Modernization of Tax Administrations and Optimal Fiscal Policies

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Abstract

Since Sandmo (1981), many articles analyze optimal fiscal policies in economies with tax evasion. Surprisingly, all of them assume that the cost of enforcing the tax law is exogenous. However, governments often invest resources to reduce these enforcement costs. In a very simple model, we incorporate such investments in the analysis of an optimal fiscal policy. We characterize their optimal level and we show numerically how they interact with the other dimensions of the optimal fiscal policy.

Keywords: Tax administration - Tax rates - Tax evasion - Enforcement - Audit costs.

JEL Codes: D82 - H26 - H83

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1 Introduction

This paper analyzes optimal fiscal policies in economies with tax evasion. For most governments, tax evasion is a problem because it threatens the equity and the efficiency of their fiscal policies [Cowell (1985), Skinner and Slemrod (1985)]. For this reason, governments react and adopt actions to assure compliance with the tax law. For example, audits are conducted to verify whether tax liabilities have been met, and if it is not the case, evaders are penalized.

But this ‘enforcement approach’ is not sufficient to deal with tax evasion. As the public finance literature shows, the fight against tax evasion cannot be isolated from the design of the fiscal policy. Why? Because the extent of tax evasion depends not only on the parameters that characterize the enforcement policy carried out by the tax administration (e.g. frequency of audits, level of fines) but also upon the structure of the tax law (e.g. tax rates). Therefore, as suggested by Allingham and Sandmo (1972) and then emphasized by Kolm (1973), the design of optimal fiscal policies or ‘optimal tax systems’ (in Slemrod’s (1990) terminology) should also include all instruments that help to enforce the tax law.

Since Sandmo (1981), many articles analyze, in different settings, optimal tax-enforcement policies. Looking carefully to these articles, we can observe that they share a common feature: the cost of a single audit or the shape of the audit cost function is exogenous. This assumption seems unsatisfactory, for two reasons. From a theoretical point of view, in modern microeconomic models costs are often endogenous because agents adopt decisions to change them [see Tirole (1988)]. Moreover, adopting such decisions is not only a theoretical possibility but, as the following paragraph shows, also has a strong empirical support: governments do invest resources to reduce the enforcement cost of the tax law.

In 1998, the US Congress created an special ‘Information Technology Investment’ account, to fund IRS modernization activities [The President’s Commission to Study Capital Budgeting (1998)]. With these funds, in 1999, the IRS launched ‘Business Systems Modernization’, an ambitious multianual project to modernize its information technology infrastructure [see IRS (2000)]. One of the pillars of this (ongoing) project is the change of its main data system, called the Master File system. This system was developed in the 1960’s and consists of large tape files stocked in one of the IRS’s computer centers in Martinsburg (West Virginia). It stores the taxpaying histories of 227 million individuals and corporations, including every transaction between taxpayers and the IRS for the past 40 years [see Varon (2004)]. Among others, the Master File system has an important drawback. The entire process the IRS needs to enter account data into the Master File and make the updated information available for researching taxpayer accounts can take from 4 to 6 weeks [US General Accounting Office (1999)]. Because of these delays, IRS employees frequently have inconsistent and out-of-date information about a given taxpayer. Under these circumstances, conducting timely audits and going after tax evaders is extremely difficult. The Business Systems Modernization project will replace the Master File system with the Customer Account Data Engine (CADE), which will allow IRS’s employees to post changes and update taxpayers accounts and

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1 One can get a good idea about the quantitative importance of tax evasion by looking at the US example. The Internal Revenue Service (IRS) estimated a tax gap for tax year 2001 of USD 345 billion (IRS, 2006). This amounts to almost 15% of total tax revenue, which is far from being a negligible figure.


3 In some models [e.g. Cowell (1985)], the cost of a single audit is a function of the number of audits conducted.
returns from their desks. As a result, according to IRS (2005), audit costs should decrease.⁴

These investments made by governments to modernize their tax administration have been either mentioned informally [e.g. Snavely (1988) and Slemrod and Yitzhaki (2002)] or studied empirically [e.g. Hunter and Nelson (1996)]. But, to our knowledge, they have not been incorporated into the formal analysis of optimal fiscal policies so far.⁵ This is precisely the purpose of our paper. We address this issue in a simple three-stage model, with two classes of active agents: a government and individuals. Each individual can be poor or rich; the rich being the only to earn a taxable income. The government follows a social welfare criterion that incorporates aversion to inequality. In order to maximize its criterion, the government designs a fiscal policy, to be implemented by the tax administration. In the first stage of the model, the government invests resources to ‘modernize’ the tax administration. In the second stage, the government designs the tax law, which specifies the tax owed by the rich and the enforcement policy to be conducted later by the tax administration. Finally, in the last stage of the model, the tax administration collects taxes and enforces the tax law, as follows. As incomes are private information, individuals are requested to report them. Then, the tax administration audits reports with a frequency previously set by the government. If an audit discovers that a taxpayer has misreported, the tax administration taxes the evader according to his true income and imposes him an additional fine. With all revenues collected (taxes and fines, net of investment and audit costs), the government finances the provision of a public good. As in many other contributions to the literature on tax evasion, we assume that audits are perfect but costly.

The main novelty of this paper is a particular assumption concerning the audit technology: the audit cost decreases with the investment made by the government in the first stage, at a decreasing rate. This is precisely the meaning that the expression ‘modernization of the tax administration’ has in our paper.

In order to have a benchmark, we derive first the optimal fiscal policy under full information, when enforcement is not necessary. Then we move to asymmetric information and we solve the model backwards. As the government can commit to the tax and to the enforcement policy to be conducted in the third stage, in the second stage we characterize the optimal tax law adopting a mechanism design approach. Depending upon the value of the audit cost, two regimes emerge. In the first regime, when the audit cost is low, the tax administration only audits individuals that have reported to be poor. In order to attenuate the stake for evasion of the rich, the optimal tax is downwardly distorted with respect to the full information optimal tax. We show that the optimal tax and the audit probability decrease with the audit cost. As it is usual in this kind of models, the optimal fine has only a deterrent role and is maximal. In the second regime, the audit cost is so high that the government prefers not to tax and so no enforcement takes place. Anticipating these decisions, in the first stage, the government chooses whether to invest in the tax administration to reduce its audit cost and, if so, how much to invest. This choice has an impact not only on the expected social welfare (because tax revenues are allocated to investment instead of being allocated to the public good) but can also fix under which regime the government and the tax administration will be afterwards. Although we can show that an optimal investment exists, we cannot completely characterize it in general. Nevertheless, when the optimal investment is strictly positive, we can derive some comparative static results. Finally, we simulate the model

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⁴ Snavely (1988) shows that, by the end of the 80’s, many US States had already invested in computer technology to improve auditing.

⁵ Alm (1996) asserts that there is a lack of analysis regarding the shape of the audit cost function. In the conclusion of their survey, Slemrod and Yitzhaki (2002) consider the rigorous study of the tax collection technology as one of the main venues for future research in the economic literature on tax evasion.
to identify the optimal investment, to quantify the other components of the optimal fiscal policy and to confirm the comparative static results above mentioned.

The main results that emerge from the simulations are the following. First, when the audit technology improves, both the optimal investment and the aggregate audit cost, as percentages of the tax collection, globally decrease. Nevertheless the behavior of the absolute levels of these two variables is not monotonic. Second, when inequality in pre-tax income distribution increases, the optimal investment and the aggregate audit cost are first complements and then substitutes; both becoming a smaller part of the government’s budget. Third, an utilitarian government adopts similar decisions (in a quantitative sense) than a Rawlsian government.

1.1 Related literature

From a methodological point of view, our model is a simplified version of Mookherjee and Png (1989) and Pestieau et al. (2004). These authors assume that pre-tax incomes are exogenous and that taxes are lump-sum, like we do. Moreover, they also assume that individuals are risk averse. Finally, adopting the mechanism design approach, they characterize optimal tax-enforcement policies. But, besides the fact that these authors take the audit cost as given, our model differs with theirs on two important aspects. First, as many others [e.g. Marhuenda and Ortuño-Ortín (1997) and Chander and Wilde (1998)], we rule out of the model rewards for truthful reports, whereas Mookherjee and Png (1989) find that they are indeed optimal deterrents for tax evasion. Second, we include in the model the provision of public goods, whereas Mookherjee and Png (1989) and Pestieau et al. (2004) follow the traditional ‘optimal taxation’ approach that fixes exogenously the government’s revenue requirement. Regarding the results, some of ours are already in these two articles, like the optimality of maximal fines, the characterization of the optimal audit probability from the binding incentive compatibility constraint and the existence of two regimes. But neither of them obtain explicit solutions for the tax law and formally show how it varies with the parameters of the model, like we do.

Incorporating the provision of public goods into the design of optimal fiscal policies in a context of tax evasion makes our paper similar to Kolm (1973), Gottlieb (1985), Usher (1986), Kaplow (1990), Falkinger (1991), Mayshar (1991), Sanchez and Sobel (1993) and Balestrino and Galmarini (2003). However these articles do not include investments to modernize the tax administration in their analysis, whereas it is our main contribution. Moreover, unlike in Falkinger (1991), in our model the level of the public good is always below the full information level.

We are not the first to address the issue of the tax administration’s technology; Usher (1986) and Mayshar (1991) have already dealt with this item. These authors characterize the optimal amount of resources spent by governments in tax enforcement activities. To do that, they rely on reduced-form functions and assume a positive relation between these resources and the cost borne by individuals to escape from being detected (Usher)/the maximal tax collection (Mayshar). While these positive relations seem logical, our numerical results show that, when one incorporates investments that improve the cost structure of the tax administration in the model, the signs of these relations become less evident.

Finally, our model sheds new light on the question of the size and the composition of the tax administration’s budget. Like many others [e.g. Slemrod and Yitzhaki (1987) and Slemrod

\[6\] Border and Sobel (1987) also find optimal tax enforcement policies, but in a context where individuals are risk neutral.

\[7\] In Mayshar’s (1991) words, these reduced-form functions are ‘black boxes’.

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(1994)], we take the budget of the audit section for the budget of the entire tax administration. By incorporating the investment as another element of the tax administration’s budget, we can show first that the relation between the audit probability and the tax administration’s budget is not so direct as in Slemrod and Yitzhaki (1987). Second, we can argue about the composition of this budget, as in Wertz (1979). In particular, we can see whether the investment is substitute or complement to the aggregate audit cost, the other element of the tax administration’s budget.

The remainder of this paper is organized as follows. Next section describes the model. Section 3 shows the optimal fiscal policy under full information. Section 4 analyzes the optimal fiscal policy under asymmetric information. Section 5 presents the numerical results of the paper. Finally, Section 6 concludes. All proofs appear in the Appendix.

2 The model

We formalize the design and the implementation of a fiscal policy in a simple three-stage model, with two classes of active agents: a government and individuals.

There is a continuum of individuals of measure 1. Each individual can be of two different types $\iota \in \{p, r\}$. A ‘poor’ individual ($\iota = p$) has no income whereas a ‘rich’ individual ($\iota = r$) earns a strictly positive taxable income $y$. Types are random variables, identically and independently distributed according to the (commonly known) probability distribution $(\mu, 1 - \mu)$, where $\mu = \Pr[\iota = r] > 0$. Each individual privately knows his type.

Poor individuals only benefit from a public good, provided by the government, in quantity $G$. Their ex-post welfare is thus

$$w_p = G$$

Rich individuals also derive utility from consumption of a private good $q$, the price of which is normalized to 1. So their ex-post welfare is

$$w_r = u(q) + G$$

where the utility function $u$ is strictly increasing, concave and verifies the Inada conditions. In order to obtain interior solutions, we also assume

$$0 < u_q(y) < \frac{1}{\alpha p}$$

where $\alpha$ and $p$ are parameters defined later.

The government follows a welfarist criterion $W$ that can be represented by a weighted sum of the individuals’ welfares, as follows

$$W = \mu w_r + (1 - \mu) w_p - \mu(1 - \alpha)(w_r - w_p)$$

$$= \alpha \mu u(q) + G$$

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8Measurability issues arise in probability spaces with a continuum of iid random variables, as it is the case in our setting. In spite of this, we will adopt throughout the paper the usual abuse of the Law of Large Numbers. Hence $\mu$ also represents the proportion of rich individuals in the population.

9Throughout the paper, subscripts of functions denote partial derivatives.
where \(0 \leq \alpha \leq 1\) is a parameter that measures preferences for redistribution. To be more specific, the government is averse to inequality, with a degree of aversion proportional to \((1 - \alpha)\). In order to maximize its criterion, the government designs a fiscal policy, to be implemented by the tax administration.

The order of events, and relevant features of the model in more detail, are as follows.

1. In the first stage, the government invests \(I\) to ‘modernize’ the tax administration.

2. In the second stage, the government designs the tax law, which specifies the tax \(t \geq 0\) owed by rich individuals (from now on, taxpayers) and the enforcement policy to be conducted afterwards by the tax administration. The enforcement policy consists specifically of an audit probability \(\pi \in [0, 1]\) and a fine for evaders \(f \geq 0\). The unique restriction to this tax law is limited liability of taxpayers.

3. In the third stage, the tax law is implemented. As the tax administration does not observe types \(\iota\), individuals are requested to report them, e.g. by filling in an income tax form. We denote such reports by \(\tilde{e}\). Then, following the enforcement policy previously designed by the government, the tax administration audits each report \(\tilde{e}\) with probability \(\pi\).

Each audit costs \(C = c(I, \delta) \geq 0\), where \(\delta\) is a positive parameter.\(^{10}\) Let \(C_0 = c(0, \delta)\) denote the initial audit cost, before an investment decision has been made. This level \(C_0\) is an exogenous function of the stock of capital the tax administration is endowed with at the beginning of the first stage and/or other exogenous parameters that are related to the difficulty in auditing (e.g. the percentage of farmers in the population).\(^{11}\) The main assumption of the paper is the following: the audit cost decreases with investment \(I\), at a decreasing rate. This is precisely the meaning that the expression ‘modernization of the tax administration’ has in our paper. Formally, the audit cost function \(c\) verifies

\[
c_I < 0, \quad c_I > 0 \quad \text{and} \quad \lim_{I \to \infty} C = 0
\]

The cost function \(c\) also verifies that the higher the parameter \(\delta\), the higher the capacity of any level of investment to reduce the initial audit cost: \(c_\delta < 0\). This is why we call \(\delta\) the ‘investment productivity’.

When a taxpayer is not audited, he pays the tax that corresponds to his report \(\tilde{e}\). But if he is audited, the tax administration discovers his true type \(\iota\). And, if a misreport is detected, the evader has to pay the tax that he legally owes plus the additional fine \(f\).\(^{12}\) With all revenues collected (taxes and fines, net of investment and audit costs), the government finances the provision of the public good \(G\), which has a unit cost equal to \(p\).

The assumptions of the model and its timing deserve some comments.

\(^{10}\) For the sake of simplicity, we assume that tax collection is costless, both for taxpayers and for the tax administration.

\(^{11}\) It is well known that auditing income generated in agricultural activities is very difficult, either in developed or developing countries.

\(^{12}\) Like Marhuenda and Ortuño-Ortín (1997) and Chander and Wilde (1998), we rule out of the model rewards for truthful reports.
To simplify the model at the maximum\textsuperscript{13}, we assume that the government makes all decisions. The tax administration is simply a ‘machine’ that follows the government’s directives, unlike in Cremer et al. (1990) and in Sanchez and Sobel (1993), where the tax administration decides the audit strategy. In spite of this, and for the sake of realism, we follow these authors in separating (only formally) the tax administration from the government.

Regarding full commitment to the enforcement policy, this assumption generates a framework where, a priori, investments have the lowest value (i.e. they do not have a commitment value). We could have adopted two other assumptions: partial commitment [Melumad and Mookherjee, (1989)] or no commitment [Graetz et al. (1986)]. These other assumptions, although more realistic, do not enrich the model substantially. In fact, they complicate unnecessarily the computations, without modifying qualitatively the main results regarding the optimality of investing to modernize the tax administration.

Concerning the investment decision, two observations can be made. First, and related to the previous comment, the fact that the government chooses its level replicates the US Congress creating an special account for the IRS, with funds to be used only for the modernization of the IRS’s information technology. Second, the lag between the investment decision and then the design of the tax law is clearly a shortcut, reflecting the long-term character of these investments (made once, with an impact that carries over many years) with respect of the sort-term of the tax law (designed yearly).

The reason for assuming that the government taxes only the rich and that individual welfares are quasi-linear in the public good is purely technical. These two assumptions enable us to obtain necessary and sufficient conditions to characterize the optimal tax law\textsuperscript{14} and non-ambiguous signs in the comparative statics, at the second stage. And all this is necessary for computing the optimal investment and the comparative statics, at the first stage.

Finally, we assume that the government can choose any fine, with no restriction other than limited liability. Therefore, we do not follow the literature that considers that ‘the punishment should fit the crime’. In spite of that, limited liability rules the beckerian-type result ‘hanging evaders with probability zero’ out of the model.

The goal of the paper is to characterize the optimal fiscal policy, which is the 4-uple \( \{I, t, f, \pi_t\} \). Before doing that, and in order to have a benchmark, we present the optimal fiscal policy under full information. In this case, the tax administration observes incomes and thus audits are useless. Anticipating this, the government does not need to invest in the tax administration and simply sets the tax that solves the following problem, where private consumption has been replaced, using the taxpayers’ budget constraint, by their disposable income

\[
\mathcal{P}^* \left\{ \begin{array}{l}
\max_{t,G} \alpha \mu u(y - t) + G \\
\text{subject to} \\
0 \leq t \\
t \leq y \quad (LL) \\
pG = \mu t \quad (B)
\end{array} \right.
\]

\textsuperscript{13}Vakneen and Yitzhaki (1989) comment on the necessity of simplifying this kind of problems.

\textsuperscript{14}In a more general model, Cremer et al. (1990) cannot find such sufficient conditions.
We denote by \((LL)\) the limited liability constraint and by \((B)\), the government’s budget constraint. The first-order condition
\[ \alpha u_q(y - t^*) = \frac{1}{p} \]
characterizes the optimal full information tax \(t^*\). At the optimum, the government taxes the rich in order to equalize their social marginal utility of consumption with the social marginal utility of the last $ spent in the public good. Due to the properties of the utility function \(u\), \(t^*\) verifies \(0 < t^* < y\). We denote by \(G^* = \frac{\mu t^*}{p}\) the optimal provision of public good.

3 Optimal fiscal policy under asymmetric information

In this section, we characterize the optimal fiscal policy under asymmetric information, when the tax administration can only observe incomes at a cost, by auditing reports. In order to do this, we solve the model backwardly.

3.1 The optimal tax law

As we mentioned before, the government can commit to the audit probability \(\pi\) when it designs the tax law. Therefore, in our setting, the Revelation Principle applies and the optimal tax law can be easily characterized adopting a mechanism design approach. According to Mookherjee and Png (1989), the tax administration does not need to audit a taxpayer that has reported to be rich. So, from now on, \(\pi\) will denote the probability of auditing an announcement \(\tilde{\tau} = p\). The optimal tax law \((t, f, \pi)\) solves the following problem, where again private consumptions have been replaced by taxpayers’ disposable income, but now at each possible final state.

\[
\mathcal{P}_1 = \begin{cases} 
\max_{t, f, \pi, G} & \alpha u(y - t) + G \\
\text{subject to} & 0 \leq t \\
& t + f \leq y \\
& u(y - t) \geq \pi u(y - t - f) + (1 - \pi) u(y) \quad (IC) \\
& pG = \mu t - (1 - \mu)\pi C - I \quad (B') 
\end{cases}
\]

Now we denote by \((LL')\) the after audit limited liability constraint\(^{15}\) and by \((IC)\), the incentive compatibility constraint\(^{16}\). With respect to the full information setting, the government’s budget constraint \((B')\) now incorporates the aggregate audit cost \((1 - \mu)\pi C\) and the investment \(I\).\(^{17}\) As it is usual in this kind of models, the fine \(f\) does not enter in the maximand of the problem \(\mathcal{P}_1\) because it has only a deterrent role.

\(^{15}\)As \(f \geq 0\), imposing after audit limited liability constraint also ensures \(t \leq y\).

\(^{16}\)Since the number of taxpayers is very large, none of them considers the impact of their non-compliance with the tax law on the amount of public good. So the public good does not appear in the incentive-compatibility constraint.

\(^{17}\)We do not impose the net tax collection to be positive. If \(I = 0\), it is straightforward to realize that the net tax collection should be positive at the optimum. If not, the option of not taxing the rich and not enforcing the tax law would dominate. If \(I > 0\), in the next section we explain why imposing this constraint is without any loss of generality.
At the optimum, \((LL')\) binds: increasing the fine \(f\) up to its maximal legal level relaxes \((IC)\). Moreover, as in other Principal-Agent models of tax-enforcement policies [e.g. Mookherjee and Png (1989)], \((IC)\) also binds: the government sets an audit strategy that makes a potential evader to be indifferent between truthfully reporting his type and misreporting. Computing the first-order condition of the problem \(P_1\) and rearranging, we obtain the expression that characterizes the optimal tax at an interior solution

\[
\alpha u_q(y - t) = \frac{1}{p} \left[ 1 - \frac{(1 - \mu)C}{(1 - \mu)C + \alpha \mu pu(y)} \right]
\]

Under asymmetric information, the taxpayers’ social marginal utility of consumption again equals the social marginal utility of the expenditure in the public good. Now, due to the necessity of auditing reports in order to collect taxes, this expenditure is less than the tax collection. Therefore, by concavity of the utility function \(u\), the optimal tax \(t\) is lower than the optimal full information tax \(t^*\). We gather these results in the next proposition, where we completely characterize the optimal tax law and the optimal provision of public good. In particular, we explain in detail how the solutions vary with the audit cost \(C\) because it is through this variable that the investment decision has an impact on the optimal tax law and, ultimately, on the expected welfare. And understanding this will be important to determine, in the next section, the optimal investment.

**Proposition 1** Let \(\overline{C} = \frac{[1 - \alpha \mu u_q(y)] \mu u(y)}{(1 - \mu)u_q(y)}\). Under asymmetric information, the following two regimes emerge.

- **Regime \(RA\)**: when \(C < \overline{C}\), the optimal tax is a continuous and strictly decreasing function of \(\overline{C}\) that verifies

  \[
  \lim_{C \to 0} t = t^* \quad \text{and} \quad \lim_{C \to \overline{C}} t = 0.
  \]

  The optimal fine is \(f = y - t\). The tax administration audits only individuals that have reported to be poor. The optimal audit probability \(\pi = 1 - \frac{u(y - t)}{u(y)}\) decreases with \(C\) and verifies

  \[
  \lim_{C \to 0} \pi = \pi^{\text{Max}} = 1 - \frac{u(y - t^*)}{u(y)} \quad \text{and} \quad \lim_{C \to \overline{C}} \pi = 0.
  \]

  Finally, the public good \(G\) also decreases with \(C\) and verifies

  \[
  \lim_{C \to 0} G = G^* \quad \text{and} \quad \lim_{C \to \overline{C}} G = 0.
  \]

- **Regime \(R^\text{NA}\)**: when \(C \geq \overline{C}\), the government does not tax and the tax administration does not audit. So, the public good is not provided.

This proposition is most easily understood with reference to Figures 1, which show the profiles of the different components of the optimal tax law \((t, f, \pi)\) and the optimal provision of public good.

Insert Figures 1 here
Like Pestieau et al. (2004), we also find two regimes, whose emergence depends upon whether the value of the audit cost $C$ is above or below the threshold $C^*$.\footnote{But unlike them, we do have to characterize formally the threshold $C^*$ because it will play a crucial role in the analysis of the first stage of the model.} In Figure 1(a), we can observe that under regime $R^A$ the optimal tax is below $t^*$ and decreases with the audit cost $C$. In order to understand why, let’s assume first that the audit cost is zero. The government sets the full information optimal tax $t^*$, charges the tax administration to audit with probability $\pi_{\text{Max}}$ and to punish evaders with the fine $f = y - t^*$. Now consider an increase in the audit cost $C$. This makes the aggregate audit cost $(1 - \mu)\pi C$ to increase, causing a decrease in the provision of the public good, with its consequent welfare loss. What should be the optimal reaction of the government? To reduce the tax $t$ and to increase the fine $f$, while keeping their sum constant, equal to $y$. Even if this change reduces the tax collection and, a priori, decreases further the provision of the public good, it has two other effects that attenuate the above mentioned welfare loss. First, the decrease in $t$ reduces the stake for evasion. Therefore, $\pi$ can decrease from $\pi_{\text{Max}}$, counter ing the impact of the initial increase in $C$ on the aggregate audit cost. Second, reducing the tax makes private consumption of the rich to increase. A similar argument can be used to explain why, due to subsequent increases in the audit cost $C$, the government distorts more the optimal tax $t$.\footnote{The distortion is measured by the difference $t^* - t$.}

This is not the end of the story. For a ‘sufficiently high’ value of the audit cost $C$, namely $C^*$, the optimal tax $t$ converges to 0, as we can see in Figure 1(a). Then, when $C \geq C^*$, the audit cost is so high that enforcement is prohibitive and thus $\pi = 0$, as Figure 1(c) shows. When this is the case, the unique incentive-compatible tax is 0 and the fine is irrelevant. So no redistribution takes place.\footnote{In a different setting, Pestieau et al. (1994) present the intuition of a similar result.}

Figure 1(d) presents the optimal provision of the public good $G$. At this stage, we can already affirm that the provision of public good when tax evasion is an issue is below the full information level $G^*$. Falkinger (1991) does not find such a clear-cut result because his model is more general.

### 3.2 The optimal investment

In the first stage, anticipating its future fiscal choices, the government chooses whether to invest to modernize its tax administration and, if so, how much to invest. This decision has two different impacts. On the one hand, it affects the expected social welfare because the government allocates tax revenues to investment $I$ instead of allocating them to the public good $G$. On the other hand, as investment changes the value of the audit cost $C$, this decision can also fix under which regime the government will design the tax law and the tax administration will operate. In order to address the choice of regime in terms of the variable $I$, let $\mathcal{T}$ denote the solution of the implicit equation $c(I, \delta) = C$.\footnote{Due to the properties of the cost function $c$, $\mathcal{T}$ is unique.} Thus, the expected welfare can now be written as a function of investment $I$, as follows

\[
\text{when } C_0 \leq C^* \quad \mathbb{E}W = \mathbb{E}W^A = \alpha \mu u(y - t) + \frac{1}{p} \left[\mu t - (1 - \mu) \pi C - I\right] \quad \text{for } I \geq 0
\]

\[
\text{when } C_0 > C^* \quad \mathbb{E}W = \begin{cases} 
\mathbb{E}W^{\mathcal{N}A} = \alpha \mu u(y) & \text{for } I = 0 \\
\mathbb{E}W^A = \alpha \mu u(y - t) + \frac{1}{p} \left[\mu t - (1 - \mu) \pi C - I\right] & \text{if } I \geq \mathcal{T} > 0
\end{cases}
\]
where the superscripts indicate the corresponding regime. The expression of $E^{WA}$ takes into account that, as the government raises no tax revenues, investment in the tax administration cannot be afforded. As we can see, the value of the initial audit cost $C_0$ is now important to characterize the expected welfare. When $C_0 \leq C$, only regime $RA$ emerges because, no matter the investment decision, $C \leq C$. This is not the case when $C_0 > C$: according to the value of the investment, both regimes $RNA$ or $RA$ can occur.

In order to solve for the optimal investment, we proceed as follows. First, we find the investment that maximizes $E^{WA}$. If it exists, we denote it by $I_A$. Second, when it is pertinent to do so, we compare $E^{WA}(I_A)$ with $E^{WA}$ to take the overall maximum.

Under regime $RA$, the optimal investment $I_A$ is the solution to the following problem

$$\begin{align*}
\mathcal{P}_2 &= \left\{ \begin{array}{l}
\text{Max}_I \quad \alpha \mu u(y - t) + \frac{1}{p} [\mu t - (1 - \mu) \pi C - I] \\
\text{subject to } \quad t = t(C), \quad \pi = \pi(C) = 1 - \frac{u(y - t(C))}{u(y)} \\
C = c(I, \delta) \\
\max\{0, \bar{T}\} \leq I \\
I \leq \mu t - (1 - \mu) \pi C
\end{array} \right. \\
\end{align*}$$

where $t(C)$ and $\pi(C)$ are the solutions to the problem $\mathcal{P}_1$. From Proposition 2, they are uniquely defined and continuous decreasing functions of the audit cost $C$. The last two inequalities characterize the constraint set. The first inequality reflects that the lowest value of $I$ supporting regime $RA$ is not unique because it depends upon the initial audit cost $C_0$, as it is clear from (3). The second inequality shows the resource constraint of the government, at this initial stage.

A general characterization of the solution to $\mathcal{P}_2$ is difficult, for the following reasons. First, when $\max\{0, \bar{T}\} = \bar{T}$, the constraint set may be empty. Indeed, under some parametric configurations [e.g. high initial audit cost $C_0$ and low investment productivity $\delta$], no investment fulfills the resource constraint. Second, even if the constraint set is non-empty and we can prove that the problem $\mathcal{P}_2$ has a maximum, it is often difficult to find it with the usual techniques. As the expected welfare $E^{WA}$ is not always concave, its profile can have many critical points. And the problem with this is that the sign of

$$\frac{\partial^2 E^{WA}}{\partial I^2} = -\frac{(1 - \mu)}{p} \left( \frac{\partial \pi}{\partial I} \frac{\partial t}{\partial C} (c_1)^2 + \pi c_{II} \right)$$

evaluated at the critical points, cannot be obtained analytically.

Finally, even if one succeeds in identifying $I_A$, the comparison between $E^{WA}$ and $E^{WA}(I_A)$ is not straightforward because it is a comparison of levels. We illustrate all these difficulties in the following figures.

Insert Figures 2 here

Figure 2(a) exhibits an example where $\bar{T} > 0$ and the resource constraint is never satisfied under regime $RA$. Irrespective of the value $I \geq \bar{T}$, the expected net tax collection is always negative. Hence, the unique solution is to invest nothing and let regime $RNA$ to emerge. Figure 2(b) plots

22By construction, this can never happen when $\max\{0, \bar{T}\} = 0$.  

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the expected welfare $E \mathbb{W}^A$ as a function of $I$, under a parametric configuration where only regime $R^A$ emerges. We can see that the curve is not concave and has two critical values. Figure 2(c) shows another profile of the expected welfare $E \mathbb{W}$, this time under a parametric configuration such that $T > 0$ and so both regimes emerge. First, we can see that under regime $R^A$, not all levels of investment $I \geq T$ make the resource constraint to hold. This only happens when $I \in [I_1, I_2]$. Second, even if we can identify $I_A = 13.67$, it is clearly not the overall maximum of the expected social welfare $E \mathbb{W}$. In fact, the maximum is achieved when $I = 0$. These difficulties push us to simulate the model in the next section.

But before doing that, we can prove the following results that apply when the optimal investment $I = I_A > 0$.

**Lemma 1** If the government invests a strictly positive amount of money to modernize its tax administration, the resource constraint does not bind.

Under regime $R^A$, the resource constraint never binds at the optimum because taxing the rich to spend only on the modernization of the tax administration is clearly dominated, in welfare terms, by not investing. This explains why, in the problem $\mathcal{P}_1$, we did not impose the expected net tax collection to be positive when $I > 0$.

**Lemma 2** If the government invests a strictly positive amount of money to modernize its tax administration, the optimal tax $t$, the optimal audit probability $\pi$ and the level of the public good $G$ are higher with respect to a model where such investment is not analyzed.

If the optimal investment is strictly positive, each audit costs less than $C_0$. Therefore, by Proposition 2, the optimal tax and audit probability can increase with respect to a model where investment in the tax administration is not considered and so $C = C_0$. If this is so, optimality implies that the provision of the public good must increase, to compensate for the lower consumption of taxpayers. Next, we present the following comparative statics results.

**Proposition 2** If the government invests a strictly positive amount of money in the tax administration, the optimal investment $I$ increases with taxable income $y$ and with the degree of aversion to inequality $(1 - \alpha)$, but decreases with the cost of the public good $p$. With respect to the other parameters of the model, the change in the optimal investment $I$ is ambiguous.

An optimal strictly positive investment $I_A$ is characterized by the following first-order condition

$$(1 - \mu) \pi c_I = 1$$

(4)

where the lhs of (4) is the marginal saving in the aggregate audit cost (MSAAC) and the rhs, the marginal cost of investment (MCI), which is always 1. From (4), we can realize that the total effect on the optimal investment of a change in one parameter is the result from the combination of two potential effects on the MSAAC. First, there is a direct effect that occurs when the parametric change affects only $(1 - \mu), \pi$ or $c_I$. Second, there is an indirect effect when the parametric change causes a variation in the optimal tax $t$, making also the optimal audit probability $\pi$ to change. So, if after a change in one parameter the MSAAC is greater (lesser) than the MCI, the government restores optimality by increasing (decreasing) the investment. Next, we explain in detail the comparative static results presented in the proposition.
When taxable income $y$ increases, the two effects go in opposite directions. On the one hand, the direct effect in $\pi$ is, due to the concavity of the utility function $u$, negative. On the other hand, the indirect effect is positive: an increase in $y$ enables the government to tax more and thus $\pi$ increases. In spite of these opposite directions, the indirect effect offsets the direct effect and thus $\pi$ increases. As the MSAAC increases, the government optimally invests more.

When the degree of aversion to inequality $(1 - \alpha)$ increases, there is only an indirect effect. Higher aversion to inequality makes the government to increase the tax. This pushes the stake for evasion upwards and so $\pi$ has to increase. As a consequence of this, the MSAAC increases and thus $I$ also optimally increases.

When the cost of the public good $p$ increases, there is only an indirect effect: the government prefers to tax less. Therefore, the stake for evasion diminishes and thus $\pi$ decreases, leading to a decrease in the MSAAC. So $I$ optimally decreases.

When either the investment productivity or the initial audit cost $C_0$ increase, the two effects may appear. Specifically, in both cases, these effects depend upon the sign and the value of $c_{1\delta}$ and $c_{IC_0}$ at the optimum. As there is no justification to assume a priori any sign for these derivatives, the result will depend upon the particular specification of the audit cost function $C$.

Finally, when the fraction of rich individuals in the population $\mu$ increases, the two effects go in opposite directions. On the one hand, the direct effect is clearly negative: when $\mu$ increases, the MSAAC decreases. On the other hand, the indirect effect is positive: an increase in $\mu$ makes the government to tax more. As $t$ increases, so do $\pi$ and the MSAAC. Hence the total effect is ambiguous.

4 Numerical results

The numerical simulations\textsuperscript{23} will help us in several ways: to identify the optimal investment $I$, to quantify the other components of the optimal fiscal policy and to obtain or confirm the comparative static results presented in Proposition 3. Throughout these simulations, we adopt the following functional specifications:

- Utility function:
  \[ u(x) = \frac{1 - \gamma}{\gamma} \left( \frac{x}{1 - \gamma} \right)^{\gamma}, \quad \text{with } 0 \leq \gamma \leq 1 \]
  The utility function is assumed to be isoelastic, as in Pestieau et al. (2004).

- Audit cost function
  \[ c(I, \delta) = \frac{C_0}{1 + \delta I}, \quad \text{with } 0 \leq \delta \]
  This audit cost function generates cross effects (i.e. $c_{1\delta}$ and $c_{IC_0}$ are both non null). This enables us to confirm that some of the comparative static results found in Proposition 3 can be ambiguous.

\textsuperscript{23}The simulations have been done with Matlab 7.0. All codes and results are available upon request from the authors.
We also adopt the following baseline parameters

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Table 1: Baseline parameters

Before going to the numerical results of the model, the next figure shows an example that illustrates why, in what follows, the profile of the optimal investment $I$ can present a jump after a change in one parameter. Figure 3 draws, for different values of the investment productivity $\delta$, the profile of the expected welfare $E[W]$ as a function of investment $I$.

Insert Figure 3 here

With our baseline parameters, only regime $R^A$ emerges. When $\delta$ increases, the profile $E[W]$ changes in two different ways. First, the set of values of $I$ that verifies the resource constraint increases. That is the reason why the lowest two curves are not defined for all values of $I$. Second, the shape of the curves changes, from a monotonic decreasing curve to a non-monotonic one. The consequence of this last change is immediate to see. When $\delta < 0.094$, the optimal investment $I = 0$. But when $\delta = 0.094$, the profile of the expected welfare has two maximums. Then, when $\delta$ increases, the optimal investment $I$ becomes strictly positive. Hence, due to the non-concavity of $E[W]$, the optimal investment can be non-continuous in $\delta$.

The next paragraphs explain in detail, with reference to the corresponding figures, the impact of changes in each parameter on the endogenous variables of the model. For each parameter of the model, we choose an interval of variation that contains the corresponding baseline parameter.

4.1 Change in the investment productivity when $\delta \in [0, 2]$}

Given our baseline parameters, $C_0 = 120 < \overline{C} = 128.62$. Hence, throughout this subsection, only regime $R^A$ prevails.

Insert Figures 4 here

Figure 4(a) shows that, when $\delta < 0.094$, the investment productivity is so low that it is too costly, in welfare terms, to reduce the audit cost $C$. Therefore, the government does not invest and $C$ remains at its initial level $C_0$. When $\delta = 0.094$, the government starts investing: $I$ jumps upward to 7.94 and so $C$ jumps downward, to 68.71. Figure 4(a) also confirms that the optimal investment $I$ can be non monotonic in $\delta$. When $\delta \in [0.094, 0.18]$, an increase in $\delta$ makes the indirect effect to be positive and more important than the negative direct effect, because, due to the fact that the audit cost is still high, $t$ and $\pi$ can increase a lot in relative terms. So the MSAAC increases above the MCI, implying that $I$ optimally increases. When $\delta = 0.18$, the optimal investment $I = 10.64$ is maximal. But then, as the investment productivity is sufficiently high, the indirect effect is lower and the negative direct effect starts dominating, making the MSAAC to decrease below the MCI. In other words, it becomes worthless to maintain the increasing profile of investment and so $I$ starts to decrease. This decrease is slow: $\delta$ has to increase more than 1000% to reduce $I$ by 50%. As we can see in Figure 4(b), in spite of the non-monotonicity of $I$, the audit cost monotonically decreases with $\delta$. When $\delta \in [0.094, 0.18]$, the combined increases in $\delta$ and $I$ push the audit cost downward. When $\delta > 0.18$, the increase in $\delta$ dominates the decrease in $I$ and thus $C$ continues to decrease, although at a slow rate. At the end, $C$ decreases 83.53%, even more than
I. Figures 4(c) and 4(d) show the profiles of the optimal tax \( t \) and the optimal audit probability \( \pi \). When \( \delta < 0.094 \), as the audit cost remains at its initial level \( C_0 \), the government sets a low tax \( t = 12.51 \). Then, when \( \delta = 0.094 \), \( t \) jumps upward and sharply, to 73.07. Then it monotonically increases 35%. As a consequence of this increase, the tax distortion \( (t^* - t) \) decreases. Similarly, \( \pi \) jumps upward at \( \delta = 0.094 \) and then monotonically increases 84%, more than the increase in \( t \). Figure 4(e) presents the profile of the expected tax collection, which is different from the profile of the optimal tax only by a multiplicative constant \( \mu \). The following three figures 4(f), 4(g) and 4(h) show the profiles of the three possible public expenditures, as percentages of the expected tax collection. Despite the fact that the expected tax collection monotonically increases when \( \delta > 0.094 \), the shape of the investment as a share of the tax collection is similar to the shape of this variable in absolute terms, as shown in Figure 4(a). At \( \delta = 0.094 \), this share jumps from 0% to 29.45%, then increases up to 31% when \( \delta = 0.13 \), to finally decrease and reach 13%. Concerning the aggregate audit cost, it jumps downward at \( \delta = 0.094 \), from 6.99% to 6.86% and then monotonically decreases, to reach 14.3%. The expenditure in the public good shows an imperceptible downward jump at \( \delta = 0.094 \) (from 6.99% to 6.86%) and then monotonically increases, to attain 72.67%. Surprisingly, from \( \delta = 0.094 \), the investment productivity \( \delta \) has to increase almost 448% in order to make the expenditure in the public good reach more than 50% of the expected tax collection. Finally, Figure 4(i) presents the profile of the expected welfare, which is continuous in \( \delta \) and, from \( \delta = 0.094 \), monotonically increasing, albeit at a decreasing rate. Although this increase seems to be very important in absolute terms (from 2.99 to 29, a 869%), a 1% increase in the investment productivity only yields, on average, a 0.42% increase in expected welfare.

4.2 Change in the initial audit cost when \( C_0 \in [0,300] \)

Insert Figures 5 here

In Figure 5(a), when \( C_0 < 8 \), the initial cost \( C_0 \) is so low that, even under regime \( R^A \), it never pays (in welfare terms) to invest in the tax administration. So the optimal investment \( I = 0 \) and thus the audit cost \( C = C_0 \), which is obviously increasing in this parameter. When \( C_0 = 8 \), the government starts investing in the tax administration because, as \( C_0 \) is relatively high, the MSAAC is now equal to the MCI. Contrary to the analysis in the previous subsection, the optimal investment does not jump upward at this point. Then \( I \) increases with \( C_0 \), at a decreasing rate. Thus, the increase in \( C_0 \) dominates the increase in \( I \), yielding the audit cost to increase from 0 to 51.6, as we can see in Figure 5(b). Then, when \( C_0 = 203 \), it becomes too expensive, again in welfare terms, to invest to attenuate the increase in the audit cost. Hence, investment jumps downwardly, from its maximum 13.34 to 0, yielding an upward jump in the audit cost \( C \) from 51.6 to 203. At this point, the audit cost verifies \( C > C^* \) and thus regime \( R^{NA} \) emerges onwards. Figure 5(c) shows that the optimal tax \( t \) decreases, starting from \( t^* = 99.87 \) and reaching 82.17; and when \( C_0 = 203 \), it jumps downwardly to 0. The profile of the optimal audit probability \( \pi \) is also decreasing, starting from \( \pi^{Max} = 0.965 \) and attaining 0.58, then when \( C_0 = 203 \), it also jumps downward to 0. Figures 5(f), 5(g) and 5(h) show the profiles of the three different public expenditures.\(^{24}\) As \( I \) increases with \( C_0 \) while the expected tax collection decreases, the percent of investment in the tax collection obviously increases and can even attain 40.59%. Figure 5(g) shows that the aggregate audit cost also increases, from 1.44% to 54.4%. From the previous two figures, it is straightforward to see in

\(^{24}\)As the expected tax collection is 0 when \( C_0 > 203 \), these profiles are not defined for such values of the initial audit cost.
Figure 5(h) that the amount of money spent in the public good decreases with $C_0$, from 98.56% and reaches a minimum of 0.05% at $C_0 = 203$. Finally Figure 5(i) depicts the profile of the expected welfare, which starts at $E\mathcal{W}^* = 40.05$ and then monotonically decreases to attain $E\mathcal{W}^{NA} = 2.83$. When investment is strictly positive, a 1% increase in the initial audit cost only yields, on average, a 0.03% decrease in expected welfare.

### 4.3 Change in the aversion to inequality when $(1 - \alpha) \in [0, 1]$

Insert Figures 6 here

Figures 6(a) and 6(b) confirm the comparative statics presented in Proposition 3. When the aversion to inequality increases, the government would like to increase the tax. Hence, by incentive compatibility, the optimal audit probability $\pi$ increases, leading to an increase in the MSAAC. Thus, when $(1 - \alpha)$ increases from 0 to 1, the government invests 9.4% more, starting at 10.03 and reaching 10.97. As a consequence of this increase, the audit cost $C$ monotonically decreases 6%, from 37.4 to 35.15. Such decrease in the audit cost re-enforces the above mentioned wish to tax more the rich, making the optimal tax $t$ and the optimal audit probability $\pi$ to monotonically increase, albeit slowly in both cases (6.05% and 13.34%, respectively). Regarding the three different public expenditures, we observe that, in spite of the fact that the expected tax collection monotonically increases 6%, from 35.08 to 37.2, the profile of the investment as a percentage of the tax collection always increases. Interestingly, the aggregate audit cost is not monotonic in $(1 - \alpha)$. First, it decreases from 41.55% to 41.54% because the decrease in the audit cost $C$ dominates. But then, due to the increase in the optimal audit probability $\pi$, it increases to reach 41.7%. Finally, the amount spent in the public good monotonically decreases with $(1 - \alpha)$, from 29.84% to 28.8%. In Figure 6(i), we can see that the expected welfare monotonically decreases, from 12.43 to 10.73. All these figures show that an utilitarian $(1 - \alpha = 0)$ or Rawlsian $(1 - \alpha = 1)$ government adopt qualitatively different decisions. But, with our baseline parameters, these differences are in fact quantitatively not very significant.

### 4.4 Change in the proportion of rich individuals when $\mu \in [0, 1]$

Insert Figures 7 here

When the proportion of rich individuals $\mu < 0.31$, $C$ is very low. Given the baseline parameters $C_0$ and $\delta$, a great investment would be necessary to make $C < \bar{C}$. In fact, such an amount of resources is prohibitive, so $I = 0$. In other words, there are so many poor that the aggregate audit cost would be very high. Hence it does not pay to enforce the tax law and thus regime $R^{NA}$ emerges. When $\mu = 0.31$, $I$ jumps upward to 10.09 and thus regime $R^{NA}$ shifts towards regime $R^A$. Figure 7(a) also confirms that the optimal investment $I$ can be non monotonic in $\mu$. From $\mu = 0.31$, $I$ increases with $\mu$. As there are relatively more rich in the population, the government increases the tax (with the consequent increase in the optimal audit probability $\pi$). Hence the indirect effect offsets the direct effect and so the MSAAC increases, making the government to invest more. When $\mu = 0.39$, optimal investment $I = 10.51$ attains its maximum. Then, when $\mu$ increases, the direct effect starts dominating. As there are fewer poor, the MSAAC decreases bellow the MCI, making $I$ to monotonically decrease. When $\mu = 0.96$, then $I = 0$ again. There are almost no poor, so the aggregate audit cost becomes negligible. So does the MSAAC and thus investment is worthless, even under regime $R^A$. Concerning the profile of the audit cost $C$, it is inversely related to the profile.
of $I$: first $C$ jumps downward from $C_0 = 120$ to 37.51, then it decreases slightly to reach 36.22 at $\mu = 0.4$ and finally, it increases to attain again $C_0$. Figure 7(c) shows the profile of the optimal tax $t$. When regime $R^{NA}$ prevails, $t = 0$; then $t$ jumps upward to 80.98. After that, the effect of an increase in $\mu$ prevails over the effect through the increase in the audit cost $C$. Hence, $t$ monotonically increases and converges to $t^*$. The optimal audit probability $\pi$ behaves similarly in Figure 7(d). First, there is the upward jump from 0 to 0.56, and then the increase, to reach $\pi^{Max} = 0.965$. From Figures 7(a), 7(b) and 7(d), we realize that investment and enforcement expenditures can be substitutes or complements, depending upon the parameters of the model. Figure 7(e) depicts the profiles of the expected tax collection under full and asymmetric information. When $\mu = 0$, both are equal. Under regime $R^{NA}$, the difference between the tax collection under both informational settings increases (because the expected tax collection under asymmetric information is always 0). When the government starts investing in the tax administration, the expected tax collection jumps upward to 25.91 and then increases, converging to the expected tax collection under full information when $\mu = 1$. Figures 7(f), 7(g) and 7(h) show the profiles of the three different public expenditures as percentages of the expected tax collection.\(^{25}\) In spite of the fact that the optimal investment $I$ first increases and then decreases, as the expected tax collection sharply increases when $\mu \geq 0.31$, the share of the amount invested always decreases, from 39.27% to 0% when $\mu = 0.96$. The aggregate audit cost presents a similar shape: it decreases from 55.81% to 1.16% when $\mu = 1$. The amount of the public good monotonically increases, from the minimum 5.92% to the maximum 98.84% when $\mu = 1$. Last, Figure 7(i) depicts the profile of the expected welfare, both under full and asymmetric information. When $\mu = 0$, $EW = EW^*$. As $\mu$ increases, so does the difference $EW^* − EW$, due to a higher increase in $EW^*$. But then, when $\mu \geq 0.31$, such difference decreases again, vanishing in the limit when $\mu = 1$.

4.5 Change in taxable income when $y \in [0, 200]$

Insert Figures 8 here

Figures 8(a) and 8(b) present the profiles of the optimal investment $I$ and the resulting audit cost $C$. When taxable income $y < 75$, $C$ is very low. So, due to the baseline parameters, it is very likely that $C > C$. In other words, the taxable income is so low, with respect to the audit cost, that it does not pay to enforce the tax law and thus regime $R^{NA}$ emerges. In that case, $I = 0$. But when $y$ increases, so does $C$. Hence, when $y = 75$, there is a shift of regimes and the government starts investing in the tax administration: $I$ jumps upward to 8.92. Then $I$ increases with $y$, reaching 12.16. The indirect effect, which makes the government to increase the tax (with the consequent increase in $\pi$) always offsets the direct effect (that makes $\pi$ to decrease). So the MSAAC increases, making the government to invest more, albeit slightly. Indeed, from $y = 75$, an increase of 170% in taxable income only induces the government to invest 36.3% more. Concerning the profile of the audit cost $C$, it is inversely related to the profile of $I$. First $C$ jumps downward, from $C_0 = 120$ to 40.5 and then decreases slowly, to 32.65. Regarding the optimal tax, if $y < 75$, the government does not tax. At $y = 75$, the government starts taxing the rich and so $t = 40.08$. Then, the optimal tax monotonically and sharply increases 388%, converging to $t^*$ (which is also increasing in $y$). This increase is more important than the increase in taxable income, which equals 166.7%. Figure 8(d) shows the profile of the optimal audit probability $\pi$, which jumps upward from 0 to 0.554 and then monotonically increases, to attain 0.85. The profile of the expected tax

\(^{25}\)As the expected tax collection is 0 whenever $\mu < 0.31$, these profiles are not defined there.
collection in Figure 8(e) is similar to the profile of $t$. As soon as regime $R^A$ emerges, the expected tax collection increases 225%, again more than the increase in taxable income. The next figures show the public expenditures. Although the optimal investment and the optimal audit probability increase, the increase in the expected tax collection is so important that investment and aggregate audit cost, as percentages of the expected tax collection, both decrease, from 37.11% to 15.5% and from 56% to 21.3% respectively. Obviously, the share of the expenditure in the public good increases with taxable income, from 6.85% to 43.12%, a surprisingly 529%. Finally, Figure 8(i) depicts the profile of the expected welfare. First, we can see that it increases almost negligibly. But then the increase is more important. In spite of this, the difference $\mathbb{E}W^* - \mathbb{E}W$ does not vanish because the expenditures in investment and enforcement are never negligible.

4.6 Change in the cost of the public good when $p \in [0,5]$  

With the chosen baseline parameters, we can see that $C < \overline{C}$. Hence, in this subsection, again regime $R^A$ prevails.

Insert Figures 9 here

Figure 9(a) confirms the result presented in Proposition 3. When the cost of the public good increases, the government reduces the tax. Hence, by incentive compatibility, the optimal audit probability decreases, leading to a decrease in the MSAAC. Thus the government invests less. The decrease in optimal investment, from 10.97 to 8.31 (24%), does not seem important, given the fact that the public good ends up costing 5 times more than the private good. Clearly, this decrease in investment also yields a slight increase in the audit cost $C$ by 20.7%, from 35.14 to 42.4. This increase in the audit cost reinforces the above mentioned wish to tax less, making $t$ to decrease monotonically, from 93.03 to 75.5. So does $\pi$, from 0.73 to 0.5. Figure 9(e) depicts the profile of the expected tax collection, which almost replicates the shape of the profile of $t$. Concerning the three different public expenditures, and in spite of the fact that the expected tax collection monotonically decreases, the profile of investment as percentage of the expected tax collection always decreases. In Figure 9(g), we can observe that the aggregate audit cost first decreases, from 41.7% to its minimum 41.54% and then increases slightly, to reach 42.57%. This reflects the opposite effects of the increase in the audit cost and the decrease in the audit probability. As in this subsection the cost of the public good changes, and thus it does not equal the price of the private good, we need to differentiate the expenditure and the physical amount of the public good. Figure 9(h) surprisingly shows that, as a consequence of the previous two figures, the expenditure in public goods increases, when $p \leq 3.65$, from 28.8% to 30.16% of the expected tax collection and then decreases. Figure 9(i) presents the profile of the physical amount of the public good, which monotonically decreases, from very high quantities when the cost is very low, converging to 0, when the public good becomes 5 times more expensive than the private good. Finally, Figure 9(j) shows that the expected welfare monotonically decreases, from very high values when the cost of the public good is very low (due to its very high provision) to very low ones, again converging to 0. Also the difference $\mathbb{E}W^* - \mathbb{E}W$ vanishes.

5 Conclusion

There is a large list of contributions that have analyzed optimal tax-enforcement policies under the threat of tax evasion. Surprisingly, all assume that the audit cost function is exogenously
given. In practice, governments invest many resources to modernize their tax administration and thus, to change audit costs. This paper is a first step towards the incorporation, in the theory of optimal fiscal policies, of these investment decisions. In a very simple model, we have been able to characterize the optimal tax-enforcement policy, adopting a mechanism design approach. As many other contributions to the costly-state verification literature, the optimal fine for evaders is maximal and the optimal audit probability is such that evasion is deterred. However, in order to attenuate the stake for evasion, the government optimally distorts taxes downward, distortion with respect to the full information optimal tax. Then we analyze the optimal investment. Unfortunately, due to a fundamental non-concavity of this last problem, we cannot completely characterize it. So we simulate the model. This enables us not only to identify the solutions but also to study how the optimal investment interacts with the other components of the optimal fiscal policy. Clearly this model suggests the need to incorporate investments in the tax administration into the currently used definitions of ‘tax effort’.

In spite of its simplicity, the model generates some testable implications. Moreover, the model can be easily generalized to a dynamic setting, to analyze the path of the different elements of the optimal fiscal policy. All these are interesting venues for future research.
References


6 Appendix

6.1 Characterization of the optimal tax law \((t, f, \pi)\)

**First-order condition**

Formally, the government solves the following problem

\[
\begin{align*}
\mathcal{P}_1 \text{ } & \begin{cases} 
\text{Max} & \alpha \mu u(y - t) + G \\
\text{subject to} & \\
0 \leq t & \\
t + f \leq y & (LL') \\
u(y - t) \geq \pi u(y - t - f) + (1 - \pi) u(y) & (IC) \\
pG = \mu t - (1 - \mu) \pi C - I & (B')
\end{cases}
\end{align*}
\]

At the optimum, \((LL')\) binds: increasing the fine \(f\) up to its maximal legal level relaxes the incentive compatibility constraint, with no other impact in the aggregate welfare. Moreover, as it is usual in Principal-Agent models of tax enforcement-policies like Mookherjee and Png (1989), \((IC)\) also binds: the government sets an audit strategy that makes a potential evader to be indifferent between truthfully reporting his type and misreporting. From this binding constraint and replacing \(f\) by \(y - t\), we can obtain the value of the optimal audit probability, as follows

\[
\pi = 1 - \frac{u(y - t)}{u(y)}
\]

Moreover, we can also obtain the value of \(G\) from \((B')\). So, replacing \(\pi\) and \(G\), the maximand of \(\mathcal{P}_1\) becomes

\[
\alpha \mu u(y - t) + \frac{1}{p} \left( \mu t - (1 - \mu) \left[ 1 - \frac{u(y - t)}{u(y)} \right] C - I \right)
\]

which is a strictly concave function of \(t\).\(^{26}\)

Differentiating \((5)\) with respect to \(t\), we obtain the necessary and sufficient first-order condition that characterizes an interior optimal tax \(t\), as follows

\[
\alpha u_q(y - t) = \frac{1}{p} \left( 1 - \frac{(1 - \mu)C}{(1 - \mu)C + \alpha \mu u(y)} \right)
\]

By strict concavity of \((5)\), the optimal tax is unique. Moreover, as the utility function \(u\) is concave, the optimal tax verifies \(t \leq t^*\).

**Comparative statics**

\(^{26}\)Indeed, the second derivative of \((5)\) with respect to \(t\) is

\[
u_{qq}(y - t) \left( \alpha \mu + \frac{(1 - \mu)C}{\mu u(y)} \right) < 0
\]
By the Maximum theorem [Berge (1963)], the optimal tax \( t \) is a continuous function of \( C \), and so are \( \pi \) and \( f \). In order to completely characterize the optimal tax law \((t, f, \pi)\), let’s compute \( \frac{\partial t}{\partial C} \).

From (6), we can apply the Implicit Function theorem and show that

\[
\frac{\partial t}{\partial C} = \frac{\mu(1-\mu)u(y)}{S} < 0 \quad \frac{\partial \pi}{\partial C} = \frac{\partial t}{\partial C} = \frac{u_q(y-t)}{u(y)} \frac{\partial t}{\partial C} < 0
\]

\[
\frac{\partial G}{\partial C} = \left( \frac{\alpha u^2 p(y)}{(1-\mu)C + \alpha \mu p(y)} \right) \frac{\partial t}{\partial C} > (1-\mu) < 0
\]

where \( S = u_{qq}(y-t).[(1-\mu)C + \alpha \mu pu(y)]^2 \). We can also compute\(^\text{27}\)

\[
\frac{\partial t}{\partial \alpha} = \frac{p[\mu u(y)]^2}{S} < 0 \quad \frac{\partial \pi}{\partial \alpha} = \frac{\partial t}{\partial \alpha} \frac{\partial t}{\partial \alpha} < 0
\]

\[
\frac{\partial t}{\partial \mu} = \frac{-u(y)C}{S} > 0 \quad \frac{\partial \pi}{\partial \mu} = \frac{\partial t}{\partial \mu} \frac{\partial t}{\partial \mu} > 0
\]

\[
\frac{\partial t}{\partial y} = \frac{-1}{u_{qq}(y-t)} \left[ \frac{\mu (1-\mu)u_q(y)C}{[(1-\mu)C + \alpha \mu pu(y)]^2} - u_{qq}(y-t) \right] > 0
\]

\[
\frac{d\pi}{dy} = \frac{\partial \pi}{\partial y} + \frac{\partial \pi}{\partial t} \frac{\partial t}{\partial y} = \frac{-u_q(y-t)}{u(y)} \frac{\mu(1-\mu)u_q(y)C}{S} + \frac{u(y-t)u_q(y)}{[u(y)]^2} > 0
\]

\[
\frac{\partial t}{\partial p} = \frac{\alpha[\mu u(y)]^2}{S} < 0 \quad \frac{d\pi}{dp} = \frac{\partial \pi}{\partial t} \frac{\partial t}{\partial p} < 0
\]

**Domain of positive taxation**

Let’s find the parametric region (with respect to the audit cost \( C \)) where \( t \geq 0 \). From (6), we can find the value \( \overline{C} \) that generates the limit case \( t = 0 \). This value is implicitly characterized by the following expression

\[
\alpha u_q(y) = \frac{1}{p} \left[ 1 - \frac{(1-\mu)C}{(1-\mu)C + \alpha \mu pu(y)} \right]
\]

After some manipulations, we obtain

\[
\overline{C} = \frac{[1-\alpha pu_q(y)]\mu u(y)}{(1-\mu)u_q(y)}
\]

\(^{27}\)The effects of the parametric changes on \( f \) have the opposite signs than the analyzed effects on \( t \).
Finally, with all these results, we can easily derive that \( \lim_{C \to 0} t = t^* \) and \( \lim_{C \to C} t = 0 \). Moreover, it is straightforward to see that
\[
\lim_{C \to 0} f = y - t^* \quad \text{and} \quad \lim_{C \to C} f = 0
\]
\[
\lim_{C \to 0} \pi = \pi_{Max} \equiv 1 - \frac{u(y - t^*)}{u(y)} \quad \text{and} \quad \lim_{C \to C} \pi = 0
\]
\[
\lim_{C \to 0} G = G^* \quad \text{and} \quad \lim_{C \to C} G = 0 \quad \square
\]

### 6.2 Characterization of an (interior) optimal investment \( I_A \)

Here, we adopt a parametric configuration such that the constraint set is not empty. Under this circumstance, the constraint set is clearly bounded by 0 and \( \mu t^* \) (i.e. the tax collection under full information). Moreover, this set is also closed because it is defined by weak inequalities and the functions \( t(C) \) and \( \pi(C) \) are continuos in \( I \). Hence, the constraint set is compact. In addition, the maximand in \( P_2 \) is also continuos in \( I \). So, by the Weierstrass theorem [Takayama (1985)], the problem \( P_2 \) has a maximum.

**First-order condition**

To find the (interior) optimal investment \( I_A \), the government solves the following problem

\[
P_2 \begin{cases} 
    \text{Max} & I \alpha u(y - t) + \frac{1}{p} \left[ \mu t - (1 - \mu) \pi C - I \right] \\
    \text{subject to} & t = t(C), \quad \pi = \pi(C) = 1 - \frac{u(y - t(C))}{u(y)} \\
    & C = c(I, \delta)
\end{cases}
\]

The first-order condition for an interior solution of problem \( P_2 \) is given by

\[
\frac{\partial \mathcal{EW}^A}{\partial I} = \frac{\partial t}{\partial C} c_I \left( -\alpha \mu u_q (y - t) + \frac{1}{p} \left[ \mu - (1 - \mu)C \left( \frac{u_q(y - t)}{u(y)} \right) \right] \right) + \frac{1}{p} [(1 - \mu) \pi c_I + 1] = 0
\]

Using (6), the term in parenthesis \( A \) vanishes, so we can rewrite (8) as

\[
\frac{\partial \mathcal{EW}^A}{\partial I} = -\frac{1}{p} [(1 - \mu) \pi c_I + 1]
\]
From (9) and assuming that $Z = -(1 - \mu) \left( \frac{\partial \pi}{\partial t} \frac{\partial c_I}{\partial C} (c_I)^2 + \pi c_{II} \right) < 0$ (i.e. that the solution verifies second-order condition), we can apply the Implicit Function theorem and compute

$$\frac{\partial I}{\partial \delta} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial C} c_I^3 c_I + \pi c_{II} \right) \geq 0 \quad \frac{\partial I}{\partial C_0} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial C} c_{C_0} c_I + \pi c_{IC_0} \right) \geq 0$$

$$\frac{\partial I}{\partial \mu} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial \mu} c_I - \pi c_I \right) \geq 0 \quad \frac{\partial I}{\partial y} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial t} c_I + \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial y} c_I \right) > 0$$

$$\frac{\partial I}{\partial \alpha} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial \alpha} c_I \right) < 0 \quad \frac{\partial I}{\partial p} = \frac{1 - \mu}{Z} \left( \frac{\partial \pi}{\partial t} \frac{\partial c}{\partial p} c_I \right) < 0$$
Figure 2(a)

Figure 2(b)

Figure 2(c)
Figure 3: Changes in the profile of $\mathbb{EW}$ when $\delta$ varies.
Figure 6(a)

Figure 6(b)

Figure 6(c)

Figure 6(d)

Figure 6(e)

Figure 6(f)

Figure 6(g)

Figure 6(h)

Figure 6(i)