WHAT DRIVES REPLACEMENT OF DURABLE GOODS AT THE MICRO LEVEL?

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Abstract

Technological innovations have contributed over the years to an increasing stock of durable goods—those products that are not immediately consumed but provide a stream of services over a long period of time. Indeed, virtually every household in a modern economy owns a refrigerator, a personal computer and an automobile. Given the inter-temporal nature of replacement decisions, the existing literature has resorted to the technique of dynamic programming, and most recently to the theory of stochastic processes.

This article focuses on micro replacement decisions. We survey some representative models of the recent literature, and discuss their empirical testability. In addition, we study replacement of home appliances in the United States, and construct a test statistic that leads to conclude that replacement decisions might be correlated across appliances. Finally, we enrich our analysis by developing a theoretical model in which replacement decisions are interdependent.

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1 Introduction

Technological innovations have contributed over the years to an increasing stock of durable goods—products that are not immediately consumed but provide a stream of services over a long period of time. Indeed, virtually every household in the United States, and to a great extent in the rest of the world, owns or has access to a microwave oven, a cloth washer, a computer, among many other durable goods (see Figures 1 a and b). Despite the rich theoretical body of knowledge existing in different fields (e.g., economics and operations research) to analyze durable goods purchases, only in the past few years have applied researchers succeeded in identifying the forces behind replacement of durable goods.

[Figures 1a and b about here]

Given that time plays an essential role in replacement decisions, dynamic programming has naturally arisen as an adequate mathematical tool to tackle the replacement problem (e.g., Beckmann, 1968; Kamien and Schwartz, 1971; Bertsekas, 1976; Sargent, 1987). The most recent literature has also resorted to the theory of stochastic processes, and characterized the physical decay of a durable good as a Markov process in either discrete time or continuous time (e.g., Rust 1985, 1987; Ye 1990; Dixit and Pindyck 1994; Mauer and Ott, 1995; Huei Yeh, 1997).

However, even though enormous progress has been made on the theoretical ground, most empirical studies of acquisition and replacement of durable goods do not derive from a consumer or firm's optimization process. Instead, they present ad-hoc statistical models developed from the techniques of discrete choice and duration analysis. Exceptions, among others, are the work of Dubin and McFadden (1984), Rust (1987), Caballero and Engle (1991, 1999), Cooper, Haltiwanger, and Power (1999), Lai, Leung, Tao, Wang (2000), and Martin (2001).

The aim of this article is threefold. First, to present some micro replacement models—which are somehow representative of the most recent literature, and discuss their empirical testability. In addition, to briefly refer to the state of the art of demand for durable goods modeling in the field of macroeconomics. Second, to study replacement of home appliances in the United States—particularly, refrigerators and water heaters, and test whether replacement decisions might be correlated across appliances. Finally, to develop a richer framework where households do not replace a durable good in isolation, but where they consider replacing several items simultaneously.

The main contributions of this article are the following. First, to give an overview of how the micro and macroeconomists have tackled the replacement problem. Second, to present an empirical application where the interdependence of replacement decisions is tested. Finally, to obtain an empirical testable model for multiple replacement decisions. The author of this paper is not aware of any study of similar characteristics.

This article is organized as follows. Section 2 goes through micro models by Rust (1985, 1987), Ye (1990), and Mauer and Ott (1995). Section 3 briefly discusses new developments in the demand for durable goods in macroeconomics and other fields. Finally, Section 4 focuses on replacement of home appliances. Specifically, Section 4.1 studies replacement of refrigerators and water heaters, using data from the U.S. Residential Energy Consumption Survey (RECS). Section 4.2 develops a test statistic for interdependence of replacement decisions, and applies it to the estimation results of Section 4.1. Section 4.3

develops a theoretical model for multiple replacement decisions. Finally, the main conclusions are presented..

2 Replacement Decisions in a Dynamic Context

The models surveyed in this section deal with replacement decisions under uncertainty in an infinite-time horizon. In particular, we go through some recent papers where the replacement problem has been tackled by resorting to the theory of stochastic processes and stochastic calculus: Rust (1985, 1987), Ye (1990), and Mauer and Ott (1995). Rust studies the existence of a stationary equilibrium in a market for a durable asset, which physically deteriorates according to a discrete-time Markov process. A similar set-up, but in continuous time, is postulated by Ye. Mauer and Ott in turn show that uncertainty about the arrival of technological innovation may lead to a significant decrease in replacement investment.

We should point out at the outset that the set of models presented in this section is by no means exhaustive. However, these models share some interesting features. In particular, all of them determine the optimal replacement time by the stopping time technique, and assume that equipment physical decay can be characterized as a Markovian process. The concept of stopping time will be particularly useful to the empirical application presented in Section 4. A more complete overview of micro and macro models developed to analyze the demand for durable goods is presented in Section 3.

2.1 Some Micro Replacement Models

Early attempts to analyze the replacement problem have modeled the optimal time until preventive maintenance of a machine must be performed. For example, Beckmann (1968) focuses on a firm that each period decides to either spend on preventive maintenance that will make its current equipment "as good as new;" or to wait until failure occurs, and purchase new equipment. Another paper along the same lines is Kamien and Schwartz (1971).

The most recent literature introduces randomness in the replacement problem by assuming that the physical condition of the piece of equipment deteriorates according to some continuous or discrete stochastic process. For example, Rust (1985, 1987) assumes that a durable good can be described as a discrete Markov process, whose physical condition at time t is described by a nonnegative real number.[2] Continuous Markov processes are analyzed by Ye (1990) and Mauer and Ott (1995), among others.

Specifically, Rust (1985) studies the existence of a stationary equilibrium in a market for a stochastically deteriorating durable asset. His model tracks the trading process of a durable good from its production in the primary market through its sequence of owners in the secondary market, until it is scrapped.

Each durable good is represented as a discrete time Markov process whose state at time t is described by z_t, a nonnegative real number. The level of z_t indicates the degree of physical deterioration of the piece of equipment. The consumer holds at most one durable per period over an infinite time horizon, and chooses an optimal durable selection and replacement policy to maximize the expected utility of owning an infinite sequence of durable goods.

At the beginning of each period, the consumer faces two choices: either to continue with his/her current piece of equipment or to scrap it and replace it. Under a set of assumptions, Rust shows that the optimal replacement policy takes the form: "replace if z_t is greater or equal than z^*, an optimal stopping barrier; do not replace otherwise". Furthermore, the author shows the existence and uniqueness of a stationary equilibrium, in which the distribution of asset lifetimes is the first time passage distribution to the optimal stopping

[2]Kristensen (1994) presents a survey of Markov decision programming techniques applied to the animal replacement problem.

barrier $\overset{*}{z}$. Equilibrium rental rates and durable prices are shown to embody the functional form of population distribution of preferences and technological features of durable goods.

In latter work, Rust (1987) develops a statistical specification for a model of bus engines replacement, similar in nature to that briefly described above. Based upon data for the time-period December, 1974-May, 1985, the author tests whether the decisions on bus engine replacement of the "Madison Metropolitan Bus Company" can be described by an optimal stopping rule.

Rust's base model assumes that the state variable, z_t, is the accumulated mileage since last replacement on the bus engine at time t, and expected per period operating costs are given by an increasing, differentiable function of z_t, $c(z_t, \theta_1)$, where θ_1 is some parameter. Each month, the following discrete decision is faced: (i) perform maintenance on the current bus engine and incur operating costs $c(z_t, \theta_1)$, or (ii) scrap the old bus engine for $\underline{P}$, install a new (or rebuilt) bus engine at cost $\overline{P}$, and incur operating costs $c(0, \theta_1)$. If i_t denote the replacement decision at t, i_t=0 (keep), i_t=1 (replace), then the stochastic process governing $\{i_t, z_t\}$ is the solution for the following regenerative optimal stopping problem[3]:

$$V_\theta(z_t) = \sup_\Pi E\left(\sum_{j=t}^{\infty} \beta^{j-t} u(z_j, f_j, \theta_1) \middle| z_t \right) \qquad (1)$$

where the agent's utility function, u, is given by:

$$u(z_t, i_t, \theta_1) = \begin{cases} -c(z_t, \theta_1) & \text{if } i_t = 0 \\ -(\overline{P} - \underline{P} + c(0, \theta_1)) & \text{if } i_t = 1 \end{cases} \qquad (2)$$

and Π is an infinite sequence of decision rules given by $\Pi=\{f_t, f_{t+1},...\}$, where each f_t specifies the replacement decision at time t as a function of the entire history of the stochastic process

[3] Once the bus engine is replaced, the system regenerates to state x_t=0.

$i_t=f_t(z_t, i_{t-1}, z_{t-1}, i_{t-2}, z_{t-2},...\}$. The expectation in (1) is taken with respect to the controlled stochastic process $\{z_t\}$, whose probability distribution is defined from Π and the transition probability $p(z_{t+1}|z_t, i_t, \theta_2)$:

$$p(z_{t+1}|z_t, i_t, \theta_2) = \begin{cases} \theta_2 \exp(\theta_2 (z_{t+1} - z_t)) & \text{if } i_t = 0, z_{t+1} \geq z_t \\ \theta_2 \exp(\theta_2 z_{t+1}) & \text{if } i_t = 1, z_{t+1} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Expression (3) states that, if the decision is to keep, then accumulated mileage z_{t+1} is drawn from the exponential distribution, $1-\exp\{\theta_2(z_{t+1}-z_t)\}$. If the decision it is to replace, z_t regenerates to state 0, and then z_{t+1} represents a draw from the exponential distribution $1-\exp(\theta_2(z_{t+1}-0))$.

The function $V_\theta(z_t)$ is the value function and is the unique solution for Bellman's equation given by:

$$V_\theta(z_t) = \max_{i_t \in C(z_t)} (u(z_t, i_t, \theta_1) + \beta EV_\theta(z_t, i_t)) \tag{4}$$

where $C(z_t)=\{0, 1\}$, and $EV_\theta(z_t, i_t) = \int_0^\infty V_\theta(y) p(dy | z_t, i_t, \theta_2)$.

The solution for this maximization problem is given by the optimal stationary Markov replacement policy $\Pi=(f, f, ...)$, where f is given by:

$$i_t = f(z_t, \theta_1, \theta_2) = \begin{cases} 1 & \text{if } z_t \geq \gamma(\theta_1, \theta_2) & \text{replace} \\ 0 & \text{if } z_t < \gamma(\theta_1, \theta_2) & \text{do not replace} \end{cases} \tag{5}$$

and $\gamma(\theta_1, \theta_2)$ represents a threshold value of mileage or the optimal stopping barrier.

Rust's data refute the assumption of monthly mileage data being exponentially distributed. Therefore, he considers a class of more general dynamic discrete choice models, which do not necessarily have a closed-form solution for the agent's stochastic control problem. In order to estimate such models, via maximum likelihood, Rust resorts to a "nested fixed point" algorithm. He concludes that this algorithm can be a practical, efficient, and a numerically stable method, and that the data seem consistent with his regenerative optimal stopping model.

Similar in nature to Rust's (1985, 1987), Ye (1990)'s replacement model assumes that the instantaneous maintenance and operation cost increases stochastically with physical deterioration. One appealing feature of Ye's set-up is that it gives rise to a parsimonious structural model that can be fitted to real data. We illustrate this point in Section 4.1 by presenting replacement models of refrigerators and electric water heaters.

Ye assumes that in every instant of time the consumer or firm must decide either to continue paying a rising maintenance and operation cost for the deteriorating piece of equipment; or, to sell it in the secondary market, and pay a fixed cost to purchase a new piece of equipment with a guaranteed low initial maintenance and operation cost. The objective function in this model is the expected total discounted cost of maintenance and operation as well as of purchasing. As in Rust's work, the optimal replacement rule is defined as a stopping barrier.

The instantaneous maintenance and operating cost is represented by x_t. This may also be indicative of the state of the equipment. In particular, as in Rust's set-up, a higher x_t indicates a more physically deteriorated piece of equipment. The evolution of x_t is described by an arithmetic Brownian motion with constant drift, b, and instantaneous volatility, σ, where $b>0$ and $\sigma \geq 0$:

$$dx_t = b\,dt + \sigma\,dW_t \qquad (6)$$

where dW_t represents an increment of a standard Wiener process.

The expected discounted total cost of obtaining the required service from this piece of equipment is given by:

$$K(x)=E[\int_0^{\infty} e^{-rs}x_s ds \mid x_0 = x] \tag{7}$$

where x_s evolves according to (6), r is the discount rate, and x_0 represents the state of the piece of equipment at time zero, which does not necessarily equal that of a new one, x^*.

The installation cost of new equipment is a fixed amount, $\tilde{C}$, and the scrap value of the previous equipment is zero. When x reaches $\bar{x}$, an upper barrier, replacement takes place and the following condition is satisfied:

$$K(\bar{x}) = \tilde{C} + K(x^*) \tag{8}$$

That is, the total cost right before replacement, K(x), equals the total cost after replacement, $K(x^*)$, plus the cost of installing a new piece of equipment, $\tilde{C}$.

The function K(x) is assumed bounded to avoid the problem of explosive behavior:

$$\lim_{x\to\infty} |K(x)| < \infty \tag{9}$$

Ye shows that the solution of K(x) is:

$$K(x) = \frac{e^{\lambda x}}{e^{\lambda\bar{x}} - e^{\lambda x^*}}[\tilde{C} - \frac{\bar{x} - x^*}{r}] + \frac{x}{r} + \frac{b}{r^2} \tag{10}$$

where λ is the positive root of the characteristic equation $(1/2)\sigma^2 p^2+bp-r=0$. The optimal upper barrier is unique, and can be found from the condition $K'(\bar{x})=0$:

$$1 + \lambda(r\tilde{C} + x^* - \bar{x}) = \exp(\lambda(x^* - \bar{x})) \tag{11}$$

In addition, Ye shows that the total operation cost, K(x), at the optimal upper barrier, is increasing in $\tilde{C}$, x^*, and b, and decreasing in r. The derivatives of $\bar{x}$ with respect to $\tilde{C}$, a, b, and x^*, and r are, however, generally ambiguous.

An issue that is tackled neither by Rust or Ye is the existence of technological change. This can be an important determinant of replacement for some durable goods, such as personal computers[4] and automobiles. Based upon a theoretical framework similar in nature to Rust and Ye's, Mauer and Ott (1995) analyze equipment replacement in the presence of uncertainty about the arrival of technological innovation. Their base model considers a firm that operates a machine that produces a fixed level of output for a given maintenance and operation cost. The before-tax cost, C_t, evolves according to a geometric Brownian motion:

$$dC_t = \alpha C_t dt + \tilde{\sigma} C_t dW_t \tag{12}$$

where α and $\tilde{\sigma}$ are the instantaneous drift and the volatility rate, respectively, with $\alpha>0$ y $\tilde{\sigma}\geq 0$, and dW_t represents the increment of a standard Wiener process. All equipment has the same initial maintenance and operation cost $C_N>0$, which subsequently evolves according to (12).

The net purchase price of new equipment is $P(1-\varphi)$, where $\varphi \in [0,1)$ denotes the investment tax credit, and P is the purchase price of a new piece of equipment. For tax purposes, it is assumed that a piece of equipment depreciates exponentially over time at the rate $\delta\geq 0$. Thus, if the piece of equipment was purchased at time zero, its remaining book value at time t is given by $P(1-\varphi)e^{-\delta t}$. For convenience, the elapsed time is modeled as a function of C_t. As a proxy for t, the authors use the expected first passage time from C_N to C_t, E(t). For a geometric Brownian motion, this is given by:

[4] The number of personal computers (PCs) in U.S. households has risen from zero in 1976, when the first 200 Apple I PCs were manufactured, to nearly 43 million in 1997, when 35 percent of all U.S. households had at least one PC (source: U.S. Department of Energy).

$$E(t) = \frac{1}{\alpha - \tilde{\sigma}^2/2} \ln\left(\frac{C_t}{C_N}\right) \tag{13}$$

Using this result, the tax book value can be approximated by $P(1-\varphi)(C_t/C_N)^{-\delta/Z}$, with $Z/\alpha-\tilde{\sigma}^2/2>0$. Then the depreciation tax shield of the machine over the time interval [t, t+dt] equals:

$$\tau\delta P(1-\varphi)\left(\frac{C_t}{C_N}\right)^{-\frac{\delta}{Z}} dt \tag{14}$$

where $\tau \in [0,1)$ is the corporate tax rate.

At some unknown level of C_t, $\overline{C}$, the firm discontinues operation of the piece of equipment, and replace it with a stochastically equivalent one. If V(C) represents the discounted expected value of the after-tax cost from the optimal replacement policy, then:

$$V(C) = \min_{\overline{C}} E\{\int_0^\infty e^{-rt}[C_t(1-\tau) - \tau\delta P(1-\varphi)\left(\frac{C_t}{C_N}\right)^{-\frac{\delta}{Z}} dt \,|\, C_0 = C]\} \tag{15}$$

where C_t evolves according to equation (12), r denotes the discount rate[5], and C_o is the state of the piece of equipment at time zero.

For given values of the parameters r, α, $\tilde{\sigma}^2$, τ, φ, C_N and P, and a functional specification of the relation between salvage value and cost, a solution of $\overline{C}$ can be obtained (see Mauer and Ott for details). On the other hand, the expected replacement cycle, $\overline{T}$ (years), can be computed, once $\overline{C}$ is determined:

$$\overline{T} = \frac{\ln(\overline{C}) - \ln(C_N)}{\alpha - \tilde{\sigma}^2/2} - \frac{\tilde{\sigma}^2}{2(\alpha - \tilde{\sigma}^2/2)^2}\left[1 - \left(\frac{\overline{C}}{C_N}\right)^{1-\frac{2\alpha}{\tilde{\sigma}^2}}\right] \tag{16}$$

This expression yields the mean time it takes for C_t to reach $\overline{C}$, conditional on having started at C_N.

Mauer and Ott carry out some sensitivity analysis on the optimal replacement policy. They specify the relation between salvage value and cost as $S(C_t)=\kappa C_t^{-1}$, with $\kappa>0$. They find, for example, that an increase in the volatility of equipment cost $\tilde{\sigma}$, makes the firm replace less often.[6] Hence, $\overline{C}$, $\overline{T}$ and $V(C_N)$ increase. Similarly, for an increase in the price of new equipment, P.

Mauer and Ott also explore the effect on the firm's replacement decisions of uncertainty about a technological change that lowers the initial maintenance and operation cost of new equipment, C_N. It is assumed that the technological breakthrough follows a Poisson process. Although the firm will not replace the existing equipment right after the innovation takes place, the value function V(.) will change because the firm rationally anticipates that at the next replacement C_N will be lower.

The authors carry out some simulations to measure the impact of changes in C_N and the parameter of the Poisson process ρ on the expected replacement cycle. They find that, for a technological breakthrough that lowers the initial maintenance and operating cost by 10 percent, a jump of ρ from 0 to 0.5 increases the expected replacement cycle by over 40 percent. Intuitively, the firm keeps the deteriorating equipment longer as ρ increases, hoping that technological uncertainty about a reduction in initial operating and maintenance cost will be rapidly resolved.

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[5] More specifically, Mauer and Ott consider a real options set-up, in which the risk of operation cost dW can be spanned by traded financial assets. And, therefore, r is a riskless rate. In Ye's model no point is made as to whether r is a riskless rate or not. So one can simply assume that agents are risk neutral.

3 Other Studies on the Demand for Durable Goods

Given that durable goods provide a stream of services over time, the literature has typically modeled the demand for these goods in the context of an inter-temporal utility maximization process. An example of such an approach is Parks (1974). The author develops a continuous-time model in which at t=0 an individual must choose a consumption and investment path for his/her indefinite future, given a certain income stream and known price paths for a perishable consumer good and for a durable good.[7] Other references of this sort of models are Malcomson (1975) and van Hilten (1991)—an extension of Malcomson's work. The most recent literature has postulated that the demand for durable goods can be modeled as a dynamic programming problem, in which changes in the stock of capital take place at optimal stopping times, as previously discussed..

In the field of macroeconomics, the demand for durable goods has been the focus of several articles. In particular, one topic that has been extensively studied is the impact of transaction costs on the frequency of purchase.[8] Early studies postulated that changes in the aggregate stock of capital could be well explained by the stock adjustment model. Specifically, this assumes the existence of quadratic adjustment costs, so that the change in the stock of capital in period t is a fraction η of the gap between the current and the desired stock. Bar-Ilan and Blinder (1992) point out that this approach has two flaws. First, empirical studies have come up with estimated values of η that seem improbably low to represent speed of adjustment. Second, the assumption that marginal adjustment costs are zero at zero and increasing thereafter seems, in general, hard to believe.

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[6] In the spirit of the real options literature, an increase in volatility makes the option of waiting more valuable.
[7] For an example in discrete time, see Roberts (1978).
[8] Explicit transaction costs, such as large commissions, or implicit transaction costs, such as the search for information on performance characteristics and prices of heterogeneous durable goods.

In the last few years the literature has moved away from the stock adjustment model and emphasized the lumpiness of expenditure on durable goods (e.g., Bar-Ilan and Blinder, 1992; Caballero and Engel, 1991, 1999; Caballero, 1994; Cooper, Haltiwanger, and Power, 1999). In particular, individuals and firms seem to update their durable stock infrequently, and when they do so, their expenditures are large.

One possible explanation is that, durable goods are replaced according to an (S, s) policy (see, for example, Caballero and Engel, 1991, 1999). That is, when the durable stock depreciates to some lower bound, s, a purchase is made to increase the durable stock to S, the upper bound. If the stock remains above s, it is optimal to do nothing. And, therefore, agents' behavior is strictly inertial within the band (S, s). In the limit as transaction costs vanish, the levels of s and S should coincide. An alternative, but somehow similar, approach is presented by Martin (2001). Based upon Grossman and Laroque (1990)'s set-up, the author develops a model where agents employ an optimal stopping rule for the purchase of durable goods defined by two boundaries, and an optimal returning point.

The fields of marketing and economic psychology have also contributed to the analysis of the acquisition of consumer durable goods. This literature has focused primarily on information search and decision making, planning of purchases and the acquisition sequence of durable goods, post-purchase behavior (disposition), and dissatisfaction and complaint behavior. Progress has been also made on determining what factors affect replacement decisions.

In particular, several studies have looked at the importance of demographic and lifestyles variables, perceived obsolescence, styling and fashion, prices, environmental awareness, and uncertainty, among other variables, on the likelihood of replacement. See, for example, Hoffer and Reilly, 1984; Bayus, 1988, 1991; Antonides, 1990; Bayus and Gupta, 1992; Cripps and Meyer, 1994; Marrel, Davidsson and Garling, 1995.

4 An Empirical Application of Replacement of Home Appliances

This section is divided into three parts. In section 4.1 we estimate replacement models for refrigerators and water heaters using a sample of U.S. households from the Residential Energy Consumption Survey (RECS). Based upon our estimation results of Section 4.1, in Section 4.2 we test the dependence of replacement decisions across appliances. Finally, in section 4.3 we develop a more general model where households consider the replacement of more than one appliance at a time.

4.1 A Replacement Model

Fernandez (2000) shows that the probability density function (p.d.f.) of the first passage time for the stochastic process x_t, T, is given by:

$$g_T(t|b, \sigma, \bar{x}, x^*) = \frac{\bar{x} - x^*}{\sigma\sqrt{2\pi t^3}} \exp\left(\frac{-(\bar{x} - x^* - bt)^2}{2\sigma^2 t} \right) \qquad t \geq 0 \qquad (17)$$

where $\bar{x}$ is given by the implicit function $H(\bar{x}, b, \Phi^2) \equiv 1 + \lambda(r\tilde{C} + x^* - \bar{x}) - \exp[\lambda(x^* - \bar{x})] = 0$, 8 is the positive root of the characteristic equation $(1/2)\sigma^2 p^2 + bp - r = 0$, and the parameters b and σ^2 come from the dynamics of x_t, $dx_t = bdt + \sigma dW_t$, as described in Section 2.1.

In order to calibrate our model, we take a sample from the Residential Energy Consumption Survey (RECS). The RECS is a statistical survey of the U.S. Department of Energy that collects energy-related data for occupied primary housing units in the 50 states and District of Columbia. Conducted triennially since 1978, it provides information on the use of energy in residential housing units in the United States. This information includes the physical characteristics of the housing units, the appliances utilized, including space heating and cooling equipment, demographic characteristics of the household, the types of fuels used, and other information that relates to energy use. The RECS also provides energy consumption and expenditures data for natural gas, electricity, fuel oil, liquefied petroleum gas (LPG), and kerosene.

Data for the RECS are obtained from three different sources: on-site 30-minute personal interviews conducted in the housing unit; telephone interviews with the rental agents of those rented housing units that have any of their energy use included in their rent; and, mail questionnaires mailed to the housing units' energy suppliers asking them to provide the units' actual energy consumption. Our sample was taken from the RECS 1990, which contains approximately 5,100 households, out of which 3,398 are homeowners.

One shortcoming of the RECS is that it does not provide information on replacement times. It only records current equipment ages in intervals. For example, category 01=equipment is less than 2 years old; category 02=equipment is between 2 and 4 years old, etc. Therefore, the model parameters cannot be estimated directly from the p.d.f. of replacement times. Instead, the p.d.f. of equipment age, U, must be used. It can be shown that an asymptotic approximation for the p.d.f. of U can be obtained from the renewal theorem (see Fernandez, 2000, for the technical details):

$$f_U(u|b, \sigma, \bar{x}, x^*) = \frac{b}{\bar{x}-x^*}\left[\mathbf{F}\left(\frac{\bar{x}-x^*-bu}{\sigma\sqrt{u}}\right) - \exp\left(\frac{2(\bar{x}-x^*)b}{\sigma^2}\right)\mathbf{F}\left(\frac{-(\bar{x}-x^*)-bu}{\sigma\sqrt{u}}\right)\right] \quad u \geq 0 \qquad (18)$$

where $\Phi(.)$ represents the cumulative distribution function of a standard normal, and $\bar{x}$ is the solution for $H(\bar{x}, b, \Phi^2)=0$.

Characteristics of household 'i' are incorporated into the model through $(\bar{x}-x^*)_i/\sigma_i$:

$$\frac{(\bar{x}-x^*)_i}{\sigma_i}=\exp(\mathbf{b}'\mathbf{z}_i) \tag{19}$$

where $\mathbf{b}$ is a vector of parameters, and $\mathbf{z}_i$ represents a vector of household characteristics. This functional form ensures the non-negativity of $(\bar{x}-x^*)_i/\sigma_i$. For simplicity, the ratio b_i/σ_i is assumed to be constant across households, and equal to b/σ. Under these extra assumptions, an asymptotic approximation to the likelihood function of U_i, the age of current equipment of household 'i', is given by:

$$f_{U_i}(u_i|\tilde{b},\beta,\mathbf{z}_i)=$$

$$\tilde{b}\exp(-\beta'\mathbf{z}_i)\left[\mathbf{F}\left(\frac{\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)-\exp\left(2\tilde{b}\exp(\beta'\mathbf{z}_i)\right)\mathbf{F}\left(\frac{-\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)\right], \tag{20}$$

where $\tilde{b}\equiv b/\sigma$.

For a sample of n independent observations, the likelihood function of equipment age is given by[9]:

$$f_{U_1,U_2,\ldots,U_n}(u_1,u_2,\ldots,u_n|\tilde{b},\mathbf{b},\mathbf{z}_1,\ldots,\mathbf{z}_n)=$$

$$\prod_{i=1}^{n}\tilde{b}\exp(-\mathbf{b}'\mathbf{z}_i)\left[\Phi\left(\frac{\exp(\mathbf{b}'\mathbf{z}_i)-\tilde{b}u_i}{ó\sqrt{u_i}}\right)-\exp\left(2\tilde{b}\exp(\mathbf{b}'\mathbf{z}_i)\right)\Phi\left(\frac{-\exp(\mathbf{b}'\mathbf{z}_i)-\tilde{b}u_i}{ó\sqrt{u_i}}\right)\right] \tag{21}$$

Estimates of $(\bar{x}-x^*)_i$, b_i, Φ_i for household 'i' can be obtained from equation (22), once the estimates for $\tilde{b}$ and $\mathbf{b}$ become available:

$$1+\lambda_i(r\tilde{C}_i+(x^*-\bar{x})_i)=e^{\lambda_i(x^*-\bar{x})_i} \qquad i=1,2,\ldots,n \tag{22}$$

where λ_i represents the positive root of the characteristic equation $(1/2)\sigma_i^2p^2+b_ip-r=0$.

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[9] When estimating the likelihood function we took account of the discreteness of the data. That is to say, that it is necessary to compute the probability of falling into each category.

Our application deals with replacement of refrigerators and water heaters. We first estimate separate replacement models for each appliance, and then test whether replacement decisions are correlated. Figure 2 illustrates how replacement sales have become a sizeable share of total annual shipments of refrigerators and electric water heaters. Indeed, this holds for all consumer durable goods with high market penetrations.

[Figure 2 about here]

Based on the annual average operation costs of new refrigerators from the RECS, and on the "Consumer Reports" (December 1992), we estimated the average price of a new refrigerator in 1990 to be $1,355. Our estimate of the annual operating and maintenance costs of a new refrigerator is $105, which corresponds with the annual operation cost of equipment aged two years or less reported in the RECS. Our estimate of the average price of a new electric water heater in 1990 is $662, and it is based on information provided in the RECS 1990 and the "National Construction Estimator" (1990). From the RECS 1990 its annual operating and maintenance cost is estimated to be $241.

The regressors in the replacement model of refrigerators are a constant, the age of the head of the household (per 10 years), monthly income (per $10,000), a dummy variable that takes on the value of 1 if the household lives in an urban area and 0 otherwise, the size of the refrigerator (cubic feet), family size (number of members), and a dummy variable that takes on the value of 1 if the household has a poor credit rating and 0 otherwise. Households are classified as having a poor credit rating in case they have received aid in terms of food stamps, unemployment benefits or income from AFDC (Aid to Families with Dependent Children) during the 12 months prior to the conduction of the survey.

Table 1 presents the estimation results for the refrigerator data. The exogenous variables that are statistically significant at the 5 percent level are the age of the head of the household and the size of the refrigerator. In particular, the older the head of the household, the less likely he/she will replace his/her piece of equipment. It is possible that older people have higher discount rates or, alternatively, that their preferences may change more slowly. By contrast, a greater refrigerator size accelerates replacement. This may be due to the fact that size is highly correlated with operating costs, after controlling for income, family size, and electricity rate, among others factors, as Table 2 shows. Although income and family size are not statistically significant at the conventional levels, they have the expected sign. That is, as income and family size increase, the gap between $\bar{x}$ and x^* shrinks making replacement more likely.

[Tables 1 and 2 about here]

The estimation results for water heaters are presented in Table 3. The regressors are in this case a constant, age of the head of the household, monthly income, a dummy variable for those households that live in an urban area, a dummy variable for those households for which natural gas is available in their neighborhood, the tank size of the water heater (gallons), family size, and a dummy variable for those households with a poor credit rating. As we see, the regressors statistically significant at the 5 and 10 percent levels are the age of the head of the household, natural gas availability, tank size, and the poor credit rating dummy.

[Table 3 about here]

As before, replacement is less likely as the head of the household becomes older. Natural gas availability and a poor credit rating have the same effect. In particular, natural gas availability might delay replacement because of differentials in operation costs between natural gas and electric powered equipment. Indeed, those households that would like to reduce operation costs by switching from electric to natural gas equipment cannot do it when natural gas is not available in their neighborhoods. Consequently, replacement of electric equipment becomes less likely. A poor credit rating delaying replacement is self-evident. Like in the case of refrigerators, a larger equipment size makes replacement more likely because of its high and positive correlation with operation costs.

Table 4 presents estimates for the difference between the threshold operation cost $\bar{x}$ and the operation cost of new equipment x^*, equipment lifetime, the drift and the standard deviation of the arithmetic Brownian process, b and σ, respectively, and for the total expected discounted costs for both appliances. Our estimate of the expected lifetime of a refrigerator is approximately 16.5 years. If we start up with new equipment, the expected total discounted cost amounts to $4,271.87. We should point out that our lifetime estimate is fairly close to that of the industry: an average lifetime of 16 years with a range of 10 (low)-20 (high) years (source: "A Portrait of the U.S. Appliance Industry 1992", Appliance, September 1992, Dana Chase Publications). For water heaters, our estimates of the expected lifetime and expected total cost are 13.7 years and $5,526.6, respectively. Like for refrigerators, our estimate of equipment lifetime is quite close to that given by the industry in 1992: 14 years with a range of 10 (low)-18 (high) years.

[Table 4 about here]

The overall fit for both models is quite good, as Table 5 shows. The percent error for all age categories of refrigerators is below 10 percent, being the greatest for equipment that are less than 2 years old, and between 5 and 9 years old. For water heaters in turn, the greatest percent error is below 5 percent in absolute value. Finally, Table 6 shows the impact of marginal changes in the value of the regressors on the probability of replacement over time. For example, a cubic-foot increase in refrigerator size leads to an increase of 9.93 per cent in the probability of replacement within 20 years. The overall probability of replacement is very small within the first 9 years, as we might have expected.

[Tables 5 and 6 about here]

4.2 Are Replacement Decisions Independent?

In the previous section we modeled household decisions to replace a given set of appliances independently. However, it may be the case that such decisions are indeed correlated. Theoretically, the demand for durable goods is derived from a utility function that depends on the services these goods provide over time. Therefore, it would not be surprising to observe some degree of either substitution or complementarity in replacement decisions of different durable goods.

In order to test the hypothesis of independent replacement decisions for an individual household, we constructed a set of generalized residuals (Gourieroux and Monfort, 1987) based on the difference between observed and expected elapsed duration.

Equation (18) is derived from the fact that an asymptotic approximation of the p.d.f. of elapsed duration U, is given by:

$$f_U(u) = \frac{S_T(u)}{\mu} \qquad u \geq 0 \tag{23}$$

where $S_T(u)$ is the survival function of completed duration or equipment lifetime, and μ is the expected lifetime (see, for example, Lancaster, 1990, pages 91-93).

From (23), the kth moment around the origin of elapsed duration (current equipment age) is given by:

$$m_k' = E(u^k) = \int_0^\infty w^k \frac{S_T(w)}{\mu} dw = \frac{1}{k+1}\left(w^k \frac{S_T(w)}{\mu} \right)_0^\infty + \int_0^\infty w^{k+1} \frac{g_T(w)}{(k+1)\mu} dw$$

$$= \frac{1}{(k+1)\mu} \int_0^\infty w^{k+1} g_T(w) dw = \frac{\mu_{k+1}'}{(k+1)\mu} \qquad (24)$$

by integration by parts. The term μ_{k+1}' is the (k+1)th moment around the origin of completed duration t, where $\mu_1 = E(t) \equiv \mu$.

From equation (24), and from the fact that for our model the moment generating function of completed duration is $M_T(t) = \exp\left[\frac{\bar{x} - x^*}{ó\sigma^2} (b - \sqrt{b^2 - 2t\sigma^2}) \right]$, the expectation of elapsed duration is given by:

$$E(U) = \frac{\mu_2'}{2\mu} = \frac{(\bar{x} - x^*)\sigma^2 / b^3 + (\bar{x} - x^*)^2 / b^2}{2(\bar{x} - x^*)/b} = \frac{\sigma^2 + (\bar{x} - x^*)b}{2b^2} \qquad (25)$$

given that it can be shown that the variance and expected value of completed duration are $\frac{(\bar{x} - x^*)\sigma^2}{b^3}$ and $\frac{\bar{x} - x^*}{b}$, respectively.

In order to test the independence of replacement decisions, we utilized a score test of the form:

$$\xi_n = \frac{\left(\sum_{i=1}^{n} \hat{\omega}_{1i} \hat{\omega}_{2i} \right)^2}{\sum_{i=1}^{n} (\hat{\omega}_{1i} \hat{\omega}_{2i})^2} \xrightarrow{d} \chi^2(1) \qquad (26)$$

where $\hat{\omega}_{1i}$ and $\hat{\omega}_{2i}$ represent the estimates of the generalized residuals of appliance 1 (refrigerator) and 2 (water heater) for household i, respectively. This test is asymptotically distributed as chi-square with 1 degree of freedom (see Gourieroux and Monfort for more details).

Each residual is computed as the difference between the observed equipment age and its expected value, evaluated at the parameter estimates of Section 4.1:

$$\hat{\omega}_{mi} = u_i - \frac{\hat{\sigma}_i^2 + (\widehat{\bar{x} - x^*})_i \hat{b}_i}{2\hat{b}_i^2} \qquad \text{m=1, 2; i=1, 2,.., n} \qquad (27)$$

In order to compute $\hat{\omega}_{1i}$ and $\hat{\omega}_{2i}$, we had to approximate equipment age. As described earlier, our data set only provides equipment age in intervals. Therefore, for the first category, "less than 2 years old", we took u=1 year. That is, the mid-point between 0 and 2. Similarly, for the categories "2-4 years old", u=3 years; "5-9 years old", u=7; "10-19 years old", u=14.5 years. The fifth age category, however, gathers all equipment aged 20 years or older. In this case, we had to estimate at which age the survival probability becomes negligible for each appliance. Based upon our estimation results of Section 4.1, we found that at age 60 the probability of survival of a water heater equals 0.055 percent (5.5e-4), whereas at age 55 the probability of survival of a refrigerator is 0.09 percent (9 e-4). Therefore, for the last age category, we took u=40 for water heaters, and u=37.5 for refrigerators.

Only those households that own both appliances are considered in our computations. We found positive correlation among residuals of refrigerators and water heaters: 11.6 percent, and $._n$=20.8, being the 95-percent critical value for a Π^2 (1) equal to 3.84. That is, we reject at the 95-percent confidence level the null hypothesis of independence of the residuals of the replacement models of refrigerators and water heaters. In addition, the positive correlation between the residuals indicates that unobservable factors that either accelerate or delay replacement of one appliance will also affect the other in the same direction.

4.3 Estimating Replacement Decisions Simultaneously: Minimum and Maximum Stopping Times

Given the evidence of the previous section, we now model replacement decisions of a set of appliances simultaneously rather than each one in isolation. In order to do so, we resort to the concept of stopping time. Although this has been already used in previous sections of this paper, we now provide a formal definition.

Let t_1, t_2, ..., be a sequence of independent random variables. An integer-valued random variable T is said to be a stopping time for the sequence t_1, t_2, .., if the event {T=n} is independent of t_{n+1}, t_{n+2}, for all n=1, 2, ... This means that we observe the t_n's in sequential order and N denotes the number observed before stopping. If T=n, then we have stopped after observing t_1, t_2, ..., t_n and before observing t_{n+1}, t_{n+2}, ...(see Ross, 1996, page 104).

Let us now consider two independent stopping times, T_1 and T_2, with corresponding cumulative density functions F_{T_1} and F_{T_2}, and density functions f_{T_1} and f_{T_2}. For example, let us think of two home appliances whose times of either technical failure or obsolescence are independent of one another. This is the assumption in Section 4.1. But, how do we reconcile the assumption of independent stopping times with the evidence in Section 4.2? One way to go about it is by thinking that, although stopping times are independent, households replace their appliances jointly.

For instance, a household might wait and replace its obsolete microwave oven until the cutting-edge technology of refrigerators becomes available at the market place.[10] Or, alternatively, the household might decide to replace its microwave oven and refrigerator at once, as soon as the technology of the former falls behind the new trends.

[10] This might be also the case if the durable goods present some degree of substitution. For example, the replacement of a stereo system might not be as urgent given that audio CDs can be played on the personal

If that is so, then we can find the minimum and maximum bounds for the replacement time of both appliances. Let Z_1=max(T_1, T_2) and Z_2=min (T_1, T_2). The probability density function of Z_1 and Z_2 are given by:

$$F_{Z_1}(z) = P(\max(T_1, T_2) \leq z)$$

$$= P(T_1 \leq z, T_2 \leq z)$$

$$= P(T_1 \leq z)\, P(T_2 \leq z)$$

by independence of T_1 and T_2.

Then:

$$F_{Z_1}(z) = F_{T_1}(z) F_{T_2}(z) \qquad (28)$$

From (28) it follows that the probability density of Z_1 is:

$$f_{Z_1}(z) = f_{T_1}(z) F_{T_2}(z) + F_{T_1}(z) f_{T_2}(z) \qquad z \geq 0 \qquad (29)$$

Similarly, for the minimum:

$$F_{Z_2}(z) = P(\min(T_1, T_2) \leq z)$$

$$= 1 - P(\min(T_1, T_2) > z)$$

$$= 1 - P(T_1 > z, T_2 > z)$$

$$= 1 - P(T_1 > z)\, P(T_2 > z)$$ by independence of T_1 and T_2.

$$= 1 - (1 - F_{T_1}(z))(1 - F_{T_2}(z))$$

$$= F_{T_1}(z) + F_{T_2}(z) - F_{T_1}(z)\, F_{T_2}(z)$$

But from (28) $F_{Z_1}(z) = F_{T_1}(z) F_{T_2}(z)$, then:

$$F_{Z_2}(z) = F_{T_1}(z) + F_{T_2}(z) - F_{Z_1}(z) \qquad (30)$$

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computer. On the other hand, a household's postponing the purchase of new durable goods might be indicative of borrowing constraints.

Therefore, the density function of the minimum is given by:

$$f_{Z_2}(z) = f_{T_1}(z) + f_{T_2}(z) - f_{Z_1}(z) \qquad z \geq 0 \qquad (31)$$

One way to model joint replacement decisions is by assuming that replacement will take place somewhere between the minimum and the maximum stopping times (Figure 3). Therefore, we can define a new random variable $W=Z_2-Z_1$ that denotes the time elapsed between the minimum and the maximum stopping times. Intuitively, households might replace both appliances right after anyone of them either becomes obsolete or breaks down. Or, alternatively, they might as well wait until both appliances render inadequate to their needs. The exact time at which households will replace both appliances is therefore random, and will be located somewhere between Z_1 and Z_2.

[Figure 3 about here]

Now, in order to determine the distribution function of the W, we make use of convolutions:

$$F_W(w) = \Pr(Z_2 - Z_1 \leq w) = \int_0^{\infty}\left(\int_0^{w+Z_1} f_{Z_1Z_2}(z_1,z_2)\,dz_2\right)dz_1$$

$$\Rightarrow \qquad f_W(w) = \frac{d}{dw}F_W(w) = \int_0^{\infty}\left(\frac{d}{dw}\int_0^{w+z_1} f_{Z_1Z_2}(z_1,z_2)\,dz_2\right)dz_1$$

$$= \int_0^{\infty} f_{Z_1Z_2}(z_1, w+z_1)dz_1 \qquad \text{by Leibnitz's rule}$$

Now, given that both Z_1 and Z_2 are stopping times, Z_2 is independent of Z_1. Therefore, the distribution function of W boils down to:

$$f_W(w) = \int_0^{\infty} f_{Z_1}(z_1) f_{Z_2}(w+z_1)dz_1 \qquad w \geq 0 \qquad (32)$$

where $f_W(w)$ is the convolution of $f_{Z_1}(z_1)$ and $f_{Z_2}(w)$.

It remains to characterize $f_W(w)$ based upon the distribution functions of our model of Section 4.1. According to equation (17), T_1 and T_2 are distributed as:

$$f_{T_j}(t_j | b_j, \sigma_j, \bar{x}_j, x_j^*) = \frac{\bar{x}_j - x_j^*}{\sigma_j\sqrt{2\pi t_j^2}} \exp\left(\frac{-(\bar{x}_j - x_j^* - b_j t_j)^2}{2\sigma_j^2 t_j}\right) \qquad t_j \geq 0,\ j=1,\ 2. \qquad (33)$$

with cumulative distribution functions:

$$F_{T_j}(t_j | b_j, \sigma_j, \bar{x}_j, x_j^*) = 1 - \Phi\left(\frac{\bar{x}_j - x_j^* - b_j t_j}{\sigma_j \sqrt{t_j}}\right) + \exp\left(\frac{2(\bar{x}_j - x_j^*) b_j}{\sigma_j^2}\right) \Phi\left(\frac{-(\bar{x}_j - x_j^*) - b_j t_j}{\sigma_j \sqrt{t_j}}\right)$$

$$t_j \geq 0,\ j=1,\ 2. \qquad (34)$$

where $\Phi(.)$ represents the cumulative distribution function of a standard normal, and $\bar{x}_j$ is given by the implicit function $H(\bar{x}_j,\ b_j,\ \sigma_j^2) \equiv 1 + \lambda_j(r\tilde{C}_j + x_j^* - \bar{x}_j) - \exp[\lambda_j(x_j^* - \bar{x}_j)] = 0$, λ_j is the positive root of the characteristic equation $(1/2)\sigma_j^2 p^2 + b_j p - r = 0$, and the parameters b_j and σ_j^2 come from the dynamics of x_{jt}, $dx_{jt} = b_j\, dt + \sigma_j\, dW_{jt}$.

From (29), the distribution function of the maximum is given by:

$$\begin{aligned} f_{Z_1}(z) = &\left(\frac{\bar{x}_1 - x_1^*}{\sigma_1\sqrt{2\pi z^2}} \exp\left(\frac{-(\bar{x}_1 - x_1^* - b_1 z)^2}{2\sigma_1^2 z}\right)\right) \\ &* \left(1 - \Phi\left(\frac{\bar{x}_2 - x_2^* - b_2 z}{\sigma_2\sqrt{z}}\right) + \exp\left(\frac{2(\bar{x}_2 - x_2^*) b_2}{\sigma_2^2}\right)\Phi\left(\frac{-(\bar{x}_2 - x_2^*) - b_2 z}{\sigma_2\sqrt{z}}\right)\right) \\ + &\left(1 - \Phi\left(\frac{\bar{x}_1 - x_1^* - b_1 z}{\sigma_1\sqrt{z}}\right) + \exp\left(\frac{2(\bar{x}_1 - x_1^*) b_1}{\sigma_1^2}\right)\Phi\left(\frac{-(\bar{x}_1 - x_1^*) - b_1 z}{\sigma_1\sqrt{z}}\right)\right) \\ &* \left(\frac{\bar{x}_2 - x_2^*}{\sigma_2\sqrt{2\pi z^2}} \exp\left(\frac{-(\bar{x}_2 - x_2^* - b_2 z)^2}{2\sigma_2^2 z}\right)\right) \qquad z \geq 0 \qquad (35) \end{aligned}$$

In turn, from (31), the distribution function of the minimum is given by:

$$f_{Z_2}(z) = \left(\frac{\overline{x}_1 - x_1^*}{\text{ó}_1 \sqrt{2\pi z^2}} \exp\left(\frac{-(\overline{x}_1 - x_1^* - b_1 z)^2}{2\sigma_1^2 z} \right) \right) + \left(\frac{\overline{x}_2 - x_2^*}{\text{ó}_2 \sqrt{2\pi z^2}} \exp\left(\frac{-(\overline{x}_2 - x_2^* - b_2 z)^2}{2\sigma_2^2 z} \right) \right)$$

$$- f_{Z_1}(z) \qquad z \geq 0 \qquad (36)$$

where $f_{Z_1}(z)$ is given by (35).

Now suppose we have a cross section of n independent pairs of stopping times for n households. Then the likelihood function of the sample is given by:

$$\prod_{i=1}^{n} f_{W_i}(w_i) = \prod_{i=1}^{n} \left(\int_0^{\infty} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1 \right) \qquad w_i \geq 0,\ i=1, 2, \ldots n$$

$$\Leftrightarrow \qquad \sum_{i=1}^{n} \ln(f_{W_i}(w_i)) = \sum_{i=1}^{n} \ln\left(\int_0^{\infty} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1 \right) \qquad (37)$$

where $w_i = Z_{2i} - Z_{1i}$, $Z_{2i} = \max(T_{1i}, T_{2i})$ and $Z_{1i} = \min(T_{1i}, T_{2i})$.

How do we go about approximating $\int_0^{\infty} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1$? We know, from our estimation results of Sections 4.1 and 4.2, that the probability mass for a stopping time greater than some constant M, large enough (say M=60), goes to zero. Therefore the above improper integral can be suitably truncated:

$$\int_0^{\infty} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1 \approx \int_0^{M} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_i) dz_1$$

Therefore the log likelihood function of the sample becomes:

$$\sum_{i=1}^{n} \ln(f_{W_i}(w_i)) \approx \sum_{i=1}^{n} \ln\left(\int_0^{M} f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1 \right) \qquad (38)$$

In turn the integral $\int_0^M f_{Z_{1i}}(z_{1i}) f_{Z_{2i}}(w_i + z_{1i}) dz_{1i}$ can be approximated by some numeric method, such as the trapezoidal rule:[11]

$$\int_0^M f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1) dz_1 \approx \frac{\Delta z_1}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{m-2} + y_m\right) \tag{39}$$

where $\Delta z_1 = \frac{M}{m}$, $y = g(z_1) \equiv f_{Z_{1i}}(z_1) f_{Z_{2i}}(w_i + z_1)$, and $g(z_{1k})=g(k\Delta z_1)$, k=0, 1, 2, ..., m.

As in Section 4.1, the parameters of the distributions of Z_{1i} and Z_{2i}, i=1, 2,..., n, may be modeled as functions of household characteristics and appliances features.

5 Summary and conclusions

In this paper we focused on micro replacement decisions. We surveyed some representative models of the recent literature, and discuss their empirical testability. In particular, we went through papers where the replacement problem has been tackled by the theory of stochastic processes and stochastic calculus: Rust (1985, 1987), Ye (1990), and Mauer and Ott (1995). In addition, we briefly referred to the state of the art of demand for durable goods modeling in macroeconomics and other fields.

The core of this paper is the empirical results for replacement of home appliances in the United States, and the theoretical model of multiple replacement decisions. Based upon individual replacement models for electric water heaters and refrigerators, we concluded that demographics and appliance features might either accelerate or delay replacement. In addition, we constructed a test statistic that led us to conclude that replacement decisions might be correlated across appliances. Based upon this evidence, we enriched our model by allowing households to replace a set of appliances simultaneously rather than each one in

[11] The area of the first trapezoid is $1/2(y_0+y_1)\Delta z_1$, the area of the second trapezoid is $1/2(y_1+y_2)\Delta z_1$, etc. up to

isolation. Although the estimation process of this extension may be computationally intensive, it is still tractable.

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the area of the nth trapezoid, which is $1/2(y_{n-1}+y_n)\Delta z_1$.

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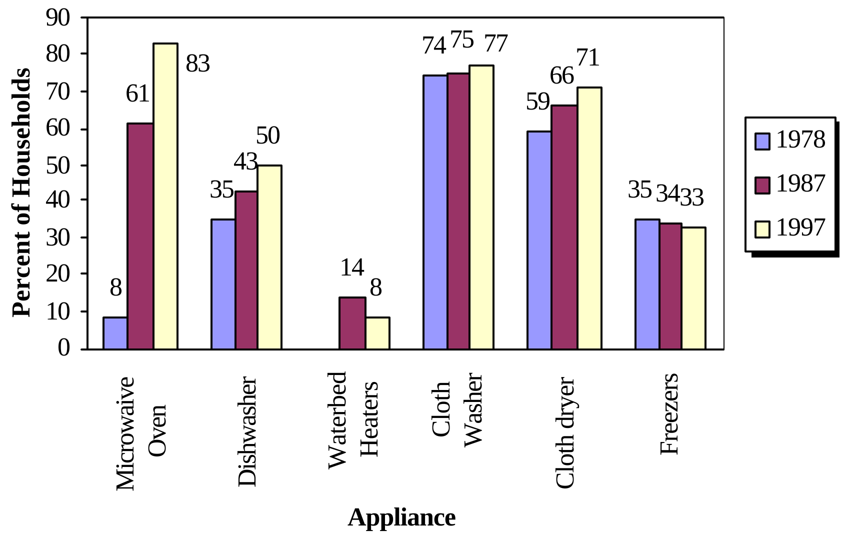
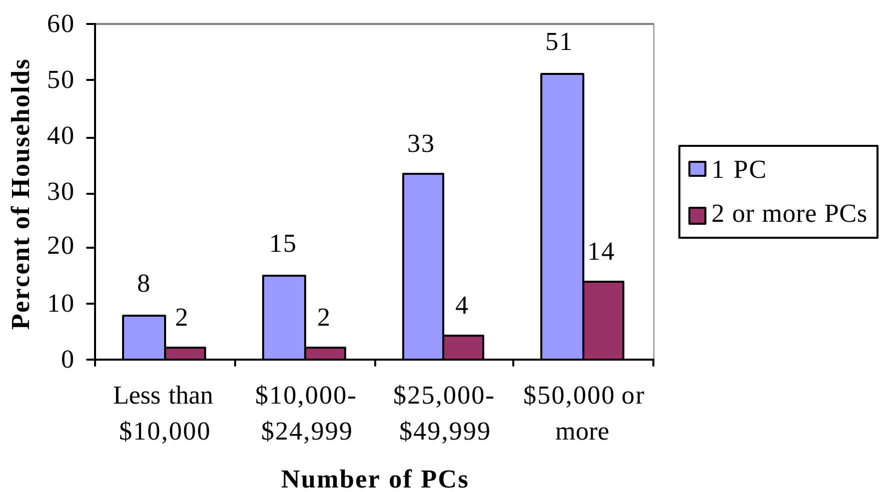
FIGURES

Figure 1a Penetration of Home Appliances



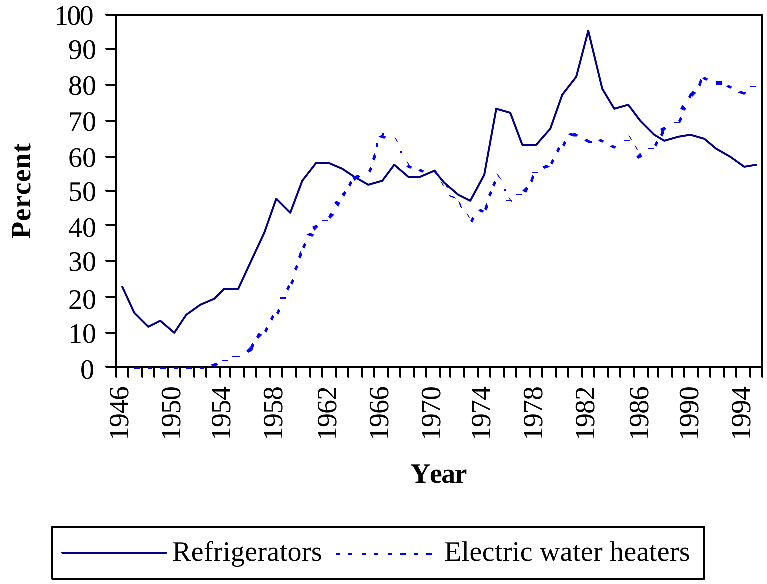
Source: U.S. Department of Energy.

Figure 1b Number of PCs by Annual Household Income in the United States, 1997



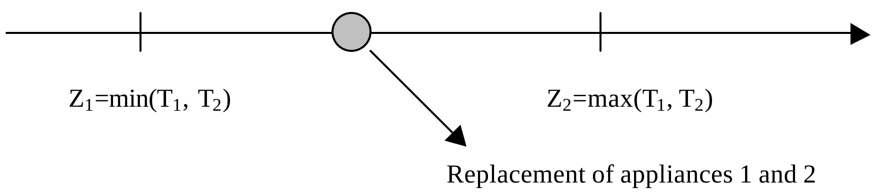
Source: U.S. Department of Energy.

Figure 2 Estimated Annual Replacement Units as a Percentage of Total Annual Shipments



Source: Own elaboration based upon the distribution of lifetime equipment calibrated with the RECS data, and data on annual shipments of appliances from the *Statistical Abstract of the United States*, various issues.

Figure 3 Joint Replacement Decisions



TABLES

Table 1. Replacement Model for Refrigerators

Regressor	Parameter estimate	Standard error	Asymptotic t-statistic
Constant	2.529	0.145	17.444*
Age head of household (per 10 years)	0.099	0.013	7.679*
Monthly income (per $10,000)	-0.006	0.010	–0.538
Urban area dummy (=1 if yes)	0.046	0.040	1.149
Family size (number of members)	–0.010	0.015	–0.703
Refrigerator size (cubic^3)	–0.040	0.006	7.218*
Poor credit rating dummy (=1 if yes)	–0.067	0.082	–0.819
Standardized drift, b/σ	0.624	0.032	19.375*

Log of likelihood function at convergence =–3,612
Number of observations = 2,440

*: Statistically significant at 5% level for H_0: $\beta=0$ against H_1: $\beta\neq0$.

Table 2. Refrigerator Monthly Operating Cost Modeled as a Linear Function of Exogenous Regressors

Regressor	Parameter estimate	Standard error	t-statistic
Constant	-48.145	7.567	–6.362*
Monthly income (per $10,000)	1.847	0.679	2.721*
Urban area dummy (=1 if yes)	7.831	2.707	2.893*
Family size (number of members)	–0.634	0.831	–0.764
Refrigerator size (feet^3)	5.053	0.348	14.541*
Average electricity rate ($/kwh)	0.947	0.056	16.906*

R^2=0.194, Adjusted R^2= 0.193
Number of observations=2,440

*: Statistically significant at 5% level for H_0: $\beta=0$ against H_1: $\beta\neq0$.

Table 3 Replacement Model for Electric Water Heaters

Regressor	Parameter estimate	Standard error	Asymptotic t-statistic
Constant	1.249	0.179	6.939*
Age head of household (per 10 years)	0.137	0.018	7.449*
Monthly income (per $10,000)	-0.069	0.140	–0.491
Urban area dummy (=1 if yes)	-0.064	0.059	–1.086
Natural gas availability (=1 if yes)	0.219	0.057	3.830*
Tank size (gallons)	–0.004	0.002	–1.757**
Family size (number of members)	0.146	0.090	1.617
Poor credit rating dummy (=1 if yes)	0.039	0.021	1.866**
Standardized drift, b/Φ	0.516	0.025	20.615*

Log of likelihood function at convergence =–2,612.9
Number of observations = 1,057

* : Statistically significant at 5% level for H_0: $\beta=0$ against H_1: $\beta\neq0$.
** : Statistically significant at 10% level for H_0: $\beta=0$ against H_1: $\beta\neq0$.

Table 4. Estimates of $\bar{x}-x^*$, b, Φ, Expected Equipment Lifetime and Total Discounted Cost

Estimates	Mean		Standard deviation	
	Refrigerator	Water heater	Refrigerator	Water heater
$\bar{x}-x^*$ ($)	243.2	139.9	42.5	26.1
b ($)	16.3	11.3	6.5	4.7
Φ ($)	26.1	21.8	10.4	9.2
Expected lifetime (years)	16.5	13.7	4.3	3.4
Total discounted cost ($)	4,271.6	5,539.3	596.8	309.6

Table 5. Fitted and Actual Frequency for Each Age Category

Age Category	Fitted		Actual		Percent error	
	Refrigerator	Water heater	Refrigerator	Water heater	Refrigerator	Water heater
Less than 2 years old	0.124	0.156	0.136	0.153	8.8	–1.9
2-4 years old	0.185	0.228	0.187	0.227	1.1	–0.4
5-9 years old	0.289	0.289	0.270	0.299	–7.0	3.3
10-19 years old	0.311	0.242	0.318	0.234	2.2	–4.7
Over 20 years old	0.090	0.084	0.088	0.086	–2.3	2.3

Table 6 Impact on the Probability of Replacement due to Marginal Changes in the Regressors

Regressor	Time Period (years) 1-3		7-9		1-20	
	Refrigerator	Water heater	Refrigerator	Water heater	Refrigerator	Water heater
Age of head of household (per 10 years)	–5.61e-4	–0.065	–0.033	–0.062	–0.256	–0.366
Monthly income (per $10,000)	1.07e-6	0.003	0.002	0.031	0.016	0.285
Equipment size (*)	2.17e-4	1.90e-4	0.013	0.002	0.099	0.011
Probability of replacement	4.4e-4	0.011	0.088	0.131	0.730	0.796

Notes: Marginal impacts are evaluated at sample means. (*): Equipment size is measured in feet for refrigerators and in gallons for water heaters.