 |
| Expected Depreciation ${ }^{(3)}$ | $3.26 \mathrm{E}-04$ | $2.2 \mathrm{E}-04$ | 1.451 | 0.147 |
| Conditional Variance Equation $\left(\mathrm{h}_{\mathrm{t}}^{2}\right)$ |  |  |  |  |
| Constant | $1.39 \mathrm{E}-04$ | $4.17 \mathrm{E}-05$ | 3.321 | 0.001 |
| $\varepsilon_{\mathrm{t}-1}^{2}$ | 0.885 | 0.189 | 4.687 | 0.000 |
| $\varepsilon_{t-2}^{2}$ | -0.697 | 0.211 | -3.305 | 0.001 |
| $\mathrm{h}_{\mathrm{t}-1}^{2}$ | 0.819 | 0.109 | 7.463 | 0.000 |
| Spread | $4.98 \mathrm{E}-04$ | $1.57 \mathrm{E}-04$ | 3.179 | 0.002 |
| Log likelihood | 129.923 | Akaik | info criterion | -3.349 |
| Engle Test (4 lags) | 8.609 (p-value: 0.072) | Sch | arz criterion | -2.999 |

Notes (1) The spread is measured by the difference of the interest rates paid on inflation-linked 20-year Central Bank bonds (PRC) and inflation-linked 90-day Central Bank bonds. (2) Country risk is measured by the difference between the U.S. dollar denominated 90-365 day deposit rate and the 180-day LIBOR rate. (3) Expected depreciation of the nominal exchange rate is measured from the uncovered parity of interest rates. The equation is estimated by the method of quasi maximum likelihood. The errors of the equation are not leptokurtic, according to a Jarque-Bera test, so the standard errors were not computed by BollerslevWooldrige's method.

## FIGURES

Figure 1 Excess Return of 90-365 day Non-indexed Deposits and Spread between 90-365 day and 30-89 day Non-indexed Deposits Rates


Figures 2 (a), (b) Distribution Functions of the Excess Return of 90-365 Day Non-indexed Deposits and of the Interest Rates Spread between 90-365 Day and 30-89 Day Non-indexed Deposits
(a)


Kernel Density (Normal, $\mathrm{h}=0.0174$ )

Excess return
(b)


Note: The bandwidth was chosen according to Silverman's rule, number of points=100.

Figure 3 Volatility of the Excess Return on 90-365 day Non-indexed Deposits Estimated according to a TARCH-Model and the Naive Method


Note: The naive method calculates the standard deviation as the absolute value of the difference of the excess return (y) between time $t$ and time $t-1,\left|y_{t}-y_{t-1}\right|$.

Figure 4 Forecast of the Excess Return of 90-365 day Non-indexed Deposits


Notes: Indicators of the forecast of the excess return: Root-mean quadratic error $=0.048$, mean absolute error $=0.031$; Theil inequality coefficient $=0.446$; bias proportion $=0.089$; variance proportion $=0.157$.

Figure 5 Excess-return for 360-Day Indexed Deposits over 180-Day Indexed Deposits


Figure 6 Excess Return of 60-Day Non-Indexed Deposits over 30-Day Non-Indexed Deposits


Figure 7 Expectations of Future Inflation
(a) Estimated expected inflation

(b) Volatility of expected inflation


Figure 8
Returns on Central Bank of Chile Indexed Bonds with Maturities Ranging from 90 Days to 20 Years


Figure 9 Excess Returns of Central Bank of Chile 10-Year and 20-Year Indexed Bonds



Figure 10 Interest Rates Spread of Long-Maturity Indexed Bonds issued by the Central Bank of Chile


Note: the sample period covers February 1993-June 2001.
Figure 11 Monetary Policy Interest Rate in Chile: January 1995-August 2002


Source: Central Bank of Chile. Until August 2001 the Central Bank of Chile's stance-of-monetary policy rate was linked to the UF: However, thereafter the rate has become $\mathrm{Ch} \$$-denominated. In order to make figures comparable, we deflated the nominal rate by actual inflation.

Figure 12 Term structure of Interest Rate obtained from Central Bank Bonds

(b) January 1999-July 2001



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[^1]:    ${ }^{2}$ The one that states that the forward interest rate is an unbiased estimate of the expected value of the future short interest rate.
    ${ }^{3}$ The difference between the forward rate and the expected future short interest rate.
    ${ }_{5}^{4}$ An expected return in excess of that on a risk-free security.
    ${ }^{5}$ The ARCH model states that the conditional variance is a lineal function of square innovations or past surprises.

[^2]:    ${ }^{6}$ Due to a more expansive monetary policy during the beginning of 1999 , the Central Bank stopped bond issues from the end of February 1999 to the end of April 1999, approximately. Therefore, there is no data available for that time period.
    ${ }^{7}$ In August 2002, approximately, the Central Bank of Chile started issuing a wider variety of nominal bonds aimed at reducing the indexation existing in the Chilean economy.

[^3]:    ${ }^{8}$ According to Figure 1, in the last four months of 1990 the volatility of the excess return was extremely high, possibly due to the economic slowdown Chile went through in the early 1990's. Therefore, we consider an alternative specification in which we included in the models of Tables 3 and 4 a dummy variable that took on the value of 1 for the time period September-December 1990, and 0 otherwise. This dummy variable turned out to be statistically significant only at the 10 percent level or higher, so we do not report our computations.

[^4]:    ${ }^{9}$ Our data shows that the Box-Ljung statistic is highly significant, at a 95 -confidence level, for up to 12 lags of the spread.

[^5]:    ${ }^{10}$ We tried alternative specifications in which we included the spread in the mean equation, and expected inflation in the variance equation. However, the best fit was obtained from the specification reported in Table 7.
    ${ }^{11} 90$-day inflation linked bonds stopped being issued by the Central Bank of Chile in August 2001.

