Ideology as Opinion: A Spatial Model of Common-value Elections

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August 2013

Abstract

This paper analyzes a spatial model of elections, in which ideological differences stem not from competing voter incentives, but from a spectrum of opinions regarding which policy will best achieve voters’ common objective, in essence synthesizing the canonical approaches of Condorcet and Downs. The model provides novel explanations for key empirical features of elections, such as candidate polarization, minor parties, electoral mandates, and the correlation between voter information, ideology, and participation. In some cases, familiar behavior has dramatically different welfare implications. For example, office-motivated candidates converge to the political center, but this harms voters if the optimal policy lies elsewhere.

JEL Classification Number D72, D82

Keywords: Voting, Elections, Ideology, Median Voter, Information Aggregation, Polarization, Platform Convergence, Jury Theorem, Public Opinion, Swing Voter’s Curse, Turnout, Abstention, Roll-off

1 Introduction

Empirical studies make abundantly clear that the central determinant of political behavior is ideology. In the United States, liberals vote Democrat, conservatives vote Republican,

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and moderates might vote either way. Theoretical models have fully embraced this fact, and the median voter model of Hotelling (1929) and Downs (1957) remains dominant in political economic theory. But what determines whether an individual is liberal or conservative? Typically, ideology is modeled without explanation, as an exogenous taste parameter. Exceptions to this derive ideology from wealth or income: in Bergstrom and Goodman (1973), this determines the demand for a public good; in Romer (1975) and Meltzer and Richard (1981), it determines the demand for redistribution from the rich to the poor. Specifically, liberal policies favor the poor, and conservative policies favor the rich.

Empirically, however, attributing ideological differences to income is somewhat problematic. Environmental preservation and defense spending are both textbook examples of public goods, for example, but the former is traditionally favored by liberals, and the latter by conservatives. Similarly, at any given time only a minority of the electorate benefits directly from redistributive programs such as minimum wages, food and health care assistance, unemployment insurance, or even social security and public education, but these programs nevertheless remain quite popular. The correlation between income and partisanship is positive, but much weaker than simple spatial models assume. For example, Figure 1 shows the Republican vote share (excluding minor parties) in the 2008 U.S. presidential election, broken down by income, according to American National Election Studies (ANES) survey data. Contrary to the predictions of a simple median voter model, 35% of voters with below-median income voted Democrat, and 44% of wealthier citizens voted Republican. Regressing vote choice on all 25 income categories only produces an $R^2$ of 0.07.\footnote{Piketty (1995) and Gelman et al. (2007) find similar trends in previous election years and internationally, and Mueller (2003, ch. 14.4) cites related evidence from various ballot initiatives.}
The basic premise of this paper is that voters actually hold common objectives, and that ideology merely reflects differences of opinion regarding how these can be achieved. Indeed, the broad goals of most policies—such as world peace, economic stability and prosperity, and reducing crime, poverty, pollution, and corruption—hold unanimous appeal, but at the same time are highly complex, making honest disagreements inevitable. Voters all value national security, for example, but may disagree whether military strength will foster peace, by intimidating opposition, or hinder peace, by provoking enemies. Similarly, citizens may share a concern for the poor, but disagree whether these are helped or harmed by, say, minimum wage laws, which provide resources but may also reduce employment. More broadly, voters unanimously desire to improve social welfare, but may disagree whether this is best accomplished by improving national security or by helping the poor. In essence, the prioritization of individual public goods can itself be viewed as a public good.

An immediate objection to this paradigm is that most policies have redistributive consequences, which are inherently zero-sum. Empirically, however, voters seem to support policies that they believe are best for society, even when they do not benefit personally. As Section 6.1 discusses, this could reflect altruistic or ethical considerations. Whatever the reason, the language of public goods is pervasive, except that what one side views as a public good, the other views as a public bad. To liberals, redistribution from rich to poor creates a better society for all, by satisfying an intrinsic human desire for fairness, providing a social “safety net” against uninsurable risks, and preserving democracy from the undue influence of wealthy elites. To conservatives, redistribution takes unfair advantage of a wealthy minority, squelches incentives for effort, investment, caution, and innovation, and makes democracy vulnerable to abuses by political elites. Fong (2001) finds that preferences for redistribution correlate with voters’ beliefs regarding the prevalence of poverty, and about the relative importance of luck and effort for economic success, more strongly than with demographic variables.

Many citizens relate to both sides of an issue, at least to some extent. Over time, voters learn more about policy effects and institutional details, and ideologies drift, especially for young voters (Jennings and Markus, 1984). Change can also occur quite suddenly, such as after market crashes, business or government scandals, or wars or acts of terrorism.

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2 This is consistent with survey evidence from Berelson, Lazarsfeld, and McPhee (1954, ch. 7).
3 Many individuals continue voting in a state by absentee ballot, for example, even after moving away, suggesting that they care about policies that do not directly affect themselves.
4 See also Alesina and Angeletos (2005).
5 In the wake of the 2008 financial crisis, for example, former U.S. Federal Reserve chairman Alan Greenspan famously testified before Congress of being suddenly convinced that the ideology motivating his earlier efforts at financial deregulation had been “wrong” (Andrews, Edmund L. 10/23/2008. “Greenspan Concedes Error on Regulation”, New York Times.)
On specific policy questions, voter opinions are highly volatile (Magleby 1984, ch. 9), and information provided during field experiments appreciably alters voter allegiances. In fact, many voters make deliberate efforts to change one another’s opinions, via debate, endorsements, and policy research: in 2008, 43% of ANES respondents reported trying to convince others “why they should vote for or against one of the parties or candidates”. Such efforts appear fruitful, as political conversions are most frequent among voters who interact and discuss politics with those of opposing views (Berelson, Lazarsfeld, and McPhee 1954, ch. 7). If political positions derived from immutable preferences, however, persuasive efforts would be futile.

In conceptualizing elections as mechanisms for information aggregation, rather than preference aggregation, this paper follows the classic tradition of Condorcet (1785). Rather than a binary policy decision, however, the model below includes a continuum of policy possibilities, as in Hotelling (1929) and Downs (1957). One of these policies is ultimately optimal, and voters would unanimously prefer this policy to any other, except that they disagree regarding its location. A voter’s ideology specifies the policy that he personally believes to be optimal, modeled as a private signal that is correlated with the truth.

In some cases, uncertainty is quantitative. The optimal level of military spending, for example, depends on the unknown strength of an enemy. In other cases, uncertainty is binary, hinging on the truth or error of a particular fact or theory. During a recession, for example, Keynesian macroeconomic theory recommends a large policy of economic “stimulus” spending, which classical economics views as ineffective and wasteful. Just as macroeconomists sort into philosophical camps (e.g. Keynesian and classical), voters self-identify as liberals or conservatives, according to their philosophies on the proper role of government. Consistent with empirical evidence, uncertain voters tend to be ideologically moderate, reluctant to commit to either extreme.

With a continuum of policy alternatives, it is infeasible for voters to choose policies directly. Instead, they must elect representatives to choose policies on their behalf. If candidates are committed to policy platforms, however, then voters’ choice is again binary.


6 For example, see Gilens (2001), Luskin, Fishkin, and Jowell (2002), Luskin and Fishkin (2005), and Banerjee et al. (2010).

7 Common values might also explain the empirical rarity of voting cycles (Feld and Grofman, 1992), which Arrow’s (1950) impossibility theorem suggests should be ubiquitous, as well as evidence discussed in McMurray (2013a), that voters on both sides of an issue tend to believe that they hold the majority view. An informational model also mirrors the obvious etymological roots of ideology in ideas and logic, as opposed to class conflict, and might explain why politics is so frustrating: a voter might acquiesce to a policy that benefits his peers at his own expense, but finds little solace when policies simply seem misguided.

8 Throughout this paper, masculine pronouns refer to voters and feminine pronouns refer to candidates.
As in Condorcet’s (1785) classic jury theorem, then, voters tend to elect the candidate whose platform is truly superior. This generates a swing voter’s curse, as in Feddersen and Pesendorfer (1996), so that citizens with limited information abstain from voting, which provides a plausible rationale for casting incomplete ballots and can explain the empirical correlation between information, ideology, and participation.

Since each citizen votes for the candidate whose platform is closer to the policy that he privately believes to be optimal, candidates who are office motivated attract votes by moving to the political center, just as in Hotelling (1929) and Downs (1957). Anticipating that they will be elected in opposite states of the world, however, truth motivated candidates adopt polarized positions, a result that differs from existing literature but is consistent with empirical evidence. The winning candidate’s margin of victory conveys further information about the state of the world, and so can be interpreted as a mandate from voters. An uncommitted candidate adjusts her policy position accordingly, in which case voting takes on a signaling role, as in Razin (2003). The pivotal calculus underlying the swing voter’s curse is then irrelevant, but citizens who lack information still abstain, to avoid strengthening a mandate in the wrong direction, which can explain why participation patterns are similar for voting and for other forms of political activism. Votes for losing candidates also influence the election outcome, providing a possible rationale for multiple parties.

In addition to making novel behavioral predictions, an information model highlights new normative implications of familiar behavior, such as polarization and flexibility, minor parties, and voter abstention. In standard models, for example, competition between candidates benefits voters, by fostering compromise between the competing interests on the left and the right. In an information setting, however, compromise can produce unattractive policies, such as a moderate-sized economic stimulus, which no one believes to be optimal. The informational analysis of mandates also substantially strengthens Condorcet’s (1785) original jury theorem, because large elections not only identify the better of two policies, but identify the optimal policy from within an entire continuum of possibilities.

Various specifications of the model below tie into a number of existing strands of literature, which are therefore discussed within the context of each version of the model, rather than collected into a single section. The basic model is introduced in Section 2, and Sections 3 and 4 analyze equilibrium behavior under the assumptions that candidates are committed and uncommitted, respectively. Section 5 then revisits each specification, in turn, to analyze voter welfare. Section 6 discusses how the model can be extended to incorporate heterogeneous preferences and beliefs, and Section 7 concludes. Regression tables and technical proofs are presented in the appendix.
2 The Model

An electorate consists of $N$ citizens where, following Myerson (1998), $N$ is drawn from a Poisson distribution with mean $n$. Together, these citizens must choose and implement a policy from the interval $\mathcal{X} = [-1, 1]$ of alternatives, which will provide a common benefit to every citizen. Let $z \in \mathcal{Z}$ denote an unknown state of the world, which designates the policy that in truth is best for society. The domain $\mathcal{Z} \in \{\mathcal{Z}_Q, \mathcal{Z}_B\}$ of the state variable may be the entire policy interval $\mathcal{Z}_Q = \mathcal{X}$, so that any policy might be optimal. In that case, uncertainty is said to be quantitative, as in the unknown strength of an enemy or the socially optimal level of redistribution. In the example of Keynesian versus classical macroeconomic theory, which determine the optimal level of economic stimulus, uncertainty is binary. In that case, other policies are still feasible but the optimal policy lies at one of the two extremes, $\mathcal{Z}_B = \{-1, 1\}$. All of the results below apply to either specification of the model. Ex ante, assume that every policy in $\mathcal{Z}$ is equally likely to be optimal. The prior density (or mass) function of $z$ is therefore

$$f(z) = \begin{cases} \frac{1}{2} & \text{if } z \in \mathcal{Z} \\ 0 & \text{otherwise} \end{cases}.$$  \hspace{1cm} (1)

If policy $x \in \mathcal{X}$ is implemented in state $z$ then each citizen receives utility $u(x|z)$, which declines quadratically with the distance between $x$ and $z$.

$$u(x|z) = -(x - z)^2.$$  \hspace{1cm} (2)

This specification is standard both in political economics and in statistical theory, and is convenient because, conditional on information $\Omega$, the expectation of (2) is maximized at the conditional expectation $E(z|\Omega)$ of the state variable, and declines quadratically with the distance from this expectation. The concavity of (2) implies that voters are risk averse, and thus favor moderate policies, which are less risky than policies at either extreme, even in the binary model, where interior policies are never actually optimal. Ex ante, the optimal policy in either version of the model lies exactly at the center, $E(z) = 0$.

A citizen’s private opinion regarding the location of the optimal policy is represented by a private signal $s_i \in \mathcal{S}$ which is positively correlated with $z$, and has the same domain

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9 Alternatively, $z$ may be interpreted as the policy that is best, conditional on all available (but still incomplete) information. In that case, even if truth is ultimately binary, the quantitative uncertainty model applies.

10 If the utility loss function were linear or convex then, in the binary uncertainty model, no voter would favor policies other than the two extremes. The single-peakedness of (2) still makes it essentially concave in that case, however, so in the quantitative uncertainty model, the results below would still hold.
\( \mathcal{S} = \mathcal{Z} \). If \( z \) is binary, for example, then so is \( s_i \). Conditional on \( z \), private signals are independent. Because citizens differ in expertise, their signals vary in quality. Specifically, a citizen’s expertise \( q_i \in \mathcal{Q} = [0,1] \) is first drawn independently (from other citizens, and from \( z \)) from a common distribution \( G \) which, for technical convenience, is differentiable and has a strictly positive density \( g \). Conditional on \( q_i \) and \( z \), the density (or mass) function of \( s_i \) is as follows.\(^{11}\)

\[
h(s_i | q_i, z) = \frac{1}{2} (1 + q_i s_i z).
\]

(3)

With this specification, every signal in \( \mathcal{S} \) is equally likely and the correlation coefficient between \( s_i \) and \( z \) is simply proportional to a citizen’s expertise \( q_i \).\(^{12}\) In other words, the greater a citizen’s expertise, the more informative his signal. The lowest quality signal reveals nothing: if \( q_i = 0 \) then \( s_i \) and \( z \) are independent. A citizen’s ideology \( \mathcal{I}_i = \mathcal{I}(q_i, s_i) = \mathcal{E}(z | q_i, s_i) \) is defined as the policy in \( \mathcal{X} \) that maximizes his private expectation of (2). This is the policy a citizen would implement, if left to himself. The distributions \( f, g, \) and \( h \) are common knowledge, but \( q_i, s_i, \) and therefore \( \mathcal{I}_i \) are observed only privately.

With a continuum of policy possibilities, it is infeasible for citizens to vote for policies directly. Instead, they vote (at no cost) for candidates from the set \( \mathcal{C} = \{A, B\} \). A voting strategy \( v : \mathcal{Q} \times \mathcal{S} \rightarrow \Delta(\mathcal{C}) \) specifies a probability \( v_j(q, s) \) of voting for each candidate \( j \in \mathcal{C} \), for every realization \( (q, s) \in \mathcal{Q} \times \mathcal{S} \) of private information. Votes are cast simultaneously, and a winner \( w \in \mathcal{C} \) is determined by majority rule, breaking a tie if necessary by a coin toss. The election winner takes office and implements policy. An uncommitted candidate can choose any policy, and can condition her choice on the precise electoral outcome. A committed candidate must commit to a particular policy platform before the election outcome is realized. Informally, candidates can be viewed as citizens themselves, in the spirit of Osborne and Slivinski (1996) and Besley and Coate (1997). Formally, however, candidates do not vote, and do not receive private signals of their own.\(^{13}\) Instead, they must take cues from voters. A truth motivated candidate seeks policies as close as possible to the social optimum, to maximize the expectation of (2), while an office motivated candidate seeks to maximize her probability \( \Pr(w = j) \) of being elected.

The ultimate policy outcome \( y \in \mathcal{X} \) depends on voter and candidate strategies, together

\(^{11}\)This linear specification is convenient, but I conjecture that results similar to those below would hold for general signal distributions, as long as \( s_i \) and \( z \) are affiliated in the sense of Milgrom and Weber (1982), with affiliation somehow increasing in \( q_i \).

\(^{12}\)If \( \mathcal{Z} = \mathcal{Z}_B \) then the correlation is exactly \( q_i \). If \( \mathcal{Z} = \mathcal{Z}_Q \) then \( \text{corr}(s_i, z | q_i) = \frac{1}{2} q_i \).

\(^{13}\)This is largely for simplicity. A more natural assumption is that candidates have more information than voters, not less, but if voters infer candidates’ private information from their platforms then the analysis hardly changes: when \( N \) is large, \( N \) signals and \( N + 2 \) signals convey similar information. For a spatial model with informed candidates and uninformed citizens, see Kartik, Squintani, and Tinn (2012).
with the realizations of $N$ and $z$, as well as the private information $(q_i, s_i)$ of each citizen. The equilibrium concepts used below are Bayesian Nash equilibrium and perfect Bayesian equilibrium. With the assumption of Poisson population uncertainty, such equilibria are necessarily symmetric, in that citizens each play the same voting strategy.\footnote{In games of Poisson population uncertainty, the finite set of citizens who actually play the game is a random draw from an infinite set of potential citizens, for whom strategies are defined (see Myerson 1998). The distribution of opponent behavior is therefore the same for any two individuals within the game (unlike a game between a finite set of players), implying that a best response for one citizen is a best response for all.}

## 2.1 Information and Ideology

By Bayes’ rule, the posterior density (or mass) function of $z$, given private information, is the same expression as (3):

$$f(z|q_i, s_i) = \frac{1}{2} \left( 1 + q_i s_i z \right). \tag{4}$$

Therefore, the conditional expectation of $z$ is $\mathcal{I}_i = q_i s_i$ in the binary model and $\mathcal{I}_i = \frac{1}{3} q_i s_i$ in the quantitative model. In both cases, the sign of $\mathcal{I}_i$ is the same as the sign of $s_i$. That is, citizens who believe the optimal policy to be negative have ideologies on the left, while those who think it is positive have ideologies on the right. How extreme an individual’s ideology is depends on $q_i$ and $s_i$ together. A moderate realization of $s_i$ suggests that $z$ is moderate as well, so $\mathcal{I}_i$ is of course moderate in that case. But $\mathcal{I}_i$ is also moderate if $q_i$ is low, even if $s_i$ is extreme, because extreme policies are risky, in that mistakes produce severe disutility. Conditional on a particular signal realization, therefore, more confident individuals are more extreme, as Remark 1 now states.

**Remark 1** For any $s$, ideological intensity $|\mathcal{I}(q, s)|$ increases in expertise $q$.

The importance of expertise is most stark when uncertainty is binary, because in that case citizens all know that $z$ lies at one of the two extremes of the policy space, but because signals are noisy, each prefers a policy on the interior of the policy space. The assumption that $G$ has full support implies that ideologies can range across the political spectrum. Thus, even though truth is binary, a range of expertise induces the familiar geometry of standard spatial models. The difference, of course, is that here voter preferences are tentative: each citizen ultimately prefers whatever policy is truly optimal, even if differs from his current opinion.\footnote{In the words of Benjamin Franklin (1787) at the close of the U.S. Constitutional Convention, “I confess that there are several parts of this Constitution which I do not at present approve, but I am not sure I shall}
The prediction of Remark 1 is consistent with a variety of empirical evidence. Using ANES data, for example, Figure 2 shows the distribution of ideology among citizens with different levels of information.\textsuperscript{16} The least informed group tend to be moderate and are rarely extreme, whereas the most informed group is more polarized. Palfrey and Poole (1987), Abramowitz and Saunders (2008), and Berelson, Lazarsfeld, and McPhee (1954, ch. 9, 11) report similar findings.\textsuperscript{17} According to the regression estimate displayed in Table 1, polarization increases by a quarter-level with each level of information. Also, consistent with the result that voting behavior is determined by the product of $q$ and $s$, not just $q$ alone, Goren (1997) finds that voters with extreme policy opinions but limited political knowledge vote similarly to more knowledgeable citizens with moderate opinions.

Attributing ideology to income or some other preference parameter provides no explanation for the relationship between information and ideology shown in Figure 2. Voters who lack information might be more moderate than their true interests warrant, but they might never approve them. For having lived long, I have experienced many instances of being obliged by better information, or fuller consideration, to change opinions even on important subjects, which I once thought right, but found to be otherwise.\textsuperscript{17}

\textsuperscript{16}Ideology is self-reported, originally on a 7-point scale (pre-election survey, question E1a). This 9-point scale distinguishes moderates who later self-identified as liberal or conservative (question E1b). A citizen’s “general level of information about politics and public affairs” was assessed subjectively by an ANES interviewer, on a 5-point scale (post-election survey, question ZZ3). Zaller (1986) finds this variable to be the single most useful information item in the NES.

\textsuperscript{17}Mullainathan and Washington (2009) report that voters who were barely old enough to vote in the previous election hold more extreme views than those who were barely too young. This could be interpreted as evidence of a causal influence of information on extremism, as eligibility induced voters to learn more about politics, which led to stronger opinions.
just as easily be more extreme. Bade and Rice (2009) argue that, if anything, the correlation ought to be reversed: individuals with extreme preferences are the least responsive to information about candidates, and so should be the least motivated to acquire it.

3 Committed Candidates

This section begins the analysis of equilibrium behavior, by assuming that candidates make binding platform commitments. Taking a pair \((x_A, x_B)\) of candidate platforms as given, Section 3.1 characterizes equilibrium voting incentives. Sections 3.2 and 3.3 then endogenize the choice of platforms, under the assumptions that candidates are office motivated and truth motivated, respectively.

3.1 Voting

If citizens follow the voting strategy \(v \in \mathcal{V}\) then, in state \(z \in \mathcal{Z}\), each votes for candidate \(j \in \mathcal{C}\) with probability \(\phi(j|z)\).\(^{18}\)

\[
\phi(j|z) = \int_{\mathcal{Q}} \int_{\mathcal{S}} v_j(q,s) h(s|q,z) g(q) ds dq. \tag{5}
\]

By the decomposition property of Poisson random variables, the numbers \(N_A\) and \(N_B\) of \(A\) and \(B\) votes are independent Poisson random variables with means \(n\phi(A|z)\) and \(n\phi(B|z)\) (Myerson, 1998). By the environmental equivalence property of Poisson games, however, an individual from within the game reinterprets \(N_A\) and \(N_B\) as the numbers of \(A\) and \(B\) votes cast by his peers (Myerson, 1998); by voting himself, he can add one to either total. A citizen’s best response to \(v\) is the strategy \(v^*\) that maximizes his expectation of (2), conditional on his private information \((q,s) \in \mathcal{Q} \times \mathcal{S}\), and taking \(v\) as given. The strategy \(v^*\) is a Bayesian Nash equilibrium if it is its own best response.

Voting for candidate \(j\) only changes a citizen’s utility if his vote is pivotal (event \(piv_j\)), meaning that it reverses the election outcome. This occurs when the candidates otherwise tie and \(j\) loses the tie-breaking coin toss, or when \(j\) wins the coin toss but loses the election by exactly one vote.\(^{19}\) Accordingly, the difference \(\Delta_{AB}(q,s)\) in expected utility between

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\(^{18}\)If uncertainty is binary then, in formulas such as (5), \(\int_{\mathcal{S}}\) and \(\int_{\mathcal{Z}}\) must be interpreted as sums, rather than integrals.

\(^{19}\)A common objection to strategic voting models is that voters lack the sophistication to estimate pivot probabilities. McMurray (2013a) cites evidence in favor of strategic voting, however, and also points out that if strategic behavior is socially optimal, as it is here (Propositions 1 and 4), a voter could behave as if he were strategic, even without thinking about pivot probabilities, simply by determining optimal voting behavior from a social planner’s perspective, and behaving accordingly.
voting $B$ and voting $A$ is given by
\[
\Delta_{AB}(q, s) = \int_{q} \left[ u(x_B|z) - u(x_A|z) \right] \Pr(piv_B|z) f(z|q, s) \, dz \\
- \int_{q} \left[ u(x_A|z) - u(x_B|z) \right] \Pr(piv_A|z) f(z|q, s) \, dz \\
= \int_{q} \left[ u(x_B|z) - u(x_A|z) \right] \Pr(piv|z) f(z|q, s) \, dz \\
= \int_{q} \left[ 2(x_B - x_A) (z - \bar{x}) \Pr(piv|z) f(z|q, s) \, dz \\
= 2(x_B - x_A) \left[ E(z|piv, q, s) - \bar{x} \right] \Pr(piv|q, s),
\]
where $\bar{x} = \frac{1}{2}(x_A + x_B)$ is the midpoint between the two candidates’ platforms, and $piv = piv_A \cup piv_B$ is the event in which a vote is pivotal for either of the two candidates.

If $x_B$ is to the right of $x_A$ (which is without loss of generality) then (6) is positive if and only if $E(z|piv, q, s)$ is sufficiently high. This expectation is increasing in $qs$ which, recall, is proportional to a voter’s ideology. Accordingly, Lemma 1 below characterizes equilibrium voting as a belief threshold strategy, defined in Definition 1. This behavior is intuitive: a citizen votes $A$ if his ideology is sufficiently to the left, and votes $B$ if his ideology is sufficiently to the right. In addition to providing this characterization, Lemma 1 guarantees that an equilibrium exists. It also states that the equilibrium belief threshold $\tau^*$ has the same sign as $\bar{x}$. If candidate platforms are symmetric around zero, this means that $\bar{x} = \tau^* = 0$, so voting is sincere. That is, citizens simply vote $A$ if $s$ is negative and vote $B$ if $s$ is positive.

**Definition 1** $v \in V$ is a belief threshold strategy, with belief threshold $\tau \in [-1, 1]$, if $qs < \tau$ implies $v_A(q, s) = 1$ and $qs > \tau$ implies $v_B(q, s) = 1$.

**Lemma 1** If candidates are committed then there exists a Bayesian Nash equilibrium $v^* \in V$ in the voting subgame. If $x_A < x_B$ then $v^*$ is a belief threshold strategy, with a threshold $\tau^*$ that has the same sign as $\bar{x}$.

### 3.2 Office Motivation

Lemma 1 characterizes an equilibrium voting strategy $v \in V$ for the subgame associated with arbitrary candidate platforms $x_A$ and $x_B$. This section now proceeds by backward induction to analyze the incentives for platform selection of candidates who are office motivated. A voting strategy $\sigma : \mathcal{X} \rightarrow V$ must now specify subgame behavior for every pair $(x_A, x_B) \in \mathcal{X}$ of candidate platforms. Let $\Sigma$ denote the set of such strategies. The strategy vector $(x_A^*, x_B^*, \sigma^*)$ is a perfect Bayesian equilibrium if $\sigma^* \in \Sigma$ induces a Bayesian Nash
equilibrium in the subgames associated with every pair \((x_A, x_B) \in \mathcal{X}^2\) of candidate platforms and each candidate’s platform \(x^*_j \in \mathcal{X}\) maximizes her likelihood \(\Pr (w = j)\) of winning the election, taking \(\sigma^*\) and her opponent’s platform as given. By Lemma 1, the equilibrium response to distinct platforms \(x_A \neq x_B\) is a belief threshold strategy. Moreover, \(\tau^*\) has the same sign as \(\bar{x}\) in that case, which implies that the candidate whose platform is closer to the origin is more likely to win the election. As Theorem 1 now states, competition for office therefore drives both candidates to the political center, as in the classic median voter theorems of Hotelling (1929) and Downs (1957).\(^{20}\)

**Theorem 1 (Median Opinion Theorem)** If candidates are committed and office-motivated then \((x^*_A, x^*_B, \sigma^*)\) is a perfect Bayesian equilibrium only if \(x^*_A = x^*_B = 0\). Furthermore, such an equilibrium exists, in which \(\sigma^*(x^*_A, x^*_B)\) is a belief threshold strategy with \(\tau^* = 0\).

The proof of Theorem 1 (including Lemma 1) is much more involved than the simple proofs of canonical median voter theorems, because voters do not have a dominant strategy to vote for the candidate who seems superior: they must take into account the informational implication of a pivotal vote. Nevertheless, in equilibrium, the standard logic applies: in an effort to appeal to larger numbers of voters, candidates move their platforms toward each other, and toward the center of the policy space. Theorem 1 is labeled as the median opinion theorem to emphasize that voter ideologies, which are all-important in determining political behavior, are actually only approximations of a more fundamental preference. Thus, as Section 5 discusses further, behavioral predictions are familiar but the implications for social welfare differ dramatically.

### 3.3 Truth Motivation and Polarization

Empirically, candidates do not seem to converge to an identical position, as a median voter theorem predicts. Figure 3, reproduced from Bafumi and Herron (2010), displays estimates of the densities of ideology for members of the U.S. House and Senate, juxtaposed with that for voters. Evidently, legislators are as polarized as the most extreme voters in the electorate. Estimates in Shor (2011) exhibit similar patterns, and McCarty and Poole (1995) find presidents to be about as extreme as the average legislator.\(^{21}\)

An early explanation for such polarization was that candidates, who after all are themselves citizens, care about policy outcomes, and so are unwilling to moderate. Such rigidity might also stem from candidates’ efforts to please extreme factions that comprise their

\(^{20}\)More precisely, since voters’ ideologies are private information, platforms converge to the median of the distribution of possible median voters, as in Calvert (1985).

\(^{21}\)Collective decisions are also more polarized than individual decisions in lab experiments (Sobel 2006).
party’s base. In the Downsian setting, however, Wittman (1977) and Calvert (1985) show that *policy-motivated* candidates converge to the center in equilibrium, just like those with office motivation. To choose policy, after all, a candidate must win office first. Extreme voters within the party should actually support this moderation, since domatically insisting on extreme policies only sacrifices the election to the opposing side, which entails severe utility loss. A policy-motivated candidate will leave the center if the location of the median voter is unknown, as in the probabilistic voting literature (e.g. Calvert, 1985), since in that case she sacrifices only some probability of winning. Even then, however, she cannot stray far: if she knows that the median voter’s ideal point lies somewhere between say $-\varepsilon$ and $\varepsilon$, she must adopt a platform within that range, or lose the election with certainty. $^{22}$

There are two ways to formalize policy motivation in this common-values environment. One is to assume that candidates prefer specific policies, distinct from the social optimum. In that case, as long as candidates’ ideal points are commonly known, the analysis of Section 3.1 can apply. This section instead assumes that candidates are *truth motivated*, meaning that they prefer policies that they believe are as close as possible to $z$. With this formulation, candidates are just like other citizens except that, as Footnote 13 notes, they do not vote or receive private signals.

For simplicity, this section assumes that candidates’ platform decisions and citizens’ voting decisions occur simultaneously. An equilibrium voting strategy $v^*$ is then still a best response to candidates’ platforms, but need not condition on platforms explicitly. A

$^{22}$Duggan (2005) also shows that equilibrium need not exist in that case.
Bayesian Nash equilibrium is then a vector \((x_A^*, x_B^*, v^*)\) such that each voter and candidate strategy maximizes the expectation of (2), taking other players’ strategies as given.

Given a voting strategy \(v\), a candidate may be more likely to win the election in some states than in others. If candidate \(j\) learns that she has won the election, therefore, she updates her expectation of the optimal policy, as follows.

\[
E(z|w = j) = \frac{\int_z \Pr (w = j|z) f(z) \, dz}{\int_z \Pr (w = j) f(z) \, dz}.
\]

When she chooses her platform policy, of course, a candidate does not know the outcome of the election. If she loses, however, her platform will not matter. Thus, just as a voter presumes that his vote will be pivotal, a candidate rationally presumes that she will win the election, and chooses her platform policy accordingly.

**Lemma 2** If candidates are committed and truth motivated then \((x_A^*, x_B^*, v^*)\) is a Bayesian Nash equilibrium only if \(x_j^* = E(z|w = j)\) for \(j = A, B\).

As in other games of communication, there exists a “babbling” equilibrium in which citizens vote un informatively. Candidates infer nothing about \(z\) from the event of winning the election in that case, so each remains at the origin, which is optimal on the basis of prior information alone. Coordinating on uninformative voting seems unlikely, however, so Theorem 2 restricts attention to voting strategies that are informative, as defined in Definition 2.

**Definition 2** A strategy \(v \in V\) is informative if \(\frac{v_B(q,s)}{v_A(q,s)}\) increases in \(s\), for any \(q\).\(^{23}\)

With informative voting, candidate \(A\) is likely to win when \(z\) is low, and \(B\) is likely to win when \(z\) is high, so (7) is negative when \(A\) wins and positive when \(B\) wins. Anticipating this, candidate \(A\) chooses a platform on the left and candidate \(B\) chooses a platform on the right. As platforms diverge, voters resort to a belief threshold strategy, as in Lemma 1. Given the symmetry of the model, equilibrium platforms may be symmetric around zero.

**Theorem 2 (Candidate Polarization)** If candidates are committed and truth motivated and \(v^*\) is informative then \((x_A^*, x_B^*, v^*)\) is a Bayesian Nash equilibrium only if (i) \(x_j^* = E(z|w = j)\) for \(j = A, B\), where \(x_A^* < 0 < x_B^*\), and (ii) \(v^*\) is a belief threshold strategy. Furthermore, such an equilibrium exists, with \(x_A^* = -x_B^*\) and \(\tau^* = 0\).

\(^{23}\)A strategy would equally informative, of course, if \(\frac{v_B(q,s)}{v_A(q,s)}\) were decreasing in \(s\), for any \(q\). Thus, Theorem 2 and similar results below could be rewritten with \(A\) on the right and \(B\) on the left.
When the number of voters is large, a candidate is unlikely to win the election unless the state of the world is “on her side”. Conditioning on this event therefore substantially bolsters her confidence, which, especially in the binary model, also makes her more extreme—potentially even more extreme than any voter. Moderating is counterproductive in that case, because it moves policy in the wrong direction. It is also unnecessary, because if the state of the world favors her own platform over her opponent’s, voters will likely recognize this, and she will win the election.

Empirically, voters do seem willing to support extreme candidates. In the U.S., in fact, ideological moderates within both major parties are often viewed with disdain, pejoratively labeled DINO and RINO (Democrats- and Republicans-in-name-only). In the 2012 presidential primaries, many Republicans reported viewing Mitt Romney as the most likely candidate to beat President Barack Obama in the general election, but “not conservative enough”, and so voted for Rick Santorum, Newt Gingrich, or Ron Paul instead.24 Achen (1978) finds that extreme candidates tend to be victorious in U.S. House elections. Voters seem to want a bold and confident champion, who can convince the electorate in favor of a particular (potentially extreme) policy position. This is the popular perception of many prominent U.S. presidential candidates, such as Franklin Roosevelt, Barry Goldwater, George McGovern, Ronald Reagan, and Barack Obama.

It is worth noting that inferring information from voters gives only one of possibly many explanations for candidates to be confident in their opinions. The logic of Section 4.2 below suggests another possibility, for example, which is that confident citizens are the most likely to run for office, just as they are the most likely to participate in other political activities. Candidates might even be irrationally overconfident. The main point of Theorem 2 is simply that, while a candidate who holds extreme policy preferences might have an incentive to compromise with those whose preferences differ, a candidate who confidently believes that her policy position is optimal may be reluctant to compromise, instead trusting voters to recognize the truth of her position, and elect her to office—potentially by a large margin, as the following section discusses.

### 3.4 Electoral Margins

Many existing models of electoral competition predict exact ties in equilibrium. This may occur when candidates converge, leaving voters indifferent, or diverge but split the electorate equally. If candidates do not move to close the electoral gap, strategic voters may do so: in the costly voting model of Krasa and Polborn (2009), for example, the incentive to free-ride is stronger for majority than minority voters, producing an “underdog effect”

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by which candidates tie even though one is more popular. Empirically, however, margins of victory are often rather large. Mueller (2003, ch. 11) reports, for example, that U.S. governors have historically won reelection by an average margin of 23%. Krehbiel (1997, ch. 1) points out that large, bipartisan majorities are also common in legislative votes.

In this model, the fact that voters’ opinions are mutually correlated produces large margins of victory in expectation. In the equilibrium of Theorem 2, for example, \( \tau^* = 0 \) implies that citizens vote sincerely, and in the quantitative model, (5) reduces to

\[
\phi(B|z) = \phi(A|z) = \int Q \int_0^1 \frac{1}{2} (1 + qsz) g(q) ds dq
\]

\[
= \frac{1}{2} + \frac{1}{4} z E(q),
\]

implying that the margin of victory \( \mu(z) \) in state \( z \) is

\[
\mu(z) = | \phi(B|z) - \phi(A|z) |
\]

\[
= \frac{1}{2} |z| E(q),
\]

which on average is equal to

\[
E[\mu(z)] = \int z \frac{1}{2} |z| E(q) f(z) dz
\]

\[
= \frac{1}{4} E(q).
\]

In the binary model, by a similar derivation, \( E[\mu(z)] = E(q) \). In either case, as Remark 2 states, the expected margin of victory is positive.

**Remark 2** If candidates are committed and truth motivated and \( (x_A^*, x_B^*, v^*) \) is a Bayesian Nash equilibrium with a belief threshold strategy \( v^* \) characterized by \( \tau^* = 0 \) then the expected margin of victory \( E[\mu(z)] \) is strictly positive.

Remark 2 is noteworthy in that candidates are ex ante identical, and adopt symmetric platforms, which may even be arbitrarily close to one another. Nevertheless, a large fraction of voters favors the same candidate, or same side of an issue. This is especially likely if \( E(q) \) is high, consistent with the empirical observation that large margins seem especially common for decisions that are obvious in some sense, such as retaining public officials with clear track records of quality governance, or ballot initiatives to revise archaic government procedures or constitutional language.
3.5 Voter Participation

The analysis above assumes that every citizen must vote, while in most elections, citizens are allowed to abstain, and many do so. Accordingly, this section expands the set of candidates to \( C' = \{A, B, 0\} \), where a vote for candidate 0 represents non-voting. Let \( \mathcal{V}' \) denote the set of redefined voting strategies. An **informative** strategy is redefined in Definition 3, and Definition 4 redefines a **belief threshold strategy** in this setting, using two thresholds instead of one. A citizen abstains if \( q_s \) falls between the thresholds; if the two thresholds coincide then everyone votes.

**Definition 3** A strategy \( v \in \mathcal{V}' \) is informative if, for any \( q \), \( \frac{v_j(q,s)}{v_j(q,s)} \) increases in \( s \) whenever \( j \) precedes \( j' \) in the ordering \( \{A, 0, B\} \).

**Definition 4** \( v \in \mathcal{V}' \) is a belief threshold strategy, with belief thresholds \( \tau_1 \leq \tau_2 \), if \( v_j(q,s) = 0 \) unless \( j = A \) and \( q_s \in [-1, \tau_1] \), \( j = 0 \) and \( q_s \in [\tau_1, \tau_2] \), \( j = B \) and \( q_s \in [\tau_2, 1] \).

With these modifications to the model above, Theorem 3 revises the equilibrium characterization of Theorem 2. As before, restricting attention to informative voting strategies implies that candidates infer different information from the event of winning the election, and so adopt divergent policy platforms. In response, citizens who strongly believe the state to be negative or positive have the greatest incentives to vote \( A \) or \( B \), respectively, so equilibrium voting follows a belief threshold strategy. Once again, the symmetry of the model is such that candidate positions and voter behavior may again be symmetric around the origin.

Since voting is costless, and since each signal induces a strict preference ordering over the two policy outcomes, it may seem that every citizen should vote. Contrary to this intuition, however, Lemma 1 states that \( \tau_1^* < \tau_2^* \), implying that a positive fraction of the electorate abstain in equilibrium.

**Theorem 3 (Swing voter’s curse)** If candidates are committed and policy-motivated and \( v^* \in \mathcal{V}' \) is informative then \( (x_A^*, x_B^*, v^*) \) is a Bayesian Nash equilibrium only if (i) \( x_j^* = E(z|w = j) \) for \( j = A, B \), where \( x_A^* < 0 < x_B^* \), and (ii) \( v^* \) is a belief threshold strategy with \( \tau_1^* < \tau_2^* \). Such an equilibrium exists, with \( x_A^* = -x_B^* \) and \( \tau_1^* = -\tau_2^* \).

\(^{25}\) As in Definition 2, this ordering could be reversed, and Proposition 3 could be restated with \( A \) on the right and \( B \) on the left.
Figure 4: Voting behavior, by opinion and expertise, for a belief threshold strategy.

The logic behind Theorem 3 is Feddersen and Pesendorfer’s (1996) swing voter’s curse: since votes reflect private opinions which are correlated with the truth, the candidate with the truly superior policy is more likely to win the election by one vote than to lose by one vote. A vote for the inferior candidate is therefore more likely to be pivotal than a vote for the superior candidate, so a citizen who is uncertain which platform is superior prefers to abstain. Theorem 3 assumes a finite electorate. With binary uncertainty, however, the logic of McMurray (2013a) implies that turnout remains at a moderate level, even as $n$ grows large. A formal analysis of large elections is beyond the scope of this paper, but it seems reasonable to conjecture that the same holds with quantitative uncertainty.

Traditionally, the analysis of voter participation has focused on the cost of voting. As Feddersen and Pesendorfer (1996) point out, however, individuals often abstain even when voting is costless. After having already paid the cost to travel to the polls and wait in line, for example, many citizens vote in some races but skip others, resulting in incomplete ballots. The swing voter’s curse provides an intuitive explanation for this phenomenon: a voter has clear, strong opinions on some issues, but not others, and so delegates the latter to those who know more.

When uncertainty is quantitative, expertise is not the only determinant of voter participation: as Figure 4 illustrates, a citizen abstains when the product of $q_i$ and $s_i$ is low. Thus, citizens with moderate ideology abstain, even when they are relatively well informed. This is because such a citizen’s vote choice is sensitive to even small mistakes. By contrast, an ideological extremist votes even if his signal is quite noisy, because even if the optimal policy is less extreme than he believes it to be, it probably has the same sign.

The prediction that voter participation depends jointly on information and ideology has strong empirical support. Figure 5 displays voter participation levels, for example, by information level and ideology. According to probit regression results in Table 2, each

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26 Members of committees often abstain even though voting requires only the raise of a hand.
information level makes the average citizen 10% more likely to register as a voter, and makes the average registered voter 12% more likely to vote in the presidential primary and 5% more likely to vote in the general election. The average general election voter is also 7% more likely to vote in both the presidential and congressional races. Moving one ideological category further from the center makes the average citizen 2% more likely to register, 5% and 2% more likely to vote in primary and general elections, and 2% more likely to cast a complete ballot.

Consistent with the evidence presented in this section, Magleby (1984, ch. 6) and Wattenberg et al. (2000) find that educated and politically knowledgeable voters are the most likely to cast completed ballots, and numerous studies (cited in McMurray, 2011) document a correlation between voter turnout and information variables such as voter education, political knowledge, age, access to news media, and contact from campaign workers.\textsuperscript{27} Palfrey

\textsuperscript{27}Voter turnout is also higher in national and general elections than in local races and primaries, perhaps because partisan labels and media exposure give voters greater confidence in their opinions. Similarly, participation tends to be low for policy initiatives, which are typically non-partisan, and which many find
and Poole (1987), Keith et al. (1992, ch. 3), and Abramowitz and Saunders (2008) further find that voter participation and information are jointly correlated with ideology.\textsuperscript{28} Sobbrio and Navarra (2010) find that participation is correlated with information, even conditional on ideology.\textsuperscript{29} Lassen (2005), Banerjee et al. (2010), and Larcinese (2007) present evidence that the relationship between information and voter participation is causal.

4 Uncommitted Candidates

It has long been recognized that campaign promises are difficult to enforce. If candidates share voters’ goal of identifying good policies, enforcement may not even be desirable. In this section, therefore, candidates are truth motivated as in Section 3.3, but are uncommitted—each is free to implement any policy after taking office, and able to base this choice $y_j : \mathbb{Z}_+^2 \rightarrow \mathcal{X}$ on vote totals $N_A, N_B \in \mathbb{Z}_+$, which were unobservable at the platform stage.\textsuperscript{30} A strategy vector $(v^*, y_A^*, y_B^*)$ is thus a perfect Bayesian equilibrium if $y_A^*$ and $y_B^*$ each maximize the expectation of (2), given the voting strategy $v^*$, and $v^*$ is optimal for a voter whose peers follow $v^*$, given that candidates follow $y_A^*$ and $y_B^*$. Like Sections 3.1 through 3.3, Section 4.1 assumes that all citizens vote. Section 4.2 then re-introduces the opportunity for abstention, and Section 4.3 considers the possibility of additional candidates.

4.1 Mandates

For a truth-motivated candidate who wins the election, expected utility is still maximized by the expectation of $z$. When she had to commit to a platform before the election, however, the only information that could shape a candidate’s expectation was the presumed fact that she would receive more votes than her opponent. Now, she can observe the exact numbers $N_A$ and $N_B$ of votes cast for either candidate, and should update further, as Remark 3 now states.

**Remark 3** If candidates are truth motivated and uncommitted then, for $j = A, B$, the optimal policy response to any voting strategy is $y_j^* (a, b) = E (z | N_A = a, N_B = b) \equiv \hat{z}_{a,b}$.

\textsuperscript{28}“too long and hard to understand” (Magleby 1984, ch. 6).
\textsuperscript{29}Bade and Rice (2009) suggest a theoretical link between information and ideology, but find the additional link with participation inexplicable.
\textsuperscript{29}Those authors actually interpret their finding as evidence against the swing voter’s curse, because following Feddersen and Pesendorfer (1996), they conceptualize ideology as a preference parameter.
\textsuperscript{30}Office motivation is not considered here, because it provides no guidance once a candidate has already won the election.
For simplicity, differences between candidates are not modeled here, which actually makes the identity of the winning candidate immaterial: given the same information, the two candidates would implement the same policy. The more important implication of Remark 3, however, is simply that vote totals shape the winning candidate’s beliefs, whoever that turns out to be.

How a candidate interprets votes depends on the strategy used by voters. An individual could deviate from this strategy, but a candidate would have no way of knowing this, and so would interpret his vote the same as she interprets any other. In equilibrium, therefore, whether a voter prefers to add one to the number of $A$ votes or to the number of $B$ votes depends on the voting strategy $v$ used by his peers—as well as on his private signal, which provides information about the location of the optimal policy, and therefore about the likely numbers of votes on either side. Accordingly, the expected benefit of voting $B$ instead of $A$ is now

$$
\Delta_{AB} (q, s) = E_{z,N_A,N_B} \left[ u \left( \hat{z}_{N_A,N_B+1}, z \right) - u \left( \hat{z}_{N_A+1,N_B}, z \right) | q, s \right] = E_{z,N_A,N_B} \left[ \left( \hat{z}_{N_A,N_B+1} - \hat{z}_{N_A+1,N_B} \right)^2 + \left( \hat{z}_{N_A+1,N_B} - z \right)^2 | q, s \right]
$$

instead of (6).

As in Section 3.3, a “babbling” equilibrium exists, in which voters ignore their private information (e.g. vote randomly, or all vote for candidate $A$), and candidates ignore vote totals in choosing policy. That voters and candidates would coordinate on an equilibrium in which communication is meaningless seems unlikely, however, so like Theorem 2, Theorem 4 restricts attention to voting strategies that are informative. In that case, as Lemma 3 now states, each $A$ vote pushes the winning candidate’s beliefs to the left, and each $B$ vote pushes her beliefs to the right.

**Lemma 3 (Monotone Expectations)** If $v \in V$ is informative then $\hat{z}_{a,b}$ and $E (z|N_A = a, N_B = b, q, s)$ are decreasing in $a$ and increasing in $b$.

A voter may feel conflicted because pushing the ultimate policy outcome say, to the left, is beneficial in some states of the world but detrimental in others. If one citizen finds it optimal to vote $A$, however, then so does another who perceives the optimal policy to lie even further to the left. With similar reasoning for $B$ voters, this implies that voters follow a belief threshold strategy in equilibrium, just as before. As in earlier results, the symmetry of the model facilitates symmetric behavior in equilibrium.
Theorem 4 (Mandates) If candidates are truth motivated and uncommitted and $v^*$ is informative then $(v^*, y_A^*, y_B^*)$ is a perfect Bayesian equilibrium only if (i) $v^*$ is a belief threshold strategy with $-1 < \tau^* < 1$ and (ii) $y_j^*(a,b) = \hat{z}_{a,b}$ for all $a,b \in \mathbb{Z}_+$ and $j = A, B$, where $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$. Furthermore, such an equilibrium exists, with $\tau^* = 0$ and $\hat{z}_{a,b} = -\hat{z}_{b,a}$ for all $a,b \in \mathbb{Z}_+$.

The equilibrium behavior described in Theorem 4 provides an intuitive formalization of the popular notion of electoral “mandates”: a wholehearted endorsement of one side or the other communicates a call for extreme policy changes, while a candidate who wins only narrowly is obligated to remain more moderate. Empirical evidence of responses to electoral mandates are documented by Faravelli, Man, and Walsh (2012), as well as Conley (2001, ch. 4), Peterson et al. (2003), and Fowler and Smirnov (2007, ch. 3).

Essentially, the logic of Theorem 4 is the same as that of the common-values model of Razin (2003). With binary signals, however, that model does not exhibit the range of public opinion emphasized in Section 2.1. Also, candidate behavior in that paper is specified exogenously, rather than derived endogenously. Perhaps most importantly, Sections 4.2 and 4.3 of this paper extend the analysis of mandates to include political participation and multiple candidates, which that paper does not consider. Other models of mandates include Castenheira (2003b), Shotts (2006), and Fowler and Smirnov (2007), where votes from one election push candidate positions in future elections, by revealing the location of the median voter’s bliss point. In Castenheira (2003b), mandate dynamics can even lead voters to support minor parties. The empirical references above suggest, however, that candidates’ responses to mandates are immediate. Aside from these references, private-value models say little about mandates, focusing exclusively on the identity of the winning candidate.

The influence of electoral mandates may extend beyond the current election. In 2006, for example, severe Republican congressional losses were widely interpreted as an indictment of incumbent President George W. Bush, who immediately responded by announcing the resignation of his defense secretary, and promising to “find common ground” with Democrats on the war in Iraq and domestic issues. Political activities other than voting could convey mandates as well. Madestam, Shoag, Veuver, and Yanagizawa-Drott (2012) find evidence, for example, that congressional voting responded to public protests in 2009 by the conservative Tea Party movement.

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4.2 Voter Participation and Political Activism

Section 4.1 assumes that all citizens vote, but as Section 3.5 points out, citizens in most real-world voting environments are allowed to abstain. Theorem 3 from that section demonstrated an incentive for strategic abstention, which was supported by a variety of empirical evidence. The logic for abstention, however, was the swing voter’s curse, which stems from the comparison of pivot probabilities. When candidates respond to mandates as in Section 4.1, the identity of the winning candidate is unimportant, so the standard pivotal calculus is no longer relevant. In a sense, every vote is pivotal, in that it (slightly) changes the policy choice of the winning candidate.

Maintaining the assumptions that candidates are truth motivated and uncommitted, this section reintroduces the possibility of abstention as a vote for candidate 0, so that the set of relevant voting strategies is \( V' \). Like Theorem 4, Theorem 5 restricts attention to informative voting strategies, and shows that voters again follow a belief threshold strategy in equilibrium. Since voting is costless, every citizen receives an informative private signal, and the swing voter’s curse does not apply, it might seem that everyone should vote. To the contrary, however, equilibrium belief thresholds are distinct, meaning that a positive fraction of the electorate abstain.

**Theorem 5 (Strategic abstention)** If candidates are truth motivated and uncommitted and \( v^* \in V' \) is informative then \((v^*, y_A^*, y_B^*)\) is a perfect Bayesian equilibrium only if (i) \( v^* \) is a belief threshold strategy with \(-1 < \tau^*_1 < 0 < \tau^*_2 < 1\) and (ii) \( y^*_j (a, b) = \hat{z}_{a,b} \) for all \( a, b \in \mathbb{Z}_+^+ \) and \( j = A, B \), with \( \hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1} \). Furthermore, such an equilibrium exists, with \( \tau^*_1 = -\tau^*_2 \) and \( \hat{z}_{a,b} = -\hat{z}_{b,a} \) for all \( a, b \in \mathbb{Z}_+^+ \).

The logic of Theorem 5 is that, if a citizen abstains, the winning candidate will make a decision based solely on the opinions of his peers. If the citizen has no private information of his own, this is exactly what he prefers. By continuity, citizens with sufficiently noisy signals prefer to abstain as well. Intuitively, this is because the winning candidate cannot observe the expertise underlying each vote, and so merely treats each vote as if it were of average quality. Therefore, a citizen who knows that his own expertise is actually below average expects the winning candidate to over-react to his vote. A citizen who is only slightly below average prefers this over-reaction to the under-reaction that will occur if he abstains, but a citizen whose signal is sufficiently noisy does not. Note that, although a formal analysis of large elections is beyond the scope of this paper, this intuition suggests that turnout and abstention should remain robust in large elections, as well: presumably, no matter how large the election gets, the winning candidate will over-react to poorly-informed
voters and under-react to non-voters, thereby preserving the incentives to vote and abstain, even in the limit.\textsuperscript{33}

Theorem 5 shows that the incentive for strategic abstention is not limited to the swing voter’s curse. This is important since, as Section 4.1 discusses, the impact of a vote seems empirically to extend beyond its impact on the identity of the election winner. Fundamentally, abstention is driven by asymmetric information, combined with the common values assumption. Regardless of the details of the decision mechanism, that is, a citizen abstains because he trusts his peers to make a decision on his behalf, more than he trusts himself.

As Section 4.1 points out, many political activities other than voting might influence the beliefs of public officials, or otherwise contribute to mandates. Examples include writing letters to representatives regarding upcoming policy decisions, attending public rallies or protests, making campaign donations, and so on. One strength of a mandates model of elections is that a vote exerts a gradual impact on policy, and so can be reinterpreted as one of these other forms of political action. In contrast, standard election models focus on making and breaking ties, which is irrelevant in these settings: a public official does not suddenly reverse her policy position, for example, when the number of constituent letters on one side of an issue exceeds the opposition by one.

Empirically, the patterns of participation in a variety of political activities are indeed the same as that of voting. Figure 6 displays levels of political activism, for example, among voters grouped by information level and ideology, as in Figure 5 above. According to the probit regression estimates from Table 3, each information level makes the average citizen 11\% more likely to report trying to persuade others to vote for or against a particular candidate, 10\% more likely to have ever signed an online political petition, 7\% more likely to have ever joined a protest march, rally, or demonstration, 10\% more likely to have contacted an elected official in the previous year to support or oppose pending legislation, 6\% more likely to have endorsed a candidate by displaying a bumper sticker, yard sign, or campaign button, and 9\% more likely to have donated to a political campaign. Moving one ideological category further from the center increases the average citizen’s participation in these activities by 8\%, 5\%, 5\%, 2\%, 3\%, and 4\%, respectively.\textsuperscript{34} Relative to the low baseline rates for these activities, these effects are quite substantial.

\textsuperscript{33}Intuition from Section 5 supports this conjecture as well: voting and abstention both convey information, so to best facilitate communication, a social planner should recommend each action to a substantial fraction of the electorate. In a common-value environment such as this, voters should follow the planner’s recommendations in equilibrium.

\textsuperscript{34}Similar findings are reported by Keith et al. (1992, ch. 3), Abramowitz and Saunders (2008), and Bafumi and Heron (2010). Coupé and Noury (2004) find that knowledgeable individuals also participate more frequently in opinion polls.
Figure 6: Like voter participation, political activism increases both with information and with ideological distance from the center.
Discussions of mandates typically focus on achieving extreme policies, but citizens who receive high-quality, moderate signals may also prefer to weaken the winning candidate’s mandate. This could explain why some citizens protest against an entire field of unattractive candidates by abstaining, despite the fact that voting would help keep the worst candidates out of office. Many citizens even cast blank ballots (sometimes called “protest votes”), perhaps to distinguish themselves from citizens who abstain simply for lack of information. This might also explain why some citizens vote against a preferred candidate who is likely to win, as documented by Franklin, Niemi, and Whitten (1994), or vote for opposite parties to fill various offices within the government (i.e. “split ticket” voting), as documented by the references in Chari, Jones, and Marimon (1997).

4.3 Multiple Candidates

So far, the analysis of this paper has included only two candidates. In many elections, however, multiple candidates run for office. In the 2008 U.S. presidential election, for example, minor parties garnered over one and a half million votes; in 2000, they received nearly four million, and in 1992 they received over 20 million (Federal Elections Commission 1992, 2000, 2008). Presidential primaries often have half a dozen candidates or more, each receiving substantial vote shares.

This section expands the set of candidates to \( C^n = \{A, B, C, D, 0\} \), with election decisions made by plurality rule. Four candidates preserve a simplifying symmetry, but other numbers can be treated similarly. As in Section 4.2, abstention is allowed, and is denoted as a vote for candidate 0. Let \( V^n \) denote the set of redefined voting strategies. An informative strategy is redefined in Definition 5, and Definition 6 redefines a belief threshold strategy, using four thresholds instead of two.

**Definition 5** A strategy \( v \in V^n \) is informative if, for any \( q \), \( \frac{v_j(q,s)}{v_j(q,s)} \) increases in \( s \) whenever \( j \) precedes \( j' \) in the ordering \( \{A, B, 0, C, D\} \).

**Definition 6** \( v \in V^n \) is a belief threshold strategy, with belief thresholds \(-1 \leq \tau_1 \leq \tau_2 \leq \tau_3 \leq \tau_4 \leq 1\), if \( v_j(q,s) = 0 \) unless

\[
\begin{align*}
  j = A & \text{ and } q_s \in [-1, \tau_1] \\
  j = B & \text{ and } q_s \in [\tau_1, \tau_2] \\
  j = 0 & \text{ and } q_s \in [\tau_2, \tau_3] \\
  j = C & \text{ and } q_s \in [\tau_3, \tau_4] \\
  j = D & \text{ and } q_s \in [\tau_4, 1]
\end{align*}
\]

Like earlier definitions, this could be redefined for other permutations of the candidates, and Theorem 6 could be restated accordingly. Orderings with abstention in the center are probably the most robust, however, since they are reinforced by the swing voter’s curse if candidates are not perfectly responsive to mandates.
Like Remark 3, Remark 4 states that the winning candidate’s optimal policy choice is simply her expectation of the state, conditional on vote totals \( N_A \) through \( N_D \). Theorem 6 then states that if citizens vote informatively then they must follow a belief threshold strategy in equilibrium, as before. Each candidate expects a positive vote share in equilibrium, and a positive fraction of the electorate abstain. As before, equilibrium behavior can be symmetric around the origin. In that case, voters who believe the optimal policy is left of zero vote \( A \) or \( B \), while those who believe the optimal policy is right of zero vote \( C \) or \( D \). Votes for \( A \) or \( D \) elicit more dramatic responses from the winning candidate than votes for \( B \) or \( C \).

**Remark 4** If candidates are truth motivated and uncommitted then, for \( j = A, B \), the optimal policy response to any voting strategy is

\[
y_j^*(a, b, c, d) = E(z|N_A = a, N_B = b, N_C = c, N_D = d) \equiv \tilde{z}_{a,b,c,d}.
\]

**Theorem 6 (Multiple candidates)** If candidates are truth motivated and uncommitted, voter abstention is allowed, and \( v^* \in \mathcal{V}^w \) is informative, then \( (v^*, (y_j^*)_{j \in C^w}) \) is an equilibrium only if (i) \( v^* \) is a belief threshold strategy with \(-1 < \tau_1 < \tau_2 < \tau_3^* < \tau_4^* < 1 \) and (ii) \( y_j^*(a, b, c, d) = \tilde{z}_{a,b,c,d} \) for all \( a, b, c, d \in \mathbb{Z}_+ \) and for \( j = A, B, C, D \), with \( \tilde{z}_{a+1,b,c,d} < \tilde{z}_{a,b+1,c,d} < \tilde{z}_{a,b,c,d+1} \). Furthermore, such an equilibrium exists, with \( y_A^*(a, b, c, d) = -y_D^*(d, c, b, a), y_B^*(a, b, c, d) = -y_C^*(d, c, b, a), \tau_1^* = -\tau_4^*, \) and \( \tau_2^* = -\tau_3^* \).

The result that every candidate receives votes in expectation differs from standard literature, where Duverger’s law predicts that voters ignore all but two front-runners, since votes for losing candidates are wasted. Here, the policy choice of the winning candidate is influenced by the numbers of votes each of the losing candidates received, so no vote is wasted.\(^{36}\) Candidate \( C \) might win the election, for example, and will likely choose a policy to the right of zero, but every \( D \) vote pushes her further to the right, while every \( B \) vote and especially every \( A \) vote makes her less extreme. It would be possible, in fact, for \( C \) to win the election but implement a policy left of zero, if \( A \) and \( B \) votes collectively outnumber \( C \) and \( D \) votes.

Especially in the binary model, where the optimal policy is known ex ante to lie at one of the extremes of the policy space, it might seem surprising that anyone votes for candidates \( B \) or \( C \), when voting for \( A \) or \( D \) would have greater impact on the policy outcome. This can be

---

\(^{36}\)In some jurisdictions, minor parties can even endorse major party candidates, thereby allowing a citizen to signal support of the minor party’s platform without worrying about spoiling the election for the better of the two major candidates. In the 2006 New York state governor’s race, for example, residents could vote for Eliot Spitzer via the Democratic, Independence, or Working Families party lines, and for John Faso on the Republican or Conservative party lines; ultimately, each candidate received more than 10% of his votes from minor party lines (New York State Board of Elections, 2006).
understood in the same light as abstention, however, as a hedge against error: the winning candidate considers every vote to be of average quality, and thus over-reacts to votes from citizens with below-average expertise. Citizens who lack confidence may therefore prefer to push the policy outcome by a small amount, rather than by a large amount. As in Theorem 5, the least confident citizens simply abstain. As in Section 3.5, citizens with moderate opinions abstain, even if they are quite confident in their information. Similarly, as Figure 7 illustrates, citizens who are confident that the optimal policy is center-left or center-right vote $B$ or $C$, just like those who have more extreme opinions but lack confidence in their opinions. The only citizens who vote $A$ or $D$ are those who are both extreme and confident. That voters with the best information and the most extreme political views support the most extreme candidates is precisely the empirical finding of Palfrey and Poole (1987).

5 Welfare

The analysis above has focused on equilibrium behavior. Returning now to each specification in turn, this section considers implications for voter welfare. With common preferences, it is uncontroversial to measure social welfare simply as the expected utility of an individual voter. Proposition 1 begins by analyzing welfare for a voting equilibrium, treating candidate platforms as exogenous. In that case, any welfare-maximizing voting strategy constitutes an equilibrium, and if the number of voters is large and uncertainty is binary then the candidate whose platform is truly superior (i.e. candidate $A$ if $z = -1$ and candidate $B$ if $z = 1$) almost surely wins the election. This then extends Condorcet’s (1785) classic jury theorem—which Krishna and Morgan (2011) laud as the “first welfare theorem of political economy”—to this spatial environment.
Proposition 1 (Jury Theorem) If candidates are committed to platforms $x_A, x_B \in \mathcal{X}$ and for every $n$ the voting strategy $v_n^* \in \mathcal{V}$ maximizes $E_{y_n,z} [u(y_n); v]$ then (i) $v_n^*$ is a Bayesian Nash equilibrium, and (ii) if $\mathcal{Z} = \mathcal{Z}_B$ then $y_n^* \rightarrow_p \arg \max_{x \in \{x_A, x_B\}} u(x|z)$.

Recall from Section 3.2 that committed, office-motivated candidates converge to the political center. In standard Downsian models, convergence represents a compromise between the competing interests of the left and right, which is appealing from a utilitarian perspective. Davis and Hinich (1968) show, for example, that policies in the center minimize the total disutility that voters suffer from a policy that is far from their bliss points. May’s (1952) axioms endorse the median voter’s ideal policy, in particular, as the one that is majority-preferred to any other (i.e. the Condorcet winner). Accordingly, the result that competition for office drives candidates—who might otherwise prefer extreme policies—toward each other and toward the political center is often viewed as the “invisible hand” of politics. In that light, the policy divergence documented by the references in Section 3.3 should be interpreted as evidence of political failure.

Proposition 2 considers the optimal combination of voter and candidate behavior. In contrast with the Downsian model, welfare is maximized in equilibrium by truth motivated, not office motivated candidates. In light of Theorem 2, this means that candidate polarization is good for voters.

Proposition 2 (Jury Theorem 2) If candidates are committed and for every $n$ the strategy vector $(x_{A,n}^*, x_{B,n}^*, v_n^*) \in \mathcal{X}^2 \times \mathcal{V}$ maximizes $E_{y_n,z} [u(y_n); x_A, x_B, v]$ then (i) if candidates are truth motivated then $(x_{A,n}^*, x_{B,n}^*, v_n^*)$ is a Bayesian Nash equilibrium, and (ii) if $\mathcal{Z} = \mathcal{Z}_B$ then $y_n^* \rightarrow_p z$.

The benefit of candidate polarization is that it tailors the policy choice to the situation: in positive states, a positive policy is chosen; in negative states, a negative policy is chosen. In pursuing votes, office motivated candidates sacrifice this flexibility. This is reminiscent of Harrington (1993), Canes-Wrone, Herron, and Shotts (2001), and Maskin and Tirole (2004), where candidates adopt inferior policies in an effort to “pander” to the premature preferences of uninformed voters. This may have motivated an American Political Science Association (1950, p. 15) manifesto, which reminded “responsible parties” that “putting a particular candidate into office is not an end in itself”, and urged the U.S. to “keep [political] parties apart”, in order to “provide the electorate with a proper range of choice”.

---

37 If utility functions are tent-shaped or quadratic, for example, then total utility is maximized at the median voter’s or mean voter’s ideal point, respectively; generically, the utilitarian optimum lies in the interior of the policy space.
Such recommendations are odd if convergence to the center is optimal for voters, but in this model can be viewed as a call for truth- rather than office-motivation.

The informational benefit of candidate divergence is also recognized by Bernhardt, Duggan, and Squintani (2009), who assume that standard spatial preferences are shifted by a random “shock”, so that the median voter prefers a menu of two policy alternatives, rather than one. While the logic is the same, the result here is more stark, especially in the binary model, because the optimal policy is not just off-center—it lies at one of the extremes of the policy space. In the example of Section 1, voters disagree over whether economic stimulus should be large or small, but the compromise achieved by office motivated candidates is a moderate-sized stimulus, which is known ex ante not to be optimal.

Proposition 3 evaluates the welfare implication of voter abstention. Since every citizen receives an informative signal, abstention implies that some information is lost in equilibrium. This may seem to vindicate efforts to increase voter participation, for example by penalizing non-voters with stigma, or even fines. Contrary to this intuition, however, it is not optimal for everyone to vote: the optimal combination of voter and candidate strategies constitutes an equilibrium, which by Theorem 3 involves some abstention.

**Proposition 3 (Jury Theorem 3)** If candidates are committed and truth motivated and for every $n$ the strategy vector $(x_{A,n}^{*}, x_{B,n}^{*}, v_{n}^{*}) \in X^2 \times V'$ maximizes $E_{y_{n},z}[u(y_{n}); x_{A}, x_{B}, v]$ then (i) $(x_{A,n}^{*}, x_{B,n}^{*}, v_{n}^{*})$ is a Bayesian Nash equilibrium, and (ii) if $Z = Z_{B}$ then $y_{n}^{*} \rightarrow_{p} z$.

The proof of Proposition 3 is virtually identical to that of Proposition 2, replacing $V$ with $V'$. The intuition is the same as that of Theorem 5 in McMurray (2013a): mandatory voting would lose information communicated by the decision to vote. Ideally, signals would all be utilized, but would also be weighted by their underlying precision, whereas standard election mechanisms weigh votes equally. Abstention provides a crude way of transferring weight to the most precise signals.

Proposition 4 considers the case of uncommitted candidates. Once again, the optimal combination of voter and candidate behavior constitutes an equilibrium. By Remark 3, equilibrium candidate behavior is responsive to vote totals, which implies that even if it is feasible to prevent candidates from deviating from platform policies chosen before the election, this is not desirable.

**Proposition 4 (Jury Theorem 4)** If candidates are uncommitted and truth motivated and for every $n$ the strategy vector $(v_{n}^{*}, y_{A,n}^{*}, y_{B,n}^{*}) \in V \times Y^2$ maximizes $E_{y_{n},z}[u(y_{n}); v, y_{A}, y_{B}]$ then (i) $(v_{n}^{*}, y_{A,n}^{*}, y_{B,n}^{*})$ is a perfect Bayesian equilibrium and (ii) $y_{n}^{*} \rightarrow_{p} z$.

As elections grow large, they provide more information. If uncertainty is binary, this makes candidates increasingly confident, and increasingly polarized, until in the limit, as
Part (ii) of Propositions 2 and 3 states, the winning candidate’s platform approaches the policy that is actually optimal. Part (ii) of Proposition 4 is even more impressive, because it applies even if uncertainty is quantitative: in the limit, electoral mandates become perfectly precise, steering the winning candidate to the best policy from within an entire continuum of possibilities.

Proposition 5 reevaluates the welfare consequences of voter abstention, this time assuming that candidates are uncommitted. As before, the optimal strategy combination constitutes an equilibrium, which by Theorem 5 involves some voter abstention.

**Proposition 5 (Jury Theorem 5)** If candidates are uncommitted and truth motivated and for every $n$ the strategy vector $(v_{n}^{*}, y_{A,n}^{*}, y_{B,n}^{*}) \in V \times \mathcal{Y}^2$ maximizes $E_{y_{n},z} \left[ u(y_{n}) ; v, y_{A}, y_{B} \right]$ then (i) $(v_{n}^{*}, y_{A,n}^{*}, y_{B,n}^{*})$ is a perfect Bayesian equilibrium and (ii) $y_{n}^{*} \rightarrow p z$.

The proof of Proposition 5 is essentially identical to that of Proposition 4, replacing $V$ with $V'$. Like Proposition 3, this can be understood in light of the observation that abstention provides a crude mechanism for transferring weight from poorly-informed votes to those that are better informed. With responsive candidates, an alternative intuition comes from viewing voters and candidates as senders and receivers in a communication game with no conflict of interest. In that sense, allowing abstention improves communication by increasing the number of available messages from two to three.

Proposition 6 considers the welfare consequence of increasing the number of candidates from two to four. Once again, the optimal strategy combination constitutes an equilibrium, in which case every candidate receives votes, by Theorem 6. Thus, it is not optimal for voters to coordinate on only two of the four candidates.

**Proposition 6 (Jury Theorem 6)** If candidates are uncommitted and truth motivated and for every $n$ the strategy vector $(v_{n}^{*}, (y_{j,n}^{*})_{j \in C''}) \in V'' \times \mathcal{Y}^4$ maximizes $E_{y_{n},z} \left[ u(y_{n}) ; v, (y_{j})_{j \in C''} \right]$ then (i) $(v_{n}^{*}, (y_{j,n}^{*})_{j \in C''})$ is a perfect Bayesian equilibrium and (ii) $y_{n}^{*} \rightarrow p z$.

The proof of Proposition 6 is essentially identical to that of Proposition 5, replacing $V \times \mathcal{Y}^2$ with $V'' \times \mathcal{Y}^4$. Like abstention, the benefit of multiple candidates can be viewed as a crude mechanism of placing greater weight on more precise signals. Alternatively, adding candidates improves communication, by raising the number of messages available to voters.\(^{38}\)

Together, Propositions 1 through 6 paint an unconventional picture of the merits and limits of democracy. The standard narrative is that political competition benefits voters, by

\(^{38}\)As McMurray (2013b) points out, multiple candidates may also be beneficial in higher dimensions, where with only two candidates, electoral margins can no longer identify the optimal policy.
fostering compromise between the extreme interests on the left and right. In that light, candidate polarization must be viewed as evidence of democratic failure, perhaps attributable to the extreme interests of candidates, extreme factions within the party’s base, or third parties. Somehow, these groups fail to recognize the competitive benefits of moderation. Voters cannot constrain candidates to implement moderate policies because campaign promises are unenforceable, or perhaps somehow because ideological moderates fail to vote in defense of their own interests. This model, by contrast, predicts almost the exact opposite: voters benefit from candidate polarization and flexibility, and from multiple parties, and abstention can be for a citizen’s own benefit.

6 Directions for Future Extension

6.1 Preference Heterogeneity

The model above assumes that voters all have identical preferences. Inevitably, however, public policies will affect different individuals differently. Moreover, while the empirical correlation between voting and income is not strong, it is also not zero. These considerations are important, because conflicts of interest sometimes eliminate results from pure common-value models. When voter utility depends both on a common value component and a private value component, for example, Feddersen and Pesendorfer (1999) show that large elections aggregate information effectively, but that this prompts citizens to vote on the basis of private values, rather than abstain. Bhattacharya (2013) shows that preference heterogeneity can also thwart information aggregation. In communication games such as Crawford and Sobel (1982), conflicts of interest reduce the ability to communicate credibly.

On the other hand, there are other ways to formulate preference heterogeneity, which dampen but do not eliminate the results above. Suppose, for example, that the policies $z_i$, $z_{i'}$, $z_j$ and $z_{j'}$ that are optimal for citizens $i$ and $i'$ and candidates $j$ and $j'$ are affiliated in the sense of Milgrom and Weber (1982). In that case, assuming that each citizen’s signal $s_i$ is affiliated with his own optimum $z_i$, citizens with positive and negative signals would still be inclined to support policies on the right and left, respectively, while those with moderate signals and those with overly noisy signals would both favor policies in the center. Those

---

39 Conceivably, the empirical importance of variables such as occupation and income for voting could also arise for informational reasons. For example, workers and managers might be more acutely aware of the pros and cons, respectively, associated with minimum wage laws, which could drive political differences independently from the incentives inherent to their positions. Similarly, Piketty (1995) and Alesina and Angeletos (2005) demonstrate how personal or local economic successes or failures might shape individuals' beliefs regarding the efficiency or inefficiency of wealth redistribution.
who lack expertise would be more inclined to vote than before, since peers may no longer share their interests, but some would still abstain, since their interests would still be affiliated with the signals of other voters. Similarly, conflicts between voters and candidates are likely to make the latter less responsive to electoral mandates, but not unresponsive.

As Section 1 indicates, voters seem to take into account the collective welfare of the electorate, in addition to their own private interests, when formulating political views. If so, then as McMurray (2013a) points out, voter differences that at first seem intractable may be substantially mitigated. For example, consider a standard formulation of altruism, in which a voter’s utility function $U_i = u_i + \alpha u_j$ places positive weight $\alpha$ on the well-being $u_j$ of each of his peers. This can be rewritten as a weighted average

$$U_i = (1 - \alpha) u_i + \alpha n \bar{u}$$

of his own well-being $u_i$ and the average well-being $\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i$ of the population. If all citizens are altruists, this second component is common across voters. Moreover, the weight on this common component is proportional to the size of the electorate, and so may be arbitrarily large, even if the altruism parameter $\alpha$ is close to zero.

Other sources of preference heterogeneity include the costs of voting and of acquiring information. As Downs (1957) and others point out, these may create a strong incentive for voters to free ride, either by casting uninformed votes or by not voting at all. Faravelli, Man, and Walsh (2012) show, however, that altruism formulated similarly to (9) together with the opportunity to influence electoral mandates help resolve the empirical puzzle of why citizens cast votes that are unlikely to be pivotal.\footnote{Many authors cite ethical motivations for costly voting, such as altruism or civic duty. Such motivations fit easily into a common-value framework such as this, since voting provides a bona fide public good. In private value settings, however, McMurray (2013a) points out that voting merely pulls policy outcomes in a voter’s own favor, so there is no clear ethical basis for voting or encouraging others to vote.}

### 6.2 Robust Disagreement

As Section 1 discusses, one strength of an information model is that it explains why political “preferences” drift over time, and why voters are able to persuade one another. As formulated above, however, voter preferences are more fragile than is realistic. With common preferences, common knowledge of the model’s structure, perfect cognition, and conditionally independent signals, that is, a voter who learns that he holds a minority opinion should immediately update his beliefs, and join the majority.\footnote{See McKelvey and Page (1986). This is related to Aumann’s (1976) well-known result that individuals cannot “agree to disagree”.

Empirically, voters are indeed
more confident in their opinions when the majority agrees with them, and even appeal to consensus wisdom in their efforts to persuade others. Nevertheless, it is also common for a minority to openly disagree with a majority consensus. An important direction for future work is therefore to explore mechanisms that sustain disagreement.

One obvious source of irreconcilability is preference heterogeneity, which Section 6.1 discusses. However, disagreements also persist on purely factual questions, where preferences are irrelevant, suggesting some sort of informational impediment as well.\textsuperscript{42} Geanakoplos (1989) and Brandenburger, Dekel, and Geanakoplos (1989) show that information structures which allow disagreement can be interpreted as cognitive errors such as forgetfulness or mistakes in using Bayes’ rule, and can be equivalently modeled simply as heterogeneous prior beliefs regarding model fundamentals. In the model above, there are various ways to specify heterogeneous beliefs. It may be, for example, that voters simply fail to update in response to others’ information, so that even though ideology is actually a matter of opinion, voters behave as if it were a preference parameter. In that case, the equilibrium analysis of Hotelling (1929) and Downs (1957) may apply, but welfare may at the same time follow the analysis of this paper. A simpler approach is to assume heterogeneous beliefs regarding the prior distribution of $z$, but by the logic of Blackwell and Dubins (1962), this would not appreciably change the analysis above, because the collective importance of private signals would outweigh any citizen’s prior.\textsuperscript{43} A third possibility is heterogeneous beliefs regarding the distributions of private signals, which Acemoglu, Chernozhukov, and Yildiz (2008) show can perpetuate disagreements about the state of the world. For example, voters (and candidates) might be overconfident about the accuracy of their own signals, as in Ortoleva and Snowberg (2012), or misinterpret information that contradicts their priors, as in Fryer, Harms, and Jackson (2013).

Extensions such as these may have important consequences for social welfare. In the present model, for example, voters and candidates only favor extreme policies when doing so is warranted by the available information, but with biased beliefs, they may be overly extreme. Properly specified, however, a generalized information structure is likely to produce similar behavior to the model above. Voters who lack confidence in their opinions, for example, will still be reluctant to support extreme policies or candidates, and will be more inclined to abstain in deference to their peers. Moderate policy stances will still give candidates a competitive electoral advantage, but those with confident, extreme opinions may still be reluctant to compromise, especially when emboldened by popular support.

\textsuperscript{42}For example, macroeconomists openly disagree over the true effects of economic stimulus, and forecast opposite trends in inflation or economic growth.

\textsuperscript{43}Blomberg and Harrington (2000) combine heterogeneous priors with public signals, emphasizing that voters with extreme and rigid prior beliefs are less sensitive to new information.
7 Conclusion

Political economic theory identifies two possible benefits of democracy. One is the ability to figure out answers to difficult questions: as Condorcet (1785) formalizes, many heads are better than one. The second is utilitarian, as in May (1952), Rae (1969), Taylor (1969), and Davis and Hinich (1968): helping the majority generally provides greater total utility than helping the minority. In the spatial setting of Hotelling (1929) and Downs (1957), helping the majority requires compromising between the extreme interests on the left and right. Happily, that is precisely the outcome of electoral competition in those models.

Given the empirical importance of ideology, existing literature focuses almost exclusively on preference aggregation, with information playing a secondary role, if any. As discussed above, however, voters do not appear simply to vote for whatever is in their own narrow interests. An information paradigm can explain why political “preferences” change over time, and why a citizen spends time and effort trying to persuade the opposition that the policies he supports would actually benefit them both. It also explains why someone who can’t decide what is optimal might abstain completely from voting or other political activities, implicitly deferring to his peers—or at least vote cautiously, avoiding either extreme in case his current opinion proves to be wrong. A voter is more confident when a large majority of the electorate agrees with him. So are candidates, which might explain why they often resist compromise, despite its apparent competitive advantage. Political compromise may be counter-productive, producing unattractive policies, such as a moderate-sized economic stimulus, which is known ex ante not to be optimal.

For simplicity, this paper maintains Condorcet’s (1785) assumptions that voters are perfectly rational and have identical preferences, that signals are free and are conditionally independent, that the model’s structure is common knowledge, and that voting is costless. Clearly, these assumptions constitute a best-case scenario for information aggregation. Section 6 conjectures that many of these assumptions can be relaxed somewhat without fundamentally changing the model’s predictions, but identifying conditions under which information aggregation succeeds or fails is an important direction for future research, where the present model can serve as a useful benchmark.

Another simplistic aspect both of this paper and of existing literature is that ideology is one-dimensional: in the real world, political decisions are complex and multi-faceted. McMurray (2013b) shows, however, that the information model above extends readily to multiple dimensions: in equilibrium, issues are bundled together so that voting appears one-dimensional, and exhibits all the patterns of the present analysis. This is important empirically, because Poole and Rosenthal (1997) and others have documented that political ideologies do appear highly one-dimensional. It is also important theoretically, because the
non-existence of equilibrium in multiple dimensions is a well known limitation of existing literature, and thus another important contribution of the model above.

Taken together, the evidence above suggests that, whether for altruistic or for other reasons, voters seem to aggregate preferences internally, so that a citizen’s ideology reflects not what he wants for himself, per se, but rather what he perceives to be best for the group. On the question of what is best, however, voters inevitably hold a myriad of opinions. What remains for democracy, then, is to aggregate these opinions to determine the truth.

A Appendix

A.1 Tables

<table>
<thead>
<tr>
<th>Information and Ideology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: Ideological Polarization</td>
</tr>
<tr>
<td>Information</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R squared</td>
</tr>
</tbody>
</table>

Table 1: Results of regression of ideological polarization on information.
Table 2: Results of regression of voter participation on information and ideological polarization.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Registered</th>
<th>Voted in primary</th>
<th>Voted</th>
<th>Completed ballot</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td>0.100</td>
<td>0.120</td>
<td>0.051</td>
<td>0.070</td>
</tr>
<tr>
<td>Polarization</td>
<td>0.022</td>
<td>0.052</td>
<td>0.018</td>
<td>0.021</td>
</tr>
<tr>
<td>Observations</td>
<td>1,999</td>
<td>1,712</td>
<td>1,734</td>
<td>1,528</td>
</tr>
<tr>
<td>Pseudo R square</td>
<td>0.191</td>
<td>0.063</td>
<td>0.068</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Source is 2008 ANES. Marginal effects are evaluated at average values of explanatory variables. Standard errors are in parentheses. *** indicate significance at the 1% level.

Table 3: Results of regression of political activism on information and ideological polarization.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Persuaded others</th>
<th>Signed petition</th>
<th>Attended protest</th>
<th>Contacted official</th>
<th>Endorsed candidate</th>
<th>Donated money</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td>0.112</td>
<td>0.097</td>
<td>0.071</td>
<td>0.099</td>
<td>0.058</td>
<td>0.092</td>
</tr>
<tr>
<td>Polarization</td>
<td>0.079</td>
<td>0.052</td>
<td>0.045</td>
<td>0.022</td>
<td>0.030</td>
<td>0.039</td>
</tr>
<tr>
<td>Observations</td>
<td>2,005</td>
<td>2,004</td>
<td>2,004</td>
<td>2,005</td>
<td>2,006</td>
<td>2,002</td>
</tr>
<tr>
<td>Pseudo R square</td>
<td>0.089</td>
<td>0.084</td>
<td>0.069</td>
<td>0.092</td>
<td>0.044</td>
<td>0.161</td>
</tr>
</tbody>
</table>

Source is 2008 ANES. Marginal effects are evaluated at average values of explanatory variables. Standard errors are in parentheses. ** and *** indicate significance at the 5% and 1% levels.

A.2 Proofs

**Lemma 1** If candidates are committed then there exists a Bayesian Nash equilibrium \( v^* \in V \) in the voting subgame. If \( x_A < x_B \) then \( v^* \) is a belief threshold strategy, with a threshold \( \tau^* \) that has the same sign as \( \tilde{x} \).

**Proof.** As a preliminary step, it is useful to derive the formulas for the pivot probabilities...
used in (6). Since \( N_A \) and \( N_B \) are independent Poisson random variables with means \( n(\cdot) \) and \( n(\cdot) \), the joint probability of vote totals \( N_A = a \) and \( N_B = b \) is

\[
\psi(a, b | z) = \frac{e^{-n(\cdot)(a + b)}}{a!b!} [n(\cdot)]^a [n(\cdot)]^b.
\]

(10)

In terms of (10), candidate A wins the election by a margin of exactly \( m \geq 0 \) votes (alternatively, B “wins” by \(-m \) votes) with probability

\[
\pi_A(m | z) = \pi_B(-m | z) = \sum_{k=0}^\infty \psi(k + m, k | z)
\]

(11)

and B wins by \( m \geq 0 \) votes (or A “wins” by \(-m \) votes) with probability

\[
\pi_B(m | z) = \pi_A(-m | z) = \sum_{k=0}^\infty \psi(k, k + m | z).
\]

(12)

Accordingly, candidate \( j \) takes office with probability

\[
\Pr(w = j | z) = \sum_{m=1}^\infty \pi_j(m | z) + \frac{1}{2} \pi_j(0 | z),
\]

(13)

where the second term reflects the event of winning the tie-breaking coin toss, and a vote for candidate \( j \) is therefore pivotal with probability

\[
\Pr(piv_j | z) = \frac{1}{2} \pi_j(0 | z) + \frac{1}{2} \pi_j(-1 | z).
\]

(14)

If \( x_A = x_B \) then voters of all types are indifferent between election outcomes, so any voting strategy constitutes a Bayesian Nash equilibrium. For the case of \( x_A < x_B \), it is straightforward to show that

\[
E(z | piv, q, s) = \frac{\int_\mathbb{R} \frac{\Pr(piv(z), q, s) 1_{(1+qs)}dz}{\mathbb{P}(piv(z), q, s) 1_{(1+qs)}dz}}{\mathbb{P}(piv(z), q, s) 1_{(1+qs)}dz} = \frac{\bar{x} - E(z | piv)}{E(z^2 | piv) - \bar{x}E(z | piv)}.
\]

This implies that (6) is negative when \( qs < \tau_{AB}^{br} \), so a citizen prefers to vote A, and positive when \( qs > \tau_{AB}^{br} \), so a citizen prefers to vote B. Therefore, the best response to any voting strategy is a belief threshold strategy, with threshold \( \tau_{AB}^{br} \).

If \( v \) is a belief threshold strategy with \( \tau > 0 \) then (5) reduces to (16), as in Lemma A1, below. If \( \tau < 0 \) then an analogous expression applies. Either way, \( \phi(j | z) \) is a continuous function of \( \tau \), and therefore so is (15), since (10) through (14) are continuous. Thus, the best response threshold \( \tau_{AB}^{br}(\tau) \) can be viewed as a continuous function from the compact
interval \([-1, 1]\) of thresholds into itself, and by Brower’s theorem, a fixed point \(\tau^* = \tau_{AB}^b(\tau^*)\) exists, which characterizes a belief threshold strategy \(\nu^*\) that is its own best response.

It remains to show that \(\tau^*\) has the same sign as \(\bar{x}\). To see this, first consider the case in which platforms \(x_A = -x_B\) are symmetric, so that \(\bar{x} = 0\). In that case, (15) reduces to
\[
\tau_{AB}^b(\tau) = -\frac{E(z|\nu)}{E(z^2|\nu)}\big|_{z=\bar{x}},
\]
which, by Lemma A1, has the opposite sign from \(\tau\). Also, for any value of \(\tau\), (15) is increasing in \(\bar{x}\), as the derivative \(\frac{\partial \tau_{AB}^b(\tau)}{\partial \bar{x}}\) has the same sign as
\[
[E(z^2|\nu) - \bar{x}E(z|\nu)] + E(z|\nu)E(\bar{x}) - E(z|\nu)^2 > 0.
\]

The result that 0 lies between \(\tau\) and \(\tau_{AB}^b(\tau; \bar{x})\) whenever \(\bar{x} = 0\), together with the result that \(\tau_{AB}^b(\tau; \bar{x})\) increases with \(\bar{x}\), implies that \(\tau_{AB}^b(\tau; \bar{x})\) is positive whenever \(\tau < 0 < \bar{x}\), and negative whenever \(\bar{x} < 0 < \tau\). A fixed point \(\tau^* = \tau_{AB}^b(\tau^*)\), therefore, cannot be negative when \(\bar{x} > 0\) or positive when \(\bar{x} < 0\); in other words, \(\tau^*\) and \(\bar{x}\) have the same sign. ■

**Lemma A1** If \(\nu\) is a belief threshold strategy with threshold \(\tau\) then \(\psi(k, k|z) - \psi(k, k|-z)\) (for any \(z > 0\)) and \(E(z|\nu)\) have the same sign as \(\tau\).

**Proof.** Let \(\tau > 0\) (analogous arguments apply if \(\tau \leq 0\)). In that case, \(\nu\) specifies that citizens vote \(A\) if \(q < \tau\) or \(s = -1\), or both, so expected vote shares \(\phi(j|z)\) reduce from (5) to the following.

\[
\begin{align*}
\phi (A|z) &= G(\tau) + \int_{\tau}^{1} (1 + qz) g(q) \, dq, \\
\phi (A|z) &= G(\tau) + \int_{\tau}^{1} (1 - qz) g(q) \, dq, \\
\phi (B|z) &= G(\tau) + \int_{\tau}^{1} (1 + qz) g(q) \, dq, \\
\phi (B|z) &= G(\tau) + \int_{\tau}^{1} (1 - qz) g(q) \, dq.
\end{align*}
\]

From these, it is straightforward to verify that \(\phi(A|z)\phi(B|z) > \phi(A|-z)\phi(B|-z)\), which implies that a \(k\)-vote tie is more likely in state \(z\) than in state \(-z\),

\[
\begin{align*}
\psi(k, k|z) - \psi(k, k|-z) &= \frac{e^{-n(\phi(A|z) - n(\phi(B|z)))k} (n\phi(A|z))^k (n\phi(B|z))^k - e^{-n(\phi(A|-z) - n(\phi(B|-z)))k} (n\phi(A|-z))^k (n\phi(B|-z))^k}{k!k!} \\
&= \frac{e^{-n(\phi(A|z) - \phi(B|z))k} ([\phi(A|z)\phi(B|z)]^k - [\phi(A|-z)\phi(B|-z)]^k)}{k!k!} > 0,
\end{align*}
\]

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implying in turn that a vote is more likely to be pivotal in state \( z \) than state \(-z\),

\[
\operatorname{Pr}(\text{piv}|z) - \operatorname{Pr}(\text{piv}|z) = \left\{ \left[ \frac{1}{2} \pi_A(0) - \frac{1}{2} \pi_A(1) \right] + \left[ \frac{1}{2} \pi_B(0) - \frac{1}{2} \pi_B(1) \right] \right\}
\]

\[= \left\{ \left[ \frac{1}{2} \pi_A(0) - \frac{1}{2} \pi_A(1) \right] + \left[ \frac{1}{2} \pi_B(0) - \frac{1}{2} \pi_B(1) \right] \right\}
\]

\[= \frac{1}{2} \sum_{k=0}^{\infty} \left\{ \psi(k, k|z) \left[ 1 + \frac{\phi(A|z)}{k+1} + 1 + \frac{\phi(B|z)}{k+1} \right] - \psi(k, k|z) \right\}
\]

\[= \frac{1}{2} \sum_{k=0}^{\infty} \left[ \psi(k, k|z) - \psi(k, k|z) \right] \left( 2 + \frac{1}{k+1} \right) > 0,
\]

and therefore that the conditional expectation of \( z \) is positive:

\[
E(z|\text{piv}) = \frac{\int_0^1 (z) \operatorname{Pr}(\text{piv}|z) f(z) + (-z) \operatorname{Pr}(\text{piv}|z) f(-z) dz}{\int_0^1 [\operatorname{Pr}(\text{piv}|z) f(z) + \operatorname{Pr}(\text{piv}|z) f(z)] dz}
\]

\[= \int_0^1 z [\operatorname{Pr}(\text{piv}|z) - \operatorname{Pr}(\text{piv}|z)] f(z) dz
\]

\[= \frac{\int_0^1 \operatorname{Pr}(\text{piv}|z) + \operatorname{Pr}(\text{piv}|z) f(z) dz}{\int_0^1 [\operatorname{Pr}(\text{piv}|z) + \operatorname{Pr}(\text{piv}|z)] f(z) dz}
\]

\[> 0.
\]

\[\blacksquare
\]

**Theorem 1 (Median Opinion Theorem)** If candidates are committed and office-motivated then \( (x^*_A, x^*_B, \sigma^*) \) is a perfect Bayesian equilibrium only if \( x^*_A = x^*_B = 0 \). Furthermore, such an equilibrium exists, in which \( \sigma^*(x^*_A, x^*_B) \) is a belief threshold strategy with \( \tau^* = 0 \).

**Proof.** By Lemma 1, there exist Bayesian Nash equilibrium voting strategies in the subgames associated with every pair \( (x_A, x_B) \in X^2 \) of candidate platforms. Let \( \sigma^* \) specify one such strategy in every subgame. In the case of \( x_A = x_B = 0 \), any voting strategy is consistent with equilibrium, so let \( \sigma^*(0, 0) = 0 \). By the symmetry of the model, this implies that \( \operatorname{Pr}(w = A) = \operatorname{Pr}(w = B) = \frac{1}{2} \). For any other platform pair, Lemma 1 implies that \( \sigma^*(x_A, x_B) \) specifies a belief threshold strategy, with \( \tau \) having the same sign as \( \bar{x} \). In that case, the candidate who is closer to zero is more likely to win the election: if \( \bar{x} > 0 \) and \( \tau > 0 \), for example, then (5) reduces to (16), so the total probability \( \phi(A) = \frac{1}{2} \phi(A|1) + \frac{1}{2} \phi(A|1) \) of a vote for \( A \) therefore exceeds the total probability \( \phi(B) = \frac{1}{2} \phi(B|1) + \frac{1}{2} \phi(B|1) \) of a vote for \( B \), and it is straightforward to show that \( \operatorname{Pr}(w = B) < \operatorname{Pr}(w = A) \). Thus, no candidate wishes to deviate when \( x_A^* = x_B^* = 0 \), and for any other platform pair, the candidate who is weakly more extreme can benefit from moving to origin.  

\[\blacksquare
\]
**Lemma 2** If candidates are committed and truth motivated then \((x_A^*, x_B^*, v^*)\) is a Bayesian Nash equilibrium only if \(x_j^* = E(z|w = j)\) for \(j = A, B\).

**Proof.** The expectation of (2) can be rewritten as

\[
E_{z,w} [u(x_w)] = E_z \left\{ \sum_{j \in C} \left[ -(x_j - z)^2 \right] Pr(w = j|z) \right\},
\]

(17)

where \(Pr(w = j|z)\) depends implicitly on \(v\) through its dependence on \(\phi(j|z), \psi(a,b|z),\) and \(\pi_j(m|z)\). Differentiating with respect to \(x_j\) therefore yields

\[
2E_z [Pr(w = j)(z - x_j)] = 2Pr(w = j)[E(z|w = j) - x_j],
\]

which is zero if and only if \(x_j = E(z|w = j)\). The second derivative \(-2Pr(w = j)\) is negative, establishing this as a maximum. ■

**Theorem 2** (Candidate Polarization) If candidates are committed and truth motivated and \(v^*\) is informative then \((x_A^*, x_B^*, v^*)\) is a Bayesian Nash equilibrium only if (i) \(x_j^* = E(z|w = j)\) for \(j = A, B\), where \(x_A^* < 0 < x_B^*\), and (ii) \(v^*\) is a belief threshold strategy. Furthermore, such an equilibrium exists, with \(x_A^* = -x_B^*\) and \(\tau^* = 0\).

**Proof.** That \(x_j^* = E(z|w = j)\) follows from Lemma 2. This expectation is a weighted average \(E_{(N_A, N_B)}[E(z|N_A, N_B)|w = j]\) of the expectations \(E(z|N_A, N_B)\) associated with particular vote totals. The symmetry of the model is such that \(E(z|N_A, N_B) = 0\) whenever \(N_A = N_B\), and this together with Lemma 3 below implies that \(E(z|N_A, N_B)\) is positive when \(N_A < N_B\) and negative when \(N_A > N_B\). The event of \(w = A\) is a composition of events in which \(N_A \geq N_B\), so \(E(z|w = A) < 0\). Similarly, \(E(z|w = A) > 0\) when \(w = B\). That \(v^*\) is a belief threshold strategy then follows from Lemma 1, since platforms are distinct.

If \(\tau^* = 0\) then (16) reduces so that \(\phi(A|z) = \phi(B|z)\) for any \(z\). From (10) through (15), this implies that \(\psi(a,b|z) = \psi(b,a|z)\), \(\pi_A(m|z) = \pi_B(m|z)\), \(Pr(piv_A|z) = Pr(piv_B|z)\), \(E(z|piv) = 0,\) and therefore \(\tau_{AB}^{br} = 0\). Also, \(Pr(w = A|z) = Pr(w = B|z)\), implying that \(x_A^* = E(z|w = A) = -E(z|w = B) = -x_B^*\). ■

**Theorem 3** (Swing voter’s curse) If candidates are committed and policy-motivated and \(v^* \in V'\) is informative then \((x_A^*, x_B^*, v^*)\) is a Bayesian Nash equilibrium only if (i) \(x_j^* = E(z|w = j)\) for \(j = A, B\), where \(x_A^* < 0 < x_B^*\), and (ii) \(v^*\) is a belief threshold strategy with \(\tau_1^* < \tau_2^*\). Such an equilibrium exists, with \(x_A^* = -x_B^*\) and \(\tau_1^* = -\tau_2^*\).

**Proof.** If candidates choose distinct platforms \(x_A < x_B\) and a citizen’s peers vote according to the strategy \(v\) then, given his private information \((q,s) \in Q \times S\), the difference \(\Delta_{AB}(q,s)\)
in expected utility between voting $B$ and voting $A$ is given by (6) as before, and is positive if and only if $qs$ exceeds (15). Similarly, the benefit of voting $B$ instead of abstaining is given by,

$$
\Delta_{0B} (q, s) = \int_{\mathcal{Z}} [u(x_B|z) - u(x_A|z)] \Pr (piv_B|z) \frac{1}{2} (1 + qsz) dz
$$

$$
= 4(x_B - x_A) \left[ E(z|piv_B, q, s) - \bar{x} \right] \Pr (piv_B, q, s),
$$

which is positive if and only if $qs$ exceeds

$$
\tau_{0B}^{br} = \frac{\bar{x} - E(z|piv_B)}{E(z^2|piv_B) - \bar{x}E(z|piv_B)},
$$

and the benefit of abstaining instead of voting $A$ is given by

$$
\Delta_{A0} (q, s) = \int_{\mathcal{Z}} [u(x_B|z) - u(x_A|z)] \Pr (piv_A|z) \frac{1}{2} (1 + qsz) dz
$$

$$
= 4(x_B - x_A) \left[ E(z|piv_A, q, s) - \bar{x} \right] \Pr (piv_A, q, s),
$$

and is positive if and only if $qs$ exceeds

$$
\tau_{A0}^{br} = \frac{\bar{x} - E(z|piv_A)}{E(z^2|piv_A) - \bar{x}E(z|piv_A)}.
$$

Thus, the best response to $v$ is a belief threshold strategy, with thresholds $\tau_1^{br} = \min \{ \tau_{A0}^{br}, \tau_{AB}^{br} \}$ and $\tau_2^{br} = \max \{ \tau_{0B}^{br}, \tau_{AB}^{br} \}$.

To see that $\tau_1 < \tau_2$ in equilibrium, consider a belief threshold strategy in which no one abstains: $\tau_1 = \tau_2 \equiv \tau$. In this case, the expected state is higher when an $A$ vote is pivotal than when a $B$ vote is pivotal. This is because $E(z|N_A = k, N_B = k + 1, q, s) > E(z|N_A = N_B = k, q, s)$ for any $k$ (by Lemma 3), implying that $E(z|N_B = N_A + 1, q, s) > E(z|N_A = N_B, q, s)$. Since $E(z|piv_A, q, s)$ is a weighted average of these two expectations, this implies that $E(z|piv_A, q, s) > E(z|N_A = N_B, q, s)$. Similarly, $E(z|piv_B, q, s) < E(z|N_A = N_B, q, s)$, and by transitivity $E(z|piv_B, q, s) < E(z|piv_A, q, s)$. If a voter is indifferent between voting $A$ and $B$, it must be that $\Delta_{AB} (q, s) = \Delta_{A0} (q, s) + \Delta_{0B} (q, s) = 0$, implying that $\Delta_{A0} (q, s)$ and $\Delta_{0B} (q, s)$ have opposite signs. Specifically, $E(z|piv_B, q, s) < E(z|piv_A, q, s)$ implies that $\Delta_{A0} (q, s) > 0 > \Delta_{0B} (q, s)$, as (18) and (20) make clear. But this is equivalent to $\tau_{A0}^{br} < qs < \tau_{0B}^{br}$, implying that the threshold pair $(\tau, \tau)$ does not characterize its own best response.

If $\tau_1 = -\tau_2$ then the symmetry of the model is such that $\phi (A|z) = \phi (B|z)$, $\psi (a, b|z) = \psi (b, a|z)$, $\pi_A (m|z) = \pi_B (m|z)$, and $\Pr (piv_A|z) = \Pr (piv_B|z)$ for all $z$, and therefore $E(z|piv_A) = -E(z|piv_B)$ and $E(z^2|piv_A) = E(z^2|piv_B)$, implying that $\tau_{A0}^{br} = -\tau_{0B}^{br}$. Since the best response to a belief threshold strategy with thresholds $(-\tau_2, \tau_2)$ is another with
thresholds \((-\tau_{0B}^x, \tau_{0B}^x)\), (19) can be reinterpreted as a continuous function from the compact set \([0, 1]\) of positive thresholds into itself. A fixed point \(\tau^*\) exists by Brouwer’s theorem, and the threshold pair \((-\tau^*, \tau^*)\) characterize a belief threshold strategy \(v^*\) that is its own best response. \(\Pr(w = A|z) = \Pr(w = B|z)\) for any \(z\) in that case as well, implying that \(x_A^* = E(z|w = A) = -E(z|w = B) = -x_B^*\). ■

**Lemma 3 (Monotone Expectations)** If \(v \in \mathcal{V}\) is informative then \(\hat{z}_{a,b}\) and \(E(z|N_A = a, N_B = b, q, s)\) are decreasing in \(a\) and increasing in \(b\).

**Proof.** If \(v \in \mathcal{V}\) is informative then it is straightforward to verify from (5) that \(\phi(A|z)\) and \(\phi(B|z)\) are decreasing and increasing in \(z\), respectively. Accordingly,

\[
E(z|N_A = a, N_B = b + 1, q, s) = \frac{\int z\psi(a, b + 1|z) f(z|q, s) \, dz}{\int \psi(a, b + 1|z) f(z|q, s) \, dz} > \frac{\int z\psi(a, b|z) \frac{\phi(B|z)}{b + 1} f(z|q, s) \, dz}{\int \psi(a, b|z) \frac{\phi(B|z)}{b + 1} f(z|q, s) \, dz} = E(z|N_A = a, N_B = b, q, s)
\]

for any \(a, b \in \mathbb{Z}_+\). That \(E(z|N_A = a + 1, N_B = b, q, s) < E(z|N_A = a, N_B = b, q, s)\) follows from a symmetric argument. The special case of \(q = 0\) proves that \(\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}\). ■

**Theorem 4 (Mandates)** If candidates are truth motivated and uncommitted and \(v^*\) is informative then \((v^*, y_A^*, y_B^*)\) is a perfect Bayesian equilibrium only if (i) \(v^*\) is a belief threshold strategy with \(-1 < \tau^* < 1\) and (ii) \(y_j^*(a, b) = \hat{z}_{a,b}\) for all \(a, b \in \mathbb{Z}_+\) and \(j = A, B\), where \(\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}\). Furthermore, such an equilibrium exists, with \(\tau^* = 0\) and \(\hat{z}_{a,b} = -\hat{z}_{b,a}\) for all \(a, b \in \mathbb{Z}_+\).

**Proof.** For candidates, optimal policy functions are given by Remark 3. That \(\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}\) follows from Lemma 3. To derive optimal voting behavior, first rewrite equation (8) as

\[
\Delta_{AB}(q, s) = E_{N_A, N_B}\left\{E_z\left[\left(\frac{\hat{z}_{N_A, N_B + 1} - \hat{z}_{N_A + 1, N_B}}{2}\right) [N_A, N_B, q, s]\right]\right\} = E_{N_A, N_B}\left\{\frac{\hat{z}_{N_A, N_B + 1} - \hat{z}_{N_A + 1, N_B}}{2} [E(z|N_A, N_B, q, s) - \frac{\hat{z}_{N_A + 1, N_B} + \hat{z}_{N_A, N_B + 1}}{2}]\right\}.
\]

(22)
\( E(z|N_A, N_B, q, s) \) is increasing in \( qs \) for any vote totals \( N_A \) and \( N_B \), so (8) increases in \( qs \) as well. This implies the existence of a belief threshold \( \tau_{AB}^{br} \in [-1, 1] \), such that (8) is positive if and only if \( qs > \tau_{AB}^{br} \). Thus, \((v^*, y_A^*, y_B^*)\) is a perfect Bayesian equilibrium only if \( v^* \) is a belief threshold strategy.

To see that \( \tau_{AB}^* < 1 \) in equilibrium, suppose to the contrary that \( \tau_{AB} = 1 \). In that case, a candidate infers that \( qs = 1 \) exactly for any \( B \) voter, since all other types vote \( A \). In other words, \( \hat{z}_{N_A,N_B+1} = E(z|N_A, N_B, q, s) \). From the voter’s perspective, then, the candidate responds optimally to his information, and (22) simplifies to

\[
\Delta_{AB}(q, s) = E_{N_A,N_B} \left[ (\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A+1,N_B}) \left( \hat{z}_{N_A,N_B+1} - \frac{\hat{z}_{N_A+1,N_B} + \hat{z}_{N_A,N_B+1}}{2} \right) \right]
\]

\[
= \frac{1}{2} E_{N_A,N_B} \left[ (\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A+1,N_B})^2 \right]
\]

\[
> 0.
\]

That \( \tau_{AB}^* > -1 \) in equilibrium follows from a symmetric argument.

It is straightforward to show that if voting is symmetric (i.e. \( \tau_{AB} = 0 \)) then candidates’ responses to vote totals are symmetric, as well (i.e. \( \hat{z}_{a,b} = -\hat{z}_{b,a} \) for all \( a, b \in \mathbb{Z}_+ \)). This produces symmetric voter incentives (i.e. \( \tau_{AB}^{br} = 0 \)), so that the same voting strategy is its own best response, thus constituting an equilibrium. \( \blacksquare \)

**Theorem 5 (Strategic abstention)** If candidates are truth motivated and uncommitted and \( v^* \in \mathcal{V}' \) is informative then \((v^*, y_A^*, y_B^*)\) is a perfect Bayesian equilibrium only if (i) \( v^* \) is a belief threshold strategy with \(-1 < \tau_1^* < 0 < \tau_2^* < 1 \) and (ii) \( y^*_j (a, b) = \hat{z}_{a,b} \) for all \( a, b \in \mathbb{Z}_+ \) and \( j = A, B \), with \( \hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1} \). Furthermore, such an equilibrium exists, with \( \tau_1^* = -\tau_2^* \) and \( \hat{z}_{a,b} = -\hat{z}_{b,a} \) for all \( a, b \in \mathbb{Z}_+ \).

**Proof.** Analogous to (22), the expected benefit of voting \( B \) instead of abstaining can be written as

\[
\Delta_{0B}(q, s) = E_{N_A,N_B} \left[ (\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A,N_B}) \left( E(z|N_A, N_B, q, s) - \frac{\hat{z}_{N_A,N_B} + \hat{z}_{N_A,N_B+1}}{2} \right) \right],
\]

which is increasing in \( qs \) since \( E(z|N_A, N_B, q, s) \) is increasing in \( qs \), and since \( \hat{z}_{N_A,N_B+1} > \hat{z}_{N_A,N_B} \) for any \( N_A, N_B \in \mathbb{Z}_+ \). An analogous expression for \( \Delta_{00}(q, s) \) is increasing in \( qs \), as well. Together, this implies the existence of belief thresholds \( \tau_1^{br} = \min \{ \tau_{AB}^{br}, \tau_{AB}^{br} \} \) and \( \tau_2^{br} = \max \{ \tau_{AB}^{br}, \tau_{AB}^{br} \} \) such that a citizen votes \( A \) if \( qs < \tau_1^{br} \) and votes \( B \) if \( qs > \tau_2^{br} \).

To see that \( \tau_1^{br} < 0 < \tau_2^{br} \) so that abstention occurs in equilibrium, consider a citizen with no expertise, so that \( q = 0 \). Such a citizen possesses no more information than a candidate, so for any vote pair of vote totals, he prescribes the same policy that a candidate would
choose on her own: \( E(z|N_A, N_B, q, s) = E(z|N_A, N_B) = \hat{z}_{a,b} \). Accordingly, (23a) reduces as follows.

\[
\Delta_{0B}(q, s) = E_{N_A, N_B} \left[ (\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A,N_B}) \left( \hat{z}_{N_A,N_B} - \frac{\hat{z}_{N_A,N_B} + \hat{z}_{N_A,N_B+1}}{2} \right) \right]
\]

\[
= -\frac{1}{2} E_{N_A, N_B} \left[ (\hat{z}_{N_A,N_B+1} - \hat{z}_{N_A,N_B})^2 \right]
\]

< 0.

Thus, \( \tau_{0B}^{br} > 0 \), and by a similar derivation, \( \tau_{A0}^{br} < 0 \). That \( \tau_{0B}^{br} < 1 \) and \( \tau_{A0}^{br} > -1 \) follow from logic analogous to that of Theorem 4.

**Theorem 6 (Multiple candidates)** If candidates are truth motivated and uncommitted, voter abstention is allowed, and \( v^* \in V'' \) is informative, then \( (v^*, y_A, y_B, y_C, y_D) \) is an equilibrium only if (i) \( v^* \) is a belief threshold strategy with \(-1 < \tau_1^* < \tau_2^* < \tau_3^* < \tau_4^* < 1\) and (ii) \( y_j^*(a, b, c, d) = \hat{z}_{a,b,c,d} \) for all \( a, b, c, d \in \mathbb{Z}_+ \) and for \( j = A, B, C, D \), with \( \hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1} \). Furthermore, such an equilibrium exists, with \( y_A^*(a, b, c, d) = -y_D^*(d, c, b, a) \), \( y_B^*(a, b, c, d) = -y_C^*(d, c, b, a) \), \( \tau_1^* = -\tau_4^* \), and \( \tau_2^* = -\tau_3^* \).

**Proof.** This proof is the same as those of Theorems 4 and 5, with only minor modifications. With four candidates, the outcome probabilities in (10) extend naturally to

\[
\psi(a, b, c, d|z) = \prod_{j=a,b,c,d} e^{-n\phi(jz)} \frac{e^{-n\phi(jz)} - [n\phi(jz)]^j}{j}
\]

(24)

Since \( v^* \) is informative, \( \hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1} \) follows from logic identical to that of Lemma 3. The expected benefit \( \Delta_{CD}(q, s) \) of voting \( D \) instead of \( C \) is analogous to (22),

\[
\Delta_{CD}(q, s) = E_{(N_j)_{j \in C''}} \left\{ E_z \left[ \left( \hat{z}_{N_A,N_B,N_C,N_D+1} - \hat{z}_{N_A,N_B,N_C+1,N_D} \right) \left( z - \hat{z}_{N_A,N_B,N_C+1,N_D} + \hat{z}_{N_A,N_B,N_C,N_D+1} \right) \right] \right\}
\]

\[= E_{(N_j)_{j \in C''}} \left\{ \left( \hat{z}_{N_A,N_B,N_C,N_D+1} - \hat{z}_{N_A,N_B,N_C+1,N_D} \right) \left( z - \hat{z}_{N_A,N_B,N_C+1,N_D} + \hat{z}_{N_A,N_B,N_C,N_D+1} \right) \right\}, \quad (25)
\]

and is increasing in \( qs \) since \( E_z(z|N_A, N_B, N_C, N_D, q, s) \) is increasing in \( qs \). Thus, (25) is positive if and only if \( qs \) exceeds a threshold \( \tau_{CD}^{br} \). Similar thresholds \( \tau_{j,j'}^{br} \) exist for every \( j, j' \in C''' \), and best-response thresholds can be constructed from these. For example, \( \tau_4^{br} = \max \{ \tau_{AD}^{br}, \tau_{BD}^{br}, \tau_{CD}^{br} \} \).

The inequalities in (i) are more numerous than before, but are strict for the same reason as in Theorem 5. Suppose, for example, that \( \tau_3 = \tau_4 \), so that a citizen votes \( C \) with
zero probability. In that case, a citizen whose ideology $qs$ coincides exactly with the two thresholds can perfectly convey his private information to the winning candidate, by voting $C$. This he prefers to do, since the winning candidate shares his preferences, and will utilize his information optimally. By continuity, then, citizens with $qs$ sufficiently close to the thresholds prefer to vote $C$, as well, implying that $\tau_{AC}^{br}, \tau_{BC}^{br}, \tau_{0C}^{br} < \tau_3$ and $\tau_{CD}^{br} > \tau_3$, so the proposed strategy is not an equilibrium.

As in the results above, if $\tau_1 = -\tau_4$ and $\tau_2 = -\tau_3$ and candidates respond optimally then $y_A^* (a, b, c, d) = -y_D^* (d, c, b, a)$ and $y_B^* (a, b, c, d) = -y_C^* (d, c, b, a)$, which in turn implies that $\tau_{AB}^{br} = -\tau_{CD}^{br}$ and $\tau_{B0}^{br} = -\tau_{CD}^{br}$, so $\tau_1^{br} = -\tau_4^{br}$ and $\tau_2^{br} = -\tau_3^{br}$. The best response to the belief threshold strategy characterized by $(-\tau_4, -\tau_3, \tau_3, \tau_4)$ is the belief threshold strategy characterized by $(-\tau_4^{br}, -\tau_3^{br}, \tau_3^{br}, \tau_4^{br})$, so the threshold pair $(\tau_3^{br}, \tau_4^{br})$ can be viewed together as a single continuous function from the compact set $\{(\tau_3, \tau_4) : 0 \leq \tau_3 \leq \tau_4 \leq 1\}$ of threshold pairs into itself. A fixed point exists by Brouwer’s theorem, which defines a belief threshold strategy that is its own best response.  

**Proposition 1 (Jury Theorem)** If candidates are committed to platforms $x_A, x_B \in X$ and for every $n$ the voting strategy $v_n^* \in V$ maximizes $E_{y_n, z} [u (y_n); v]$ then (i) $v_n^*$ is a Bayesian Nash equilibrium, and (ii) if $Z = Z_B$ then $y_n^* \rightarrow_p \arg \max_{x \in \{x_A, x_B\}} u (x|z)$.

**Proof.** Part (i) follows from McLennan (1998): since voters share a common interest and the strategy $v_n^*$ is socially optimal, it is also individually optimal, and thus constitutes an equilibrium.

To see Part (ii), let $v'$ be the (potentially non-equilibrium) belief threshold strategy with $\tau = 0$, and for every $n$ let $y_n'$ denote the policy outcome produced by this strategy. Voting is informative in that case, so $z = 1$ and $x_A < x_B$ together imply, by the law of large numbers, that $\lim_{n \rightarrow \infty} \Pr (w = A|z) = -1; v') = -1$. Similarly, $\lim_{n \rightarrow \infty} \Pr (w = B|z = 1; v') = 1$. This implies that $y_n' \rightarrow_p \arg \max_{x \in \{x_A, x_B\}} u (x|z)$, and since $y_n^*$ provides weakly greater welfare than $y_n'$, (ii) holds as well. 

**Proposition 2 (Jury theorem 2)** If candidates are committed and for every $n$ the strategy vector $(x_{A,n}^*, x_{B,n}^*, v_n^*) \in X^2 \times V$ maximizes $E_{y_n, z} [u (y_n); x_A, x_B, v]$ then (i) if candidates are truth motivated then $(x_{A,n}^*, x_{B,n}^*, v_n^*)$ is a Bayesian Nash equilibrium, and (ii) if $Z = Z_B$ then $y_n^* \rightarrow_p z$.

**Proof.** Part (i) follows from McLennan (1998): since voters and candidates have common values and the strategy vector $(x_{A,n}^*, x_{B,n}^*, v_n^*)$ is socially optimal, it is also individually optimal, and thus constitutes an equilibrium.
To see Part (ii), define $v'$ and $y_{n}'$ as in the proof of Proposition 1. Voting is informative in that case, so $z = 1$ and $x_A < x_B$ together imply, by the law of large numbers, that $\lim_{n \to \infty} \Pr (w = A | z = 1; v') = 1$. Similarly, $\lim_{n \to \infty} \Pr (w = B | z = 1; v') = 1$. Therefore, $\lim_{n \to \infty} E (z | w = A; v') = 0$ and $\lim_{n \to \infty} E (z | w = B; v') = 1$. By Lemma 2, these expectations are the platforms $x_A^{br}$ and $x_B^{br}$ candidates adopt in response to $v'$, so $\lim_{n \to \infty} x_{A,n}^{br} = -1$ and $\lim_{n \to \infty} x_{B,n}^{br} = 1$. Since candidate platforms diverge toward the extremes of the policy interval and the election outcome follows the state, $y_{n}' \to_p z$. Since $y_{n}'$ provides weakly greater welfare than $y_{n}^*$, (ii) holds as well. $\blacksquare$

Proposition 4 (Jury Theorem 4) If candidates are uncommitted and truth motivated and for every $n$ the strategy vector $(v_{n}', y_{A,n}^*, y_{B,n}^*) \in V \times Y^2$ maximizes $E_{y_{n}, z} [u (y_{n}) ; v, y_A, y_B]$ then (i) $(v_{n}', y_{A,n}^*, y_{B,n}^*)$ is a perfect Bayesian equilibrium and (ii) $y_{n}' \to_p z$.

Proof. Part (i) follows from McLennan (1998), as in the proof of Proposition 2. By Remark 3, this implies that $y_{j,n}' = \bar{z}_{a,b}$ for all $n$ and for $j = A, B$. Define $v'$ as in the proof of Proposition 1, and let $y_{n}'$ denote the policy outcome when citizens follow $v'$ and candidates follow $y_{j,n}'$. As $z$ increases from $-1$ to 1, the distribution $h (s | z)$ of signals increases in the sense of first-order stochastic dominance, so the expected vote shares $\phi (A | z)$ and $\phi (B | z)$ of candidates $A$ and $B$ decrease and increase, respectively, implying that the expected margin $\frac{\phi (A | z)}{\phi (A | z) + \phi (B | z)}$ increases as well. As the electorate grows large, the actual margin $\frac{N_A}{N_A + N_B}$ converges in probability to $\frac{\phi (A | z)}{\phi (A | z) + \phi (B | z)}$, thus perfectly revealing $z$. The policy $y_{n}'$ that the winning candidate chooses to implement in response to vote totals, therefore, converges in probability to $z$ and welfare converges to zero, which is the maximum possible. The vector $(v_{n}', y_{A,n}^*, y_{B,n}^*)$ is weakly better, which is only possible if (ii) holds as well. $\blacksquare$

References


