Contagion in an Economy with Heterogeneous Beliefs

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February 2014

Abstract

To the light to the financial crisis of 2007-2008, this article examines contagion from the perspective of emerging markets economies. Following Gertler and Kiyotaki (2010) and Cogley et al. (2012), I develop a theoretical model with heterogeneous beliefs that represents the case of a small economy, which is partially integrated to the rest of the world. When the economy is liquidity constrained and the rest of the world is fully integrated, I find that a bad news about the state of the international economy can lead to abrupt falls in foreign credits, to a rise in foreign interest rates, and to a change in agents’ beliefs. Although these three situations may have adverse effects in the economy, the distress can be exacerbated due to a discrepancy in beliefs.

JEL CLASSIFICATION : E44, D53, F34, F44

Key Words: Contagion, investment decisions, volatility, diverse beliefs

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1 Introduction

Economists have attributed the crisis of 2007-2008 to a combination of factors: highly leveraged financial institutions, hugely expansionary monetary policy in earlier years, and large drops in asset prices (Caballero and Kurlat, 2009; Gertler and Kiyotaki, 2010; Faia, 2010). Nevertheless, it is not completely clear how mechanisms spread contagion across financial markets and between those markets and the real economy.

Before the crisis, many theories predicted that financial crises resulted from incomplete financial markets. Scholars thus concluded that less developed financial markets were more prone to suffering distress, while more integrated markets, possessed of better liquidity and risk sharing, were expected to be less exposed to contagion. The facts showed the opposite. The crisis arose in the U.S., which ranked first in global indices of financial development (Mendoza et al., 2008) and quickly spread to the main financial centers of Europe, which were highly integrated with the U.S. financial market (see Figure 1). Considering that financial integration has increased over the past several decades in developed and developing countries, one might expect that most or all countries should have been exposed equally to contagion. However, with few exceptions, financial markets in developing countries did not suffer much distress and few were as troubled as Europe and the U.S. A puzzle thus results: Contrary to expectations, did incomplete markets benefit from their lower degree of financial integration? More generally, what transmission mechanisms explain these unexpected observations?

The motivation of this paper is to examine the contagion mechanisms in economies with different degrees of financial integration. The recent literature provides diverse explanations for contagion in financial markets. Most of this literature assumes the perspective of mature economies, in particular, the cases of the U.S. and Europe. Comparatively few scholars consider the perspectives of emerging economies, whose financial markets are partially integrated with global markets. To understand spillover from the point of view of emerging economies is crucial, both at the level of individual countries and across borders. In the past, crises in developing countries were connected to imbalances in mature economies. For example, the boom of real estate in Japan during the 1990s created conditions for the Asian crisis of 1997-1998. On the other hand, distress in emerging economies during the 1980s and 1990s spread to mature economies.

Even though contagion was less for developing than for developed countries in the recent crisis, most countries suffered large capital losses. In addition, developing countries have been exposed to "second round" effects caused by the debt crisis in Europe after 2009.

In an increasingly integrated world, spillover mechanisms move in different directions. Developing countries need foreign capital to develop, but closer larger financial integration with global markets might be risky. Are capital controls

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1Mendoza et al. (2008).
2See, for example, Caballero and Krishnamurthy (2008), Geanakoplos (2009).
or more financial regulation the solution to this dilemma? What is the role of financial structure?

To answer these questions I develop a theoretical model in the spirit of macro models of general equilibrium (Cooley and Prescott, 1995; Ljungvist and Sargent, 2000) in which agents have different beliefs about the return of the risky assets. I consider the case of a small economy, financially integrated with the rest of the world, whose domestic agents have different beliefs (and thus different risk perceptions) about the state of the foreign economy.

The introduction of heterogeneities in macro models as analytical tools has been common in the literature during the past two decades, especially with regards to the behavior of financial markets. Considering agents with different attitudes toward risk, heterogeneous discount factors and endowments of wealth, and idiosyncratic productivity shocks, authors explain phenomena such as banking crises, bubbles, and financial crashes that spillover to the real economy. The impetus for modeling agents with heterogeneous beliefs comes from the observation that big crashes in financial markets seem to be driven by some “irrational behavior” or “exuberances”. For example, Caballero and Krishnamurthy (2008) attribute large flight to quality episodes to the behavior of heterogeneous agents dominated by great uncertainty about the return of risky assets. The more agents differ in their beliefs, the more “irrational” their market decisions seem to be. Geanakoplos (2009) on the other hand, points to the leverage cycle3, suggesting that the combination of large leverage and differences in beliefs might exacerbate contagion. Similarly, Cao (2012) and Cogley et al. (2012) attribute large volatility in financial markets to the behavior of agents with heterogeneous beliefs. When information about risk and the state of the economy differ across agents, the economy can experience wealth distributions in equilibrium, which contribute to generate more distress. In both cases, the role of financial structure matters. When financial markets are complete, agents with incorrect beliefs lose wealth and exit the market. In contrast, when markets are financially constrained, these agents survive and can accumulate wealth, creating asset prices volatility.

Allen and Gale (1998) and Geanakoplos (2009) on the other hand, prove that the both type of market structure in financial markets and the way to complete the markets matter for contagion. In this paper, I consider financial structure to be given by a country’s degree of financial integration with the rest of the world. I then examine contagion under different types of financial structures: a highly integrated economy, a partially integrated economy and a relatively non-integrated economy. My contribution is in providing the perspective of an emerging market economy and reducing the bridge between the macro and the micro perspective. To continue with this research, I analyze the contagion literature in the next section. In Section Three I develop a theoretical model for contagion. Section Four is devoted to examine contagion for the small economy and in Section Five, I conduct a numerical exercise. Section Six concludes.

3Where asset prices tend to explode during economic expansions and tend to collapse in contraction. Consequently, leverage increases in times of booms and decreases in slumps.
2 The Literature of Contagion

There is a vast literature explaining contagion in financial markets, both from the micro and the macro perspective. The micro literature examines how distress propagates across financial markets; the macro literature seeks to understand the mechanisms allowing for spillover between financial markets and the real economy.

The most representative micro literature is Diamond and Dyvig (1983), Glosten and Milgrom (1985), Bikhchandani et al. (1992), Avery and Zemsky (1998), Allen and Gale (1998a,b,c), Geanakoplos (2003), Fostel and Genakopolos (2008), and Geanakoplos (2009). A good summary of the macro literature can be found in Bernanke and Gertler (1989), Bernanke et al. (1996), Kiyotaki and Moore (1997), Holmstrom and Tirole (1998), Corsetti et al. (1998); Bernanke et al. (1999), Caballero and Khishnamurthy (2008), Faia (2010), and Gertler and Kiyotaki (2010). In most cases, authors attribute crises and contagion to problems such as imperfect information, non-linearities and other market failures that diverts the economy from optimal allocations

2.1 The Microeconomic Perspective of Contagion

Diamond and Dyvig (1983) focus on bank runs and attribute contagion to uncertainty created about the soundness of the financial system. If agents have different discount factors, they can take advantages of this heterogeneity, withdrawing money from their bank accounts according to their liquidity needs. However, when the behavior of agents is driven by self-fulfilling prophecies, a liquidity shock may trigger a bank run, with spillover effects affecting the entire system. Bikhchandani et al. (1992) explain crowd behavior during booms and busts by arguing that periods of high uncertainty can end in informational cascades if agents distrust the quality of their private information. Thus, it can be optimal for agents to ignore their own information about the state of the market and follow the behavior of others (convergence in behavior). Even though informational cascades do not explain crises, they help understand contagion during great financial distress. Similarly, Avery and Zemsky (1998) relate the phenomena of herding and bubbles in financial markets to different types of uncertainty: about the value of assets, about the frequency of crashes, and about the accuracy of traders information (composition uncertainty). For them, the type of market uncertainty seems to be key for explaining the severity of contagion. Whereas Diamond and Dyvig (1983) consider crises to be like “sunspots”, in which panic is triggered without connection to any economic facts, the literature on herding and informational cascades links contagion to bad signals about the asset values and market conditions. Allen and Gale (1998a) argue that panics and bank runs are part of the business cycle, and so runs can be triggered by unexpected events in the real sector, as when agents perceive the returns of their investment to be unusually low. In this context, bank runs can be first-best efficient, allowing

\footnote{Knight (1957).}
optimal risk sharing between depositors with different time discounting and risk
tolerance. Allen and Gale (1998b), on the other hand, point out that bubbles
and crises can arrive when high levels of uncertainty are present in the financial
or the real market. Accordingly, large credit expansions might lead agents to
take on excesses of risk, causing bubbles in the prices of assets (like real estate
or stocks). A sudden price collapse may trigger a wave of defaults in firms and
other agents who acquired debt to buy the assets at inflated prices. When this
phenomenon is accompanied by large uncertainties about credit expansion, the
magnitude of the bubble may increase and cause financial fragility. For Allen
and Gale (1998c), contagion stems from risk sharing in the banking system,
where banks share claims on different economic sectors. The main device for
spillover is the overlap among claims maintained by banks. Thus, a shock in
one sector can spread to another sector and finally to the whole market. There
is a trade-off between the benefits that a bank receives for having access to liq-
uidity in the banking system and the social costs of contagion. The structure
of the financial market matters for contagion: complete markets with perfectly
integrated institutions would be less prone to suffer contagion than incomplete
and less integrated markets.

More recent literature has introduced other types of “markets imperfec-
tions”, such as heterogeneities in beliefs. Geanakoplos (2003) suggests that a
liquidity crisis begins when agents with different priors receive bad news about
their assets returns. This bad news increases the probability of default and redis-
tributes wealth across agents, triggering a liquidity crisis. When borrowers who
have used the assets as collateral for credits become constrained, there is panic in
the market and the price of assets collapses. The mechanisms of contagion here
are given by the correlation between collateralized assets and output, and by
the correlation among agents’ beliefs, especially if they were cross-collateralized.
Heterogeneities in beliefs thus create conditions for spillover.

Similarly, Fostel and Geanakoplos (2008) explain contagion in emerging mar-
kets as the result of heterogeneities in beliefs. For them, spillover can be consid-
ered a portfolio effect caused when bad news in the global economy reduces the
expected payoffs of the high-yield investment and increases uncertainty about
emerging market payoffs. Consequently, the disagreement about the probability
of default increases\(^5\) across agents, exacerbating the initial distress. This may
occur even when leverage is not so high, contradicting earlier scholars’ claims,
although, contagion still occurs in the context of incomplete markets and het-
ogeneous investors.

Looking back to the leverage argument, Geanakoplos (2009) argues that
an excess of leverage lies behind many episodes of financial distress, such as
the crisis of 2007-2008. In particular, he points to the leverage cycle in which
leverage is high during economic expansions and low during contractions. In
these movements, the price of assets soars during booms and collapses during
slumps. This phenomenon would be exacerbated by agents’ disagreement about

\(^{5}\) Pessimistic agents become more pessimistic, in opposition, optimistic agents remain op-

5
the state of the economy. In opposition with the hypothesis of “irrational exu-
berance”, Geanakoplos (2009) suggests that high rates of leverage may trigger
agents panic when the prices of assets collapse. In the worst circumstances,
this panic could end in successive flights to collateral that spills over the whole
economy\textsuperscript{6}. Like many others authors, Geanakoplos (2009) believes that market
structure matters and that crises are characteristic of incomplete markets.

2.2 The Macroeconomic Perspective of Contagion

The macro literature has mainly focused on identifying the mechanisms of con-
tagion between financial markets and the real economy. How financial shocks
affect investment decisions and the rates of return of capital has puzzled au-
thors for years. Until the early 1980s, the predictions of Modigliani and Miller
(1958, 1961) and others, arguing that the structure of capital was irrelevant
for the return of capital and the value of firms, dominated the macro liter-
ature. Consequently, the financial decisions of firms were irrelevant for real
equilibrium allocations and firms’ value (Lucas, 1978). However, Bernanke and
Gertler (1989) found that when credits markets were characterized by asymmet-
ric information and agency problems, the Modigliani-Miller theorem no longer
applied. By introducing the concept of an investment (financial) accelerator,
they demonstrated that adverse shocks to the value of assets might affect
the balance sheets of borrowers and be amplified through the investment demand to
the entire economy. In the presence of asymmetries of information\textsuperscript{7}, contagion
is caused by agency costs inversely related to the borrower’s net worth. Lat-
ter, Gottardi (1995) demonstrated that if markets were incomplete and there
was only one type of security, a change in the capital structure of a firm would
modify both the real equilibrium allocation and the value of the firm. Con-
sequently, shocks and financial distress could affect the return of capital and
agents’ investment decisions.

From another perspective, Kiyotaki and Moore (1997) attribute contagion
to the dual role of durable assets, in which agents can use assets as both pro-
duction factors and as collateral for loans. Thus the interaction between assets
values and credit restrictions can transmit shocks across different sectors and
explain the dissemination of credit crunches due to problems of moral hazard
that constrain capital for firms and financial intermediaries. Having private in-
formation, firms could be tempted to reduce the return of investment projects.
To reduce this possibility, banks can supervise and contribute their own cap-
tal to the projects. However, banks can be tempted to shrink investment in
order to obtain a private benefit. Thus, banks’ risky actions expose the entire
economy to this behavior when adverse shocks arrive. Combining this approach
and returning to the previous concept of a financial accelerator, Bernanke et

\textsuperscript{6}As it occurs during 2007-2008.

\textsuperscript{7}Between entrepreneurs who control the results of investment projects and lenders who only
observe their returns.
al. (1999) formulated a dynamic general equilibrium model in which endogenous developments in credit markets could propagate and amplify shocks in the macroeconomy. Among other findings, they identify two main transmission mechanisms: links between the premium of external finance and the opportunity cost of internal funds, and the net worth of potential borrowers. Like Kiyotaki and Moore (1997), these authors highlighted the dual role of assets as production factors and collateral for loans, which would become powerful channels for spillover.

As in the micro perspective, the macro literature relates contagion to problems of information, incomplete markets, and uncertainties that exacerbates agents’ response to adverse shocks. To identify the type of uncertainty present in the market during these episodes is key to understanding the phenomenon. In this context, Hansen and Sargent (2008) and others differentiate between risk and uncertainty, where the first concept corresponds to unknown situations where probabilities are measurable and the second one to uncertainty a la Knight (1957), where probabilities are unmeasurable (Epstein and Wang, 1994). One application of this distinction is in Caballero and Krishnamurthy (2008). They argue that many large flight to quality episodes that involve contagion are triggered by unusual or unexpected events in which Knightian uncertainty drives the agent’s behavior. In these events, if agents are liquidity constrained and uncertainty a la Knight coexists with risk in the market, a decrease in aggregate liquidity or a rise in uncertainty can produce a flight to quality episode in which agents consider their worst case scenario among other possible outcomes. As a result expectations can change from rational to “irrational”. More recent literature such as Mendoza et al. (2008) and Mendoza and Quadrini (2010) attributes contagion from financial markets to the real economy to abrupt changes in agents expectations in a context of incomplete markets. Cogley et al. (2012) on the other hand, explain large volatility in financial markets to heterogeneities in agents beliefs about the law of motion of an exogenous endowment and incomplete markets. When markets are complete, less informed agents lose wealth and eventually exit the market. However, when markets are incomplete, less informed agents survive and the difference in agents beliefs lead to changes in wealth distributions and greater volatility in financial markets. Both conditions have been recognized in the literature as conditions for financial crises or financial distress. Although, Cogley et al. (2012) do not explain contagion their model highlight the relationship between market structure and agents beliefs along the line of Blume and Easley (2006).

Both the micro and the macro literatures highlight the role of market failures and the behavior of assets prices during financial distress. High degrees of uncertainty that prevails in these events could contribute to deepening these episodes. Nevertheless, there is not much agreement about the type of uncertainty present in the market during these episodes nor the mechanisms propagating contagion, especially in the case of small economies partially integrated to the international financial markets.
3 The Model

There is an open economy, populated by infinite-lived households, a large number of firms, and banks. To finance investment projects and share risk, the economy can borrow abroad and (eventually) buy foreign assets. The rest of the world is conceived as a conglomerate\(^8\) of \(N\) countries that share risk through financial integration. Households are homogeneous except in their beliefs about the state of the foreign economy, which is represented by the stochastic variable \(s_t\). This variable is defined as a first order Markov process with transition matrix \(\Pi = \{\pi_{lk}\}\), for \(\pi_{lk} = \Pr\{s_{t+1} = k/s_t = l\}\) and \(S\) being the set of all possible states of the foreign economy. Let \(\Omega\) denotes the set of all histories of states and the partial history is represented by \(s^t = (s_1, ..., s_t)\). At any time \(t\), households dispose a proportion of aggregate wealth \(w\) that devote to consumption and saving. Firms produce final goods that are devoted to household consumption and capital formation. In addition, firms produce capital goods they use as inputs for production. To create new capital, firms finance investment projects using their own resources and borrowing from banks. These intermediaries sell bank deposits to households, maintain their own resources, and obtain funds from the rest of the world. Eventually, banks can also buy foreign assets to share risk with the global economy.

The degree of financial integration of the domestic economy with the conglomerate of countries (the rest of the world) will depend on domestic conditions and the structure of the global financial markets. Following Allen and Gale (1998c), if the rest of the world is perfectly integrated, the degree of financial integration of the home economy depends only on domestic conditions. In other type of financial structure, financial integration will depend on both the structure of its partners and domestic conditions.

3.1 Households

There is a continuum of households of type \(i = (1, 2)\), which have different perceptions about the state of the global economy. The idea is that there is a group of countries with better perceptions about the state of nature. Beliefs of households of type \(i = 1\) are closer to the "true" state and beliefs of agents of type \(i = 2\) are less accurate. In this sense, expectations of the first type of agents can be closer to rational expectations and those of the second type of agents are biased. Despite that, agents of type \(i = 1\) are more optimistic, such that when their perceptions contain mistakes they overestimate the probability of good states and underestimate the probability of bad states. Conversely, agents of type \(i = 2\) will underestimate the probability of good states and will overestimate the probability of bad states. For sake of simplicity I assume there is not learning in this economy, however, agents of type \(i = 1\) have better information than agents of type \(i = 2\). These perceptions can be understood as innate, the result of fast learning, or just the result of information of better

\(^8\)This conglomerate can represent the European Union, or the group formed by the U.S. and Europe.
quality. Whatever the cause, the error perceptions will be larger for the second group of agents. As with the discussion in Blume and Easley (2006), results do not differ dramatically assuming that agents differ in beliefs because some of them learn slower or just because their perceptions are biased by nature.

When agents’ beliefs are correct on average\(^9\), the transition matrix \( \Pi \) for agent \( i \) satisfies \( \pi_i(s) = \pi(s) \pm \kappa \), with \( \kappa \geq 0 \). The financial structure follows Lucas (1978), Gertler and Kiyotaki (2010), and Cogley et al. (2012), where households consume final goods, trade Arrow-securities, and take bank deposits at market interest rates. For simplicity, we assume that there is no income from labor and households receive income from firms and banks profits. In addition, for precautionary (and speculation) motives households can invest in financial assets. The agents’ preferences are ordered in a stream of consumption plans \( c = \{ c(s^t) : \forall t, \forall s^t \in H^t \}_{t=0}^{\infty} \), represented by

\[
E^i = \sum_{t=0}^{\infty} \beta^t u(c^i(s^t)), \quad \beta \in (0, 1) \tag{1}
\]

where \( E^i \) denotes the expectations of agent \( i \) about the state of the global economy, which are based on \( \pi_i(s) \). The utility function \( u(c) \) is continuous and concave, satisfying \( u’(c) > 0 \), \( u''(c) < 0 \), and \( \lim_{c \to 0} u’(c) = \infty \). The sequence of budget constraints for agent \( i \), expressed in units of tradable goods, follows

\[
c^i(s^t) + \sum_{s^{t+1}} Q(s^{t+1}) a^i(s^{t+1}) + D^i(s^{t+1}) = w^i(s_t) + a^i(s^t) + R^D(s^t) D^i(s^t), \text{ for all } t, s^t \tag{2}
\]

where \( c^i(s^t) \) is consumption of agent \( i \), \( a^i(s^t) \) is the amount of Arrow securities that the agent trades in history \( s^t \), and \( D^i(s^t) \) is the amount of bank deposits, which can be considered risk-free assets (see Gertler and Kiyotaki (2010)). The expression \( w^i(s_t) \) on the right of the budget constraint represents the amount of income (in units of consumption goods) that agent \( i \) receives in period \( t \) and state \( s_t \). This income comes from firms’ and banks’ profits, \( \Gamma^f \) and \( \Gamma^b \), respectively, and it is distributed according to \( \phi_i \in \Phi \equiv (0, 1) \). Thus for each agent \( i \), \( w^i(s_t) = \phi^i_w(s_t) \), with \( w(s_t) = \Gamma^f(s_t) + \Gamma^b(s_t) \). Following Tsyrennikov (2011) and Cogley et al. (2012), \( Q(s^t) \) represents the Arrow prices in history \( s^t \) and \( R^D(s^t) \) is the interest rate of bank deposits in this history. The rate \( R^D \) is assumed to be exogenous and considered a free-risk interest rate. To avoid Ponzi games, agent \( i \) must satisfy

\[
-a^i(s^{t+1}) \leq B^i(s^{t+1}) \tag{3}
\]

In addition, he needs satisfy

\[
-D^i(s^{t+1}) \leq w^i \tag{4}
\]

The amount \( B^i(s^{t+1}) \) in constraint (3) is the natural debt limit for agent \( i \) in history \( s^{t+1} \) and \( w^i \) in constraint (4) represents a minimum household’s wealth.

\(^9\)Tsyrennikov (2011).
that limits borrowing from banks. Under these two constraints, households are
allow to borrow until they can afford their debts.

3.2 Growth and Wealth Distribution

The rate of growth $\mu \geq 0$ of aggregate wealth depends on the state of the rest
of the world, where $\mu = \mu(s_t)$ is the rate of growth at state $s_t$. Accordingly,
aggregate wealth evolves as $w(s^t) = w_0 \mu(s_1) \ldots \mu(s_t)$. Thus, when the rest of the
world is in a good shape there will be an expansion in the wealth of the domestic
economy and when the rest of the world is experiencing a contraction period,
wealth will be reduced. Although, the coincidence between the real business
cycles of developed and developing countries has diverged in the recent years,
the evidence shows that this situation is temporary and does not represent the
typical pattern for small open economies, especially in the case of small export
economies. In addition to the rate of aggregate growth, the state of the foreign
economy can represent foreign liquidity. In this sense, a good state would be
accompanied by certain abundance of foreign funds and a bad state would be
followed by scarcity of foreign funds. In line with (Tsyrennikov, 2011), wealth
distribution is given by $\phi^i(s_t)$ for agent $i$ and it will evolve with the
state of the foreign economy (the rest of the world). In such a way, at any time $t$ and history $s_t$, income of agent $i$ in state $s_t$ will be $w^i(s^t) = \phi^i(s_t)w(s^t)$. I
assume here that the rate of growth $\mu(s_t)$ and the variable of income distribution $\phi^i(s_t)$ are independent stochastic processes.

3.3 Firms

Firms produce final goods they sell to households and capital goods for their
own consumption in history $s^t$. Despite the fact that households differ in beliefs
about the state of the foreign economy, firms are completely homogeneous, and
each firm makes the same decisions as every other. To produce final goods,
firms use the following technology

$$Y(s^t) = A(s_t)K(s^t)^\alpha$$

where $Y(s^t)$ is the output obtained in production for the average firm in history
$s^t$, $A_t$ is history-contingent productivity$^{10}$, and $K(s^t)$ is the stock of capital
owned by firms in history $s^t$, with $0 < \alpha < 1$. As in the standard literature, the
production function in equation (6) is positive, continuous, and capital satisfies
the INADA conditions, such that $\lim_{K \to 0}f_K = \infty$ and $\lim_{K \to \infty}f_K = 0$, with
$f(K(s^t)) = A(s_t)K(s^t)^\alpha$. For simplicity, we assume that both capital and final
goods are tradable. To produce capital in each state $s_t$, firms utilize domestic
inputs $I^h$ and foreign inputs $I^f$, according to the aggregation technology$^{11}$

$$K(s^t) = \psi(I^h(s^t))^\gamma (I^f(s^t))^{1-\gamma}$$

\( K(s^t) \) here is the amount of capital produced by the firm in history \( s^t \), \( I^H(s^t) \) is the amount of domestic inputs and \( I^F(s^t) \) is the amount of foreign inputs used in capital production, with \( \psi = 1/\gamma(1 - \gamma)(1 - \gamma) \) and \( 0 < \gamma < 1 \). Accordingly, the price index of one unit of capital is a weighted average of prices of domestic inputs and prices of foreign inputs, \( P = (P^H)^\gamma(P^F)^{1-\gamma} \), where \( S = P^F/P^H \) is the real exchange rate of the domestic economy (see Obstfeld and Rogoff (2000)).

For simplicity, I consider that all prices are normalized at unity. Assuming that firms maintain a stock of domestic inputs \( X \) and a total amount of resources \( FW \) at time \( t \), the firm finances its acquisitions of foreign inputs with bank loans. Thus, to acquire foreign inputs the firm faces the following flows of budget constraints in history \( s^t \)

\[
(1 - \gamma)K(s^t) = NW(s^t) + L(s^t)
\]

The expression on the left of equation (7) is the demand for foreign inputs in history \( s^t \) \( (I^F(s^t) = (1 - \gamma)K(s^t)) \) under the assumption that the real exchange rate is equal to one, \( S = 1 \). The expression on the right is the “supply” of foreign inputs, composed by the net amount of foreign inputs \( NW(s^t) = FW(s^t) - X(s^t) \) maintained by the firm at history \( s^t \), and bank loans \( L(s^t) \). In line with the standard neoclassical model, the dynamics of capital is given by \( K_{t+1} = (1 - \delta)K_t + I_t \), where \( I \) is aggregate investment and \( \delta \) is the rate of capital depreciation in this economy. Because goods’ producer firms are homogeneous, aggregate output \( Y \) is the sum of output produced by the continuum of firms normalized at unitary, thus for any story \( s^t \), the aggregate output will be \( Y(s^t) = \int^1 Y(s^t) dj \).

### 3.4 The Rest of the World

In the spirit of Allen and Gale (1998c), the rest of the world is conceived as a conglomerate of \( N > 1 \) countries (e.g. Europe and the U.S.) that shares financial risk through financial intermediaries. Assuming that banks are identical, the degree of financial integration across the conglomerate will depend on the banks’ capacity to exchange financial assets. Accordingly, the conglomerate will be fully integrated if any of its members can hold deposits \( z \) and/or securities from another member (see Allen and Gale (1998c)). In addition, households in each country can trade Arrow securities \( a^* \) to share aggregate risk. Under these two conditions, financial markets in the foreign economy (rest of the world) will be complete. However, when interbank transactions are restricted for some members of the conglomerate (because of transaction or informational costs) and/or agents are not allowed to trade Arrow securities, financial markets will be incomplete. Even though that banks may share risk in both types of structures (complete and incomplete markets), the nature of financial integration between the conglomerate and a third country (small economy) will be different. In the first case, when the conglomerate is fully integrated and foreign markets are complete, the financial structure of the small economy and its degree of financial integration with the rest of the world may depend on the domestic conditions of
this economy. In the second case, the degree of financial integration will depend on both the market structure across the conglomerate and the market conditions of the small economy. The nature of contagion would differ between these two cases. When markets are complete in the conglomerate, adverse shocks may spillover to the small economy, but a crisis would be only a possible outcome among others of no crisis. When markets are incomplete, there are certain structures where contagion always leads to a crisis in the small economy. For a benchmark purpose, I assume that the conglomerate of countries is perfectly integrated with complete markets, but the small economy just can borrow from these countries.

3.5 Banks

There are a large number of homogeneous banks that belong to households. Although banks are prices takers, they can obtain positive profits in state \( s_t \) because of agency problems. If so, profits are distributed among households according to the rule \( \phi^b \in \Phi \) for each agent \( i \). Following Gertler and Kiyotaki (2010), to finance lending, banks can raise funds in the domestic economy and in the global economy (rest of the world). In the domestic economy, banks can obtain deposits from households and they can also use their own funds. In the global economy, banks obtain funds from foreign banks (international interbank market). Depending on the degree of financial integration between the domestic economy and the conglomerate of banks, domestic banks might also buy foreign assets to share risk with the rest of the world. The flow-of-funds constraint for a representative bank in history \( s_t \) is

\[
L(s_t) = BW(s_t) + D(s_t) + F(s_t)
\]

Here, \( L(s) \) denotes the amount of bank lending to firms expressed in tradable goods, \( BW(s_t) \) is the bank’s net worth, which is composed of non-tradable goods, \( D(s_t) \) is the amount of bank deposits expressed in tradable goods, and \( F(s_t) \) is the debt acquired with the conglomerate of foreign banks (in tradable goods). Thus, at any history \( s_t \), the bank has a net flow of profit \( \Gamma^b(s_t) = R^L(s_t)L(s_t) - (R^D(s_t)D(s_t) + R^F(s_t)F(s_t)) \), with \( R^L(s_t) \) being the domestic rate of bank lending, \( R^D(s_t) \) corresponds to the interest rate of bank deposits, and \( R^F(s_t) \) is the foreign interest rate. Even though the rest of the world can be fully integrated, in line with Allen and Gale (1998c), I assume that the bank in the small economy is borrowing constrained because of the probability of bank default. Denoting by \( \varepsilon R^L(s_t)L(s_t) \) the amount of foreign debt susceptible to be defaulted, the banks’ incentive compatibility constraint will be

\[
(1 - \varepsilon)R^L(s_t)L(s_t) \geq R^D(s_t)D(s_t) + R^F(s_t)F(s_t)
\]

with \( \varepsilon \) being a random variable distributed around \((0, 1)\). Note that although \( \varepsilon \) can be zero and the law of one price applies to the interest rates and the payoff of the risky asset, the representative bank can obtain a positive profit in state \( s_t \) if the bank net worth is positive. In this sense, such as in Gertler and Kiyotaki (2010), we can interpret the flow of bank net worth as a flow of past profits.
3.6 Competitive Equilibrium and Market Clearing

A competitive equilibrium in this economy is defined as a set of individual choices for consumption, investment, and portfolio plans that maximize subjective expected utility functions and constraints (2), (3), and (4) are satisfied. Asset prices adjust to individual expectations. A collection of choices for capital and lending that maximize firm’s profit according to subjective expectations for the average individual and satisfies constraints (5), (6), and (7). A set of bank choices for lending, deposits, foreign funds, and interest rates that maximize the bank’s profit based on subjective expectations of the average individual and satisfies restrictions (8) and (9), and consider the foreign interest rate $R^F$ as given. Clearing markets in this competitive equilibrium implies that market clearing for final and intermediate goods, and market clearing in financial markets.

3.6.1 Financial markets

Financial markets clear when the market for Arrow securities clears, the demand for bank deposits ($D(s^t)$) equals supply ($\bar{D}(s^t)$), the demand for domestic lending ($L(s^t)$) equals supply ($\bar{L}(s^t)$), and the demand for foreign lending ($F(s^t)$) equals supply of foreign funds ($F^*(s^t)$), thus

i) Clearing the market for Arrow securities implies: $a^1(s^t) + a^2(s^t) = 0$, at any time $t$ and history $s^t$.

ii) Clearing the market for bank deposits: $D(s^t) = \bar{D}(s^t)$, at any time $t$ and history $s^t$.

iii) Clearing the market for domestic lending: $L(s^t) = \bar{L}(s^t)$, at any time $t$ and history $s^t$.

iv) Clearing the market for foreign lending: $F(s^t) = F^*(s^t)$, at any time $t$ and history $s^t$.

3.6.2 The market for final goods

The market for final goods clear when aggregate consumption ($c(s_t)$) plus investment in capital goods ($D(s_{t+1})$) equals aggregate output $Y(s_t)$ in state $s_t$

$$c(s_t) + D(s_{t+1}) = Y(s_t)$$

where $c(s) = c^1(s) + c^2(s)$ (see Proof 1 in the Appendix).

3.6.3 The market for intermediate goods

Clearing conditions in the market for intermediate goods requires that both markets for tradable and non tradable goods clear at time $t$ and history $s^t$. Based on definitions of sections 3.2, 3.3 and 3.5, and constraints (6), (7), and (8), market clearing for tradable and non tradable goods follows

i) Non tradable inputs: $\gamma K(s^t) = I^H(s^t) = X(s^t) + BW(s^t)$, for all $t$ and history $s^t$.

ii) Tradable inputs: $(1 - \gamma)K(s^t) = I^F(s^t) = NW(s^t) + D(s^t) + F(s^t)$, for all $t$ and history $s^t$. 

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where $I^H(s^t) = \gamma K(s^t)$ is the demand for non tradable inputs and the expression on the right of (i) is the supply. Analogously, $I^F = (1 - \gamma)K(s^t)$ is the demand for tradable inputs and the expression on the right of (ii) represents the supply of these inputs. Note that the supply of nontradable inputs comes from firms' and banks' saving decisions. On the other hand, the supply of tradable inputs depends on banks' and households' decisions in the domestic economy and saving decisions in the rest of the world.

4 Contagion in the Domestic Economy

To analyze contagion between the domestic economy and the rest of the world, I will use the approach of Fostel and Geanakoplos (2008) and Fostel (2009). In this setup, a shock arrives in the domestic economy when agents receive bad news about the state of the foreign economy, $s_t$. The bad news can be materialized as a small contraction or a deep recession. In either case, growth in the domestic economy will be affected. Depending on the magnitude of the shock and the financial structure of the foreign economy, falls in growth can be accompanied by constraints in foreign liquidity. Based on the economic framework described in the previous section, contagion may be expressed through three main mechanisms: a change in agents' beliefs, together with a fall in aggregate wealth $w$; a change in foreign interest rates $R^F$; and a liquidity contraction in the supply of foreign funds $F^*$. In addition, there is a "balance sheet" effect caused by exchange rates movements (for $S \neq 1$). Thus the bad news may lead to wealth redistribution, having a portfolio effect. Greater asset price volatility is expected as a consequence. The magnitude of these two effects will depend on the structure of the small economy. In this sense, the literature suggests that when beliefs are incorrect and agents are pessimistic, they tend to lose their wealth because of their wrong predictions. Conversely, rational agents (with correct beliefs) would take advantage of this situation and become richer. At the end, pessimistic agents would leave the market and influence less asset prices than optimistic agents in equilibrium (see Cochrane (2006) for a discussion). In this model, agents with wrong beliefs (type 2) have the possibility to take bank deposits in addition to trade contingent securities. This possibility avoids market exit for pessimistic agents. In the case of foreign interest rates, the theory predicts they are procyclical (smaller during contractions and higher during expansions), which is sustained by the evidence for developed countries. Despite that, many adverse shocks in developing countries arrived in the past with abrupt rises in foreign interest rates. This effect can be captured in the model by changes in portfolio decisions that constraint foreign liquidity. For example, the adverse shock in the rest of the world can be accompanied by a change in expectations that constraints liquidity or the domestic economy becomes riskier for foreign investors.

In line with Allen and Gale (1998c), I consider that the severity of contagion will depend on the degree of financial integration between the economy and the rest of the world. In addition, it will depend on the domestic conditions of the
foreign economy. Therefore, to analyze contagion, I solve the model considering four cases: a benchmark domestic economy with a fully integrated rest of the world, a benchmark economy with incomplete foreign markets, an incomplete domestic economy with a fully integrated rest of the world, and an incomplete domestic economy with a partially integrated rest of the world.

4.1 Benchmark Economy with a Fully Integrated Rest of the World

Complete markets in the domestic economy would require that households share risk trading Arrow securities and financial markets be fully integrated with the rest of the world. However, these conditions may not represent the situation of developing countries, characterized by restrictions limiting risk sharing and international financial integration. To represent the situation of a developing country in this benchmark case, I assume that the small economy can only borrow from the rest of the world. In addition, as a benchmark exercise, I consider that domestic agents do not suffer liquidity shocks and the only uncertainty they face is the state of the foreign economy \( s_t \). I assume complete markets for the rest of the world in the spirit of Allen and Gale (1998c), where banks of country members share liquidity risk holding an amount \( z^* \) of bank deposits across the conglomerate. In addition, foreign households can trade Arrow securities \( a^* \) and save in bank deposits \( D^* \). Thus, at any date \( t \) and in history \( s^t \), the flow of budget constraints for foreign banks follows

\[
L^*(s^t) = BW^*(s^t) + (N - 1)z^*_C(s^t) + D^*(s^t) \tag{11}
\]

where \( L^* \) is foreign demand for bank’s lending, \( BW^* \) represents wealth of foreign banks, \( z^*_C \) is the amount of deposits that the representative bank holds from the rest of the banks \( (N - 1) \) in complete markets, and \( D^* \) are deposits collected from foreign households. Assuming that liquidity shocks are idiosyncratic, banks of the conglomerate are fully insured holding an amount of \( (N - 1)z^*_C(s^t) \) deposits from other banks. Thus a demand for "extra" liquidity equals to \( (N - 1)z^*_C(s^t) \) is always satisfied in complete markets without to liquidate the other bank’s assets. Under this consideration, the supply of foreign funds is constrained by \( L^*(s^t) \), with \( F^*(s^t) \leq BW^*(s^t) + D^*(s^t) - L^*(s^t) \).

4.1.1 The Problem of Households - Recursive Approach

Expressed recursively, the problem of the household can be summarized in a Bellman equation, where agent \( i \) chooses optimal consumption, investment, and portfolio plans based on his individual beliefs \( \pi^i(s) \), the sequence of budget constraints (2), and debt limits (3) and (4). In this equilibrium, the state variables are composed by the initial endowments of financial assets, the initial amount of bank deposits, a wealth distribution \( n = (n_1, n_2) \), given by \( \phi^i = \phi(s^t) \in \Phi \), and the current state of the foreign economy, \( s \). In order to guarantee

\[\text{The idea is there is a representative bank in each country of the conglomerate.}\]
a "well behaved" equilibrium, all variables are normalized by the current level of aggregate wealth \( w \) (see Cogley et al. (2012)). Thus, for any variable \( x \), \( \tilde{x} = x/w \). Defining the value function for individual \( i \) in the benchmark economy as \( V^i_{\text{ben}}(\tilde{a}, \tilde{D}, n, s) \), the household problem is characterized by the following Bellman equation

\[
V^i_{\text{ben}}(\tilde{a}, \tilde{D}, n, s) = \max_{\tilde{c}, \tilde{a}'(s')} [u(\tilde{c}) + \beta \sum_{s'} V^i_{\text{ben}}(\tilde{a}', \tilde{D}', n', s') \pi^i(s'/s)]
\] (12)

Subject to the budget constraint

\[
\tilde{c} + \sum_{s'} Q(n, s, s') \mu(s') \tilde{a}' + \tilde{D}' \mu(s') = \phi^i(n) + \tilde{a} + R^D \tilde{D}
\] (13)

the limit debt in financial assets

\[
\tilde{a}'(s') \geq -A^i(n, s, s')
\] (14)

and the limit borrowing in bank deposits

\[
-\tilde{D}'(s') \leq w(n, s, s')
\] (15)

where \( n \) and \( n' \) represent wealth distribution at time \( t \) and \( t + 1 \), respectively. In general, the superscripts with "commas" indicate \( t + 1 \). Solving the problem of agent \( i \) for \( t \) and \( t + 1 \), the first order conditions lead to the following Euler equations:

\[
E^i_s \left[ \frac{\beta u'(\tilde{a}'(s'))}{u'(\tilde{c}(s))} \mu(s)^{-1} R^D(s') \right] = 1
\] (16)

\[
E^i_s \left[ \frac{\beta u'(\tilde{a}'(s'))}{u'(\tilde{c}(s))} \mu(s)^{-1} \right] = Q(n, s, s')
\] (17)

Equation (16) establishes the standard conditions for gross return of bank deposits, considered as risk-free assets in financial markets. Conversely, equation (17), would be interpreted as the condition for risky assets, where \( Q(s) \) is the price of asset \( a \) in state \( s' \). The term \( m_{s,s'} = \frac{\beta u'(\tilde{c}(s'))}{u'(\tilde{c}(s))} \) in both equations is the stochastic discount factor or pricing kernel (see Cochrane (2005); Tsyrennikov (2011)).

4.1.2 The Problem of Firms

Firms take market prices as given, and so the representative firm chooses the amount of capital to produce and the amount of bank loans it needs in history \( s^t \), solving the problem

\[
\max \sum_{t=0}^{\infty} \sum_{s^t} m_{t+1} [\tilde{Y}(s^t) - R^L(s^t)\tilde{L}(s^t)] \pi(s^t)
\] (18)

subject to constraints (5), (6), and (7) of section 3.3. As in the household’s problem, variables are normalized with respect to the aggregate level of wealth
Thus, $\tilde{Y}(s^t)$ in equation (23) is the normalized level of output produced by the representative firm and $R^L(s^t)$ is the loan interest rate paid to banks in history $s^t$. The term $m_{t+1} = \beta u'(\tilde{c}(s^{t+1}))/u'(\tilde{c}(s^t))$ is the households’ stochastic discount factor and $\tilde{\pi}(s^t)$ is the agents’ "average" probability in history $s^t$, with $\tilde{\pi}(s^t) = \pi(s^t) + \Delta$, for $\Delta \geq 0$. When beliefs are correct on "average", $\tilde{\pi}(s^t) = \pi(s^t)$ and the error term is equal to zero ($\Delta = 0$) (see Tsyrennikov (2011)). Solving the firm’s problem, the necessary first-order condition is

$$ (1 - \gamma)R^L(s^t) = \alpha A(s^t)\tilde{K}(s^t)^{\alpha-1} $$

with $\tilde{K}(s^t) = K(s^t)/w(s_t)$ and $(1 - \gamma)$ being the parameter of the demand of foreign inputs in equation (7). The expression $\alpha A(s^t)\tilde{K}(s^t)^{\alpha-1}$ is the marginal product of capital (MPC).

### 4.1.3 The Problem of Banks

Considering the foreign interest rate $R^F$ as given and agents’ beliefs on average $\tilde{\pi}(s^t)$, banks choose a sequence of loans for firms, a sequence of bank deposits, and an optimal level of foreign debt to solve

$$ \max \sum_{t=0}^{\infty} \sum_{s^t} m_{t+1} [R^L(s^t)\tilde{L}(s^t) - (R^D(s^t)\tilde{D}(s^t) + R^F(s^t)\tilde{F}(s^t))\tilde{\pi}(s^t)] $$

subject to the bank’s flows constraints (8) and the incentive compatibility constraint (9). The solution of the bank problem then in history $s^t$ leads to the following first order conditions:

$$ R^L(s^t)(1 + \lambda_t(1 - \varepsilon)) = R^F(s^t)(1 + \lambda_t) $$

$$ R^D(s^t) = R^F(s^t) $$

where the Lagrange multiplier $\lambda_t$ in equation (21) belongs to a sequence of multipliers $\{\lambda_t\}_{t=0}^{\infty}$ on the bank-flow constraint (8) and the incentive compatibility constraint (9) in history $s^t$. Because the probability of default is zero in the benchmark economy, $\varepsilon = 0$ in equation (21) and both rates, the lending interest rate and the deposits interest rates are equal to the foreign interest rate in equilibrium, $R^L(s^t) = R^D(s^t) = R^F(s^t)$.

The difference between these two rates should be proportional to the probability of default. There is a “premium” for the domestic bank then to guarantee it has not incentives to abscond with a proportion of the bank’s revenue. Considered bank deposits as risk-free assets, the interest rate $R^D$, in equation (15), will be equal to the borrowing interest rate, $R^L$.

### 4.1.4 Contagion

The spillover effects of an adverse shock in the benchmark economy (with a fully integrated rest of the world) must be analyzed from of the point of view of

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$^{13}$See Cochrane (2005); Gertler and Kiyotaki (2010).
the three main mechanisms: change in asset prices, change in domestic interest rates due to a change in foreign interest rates, and the availability of foreign funds.

**Asset Prices**  
Regarding asset prices, equation (17) tells us that these prices will fluctuate with the stochastic discount factor $m$ and agents beliefs. When beliefs are correct, the stochastic discount factor $m$ for any agent $i$ will be equal to $\beta$ times the rate of growth of marginal utility of consumption, $\tilde{u}(s) = \frac{u'(c'(s'))}{u'(c'(s))}$, then

$$Q(n, s, s') = \beta \bar{m} \pi(s'/s) = \bar{Q}$$

where $\bar{Q}$ is the price of an Arrow security in the context of correct beliefs. In this way, the asset prices for agents type 1 then should be $e \bar{Q} = \beta \bar{m} \pi(s'/s)$, where $\pi^1(s') = \pi(s'/s)$ is the subjective probability of agents type 1 (see Cogley et al. (2012)).

When agents disagree about their beliefs, the prices of assets will fluctuate with this disagreement. Let $q(s') = \pi^2(s')/\pi^1(s')$ be the ratio of subjective probabilities between agents of type 2 and agents of type 1, which represents their disagreement. If beliefs are correct, $q(s') = 1$, given that $\pi^1(s') = \pi^2(s')$, however when beliefs are incorrect, $q(s') > 1$ and the value will depend on the agents perceptions about the different states. Because agents of type 1 are rational traders, their beliefs are correct, however, agents of type 2 are pessimistic, with $\pi^2(s') = \pi(s') + \infty$.

**Proposition 1**  
When the foreign economy is transiting toward a good state, $\mu \geq 1$, pessimistic agents underestimate this probability and $\pi^2(s') > \pi^1(s')$. Conversely, when the foreign economy is going to a bad state, $\mu < 1$, pessimistic agents overestimate this occurrence and $\pi^2(s') < \pi^1(s')$.

Reorganizing equation (17), the price of assets when agents disagree in beliefs stays as

$$Q(n, s, s') = \sum_{s'} \bar{Q} h(\tilde{u}^2, q(s'))$$

where $h(\tilde{u}^2, q(s')) = \left[ \frac{u'(c'(s'))}{u'(c'(s))} \right]^{-1} q(s')$ and $\tilde{u}^2$ being the rate of growth of marginal utility of consumption for agents of type 2. According to Proposition 1 and equation (24), the bad news about the state of foreign economy will cause a change in agents beliefs. Estimations of rational agents ($\pi^1(s')$) will be close to the true probabilities and those of pessimistic agents ($\pi^2(s')$) will be biased, overestimating the bad state, with $\pi^2(s') > \pi^1(s')$ and $q(s') > 1$. This change in beliefs will lead to a rise in asset prices and a fall in returns $R^a$. As a consequence, there will be a reallocation in agents’ portfolio and a wealth distribution across agents. In a close economy and complete market such as Cogley et al. (2012), this change in beliefs leads pessimistic agents to buy assets in the bad states and sell assets during the good states. Because, pessimistic agents overestimate the bad states, they lose wealth and eventually
leave the market in the long term. Better informed about the true probabilities, rational agents sell overvalued assets buy undervalued assets. In opposition to pessimitic agents, rational traders obtain gains with these transaction and (eventually) become rich.

In this model, agents have the possibility to acquire bank deposits in addition to trade asset securities. Considered risk free assets, banks deposits may act as a "buffer" for pessimistic agents. If banks are solvent and there are no liquidity shocks in the domestic economy, agents of type 2 can take bank deposits as alternatives of Arrow securities. Whereas $R^a \leq R^P$, this strategy may attenuate their losses keeping them in the market in the long term. Because of that, agents type 1, would be limited to take advantages from the incorrect predictions of pessimistic agents. Nevertheless, even though the buffer may protect pessimistic agents from big wealth loses, the market would be more volatile than the complete market’s case of Cogley et al. (2012).

**Interest Rates**  When foreign markets are complete and there is not risk of default in the domestic economy, the adverse shock should be accompanied by a fall in foreign interest rates, which causes a reduction in domestic interest rates, given that $R^D = R^L = R^F$. From market clearing conditions 3.6.2 and 3.6.1, we see that the fall in domestic interest rates does not affect directly the markets of final and capital goods at aggregate level. However, interest rates can affect output growth and also the dynamic of intermediate goods, at aggregate level. In the case of output growth, the fall in interest rates would offset (in part) the contraction caused by the bad state (see equation (18)). In the case of intermediate goods, equation (19) shows that there are two effects, on the one hand, the fall in interest rates leads to a rise in demand. On the other hand, output contraction produces a demand contraction. Which effect prevails, it depends on the size of these two movements. In financial markets, the interest rate reduction has no effect on bank’s profit or on liquidity. Foreign borrowing is cheaper for banks, although revenue from lending to firms is less profitable as well.

**Foreign Liquidity**  In a complete market environment, the foreign economy should be able to meet its liquidity requirements without constraints funds to the domestic economy. Maintaining the assumption of idiosyncratic liquidity shocks (as in Allen and Gale 1998c), banks can share liquidity risk across the conglomerate and the spillover effects toward the small economy should be limited. Nevertheless, when the foreign economy experiences an output contraction or a recession, there is a wealth effect that reduces consumption and saving. From equation (11) we see that the supply of foreign funds $F^*$ will be reduced and the domestic economy becomes borrowing constrained because of the adverse shock (which reduces the rate of output growth $\mu$). The magnitude of this constraint will depend on the magnitude of the shock.

Summarizing, when domestic agents receive bad news about the state of the foreign economy, asset prices will raise due to the discrepancy in beliefs. This
raise in prices has two effects: first, it leads to a portfolio reallocation across agents and possibly between financial instruments (assets and banks deposits). Because agents differ in beliefs, some of them lose wealth and others gain. There is a wealth redistribution across agents. Second, fluctuations in asset prices produces volatility in financial markets of the small economy and certain degree of financial distress. Because markets are complete, the effect is limited at aggregate level.

4.2 The Benchmark Economy with Imperfect Integrated Rest of the World

The assumption of a perfect integrated rest of the world is not much realistic. Although financial markets of developed countries are more complete and more integrated across them than financial markets of developing countries, risk sharing is not perfect. To examine then how this incompleteness affects contagion in a small economy, I will repeat the previous analysis, considering now that foreign banks are not completely insured against liquidity shocks. This fact would cause a liquidity restriction of a bigger magnitude in the domestic economy when there is bad news about the foreign economy. Maintaining the assumption that domestic financial markets are (almost) complete, the problem of households, the problem of the representative firm and the problem of banks are identical to those developed in section 4.1. However, the supply of foreign funds with incomplete markets will be different, where budget constraint (11) ex-ante stays as

\[
\hat{L}^*(s^t) = \hat{BW}^*(s^t) + \hat{D}^*(s^t) + z_{IC}(s^t) \tag{25}
\]

With \(z_{IC}(s^t)\) being the amount of interbank deposits that foreign banks hold from others members. To the difference with complete markets, I assume here that foreign banks can only hold deposits from neighbor banks (see Allen and Gale (1998c)), thus the conglomerate of banks is not fully ensured against liquidity shocks, with \(z_{IC} < z_C\). In this case, if liquidity needs are \(z_{IC}(s^t) + \zeta \geq z_{IC}(s^t)\) at time \(t\), with \(\zeta \geq 0\), the bank must liquidate some of its wealth \(\hat{BW}^*\) and the foreign funds available for the domestic economy at time \(t\) will be

\[
\hat{F}^*(s^t) \leq (1 - \zeta) \hat{BW}^*(s^t) + \hat{D}^*(s^t) - \hat{L}^*(s^t) \tag{26}
\]

Where \(\zeta\hat{BW}^*\) is the amount of resources liquidated by foreign banks when the economy faces liquidity shocks. As in Allen and Gale (1998c) I assume that liquidation obeys a pecking order, meaning that banks liquidate their own resources first and then they liquidate households deposits \(\hat{D}^*\). Variables with "hat" are normalized variables with respect to the rate of growth.

4.2.1 Contagion

With incomplete foreign markets, the bad news about the state of the foreign economy can be followed by a liquidity contraction in domestic banks. To
analyze how this fact affects the domestic economy, considering the three main mechanisms of contagion described in Section 4.1.

**Asset Prices**  Such as in the case of complete markets, when agents receive bad news about the foreign economy, asset prices will increase because of the agents’ disagreement in beliefs. Nevertheless, with incomplete foreign markets the portfolio effect would be intensified due to a scarcity of foreign funds. If agents perceive that domestic banks will be borrowing constrained, they would be less willing to save in banks deposits and would save more in Arrow securities. This effect could be more marked in the case of pessimistic agents that overestimate the probability of the bad state, leading them to greater wealth losses than in the case of complete markets. The buffer role of bank deposits could be diminished in this sense, compared with the previous case. As before, the change in asset prices will lead to more volatility in domestic financial markets.

**Interest Rates**  As in 4.1, domestic interest rates should decrease following the fall in foreign interest rates. The magnitude of the fall will depend on the size of the shock. Even though, this effect would be attenuated by the contraction in foreign liquidity.

**Foreign Liquidity**  Incomplete foreign markets can lead to a reduction in foreign liquidity for the domestic economy, affecting lending and the provision of tradable inputs (see equations (8) and (ii) in 3.6.3). From both equations we see that a reduction in foreign funds $F^*$ will reduce the lending capacity of domestic banks, diminishing the productive capacity of firms if there is not reallocation of funds in the domestic economy. Compared with the case of complete markets, contagion in the domestic economy would be higher when foreign markets are incomplete. Changes in portfolio decisions would lead pessimistic agents to greater losses in wealth, experiencing the economy a larger degree of wealth redistribution. In addition, the adverse state can be accompanied by a liquidity shock leading to an output contraction in the current and the next period. The magnitude of these effects will depend on the size of shocks (affecting growth and liquidity) and on the solvency conditions of the domestic economy, given by $FW$ and $BW$. Higher volatility and more financial distress is expected in the small economy.

### 4.3 Small Economy with Incomplete Markets and a Fully Integrated Rest of the World

Developing countries have several restrictions that limit development of financial markets. Given that some of these restrictions are caused by institutional problems, I consider that domestic banks can abscond with part of the foreign funds and $\varepsilon \geq 0$. From the first order condition (15) of the bank’s problem, we have

$$R^L(s_t) = \theta R^F(s_t)$$

\[ (27) \]
where $\theta = [(1 + \lambda_t)/(1 + \lambda_t(1 - \varepsilon))]$ is a parameter of "country risk", given by the probability of banks' default, with $\theta \leq 1$ for $\lambda_t \geq 0$ and $0 \leq \varepsilon < 1$. Because of this possibility, the lending rate will be higher than the deposit rate in equilibrium ($R^L(s_t) \geq R^D(s_t) = R^F(s_t)$), which implies that banks receive a "premium" to discourage them to default their debts. This "premium" given by $(\theta - 1) \geq 0$ increases with country risk $\theta$ and provides banks a second source of profits in state $s_t$, thus $\Gamma^\theta(s_t) = (\theta - 1)R^F(s_t)L(s_t) + R^F(s_t)BW(s_t)$. Due to this additional gain, banks would be tempted to lend as much as they can to obtain higher profits. However, the availability of bank loans is constrained by households savings and foreign resources and it will decrease with country risk, $L(s_t) \geq (D(s_t) + F(s_t))/\theta(1 - \varepsilon)$ (see equation (9)). With higher lending interest rates, firms' profits will decrease reducing households' wealth in state $s_t$, although this effect will be compensated by an increase in banks' profits. Despite of, the demand of capital will decrease, where $K(s_t) = \alpha Y(s_t)/[(1 - \gamma)\theta R^F(s_t)]$ for $\theta \geq 1$.

From market clearing conditions 3.6, we have that interest rates do not affect equilibrium at aggregate level, but an increase in interest rates for lending may reduce the firm's purchases of home and foreign inputs. Thus, capital accumulation would decrease with respect to the complete market case, pushing firms to maintain more balances of home and foreign resources, $X$ and $NW$, respectively.

### 4.3.1 Contagion

Because foreign markets are complete, the contagion mechanisms are in part similar to those analyzed in 4.1, but the final effects on the domestic economy will differ due to its lending restrictions.

**Asset Prices** With complete markets in the foreign economy, the movements in asset prices should be similar to those of 4.1.4, where the bad news about state $s'$ leads asset prices to increase following the disagreement in beliefs, with $q(s') = \pi^2(s')/\pi^1(s')$. Depending on the nature of the shock, the fall in output growth would be accompanied by a restriction in foreign liquidity, but this effect should be attenuated by the fact that foreign banks share risk and financial markets are complete. As before, the rise in assets prices will lead to a portfolio reallocation, where pessimistic agents try to save more to overcome the bad state. However, both agents would invest more in banks' deposits than in the previous cases. As they own banks and lending is more profitable now, agents would take banks deposits as speculative motives in addition to precautionary saving. This strategy would attenuate wealth redistribution, causing less losses in pessimistic agents and more losses agents in rational agents.

**Interest Rates** Such as in 4.1.4 the adverse shock in the foreign economy will be accompanied by a fall in foreign interest rates $R^F$, with $R^D = R^F$ and $R^L = \theta R^F$ for $\theta \geq 1$. This fall will reduce the costs of lending and compensate
the "excess" in interest rates given by $\theta$. The latter will benefit firms without affects banks' profits.

**Foreign Liquidity** Because foreign banks share risk, they can face liquidity shocks without liquidate assets and the supply of foreign funds would not suffer a severe constraint. However, the scarcity of foreign funds in the domestic economy would be larger than in 4.1 because in this case domestic banks are lending constrained. A change in agents' portfolio would compensate this effect, if pessimistic agents save more in bank deposits than in Arrow securities. Given a fix amount of banks deposits held by agents of type 1 (rational traders), the amount of bank deposits would increase at aggregate level, reducing trade in Arrow securities. The final effect will depend on the magnitude of these effects and changes of investment decisions of agents. Less wealth redistribution is expected than in the previous cases, but with higher volatility in domestic markets.

**4.4 Small Economy with Incomplete Markets and Imperfect Integrated Rest of the World**

This case would be the most realistic one representing the situation of financial markets around the world, where financial markets are not fully integrated and developing countries face diverse problems that limit their financial development. I consider incomplete foreign markets as in 4.2 and incomplete markets in the domestic economy as in Section 4.3. To analyze contagion under severe financial distress, I assume that the domestic economy may be affected by liquidity shocks and domestic banks can hold deposits from foreign banks. The flows-of funds for domestic banks in this case will be

$$\hat{L}(s^t) = \hat{B}W(s^t) + \hat{D}(s^t) + \hat{F}(s^t) + \rho z_{IC}(s_t)$$

(28)

Where $0 \leq \rho \leq 1$, represents the degree of financial integration between the domestic economy and the rest of the world and $z_{IC}(s_t)$ is the amount of foreign deposits that domestic banks hold to prevent liquidity shocks. Accordingly, if $\rho = 0$, the economy is not integrated and just can borrow from foreign banks ($F \geq 0$), and if $\rho = 1$, the economy is fully integrated with a member (or group of members) of the conglomerate ($F \geq 0$). The most representative case for a developing country is to assume $0 < \rho < 1$.

**4.4.1 Contagion**

In addition to receive bad news about the rest of the world, the domestic economy can face liquidity shocks. Contagion in this case will depend on the value of $\rho$. If the bad news arrives with a liquidity shock in the foreign economy and $\rho = 0$, the situation is similar to that in 4.2. For $\rho > 0$, contagion will depend on the nature of the shock and the degree of financial integration ($\rho$). When $\rho = 1$, the economy is fully insured against liquidity risk with a neighbor
although it receives also the full impact of the shock (because foreign markets are incomplete). In this case, contagion could arrive from the rest of the world. When $\rho < 1$, the shock would be smoothened, but the economy is less insured from its own liquidity shocks.

**Asset Prices** In the previous cases, the bad news about the state of the foreign economy led to a rise in asset prices, to a portfolio reallocation and to a wealth redistribution across agents. In all cases, agents have the possibility to smooth the effect of change in prices saving in bank deposits. In this case, this choice can be restricted by the possibility to receive liquidity shocks. If agents perceive that domestic banks are exposed to liquidity risk from abroad, they will reduce the amount of bank deposits and eventually they would panic, triggering a bank run. The less agents save in bank deposits the more financial markets look like those in Cogley et al. (2012).

**Interest Rates** Dynamics of interest rates are similar to those of the previous cases. Even though, reductions in foreign interest rates can be offset by liquidity restrictions in foreign markets.

**Foreign Liquidity** When $\rho = 1$ and markets are incomplete a liquidity shock that demands an amount $(1 + \zeta)z_{IC}(s_t) \geq z_{IC}(s_t)$ of foreign resources will produce a severe scarcity in the foreign liquidity, $F^*$. In addition, domestic banks would be obliged to liquidate part of their assets, reducing lending and domestic liquidity. This would cause panic in households, triggering a liquidity crisis. To prevent this scenario, rational agents could reduce exposition, taking less bank deposits. In both cases, the small economy would suffer a severe liquidity contraction and financial distress.

When $\rho < 1$, the foreign liquidity shock would have less effect in the domestic economy. In this case, it can experience a liquidity contraction but without severe financial distress. Agents can lose wealth, without panic. However, although the economy would be "protected" from foreign liquidity shocks, it could be more exposed to idiosyncratic shocks.

## 5 Numerical Exercise

I conduct here a numerical exercise to evaluate the analytical properties of the model. For simplicity, I consider an economy with two agents, where agent 1 is optimistic and agent 2 is pessimistic. In line with Cogley et al. (2012), there are three states for the global economy, $S = \{\text{good, medium, bad}\}$, thus I call $s_1 = \text{good}$ to an expansionary period (high productivity), $s_2 = \text{medium}$ to a moderate contraction (low productivity) and $s_3 = \text{bad}$ to a recession period (small productivity). For simplicity, I assume that the two agents are symmetric in period zero, implying they have equal participation in aggregate output and coincide in beliefs. Thus, $\phi^1 = \phi^2 = 0.5$ and $\pi^1(s_0) = \pi^2(s_0)$ for the two agents.
On the other hand, I assume that both agents start at time zero with no endowment of the Arrow security and equal endowment of wealth (aggregate output). At period one, agents' beliefs differ and agents can buy or sell the security. As a benchmark, in this first exercise, I assume that agents save in capital and there are no financial intermediaries. The utility function follows $u(c) = \frac{(c^{1-\sigma} - 1)}{1-\sigma}$, with $\sigma > 0$ being the coefficient of risk aversion. The parameters values are represented in Table 1.
Table 1
Parameter Values and Initial Conditions
\[
\begin{array}{ccc}
\alpha & 0.35 & 0.35 & 0.35 \\
\beta & 0.95 & 0.95 & 0.95 \\
\sigma & 1.0 & 1.5 & 2.0 \\
\gamma & 0.6 & 0.6 & 0.6 \\
\phi_1 & 0.50 & 0.50 & 0.50 \\
\phi_2 & 0.50 & 0.50 & 0.50 \\
\end{array}
\]

Such as in Cogley et al. (2012) we consider the following “true” transition matrix:
\[
\Pi = \begin{bmatrix}
0.917 & 0.0747 & 0.0083 \\
0.500 & 0.4500 & 0.050 \\
0.500 & 0.4500 & 0.050 \\
\end{bmatrix}
\]

where the unconditional probabilities to reach the good, medium, and bad states are equal to \(P_g = 0.8576\), \(P_m = 0.1281\), and \(P_b = 0.0142\), respectively. This implies that the good states are more frequent for the global economy than the bad states. For a benchmark purpose, I consider that both agents are biased with respect to the “true” probabilities, although errors in predictions of agents of type 1 are smaller than those of agents of type 2. In this regard, the optimistic overestimates the good states and underestimates the bad states, while the pessimistic agent, underestimates the good states and overestimates the bad states. The vector of states is generated from the transition matrix \(\Pi\), considering 100 periods of time. Given the assumptions above, households finance their consumption expenditures with shares of aggregate wealth. Because agents are symmetric at period zero and we have CRRA preferences, I can apply the Negishi (1960) method to obtain the security price at each state \(s\). Thus, the price \(Q\) can be expressed as a function of the ratio of discrepancy in beliefs between agents of type 2 and agents of type 1 \((Q(\pi_2(s)/\pi_1(s)))\). Table 2 summarizes the main findings of this exercise, where I obtained a sequence of prices in each state, \(Q(s_1)\), \(Q(s_2)\), and \(Q(s_3)\). The first column of the table shows the standard deviations of these prices considering a coefficient of risk aversion equal to one \((\sigma = 1.0)\), the second one, considers \(\sigma = 1.5\), and the third one, \(\sigma = 2.0\), which has been used in the literature. To compare results, I include the standard deviation of the risk-free interest rates, \(R\), which meet condition (19) in equilibrium. The "shock" of the foreign economy was generated with function a Markov process, using the Matlab function Shock. As Table 2 shows, the assets prices are more volatile than the interest rate \(R\). In this sense, although abrupt changes in foreign interest rates can generate distress in the domestic economy and become a source of contagion, the results suggest that the discrepancy in beliefs would deepen this distress. I use different values of risk aversion to compare results. As figures 3 and 4 show in the Appendix, volatility seem to increase with a higher rate of risk aversion. However, when I use \(\sigma = 2.0\), volatility decreases in the bad state (see Figure 4 and Figure 5 in
the Appendix). Even though, results do not change dramatically (on average) using different coefficients of risk aversion, the gap between states increases with $\sigma = 1.5$. This has been found in other studies and it would be indicating that agents discrepancy is exacerbated in the bad states, such as it is observed in the figures in the Appendix. In Figure 2, from the initial value at $1/\beta$, the lending interest rate falls and fluctuates widely when the economy receives adverse shocks (see Figure 2 in the Appendix). However, the path of price $Q$ is more erratic and this behavior is pronounced in states $s_2$ and $s_3$ (see figures 3, 4 and 5 of the Appendix).

To evaluate the behavior of asset prices when there is a change in agents’ beliefs, I repeat the previous exercise, but now keeping production and investment decisions fixed. The results of this exercise are displayed in figures 6, 7, and 8 of the Appendix. Accordingly, Figure 6 shows the evolution of the price of the Arrow security ($Q$) in the good state (growth equals to 1.05) for the first row of Matrix II. Figure 7 shows the results of simulating the path of $Q$ in the medium state (productivity equals to 0.98) for the second row of Matrix II and Figure 8 shows the path of $Q$ obtained in the bad state (productivity equals to 0.90). The transition probabilities in the last case were taken from the third row of Matrix II. Starting from the "true" probabilities at time 0, the price of the asset (Arrow security) changes with agents’ beliefs, where the level of the price decreases in the good states and it increases when the state of the economy is worsening and also it augments the discrepancy between the two agents. These results are interesting because they represent situations where uncertainty exacerbates agents’ disagreement about the state of the economy leading to higher distress. Even though, there is not a direct source for contagion, the results suggest that the distress can be spread because of agents’ beliefs, such as occurred during the crisis of 2007-2008. For a richer analysis I will calibrate the model in a next step.

<table>
<thead>
<tr>
<th>Variable</th>
<th>St. Dev</th>
<th>St. Dev</th>
<th>St. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 1.0$</td>
<td>0.0456</td>
<td>0.0499</td>
<td>0.0658</td>
</tr>
<tr>
<td>$\sigma = 1.5$</td>
<td>0.0731</td>
<td>0.1669</td>
<td>0.3112</td>
</tr>
<tr>
<td>$\sigma = 2.0$</td>
<td>0.0683</td>
<td>0.0793</td>
<td>0.1037</td>
</tr>
</tbody>
</table>

| $Q(s_1)$ | 0.0882 | 0.0942 | 0.0378 |

### 6 Preliminary Conclusion

With the recent crisis experience in mind, the main goal of this article is to examine contagion during times of financial distress in economies with different degree of financial integration. For this purpose I develop a theoretical model in the spirit of Gertler and Kiyotaki (2010) and Cogley et al. (2012) that represents
the case of a small economy that can borrow from the rest of the world. I analyze contagion under four financial structures, a fully integrated rest of the world and a benchmark economy with (almost) complete markets, incomplete markets in the foreign economy and the benchmark economy, a fully integrated rest of the world and incomplete markets in the domestic economy, and incomplete markets in both the domestic economy and the rest of the world. I consider agents with heterogeneous beliefs, I assume that the small economy is liquidity constrained because of the possibility of banks default. In this context, I find at least three sources of contagion when the economy receives bad news about the state of the economy in the rest of the world. The first source is a change in agents beliefs that causes asset prices to deviate from their equilibrium of full information. This movement in asset prices lead to a portfolio reallocation and to a wealth redistribution across agents. Although, agents have the possibility to take banks deposits that act as a "buffer", this resort is limited by market imperfections. The second source is an abrupt change foreign interest rates that can have adverse consequences on output and welfare and creates volatility. The third source is a liquidity constraint from foreign funds that reduces investment, output and welfare in the small economy, and eventually can be a source of severe distress if financial markets are incomplete. Conducting a numerical exercise, I find that volatility increases when agents disagree about the state of the foreign economy. Thus, comparing the path of foreign interest rates and the path of asset prices when the economy receives an adverse shock, the results show higher volatility in the latter than in the former. On the other hand, agents disagreement seems to increase in the bad states of the economy, following a rise in the asset price.

References


Appendix

**Proof.** Final goods are devoted to household’s consumption and investment for firms. Given that consumption and investment functions are additive, households’ consumption at aggregate level is the sum of consumption of agents of type 1 and consumption of agents of type 2, with \( c(s^t) = c_1(s^t) + c_2(s^t) \). Thus aggregate budget constraint in the domestic economy will be given by

\[
c(s^t) + \sum_{s_{t+1}} Q(s^{t+1})(a_1(s^{t+1})+a_2(s^{t+1})) + D(s^{t+1}) = w(s_t) + a_1(s^t) + a_2(s^t) + R^D(s^t)D(s^t)
\]

(29)

where, \( D(s^t) = D_1(s^t) + D_2(s^t) \) is the total amount of deposits in history \( s^t \) for all \( t \) and \( s^t \), and \( w(s_t) = \phi_1(s^t)w(s_t) + (1 - \phi_1(s^t))w(s_t) \) is aggregate wealth (household’s income), with \( w(s_t) = (Y(s_t) - R^d(s^t)L(s^t)) + (R^d(s^t)L(s^t) - (R^D(s^t)D(s^t))) \) according to definitions of sections 3.1, 3.3, and 3.5 (pages 9, 11, and 12). \( Y(s_t) \) in the last expression represents aggregate output in state \( s_t \) and \( L(s^t) \) is the total amount of banks loans in history \( s^t \).
Figure 1 – Financial Integration between Europe and the U.S. (1990 – 2007)

Figure 2 – Interest Rates (R)

Risk - Free Interest Rate

Interest Rate - Standard Deviation

Time
Figure 3 – Asset Prices with $\sigma = 1$

- Asset Prices (Q(s1))
- Asset Prices (Q(s2))
- Asset Prices (Q(s3))
Figure 4 – Asset Prices with $\sigma = 1.5$
Figure 5 – Asset Prices with \( \sigma = 2.0 \)
Figure 6 – Asset Prices in the Good State (s = 1.05)
Figure 7 – Asset Prices in a Contraction ($s = 0.98$)
Figure 8 – Asset Prices in the Bad State (s = 0.90)