Technical Appendix

Appendix A: Sampling common demand shocks and preference coefficients (independent samples case)

In this Appendix, we describe the procedure to sample common demand shocks and preference coefficients for the independent samples case assuming that the utility function of each consumer includes brand intercepts and that data from R consumers in each period are considered. We first analyze the case in which x_t and ξ_t are not correlated and then we relax this assumption.

A1. No endogeneity

Let α_{it} denote the J vector of brand intercepts for the i^{th} consumer the t^{th} period random sample and β_{it} the remaining preference coefficients in θ_{it}. Denoting by m the number of independent variables, then θ_{it} is a vector with m components. Define $\tilde{\alpha}_{ijt} = \alpha_{ijt} + \xi_{jt}$ and let $\tilde{\theta}_{ijt} = (\tilde{\alpha}'_{it}, \beta'_{it})'$. Accordingly, the probability that the i^{th} consumer in the t^{th} period random sample chooses alternative j can be computed as follows:

$$p_{ijt}(\tilde{\theta}_{ijt}) = \frac{e^{\tilde{\theta}'_{ijt} x_{jt}}}{\sum\limits_{k=1}^{J} e^{\tilde{\theta}'_{ikt} x_{kt}}}. \tag{25}$$

In addition, let $\tilde{\xi}_t = (\xi'_t, 0'_{m-J})'$, where 0_{m-k} denotes a column vector with $m - J$ zeros and decompose D in blocks as follows:

$$D = \begin{bmatrix} D_{\alpha\alpha} & D_{\alpha\beta} \\ D'_{\alpha\beta} & D_{\beta\beta} \end{bmatrix},$$

where $D_{\alpha\alpha}$ is matrix with J rows and columns, $D_{\beta\beta}$ is a matrix with $m - J$ rows and columns, and $D_{\alpha\beta}$ is a matrix with J rows and $m - J$ columns. In addition, denote the first J components of $\overline{\theta}$ by $\overline{\alpha}$ and the remaining $m - J$ components by $\overline{\beta}$.

The method proposed here relies on the following two results. First, it is easy to verify that conditioning on $\overline{\theta}$, D and ξ_t, the prior probability of $\tilde{\theta}_{it}$ corresponds to a multivariate normal with mean $\overline{\theta}+\tilde{\xi}$ and variance D. This result will be used to sample $\tilde{\theta}_{it}$ from its full-conditional posterior distribution.

Second, using the properties of multivariate random variables, it can be shown that $\tilde{\alpha}_{jt} - \overline{\alpha} - D_{\alpha\beta} D_{\beta\beta}^{-1}(\beta_{it} - \overline{\beta})$ follows a multivariate normal distribution with mean ξ_t and variance-covariance matrix equal to $D_{\alpha\alpha} -$

$D_{\alpha\beta}D_{\beta\beta}^{-1}D'_{\alpha\beta}$. This results will be used to sample ξ_t from its full-conditional posterior distribution.

Accordingly, a Gibbs sampler can be implemented as follows:

1. Sample $\tilde{\theta}_{it}$ from its full-conditional posterior distribution using a MH step:

   1a) Generate $\tilde{\theta}^*_{it}$ from a multivariate normal with mean $\overline{\theta}^{(k)} + \tilde{\xi}^{(k)}$ and variance $D^{(k)}$.

   1b) Accept $\tilde{\theta}^*_{it}$ with MH probability $\alpha_{\mathrm{MH},\tilde{\theta}_{it}} = \frac{p_{iy_{it}t}(\tilde{\theta}^*_{it})}{p_{iy_{it}t}(\tilde{\theta}^{(k)}_{it})}$, otherwise set $\tilde{\theta}^{(k+1)}_{it} = \tilde{\theta}^{(k)}_{it}$.

2. Sample ξ_t directly from its full-conditional posterior distribution by generating $\xi_t^{(k+1)}$ from a multivariate Normal distribution with mean A_t and variance B, which are defined as follows:

$$\begin{aligned} A_t &= B(RC^{-1}E_t) \\ B &= \left(RC^{-1} + \Sigma^{(k)^{-1}}\right)^{-1} \end{aligned}$$

where:

$$\begin{aligned} C &= D^{(k)}_{\alpha\alpha} - D^{(k)}_{\alpha\beta}\left(D^{(k)}_{\beta\beta}\right)^{-1} D^{(k)\prime}_{\alpha\beta}, \\ E_t &= \frac{1}{R}\sum_{i=1}^{R}\left(\tilde{\alpha}^{(k)}_{it} - \overline{\alpha}^{(k)} - D^{(k)}_{\alpha\beta}\left(D^{(k)}_{\beta\beta}\right)^{-1}\left(\beta^{(k)}_{it} - \overline{\beta}^{(k)}\right)\right). \end{aligned}$$

Finally, $\overline{\theta}$ and D can be easily updated using standard conjugate methods by noting that $\theta_{it} = \tilde{\theta}_{it} - \tilde{\xi}_t$ and that each θ_{it} is i.i.d. multivariate normal with mean $\overline{\theta}$ and variance D. Similarly, Σ can also be sampled using standard conjugate methods by noting that each vector ξ_t is i.i.d. multivariate normal with zero mean and variance-covariance matrix equal to Σ.

A2. Endogeneity

Under the assumptions in §4, ξ_t is allowed to be correlated with $x_{jt,4}$. Consequently, the procedure in **A1** for sampling ξ_t and Σ from their full-conditional posterior distribution must be generalized.

First, decompose Σ in blocks as follows:

$$\Sigma = \begin{bmatrix} \Sigma_{\eta\eta} & \Sigma_{\eta\xi} \\ \Sigma'_{\eta\xi} & \Sigma_{\xi\xi} \end{bmatrix}.$$

Then, redefine A_t and B according to the following expressions:

$$A_t = B\left(RC^{-1}E_t + \Sigma_{\xi|\eta}^{(k)}{}^{-1}F_t\right)$$
$$B = \left(RC^{-1} + \Sigma_{\xi|\eta}^{(k)}{}^{-1}\right)^{-1}$$

where:

$$\begin{aligned}
C &= D_{\alpha\alpha}^{(k)} - D_{\alpha\beta}^{(k)}\left(D_{\beta\beta}^{(k)}\right)^{-1}D_{\beta\alpha}^{(k)}, \\
\Sigma_{\xi|\eta}^{(k)} &= \Sigma_{\xi\xi}^{(k)} - \Sigma_{\xi\eta}^{(k)}\left(\Sigma_{\eta\eta}^{(k)}\right)^{-1}\Sigma_{\xi\eta}^{(k)\prime}, \\
E_t &= \frac{1}{R}\sum_{i=1}^{R}\left(\tilde{\alpha}_{it}^{(k)} - \overline{\alpha}^{(k)} - D_{\alpha\beta}^{(k)}\left(D_{\beta\beta}^{(k)}\right)^{-1}\left(\beta_{it}^{(k)} - \overline{\beta}^{(k)}\right)\right), \\
F_t &= \Sigma_{\xi\eta}^{(k)}\left(\Sigma_{\eta\eta}^{(k)}\right)^{-1}\eta_t^{(k)},
\end{aligned} \tag{26}$$

Therefore, each ξ_t is simulated from its posterior distribution by drawing a vector from a multivariate normal distribution with mean A_t and variance-covariance matrix B.

Finally, the updating of Σ can be implemented using standard conjugate methods by noting that each vector (η_t, ξ_t) is i.i.d. multivariate normal with zero mean and variance-covariance matrix equal to Σ. Similarly, the updating of $\delta = (\delta_1, ..., \delta_J)'$ can be implemented using conjugate methods for linear regression by noting that each vector $(\eta_t - \Sigma_{\eta\xi}\Sigma_{\xi\xi}^{-1}\xi_t)$ is distributed according to a multivariate normal distribution with zero mean and variance-covariance matrix $(\Sigma_{\eta\eta} - \Sigma_{\eta\xi}\Sigma_{\xi\xi}^{-1}\Sigma_{\xi\eta})$, where $\eta_{jt} = x_{jt,4} - w_{jt}{}'\delta_j$.

Appendix B: Additional Results

Sections 2 and 3 in the manuscript present simulation results for only one of the three different cases under study (i.e., the correlation case). Results for the low and high heterogeneity cases are presented in this appendix in Table A1 and Table A2, respectively.

Table A1

Results: Estimated posterior mean, standard deviation and quantiles for $\overline{\theta}$ and D (low heterogeneity).

Demand System	Method	R (ind. samples), B (panel)		$\overline{\theta}_1$	$\overline{\theta}_2$	$\overline{\theta}_3$	D_{11}	D_{22}	D_{33}	D_{12}	D_{13}	D_{23}
Ind. Samples	Full Sample	250	mean	0.969	0.954	-0.949	0.777	0.835	0.941	-0.068	-0.018	0.054
			std.dev.	0.054	0.058	0.043	0.323	0.410	0.116	0.197	0.101	0.099
			2.5%	0.868	0.844	-1.038	0.375	0.311	0.727	-0.443	-0.225	-0.133
			50.0%	0.966	0.953	-0.948	0.683	0.756	0.936	-0.077	-0.015	0.047
			97.5%	1.085	1.067	-0.871	1.636	1.761	1.176	0.343	0.158	0.247
Ind. Samples	Subsampling	50	mean	0.965	0.890	-1.028	1.324	1.864	0.746	0.192	-0.027	-0.087
			std.dev.	0.142	0.142	0.102	0.813	1.289	0.229	0.636	0.207	0.225
			2.5%	0.732	0.635	-1.265	0.409	0.484	0.414	-0.808	-0.476	-0.588
			50.0%	0.950	0.880	-1.016	1.128	1.469	0.710	0.079	-0.016	-0.068
			97.5%	1.298	1.200	-0.863	3.370	5.359	1.326	1.760	0.368	0.300
Panel	Gibbs	2	mean	1.011	1.086	-0.959	1.691	0.773	1.085	0.089	-0.013	0.087
			std.dev.	0.110	0.095	0.086	0.686	0.306	0.192	0.292	0.172	0.127
			2.5%	0.803	0.908	-1.137	0.621	0.340	0.760	-0.437	-0.379	-0.165
			50.0%	1.007	1.082	-0.956	1.576	0.718	1.068	0.070	-0.006	0.087
			97.5%	1.238	1.285	-0.801	3.246	1.532	1.510	0.699	0.305	0.333
Panel	MH	10	mean	1.001	1.078	-0.958	1.697	0.742	1.081	0.056	-0.018	0.085
			std.dev.	0.105	0.090	0.084	0.705	0.325	0.185	0.263	0.166	0.124
			2.5%	0.803	0.915	-1.132	0.638	0.315	0.768	-0.402	-0.361	-0.165
			50.0%	0.997	1.073	-0.955	1.575	0.667	1.063	0.034	-0.010	0.085
			97.5%	1.221	1.272	-0.802	3.352	1.564	1.489	0.648	0.292	0.335
True Values				1.000	1.000	-1.000	1.000	1.000	1.000	0.000	0.000	0.000

Table A2

Results: Estimated posterior mean, standard deviation and quantiles for $\overline{\theta}$ and D (high heterogeneity).

Demand System	Method	R (ind. samples), B (panel)		$\overline{\theta}_1$	$\overline{\theta}_2$	$\overline{\theta}_3$	D_{11}	D_{22}	D_{33}	D_{12}	D_{13}	D_{23}
Ind. Samples	Full Sample	250	mean	0.568	0.603	-1.267	4.476	3.048	2.440	-2.303	-0.733	0.505
			std.dev.	0.329	0.288	0.293	2.617	1.814	1.552	2.214	1.378	1.157
			2.5%	-0.043	0.001	-1.818	0.731	0.980	0.466	-8.072	-3.748	-1.549
			50.0%	0.520	0.567	-1.259	4.141	2.524	2.113	-2.077	-0.564	0.402
			97.5%	1.155	1.096	-0.722	9.955	8.262	6.200	0.605	1.605	3.043
Ind. Samples	Subsampling	50	mean	1.032	1.041	-1.565	4.130	4.411	3.338	0.765	-1.614	0.750
			std.dev.	0.709	0.649	0.539	4.900	6.612	3.341	5.006	3.183	1.520
			2.5%	0.261	0.326	-2.732	0.573	0.601	0.557	-3.939	-10.771	-1.636
			50.0%	0.892	0.909	-1.540	2.417	2.311	2.179	-0.128	-0.730	0.486
			97.5%	3.074	2.727	-0.571	18.348	27.354	13.001	13.764	2.306	4.754
Panel	Gibbs	2	mean	1.149	1.224	-1.191	2.810	3.964	2.386	0.757	-0.693	-1.587
			std.dev.	0.308	0.331	0.240	2.144	2.442	1.218	1.348	0.804	1.331
			2.5%	0.671	0.688	-1.684	0.585	0.947	0.688	-1.209	-2.528	-5.040
			50.0%	1.090	1.169	-1.184	2.130	3.480	2.152	0.410	-0.581	-1.217
			97.5%	1.817	1.938	-0.743	8.618	9.805	5.268	3.709	0.645	0.1093
Panel	MH	10	mean	1.126	1.178	-1.293	2.757	4.780	2.622	0.622	-0.802	-2.104
			std.dev.	0.280	0.290	0.274	2.137	3.324	1.365	1.173	1.005	1.615
			2.5%	0.677	0.714	-1.857	0.629	1.053	0.754	-0.982	-3.531	-6.141
			50.0%	1.092	1.143	-1.279	1.992	3.608	2.406	0.400	-0.631	-1.735
			97.5%	1.756	1.822	-0.801	8.433	13.18	5.859	3.377	0.612	0.061
True Values				1.000	1.000	-1.000	3.000	3.000	3.000	0.000	0.000	0.000