Aggregate Implications of Employer Search and Recruiting Selection*

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Abstract

In this paper I develop a general equilibrium model of employer search with recruiting selection and heterogenous workers and characterize its equilibrium. Departing from the standard search model, firms simultaneously meet several applicants and choose the best candidate. In line with the evidence, the worker hazard rate increases in her productivity. The model explains evidence on negative duration dependence and unobserved heterogeneity of the hazard rate. Consistent with evidence showing that workers with longer unemployment spells have lower permanent incomes, in a frictional labor market recruiting selection amplifies the permanent income and welfare inequality because low wage workers go through longer and more volatile unemployment spells, and have less valuable outside options to bargain with.

The calibration targets the unemployment rate, median unemployment duration and mean and standard deviation of log earnings in CPS data. The model can qualitatively mimic that the mean wage decreases in unemployment duration, and that both mean and variance of the unemployment duration decrease in wages, but the magnitudes are exaggerated. To show that the stark trade-off between equality and efficiency arising in the model, I also perform a counterfactual experiment changing screening costs.

Keywords: Employer search, Nonsequential search, Recruiting selection, Duration dependence, Hazard rate heterogeneity, Inequality amplification.

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1 Introduction

Anyone who has ever searched for a job have personally experienced that prospective employers usually interview several applicants before making a hiring decision. This is not just anecdotal evidence: the National Employer Survey 1997 (NES97) data shows that 6.7 applicants are interviewed to fill a vacancy on average. This feature is absent in most popular models of the job search process. This paper embeds this feature into an otherwise standard stationary general equilibrium search model of the labor market and characterizes its equilibrium.

I account for the fact that employers interview several candidates before filling a vacancy by introducing a recruiting selection process, a nonsequential employer search strategy. Firms simultaneously meet various heterogenous workers to fill vacancies and select the best one. In contrast, I show that the standard sequential search model with heterogenous workers -which is a particular case of the recruiting selection model when only one random applicant is interviewed- cannot explain this characteristic of real labor markets in general equilibrium.

The model generates relevant macroeconomic implications besides being a plausible vacancy-filling mechanism. The recruiting selection mechanism qualitatively replicates and provides a simple explanation for cross-sectional CPS data features of wages and unemployment durations in a general equilibrium framework, namely that the expected wage decreases in unemployment duration, and that both the mean and variance of the unemployment duration decreases in wages. The benchmark models of sequential search (McCall 1970; Mortensen and Pissarides 1994) do not readily produce these features.

Several papers\(^1\) have documented the empirical relevance of nonsequential search employer strategies. The main findings are that firms fill job openings by choosing an applicant from a pool that is formed shortly after the posting of the vacancy, and that almost no applications arrive during the rest of the vacancy duration. This literature sees vacancy durations as a selection periods and not as periods of search. Even though this is not the first work introducing some kind of hiring decision based on workers' characteristics\(^2\), developing this idea in a general equilibrium model and analyzing its quantitative implications seems to be a novel contribution.

The economic environment of the model is maintained deliberately simple to highlight the effect of recruiting selection. There is no aggregate uncertainty and jobs are homogeneous. Workers are risk-averse and heterogeneous only in time-invariant productivity. Workers’ unique decision is whether to submit a costly application per period

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(participation decision). Since all vacant jobs are ex ante identical, in a symmetric equilibrium all firms receive the same number of workers in expectation.

To fill a vacancy, firms observe the number of applicants received and decide how many to interview considering the related costs. Then, the employer perfectly observes the screened applicants’ productivities and picks the best candidate. Not chosen applicants remain into the unemployed pool. In the analyzed equilibrium, the best workers get jobs faster on average and the “quality” of the pool of unemployed progressively decreases as the duration of unemployment increases. Therefore, the hazard rate -or probability of finding a job- decreases in productivity and the average productivity of workers is negatively correlated to the duration of unemployment.

Once a worker is chosen, the solution of an axiomatic Nash-bargaining problem determines the wage. The value of the job only depends on the worker’s productivity. In the steady-state of the model, the joint distribution of productivities and unemployment durations, and the hazard rates are equilibrium objects that arise from the employers’ recruiting selection technology, worker-selection strategies of firms and the exogenous distribution of workers’ productivity types.

Two opposite forces play a role in the wage determination of the model. On one hand, productivity increases the created surplus; on the other, more productive workers have a greater outside option value because they can be hired more easily if they apply again. Because wages and hazard rates are jointly determined in equilibrium, it is not readily clear that the most productive workers are also the most profitable. I show conditions for the existence of the symmetrical “coincidence ranking” equilibrium in which the productivity and profitability rankings are the same for all workers.

The presented model can help to understand several other empirical facts. Using Mixed Proportional Hazard models, most of the work on empirical unemployment duration models has tested for (conditional) negative duration dependence of the hazard rate, i.e. the effect of elapsed duration reduces the probability of finding a job controlling for worker heterogeneity. Negative duration dependence is not a robust finding. There are several empirical studies concluding that after controlling for observable variables and allowing for unobserved (idiosyncratic) heterogeneity, the negative duration dependence disappears. Additionally, variables that are usually associated to higher earnings are also related to higher hazard rates. The recruiting selection model can explain these findings because (i) high productivity workers find jobs more easily than do low productivity ones and (ii) once the hazard rate is adjusted for productivity determinants, the negative duration dependence pattern is revealed as spurious. More-

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3 Abbring et al. (2002), Cockx and Dejemeppe (2005) and some other reported by Machin and Manning (1999)
over, if productivity cannot be fully explained by observables, the model predicts that some negative duration dependence may show up in the data.

A closely related finding is that the unobserved heterogeneity component explains a great deal of hazard rate variation after controlling by elapsed duration and observables\(^4\). Instead of linking the unobserved heterogeneous component to some unspecified idiosyncratic feature that affects the job finding rate, the recruiting selection model establishes a clear relationship between worker’s productivity and her hazard rate.

Related to this evidence, several empirical studies find that conditional on heterogeneity, re-employment wages exhibit significant negative duration dependence. Additionally, individuals who have experienced long unemployment durations tend to go through long unemployment spells frequently\(^5\). Stewart (2007) also finds that not only past unemployment duration causes recurrence of unemployment spells, but also does low wages. The bottom line is that workers with long unemployment spells have low permanent income.

The model provides a simple explanation for all these phenomena: less productive workers have lower chances of being hired due to recruiting selection. In a labor market with search frictions, the recruiting selection model amplifies the \textit{ex ante} productivity inequality of the population. In contrast to a sequential search model in which all workers are affected in the same way by the search frictions, in a model of recruiting selection the low productivity workers experience longer unemployment spells and earn lower wages because their outside option is less valuable. Additionally, in the model the variance of the unemployment spells is also decreasing in productivity, so that low productivity individuals are exposed to greater risks. Since these workers also marginally value their consumption the most, the recruiting selection labor market is even more powerful to amplify the \textit{ex ante} distribution of welfare given the distribution of productivities.

Moreover, in the model there is a trade-off between equality and efficiency. If the recruiting technology makes optimal for the employer to interview more applicants, the \textit{ex post} assignment of jobs is more efficient on average. A change like that will decrease the chances of being employed of low productivity workers, their permanent income and welfare, whereas the opposite occurs to high productivity workers.

The calibration of the model targets the unemployment rate, median of the unemployment duration, and mean and variance of the previous year wages of employed workers in CPS data 1985-2006. The theoretical predictions of the model qualitatively


\(^5\)Corcoran and Hill 1985; Omori 1997; Arulampalam et al. 2000)
hold in the data using a log-normal exogenous productivity distribution and reasonable parameters for preferences. In quantitative terms, the model exaggerates the patterns observed in the data: mean wages decreases in unemployment duration too sharply, and the mean and variance of unemployment duration show a diminishing pattern that is too strong. The fit to the data improves in some dimensions by using a wider definition of unemployment. I also perform a counterfactual experiment to illustrate the mentioned trade-off between equality and efficiency which suggests that switching steady-states from the benchmark recruiting selection to the sequential search case would greatly improve equality, especially in terms of welfare, but the cost in inefficiency would be a reduction of 34% of the mean wage and of 53% of the mean welfare measured in terms of consumption.

The paper is organized as follows. Section 2 discusses some related literature. Section 3 describes the model in detail, characterizes the most relevant equilibrium of the model, compares the model to the special sequential case and discusses its implications. Section 4 evaluates the empirical consequences of the model using CPS matched data and runs a counterfactual experiment. The last section presents some conclusions and potential extensions of the model.

2 Related Literature

Only few attempts have been made in the literature to address the fact that employers interview multiple candidates to fill vacancies. In their work Barron, Bishop, and Dunkelberg (1985) empirically analyzes employer’s recruiting behavior for the US using data from the Employment Opportunity Pilot Projects (EOPP). They find that employers interview 6.3 candidates per vacancy (very close to NES97 data) and spend a significant amount of time in recruiting activities. The fact that 90% of the employers make only one job offer, makes the authors conclude that employer search can be described as a selection process of applicants. Using the same dataset, Burdett and Cunningham (1998) deal with the related issue of vacancy durations using a sequential employer search model partial equilibrium model. Nevertheless, as discussed later on, the sequential environment cannot replicate the magnitude of received applications observed in the NES97 data in general equilibrium.

Dutch economists have contributed to the literature of empirical research on employer’s search strategies taking advantage of an interesting dataset that gathers information about job openings in two periods. This literature initiated by van Ours and Ridder (1992) has found that employers search nonsequentially, especially when trying to fill qualified positions. Their conclusion is based on the fact that almost all
applicants arrive shortly after the vacancy is posted, but the actual hiring takes place after some time, presumably a selection period. As pointed out by several authors, the prevalent focus on worker’s search instead of employers’ search seems to be explained by the lack of data of recruitment processes (van Ours and Ridder 1993; Burdett and Cunningham 1998; Weber 2000). The same authors have highlighted the necessity of more research in employer’s search. For instance, several phenomena observed in aggregate labor markets, such as shifts on Beveridge curve can be explained for changes in employers’ search behavior instead of workers’.

The model is also related to the literature of nonsequential search models, whose tradition goes back to the seminal work of Stigler (1961) and the papers of Wilde (1977), Burdett and Judd (1983), Morgan (1983) and the empirical implementation of Stern (1989). Morgan and Manning (1985) show that nonsequential search strategies generally dominate sequential search. More recent contributions in this area are Acemoglu and Shimer (2000), Kandel and Simhon (2002) and Albrecht, Gautier, and Vroman (2006) among others. The application of nonsequential search has been focused on the worker’s side: after submitting multiple applications, the workers choose the best offer received from employers. The labor-demand side approach taken in this paper as well as the general equilibrium considerations analyzed seem not to be addressed by the previous literature.

The empirical facts about unemployment duration studies come from papers using the Mixed Proportional Hazard model which decomposes the hazard rate into three multiplicative components: a baseline hazard common to all individuals, and observable and unobservable heterogeneity components (van den Berg 2001 for a survey). Although evidence of duration dependence of the hazard rate can be found in the literature (van den Berg and van Ours(1996, 1999), Addison and Portugal (2003), Guell and Hu (2006) among many others), the findings are not generally robust. Machin and Manning (1999) assert that after controlling for readily observable workers’ characteristics, the negative duration dependence pattern disappears in most European studies. Most of the studies show that unobserved heterogeneity component explains a high proportion of the hazard variation. The inclusion of the unobserved component plays the important role of preventing the econometrician from finding spurious negative hazard duration dependence because individuals with intrinsic low hazard rates (or longer unemployment spells) tends to be over-represented in the stock of unemployed.

Some other approaches have been followed to explain the empirical facts addressed in this paper. One explanation for hazard rate negative duration dependence is the so-called “stigma” effect, which is the fact that firms tend not to hire workers of long unemployment spell because their duration signals low productivity (Vishwanath 1989;
Lockwood (1991) and Kollman (1994) show how stigma effect arises in general equilibrium. Firms test applicants to fill a vacancy but the screening is imperfect because low productivity worker can pass with some probability. If a worker has been rejected several times is much more likely that she is of low type and therefore, her unemployment duration becomes a bad signal. Stigma effect arises due to an information externality, which is a side-effect of the firms’ hiring process. In contrast to these models, the informational externality channel is shut down in order to focus on the implications of pure recruiting selection. As the latter shows, all stigma effect models are built on some recruiting selection process. To really assess the theoretical and empirical importance of stigma effect it is necessary to understand the consequences of pure recruiting selection in general equilibrium.

Another candidate to explain negative duration dependence of the hazard rate in the literature is human capital depreciation. Wages decay as unemployment spell increases because workers become less productive. For the same reason, firms are reluctant to hire long unemployment duration individuals. Heterogeneity in hazard and wages can be attained by assuming some heterogenous distribution of human capital across the population. The downside of this theory is the striking amount of human capital depreciation needed to replicate the important decay of wages observed in data. For instance, Keane and Wolpin (1997) estimate a 30.5% yearly depreciation rate of human capital when white collars are nonemployed. Machin and Manning (1999) (p. 3119) show an somewhat skeptical view of this explanation arguing there is lack of direct evidence about human capital depreciation and referring to an employer’s surveys in which long-term unemployed workers are assessed not be worse than the average recruit.

In most random sequential search models hazard rate dispersion may arise due to heterogeneity in reservation wages. It follows that workers of high reservation wage should have larger unemployment spells. Assuming rational expectation agents, a high reservation wage must be associated to high re-employment wages. In turn, higher earnings are related to high education, race and gender characteristics, all of which are features linked to high hazard rates in the data. Therefore, generating hazard rate heterogeneity by assuming dispersion of reservation wages generates counterfactual implications unless a third variable, such as the unemployment benefits, is a more important determinant of reservation wages. The evidence from Hogan (2004) suggests this is not the case. He finds that unemployment benefits do not significantly impact reservation wages after controlling for previous wages and re-employment wages.

Several authors study facts that are related to the relationship between permanent income and duration of unemployment. Addison and Portugal (1989), Belzil (1995), Rao Sahib (1998) and Christensen (2002) document negative duration dependence of
re-employment wages after controlling by other elements that may cause a similar effect such as unemployment benefits exhaustion. Other papers examine the earnings loses suffered by a worker after a displacement or separation, without focusing on the specific role of unemployment duration. For instance, Ruhm (1991), Kletzer (1998) and Farber (2003) report that displaced workers even experience substantial earning loses, even various years after the separation. Corcoran and Hill (1985), Omori (1997) and Arulampalam, Booth, and Taylor (2000) find that long unemployment spells tend to occur more frequently to individuals that have been unemployed before. Stewart (2007) finds that individuals with previous low wages tend to be unemployed longer, and that individuals that experienced longer unemployment spells tend to have lower wages. Thus, low productivity workers seem to be “trapped” in a vicious circle of long unemployment spells and low wages. A paper of Gonzalez and Shi (2007) address this issue theoretically by setting a model in which reservation wages decrease in unemployment duration due to the effect of individual learning about job finding probabilities. In contrast, this paper focuses on the effects of recruiting selection on the hazard rate, keeping aside informational issues.

3 The Model

In the model time is discrete and there is a continuum of homogenous risk-neutral firms or employers that post *ex ante identical* job vacancies. Employers’ hiring technology generates a pool of applicants to detect the best candidate. Firms make three decisions: (i) whether to create new vacant jobs, (ii) how many of the arrived applicants to screen and (iii) which applicant to hire, if any. Differently from sequential search models, a single vacancy can contemporaneously meet various unemployed workers. The cost of creating a vacancy is the fixed amount $\kappa$ and the cost of screening $C(\cdot)$ is increasing in the number of applicants. Although any number of applications could arrive to fill a vacancy, the firm is not committed to interview all the candidates who show up.

To keep the focus on the consequences of recruiting selection, once the firm pays the cost of screening an applicant, her productivity is totally revealed to the firm. As a consequence, previous employment history of the worker has no informational value for a prospective employer once the interview has taken place.

There is a fixed mass of size 1 of workers. Their unique decision is whether to submit a costly application per period (participation decision). At birth, each worker draws a productivity $\theta$ from the exogenous distribution $f_\theta(\theta)$. The other workers’ characteristic is unemployment duration $\delta$. Conventionally, $\delta = 0$ represents an employed worker and $\delta > 0$ a worker who became unemployed $\delta$ periods ago.
The general state of the economy is denoted by \( X = (A, V, F(\theta, \delta)) \), where \( A \) represents the total number of unemployed applicants in the economy, \( V \) is the aggregate vacancies\(^6\) and \( F(\theta, \delta) \) stands for the joint distribution of productivities and unemployment durations in the economy. All of these components are endogenously determined later on.

The timing of the events is as follows. Every period, having observed the aggregate state \( X \), potential employers optimally create vacancies and receive \( N \) applications which are randomly drawn with mean number of applicants \( \lambda \). At the beginning of the period, unemployed workers receive some exogenous income \( b \) and decide whether to send a costly application. By doing so, a worker is randomly allotted into some firm’s applicant pool. Workers not applying this period have to decide again whether to send a new application next period.

Having received \( N \) applications, the employer optimally decides to interview \( i \leq N \) applicants the screening cost \( C(i) \). The employer learns the productivities of interviewed applicants and makes a job offer to the most profitable worker in the pool. Conditional on the workers’ productivity type, the firm and the chosen worker bargain on a constant wage contract. Rejected candidates remain into the pool of unemployed and increase their unemployment spells by one.

Each employed worker deterministically produces according to her true productivity \( \theta \) at the end of each period and receives the already bargained wage. No commitment issues arise because both sides perfectly observe productivity. The workers may be hit by an exogenous separation shock with probability \( \eta \). If a worker gets fired, she joins the pool of unemployed workers and decides whether to submit a new application next period.

This paper only analyzes the symmetric steady-state equilibrium of this economy. Dealing with transitional dynamics would substantially increase the complexity of the analysis and is beyond the scope of this paper. Henceforth, to ease the notation the aggregate state \( X \) is dropped.

### 3.1 Workers’ Problem

At the beginning of every period if a worker of productivity \( \theta \) chooses to send a costly application, she pays a share \( z \) of her exogenous unemployment income \( b \) and faces an equilibrium probability \( \pi(\theta) \) of being hired (hazard rate), in which case she gets the value of being employed \( W(\cdot) \) earning a wage \( \tilde{w}(\theta) \) bargained with a prospective employer. Having received no offer, the worker increases her unemployment spell by one.

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\(^6\)Since jobs are ex ante identical the specific assignment of workers to jobs is not relevant.
unit of time and faces the participation decision again. In case the worker chooses not to send the application, she cannot get a job and consumes her whole unemployment income $b$.

Workers value consumption of every period according to an increasing, concave and $C^2$ utility function $u(\cdot)$ and have a constant discount factor $\beta$. Hence, an unemployed worker’s lifetime utility is $Q(\theta, \tilde{w}(\theta)) = \max\{Q_a(\theta, \tilde{w}(\theta)), Q_n\}$, where $Q_a(\cdot)$ stands for the value of submitting an application and is given by

$$Q_a(\theta, \tilde{w}(\theta)) = u(b(1 - z)) + \beta \pi(\theta) W(\tilde{w}(\theta)) + \beta(1 - \pi(\theta)) Q(\theta, \tilde{w}(\theta))$$

and $Q_n$ is the value of not submitting an application given by

$$Q_n(\theta, \tilde{w}(\theta)) = u(b) + \beta Q(\theta, \tilde{w}(\theta))$$

In case the worker is hired, she produces her productivity $\theta$ and receives a bargained wage $w(\theta)$. Before the period ends, the worker faces the exogenous chance of separation, $\eta$. Note that $w(\theta)$ is the wage bargained with the employer who is making an offer, as opposed to $\tilde{w}(\theta)$, which is the wage to be bargained with some other prospective employer. The value of a type $\theta$ worker employed at wage $w(\theta)$ is

$$W(w(\theta)) = \frac{u(w(\theta)) + \beta ((1 - \eta)W(w(\theta)) + \eta Q(\theta, \tilde{w}(\theta)))}{1 - \beta (1 - \eta)}$$

In the analyzed steady-state the hazard rate $\pi(\theta)$ does not change because the aggregate state $X$ is always the same. It follows that a worker of productivity type $\theta$ will always make the same optimal decision. Hence, by substituting (1) and rearranging, the utility of applying can be written as

$$Q_a(\theta, \tilde{w}(\theta)) = \frac{1}{1 - \beta} [S(\theta) u(b(1 - z)) + (1 - S(\theta)) u(\tilde{w}(\theta))]$$

with $S(\theta) \equiv \frac{\beta^{-1} - 1 + \eta}{\beta^{-1} - 1 + \eta + \pi(\theta)}$

Using the same reasoning, the lifetime utility of not applying is expressed by

$$Q_n = u(b)/(1 - \beta)$$

### 3.2 Firms Problem

A firm that posts a vacancy pays a cost $\kappa$ and receives a random number of $N$ applicants, where $N$ follows distribution a Poisson distribution with mean $\lambda$. Once the number of
applicants is realized, the firm optimally decides about \( i \) the number of applicants to interview out of the \( k \) arrived. In doing this, it considers the cost screening function \( C(i) \), which is increasing in \( i \), convex and satisfies \( C(0) = 0 \). Then, the employer chooses the most profitable worker within the screened applicants. If no application arrives, the firm can post a vacancy again to create a new pool of applications next period. Equation (4) states the value of posting a vacancy \( P \) and equation (5) represents the lifetime profits for a firm that has received \( k \) applicants.

\[
P = -\kappa + \beta \sum_{k=0}^{\infty} Pr(N = k) \max \{ P, G(k) \} \quad \text{with} \quad (4)
\]

\[
G(k) = \max_{i \leq k} \left\{ \mathbb{E} \left[ \max_{j} \{ J(\theta) \}_{j=1}^{i} | \delta > 0, a = 1, N = i \right] - C(i) \right\} \quad \text{with} \quad (5)
\]

When a worker is hired, she produces her productivity \( \theta \) and is paid the bargained wage \( w(\theta) \). If the exogenous separation shock hits the match with probability \( \eta \), the firm can post vacancies again. If the match survives, production takes place and wages are paid again in the same way. Hence, the Value Function \( J(\theta) \) represents the value obtained by a firm hiring a worker of productivity \( \theta \).

\[
J(\theta) = \theta - w(\theta) + \beta((1 - \eta)J(\theta)) + \eta P
\]

\[
= \frac{\theta - w(\theta) + \beta \eta P}{1 - \beta(1 - \eta)} \quad \text{with} \quad (6)
\]

Employers post vacancies in a free-entry market, so that \( P = 0 \) in equilibrium. Notice that no applicant whose value to the firm is lower than \( P \) will ever submit a costly application. It is optimal for the worker not to apply because the firm would never choose to hire her. Hence, in equilibrium, the firm will never choose to post a vacancy again if some applicant arrived.

To solve problem in (4) it is convenient to ease notation. The expected value of hiring the best worker out of \( k \) arrived applicants from the pool of unemployed (i.e, \( \delta > 0 \) and \( a(\theta) = 1 \)) is denoted \( \tilde{J}(k) = \mathbb{E}[J(\theta)|\delta > 0, a(\theta) = 1, N = k] \). Concavity of \( \tilde{J}(k) \) is intuitive: as the number of applicants increases it becomes progressively harder for a marginal applicant to be better than all the others. This result is formally established in Lemma 1.

**Lemma 1** The expected lifetime profit conditional on receiving \( k \) applicants, \( \tilde{J}(k) \), is strictly increasing and strictly concave in \( k \), that is, \( \tilde{J}(k) - \tilde{J}(k-1) < \tilde{J}(k-1) - \tilde{J}(k-2) \).
Proof. See Appendix ■

With Lemma 1, the following result is demonstrated

Under mild conditions, the optimal firms policy to solve problem (5) is to set \( g^*(k) = \arg\max G(k) = \min\{k, i^*\} \). The employer would interview all the applicants if their number is not greater than \( i^* \) and if the actual number of applicants surpasses \( i^* \), the firm randomly interview only \( i^* \) of them. The intuition of this is simple. Provided the function \( \tilde{J}(k) \) lies above the screening cost function \( C(k) \) for \( k = 1 \), the convex cost function grows at a higher rate and surpasses the expected profits function \( \tilde{J}(k) \) at some point. Since the difference between the two is a concave function, a maximum is reached at some finite number of applicants. If there is no cost of screening, then all applicants are interviewed, so \( i^* = +\infty \). In case that \( C(k) > \tilde{J}(k) \) for all \( k \), then no applicants are interviewed, so \( i^* = 0 \).

Lemma 2 If the expected profit obtained for interviewing one applicant is positive and finite, i.e. \( 0 < \tilde{J}(1) - C(1) \equiv M(1) < \infty \), and \( C(\cdot) \) is strictly increasing, then there is a finite integer \( i^* > 0 \) such that a firm receiving \( k \geq i^* \) applicants optimally screens \( i^* \) of them, while a firm receiving \( k < i^* \) applicants optimally screens all of them.

Proof. See Appendix ■

Using the results from Lemma 2, and the free-entry condition in (4), the average number of workers per vacancy \( \lambda \) is determined by

\[
0 = -\kappa + \beta \sum_{k=0}^{\infty} \frac{\exp(-\lambda)\lambda^k}{k!} \max\{\tilde{J}(k) - C(k), \tilde{J}(i^*) - C(i^*)\}
\]

(7)

The following proposition shows that the former equation has a unique solution if the profits obtained are sufficiently high to pay the cost of posting a vacancy. The result follows because the vacancy posting value \( P \) is strictly increasing in \( \lambda \).

Proposition 3 There is a unique mean number of applicants per vacancy \( \lambda \) that is consistent with the free-entry condition if \( M_i = \tilde{J}(i^*) - C(i^*) > \kappa/\beta \).

Proof. See Appendix ■

3.3 Wage determination

A natural framework for wage determination is Nash bargaining. Once the firm has chosen some worker, both sides negotiate about how to split the generated surplus. For the worker, the outside option is the value of being unemployed next period. For the firm, the outside option is to post vacancies again under aggregate conditions \( X \)
because it is assumed that no recall of other applicants is possible. Since firms are identical, all of them offer the exactly same wage to the same productivity type. As a consequence, bargained wages are never turned down in equilibrium for two reasons. First, a worker can send only one application per period. If a worker could contact two or more firms at the same time, there would be a positive chance that a worker turn down all the offers but a randomly picked one. Such situation, though interesting, is not analyzed in this paper. Secondly, because the value of a match is completely determined by the productivity of the worker, the expected utility obtained from a forthcoming match is the same obtained at the current match. The Nash axiomatic solution solves the following problem.

$$\max_{w(\theta)} \{ (\beta(W(w(\theta)) - Q(\theta, \tilde{w}(\theta))))^\alpha (\beta((J(\theta) - P))^{1-\alpha} \}$$

subject to $W(w(\theta)) - Q(\theta, \tilde{w}(\theta)) \geq 0$ and $J(\theta) - P \geq 0$

Substituting equations (1), (2), (6), using the free entry condition $P = 0$ and assuming an interior solution the first order condition is

$$\frac{\alpha}{1-\alpha}(\theta - w(\theta))u'(w(\theta)) = u(w(\theta)) - (1 - \beta)Q_\alpha(\tilde{w}(\theta), \theta)$$

$$= u(w(\theta)) - S(\theta)u(b(1 - z)) - (1 - S(\theta))u(\tilde{w}(\theta)) \quad (8)$$

Being ex ante identical, all employers bargain in the same way. For this reason, $w(\theta) = \tilde{w}(\theta)$ in equilibrium. The worker wants to keep the match (i.e, the solution is interior) only if $w(\theta) > b(1 - z)$. Because of the free-entry condition, employers will hire the worker as long as $\theta > w(\theta)$. Thus, the match is jointly preferred to outside options for both worker and employer if and only if $\theta > w(\theta) > b(1 - z)$.

To gain insight about the factors intervening in wage determination, substitute that $w(\theta) = \tilde{w}(\theta)$ into (8) and do some algebra to get

$$(\theta - w(\theta))B(\theta) = u(w(\theta)) - u(b(1 - z)) \quad (9)$$

with $B(\theta) = \frac{\alpha}{1-\alpha}u'(w(\theta))S(\theta)^{-1} = \frac{\alpha}{1-\alpha}u'(w(\theta)) \left(1 + \frac{\pi(\theta)}{\beta^{-1} - 1 + \eta}\right)$

The expression $B(\theta)$ stands for the overall worker’s bargaining power. The productivity type $\theta$ has two opposite effects. On one hand, a lower productivity increases the

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7 The restrictions is used for simplicity reasons. In a more general framework, firm’s outside option depends on the profitability of the second-best worker among the applicants.
worker’s share of the surplus because a low wage increases the marginal utility of the worker. On the other hand, a lower productivity has a small job finding probability or hazard rate \( \pi(\theta) \), which implies a reduced value of the outside option. A low parameter \( \alpha \) is clearly associated to a low bargaining power.

It is not clear that the greater the worker’s productivity, the higher the profits. Suppose that the productivity and profitability ranking of workers within any application pool coincide. On one hand high-productivity workers demand a lower share of the surplus due to risk-aversion; on the other, they have a greater chance of being hired. The complexity of the problem arises because the profitability ranking determine the hazard rate in equilibrium, which in turn affects the worker’s outside option value. To have an equilibrium it must be true that the profitability obtained in the wage bargaining generates a ranking that does not change once the effect of the hazard rates is taken into account. A natural equilibrium to look at is the ”coincidence ranking” equilibrium, in which profitability is strictly increasing in productivity. Conditions for the existence of such equilibrium are discussed in Section 3.7.

### 3.4 Matching and Equilibrium probabilities

Due to their ex ante homogeneity all the firms make the same optimal decisions under aggregate state \( X \). The total number of applications in the economy, \( A \), equals the number of unemployed workers who send an application, which is computed as

\[
A = \sum_{i=1}^{\infty} \int a(\theta)f(\theta, \delta = i)d\theta
\]  

(10)

The total number of employed workers is denoted

\[
L = \int f(\theta, \delta = 0)d\theta
\]  

(11)

The unemployment rate equals \( U = \frac{A}{A+L} \) whereas the participation rate is \( P = A + L \). The matching process adopted departs from the urn-ball approach in the literature (see Petrongolo and Pissarides (2001)) in which a worker’s application may not be matched due to a coordination failure: since each urn can accept only one ball, the unlucky balls sorted into already occupied urns remain unemployed. On the contrary, in the present model vacancies can receive any number of balls, but unemployed workers arise because of the recruiting selection of firms.

Since jobs are ex ante identical to workers, the probability that a single application arrives to a vacancy is \( \frac{1}{P} \). Applications are randomly sorted into the vacancies because
jobs are ex ante identical. Thus, the number of applications arrived to a vacancy $N$ follows a binomial distribution.

$$Pr(N = k) = \binom{A}{k} \left( \frac{1}{V} \right)^k \left( 1 - \frac{1}{V} \right)^{A-k}$$

The latter probability does not converge to 0 for a finite $k$ if there is a stable market tightness $\varphi = V/A$. Taking limits as $A$ goes to infinity, $N$ converges to a Poisson distribution with mean $\lambda = A/V$.

From the workers’ point of view, the chance of being in an application pile of size $k$ equals the probability of that $k-1$ workers arrive to the same pile. Hence, conditional on a worker being there, the chance of being in an application pile of size $k$ when there are $V$ vacancies equals

$$\left( \frac{A-1}{k} \right) \left( \frac{1}{V} \right)^k \left( 1 - \frac{1}{V} \right)^{A-1-k}$$

As $A \to \infty$, the probability of being in an application pile of size $k$ equals

$$\frac{\lambda^{k-1} \exp(-\lambda)}{(k-1)!}$$

To derive the probabilities of being hired, it is assumed a coincidence ranking equilibrium, that is $J'(\theta) > 0, \forall \theta$. Besides, since workers will accept any offer from an employer receiving an offer and being hired are equivalent in this model.

If the worker is the only applicant for that vacancy, she gets hired for sure. If she arrives to a pool with one competitor, she gets hired as long as her type is greater than the other applicant’s. Since both workers are independently drawn from the distribution of unemployed workers $\tilde{F}(\theta) \equiv F(\theta|\delta > 0, a(\theta) = 1)$, the chance of being hired is just $\tilde{F}(\theta)$. Generalizing, the chance of being hired conditional on that the number of applicants is $k$ is $\tilde{F}(\theta)^{k-1}$.

In case the number of applicants exceeds $i^*$, the chance of being randomly selected into the screened group is $i^*/k$. Thus, the chance of being recruited conditional on $k$ applicants arriving is $\tilde{F}(\theta)^{i^*} i^*/k$. The equilibrium probability of receiving an offer is obtained by using Bayes’ total probability law

$$\pi(\theta) = \sum_{k=1}^{i^*} \frac{\lambda^{k-1} \exp(-\lambda)}{(k-1)!} \tilde{F}(\theta)^{k-1} + \sum_{k=i^*+1}^{\infty} \frac{\lambda^{k-1} \exp(-\lambda)}{(k-1)!} \tilde{F}(\theta)^{i^*-1} \frac{i^*}{k}$$

After some algebra equation (13) is obtained. Its first component indicates the chance of being hired when all arrived applicants are screened; the second term shows
the effect of the limited screening in case the application pile size exceeds $i^*$.

$$\pi(\theta) = \exp\left(-\lambda(1 - \tilde{F}(\theta))\right) + \sum_{k=i^*}^{\infty} \frac{\lambda^k \exp(-\lambda)}{k!} \left( \frac{i^*}{k+1} - \tilde{F}(\theta)^k \right)$$

A relevant result is that the average hazard rate does not depend on $i^*$.

**Lemma 4** The expected value of the hazard rate $E[\pi(\theta)]$ equals $\frac{1 - \exp(-\lambda)}{\lambda}$ for all values of $i^*$.

**Proof.** See Appendix ■

The value of $i^*$ modifies the hazard function. The lower the value, the flatter becomes the shape of the hazard rate across productivities. Intuitively, when only few applicants are interviewed, low productivity workers are more likely to be hired instead of better candidates because randomness plays a greater role in selection. Since the mean hazard rate only depends on the average number of people interviewed, the result in Lemma 4 allows the model to accommodate different shapes of the hazard rate and unemployment duration without affecting the unemployment rate $U$.

By looking at the flows of workers going in and out of unemployment, the law of motion of the mass of unemployed workers evolves according to

$$A' = A - (1 - \exp(-\lambda))V + \eta L$$

The latter equation indicates that the mass of unemployed workers next period is the mass of unemployed today less the mass of jobs created, plus the mass of jobs destroyed and the number of newborns that participate in the labor market. The job creation term stands for the number of vacant jobs that have received at least one applicant. The job destruction term is just the number of employed workers who had a separation shock. In steady state equilibrium, $A' = A = A^*$ and $\lambda = A/V$. A double equality arises by using Lemma 4

$$\frac{\eta(1 - U^*)}{U^*} = \frac{1 - \exp(-\lambda)}{\lambda} = E[\pi(\theta)]$$

Therefore, the model establishes a triple equality between inflows and outflows from unemployment, the separation rate and the average hazard rate or job finding rate. Moreover, we can see from these equalities that if $0 < \lambda^* < \infty$, the unemployment rate is bounded away from 0 and from 1. There is positive relationship between the mean number of applicants and the equilibrium unemployment rate. When $\lambda$ is large, the number of workers who are hired is small.
3.5 Productivities distribution Law of Motion

At every period, there is a distribution \( F(\theta, \delta) \) of workers, defined on a \( \sigma \)-algebra \( \Theta \times \Delta \), that essentially evolves according to the hiring decisions of firms and the exogenous separation shocks. The latter is summarized by the operator \( F' = T(F(\theta, \delta)) \) in which the prime represents the measure in the next period. In the steady-state equilibrium of the economy, there is a unique invariant measure \( F^*(\theta, \delta) \) that satisfies \( F^*(\theta, \delta) = T(F^*(\theta, \delta)) \). The marginal distribution of productivities is exogenous, time-invariant and obviously satisfies \( f_\theta(\theta) = \sum_{i=0}^{\infty} f(\theta, \delta = i) \), where \( f(\theta, \delta) \) represents the density of \( F \).

The density of productivity types and unemployment durations \( f(\theta, \delta) \) evolves according to the following law of motion, which implicitly define the operator \( T(\cdot) \).

\[
\begin{align*}
    f(\theta, \delta + 1) &= (1 - \pi(\theta)a(\theta))f(\theta, \delta) \quad \forall \delta \geq 1 \quad (15)
\end{align*}
\]

The equation (15) shows that in equilibrium the individuals with characteristics \( (\theta, \delta + 1) \) are workers of characteristics \( (\theta, \delta) \) who were not hired in the previous period. Workers who do not apply, so that \( a(\delta) = 0 \), have no chance of being hired. The law of motion for agents with \( \delta = 0 \) or \( \delta = 1 \) is given by

\[
\begin{align*}
    f(\theta, 1) &= \eta f(\theta, 0) \quad (16) \\
    f(\theta, 0) &= (1 - \eta)f(\theta, 0) + \pi(\theta)a(\theta) \sum_{i=1}^{\infty} f(\theta, \delta = i) \quad (17)
\end{align*}
\]

In equation (16), the density of workers with \( \delta = 1 \) only consists on exogenously separated individuals. In (17), the number of the employed is simply calculated by adding (i) not separated employed individuals in the previous period and (ii) individuals that applied and were hired in the previous period. Doing some algebra and recalling that \( f_\theta(\theta) \) is the marginal distribution of productivities, equations (17) and (15) can be written as\(^8\)

\[
\begin{align*}
    f(\theta, 0) &= \frac{f_\theta(\theta)\pi(\theta)a(\theta)}{\eta + \pi(\theta)a(\theta)} \quad (18) \\
    f(\theta, \delta + 1) &= \eta f(\theta, 0)(1 - \pi(\theta))^\delta \quad (19)
\end{align*}
\]

Using the previous densities, the operator \( (TF)(\theta, \delta) \) is defined by simply integrat-

\(^8\)Notice that \( f_\theta(\theta) - f(\theta, 0) = \sum_{i=1}^{\infty} f(\theta, \delta = i) \)
ing over the arguments of these functions

\[(TF)(\theta,0)) = \int_{-\infty}^{0} \frac{f_{\theta}(v)\pi(v)a(v)}{\eta + \pi(v)a(v)} dv \tag{20}\]

\[(TF)(\theta,1)) = (1 + \eta)F(\theta,0) \tag{21}\]

\[(TF)(\theta,\delta + 1)) = F(\theta,\delta) + \eta \int_{-\infty}^{\theta} (1 - \pi(v)a(v)) f(v,0) dv \quad \forall \delta \geq 1 \tag{22}\]

To obtain an expression for the density of the unemployed workers we can simply use (18) and basic properties of probabilities

\[f(\theta,\delta > 0) = f_{\theta}(\theta) - f(\theta,0) = \frac{\eta f_{\theta}(\theta)a(\theta)}{\eta + \pi(\theta)}\]

Thus, the following result holds

\[f(\theta|\delta > 0, a(\theta) = 1) = \tilde{f}(\theta) = \frac{\eta f_{\theta}(\theta)}{f(\delta > 0|a(\theta) = 1)(\eta + \pi(\theta))}\]

Since the equilibrium unemployment rate \(U = f(\delta > 0|a(\theta) = 1)\) is well-defined for a positive finite \(\lambda\), we obtain the CDF of productivities of unemployed workers by integrating the previous equation

\[\tilde{F}(\theta) = \int_{-\infty}^{\theta} f(v|\delta > 0, a(\theta) = 1) dv = \int_{-\infty}^{\theta} \frac{\eta f_{\theta}(v)}{\eta + \pi(v)} dv \tag{23}\]

This is a Volterra nonlinear integral equation whose solution is the endogenous cumulative distribution function of the unemployed workers. The next Proposition establishes its existence, uniqueness and other properties via a fixed point argument.

**Proposition 5** There exists a unique steady-state differentiable measure of productivities and durations \(F^*(\theta,\delta)\) that is a fixed point of the operator \((TF)(\theta,\delta))\) provided that \(f_{\theta}(\theta)\) is continuous and \(\sup_{\theta} f_{\theta}(\theta) < \infty\).

**Proof.** See Appendix

### 3.6 Stationary Symmetric Recursive Competitive Equilibrium

The Stationary Symmetric Recursive Competitive Equilibrium of this model equilibrium is defined as

i. A set of value functions \(Q(\theta,\tilde{w}(\theta)), Q_{a}(\theta,\tilde{w}(\theta)), Q_{n}, W(w(\theta)), P\) and \(J(\theta)\) defined in equations (2), (3), (1),(4) and (6).
ii. Policy functions $a^*(\theta)$ and $g^*(k)$ that solves the worker’s application problem in $Q(\theta, \tilde{w}(\theta))$ and the firm’s number of screened applicants in $G(k)$.

iii. An aggregate state $X$ consistent with individuals’ behavior and free entry $P = 0$, $A$ is defined according to equation (11).

iv. Equilibrium hazard rate functions $\pi(\theta)$ as described in equation (13).

v. A wage schedule $w(\theta)$ that solves condition (8).

vi. An equilibrium distribution $F(\theta, \delta)$ which is a fixed point of the operator $F' = (TF)(\theta, \delta)$ described by equations (20),(21) and (22).

3.7 Characterization of the equilibrium

The equilibrium of the model generates several features of the cross-sectional distribution of wages and unemployment durations that are characterized in the following propositions\(^9\). In the first result, the model’s equilibrium shows wage negative duration dependence. Workers who have been unemployed for a long time expect low wages, which is a direct consequence of the recruiting selection process.

**Proposition 6** In the economy under study expected wages decrease in unemployment duration

1. For all $\delta > 0$ and $a(\theta) = 1$, $E[w(\theta)|\delta] > E[w(\theta)|\delta + 1]$.

2. If $\delta = 0$, $E[w(\theta)|\delta] = E[w(\theta)|\delta + 1]$.

**Proof.** See Appendix □

The second result shows that expected durations are longer for individuals of low productivity.

**Proposition 7** The expected duration conditional on the productivity type is

$$E[\delta|\theta] = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)}$$

which is a differentiable and strictly decreasing function in $\theta$.

**Proof.** See Appendix □

Since for a given worker the probability of leaving unemployment is $\pi(\theta)$, the duration of unemployment follows a geometric distribution conditional $\theta$. For that reason,

\(^9\)The propositions obviously hold only for individuals in the labor force ($a(\theta) = 1$)
Proposition 7 shows that the unconditional duration of unemployment is the probability of being unemployed \( \frac{\eta}{\eta + \pi(\theta)} \) multiplied by the expected duration conditional on being unemployed \( \frac{1}{\pi(\theta)} \). Using the same rationale, the variance of the duration conditional on being unemployed is \( \frac{1}{\pi(\theta)^2} \). The next result shows a result about the unconditional variance of unemployment durations

**Proposition 8**  
The variance of the unemployment duration conditional on the productivity type is

\[
\text{Var}[\delta|\theta] = \frac{\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta + \pi(\theta))^2\pi(\theta)^2},
\]

which is a differentiable and strictly decreasing function in \( \theta \).

**Proof.** See Appendix □

A pending point was to show some guidelines the parametrization the model should have to generate a symmetric “Coincidence ranking” equilibrium, which holds if \( J'(\theta) > 0 \) for all \( \theta \). Unfortunately, sufficient conditions that only rely on primitives are not possible to obtain, because they depend on \( \tilde{F}(\theta) \), which does not have an analytical solution. Verifcation of the equilibrium’s existence can be done by checking if \( J'(\theta) > 0 \) for all \( \theta \) in a computed numerical solution. Nevertheless, it is possible to get some insights about the kind of primitives that generate such equilibrium, which in turn, enhances the understanding of the model. Then, in the Appendix B I analyze \( J'(\theta) \) to find out under what conditions the model has an equilibrium. A first conclusion is that the wage function \( w(\theta) \) is always increasing in productivity, regardless the preferences, the distribution of productivities and firms’ screening technology. Secondly, I show that for the specific case in which screening applicants is free so that \( i^* = +\infty \), the coincidence ranking equilibrium exists if the following inequality holds

\[
\lambda \tilde{f}(\theta) < \frac{1 - \alpha}{\alpha(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right)
\]

The condition (26) tells us that in order to have a coincidence ranking equilibrium it is necessary high enough dispersion of productivity for unemployed workers, i.e. low values for the density of the unemployed. The latter in turn, depends on the population’s productivity dispersion. Intuitively, the relative productivity ranking of workers is what matters to determine the hazard rate of workers. In case the dispersion of productivities is small, a great difference in hazard rates can be associated to a minor increase in productivity. The outside option of a worker rises much more significantly than her productivity does, making unprofitable for the firm to hire high productivity workers. On the other hand, workers’ risk aversion play a role, too. In order to have
a coincidence ranking equilibrium, the effect of low productivity dispersion may be overcome by the fact that workers do not appreciate marginal wage raises too much.

Additionally, a high $\lambda = A/V$ -or equivalently a high unemployment rate- make it difficult for a coincidence ranking equilibrium to exist, ceteris paribus. Intuitively, if too many applicants show up to fill a vacancy in expectation, available jobs are scarce and it is extremely easy for very good workers to get hired. Since the top applicant’s outside option is too high, firms have to yield almost all the surplus to the worker, which creates incentives to hire someone else. The greatest difficulty of having a coincidence ranking equilibrium is precisely achieved when $i^* = +\infty$, because top applicants have the highest outside option value. Hence, although condition (26) is derived for a special case $i^* = +\infty$, if the equilibrium exists under that scenario, it surely exists -ceteris paribus- for a finite $i^*$.

A final comment: the fact that the employer’s outside option is just posting vacancies again is the critical assumption that makes difficult to establish the coincidence ranking equilibrium existence. If the employer’s outside option were the value associated to the second-best worker within the applicants, the best applicant will have to yield at least the value the employer would have obtained from the second-best worker. This possibility is certainly interesting and may become a subject of future research.

3.8 Discussion

3.8.1 Sequential vs. Nonsequential search

The sequential model is a particular case of the analyzed framework when the marginal cost of interviewing is high enough to make $i^* = 1$. This special case is inconsistent with the fact that several applicants are interviewed to fill a vacancy. In this case, the firms’ problem collapses to a simpler hiring problem. Employers receive at least one application with probability $\exp(-\lambda)$, randomly select one worker and obtain the average lifetime profits from an unemployed worker.

$$
P = -\kappa + \beta \left[ \exp(-\lambda)P + (1 - \exp(-\lambda))\mathbb{E}[J(\theta)\delta > 0, a = 1] \right]
$$

Firms’ optimal hiring policy consists on making offers to applicants of productivity above some threshold $\theta$. Above that reservation productivity, all workers face the same hazard rate; for all productivities below $\theta$, there is no chance of being hired. Consequently, in equilibrium no worker below that threshold submits a costly application. Thus, the partial equilibrium model of Burdett and Cunningham (1998) cannot explain why an arriving candidate is not hired if the cost of application is arbitrarily low but positive. On the other hand, if sending an application is free ($z = 0$), unprofitable
applicants may arrive to the firm and multiple workers can be interviewed before the vacancy is filled. However, the number of candidates interviewed in the data demonstrate that this situation is implausible. According to NES97 data, roughly 6 out of 7 applicants are not hired for any firm in the economy. Consequently, the steady-state unemployment rate would be enormous.

Moreover, the fact that hazard rates are constant regardless the workers’ productivity is problematic on its own. In empirical studies of duration analysis the presence of hazard rate unobserved heterogeneity is pervasive (see for instance, Machin and Manning (1999), van den Berg (2001)). These findings typically derive from the estimation of Mixed Proportional Hazard models in which the hazard rate is decomposed into three multiplicative components: a baseline hazard common to all individuals, observable and unobservable heterogeneity. Moreover, the model helps to understand the nature of unobserved heterogeneity. Rather than simply assuming this heterogeneity, in the recruiting selection model it arises naturally from the interaction of workers’ heterogeneous productivity with the recruitment technology of firms if the econometrician cannot fully observe worker’s productivity.

The closely related issue of negative duration dependence of the hazard rate can also be understood as an incomplete observation of the productivity of the worker \( \theta \). The predictions of the recruiting selection model are consistent with the empirical work in this area typically since covariates that are typically associated to higher earnings are also associated to higher hazard rates (with the exception of age). Moreover, empirical evidence (Machin and Manning 1999; Abbring et al. 2002; Cockx and Dejemeppe 2005) report that after controlling for worker’s observables and unobserved heterogeneity, the negative duration dependence pattern vanishes, which supports the idea that the composition of workers’ productivities decays as unemployment duration increases.

An augmented version of the sequential search model with heterogenous workers that is nested into the recruiting selection model, might replicate the number of applicants per vacancy observed in the data by introducing randomness in the selection process (employer imperfectly learns \( \theta \)) or in the match surplus (match productivity shock or match heterogeneity). The major technical difficulty of these extensions is that the worker turn down offers if the received shocks are not good enough. Although intuition suggests that such models may display properties that are similar to a recruiting selection model, the ultimate test for these theories is possibly empirical.

### 3.8.2 Permanent Income and Welfare

Computing permanent incomes using the model is simple. The discounted lifetime earnings conditional on being employed and on being unemployed are denoted \( Y^E(\theta) \)
and $Y^U(\theta)$ respectively, can be written recursively as

\[
Y^E(\theta) = w(\theta) + \beta(1 - \eta)Y^E(\theta) + \beta\eta Y^U(\theta) 
\] (27)
\[
Y^U(\theta) = b(1 - z) + \beta \pi(\theta)Y^E(\theta) + \beta(1 - \pi(\theta))Y^U(\theta) 
\] (28)

After doing the same algebra used to derive equations (2), the permanent income conditional on unemployment $y^U$ is

\[
y^U(\theta) = S(\theta) b(1 - z) + (1 - S(\theta)) w(\theta) 
\] (29)

where $S(\theta)$ is defined as in (2). It follows that the permanent income for an employed worker is

\[
y^E(\theta) = \frac{(1 - \beta) w(\theta) + \beta \eta y^U(\theta)}{1 - \beta(1 - \eta)} 
\] (30)

The unconditional long-run probability of being employed is $p^E(\theta) = \frac{\pi(\theta)}{\pi(\theta) + \eta}$ while the one of being unemployed is $p^U(\theta) = 1 - p^E(\theta)$. Therefore, the unconditional permanent income is

\[
y(\theta) = p^E(\theta)y^E(\theta) + (1 - p^E(\theta))y^U(\theta) 
\] (31)

Using an analogous logic, the unconditional welfare (certain equivalent) measured in terms of consumption is

\[
we(\theta) = u^{-1}\left((1 - \beta) \left(p^E(\theta)W(w(\theta)) + (1 - p^E(\theta))Q(\theta, w(\theta))\right)\right) 
\] (32)

In a sequential search model the ex ante inequality is simply determined by the workers productivity because all of the applicants have the same chance of being hired. In contrast, in the recruiting selection case individuals of low productivity forgo more labor earnings during their lifetimes than their highly productive counterparts do. Moreover, the joint effect of search frictions and Nash bargaining creates another channel to intensify inequality in that high productivity workers obtain higher wages because their outside options are more valuable. The inequality amplification effect is even greater in terms of welfare due to the concavity of the utility function. Low productivity workers not only have greater unemployment duration, but also face greater uncertainty about the length of their unemployment spells, according to Proposition 8.

An alternative explanation that may create similar effects on permanent income and welfare is the existence of endogenous separation, that is, once an idiosyncratic shock hits the worker-job match, low-quality workers are more likely to be fired. That would explain why the pool of unemployed workers has lower average productivity than the employed group as it does the model outlined here. However, if such hypothetical model have employers using a sequential search strategy, all workers above certain productivity
threshold face exactly the same chance of being hired, as shown above. The conclusion is that endogenous separation story fails to explain why unemployment duration and wages are negatively related in cross-sectional data, but may certainly contribute to explain why there is a permanent income gap between employed and unemployed workers. Endogenous separation explains differences in permanent income due to the labor market frictions in terms of reoccurrence of unemployment whereas recruiting selection mechanism generates consistently longer and more volatile unemployment spells. The evidence is mixed regarding occurrence dependence of unemployment. For instance, Heckman and Borjas (1980) and Choi and Shin (2002) find that previous joblessness does not seem to affect future unemployment (occurrence dependence).

The recruiting selection model also creates a tension between efficiency and equality. Suppose there is a compensated screening cost reduction so that the number of vacancies posted $V$ does not change, but the maximum number of interviewed applicants per vacancy $i^*$ increases. As discussed in Section 3.4, in this case the shape of the hazard rate turns steeper because it is more likely that high productivity workers are detected in application pools by employers. Two consequences arise. First, the assignment of workers to job is more efficient because the productivity of hired workers is higher in expectation. Secondly, since low productivity workers heavily rely in good luck to exit from unemployment, the improved employer screening makes more difficult for them to find a job. Screening technology plays a significant role in both augmenting efficiency and permanent income inequality. Differently from the case of temporary earnings loses due to job displacement, the implications for public policies in this case are different. In the case of job displacement some social insurance mechanism can help individuals to smooth consumption over time. In contrast, low permanent income is permanently tied to longer and more volatile unemployment spells in the recruiting selection model so, social insurance can probably play a more modest role.

4 Empirical Assessment

4.1 The data

Two interrelated issues arise when the model is taken to the data. One one hand, the model generates macroeconomic predictions that should have a sound empirical counterpart in aggregate data of the whole economy or, at least, in a relatively closed well-defined labor market. This consideration motivates the use of a large representative dataset such as the US Current Population Survey (CPS). On the other hand, the problem of the approach is that the re-employment wages of unemployed workers at the
time of the survey are not observed due to the cross-sectional nature of the data. Even though using longitudinal data can partially circumvent this problem, other available datasets for the US such as NLSY79 or PSID do not have enough observations or are not representative enough of the whole US economy. The CPS is possibly the choice that minimizes the representativeness issue and provides a limited longitudinal structure that suffices for the empirical exercise in this paper.

The wages are measured as last year weekly earnings of workers reported in the CPS Earnings Study. All wages are detrended and expressed in dollars of 2000\(^{10}\). Since the model represents an economy in its steady-state, the distribution unemployment duration \(\delta\) is, at the same time, the ongoing duration of individuals unemployed at some point in time, and also the distribution of completed unemployment durations of individuals hired at certain point in time. The CPS provides measures of the ongoing duration in weeks (from Earnings Study) as well as the number of weeks unemployed last year in one stretch (from March Supplement). The reasonable data counterpart of the predicted relationship between (re-employment) wages and durations is the empirical jointly behavior of completed unemployment spells and re-employment weekly earnings. Focusing on ongoing durations would lead us to the problem of assigning some re-employment wage to an unemployed worker.

### 4.2 Calibration

Since a worker can only send one application per period in the model, choosing a week as time period makes this restriction less stringent. The goal is to match main features of the US labor market 1985-2006, such as the unemployment rate and the median of unemployment duration using the number of applicants interviewed from the NES97. Another goal is to mimic some moments of the weekly earnings distribution of employed workers. Having these targets, the interest is too see if the model’s predictions replicate the proper CPS counterparts. Since one element to be calibrated is the distribution of types, an infinite dimensional object, the tie-hand assumption will be that the productivity distribution of the labor force \(f_\theta(\theta|a(\theta) = 1)\) is lognormal truncated at the lowest productivity worker who earns the minimum wage.

\(^{10}\)The detrending procedure consists on subtracting the mean of log wages of the year and adding the mean of the year 2000. The results shown in this paper do not importantly change due to this procedure.
4.2.1 Targeting Unemployment and recruiting costs:

In Section 3.4 there is a double equality in the equation (14) that shows a relationship among the unemployment rate, average hazard rate, separation rate and average number of applicants interviewed. All these pieces of information come from independent sources. I follow the approach of solving the separation rate in terms of the mean number of interviewed applicants and the unemployment rate using equation (14). The number obtained for the separation rate is very close to the one reported by Shimer (2005).

\[ \hat{\eta} = \frac{(1 - \exp(-\lambda)) \mathcal{U}}{\lambda(1 - \mathcal{U})} \]  

(33)

Although endogenous, the maximum number of interviewed applicants \( i^* \) is treated as a parameter for calibration purposes. As discussed in Section 3.4, this parameter can modify the shape of the hazard rate without affecting the unemployment rate in equilibrium. The choice of \( i^* \) is therefore crucial to mimic the shape of the conditional distribution of unemployment durations \( f_\delta(\delta) \). Two approaches are followed in two different calibrations. The first is to set \( i^* \) exogenously at reasonable values using the available data. Thus, setting \( i^* = 10 \) seems a reasonable choice because it is the percentile 90 of the distribution of the number of interviewed applicants in NES97. The second approach is letting the data to decide the value of \( i^* \) in order to match the median duration, which seems the most logical target because of the truncation of unemployment spells greater than 99 weeks in the data.

The recruiting costs \( \kappa \) and the function \( C(\cdot) \), specialized as \( C(k) = \xi k \), are calibrated so that after solving the model and computing a numerical solution for the function \( \tilde{J}(k) \), the values chosen ensure that the maximum number of interviewed applicants is the desired \( i^* \) and that the targeted mean number of candidates \( \lambda^T \) is exactly achieved. Thanks to Proposition 3, there is only a unique solution to this problem.

4.2.2 Targeting unemployment benefits and minimum wage:

Assuming CRRA preferences with parameter \( \gamma \) the Nash condition becomes

\[ \theta = w(\theta) + \frac{1 - \alpha}{\alpha(1 - \gamma)} S(\theta) \left( w(\theta) - (b(1 - z))(1 - \gamma) w(\theta)^\gamma \right) \]  

(34)

The lowest type \( \theta \) earns the lowest wage \( w(\theta) \) and has the lowest hazard rate \( \pi(\theta) = \exp(-\lambda) \) because she only gets hired if she does not face any other competitor. Moreover, the type \( \theta \) must also be indifferent between being in the labor force or not, i.e.

\[ Q_a(\theta, w(\theta)) = Q_n \Rightarrow S(\theta) u(b(1 - z)) + (1 - S(\theta)) u(w(\theta)) = u(b) \]
Using these conditions, the unemployment income $b$ is expressed in terms of other parameters.

$$b = w(\theta) \left[ \frac{1 - S(\theta)}{1 - S(\theta)(1 - z)^{1-\gamma}} \right]^{\frac{1}{1-\gamma}}$$

(35)

Replacing (35) into (34) and doing some algebra

$$\theta = w(\theta) \left( 1 + \frac{1 - \alpha}{\alpha(1 - \gamma)} S(\theta) \left( \frac{1 - (1 - z)^{1-\gamma}}{1 - S(\theta)(1 - z)^{1-\gamma}} \right) \right)$$

(36)

Two additional restrictions are needed: (i) the observed minimum wage $w(\theta)$ is greater than $b(1 - z)$ and (ii) Unemployment income of an applicant $b(1 - z)$ is positive. There are three cases to analyze. If $\gamma > 1$, by algebraic manipulation of (35), the conditions are met if $z < \hat{z} \equiv 1 - S(\theta)^{\frac{1}{1-\gamma}}$. In case $\gamma < 1$, it is needed that $z > \hat{z} \equiv 1 - S(\theta)^{\frac{1}{1-\gamma}}$. Finally, the restrictions are always satisfied if $\gamma = 1$.

4.2.3 Targeting wage distribution:

To keep the calibration as simply as possible, only the mean and variance of the log earnings of employed workers will be matched. Other parameters such as the relative risk aversion $\gamma$, the exogenous bargaining power $\alpha$ and the ratio application cost to unemployment income $z$ are set at conventional values. The parameters are summarized in Table 1.

4.2.4 Solving the model:

Solving the model is basically to find a solution for the Volterra integral equation in (23), which is cumulative distribution function of unemployed workers, $\tilde{F}(\theta)$. The computational algorithm is simple, fast and accurate in a grid of points. Its description is in Appendix B. Using numerical solution, it is straightforward to compute hazard rates, wages and value functions.

4.3 Results

The main results of the model are in Table 2. Parameters in Table 1 are set to exactly match the unemployment rate, mean and standard deviation of wages of the employed workers ($\delta = 0$). Conditions for the existence of the coincidence ranking equilibrium are met for both calibrated models, i.e. $J'(\theta) > 0$ for all $\theta$, having used reasonable productivity dispersion and workers’ risk-aversion$^{11}$. $^{11}$Some non reported computations show that for relative risk aversion lower than 1, the existence of a coincidence ranking equilibrium requires a productivity dispersion that is too high.
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$: Worker’s bargaining power</td>
<td>0.5</td>
<td>0.5</td>
<td>Symmetric Nash</td>
</tr>
<tr>
<td>$\beta$: Discount rate</td>
<td>$\sqrt{0.95}$</td>
<td>$\sqrt{0.95}$</td>
<td>Standard</td>
</tr>
<tr>
<td>$\eta$: Separation rate</td>
<td>0.863%</td>
<td>0.863%</td>
<td>Obtained from eq. (33)</td>
</tr>
<tr>
<td>$\gamma$: Relative risk aversion</td>
<td>2</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>$z$: Ratio application cost to b</td>
<td>1%</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>$\bar{w}$: Minimum wage obs.</td>
<td>20.00</td>
<td>20.00</td>
<td>US$ in 2000</td>
</tr>
<tr>
<td>$b$: Unemployment income</td>
<td>18.41</td>
<td>18.41</td>
<td>from eq. (35)</td>
</tr>
<tr>
<td>$\mu_\theta$: Mean of $\theta$</td>
<td>6.6358</td>
<td>6.7769</td>
<td>Target mean and SD of log wages</td>
</tr>
<tr>
<td>$\sigma_\theta$: Std.dev of $\theta$</td>
<td>0.8193</td>
<td>0.9061</td>
<td>Target mean and SD of log wages</td>
</tr>
<tr>
<td>$i^*$: Max.interviewed applicants</td>
<td>10</td>
<td>4</td>
<td>Explained in text</td>
</tr>
<tr>
<td>$\xi$: Marg. cost per interview</td>
<td>3312.74</td>
<td>11660.3</td>
<td>To match choice of $i^*$ and $\lambda$</td>
</tr>
<tr>
<td>$\kappa$: Cost of vacancy posting</td>
<td>25974.7</td>
<td>29715.8</td>
<td>To ensure free-entry condition</td>
</tr>
</tbody>
</table>

While in model 1, the maximum number of interviewed applicants $i^*$ is fixed at 10 according to NES97 data, in model 2 the value is set so that the median unemployment duration is matched. The conclusion is that a reasonable value for $i^*$ overstates the median unemployment ongoing duration. Figure 1 displays the marginal distribution of unemployment spells. The data shows spikes at durations that are multiples of 4, which is usually attributed to recall or rounding biases of survey respondents. The first result is that the measure of workers with one or two unemployment durations is overstated by the calibration. In order to match the desired unemployment rate and $\lambda$ the separation rate is too high for the sample analyzed, in spite of being close to the mean value reported by Shimer (2005). The second relevant fact is the big spike that is observed at the truncation point at 51+ weeks. Both models predict ongoing unemployment durations that are too long compared to the CPS data. In Figure 2 displays the marginal distribution of completed unemployment spells. Table 2 also indicates that the median in this case is closer to the model’s values (12 weeks), although the shape of the curve is not well mimicked. Unemployment spells greater than 51 weeks are missing because they correspond to individuals who do not report any wages during the previous year. That explains the absence of a spike at week 51.

The excessive number of long-term unemployed in Figure 1 can be somewhat reconciled with the evidence after using an alternative measure of unemployment that include
workers who are “passive” searchers, or marginally attached to the labor force\textsuperscript{12}. Jones and Riddell (1999) analyze what is the most appropriate definition. After studying the transitions for four states: Employed, Unemployed, Marginally Attached and Not Attached, they conclude that the boundary between the Unemployed and the Marginally Attached is somewhat blur. Although the recruiting selection model does not consider individual dynamics in its simpler form, low-skilled individuals that experience some productivity reduction can qualify as ”passive” searchers. The measure of unemployment duration used in the CPS may be affected by the behavior of more productive and wealthier nonemployed individuals sorting into the “active” unemployment state and low productivity workers leaving the traditionally defined labor force. Albeit there is no exact recorded nonemployment duration of individuals marginally attached in the data, two approximated measures are constructed with the available information, called duration A. Details on the construction are provided in the Appendix D. Figure 1 shows that the alternative unemployment duration A generates a similar spike at $\delta = 51$, although the performance of the model at shorter duration worsens.

The overall fit to the distribution of log wages of employed workers seems fine\textsuperscript{13} (see Figure 3) although only the mean and variance are explicitly targeted. The model mimics the distribution of unemployed workers with 12 or less weeks of unemployment well (short-term unemployment, Figure 4). In particular, the mean of this distribution is considerably lower than the distribution of employed workers, and its dispersion is higher. However, for a wider range of durations of unemployed workers the fit worsens. According to the model, long-term unemployed workers are of very low quality and therefore their wages’ distributions progressively shift to the left. Graphical displays of the conditional densities for models 1 and 2 are in Figures 10 and 11.

Other predictions of the model are those coming from Propositions 6, 7 and 8. In order to compare those predictions to CPS data, I computed the relevant statistics in the model and in the data. For Proposition 6, the results can be seen in Figure 5. Although CPS data does show the pattern that re-employment wages decrease in unemployment duration, both models suggest that the observed decreasing pattern should be much more accentuated. The recruiting selection mechanism generates too much selection.

To make an empirical measure of the predictions of unemployment duration conditional on wages, I construct the mean duration and variance of the duration by wage percentiles. Because the expected mean and wages depend on the separation rate, which is assumed to be constant for all workers, reporting moments conditional on

\textsuperscript{12}For instance, Yashiv (2006) considered this extended definition to assess the cyclical properties of the standard matching search model.

\textsuperscript{13}Empirical densities are computed using nonparametric kernel estimator with the “rule of thumb” bandwidth suggested by Silverman (1986).
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>5.48%</td>
<td>5.48%</td>
<td>5.48%</td>
</tr>
<tr>
<td>Median ongoing duration</td>
<td>9</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>Mean ongoing duration(1)</td>
<td>16.59</td>
<td>24.45</td>
<td>19.78</td>
</tr>
<tr>
<td>Std.Dev ongoing duration(1)</td>
<td>20.81</td>
<td>21.53</td>
<td>20.40</td>
</tr>
<tr>
<td>Median completed duration</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean completed duration(2)</td>
<td></td>
<td>15.81</td>
<td></td>
</tr>
<tr>
<td>Std.Dev completed duration(2)</td>
<td></td>
<td>12.16</td>
<td></td>
</tr>
<tr>
<td>Mean log($w$) Emp.</td>
<td>6.20</td>
<td>6.20</td>
<td>6.20</td>
</tr>
<tr>
<td>Std.Dev log($w$) Emp.</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>Perc 10 log($w$) Emp.</td>
<td>5.23</td>
<td>5.16</td>
<td>5.16</td>
</tr>
<tr>
<td>Perc 25 log($w$) Emp.</td>
<td>5.80</td>
<td>5.71</td>
<td>5.70</td>
</tr>
<tr>
<td>Perc 50 log($w$) Emp.</td>
<td>6.34</td>
<td>6.26</td>
<td>6.25</td>
</tr>
<tr>
<td>Perc 75 log($w$) Emp.</td>
<td>6.83</td>
<td>6.75</td>
<td>6.75</td>
</tr>
<tr>
<td>Perc 90 log($w$) Emp.</td>
<td>7.25</td>
<td>7.15</td>
<td>7.15</td>
</tr>
<tr>
<td>Mean log($w$) Unem. $\delta \leq 12$</td>
<td>5.91</td>
<td>5.63</td>
<td>5.78</td>
</tr>
<tr>
<td>Std.Dev log($w$) Unem. $\delta \leq 12$</td>
<td>0.72</td>
<td>0.81</td>
<td>0.82</td>
</tr>
<tr>
<td>Perc 10 log($w$) Unem. $\delta \leq 12$</td>
<td>4.67</td>
<td>4.58</td>
<td>4.70</td>
</tr>
<tr>
<td>Perc 25 log($w$) Unem. $\delta \leq 12$</td>
<td>5.25</td>
<td>5.04</td>
<td>5.19</td>
</tr>
<tr>
<td>Perc 50 log($w$) Unem. $\delta \leq 12$</td>
<td>5.84</td>
<td>5.59</td>
<td>5.77</td>
</tr>
<tr>
<td>Perc 75 log($w$) Unem. $\delta \leq 12$</td>
<td>6.44</td>
<td>6.19</td>
<td>6.37</td>
</tr>
<tr>
<td>Perc 90 log($w$) Unem. $\delta \leq 12$</td>
<td>6.97</td>
<td>6.73</td>
<td>6.87</td>
</tr>
</tbody>
</table>

Notes: (1) Model-generated statistics are also computed with truncation at 51 weeks to be compared to data statistics. (2) Data is missing (truncated) after 51 weeks.
wage and conditional on wage and unemployment is important. CPS data plotted in Figures 6, 7, 8 and 9 show that the pattern of mean and variance of unemployment durations decreasing in wages is still there even if the relevant separation rates for high and low-wages workers are different. Hence, the decreasing pattern is not due to heterogenous separation rates. Even though CPS data in Figures 6 and 7 show a clear decreasing pattern, the shape of the model-generated curves imply an extremely large expected unemployment duration for low wage workers and an exceedingly low one for high-wage workers. A similar pattern for variances conditional on wages and conditional on wages and unemployment shows up in Figures 8 and 9. CPS data shows that high-wage workers tend to have less volatile unemployment spells, but the shapes created by the models clearly overstates the duration volatility for low-wage workers and understates it for high-wage workers.

Overall, the empirical assessment shows that the recruiting selection mechanism with the simplifying assumption of perfect observability of workers’ productivities and the fact that only one application is sent each period, exaggerates the patterns of the joint distribution of wages and unemployment duration. High productivity workers find jobs too easily and low productivity ones are condemned to stay unemployed for too long.

4.4 An experiment: Efficiency vs. Equality

The previous results show that the labor market generate strong inequality amplification given an exogenous distribution of productivities. Another way to see the model is that there is a trade-off between efficiency and equality. If firms are able to interview all the applicants they can always distinguish the best worker in the pool. The exactly opposite case occurs in the sequential search model. If only one applicant can be interviewed, it is less likely that the firm can hire the most productive worker within the arrived candidates. The experiment proposed here is to increase the marginal cost per interview $\xi$ so that the firm would optimally set $i^* = 1$ but allowing the vacancy-posting cost $\kappa$ to vary so that the unemployment rate remains constant in all the cases studied. When $\xi$ rises, the hiring process becomes less profitable and therefore, the number of vacancies decreases, the average number of interviewed applicants increases and the unemployment rate rises. To keep the unemployment rate $U$ constant $\kappa$ is reduced so that all the effects presented here are only driven by the quality of the assignment of workers to job. Results are presented in Table 3. The first two columns just restate results from Table 2 to ease comparison.

Figure 12 shows the dramatic effects of such experiment with model 1 as the benchmark. The permanent income function is computed according to equation (31) and
plotted using logarithmic scale. Very low productivity workers are better off in terms of permanent income than they are when there is recruiting selection. The curve of permanent income is also much flatter in the sequential search case, indicating that the inequality is much lower. Two issues play a key role to explain the observed differences between sequential and nonsequential scenarios. First, expected unemployment durations are much lower for highly productive workers in the baseline scenario; in contrast, all workers face the same expected duration in the sequential case. Secondly, due to the Nash bargaining wage determination, the outside option of high productivity workers is considerably more valuable under recruiting selection. Therefore, the wages obtained for them are higher than the ones paid under sequential search. Similar effects in permanent income are observed in Fig 13. The magnitudes are lower because the recruiting selection was already less important in model 2 with \( i^* = 4 \). In Table 3 we can see the equality-efficiency trade-off. For model 1, the low efficiency of assignment to job in the sequential case reduces the average wages in 34% and the average permanent income in 27%. However, equality improves a lot: the dispersion in wages and permanent income decrease drastically. For model 2, the number are still remarkable, which suggests that in a sequential search economy just a moderate introduction of recruiting selection can have dramatic effects in efficiency and inequality.

For the proposed welfare measure -the certain equivalent of lifetime utility- the same patterns are observed but they are even stronger. In Figures 12 and 13 a considerable part of the workers at the bottom of the distribution are better off in the sequential case than they are in the benchmark recruiting selection model. The certain equivalent curve is flatter than the permanent income curve because in the sequential search model all individuals face the same average hazard rate. Switching from recruiting selection to sequential search increases equality through two channels. First, in the sequential search environment low productivity individuals work on average the same as high productivity workers do. In contrast, under recruiting selection, the better the worker is, the most time she is employed in expectation. Secondly, in the sequential case the less productive workers face considerably less risks because the duration of their unemployment spells is less uncertain. The other side of the coin is that the most productive workers face much more volatility of their unemployment duration, which diminishes their welfare in comparison to the benchmark case.

Table 3 shows that the effects of firms’ recruiting behavior on welfare are enormous. In model 1, the mean certain equivalent decreases by 53% while the inequality radically diminishes. The dispersion of welfare falls from 0.87 log points to 0.27. The effects for model 2 are milder, but still important.

Moreover, the median unemployment duration is just 5 weeks under the sequential
case, which means that most unemployment spells are very short. This example shows that the existence of long unemployment spells may not be regarded as an inefficiency of the labor market to assign workers to jobs: it may be exactly the opposite.

Table 3: Recruiting Selection vs. Sequential search

<table>
<thead>
<tr>
<th>Parameter/Statistic</th>
<th>Model 1</th>
<th>Model 1, seq</th>
<th>Model 2</th>
<th>Model 2, seq</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^*$: Max.interviewed</td>
<td>10</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$\xi$: Marg. cost interview</td>
<td>3312.7</td>
<td>11660.3</td>
<td>36674.0</td>
<td>51453.5</td>
</tr>
<tr>
<td>$\kappa$: Cost vacancy posting</td>
<td>25974.7</td>
<td>29715.8</td>
<td>32445.7</td>
<td>39445.4</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>5.48%</td>
<td>5.48%</td>
<td>5.48%</td>
<td>5.48%</td>
</tr>
<tr>
<td>Median duration</td>
<td>16</td>
<td>5</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Mean duration(1)</td>
<td>24.45</td>
<td>6.72</td>
<td>19.78</td>
<td>6.72</td>
</tr>
<tr>
<td>Std.Dev duration(1)</td>
<td>21.53</td>
<td>6.18</td>
<td>20.40</td>
<td>6.18</td>
</tr>
<tr>
<td>Mean log($w$) Emp.</td>
<td>6.20</td>
<td>5.86</td>
<td>6.20</td>
<td>5.94</td>
</tr>
<tr>
<td>Std.Dev log($w$) Emp.</td>
<td>0.77</td>
<td>0.53</td>
<td>0.77</td>
<td>0.58</td>
</tr>
<tr>
<td>Mean log Perm. Income</td>
<td>6.08</td>
<td>5.80</td>
<td>6.09</td>
<td>5.89</td>
</tr>
<tr>
<td>Std.Dev log Perm. Income</td>
<td>0.91</td>
<td>0.53</td>
<td>0.88</td>
<td>0.58</td>
</tr>
<tr>
<td>Mean log Cert. Equiv.</td>
<td>5.66</td>
<td>5.13</td>
<td>5.55</td>
<td>5.16</td>
</tr>
<tr>
<td>Std.Dev log Cert. Equiv.</td>
<td>0.87</td>
<td>0.27</td>
<td>0.77</td>
<td>0.28</td>
</tr>
<tr>
<td>$\Delta$ log mean wage</td>
<td>-0.34</td>
<td>-0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log std.dev wage</td>
<td>-0.24</td>
<td>-0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log mean Perm. Income</td>
<td>-0.27</td>
<td>-0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log std.dev Perm. Income</td>
<td>-0.38</td>
<td>-0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log mean Cert. Equiv.</td>
<td>-0.53</td>
<td>-0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log std.dev Cert. Equiv.</td>
<td>-0.61</td>
<td>-0.49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) Model-generated statistics are also computed with truncation at 51 weeks to be compared to data statistics.

5 Conclusions

The model presented in this paper embeds a recruiting selection process into a benchmark sequential search model with heterogeneous workers. The empirical relevance of this employer’s nonsequential search strategy has been documented in several empirical
papers. Augmenting the model with this highly realistic characteristic provides a simple explanation for documented empirical facts. In the data high hazard rates and wages are positively correlated, there is substantial hazard heterogeneity after controlling for observables and the negative duration of the hazard rate is generally find to be spurious once adjusted for workers’ heterogeneity.

The model can be regarded as an alternative approach to explain several empirical phenomena that cannot be readily understood using standard search models. Although one can think about certain stochastic augmentations of the standard search model (including stochastic productivity dynamics, imperfect screening and match heterogeneity) that may also account for certain features of the data, this model provides a benchmark that is able to explains several features of the aggregate labor market through a very simple and intuitive mechanism of selection. Even though the calibration of the model overstates the features of the joint distribution of wages and unemployment durations observed in CPS data, the result seems to be driven for simplifying assumptions to make the model tractable. In particular, relaxing the assumption that firms can perfectly observe workers’ productivity and the restriction placed on the number of applications sent every period can improve the fit to the data. But economic theories cannot usually make strong points without a great deal of simplification.

The model also provides and explanation for the fact that workers with long unemployment spells have low permanent incomes. As shown, the recruiting selection labor market amplifies the ex ante productivity heterogeneity. The model also provides a novel viewpoint to address certain issues that concerns policy-makers. The probably standard view that the existence of long unemployment spells is indicative of labor market inefficiency is not supported in the model. A more efficient assignment of workers to jobs is related to longer unemployment spells on average and greater inequality in terms of welfare, permanent income and wages.

Some extensions of the benchmark case presented here can be considered. The model can readily be extended to allow for a more realistic wage setting mechanism in which other applicants productivities can be the employer outside option. Such a model would deliver a novel source of “frictional” wage dispersion, even in a scenario in which all productivities are fully observed. Other possible augmentation of the model is the introduction of aggregate uncertainty. Since the profitability of the job depends on the productivity of the top applicant -not on the average applicant as in the sequential environment-, the gains from hiring workers during a boom may be greatly amplified. This may contribute to explain why the cyclical volatility of vacancies is so pronounced.
Appendices

Appendix A: Proofs

Proof of Lemma 1.

Since the function is non-differentiable in pile size, consider the following difference
\[
\tilde{J}(k+1) - \tilde{J}(k) = \mathbb{E}[J(\theta^{k+1})|\delta > 0, a = 1, N(e^*) = k+1] - \mathbb{E}[J(\theta^k)|\delta > 0, a = 1, N(e^*) = k]
\]
where \(\theta^j\) denotes the \(j\)-th highest profitability in a group of \(k\) applicants. Writing the expectations in terms of integrals and using the notation \(F(\theta|\delta > 0, a = 1, X) = \tilde{F}(\theta)\) for the CDF and \(\tilde{f}(\theta)\) for the density, it is found that
\[
\tilde{J}(k+1) - \tilde{J}(k) = \frac{1}{k+1} \left( \int J(v)(k+1)\tilde{f}(v)\tilde{F}(v)^k dv - \int J(v)k(k+1)\tilde{f}(v)\tilde{F}(v)^{k-1}(1 - \tilde{F}(v)) dv \right)
\]
\[
= \frac{1}{k+1} \left( \mathbb{E}[J(\theta^{k+1})|\delta > 0, a = 1, N = k+1] - \mathbb{E}[J(\theta^k)|\delta > 0, a = 1, N = k+1] \right) > 0
\]

Since the first term is the expected value of the highest profitable worker among \(k+1\) applicants and the second term is the expected value of the second-highest among \(k+1\) applicants \(^{14}\), the difference is always strictly positive, which proves that \(\tilde{J}(k)\) is strictly increasing in \(k\).

To prove concavity, the function is differentiated twice
\[
\left( \tilde{J}(k+1) - \tilde{J}(k) \right) - \left( \tilde{J}(k) - \tilde{J}(k-1) \right)
\]
\[
= \int J(v)\tilde{f}(v) \left[ \tilde{F}(v)^{k-1}(-\tilde{F}(v) - k(1 - \tilde{F}(v))) - \tilde{F}(v)^{k-2}(\tilde{F}(v) - (k-1)(1 - \tilde{F}(v))) \right] dv
\]
\[
= \int J(v)\tilde{f}(v)\tilde{F}(v)^{k-1} \left[ (k+2)\tilde{F}(v)^2 - 2(k+1)\tilde{F}(v)) + k \right] dv
\]

Doing a bit of algebra, the last expression equals
\[
- \frac{1}{k+1} \left[ \int (k+2)(k+1)J(v)\tilde{F}(v)^{k-1}(1 - \tilde{F}(v))\tilde{f}(v) dv \right]
\]
\[
+ \int k(k+1)J(v)\tilde{f}(v)\tilde{F}(v)^{k-1}(1 - \tilde{F}(v)) dv \]
\[
= - \frac{1}{k+1} \left[ \mathbb{E}[J(\theta^k)|\delta > 0, a = 1, N = k+1] + \mathbb{E}[J(\theta^{k-1})|\delta > 0, a = 1, N = k] \right] < 0
\]

\(^{14}\)In general, the density function of the \(m\)-th highest value within \(k\) elements is
\[
f^m(\theta^m) = \frac{k!}{(m-1)!(k-m)!} F(\theta^m)^{m-1}(1 - F(\theta^m))^{k-m} f(\theta^m)
\]
with \(F(\cdot), f(\cdot)\) being the CDF and PDF of \(\theta\).
The last two terms are the expected value of the second-highest worker when \( k + 1 \) and \( k \) applicants arrive. Due to the negative sign at the beginning, the expression is always negative, which proves concavity. ■

**Proof of Lemma 2.**

The second assertion is almost trivial. Being the screening process free, all applicants are going to be interviewed, so \( i^* = +\infty \). For the first part, consider \( \bar{J}(1) > C(1) \). Due to the concavity of \( \bar{J}(k) \) proved in Lemma 1, it follows that \( \bar{J}(2) - \bar{J}(1) < \bar{J}(1) - \bar{J}(0) = \bar{J}(1) \). Using the convexity of the cost function \( C(\cdot) \) and denoting \( \Delta(k) = (\bar{J}(k) - C(k)) - (\bar{J}(k - 1) - C(k - 1)) \), it can be established that the marginal profit of interviewing an additional applicant is strictly decreasing as the number of applicants increases

\[
\Delta(1) > \Delta(2) - \Delta(1) > \Delta(3) - \Delta(2) > \ldots
\]

The maximum number of interviewed applicants \( i^* \) cannot be 0 because firms obtain positive profits by interviewing 1 applicant. The maximum number of applicants interviewed would be infinity \( (i^* = +\infty) \) only in case that to screen an additional applicant increases profits for any arbitrarily large \( k \). Formally,

\[
\Delta(k - 1) - \Delta(k) > \zeta > 0, \text{ for all } k
\]

In the following, to assume the latter inequality holds will lead to a contradiction.

Taking an arbitrarily large \( k \), it is true that

\[
\Delta(k - 1) - \Delta(k) > \zeta > 0 \Rightarrow \Delta(k - 1) > \zeta + \Delta(k) > \Delta(k)
\]

It also holds that

\[
\Delta(k - 2) - \Delta(k - 1) > \Delta(k - 1) - \Delta(k) > \zeta > 0 \Rightarrow \\
\Delta(k - 2) > 2\Delta(k - 1) - \Delta(k) > \zeta + \Delta(k - 1) > 2\zeta + \Delta(k)
\]

Using a similar reasoning,

\[
\Delta(k - 3) > \zeta + \Delta(k - 2) > 3\zeta + \Delta(k)
\]

Redoing the same argument \( k - 1 \) times, it is obtained

\[
\Delta(1) > (k - 1)\zeta + \Delta(k)
\]

By making \( k \to \infty \) in the inequality (37) with \( 0 < \bar{J}(1) - C(1) = \Delta(1) < \infty \), the term \( \Delta(k) \) must necessarily be negative. Therefore, the optimal maximum number of interviewed applicants must be lower than \( k \) so that the marginal screened worker generates a nonnegative change in profits. ■
Corollary 9 The value of a vacancy with $k$ applications, $M(k)$ is a nondecreasing function in $k$.

Proof.

Using the argument for the proof of Lemma 2, for any $k < i^*$ it must be true that $\Delta(k-1) - \Delta(k) > 0$, which implies that $M(k-1) - M(k-2) > M(k) - M(k-1)$. In the following, if $M(k)$ is not increasing for $k < i^*$, a contradiction arises.

Suppose $M(k-1) > M(k)$. By the previous condition, it must be true that $M(k-1) - M(k-2) > 0$ or $M(k-2) > M(k-1)$. Therefore, doing backward substitution the contradiction $M(0) = 0 > M(1)$ is obtained, which proves that $M(k)$ is increasing.

For any $k > i^*$, the employer randomly pick $i^*$ applicants to interview, so that $M(k) = \max\{\tilde{J}(k) - C(k), \tilde{J}(i^*) - C(i^*)\}$ is always nondecreasing.

Proof of Proposition 3.

Denoting $M(k) = \max\{\tilde{J}(k) - C(k), \tilde{J}(i^*) - C(i^*)\}$, the firms’ value $P$ is represented by the following equation as a function of $\lambda$

$$P(\lambda) = \beta \sum_{k=0}^{\infty} \frac{\exp(-\lambda)\lambda^k}{k!} M(k) - \kappa$$

To prove the proposition, the function $P(\lambda)$ must have a unique zero. To do this, first notice that $P(0) = -\kappa < 0$. Secondly, $P(\lambda)$ is continuous an increasing in $\lambda$ because $M(k)$ is increasing in $k$ as established in Corollary 9. To see this, simply take the derivative

$$P'(\lambda) = \beta \exp(-\lambda) \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} (M(k+1) - M(k)) \geq 0$$

Since $M_{i^*} > \kappa/\beta$ it is true that

$$\lim_{\lambda \to \infty} P(\lambda) = \beta M_{i^*} - \kappa > 0$$

This condition ensures that there is some $\lambda$ such that $P(\lambda) > 0$. Since $P(\lambda)$ is strictly increasing and continuous in $\lambda$, there exists some $\lambda^*$ such that $P(\lambda^*) = 0$ by the Intermediate Value Theorem.

Proof of Lemma 4.

Taking expectation of equation (27) yields

$$E[\pi(\theta)] = \sum_{k=1}^{\tilde{i}} \frac{\lambda^{k-1} \exp(-\lambda)}{(k)!} \int k \tilde{F}(v) \tilde{f}(v) dv + \sum_{k=\tilde{i}+1}^{\infty} \frac{\lambda^{k-1} \exp(-\lambda)}{k!} \int \tilde{i} \tilde{F}(v) \tilde{f}(v) dv$$
Since $k\tilde{F}(v)^{k-1}\tilde{f}(v)$ and $i^*\tilde{F}(v)^{i*-1}\tilde{f}(v)$ are the densities of the maximum element in a sample of size $k$ and in a sample of size $i^*$ respectively, the previous expression collapses to

$$E[\pi(\theta)] = \exp(-\lambda) \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} = \frac{1 - \exp(-\lambda)}{\lambda}$$

Proof of Proposition 5.

The Volterra nonlinear integral equation shown in the main text is restated here. The goal is to show existence, uniqueness and differentiability\textsuperscript{15}.

$$\tilde{F}(\theta) = \int_{-\infty}^{\theta} f(v|\delta > 0, a(\theta) = 1)dv = \int_{-\infty}^{\theta} \frac{\eta f_\theta(v)}{U^*(\eta + \tilde{\pi}(\tilde{F}(v)))}dv$$

Equation (??) indicates that the hazard rate $\pi(\theta)$ depends on $\tilde{F}(\theta)$. By defining $\tilde{\pi}(\tilde{F}(\theta)) \equiv \pi(\theta)$, the previous equation is rewritten in terms of an operator $O(\cdot)$.

$$O(\tilde{F})(\theta) = \int_{-\infty}^{\theta} \frac{\eta f_\theta(v)}{U^*(\eta + \tilde{\pi}(\tilde{F}(v)))}dv$$

(38)

It is necessary to establish that (38) is a contraction mapping so that there exists a unique function $\tilde{F}(\theta)$ that is a fixed point of $O(\cdot)$. A sufficient condition to show this is that the kernel function

$$K(v, \tilde{F}(v)) = \frac{\eta f_\theta(v)}{U^*(\eta + \tilde{\pi}(\tilde{F}(v)))}$$

integrated in the right-hand side of (38) satisfies a global Lipschitz condition so that there exists a finite constant $\Lambda$ for which holds

$$|K(v, z_0) - K(v, z_1)| \leq \Lambda |z_0 - z_1| \forall v \in \mathbb{R} \text{ and } z_0, z_1 \in [0, 1]$$

In order to show this, it is invoked the Mean Value Theorem

$$|K(v, z_0) - K(v, z_1)| = \frac{\eta f_\theta(v)}{U^*} \left| \frac{\tilde{\pi}'(z)}{(\eta + \tilde{\pi}(\tilde{F}(v)))^2} \right| |z_0 - z_1| \text{ for some } z \in (z_0, z_1)$$

Using that $z_0, z_1 \in [0, 1]$, an upper bound for $\tilde{\pi}'(z)$ is found as follows

$$\tilde{\pi}'(z) = \sum_{k=2}^{i^*} \frac{\exp(-\lambda)\lambda^{k-1}z^{k-1}}{(k-2)!} + i^*(i^* - 1) \sum_{k=i^*+1}^{\infty} \frac{\exp(-\lambda)\lambda^{k-1}z^{i^*-1}}{k!}$$

$$\leq \lambda \sum_{k=0}^{i^*-2} \frac{\exp(-\lambda)\lambda^{k}}{k!} + \frac{i^*(i^* - 1)}{\lambda} \sum_{k=i^*+1}^{\infty} \frac{\exp(-\lambda)\lambda^{k}}{k!}$$

$$\leq \lambda + \frac{i^*(i^* - 1)}{\lambda}$$

\textsuperscript{15}The proof adapts a standard solution in Hackbusch (1995), chapter 2.
If $i^* = +\infty$, $\tilde{\pi}(z) = \exp(-\lambda(1-z))$ so that $\tilde{\pi}'(z) = \lambda \exp(-\lambda(1-z)) \leq \lambda$. Hence, using the assumption that $\sup_{\theta} f_{\theta}(\theta) < \infty$, the Lipschitz constant is

$$\Lambda = \sup_{\theta} f_{\theta}(\theta) \frac{\lambda + i^*(i^* - 1)/\lambda}{\eta U^*}$$

Let $\mathcal{C}$ be the space of continuous functions with the special norm $\|g\| = \max_v |\exp(-\phi \Lambda v)g(v)|$ and $\phi > 1$, which is equivalent to the sup-norm $\|\cdot\|_\infty$. In that integration preserves continuity, the operator $O(\tilde{F})$ maps from the space $\mathcal{C}$ into the same space because $f_{\theta}(\theta)$ and the kernel function $K(v, z)$ are continuous.

Moreover, it follows that for all $\theta$

$$\|(O(\tilde{F}_0) - O(\tilde{F}_1))(\theta)\| = \left|\int_{-\infty}^{\theta} (K(v, \tilde{F}_0(v)) - K(v, \tilde{F}_1(v)))dv\right|$$

$$\leq \int_{-\infty}^{\theta} \Lambda|\tilde{F}_0(v) - \tilde{F}_1(v)|dv$$

$$= \int_{-\infty}^{\theta} \Lambda \exp(\phi \Lambda v) \exp(-\phi \Lambda v) \left(\tilde{F}_0(v) - \tilde{F}_1(v)\right)dv$$

$$\leq \max_{\tilde{v}} \left\{\exp(-\phi \Lambda \tilde{v})(\tilde{F}_0(\tilde{v}) - \tilde{F}_1(\tilde{v}))\right\} \left|\int_{-\infty}^{\theta} \Lambda \exp(\phi \Lambda v)dv\right|$$

$$= \|\tilde{F}_0 - \tilde{F}_1\| \|\phi^{-1} \exp(\phi \Lambda \theta)\|$$

Since the former inequality holds for all possible $\theta$, it also holds for the productivity that maximizes the value of the operator in the left-hand side. Therefore, the conclusion is that

$$\|(O(\tilde{F}_0) - O(\tilde{F}_1))(\theta)\| \leq \phi^{-1} \|\tilde{F}_0 - \tilde{F}_1\|$$

Since $\phi > 1$, the integral equation is a Contraction Mapping. Due to the Banach Fixed Point Theorem, existence, uniqueness and continuity of $\tilde{F}(\theta)$ are proven. Moreover, thanks to the Fundamental Theorem of Calculus, the density $\tilde{f}(\theta)$ also exists and is unique. It is also proven the existence and uniqueness of the hazard rate function $\pi(\theta)$. Then, using the fact that a positive $\lambda$ imply an unemployment rate bounded away from 0 and 1, we have that $f(\theta, 0) = f(\theta|\delta > 0, a(\theta) = 1) a(\theta) A^*$. Applying equations (19), (18) and (16), it is apparent that all the densities are uniquely determined. By the definition of the operator $T(\cdot)$ in equations (20), (21) and (22), there also exists a unique distribution in the economy which is a fixed point of the operator $T(F^*(\theta, \delta)) = F^*(\theta, \delta)$.

**Proof of Proposition 6.** The second claim is obvious since all separations are
exogenous. For the case $\delta > 1$, we have that
\[
\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta + 1] = \int \theta \left( \frac{f(\theta, \delta)}{f_\delta(\delta)} - \frac{f(\theta, \delta + 1)}{f_\delta(\delta + 1)} \right) d\theta
\]
Substituting (15) into the latter equation, the right-hand side becomes
\[
\int \theta \frac{f(\theta, \delta)}{f_\delta(\delta)} \left( 1 - (1 - \pi(\theta)a(\theta)) \frac{f_\delta(\delta)}{f_\delta(\delta + 1)} \right) d\theta
\]
Integrating both sides of equation (15) and considering the case when $a(\theta) = 1$ yields
\[
\int f(\theta, \delta + 1)d\theta = \int (1 - \pi(\theta)) f(\theta, \delta)d\theta
\]
\[
f_\delta(\delta + 1) = f_\delta(\delta) - f_\delta(\delta) \int \pi(\theta) \frac{f(\theta, \delta)}{f_\delta(\delta)} d\theta
\]
Hence,
\[
\frac{f_\delta(\delta)}{f_\delta(\delta + 1)} = \frac{1}{1 - \mathbb{E}[\pi(\theta)|\delta]}
\]
Substituting this expression in equation (39) yields
\[
\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta + 1] = \int \theta \frac{f(\theta, \delta)}{f_\delta(\delta)} \left( 1 - \frac{1 - \pi(\theta)}{1 - \mathbb{E}[\pi(\theta)|\delta]} \right) d\theta
\]
\[
= \mathbb{E}[\theta|\delta] - \frac{\mathbb{E}[\theta(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]}
\]
\[
= \mathbb{E}[\theta|\delta] - \frac{\text{Cov}[	heta, (1 - \pi(\theta))|\delta] - \mathbb{E}[\theta|\delta] \mathbb{E}[1 - \pi(\theta)|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]}
\]
\[
= \frac{\text{Cov}[	heta, \pi(\theta)|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} > 0
\]
Since $\pi(\theta)$ is strictly increasing in $\theta$, the covariance is positive, which proves the proposition.

**Proof of Proposition 7.** Applying the definition of conditional expectation and equation (19)
\[
\mathbb{E}[\delta|\theta] = \sum_{k=0}^{\infty} k \frac{f(\theta, \delta = k)}{f_\delta(\theta)}
\]
\[
= f(\theta|\delta = 1) \sum_{k=0}^{\infty} k(1 - \pi(\theta)a(\theta))^{k - 1}
\]
Considering the case with $a(\theta) = 1$ and using the differentiating the absolutely summable series $\pi(\theta)^{-1} = \sum_{k=0}^{\infty} (1 - \pi(\theta))^k$ gives that
\[
\sum_{k=0}^{\infty} k(1 - \pi(\theta))^{k-1} = \frac{1}{\pi(\theta)^2}
\]
Therefore, using equations (18) and (16) yields
\[
\mathbb{E}[\delta|\theta] = \frac{f(\theta|\delta = 1)}{\pi(\theta)^2} = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)}
\]
Deriving the former expression, we obtain
\[
\frac{d\mathbb{E}[\delta|\theta]}{d\theta} = -\frac{\eta\pi'(\theta)(1 + 2\pi(\theta))}{(\eta + \pi(\theta))^2\pi(\theta)^2}
\]
which proves the claim. ■

**Proof of Proposition 8.** First consider that $\mathbb{V}[\delta|\theta] = \mathbb{E}[\delta^2|\theta] - \mathbb{E}[\delta|\theta]^2$. Applying the definition of conditional expectation and equation (19)
\[
\mathbb{E}[\delta^2|\theta] = \sum_{k=0}^{\infty} k^2 f(\theta, \delta = k) = f(\theta|\delta = 1) \sum_{k=0}^{\infty} k^2 (1 - \pi(\theta)a(\theta))^{k-1}
\]
Assuming that $a(\theta) = 1$ and differentiating twice the absolutely summable series $\pi(\theta)^{-1} = \sum_{k=0}^{\infty} (1 - \pi(\theta))^k$ gives that
\[
\sum_{k=0}^{\infty} k^2 (1 - \pi(\theta))^{k-1} = \frac{2 - \pi(\theta)}{\pi(\theta)^3}
\]
Replacing this expression into (42) yields
\[
\mathbb{E}[\delta^2|\theta] = \frac{\eta(2 - \pi(\theta))}{(\eta + \pi(\theta))\pi(\theta)^2}
\]
By using equation (24) into the variance identity stated at the beginning of the proof, the expression for (25) is obtained
\[
\mathbb{V}[\delta|\theta] = \frac{\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta + \pi(\theta))^2\pi(\theta)^2}
\]
To prove that $\partial \mathcal{V}[\delta|\theta]/\partial \theta < 0$ involves tedious algebra. For this statement to be true it suffices to show that $\partial \mathcal{V}[\delta|\theta]/\partial \pi(\theta) < 0$ since $\pi'(\theta) > 0$. Hence,

$$
\frac{\partial \mathcal{V}[\delta|\theta]}{\partial \pi(\theta)} = \frac{2\eta - \eta^2 - 2\eta \pi(\theta)}{(\eta + \pi(\theta))^2 \pi(\theta)^2} - \frac{(2(\eta + \pi(\theta)) \pi(\theta)^2 + 2\pi(\theta)(\eta + \pi(\theta))^2(\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta \pi(\theta)^2)}{(\eta + \pi(\theta))^4 \pi(\theta)^4}
$$

Since the denominator of the former expression is always going to be negative, in the following the focus is only on the numerator, which after some simplifications becomes

$$
numerator = -(2\eta - \eta^2)(\eta + \pi(\theta))^2 \pi(\theta)^2 - 2\eta^2(\eta + \pi(\theta)) \pi(\theta)^2 - 2(2\eta - \eta^2)(\eta + \pi(\theta)) \pi(\theta)^3 + 2\eta(\eta + \pi(\theta)) \pi(\theta)^4 - 2\eta^2(\eta + \pi(\theta))^2 \pi(\theta)
$$

$$
= -\eta(2 - \eta)(\eta + \pi(\theta)) \pi(\theta)^3 - 2\eta(\eta + \pi(\theta)) \pi(\theta)^3 - 2\eta^2(\eta + \pi(\theta)) \pi(\theta)^2(1 - \pi(\theta)) - 2\eta(\eta + \pi(\theta)) \pi(\theta)^3(1 - \pi(\theta)) - 2\eta^2(\eta + \pi(\theta)) \pi(\theta) < 0
$$

Therefore, the numerator is always negative for any $\pi(\theta)$ between 0 and 1, which proves the proposition. ■

**Appendix B: Deriving conditions for a Symmetric Coincidence Ranking Equilibrium**

In this section I derive the condition (26). Deriving expression (9) yields

$$
(1 - w'(\theta))u'(w(\theta)) + (\theta - w(\theta))u''(w(\theta))w'(\theta) = \frac{1 - \alpha}{\alpha} [S'(\theta)(u(w(\theta)) - u(b(1 - z))) + S(\theta)u(w(\theta))w'(\theta)]
$$

Rearranging the following expression yields

$$
w'(\theta) = \frac{u'(w(\theta)) - \frac{1 - \alpha}{\alpha} S'(\theta) [u(w(\theta)) - u(b(1 - z))]}{u'(w(\theta)) + \frac{1 - \alpha}{\alpha} S(\theta)u'(w(\theta)) - u''(w(\theta))(\theta - w(\theta))}
$$

(43)

Since $u''(w(\theta)) < 0$ and

$$
S'(\theta) = -\frac{(\beta^{-1} - 1 + \eta)\pi'(\theta)}{((\beta^{-1} - 1 + \eta + \pi(\theta)))^2} < 0
$$

the conclusion is that $w'(\theta)$ is always positive. For $J'(\theta)$ to be positive, it suffices that $w'(\theta) < 1$. By substituting (9) into equation (43) and rearranging, the existence of a symmetric coincidence ranking equilibrium needs that

$$
w'(\theta) = \frac{1 + \frac{\pi'(\theta)}{\beta^{-1} - 1 + \eta + \pi(\theta)} (\theta - w(\theta))}{1 + \frac{\pi(\theta)}{\beta^{-1} - 1 + \eta + \pi(\theta)} + \gamma(w(\theta)) \frac{w'(\theta)}{w(\theta)}} < 1
$$
with $\gamma$ as CRRA parameter. The previous condition becomes
\[
\frac{\pi'(\theta)}{\pi(\theta)} < \frac{1 - \alpha}{\alpha(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right)
\]
Focusing on the special case when $i^* = \infty$ provides some intuition to analyze the result, because $\pi'(\theta)/\pi(\theta) = \lambda \tilde{f}(\theta)$. For $i^*$ finite, additional terms show up. Thus,
\[
\lambda \tilde{f}(\theta) < \frac{1 - \alpha}{\alpha(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right)
\]
which is the condition discussed in Section 3.7.

Appendix C: Computational Algorithm

The key to solve the model is to solve the Volterra nonlinear integral equation (23) whose fixed point is the cumulative distribution function of the unemployed workers. The computation is done by steps

- Generate a grid of log-productivities of $N$ points.
- Take the two largest element of the grid $\hat{\theta}_N$ and set $\tilde{F}(\hat{\theta}_N) = \tilde{F}(\hat{\theta}_{N-1}) = 1$. Obtain a guess for $\tilde{F}(\hat{\theta}_{N-2})$ using Simpson quadrature rule. Use the obtained values to solve for $\tilde{F}(\hat{\theta}_{N-3})$ and continue doing so until $\tilde{F}(\hat{\theta}_1)$.
- Solve the right-hand side of the Volterra equation using a quadrature rule and linear interpolation of $\tilde{F}$.
- Check if the result of the right-hand side of (23) equals the previous guess for $\tilde{F}$. Iterate until convergence is achieved.
- Using $\tilde{F}(\theta)$ compute the hazard rate $\pi(\theta)$ and solve Nash bargaining problem.
- Compute Value Functions $Q(\theta)$, $W(\theta)$ and $J(\theta)$. Check if the value of the match $J(\theta)$ is increasing in $\theta$ to see if the coincidence ranking equilibrium equilibrium exists.
- Using a quadrature rule compute the expected value of profits conditional in the number of applicants interviewed $\tilde{J}(k) = \int J(\theta)k\tilde{F}(\theta)^k\tilde{f}(\theta)d\theta$.
- Infer Cost $C(\cdot)$ function and $\kappa$ that generates choices of $i^*$ and $\lambda$ using FE conditions of the firms’ problem. Specifically, assuming a constant marginal cost of recruiting $\xi$, set it to the value $\tilde{J}(i^* + 1) - \tilde{J}(i^*) + \text{small constant}$. Then, set the vacancy-posting cost $\kappa$ so that the Free-Entry condition in (7) holds.
The solutions provided are computed using \( N = 2501 \) evenly spaced points of log-productivity in the range \([\theta, \overline{\theta}]\), with \( \theta \) satisfying equation (36) and \( \overline{\theta} = \mu_\theta + 6\sigma_\theta \) with \( \mu_\theta \) and \( \sigma_\theta \) as mean and standard deviation of the log-productivity distribution of the labor force. All the integration is computed numerically using Gauss-Chebyshev quadrature with 80 nodes due to its simplicity. The algorithm converges very fast and the results do not perceptively change using a larger grid.

**Appendix D: Construction of the database**

**Constructing nonemployment durations for marginally attached workers**

The goal is to recover as much as possible of the nonemployment spell for marginally attached people in the data. There is a CPS variable indicating how long ago the out-of-the-labor-force respondent worked last time, whose possible interesting values are more and less than a year ago. Individuals answering more than a year ago are included as top coded. For individuals that are out of the labor force, there is a question about how many weeks was the respondent looking for a job last year. A rough estimate for the nonemployment spell is the value of this variable plus ten weeks since the individual is interviewed in March. This variable is called duration A.

**Weights**

The CPS sampling weights were corrected so as each year of the sample would weight the same as any other, regardless the number of observations per year. For statistics involving only matched individuals an analogous correction was made for the same purpose. In any case, unweighted statistics do not differ a great deal to the weighted ones.
Figures

Figure 1: Marginal density of ongoing unemployment duration
Figure 2: Marginal density of completed unemployment duration

Figure 3: Marginal density of log wages, Employed
Figure 4: Marginal density of log wages, Unemployed $\delta \leq 12$

![Image](image1.png)

Figure 5: Expected wage conditional on duration $\mathbb{E}[w(\theta)|\delta]$

![Image](image2.png)
Figure 6: Expected duration conditional on wages $E[\delta | \theta]$

Figure 7: Expected duration conditional on wages and unemployment $E[\delta | \theta, \delta > 0]$
Figure 8: Variance duration conditional on wages $\mathbb{V}[\delta|\theta]$

Figure 9: Variance duration conditional on wages and unemployment $\mathbb{V}[\delta|\theta, \delta > 0]$
Figure 10: Densities conditional on duration, model 1

Figure 11: Densities conditional on duration, model 2
Figure 12: Welfare and Permanent Income comparison, model 1

Figure 13: Welfare and Permanent Income comparison, model 2
References


