When Should a Firm Expand Its Business?

The Signaling Implications of Business Expansion

Ana Espínola-Arredondo*  
School of Economic Sciences  
Washington State University  
Pullman, WA 99164

Esther Gal-Or†  
Katz Graduate School of Business  
University of Pittsburgh  
Pittsburgh, PA 15260

Félix Muñoz-García‡  
School of Economic Sciences  
Washington State University  
Pullman, WA 99164

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Abstract

We examine an incumbent’s trade-off between expanding her business, which increases her profits, and the information that such expansion signals to potential competitors, which attracts them to the market. Specifically, we consider a signaling game where the incumbent knows the actual realization of demand, whereas the entrant can only observe whether the incumbent decided to expand the size of her business in the past. In particular, we analyze the set of pooling and separating equilibria surviving the intuitive criterion in this signaling model. Our results can support the more predictable observation that only incumbents in good market conditions expand their businesses (separating equilibria), but also the less obvious and interesting pooling equilibria in which no firm expands her business and despite such non-expansion, entrants choosing to enter the market. This equilibrium result helps us provide an explanation for the high failure rates that new firms face when entering a market, as confirmed by multiple empirical studies.

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*Address: 205C Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: anaespinola@wsu.edu.
†Address: 368A Mervis Hall, University of Pittsburgh, Pittsburgh, PA 15260. E-mail: esther@katz.pitt.edu.
‡Address: 103G Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: fmunoz@wsu.edu.
1 Introduction

Firms regularly have to make decisions about expanding their businesses by acquiring new technologies, enlarging the size of their plant or hiring more workers. This expansion may have a direct effect on the firms’ profits by reducing costs or by facilitating serving a larger group of customers. However, it may also have an indirect effect, because of the role it plays in signaling market conditions to potential competitors. In this paper we investigate the trade-offs introduced by these two effects. We explore the signaling role that expansion plays in communicating market demand to potential entrants, and how an incumbent’s expansion decision is affected by the potential entry of competitors.

Our model is especially relevant in growing markets, where potential entrants have very little information about the state of the demand. This is valid in particular in markets that have been monopolized for a while, thus making it difficult for new companies to assess the prospects of entering the market. The potential investors, in this case, watch carefully actions taken by the incumbent in attempting to evaluate market conditions. The expansion decision of the incumbent is one such action.

Given our objective in this paper, we examine a signaling model where the incumbent privately observes the actual realization of market demand (either high or low) and decides whether to expand her business. The expansion is observed by the entrant who uses it as a signal to infer the true state of demand. After updating his beliefs of market demand the entrant decides whether to enter the market. Specifically, we analyze the set of parameter values which support pooling and separating equilibria in this signaling game.\footnote{In addition, the paper shows that, under certain parameter conditions, both pooling and separating equilibria survive the Cho and Kreps’ (1987) Intuitive Criterion, as opposed to most models on entry-deterrence in which only separating equilibria survive such a refinement.}

The first type of pooling equilibrium we derive has both high and low-demand incumbents expanding their businesses. This equilibrium can explain the expansion decision of firms in markets with overall strong levels of demand. In certain markets even the most pessimistic estimates of the demand are still sufficiently strong to support expansion of current businesses, even when they anticipate additional entry of new competitors. This is the case, for instance, for some electronic markets, such as the MP3 players. Growth expectations of this market in 2008 varied significantly, with International Data Corporation predicting a growth rate of 75% per year until 2009 (Newratings.com), and other sources being less optimistic and predicting a growth rate of 10% (see In-Stat.com, a research group focused on communication industries). Both estimates were sufficiently strong to support additional entry to the industry at the time.

In the second type of pooling equilibrium we identify, both types of incumbents do not expand their businesses, and despite this lack of a favorable signal about market demand, entrants enter the market. This type of equilibrium is supported by some stylized empirical results obtained in
the literature of industrial economics, showing that new entrants comprise a significant share of
many US industries (about 20% according to Dennis (1997)). New entrants, however, are not
only attracted to highly profitable but also to unprofitable markets. For instance, Bernardo and
Welch (1997) estimate that 75% of entrants in the US die within their first five years and Mata and
Portugal (1994) found a 50% failure rate of Portuguese firms within their first four years. The above
empirical results support the former pooling equilibrium, where entrants base their entry decision
on some preliminary, imprecise information about the demand, without being able to update this
information from the business decisions of established companies. In particular, they may choose to
invest even when actions taken the established companies give no indication of favorable prospects
in the market.\footnote{As we show in section five, incumbents’ expansion decision in this pooling equilibrium could also be supported even in the absence of entry threats, and it cannot be exclusively attributed to entry threats from potential competitors. This pooling equilibrium can nevertheless be sustained for multiple parameter values, and it can explain entrants’ behavior in expanding markets, as suggested above.}

This pattern of entry has been previously analyzed using arguments different from those we
consider in this paper. For example, some contributions to the literature have explained this entry
phenomenon using arguments related to “network externalities”; namely the expanded customer
traffic that is induced by the joint location of companies in the same area.\footnote{Eaton and Lipsey (1979), Stahl (1982) and Wolinsky (1983). For a summary of the main contributions in this literature see Fujita and Thisse (2002).} Another explanation
was offered by Camerer and Lovallo (1999) who suggest that entrants may decide to enter (even
unprofitable) markets because they are overly confident of their abilities. Our paper provides an
explanation that does not rely on behavioral reasons. It is simply the result of lack of information
on the part of potential investors that leads to their entry to sometimes unprofitable markets, and
not their overconfidence.

We also identify regions of parameter values for which separating equilibria can be supported
where only an incumbent facing high demand expands her business. In these separating (fully
informative) equilibria the entrant infers market conditions from the incumbent’s expansion deci-
sion, and enters the market only if demand is high. Empirical evidence in the banking industry
can be explained using this theoretical result. For instance, Berger and Dick (2004) show that the
expansion decision of banks already operating in particular locations (incumbent’s investment in
new branches) is highly correlated with the entry of new banks to the industry. In particular, using
data from the Federal Deposit Insurance Corporation (FDIC), they demonstrate that early entrants
in the banking industry experience high growth in their deposit market shares, which induces them
to open new branches in the relevant geographic market where they compete. Entrants then open
branches as well, and this expansion and entry process diminishes until no more banks enter into
the market.

Finally, we describe the opposite type of separating equilibrium, whereby the incumbent facing
low demand is the only one expanding her business. However this equilibrium can only be supported
if the expansion costs facing high demand incumbents are higher than those facing low demand incumbents. Note, however, that this requirement is not very realistic. It is much more likely that financial institutions offer more favorable terms to companies facing better prospects. Hence, this type of a separating strategy profile is unlikely to be supported as an equilibrium of the signaling game.

We conduct a comparative static analysis to investigate how the range of parameter values supporting each type of equilibria changes when the extent of cost reduction that is facilitated by business expansion varies. In particular, we show that the region of parameter values supporting separating equilibria expands and that supporting the first type of no-expansion pooling equilibria contracts as the extent of the cost-reduction effect is more significant. This implies, in particular, that any government policy that encourages the acquisition of cost reducing technologies will lead to a better informed entry decision by potential competitors, as they can infer more information about the prospects of markets from actions taken by their participants.

We finally examine the set of equilibria which could be supported in the absence of entry threats. In particular, we identify those equilibria that can be sustained with entry threats, but which cannot be supported without them: the pooling equilibrium in which both types of incumbents expand, and the separating equilibrium in which only the incumbent facing high demand expands. Hence, the introduction of entry threats from potential competitors induces incumbents to strategically expand their businesses under larger sets of parameter values, relative to the context without entry threats.

Several existing models in the literature analyze entry deterrence in complete information environments. Examples include Spence’s (1977) paper on capacity commitment, Dixit’s (1980) model on capacity accumulation, and Eaton and Lipsey’s (1980) analysis of how potential entry affects investment in capital by an incumbent when this capital depreciates over time. Our model, however, is closer to that of Milgrom and Roberts (1982), where an incumbent may use limit pricing (by higher production levels) as a cost-signaling device to inform potential entrants about her cost structure. In particular, given that increasing production during the pre-entry period is more costly for high-cost than for low-cost firms, they find a separating equilibrium in which only low-cost firms reduce prices (by increasing production) during the pre-entry period. Importantly, in Milgrom and Robert entrants prefer to enter markets with high-cost incumbents, while incumbents prefer to be low-cost. In contrast, in our model both incumbents and entrants prefer to operate in high-demand markets, which limits the potential for separating equilibria to arise as the unique equilibrium surviving the intuitive criterion. Instead, both separating and pooling strategy profiles can be sustained as equilibria of this game.

Fudenberg and Tirole (1986) analyze how players may want to use “signal jamming” when choosing actions that can transmit information to a potential entrant. In their model, the incumbent’s characteristics are common knowledge, while the entrant is uncertain about his own future profitability. The latter assumption is similar to our model whereas the former is not, since in our model the incumbent’s characteristics are privately observed by him.
Gal-Or (1987) finds similar results as Milgrom and Roberts (1982) for a signaling game in which the leader in an oligopolistic market uses its production decision to signal market demand to the follower. Specifically, this paper identifies the set of separating equilibria in which the leader perfectly reveals market demand to the follower, and compares both firms’ equilibrium profits with those they obtain in complete information environments. Hence, signals are used by the leader to reduce the follower’s production in the second stage. Unlike Gal-Or (1987), however, we assume that signals are used by incumbents as an entry-deterrence device, and in addition, we are interested in the existence of pooling equilibria.

On the other hand, Harrington (1986) modifies Milgrom and Roberts’ (1985) model by allowing the possibility that the entrant is uncertain about his costs after entry. Interestingly, this paper shows that when the entrant and the incumbent costs are sufficiently positively correlated then Milgrom and Roberts’ (1985) results are reversed. That is, the incumbent’s production is below the simple monopoly output in order to strategically deter entry. The intuition is that this low production now conveys information not only of the incumbent’s but also of the entrant’s costs. Since the potential entrant is uncertain of his costs and these costs are highly correlated with those of the incumbent, he infers that his costs are high, thus deterring him from the market. Our model is different from Harrington (1986) since here both firms know each others’ costs, but the entrant is uninformed about market demand.

Another related paper which is probably closest in spirit to ours is Matthews and Mirman (1983). This paper analyzes a signaling model where the incumbent sets prices that can communicate some information about market profitability to potential entrants (as in our model). However, the actual price in the market (which is also the message observed by the potential entrant) receives a random shock, since the incumbent sets its price before demand is actually realized. This assumption is different from our model, where demand is assumed to be perfectly known by the incumbent across periods.

This paper is organized as follows. In the next section, we describe the signaling model, as well as the firms’ incentives under each market condition. In Section 3 we analyze the equilibria and section 4 discusses some policy implications. Section 5 examines which equilibria are worthwhile in the absence of entry threats, relating them with those supported only under entry threats. In section 6 we conclude and offer some extensions of the model. The Appendix includes the proofs of all lemmas and propositions.

2 Model

Let us consider a monopolist (female) who operates as the incumbent in a market. This incumbent privately observes the current level of market demand, either high or low; with inverse demand function given, respectively, as \( p(Q) = a_H - bQ \) or \( p(Q) = a_L - bQ \), with \( a_H > a_L \). An entrant
(male) considers whether to enter this market (despite of not directly observing the realization of market demand) or to operate in an alternative market where demand is fully observable and given, by \((a - bQ)\) and where an existing incumbent operates. Competition in both markets is à la Cournot. The entrant would like to enter the incumbent’s market when demand is high, but operate in the alternative market when demand is low, \(^5\) i.e., \(a_H > a > a_L\). Both firms’ average and marginal costs of production are \(c > 0\), except when stated otherwise. The time structure of this incomplete information game is described as follows.

1. Nature decides the realization of market demand, either \(a_H\) or \(a_L\) with probabilities \(p\) and \(1 - p\), respectively. This realization is observed by the incumbent, but not by the entrant.

2. Observing the actual level of demand, the incumbent decides whether to expand her business.

3. Observing the incumbent’s expansion (or no expansion) decision, the entrant forms beliefs about the level of market demand. Let \(\mu (H|E)\) and \(\mu (H|NE)\) denote the entrant’s posterior beliefs about a high level of demand after he observes, respectively, an expansion or no expansion of the incumbent business.

4. Given these beliefs, the entrant chooses whether to enter the incumbent’s market or an alternative market in which demand is perfectly observed by the entrant.

We assume that expansion is costly and designate by \(C_H\) and \(C_L\) the expansion cost of the incumbent when demand is high and low, respectively. We do not impose any restrictions on the relationship between \(C_H\) and \(C_L\). In particular, when \(C_H \neq C_L\) we implicitly assume that financial institutions can observe the state of the demand facing the firm and charge her different\(^6\) financing cost contingent upon the state of demand. If, however, \(C_H = C_L\), such price discrimination is not feasible and both types of firm face the same expansion cost.

As a result of expanding her business the incumbent can reduce her unit cost of production. Specifically, we assume that when the incumbent does not expand her business, both firms’ average and marginal costs, \(c\), coincide. When the incumbent decides to expand her business, however, her per-unit cost decreases from \(c\) to \(c_1\). In contrast, the entrant cannot benefit by acquiring the new technology, since he lacks the incumbent’s experience in the industry. Note that when the entrant does not enter the incumbent’s market, the latter’s monopoly profit in case of high-demand markets is \(\frac{(a_H - c)^2}{4b}\) when she does not incur the cost-reducing expansion and changes to \(\frac{(a_H - c_1)^2}{4b} - C_H\) when she does expand her business. Similar expressions can be obtained for the case of a low-demand market. On the other hand, when the entrant enters the incumbent’s market, the firms compete

\(^5\) Considering a general payoff \(\pi\) for the entrant from operating in this alternative market (which allows for any form of market structure) would not change the direction of our results, as long as the entrant prefers to enter the incumbent’s market when demand is high, and the alternative market when demand is low.

\(^6\) If, in contrast to the entrant, financial institutions can observe the state of the demand, it is reasonable to assume that \(C_H < C_L\), since the cost of financing is likely to be lower for high demand than low demand incumbents.
as Cournot duopolists. In the absence of expansion, the profits for both the incumbent and the entrant are \( \frac{(a_H-c)^2}{9b} \). However, if the incumbent expands her business, the firms compete with different costs. In this case, the profits for the incumbent are \( \frac{(a_H-2c+c_1)^2}{9b} - C_H \), and those of the entrant are \( \frac{(a_H-2c+c_1)^2}{9b} \) when demand is high, and similar expressions can be derived in the case of a low-demand market.

After observing expansion, we assume that the entrant wants to enter when demand is high, \( \frac{(a_H-2c+c_1)^2}{9b} > \frac{(a-c)^2}{9b} \), i.e., \( a_H > a + c - c_1 \), but prefers to operate in the alternative market when demand is low, \( \frac{(a_L-2c+c_1)^2}{9b} < \frac{(a-c)^2}{9b} \), i.e., \( a_L > a + c - c_1 \). And similarly, when the incumbent does not expand her business, the entrant enters when demand is high, \( \frac{(a_H-c_1)^2}{9b} > \frac{(a-c)^2}{9b} \), i.e., \( a_H > a \), but would operate in the alternative market when demand is low, \( \frac{(a_L-c_1)^2}{9b} < \frac{(a-c)^2}{9b} \), i.e., \( a_L < a \). Finally, in the case of entry, the entrant observes market demand and chooses her optimal output, which is positive only if \( \frac{(a_K-2c+c_1)^2}{9b} > 0 \), i.e., \( a_K > 2c - c_1 \). Note that, when demand is high \( a_H > 2c - c_1 \) is satisfied since \( a_H > a + c - c_1 > 2c - c_1 \), but when demand is low the entrant produces positive amounts if \( a + c - c_1 > a_L > 2c - c_1 \), and remains inactive when \( a_L < 2c - c_1 \).

## 3 Equilibrium analysis

Recall that there are two possible effects of the expansion decision of the incumbent. The direct effect is to reduce the incumbent’s unit cost and the indirect effect is to determine the extent of information about market demand that is available to the entrant, and as a result, his entry decision. The incumbent’s benefit from expanding her business when such expansion induces entry can therefore be represented as follows.

\[
B_E^K = \begin{cases} 
\frac{\left(\frac{(a_K-c_1)^2}{4b} - \frac{(a_K-c)^2}{4b}\right)}{\frac{(a_K-c_1)^2}{4b} - \frac{(a_K-2c+c_1)^2}{9b}} & \text{for all } a_H, \text{ and if } a_L > 2c - c_1 \\
\frac{\left(\frac{(a_L-c_1)^2}{4b} - \frac{(a_L-c)^2}{4b}\right)}{\frac{(a_L-c_1)^2}{4b} - \frac{(a_L-2c+c_1)^2}{9b}} & \text{if } a_L < 2c - c_1
\end{cases}
\]

where \( K = \{H, L\} \), and the superscript \( E \) denotes an expansion that causes entry (no entry-deterrence effect). First, note that if the incumbent faces low demand which is sufficiently small, \( a_L < 2c - c_1 \), then the entrant remains inactive after entry (produces zero units). In this case, the incumbent is still the only producer in the market, and her benefits from expanding her business are restricted to the cost-reducing effect alone. In contrast, when the incumbent faces low demand but \( a_L > 2c - c_1 \), the entrant produces a positive output level after entry. The incumbent’s benefits from expansion in this case can be divided into two components: first, the purely technological

\footnote{Interestingly, note an important distinction between the above payoff structure and that in standard entry-deterrence games analyzed in the literature on signaling, such as Milgrom and Roberts (1982). In particular, in this previous paper the incumbent’s and entrant’s profits move in opposite directions as the incumbent’s private information changes. For example, the incumbent’s profits decrease in her private costs, whereas the entrant’s profits increase in the incumbent’s costs. In contrast, in our paper both firms’ profits increase with the private information concerning the state of market demand.}
benefit from the expansion, measuring the additional benefits that the incumbent obtains from the cost-reducing effect of the expansion, for a given market structure. And second, the incumbent experiences a loss in profits from the competition of new firms in the industry, for a given cost structure. On the other hand, if expansion deters entry, the incumbent’s benefit from expansion can be described as follows:

\[
B_{ED}^K = \left( \frac{(a_K - c)^2}{4b} - \frac{(a_K - c)^2}{9b} \right) + \left( \frac{(a_K - c_1)^2}{4b} - \frac{(a_K - c)^2}{4b} \right)
\]

where \( K = \{H, L\} \), and the superscript \( ED \) denotes that expansion has entry deterrence effects. Incumbent’s benefits from expanding her business can be divided again into two components: first, the incumbent protects her market relative to the case of entry, for her initial costs, \( c \), i.e., she does not have to share her market with the entrant. And secondly, she experiences a cost-reduction effect arising from her expansion decision, for a given market structure.

**Lemma 1.** In the expansion signaling game, the benefits from expansion satisfy:

1. The benefits from expansion when the incumbent achieves entry deterrence are higher than when she does not achieve deterrence, for every state of demand, i.e., \( B_{ED}^K > B_{E}^K \) for all \( K = \{H, L\} \).

2. The incumbent’s benefits from expanding her business are increasing in market demand when the expansion deters entry, i.e., \( B_{ED}^H > B_{ED}^L > 0 \) for all parameter values.

3. When expansion does not deter entry, incumbent’s benefits from expanding her business are increasing in market demand, \( B_{E}^H > B_{E}^L \), if and only if \( c_1 < \frac{26c - 5(a_H + a_L)}{16} \).

Hence, when the expansion deters entry, the high demand incumbent faces a stronger incentive to expand her business than the low demand incumbent does, \( B_{ED}^H > B_{ED}^L \). When the expansion does not deter entry, however, this result holds only if the cost-reducing effects from the expansion are strong enough, i.e., if the post-expansion marginal cost \( c_1 \) becomes low enough. Let us now analyze the equilibria in this signaling game with cost-reducing expansions. We start with the set of \((C_H, C_L)\) pairs that yield separating equilibria.

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8 We refer to this component as the benefit from the expansion when there is no threat of entry, and denote it as \( B_{NT}^K = \frac{(a_K - c_1)^2}{4b} - \frac{(a_K - c)^2}{46} \).

9 In addition, note that \( B_{E}^K \) can be negative if the cost-reduction effect is sufficiently small. That is, when the reduction of profits of the incumbent due to entry is larger than the cost-reduction effect she experiences from her expansion, then the benefits from such an expansion \( B_{E}^K \) are clearly negative. We examine this case in the following section.

10 Unlike \( B_{E}^K \), \( B_{ED}^K \) is positive for all parameter values.

11 This implies that the single-crossing property is not satisfied for all parameter values. As a consequence, both pooling and separating strategy profiles can be supported in equilibrium, and survive the Cho and Kreps’ (1987) intuitive criterion.
**Proposition 1.** In the expansion signaling game the following strategy profile can be supported as a separating PBE:

1. the incumbent chooses \((E_H, NE_L)\), and

2. the entrant selects \((In_E, Out_{NE})\) if and only if \(C_H < B^E_H\) and \(C_L > B^E_L\); and his beliefs are \(\mu (H|E) = 1\) and \(\mu (H|NE) = 0\).

To understand the intuition of proposition 1, note that in this signaling equilibrium the entrant considers that any expansion must only come from a high-demand incumbent, which leads the entrant to enter. In contrast, when the entrant does not observe any expansion, he assigns full probability to the low-demand market, and he does not enter such a market. Given these incentives of the entrant, the incumbent decides to expand her business when demand is high if and only if her expansion costs, \(C_H\), are sufficiently low, \(C_H < B^E_H\). In this specific case the expansion has no entry-deterrence effects (since it attracts potential entrants), but only cost-reduction effects. Hence, the condition \(C_H < B^E_H\) intuitively specifies that the incumbent expands when the costs from expanding her business are lower than the cost-reducing benefit arising from such an expansion. The opposite intuition is applicable for the incumbent operating in a low-demand market.

Notice the policy implications that can be derived from such a result: government programs which raise the expansion costs (e.g. financial costs) of low-demand incumbents and decrease those of high-demand incumbents enhance the transmission of information about market conditions to potential entrants, thus inducing more competition in markets with high demand, and less competition in markets with low demand. Next, we examine the set of parameter values supporting separating equilibria in which the high-demand incumbent does not expand her business while the low-incumbent does.

**Proposition 2.** In the expansion signaling game the following strategy profile can be supported as a separating PBE:

1. the incumbent chooses \((NE_H, E_L)\), and

2. the entrant selects \((Out_E, In_{NE})\) if and only if \(C_H > B^E_D\) and \(C_L < B^E_L\); and his beliefs are \(\mu (H|E) = 0\) and \(\mu (H|NE) = 1\).

Therefore, in these separating equilibria (as in that of Proposition 1), the entrant perfectly infers the state of demand from the incumbent’s actions. As a consequence, entry occurs after observing that the incumbent did not expand its business (when demand is high), but stays out of the market after observing that the incumbent expanded its business (since demand is then low). Note, however, that this type of counterintuitive separating equilibrium exists, only if \(C_H > C_L\), an assumption that is unlikely to be observed in the market.
Before analyzing the pooling PBE of this signaling game, we introduce some additional notation. In particular, let \( p^{\text{IdenC}} \) denote the probability that makes an entrant indifferent between the expected profits from Cournot competition with identical costs, \( p \frac{(a_H-c)^2}{96} + (1-p) \frac{(a_L-c)^2}{96} \), and profits from operating in the alternative market, \( \frac{(a-c)^2}{96} \); where the superscript \( \text{IdenC} \) represents the Cournot market in which both incumbent and entrant compete with identical costs structures. Similarly, the probability that makes an entrant indifferent between the expected profits from Cournot competition in which the incumbent has reduced costs, \( p \frac{(a_L-2c+c1)^2}{96} + (1-p) \frac{(a_H-2c+c1)^2}{96} \), and the profits from operating in the alternative market, \( \frac{(a-c)^2}{96} \), is denoted by \( p^{\text{DiffC}} \), where the superscript \( \text{DiffC} \) represents the Cournot market in which the incumbent and entrant compete with different cost structures.

Proposition 3. In the expansion signaling game the following strategy profile can be supported as a pooling PBE:

1. the incumbent chooses \((NE_H, NE_L)\), and
2. the entrant selects \((Out_E, In_{NE})\) if and only if \( C_H > B^{ED}_H \) and \( C_L > B^{ED}_L \); and his beliefs are \( \mu(H|NE) = p > p^{\text{IdenC}} \) and \( \mu(H|E) < p^{\text{DiffC}} \).

Thus, in these pooling equilibria the entrant enters after observing the equilibrium message of no-expansion, if in addition, the prior probability that the market is in high demand is sufficiently high, i.e., \( p \) exceeds the cutoff \( p^{\text{IdenC}} \). After observing the (out-of-equilibrium) message of expansion he does not enter, since his beliefs satisfy \( \mu(H|E) < p^{\text{DiffC}} \). That is, only if he assigns a sufficiently low probability to the market being in high demand after observing a expansion, i.e., if he is relatively pessimistic. Finally, both types of incumbents do not expand their businesses since their expansion costs, \( C_H \) and \( C_L \), are higher than the benefits from such an expansion, i.e. \( C_H > B^{ED}_H \) and \( C_L > B^{ED}_L \). Note that in this case, incumbent’s benefit from expansion comprises of both deterring the competitor (since the entrant would not enter) and reducing her unit cost. As commented above, this pooling equilibrium can rationalize the observation that due to high financing costs, small firms may decide not to expand their businesses; and in spite of this “negative” news transmitted to the entrant, additional firms join the market.\(^{13}\) Let us finally analyze the pooling equilibria for which both types of incumbents expand their businesses.

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\(^{12}\)The expression for \( p^{\text{IdenC}} \) is obtained by solving for \( p \) in this indifference condition. It is included in the appendix.

\(^{13}\)Indeed, examples abound of new entrants investing despite of no expansion by existing businesses in the area. The literature has suggested the existence of network externalities to explain such entry decisions. This paper provides an additional argument to explain entry into such “crowded” markets, even in the absence of network externalities.
Proposition 4. In the expansion signaling game the following strategy profile can be supported as a pooling PBE:

1. the incumbent chooses \((E_H, E_L)\), and

2. the entrant selects \((\text{In}_E, \text{Out}_{NE})\) if and only if \(C_H < B_H^E\) and \(C_L < B_L^E\); and his beliefs are \(\mu(H|E) = p > p^{\text{DiffC}}\) and \(\mu(H|NE) < p^{\text{IdenC}}\).

Hence, in these pooling equilibria the entrant enters only after observing the equilibrium message of expansion, if in addition, the prior probability that the market is in high demand is very high, \(p > p^{\text{DiffC}}\). Otherwise he does not enter when his out-of-equilibrium beliefs (after observing no expansion), \(\mu(H|NE)\), are highly pessimistic, and assign a very low probability to the market being in high demand, i.e., \(\mu(H|NE) < p^{\text{IdenC}}\). On the other hand, both types of incumbent expand their businesses since the costs from doing so, \(C_H\) and \(C_L\), are lower than the benefits from such an expansion. In particular, note that the benefit from this expansion is derived only from its cost-reducing effects, since the expansion attracts entrants. Therefore, the conditions \(C_H < B_H^E\) and \(C_L < B_L^E\) capture the fact that the incumbents’ costs from expanding their businesses are lower than the cost-reducing benefits arising from such an expansion. The pooling equilibrium is then supported for very restrictive parameter values and beliefs, and as we show in the comparative statics section, it is eliminated if the cost-reducing benefits from the expansion are small, i.e., if the expansion does not significantly reduce the incumbent’s marginal costs.

Finally, we show that, under certain conditions, all the equilibria described in Propositions 1-4 survive the Cho and Kreps’ (1987) intuitive criterion.

Lemma 2. The equilibria characterized in Propositions 2-3 satisfy Cho and Kreps’ (1987) for all parameter values. In contrast, the separating equilibrium described in Proposition 1 satisfies the intuitive criterion, only if \(a_L < 2c - c_1\). The pooling equilibrium identified in Proposition 4 survives the intuitive criterion only if \(a_L > 2c - c_1\).

That is, the separating equilibrium where only the high demand incumbent expands her business survives the intuitive criterion if low demand is sufficiently small, \(a_L < 2c - c_1\). Under such condition, the incumbent facing low demand does not benefit from deviating away from her equilibrium strategy, regardless of the entrant’s beliefs after observing such deviation (i.e., all off-the-equilibrium strategies are equilibrium dominated). Similarly, the pooling equilibrium in which both types of incumbents expand their businesses survives the intuitive criterion only when low demand is sufficiently high, \(a_L > 2c - c_1\). Otherwise, when \(a_L < 2c - c_1\) the entrant remains inactive after entering the market (zero production), and the incumbent facing low demand does not have incentives to expand her business in order to protect it from potential entrants.
4 Comparative statics and discussion

In the previous section, we identify different PBE in the signaling game when the incumbents’ expansion reduces her marginal costs below those of the entrant. Specifically, we find the benefits that the incumbent obtains from expanding her business when such expansion deters entry, $B_{EH}^{ED}$ and $B_{EL}^{ED}$, and when it does not, $B_{EH}^E$ and $B_{EL}^E$. Given these results, in figures 1(a) and 1(b) we illustrate the set of parameter values supporting the equilibria identified in the previous section. In particular, figure 1(a) depicts the case in which $B_{EH}^{ED} > B_{EL}^{ED}$, which holds if $c_1$ is low enough as specified in lemma 1, $c_1 < \frac{26c-5(aH+aL)}{16}$, whereas figure 1(b) represents the case that $B_{EH}^{ED} < B_{EL}^{ED}$.\footnote{From lemma 1 recall that, when expansion deters entry, the incumbent’s benefits satisfy $B_{EH}^{ED} > B_{EL}^{ED} > 0$ for all parameter values.}

![Figure 1(a). Set of equilibria for relatively low $c_1$](image1)

![Figure 1(b). Set of equilibria for relatively high $c_1$](image2)

Note that the grey $\perp$-shaped area of the figure represents parameter values for which at least one of the firms randomizes with strictly positive probability (semi-separating equilibria). We restrict our attention to equilibria in which players use pure strategies, but include the semi-separating equilibria of the game in the appendix. In the next lemma we examine how the cutoffs in figures 1(a) and 1(b) decrease in the incumbent’s post-expansion marginal costs, $c_1$.

**Lemma 3** The incumbent’s benefits from expanding her business, $B_{EH}^{ED}$ and $B_{EH}^E$, decrease in the incumbent’s (after-expansion) marginal costs, $c_1$, i.e., $\frac{\partial B_{EH}^{ED}}{\partial c_1} < 0$ and $\frac{\partial B_{EH}^E}{\partial c_1} < 0$ for all $K = \{L, H\}$. Additionally, a given increase in $c_1$ induces a greater decrease in $B_{EH}^{ED}$ ($B_{EH}^E$) than in $B_{EL}^{ED}$ ($B_{EL}^E$).

Graphically, when $c_1$ increases, cutoffs $B_{EH}^E$ and $B_{EH}^{ED}$ (in the vertical axis) shift downwards, and cutoffs $B_{EL}^E$ and $B_{EL}^{ED}$ (in the horizontal axis) shift leftward. These shifts enlarge the set supporting
the pooling PBE in which no type of incumbent expands her business. Furthermore, a given increase in \( c_1 \) induces a larger downward shift in cutoff \( B_H^E \) than the leftward shift it provokes in \( B_L^E \). That is, \( B_H^E \) is more responsive than \( B_L^E \) to the cost-reducing effects of the expansion. Similarly for \( B_H^{ED} \) and \( B_L^{ED} \). Let us now examine how the set of equilibria varies as \( c_1 \) increases.

**Lemma 4.** In the expansion signaling game,

1. If \( c_1 < 2c - a_L \) all equilibria survive the Cho and Kreps’ (1987) intuitive criterion, except for the pooling equilibrium in which both incumbents expand their business.

2. If \( c_1 > 2c - a_L \), the separating equilibria in which only the high-demand incumbent expands her business violates Cho and Kreps’ (1987) intuitive criterion. All other equilibria still survive the Cho and Kreps’ (1987) intuitive criterion, under the conditions specified in Propositions 1-4.

3. If \( c_1 \in \left( \frac{5c-a_H}{4} , \frac{5c-a_L}{4} \right) \), then \( B_H^E < 0 \) but \( B_L^E > 0 \).

4. If \( c_1 > \frac{5c-a_L}{4} \), then both \( B_H^E < 0 \) and \( B_L^E < 0 \).

Lemma 4 specifies how the set of strategy profiles described in figure 1(a) and 1(b) shrinks as \( c_1 \) increases, i.e., as the cost-reducing effect of the expansion becomes smaller.\(^{15}\)

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\(^{15}\)Recall from lemma 1 that the benefits from entry deterrence \( B_H^{ED} \) and \( B_L^{ED} \) are both positive for all parameter values.

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**Figure 2.** Set of PBEs when \( c_1 \in \left( \frac{5c-a_H}{4} , \frac{5c-a_L}{4} \right) \)

**Figure 3.** Set of PBEs when \( c_1 > \frac{5c-a_L}{4} \)

First, when \( c_1 \in \left( \frac{5c-a_H}{4} , \frac{5c-a_L}{4} \right) \) the cutoff \( B_H^E \) becomes negative while \( B_L^E \) is still positive. As a consequence, neither the separating equilibria (in the lower right-hand corner of figure 1a) nor...
the pooling equilibria (in the lower left-hand corner of the figure) can be sustained. Figure 2 above illustrates this case. (Note that the results from Lemma 4 can alternatively be interpreted in terms of successive increments in the low demand, \( a_L \), for a given cost structure). When \( c_1 \) increases beyond \( \frac{5c-a_L}{4} \) cutoff \( B_L^E \) also becomes negative. This implies that the separating equilibria in the upper left-hand corner of figure 2 cannot be supported either. As a result, the only equilibrium that can be supported in pure strategies is the pooling equilibrium in which neither type of incumbent expands but entry nevertheless takes place. This result is described in figure 3, where \( c_1 > \frac{5c-a_L}{4} \).

Finally, note that the previous explanation can also rationalize firms’ behavior in the limiting case in which expansion do not have any cost-reducing effect, i.e., when \( c_1 \rightarrow c \). Indeed, in this case, \( B_K^E \) is negative, both for the high and low-demand incumbent, and \( \lim_{c_1 \rightarrow c} B_K^{EP} = \frac{5(a_K-c)}{366} > 0 \) for all \( K = \{H, L\} \). Hence, expansions without any cost reducing effects can be analyzed as an extreme case of point 4 in the previous lemma (illustrated in figure 3), since \( c_1 \rightarrow c \) implies \( c_1 > \frac{5c-a_L}{4} \). In particular, this eliminates the possibility for any type of separating equilibria as well as the pooling PBE where both types of incumbent expand their business, as represented in figure 3. As a consequence, expansions cannot be used as a signal to deter entry, and no type of incumbent chooses to expand her business.

Our results have some important policy implications. First, we find that reducing expansion costs (e.g., financial costs) of incumbents operating in high-demand markets, \( C_H \), and increasing those of incumbents in low-demand markets, \( C_L \), enhances the likelihood that separating equilibria will arise. At such equilibria, only the high-demand incumbent expands her business and attracts new entrants, whereas a low demand incumbent does not. This type of equilibrium is desirable for two reasons. First, it reduces the failure rate of new entrants, and second, it attracts more competition to profitable markets, to the advantage of consumers.

Second, we show that separating equilibria (where only the high-demand incumbent expands her business, and attracts potential competitors) can arise only if expansion facilitates a sufficient reduction in the costs of production of the incumbent. Hence, governmental policies supporting cost-reducing technologies may help the emergence of these separating equilibria.

### 4.1 High demand firms facing lower financial costs

For generality, we did not impose any restrictions on incumbents’ financial costs from expansion, \( C_H \) and \( C_L \). However, if financial institutions are able to discern high from low demand incumbents, they are likely to reduce the financial premium they charge to incumbents operating in high demand markets (lowering \( C_H \)). Under this condition, expansion costs satisfy \( C_H < C_L \) (parameter values below the \( 45^0 \)–line).

First, note that the separating equilibrium in which only the low-demand incumbent expands cannot be supported for expansion costs satisfying \( C_H < C_L \) (see figure 4 below). All other strategy
profiles can still be supported as equilibria of this signaling game. In particular, the fully-informative separating equilibrium, where only incumbents operating in high demand markets expand their business, is most likely to be observed. This is especially the case if companies facing high demand can benefit from significantly lower financing costs than those facing low demand.

Note also that if the financial costs facing either type of incumbent are comparable, in particular if $C_H = C_L$, the equilibria lies close to the 45°-line. Moreover, if those comparable costs are relatively high (i.e., $C_H > B_{H}^{ED}$), the only pure strategy equilibrium that can survive is the pooling equilibrium where neither type of incumbent expands but entry nevertheless takes place; see parameter values for which $C_H > B_{H}^{ED}$ in figures 4 and 5. However, as pointed out earlier, this outcome is feasible only if the prior probability of high demand is sufficiently high, i.e., $p > p^{IdenC}$.

5 Is expansion worthwhile in the absence of entry threats?

In previous sections we assume that the entrant can access the market after observing incumbents’ expansion decision. If, in contrast, there is no threat of entry, the incumbent’s benefits from expanding her business are restricted to its cost-reducing effects, since the incumbent produces at a lower marginal cost, $c_1$, maintaining her monopolistic position. In particular, when there are no entry threats ($NT$), the incumbent’s benefits from expanding her business are

$$B_{K}^{NT} = \frac{(a_K - c_1)^2}{4b} - \frac{(a_K - c)^2}{4b} \quad \text{for all} \ K = \{H, L\}$$
which is positive for all parameter values, and converges to zero when \( c_1 \to c \), i.e., when the incumbents’ marginal costs before and after the expansion coincide. Note that \( B_{K}^{ED} \) exceeds \( B_{K}^{NT} \) for both high and low demand, and for all parameter values. Indeed, when the incumbent deters entry, the benefits from her expansion, \( B_{K}^{ED} \), are not only technological (reducing her marginal costs from \( c \) to \( c_1 \)) but also strategic, since she retains a larger market share by deterring entry. When expansion has no entry deterrence effects, and \( a_L < 2c - c_1 \) the entrant remains inactive after entering the market, which implies that the incumbent’s benefits from expanding her business when there are entry threats, \( B_{L}^{E} \), coincide with those when entry is not possible, \( B_{L}^{NT} \), i.e., \( B_{L}^{NT} = B_{L}^{E} \). In contrast, when \( a_L > 2c - c_1 \), the entrant produces positive amounts after entry, reducing the incumbent’s profits, i.e., \( B_{L}^{E} < B_{L}^{NT} \).

In the absence of entry threats, expansion is worthwhile for both types of incumbents if and only if \( B_{K}^{NT} > C_{K} \) for all \( K = \{H, L\} \). We refer to this case as Regime 1. If, instead, \( B_{H}^{NT} > C_{H} \) and \( B_{L}^{NT} < C_{L} \), expansion is only profitable for the incumbent facing high demand (referred as Regime 2). Finally, if \( B_{K}^{NT} < C_{K} \) holds for all \( K = \{H, L\} \), expansion is not worthwhile for any type of incumbent (Regime 3). Let us next examine which of the equilibria identified in Propositions 1-4 can be supported under the costs of expansion leading to each of these regimes.\(^{16}\) We are specifically interested in those equilibria that arise when incumbents face entry threats (Propositions 1-4), but cannot be supported in the absence of threats of entry (i.e., when the expansion benefits are only given by \( B_{K}^{NT} \)).

**Proposition 5.** In the expansion signaling game,

1. If expansion is optimal for both incumbents in the absence of entry threats, an additional equilibrium \((E_H, NE_L)\) can be sustained when incumbents face entry threats, if and only if \( a_L \in (2c - c_1, a + c - c_1) \), \( a_H \in (a + c - c_1, 5c - 4c_1) \), and expansion costs satisfy \( C_{L} \in (B_{L}^{E}, B_{L}^{NT}) \) and \( C_{H} < B_{H}^{E} \). This equilibrium, however, does not survive Cho and Kreps’ (1987) intuitive criterion.

2. When expansion is not optimal for any incumbent in the absence of entry threats, and if expansion is only optimal for the incumbent facing high demand, no additional equilibrium can be supported.

Intuitively, when both incumbents expand their businesses if they do not face entry threats, the separating equilibrium \((E_H, NE_L)\) can be supported with entry threats if high demand is relatively stronger than low demand, and if the expansion costs of the incumbent facing high demand, \( C_{H} \), are relatively lower than those of the firm facing low demand, \( C_{L} \). Indeed, under such parameter values, expansion is still worthwhile for the incumbent facing high demand, but it is not optimal.
for the low-demand incumbent, who can protect her market from potential competitors if she does not expand, i.e., she separates from the high-demand incumbent.

Note an implication of the results. Only the separating equilibrium \((E_H, NE_L)\) can be supported when incumbents face entry threats from potential competitors, but could not be sustained without entry threats. On the contrary, all other equilibria in Propositions 2-4 can be supported even when incumbents do not face entry threats. That is, incumbents’ expansion decision in such equilibria could be explained because of the cost-reducing benefit of the expansion alone.

6 Conclusions

We investigate equilibria in an expansion signaling game, where an incumbent weighs the benefits from a cost-reducing expansion against the consequence that expansions may signal market conditions to potential competitors. We identify both fully informative, separating equilibria and pooling equilibria that survive the intuitive criterion. At the separating equilibria, only an incumbent facing high demand expands her business and the entrant, upon observation of expansion, enters the market. In one type of pooling equilibrium we derive, the incumbent always expands her business and the entrant decides to enter nevertheless (when her priors are sufficiently high, i.e., “optimistic,” about market conditions). This type of pooling equilibrium would become more likely if we introduced additional “network externalities” in sales (i.e., joint location leading to enhanced customer traffic at the location) in our model. The set of parameter values supporting this type of pooling equilibrium would expand since both the profits of the entrant upon entry and the incumbent’s benefit from expanding her business, even absence entry deterrence \((B^E)\), increase with network externalities.

Another type of pooling equilibria we derive has the incumbent not expanding her business irrespective of the state of the demand, and despite lack of information about the state of the demand entry takes place. A caveat of this result, however, is that it could be supported even in the absence of entry threats. That is, the equilibrium can be explained solely because of the cost reducing benefits of the expansion. This equilibrium result can nevertheless be sustained for multiple parameter values in economic contexts such as those considered in this paper. In particular, it can explain entrants’ behavior in expanding markets, providing an explanation for the high failure rate of new businesses, without the need to rely on behavioral explanations such as those in the literature on overconfidence. In fact, introducing the reasoning of Camerer and Lovallo (1999) in our model would only strengthen our results. The set of parameter values supporting the type of pooling equilibrium in which the entrant decides to enter (despite the absence of additional favorable information) would expand if entrants were overconfident.

Different extensions of this model would enhance its predictive power in more realistic settings. In particular, we assume that the financial market can perfectly observe the incumbent’s profits, and
actual demand levels. As a result, financial institutions do not have to use the expansion decision of the firm in order to infer the state of the demand. However, if market conditions are unobservable to the financial market, the expansion decision of the incumbent would be used as a signal to transmit information to two different audiences: the entrant and financial markets. Clearly, with signaling to two audiences, financial markets would favor firms whose demand is believed to be higher, thus increasing the likelihood of the separating equilibria identified in this paper. Intuitively, one would expect firms operating in high-demand markets to have stronger incentives to separate themselves from those operating in low-demand industries, which in turn shrinks the set of parameter values supporting the pooling equilibrium identified above.

7 Appendix

7.1 Semi-separating equilibria

In the expansion signaling game in which expansions can reduce the incumbent’s costs, the following strategy profiles can be supported as the semiseparating PBE of the game:

1. If $C_H > B^{ED}_H$ and $C_L \in (B^{E}_L, B^{ED}_L)$:
   
   (a) When demand is high, the incumbent chooses not to expand, and when demand is low the incumbent expands with probability $q_L = \frac{(2c-a)-2cp_aH+pa_H^2+(p-1)(2c-a_L)a_L}{(p-1)(a-a_L)(a+a_L-2c)}$.
   
   (b) After observing expansion, the entrant does not enters; and after not observing a expansion, the entrant enters with probability $s = \frac{(a-a_L)(a+a_L-2c)}{(aH-a_L)(aH+a_L-2c)}$, and her beliefs are $\mu(H|E) = 0$ and $\mu(H|NE) = \frac{(a-a_L)(a+a_L-2c)}{(aH-a_L)(aH+a_L-2c)}$.

2. If $C_H \in (B^{E}_H, B^{ED}_H)$ and $C_L > B^{ED}_L$:
   
   (a) When demand is high, the incumbent expands with probability
   
   $q_H = \frac{a(a-2c) + 2cp_aH - pa_H^2 + (p-1)a_L(a_L-2c)}{p(a-a_H)(a+a_H-2c)}$
   
   and when demand is low the incumbent does not expand her business.
   
   (b) After observing expansion, the entrant enters; and after not observing a expansion, the entrant enters with probability $s = \frac{5(2c-a)-7c^2+16c_1-4c_1^2-2a_H(c+4c_1)+36cH}{5(c-a_H)^2}$, and her beliefs are $\mu(H|E) = 1$ and $\mu(H|NE) = \frac{(a-a_L)(a+a_L-2c)}{(aH-a_L)(aH+a_L-2c)}$.

3. If $C_H \in (B^{E}_H, B^{ED}_H)$ and $C_L < B^{E}_L$:
(a) When demand is high, the incumbent expands with probability

\[ q_H = \frac{(p-1)(a+c-a_L-c_1)(a-3c+a_L+c_1)}{p(a+c-a_H-c_1)(a-3c+a_H+c_1)} \]

and when demand is low the incumbent expands her business.

(b) After observing expansion, the entrant enters with probability

\[ \frac{(c-c_1)(c-2a_H+c_1)+9bC_H}{4(c-a_H)(c-c_1)} \] and after not observing a expansion, the entrant enters, and her beliefs are \( \mu(H|E) = \) and \( \mu(H|NE) = 1. \)

4. If \( C_H \in (B^E_H, B^{ED}_H) \) and \( C_L \in (B^E_L, B^{ED}_L) \):

(a) When demand is high, the incumbent expands with probability

\[ q_H = \frac{(a+c-a_L-c_1)(3c-a-a_L-c_1)A}{B} \]

and when demand is low the incumbent expands with probability

\[ q_L = \frac{(a+c-a_H-c_1)(3c-a-a_H-c_1)A}{B} \]

where \( A = a(a-2c) + 2apa_H - pa_H^2 + (p-1)a_L(a_L-2c) \), and \( B = p(a_H-a_L)(c-c_1) \left[ 2a^2 - 4ac + 6c^2 - 3ca_L + (a_L-2c)c_1 + a_H(2a_L+c_1-3c) \right] \).

(b) After observing expansion, the entrant enters with probability

\[ r = \frac{a_L^2(c^2-c_1^2) + 9bC_H - 2a_LF_H + 9bc^2(c_H-c_L) + a_H^2G_L + 2a_H \left[ F_L + a_L^2(c_1-c) \right]}{4D(c-c_1)} \]

and after not observing a expansion, the entrant enters with probability

\[ s = \frac{5a_H^2(a_L-c) + 5ca_L^2 + a_LJ_H + 36bcH(C_L-C_H) + a_HHL}{5D} \]

where \( D = (c-a_H)(c-a_L)(a_H-a_L) \), \( F_K = c(c-c_1)c_1 + 9bcC_K \), \( G_K = (c-2a_H+c_1)(c_1-c) - 9bC_K \), \( J_K = 8cc_1 - 9c^2 - 4c_1^2 + 36bC_K \), \( H_K = 4c_1(c_1-2c) - 5a_H^2 + 9(4^2 - 4bC_K) \), and her beliefs are \( \mu(H|E) = \) and \( \mu(H|NE) = \).

**Proof.** First case: \( C_H > B^{ED}_H \) and \( C_L \in (B^E_L, B^{ED}_L) \). In this case, the entrant’s posterior beliefs are \( \mu(H|E) = 0 \) after observing a expansion, which leads him to do not enter since \( \frac{(a_L-2c+c_1)^2}{9b} < \frac{(a-c)^2}{9b} \). In the case that the entrant observes no expansion from the incumbent, the entrant mixes if and only if his beliefs \( \gamma = \mu(H|NE) \) are such that

\[ \frac{\gamma(a_H-c)^2}{9b} + (1-\gamma)\frac{(a_L-c)^2}{9b} = \frac{(a-c)^2}{9b} \]
where \( \gamma = \mu(H|NE) = \frac{p}{p + (1-p)(1-q_L)} \). Solving for \( q_L \) in the above expression, we obtain 
\[ q_L = \frac{(2c-a) - 2cpa_H - pa_H^2 + (p-1)(2c-a)L_aL}{(p-1)(a-a_L)(a+a_L-2c)} \].

Regarding the incumbent, when demand is low, she mixes if and only if
\[
\frac{(a_L - c_1)^2}{4b} - C_L = s \frac{(a_L - c)^2}{9b} + (1-s) \frac{(a_L - c_2)^2}{4b}
\]

(where \( s \) is the probability with which the entrant enters the market after observing no expansion).

Solving for \( s \), we have 
\[ s = \frac{9(c-a_L)^2(c+c_1-2a_L)+36bc_L}{5(c-a_L)^2} \]. On the other hand, when demand is high, the incumbent decides to not expand if
\[
\frac{(a_H - c_1)^2}{4b} - C_H < s \frac{(a_H - c)^2}{9b} + (1-s) \frac{(a_H - c_2)^2}{4b}
\]

\[ \Leftrightarrow s \left[ \frac{(a_H - c)^2}{4b} - \frac{(a_H - c_1)^2}{9b} \right] < C_H \text{ because } B_{H}^{ED} < C_H \]

And hence, substituting for \( q_L \), the entrant’s beliefs are \( \mu(H|E) = 0 \) and
\[
\mu(H|NE) = \frac{(a-a_L)(a+a_L-2c)}{(a_H-a_L)(a_H+a_L-2c)}
\]

Second case: \( C_L > B_{L}^{ED} \) and \( C_H \in (B_{H}^{E}, B_{H}^{ED}) \). In this case, the entrant’s posterior beliefs are \( \mu(H|E) = 1 \) after observing a expansion, which leads him to enter since \( \frac{(a_H-2c+c_1)^2}{9b} > \frac{(a-c)^2}{9b} \). In the case that the entrant observes no expansion from the incumbent, then the entrant mixes if and only if his beliefs \( \gamma = \mu(H|NE) \) are such that
\[
\gamma \frac{(a_H - c)^2}{9b} + (1-\gamma) \frac{(a_L - c)^2}{9b} = \frac{(a-c)^2}{9b}
\]

where \( \gamma = \mu(H|NE) = \frac{p(1-qa_H)}{p(1-qa_H)+(1-p)}. \) Solving for \( q_H \) in the above expression, we obtain 
\[
q_H = \frac{a(a-2c) - 2cpa_H - pa_H^2 + (p-1)aL(a_L-2c)}{p(a-a_H)(a+a_H-2c)}
\].

Regarding the incumbent, when demand is high, she mixes if and only if
\[
\frac{(a_H - 2c + c_1)^2}{9b} - C_H = s \frac{(a_H - c)^2}{9b} + (1-s) \frac{(a_H - c_2)^2}{4b}
\]

Solving for \( s \), we have 
\[ s = \frac{5a^2_H - 7c^2 + 16cc_H - 4c^2 - 2a_H(c+4c_1) + 36bC_H}{5(c-a_H)^2} \]. On the other hand, when demand is low, the incumbent decides to not expand if
\[
\frac{(a_L - 2c + c_1)^2}{9b} - C_L < s \frac{(a_L - c)^2}{9b} + (1-s) \frac{(a_L - c_2)^2}{4b}
\]

which can be simplified to
\[
\left[ \frac{(a_L - 2c + c_1)^2}{9b} - \frac{(a_L - c)^2}{4b} \right] - sB_{L}^{NE} < C_L
\]
which holds given that: (1) the term in brackets is negative for any parameter values, and $B^E_{L} < C_L$ by definition. Hence, substituting for $q_H$, the entrant’s beliefs are $\mu (H|E) = 1$ and

$$\mu (H|NE) = \frac{(a - a_L) (a + a_L - 2c)}{(a_H - a_L) (a_H + a_L - 2c)}.$$  

**Third case**: $C_L < B^E_{L}$ and $C_H \in (B^E_{H}, B^{ED}_{H})$. In this case, the entrant’s posterior beliefs are $\mu (H|NE) = 1$ after observing no expansion, which leads him to enter since $\frac{(a_H - c)^2}{9b} > \frac{(a - c)^2}{9b}$. In the case that the entrant observes a expansion from the incumbent, then the entrant mixes if and only if his beliefs $\mu = \mu (H|E)$ are such that

$$\mu \frac{(a_H - 2c + c_1)^2}{9b} + (1 - \mu) \frac{(a_L - 2c + c_1)^2}{9b} = \frac{(a - c)^2}{9b}$$

where $\mu = \mu (H|E) = \frac{pq_H}{pq_H + (1-p)q_H}$. Solving for $q_H$ in the above expression, we obtain $q_H = \frac{(p-1)(a+c-a_L-c_1)(a-3c+a_L+c_1)}{p(a+c-a_L-c_1)(a-3c+a_L+c_1)}$. Regarding the incumbent, when demand is high, she mixes if and only if

$$r \frac{(a_H - c_1)^2}{9b} + (1 - r) \frac{(a_H - 2c + c_1)^2}{9b} - C_H = \frac{(a - c)^2}{9b}$$

(where $r$ is the probability with which the entrant enters the market after observing an expansion). Solving for $r$, we have $r = \frac{(c-c_1)(c-2a_H+c_1) + 9bC_H}{4(c-a_H)(c-c_1)}$. On the other hand, when demand is low, the incumbent decides to expand if

$$r \frac{(a_L - c_1)^2}{9b} + (1 - r) \frac{(a_L - 2c + c_1)^2}{9b} - C_L > \frac{(a - c)^2}{9b}$$

which simplifies to $(1 - r)B^E_{L} > C_L$, which is clearly satisfied since $B^E_{L} > C_L$. Hence, substituting for $q_H$, the entrant’s beliefs are $\mu (H|NE) = 1$ and

$$\mu (H|E) = \frac{(a - 3c + a_L + c_1)(c + a - a_L - c_1)}{(a_H - a_L) (a_H + a_L + 2c_1 - 4c)}.$$  

**Fourth case**: $C_L \in (B^E_{L}, B^{ED}_{L})$ and $C_H \in (B^E_{H}, B^{ED}_{H})$. In this case, when the entrant observes a expansion, he mixes if and only if his beliefs $\mu = \mu (H|E)$ are such that

$$\mu \frac{(a_H - 2c + c_1)^2}{9b} + (1 - \mu) \frac{(a_L - 2c + c_1)^2}{9b} = \frac{(a - c)^2}{9b}$$

where $\mu = \mu (H|E) = \frac{pq_H}{pq_H + (1-p)q_H}$. On the other hand, when the entrant observes no expansion, then he mixes if and only if his beliefs $\gamma = \mu (H|NE)$ are such that

$$\gamma \frac{(a_H - c)^2}{9b} + (1 - \gamma) \frac{(a_L - c)^2}{9b} = \frac{(a - c)^2}{9b}$$

where $\gamma = \mu (H|NE) = \frac{p(1-q_H)}{p(1-q_H) + (1-p)q_H}$. Solving now simultaneously for $q_H$ and $q_L$ we have $q_H = \frac{(a+c-a_L-c_1)(3c-a-a_L-c_1)}{B(a+c-a_L-c_1)(3c-a-a_H-c_1)A}$ and $q_L = \frac{(a+c-a_H-c_1)(3c-a-a_H-c_1)A}{B(a+c-a_H-c_1)(3c-a-a_H-c_1)A}$, where $A = a(a-2c) + 2cpa_H - pa^2_H + \ldots$. 

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\((p-1)a_L (a_L - 2c)\), and \(B = p(a_H - a_L)(c - c_1) \left[ 2a^2 - 4ac + 6c^2 - 3caL + (a_L - 2c)c_1 + a_H(2a_L + c_1 - 3c) \right].\) Regarding the incumbent, when demand is high, she mixes if and only if

\[
r \frac{(a_H - c_1)^2}{4b} + (1 - r) \frac{(a_H - 2c + c_1)^2}{9b} - C_H = s \frac{(a_H - c)^2}{9b} + (1 - s) \frac{(a_H - c)^2}{4b}
\]

On the other hand, when demand is low, the incumbent mixes if and only if

\[
r \frac{(a_L - c_1)^2}{4b} + (1 - r) \frac{(a_L - 2c + c_1)^2}{9b} - C_L = s \frac{(a_L - c)^2}{9b} + (1 - s) \frac{(a_L - c)^2}{4b}
\]

Solving then simultaneously for \(r\) and \(s\), we have

\[
r = \frac{a_L^2(c^2 - c_1^2 + 9bcH) - 2a_L F_H + 9bc^2(c_H - c_L) + a_H^2 G_L + 2a_H \left[F_L + a_H^2(c_1 - c)\right]}{4D(c - c_1)}
\]

\[
s = \frac{5a_H^2(a_L - c) + 5ca_H^2 + a_L J_H + 36bc_H(C_L - C_H) + a_H H_L}{5D}
\]

where \(D = (c - a_H)(c - a_L)(a_H - a_L)\), \(F_K = c(c - c_1)c_1 + 9bcK, G_K = (c - 2a_K)(c_1 - c) - 9bcK, J_K = 8cc_1 - 9c^2 - 4c_1^2 + 36bcK, H_K = 4c(c_1 - 2c) - 5a^2 + 9(c_1^2 - 4bcK)\). Finally, substituting for \(q_H\) and \(q_L\), the entrant’s beliefs are \(\mu(H|E) = \frac{(3c-a-aL+c_1)(c_1-a-c+aL)}{(aH-aL)(aH+aL+2c_1-4c)}\) and \(\mu(H|NE) = \frac{(a-aL)(a-2c+aL)}{(aH-aL)(aH+aL-2c)}\) ■

### 7.2 Proof of Lemma 1

We must show that \(B_{HE}^{ED} > B_{HE}^{E}\), so

\[
B_{HE}^{ED} - B_{HE}^{E} = \frac{9(a_H - c_1)^2 - 4(a_H - c)^2}{36b} - \frac{4(a_H + c - 2c_1)^2 - 9(a_H - c)^2}{36b}
\]

\[
= \frac{9(a_H - c_1)^2 + 5(a_H - c)^2 - 4(a_H + c - 2c_1)^2}{36b}
\]

which is always positive. Then, \(B_{HE}^{ED} - B_{HE}^{E} > 0\). Similarly for \(B_{LE}^{ED} - B_{LE}^{E} > 0\) in the case that \(a_L > 2c - c_1\). In the case that \(a_L < 2c - c_1\),

\[
B_{LE}^{ED} - B_{LE}^{E} = \frac{9(a_L - c_1)^2 - 4(a_L - c)^2}{36b} - \frac{(2a_L - c - c_1)(c - c_1)}{4b}
\]

which is positive if and only if \(-\frac{(3c-a-al-2c_1)(3c-5aL+2c_1)}{36b}\) > 0. This is satisfied if \(a_L > 3c + 2c_1\) and \(a_L < \frac{3c-2c_1}{5}\), or vice versa. Since \(\frac{3c-2c_1}{5} > 3c + 2c_1\), then it must be that \(B_{LE}^{ED} > B_{LE}^{E}\) if \(a_L \in \left(\frac{3c-2c_1}{5}, 3c + 2c_1\right)\). In particular, note that \(\frac{3c-2c_1}{5} < 3c + 2c_1 < a_c + c - c_1 < a_L\), which implies that such conditions on \(a_L\) are satisfied. Additionally, note that \(B_{HE}^{ED} - B_{HE}^{E} = \frac{(aH-aL)[8c+5(aH+aL)-18c]}{36b}\) which is positive for all parameter values. Furthermore, \(B_{LE}^{ED} = \frac{9(aL-c_1)^2 - 4(aL-c)^2}{36b} > 0\) since \(aL-c_1 > aL-c\) given that \(c_1 \leq c\). Finally, \(B_{LE}^{E} - B_{LE}^{E} = -\frac{(aH-aL)[5(aH+aL)+16c-26c_1]}{36b}\) which is positive in the case that \(aL > 2c - c_1\) if and only if \(c_1 > \frac{26c-5(aH+aL)}{16}\). In the case that \(a_L < 2c - c_1\), \(B_{HE}^{E} - B_{HE}^{E} = \frac{4c^2+26caH-5a^2+18caL-2[8(c+aH)-9aH]c+7c_1^2}{36b}\).
which is positive if and only if \( a_L < \frac{4c^2+26ac_H-5a^2_H-16(c+a_H)c_1+7c_1^2}{18(c-c_1)} \equiv \bar{a}_L \). In order to check if \( a_L < \bar{a}_L \) imposes additional constraints, let us compare \( a_L \) with respect to \( a_L < 2c - c_1 \). In particular, \( \bar{a}_L > 2c - c_1 \) if and only if \( a_H > 2c - c_1 \). From the assumption that the entrant prefers to enter when demand is high, we have \( a_H > a + c - c_1 > 2c - c_1 \). Hence, \( \bar{a}_L > 2c - c_1 > a_L \) holds for all parameter values, and the initial constraint \( 2c - c_1 > a_L \) is more restrictive than \( \bar{a}_L > a_L \). ■

### 7.3 Proof of Proposition 1

Let us show that the strategy profile \((Exp_H, NoExp_L)\) can be supported as a separating PBE of this signaling game. First, the entrant beliefs are \( \mu = \mu (H|E) = 1 \) and \( \gamma = \mu (H|NE) = 0 \). These beliefs lead him to enter after observing expansion if and only if \( \frac{(a_H-2c_1+c)^2}{9b} > \frac{(a_H-c)^2}{4b} \), which is true by definition. In contrast, the entrant does not enter after observing no expansion since \( \frac{(a_L-2c_1+c)^2}{9b} < \frac{(a_L-c)^2}{9b} \), given that \( a_L < a \). Given these responses by the entrant, the high-demand incumbent expands if and only if

\[
\frac{(a_H-2c_1+c)^2}{9b} - C_H > \frac{(a_H-c)^2}{4b}
\]

which is satisfied if and only if \( C_H < \frac{4(a_H+c-2c_1)^2-9(a_H-c)^2}{36b} = B_H^F \). On the other hand, the low-demand incumbent does not expand her business, in the case that \( a + c - c_1 > a_L > 2c - c_1 \), if and only if

\[
\frac{(a_L-2c_1+c)^2}{9b} - C_L < \frac{(a_L-c)^2}{4b}
\]

which is satisfied if and only if \( C_L > \frac{4(a_L+c-2c_1)^2-9(a_L-c)^2}{36b} = B_L^F \). And in the case that \( a_L < 2c - c_1 \), the entrant remains inactive after entry, and the low-demand incumbent does not expand if

\[
\frac{(a_L-c)^2}{4b} - C_L < \frac{(a_L-c)^2}{4b}
\]

which is satisfied if and only if \( C_L > \frac{(2a_L-c-c_1)(c-c_1)^2}{4b} = B_L^F \). Hence, this strategy profile can be supported as a separating PBE with beliefs \( \mu = 1 \) and \( \gamma = 0 \). ■

### 7.4 Proof of Proposition 2

Let us show that the strategy profile \((NoExp_H, Exp_L)\) can be supported as a separating PBE of this signaling game. First, the entrant beliefs are \( \mu = \mu (H|E) = 0 \) and \( \gamma = \mu (H|NE) = 1 \). These beliefs lead him to not enter after observing expansion since \( \frac{(a_L-2c_1+c)^2}{9b} < \frac{(a_L-c)^2}{9b} \). In contrast, the entrant enters after observing no expansion since \( \frac{(a_H-c)^2}{9b} > \frac{(a_L-c)^2}{9b} \), given that \( a_H > a \). Given these responses by the entrant, the high-demand incumbent does not expand if and only if

\[
\frac{(a_H-c_1)^2}{4b} - C_H < \frac{(a_H-c)^2}{9b}
\]
which is satisfied if and only if \( C_H > \frac{9(a_H-c_1)^2-4(a_H-c)^2}{36b} = B_H^{ED} \). On the other hand, the low-demand incumbent expands her business if and only if

\[
\frac{(a_L - c_1)^2}{4b} - C_L > \frac{(a_L - c)^2}{9b}
\]

which is satisfied if and only if \( C_L < \frac{9(a_L-c_1)^2-4(a_L-c)^2}{36b} = B_L^{ED} \). Hence, this strategy profile can be supported as a separating PBE when \( C_H > B_H^{ED}, C_L < B_L^{ED} \), and beliefs \( \mu = 0 \) and \( \gamma = 1 \). ■

### 7.5 Proof of Proposition 3

Let us show that the strategy profile \((\text{NoExp}_H, \text{NoExp}_L)\) can be supported as a pooling PBE of this signaling game. First, the entrant beliefs are \( \gamma = \mu (H|\text{NE}) = p \) and \( \mu = \mu (H|E) \in [0,1] \). These beliefs lead him to enter after observing no expansion if and only if

\[
p \frac{(a_H - c)^2}{9b} + (1-p) \frac{(a_L - c)^2}{9b} > \frac{(a - c)^2}{9b}
\]

which is satisfied if and only if \( p > \frac{(a-a_L)(a-2c+a_L)}{(a_H-a_L)(a_H+a_L-2c)} = p^{\text{IdenC}} \). On the other hand, the entrant does not enter after observing expansion (off-the-equilibrium path) if and only if

\[
\mu \frac{(a_H - 2c + c_1)^2}{9b} + (1-\mu) \frac{(a_L - 2c + c_1)^2}{9b} < \frac{(a - c)^2}{9b}
\]

which holds if and only if \( \mu < \frac{(a+a_L+c_2-3c)(a-c_1-a_L+c)}{(a_H-a_L)(a_H+a_L+2c_1-4c)} = p^{\text{DiffC}} \). Therefore, the high-demand incumbent does not expand her business if and only if \( \frac{(a_H-c_1)^2}{4b} - C_H < \frac{(a_H-c)^2}{9b} \), which implies \( C_H > \frac{9(a_H-c_1)^2-4(a_H-c)^2}{36b} = B_H^{ED} \). On the other hand, the low-demand incumbent does not expand if and only if \( \frac{(a_L-c_1)^2}{4b} - C_L < \frac{(a_L-c)^2}{9b} \), which holds if \( C_L > \frac{9(a_L-c_1)^2-4(a_L-c)^2}{36b} = B_L^{ED} \). Hence, this strategy profile can be supported as a pooling PBE when \( C_H > B_H^{ED}, C_L > B_L^{ED} \), and beliefs \( \gamma = p \) and \( \mu < p^{\text{DiffC}} \). ■

### 7.6 Proof of Proposition 4

Let us show that the strategy profile \((\text{Exp}_H, \text{Exp}_L)\) can be supported as a pooling PBE of this signaling game. First, the entrant beliefs are \( \mu = p \) and \( \gamma = \mu (H|\text{NE}) \in [0,1] \). These beliefs lead him to enter after observing expansion if and only if

\[
p \frac{(a_H - 2c + c_1)^2}{9b} + (1-p) \frac{(a_L - 2c + c_1)^2}{9b} > \frac{(a - c)^2}{9b}
\]

which is satisfied if and only if \( p > \frac{(a+a_L+c_2-3c)(a-c_1-a_L+c)}{(a_H-a_L)(a_H+a_L+2c_1-4c)} = p^{\text{DiffC}} \). On the other hand, the entrant does not enter after observing no expansion (off-the-equilibrium path) if and only if

\[
\gamma \frac{(a_H - c)^2}{9b} + (1-\gamma) \frac{(a_L - c)^2}{9b} < \frac{(a - c)^2}{9b}
\]
which holds if and only if \( \gamma < \frac{(a-a_L)(a-2c+a_L)}{(a_H-a_L)(a_H+oL+2c)} = p^{\text{IdenC}} \). Given these reactions by the entrant, the high-demand incumbent expands her business if and only if \( \frac{(a_H-2c_1+c)^2}{9b} - C_H > \frac{(a_H-c)^2}{4b} \), which implies \( C_H < B_H^E \). On the other hand, the low-demand incumbent expands, in the case that \( a + c - c_1 > a_L > 2c - c_1 \), if and only if \( \frac{(a_L-2c_1+c)^2}{9b} - C_L > \frac{(a_L-c)^2}{4b} \), which holds if \( C_L < B_L^E \). And in the case that \( a_L < 2c - c_1 \), the entrant remains inactive after entry, and the low-demand incumbent expands if

\[
\frac{(a_L-c_1)^2}{4b} - C_L > \frac{(a_L-c)^2}{4b}
\]

which is satisfied if and only if \( C_L < \frac{2(a_L-c_1)(c-c_1)}{4b} = B_L^E \). This strategy profile can be supported as a pooling PBE with beliefs \( \mu = p \) and \( \gamma < p^{\text{IdenC}} \).

### 7.7 Proof of Lemma 2

1. Let us start by checking if the separating PBE \((\text{Exp}_H, \text{NoExp}_L)\) of Proposition 1 survives the intuitive criterion. We must first construct the subset of types for which deviating is equilibrium dominated. If the high-demand incumbent deviates, the most that she can obtain by not expanding is \( \frac{(a_H-c)^2}{4b} \), which exceeds her equilibrium payoff, \( \frac{(a_H-2c_1+c)^2}{9b} - C_H \), if and only if \( C_H > B_H^E \), which is a contradiction with the parameter values supporting this separating PBE (i.e., \( C_H < B_H^E \)). Hence, the high-demand incumbent cannot profitably deviate given this equilibrium parameter values. Similarly, for the low-demand incumbent, the most that she can obtain from deviating towards expansion is \( \frac{(a_L-c_1)^2}{4b} - C_L \), which exceeds her equilibrium payoff of \( \frac{(a_L-c)^2}{4b} \) when \( C_L < \frac{(2a_L-c_1-c)(c-c_1)}{4b} = \overline{B}_L \). In addition, recall that this equilibrium is supported for \( C_L > B_L^E \). In order to show that the low-demand incumbent deviation (profitable when \( C_L < \overline{B}_L \)) is not compatible with this equilibrium parameter values \( (C_L > B_L^E) \) we must show that \( B_L^E > \overline{B}_L \). On one hand, when \( a + c - c_1 > a_L > 2c - c_1 \),

\[
B_L^E - \overline{B}_L = \frac{(5a_L + 2c - 7c_1)(2c - a_L - c_1)}{36b}
\]

where \( (5a_L + 2c - 7c_1) > 0 \) since \( a_L > c \) and \( c > c_1 \); and where \( (2c - a_L - c_1) = (c - c_1) - (a_L - c) \) is negative since \( a_L > 2c - c_1 \). Hence, this separating equilibrium violates the intuitive criterion when \( a + c - c_1 > a_L > 2c - c_1 \). On the other hand, when \( a_L < 2c - c_1 \), \( B_L^E \) becomes \( B_L^E = \frac{(a_H-c_1)^2}{4b} - \frac{(a_H-c)^2}{4b} = \frac{(2a_L-c_1-c)(c-c_1)}{4b} = \overline{B}_L \). Therefore, separating equilibrium survives the intuitive criterion for any \( a_L < 2c - c_1 \).

2. Let us now check the separating PBE \((\text{NoExp}_H, \text{Exp}_L)\) of proposition 2. First, the high-demand incumbent obtains a maximum payoff of \( \frac{(a_H-c_1)^2}{4b} - C_H \) by deviating towards expansion, which exceeds her equilibrium payoff of \( \frac{(a_H-c)^2}{4b} \) if and only if \( C_H < \frac{(2a_L-c_1-c)(c-c_1)}{4b} = \overline{B}_H \). In addition, recall that this equilibrium is supported for \( C_H > B_H^ED \). However, \( B_H^{ED} > \overline{B}_H \) given that

\[
B_H^{ED} - \overline{B}_H = \frac{5(c-a_H)^2}{36b} > 0
\]
But we cannot simultaneously have $C_H < \overline{B}_H$ and $C_H > B_{H}^{ED}$, since $B_{H}^{ED} > \overline{B}_H$. Therefore, the high-demand incumbent does not deviate towards expansion. Regarding the low-demand incumbent, she obtains a maximum payoff of $\frac{(a_l-c)^2}{4b}$ by deviating towards no expansion, which exceeds her equilibrium payoff of $\frac{(a_l-\overline{c}_l+c)^2}{9b} - C_L$, if and only if $C_L > B_{L}^{E}$, which contradicts the parameter values supporting this separating PBE (i.e., $C_L < B_{L}^{E}$). Hence, no type of incumbent deviates, and this separating PBE survives the intuitive criterion.

3. Let us now check if the pooling PBE $(\text{NoExp}_H, \text{NoExp}_L)$ from Proposition 3 survives the Intuitive Criterion. If the high-demand incumbent deviates, the highest payoff that she can obtain by expanding her business is $\frac{(a_H-c_h)^2}{4b} - C_H$, which exceeds her equilibrium payoff of $\frac{(a_H-c_h)^2}{9b}$ if and only if $C_H > B_{H}^{ED}$, which contradicts the parameter conditions supporting this pooling PBE (i.e., $C_H > B_{H}^{ED}$). Similarly for the low-demand incumbent: the highest payoff she can obtain by deviating towards expansion is $\frac{(a_L-c_l)^2}{4b} - C_L$, which exceeds her equilibrium payoff of $\frac{(a_L-c_l)^2}{9b}$ if and only if $C_L < B_{L}^{E}$, which contradicts the parameter conditions supporting this pooling PBE (i.e., $C_L > B_{L}^{E}$). Hence, no type of incumbent deviates, and this pooling PBE survives the intuitive criterion.

4. Let us finally check the pooling PBE $(\text{Exp}_H, \text{Exp}_L)$ of Proposition 4. First, the highest payoff that the high-demand incumbent can obtain by deviating is $\frac{(a_h-c_h)^2}{4b}$, which exceeds her equilibrium payoff of $\frac{(a_H-2c_l+c)^2}{9b} - C_H$ if and only if $C_H > B_{H}^{ED}$, which contradicts the parameter conditions supporting this PBE (i.e., $C_H > B_{H}^{ED}$). Similarly for the low-demand incumbent: the highest payoff she can obtain by deviating towards no expansion is $\frac{(a_l-c_l)^2}{4b} - C_L$, which, in the case that $a + c - c_1 > a_L > 2c - c_1$, exceeds her equilibrium payoff of $\frac{(a_L-c_l)^2}{9b} - C_L$ if and only if $C_L > B_{L}^{E}$. This contradicts the parameter conditions supporting this PBE (i.e., $C_L < B_{L}^{E}$). Hence, no type of incumbent deviates, and this pooling PBE survives the intuitive criterion when $a + c - c_1 > a_L > 2c - c_1$. In contrast, when $a_L < 2c - c_1$, the highest payoff she can obtain by deviating towards no expansion is $\frac{(a_l-c_l)^2}{4b} - \frac{(a_L-c_l)^2}{4b} - C_L$, which exceed her equilibrium payoff of $\frac{(a_l-c_l)^2}{4b} - \frac{(a_L-c_l)^2}{4b} - C_L$ if $C_L > \frac{(a_L-c_l)^2}{4b} - \frac{2(a_L-c_l)^2}{4b}$, which is compatible with the parameter conditions supporting this pooling PBE: $C_L < \frac{(a_l-c_l)^2}{4b} - \frac{(a_L-c_l)^2}{4b}$. Therefore, if $a_L < 2c - c_1$, this pooling PBE violates the intuitive criterion.

7.8 Proof of Lemma 3

Differentiating $B_{H}^{E} = 4(a_H-c-2c_l)^2-9(a_H-c)^2$ with respect to $c_1$, we obtain $\frac{4(a_H-2c_l+c)}{9b}$, and similarly for the low-demand incumbent in the case that $a_L > 2c - c_1$. In the case that $a_L < 2c - c_1$, we obtain $\frac{c_1-a}{2b} < 0$. Since $a_H > c$ and $a_H > c_1$, then $-\frac{4(a_H-2c_l+c)}{9b} < 0$. Similarly, differentiating cutoff $B_{H}^{ED} = \frac{9(a_H-c_h)^2-4(a_H-c)^2}{30b}$ with respect to $c_1$ we obtain $\frac{c_1-a_H}{2b} < 0$ since $c_1 > a_H$. And similarly for the low-demand incumbent. This implies $\frac{c_1-a_H}{2b} < \frac{c_1-a_L}{2b}$, or alternatively $|\frac{c_1-a_H}{2b}| > |\frac{c_1-a_L}{2b}|$. Additionally, note that $\frac{\partial B_{H}^{E}}{\partial c_1} = -\frac{4(a_H-2c_l+c)}{9b} < 0$ for any $K = \{L, H\}$ in the case that $a_L > 2c - c_1$, which implies that $-\frac{4(a_H-2c_l+c)}{9b} < -\frac{4(a_l-2c_l+c)}{9b}$. Similarly, when $a_L < 2c - c_1$, $-\frac{4(a_H-2c_l+c)}{9b} < \frac{c_1-a}{2b}$ which is also satisfied for $a_L < 2c - c_1$, or alternatively, that $\frac{\partial B_{H}^{E}}{\partial c_1} > \frac{\partial B_{L}^{E}}{\partial c_1}$.
7.9 Proof of Lemma 4

First note that for the high-demand incumbent $B_{EH}^E = 4(a_H + c - 2c_a)^2 - 9(a_H - c)^2 < 0$ if and only if $c > 5c - a_H$. And similarly for the low-demand incumbent, $B_{EL}^E < 0$ if and only if $c > \frac{5c - a_L}{4}$. (Note that $B_{EL}^E$ cannot be negative if $a_L < 2c - c_1$ since $B_{EL}^E < 0$ if and only if $c > 2a - c$, or alternatively, $c_1 - a > a - c$ which cannot be satisfied). Second, note that $\frac{5c - a_1}{4} > \frac{5c - a_H}{4}$ given that $a_L < a_H$. Third, note that $\frac{5c - a_K}{4} < c$ for any $K = \{H, L\}$ since $c < a_K$. Hence, we have shown that $c > \frac{5c - a_L}{4} > \frac{5c - a_H}{4}$. Finally, note that the separating equilibrium of proposition 1 survives the intuitive criterion if $c - c_1 > a_L - c$, which can be rewritten as $2c - a_L > c_1$. □

7.10 Proof of Proposition 5

In Regime 1, $C_K < B_{KNT}^K$ for all $K$. Let us first analyze the case in which $a + c - c_1 < 5c - 4c_1$. When $a_L \in (2c - c_1, a + c - c_1)$ and $a_H \in (a + c - c_1, 5c - 4c_1)$, we have $0 < B_{EL}^E < B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the pooling equilibrium $(E_H, E_L)$ can be supported for $C_K < B_{K}^E$, the separating equilibrium $(E_{E}H, NE_{L})$ can be sustained for $C_L < B_{EL}^E$, and the semiseparating equilibrium $(E_{E}H, NE_{E}L)$ is not supported for $C_L < B_{EL}^E$. When $a_L \in (2c - c_1, a + c - c_1)$ and $a_H > 5c - 4c_1$, we have $B_{EH}^E < 0 < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$. When $a_L < 2c - c_1$ and $a_H \in (a + c - c_1, 5c - 4c_1)$, we have $0 < B_{EH}^E < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the pooling equilibrium $(E_{H}, E_{L})$ can be supported for $C_K < B_{K}^E$ and the semiseparating equilibrium can be sustained for $C_H < B_{K}^{NT}$ and $C_L < B_{K}^{E}$. When $a_L < 2c - c_1$ and $a_H > 5c - 4c_1$, we have $0 < B_{EH}^E < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$. Let us now analyze the case that $a + c - c_1 > 5c - 4c_1$. When $a_L \in (5c - 4c_1, a + c - c_1)$ we have $B_{EL}^E < 0 < B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$. When $a_L \in (2c - c_1, 5c - 4c_1)$ we have $B_{EH}^E < 0 < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$. When $a_L < 2c - c_1$ and $a_H \in (a + c - c_1, 5c - 4c_1)$, we have $0 < B_{EH}^E < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$. In Regime 2, $C_H < B_{HNT}^H$ and $C_L > B_{HNT}^L$. Let us first analyze the case in which $a + c - c_1 < 5c - 4c_1$. When $a_L \in (2c - c_1, a + c - c_1)$ and $a_H \in (a + c - c_1, 5c - 4c_1)$, we have $0 < B_{EL}^E < B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the semiseparating equilibrium $(E_{H}, NE_{L})$ can be supported for $C_H < B_{EH}^E$ and $C_L < B_{EL}^E$, and the semiseparating equilibrium $(E_{L}, NE_{E})$ can be sustained for $C_L > B_{LNT}^L$ and $C_H \in (B_{EH}^E, B_{HNT}^H)$ When $a_L \in (2c - c_1, a + c - c_1)$ and $a_H > 5c - 4c_1$, we have $B_{EH}^E < 0 < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_L > B_{LNT}^L$ and $C_H < B_{HNT}^H$. When $a_L < 2c - c_1$ and $a_H \in (a + c - c_1, 5c - 4c_1)$, we have $0 < B_{EH}^E < B_{EH}^{NT} < B_{EH}^E$ and $0 < B_{EL}^E = B_{EL}^{NT} < B_{EL}^E$. Under these parameter values, the semiseparating equilibrium can be sustained for $C_K < B_{KNT}^E$.
sustained for $C_L > B_L^E$ and $C_H < B_H^{NT}$. Let us now analyze the case that $a + c - c_1 > 5c - 4c_1$. When $a_L \in (5c - 4c_1, a + c - c_1)$ we have $B_K^E < 0 < B_K^{NT} < B_K^{ED}$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_L > B_L^{NT}$ and $C_H < B_H^{NT}$. When $a_L \in (2c - c_1, 5c - 4c_1)$ we have $B_H^E < 0 < B_H^{NT} < B_H^{ED}$ and $0 < B_L^E < B_L^{NT} < B_L^{ED}$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_L > B_L^{NT}$ and $C_H < B_H^{NT}$. When $a_L > 2c - c_1$ we have $B_H^E < 0 < B_H^{NT} < B_H^{ED}$ and $0 < B_L^E = B_L^{NT} < B_L^{ED}$. Under these parameter values, only the semiseparating equilibrium can be sustained for $C_L > B_L^{NT}$ and $C_H < B_H^{NT}$.

In Regime 3, $C_K > B_K^{NT}$ for all $K$. Let us first analyze the case in which $a + c - c_1 < 5c - 4c_1$. For all $a_L < a + c - c_1$ and $a_H > a + c - c_1$, we have that the pooling equilibrium $(NE_H, NE_L)$ can be supported for $C_K > B_K^{ED}$, and the semiseparating can be supported for $C_L > B_L^{NT}$ and $C_H \in (B_H^{NT}, B_H^{ED})$, and for $C_H > B_H^{NT}$ and $C_L \in (B_L^{NT}, B_L^{ED})$. Similarly for the case that $a + c - c_1 > 5c - 4c_1$. For all $a_L < a + c - c_1$ we have that the pooling equilibrium $(NE_H, NE_L)$ can be supported for $C_K > B_K^{ED}$, and the semiseparating can be supported for $C_L > B_L^{NT}$ and $C_H \in (B_H^{NT}, B_H^{ED})$, and for $C_H > B_H^{NT}$ and $C_L \in (B_L^{NT}, B_L^{ED})$. ■
References


