Facility Location with Capacitated and Length-Bounded Tree Connections

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Workshop on Algorithms and Dynamics for Games and Optimization
Playa Blanca, October 17, 2013

Motivation

Design of Optical Access Networks

- determine sites for central offices with access hardware
- connect clients to central offices using optical fibres

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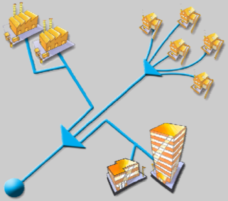
Fiber Trees



Splitting

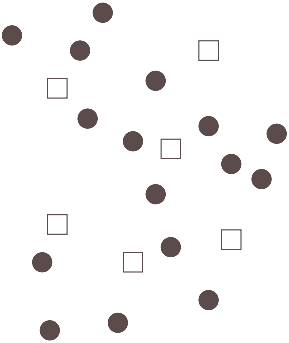
- single fiber at central office
- split into tree by splitters
- shared by many clients

Technological constraints

- clients share capacity of fiber
- limited signal strength

Outline

Problem Definition



UFL-CLT

- **input:** graph $G = (V, E)$,
  facilities $\mathcal{F} \subseteq V$, clients $\mathcal{C} \subseteq V$
  - client $v \in \mathcal{C}$: demand $d(v)$
  - facility $w \in \mathcal{F}$: opening cost $\phi(w)$
  - edge $e \in E$: length $\ell(e)$, cost $c(e)$
  - tree capacity U, length bound L
- **task:** Find facilities $F \subseteq \mathcal{F}$
  and trees $\mathcal{T}$ of minimum cost s.t.
  - every $T \in \mathcal{T}$ is rooted at $w_T \in F$,
  - every $v \in \mathcal{C}$ is served by $T_v \in \mathcal{T}$,
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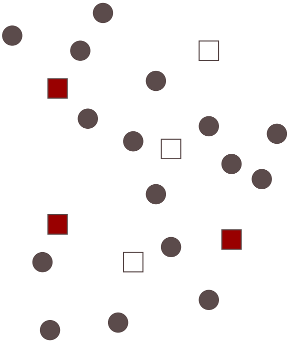

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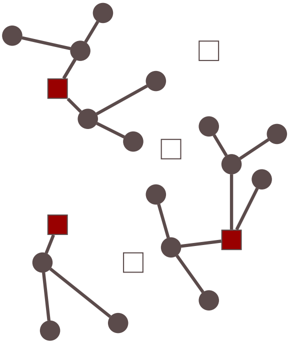

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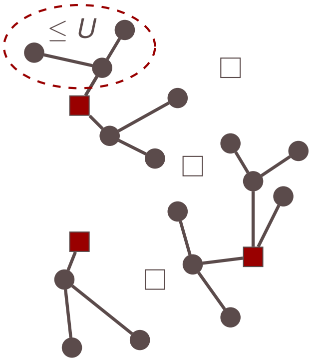

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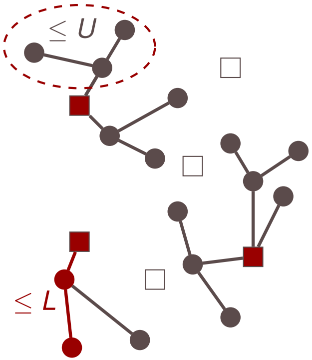

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Related Problems

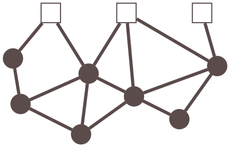
(α, β)-Approximation Algorithm

- approximates length bound by a factor of α
- approximates cost of optimal length-bounded solution by β

- Shallow-Light Trees
  - $(O(\log n), O(\log n))$-approximation for SL Steiner Tree [Marathe et al. 1998]
  - $(\alpha, 1 + \frac{2}{1-\alpha})$-Light Approximate Shortest Path Tree ($\ell = c$) [Khuller et al. 1995]
  - $(1, O(\log n))$-approximation for k-hop Spanning Tree ($\ell \equiv 1$) [Althaus et al. 2005]
- Capacitated Cable Facility Location
  - no length bound, splittable demands
  - $\rho_{\text{UFL}} + \rho_{\text{ST}}$-approximation [Ravi & Sinha 2006]

Tree Lower Bound

Construct graph G':



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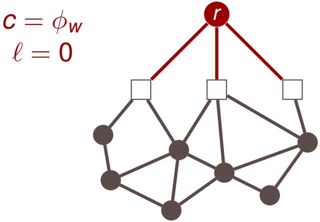
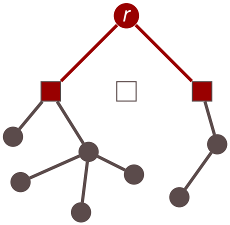


- add root vertex r
- connect r to all facilities with ϕ-cost and 0-length edges

Tree Lower Bound

Construct graph G':

$c = \phi_w$
$\ell = 0$



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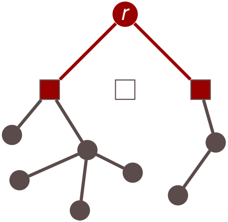

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Lemma

$c(S) \leq OPT$

UFL Lower Bound

Construct UFL instance:

- $\mathcal{C}$, $\mathcal{F}$, ϕ as before
- $\tilde{c}(v, w) = \frac{d(v)}{U} \min\{c(P) : P \text{ is a } v\text{-}w\text{-path}, \ell(P) \leq L\}$

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$\phi(F) + \sum_{v \in \mathcal{C}} \tilde{c}(v, F) \leq OPT$

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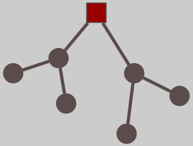

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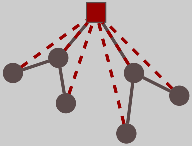
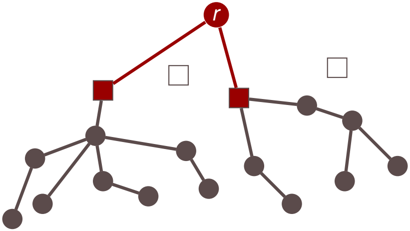


$$\sum_{v:T_v=T} \tilde{c}(v,w) \leq \underbrace{\left(\sum_{v:T_v=T} \frac{d(v)}{U} \right)}_{\leq 1} c(T)$$

Algorithm



$U = 3$, $d \equiv 1$

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1. compute $(\alpha_{\mathrm{ST}}, \beta_{\mathrm{ST}})$-approx. SLST T^*

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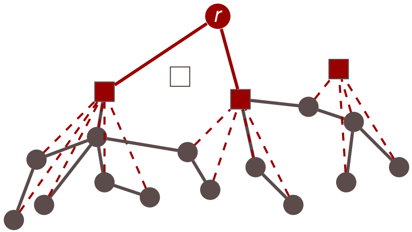

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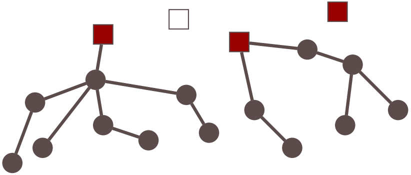

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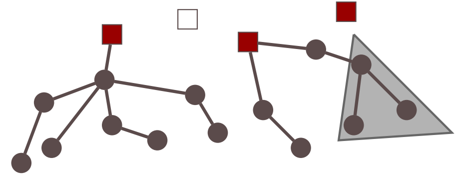

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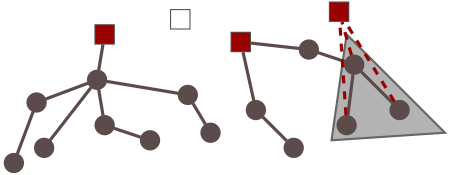

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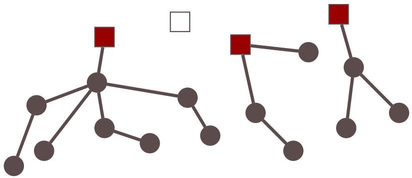

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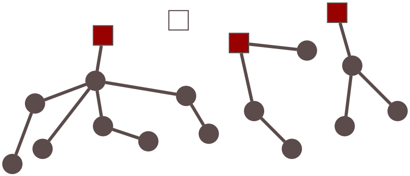

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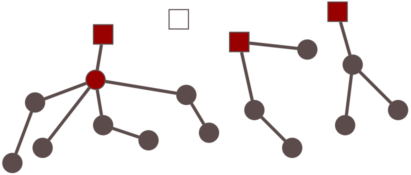

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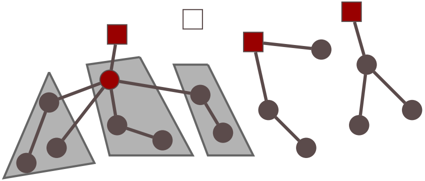

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2. partition subtrees into groups with demand between $U/2$ and U

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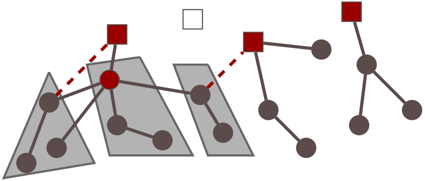

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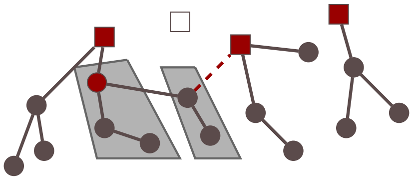

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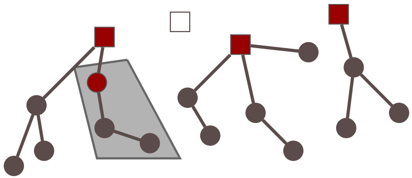

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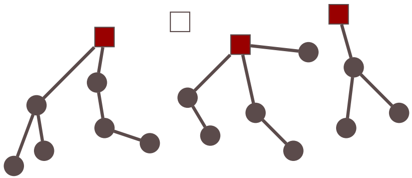

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Analysis

- returned solution $(\mathcal{T}, F)$
- initial tree T^*, UFL solution F^*

capacity: ?

length: ?

cost: ?

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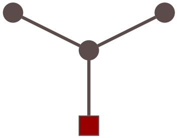

capacity: $d(T) \leq U$ for all $T \in \mathcal{T}$

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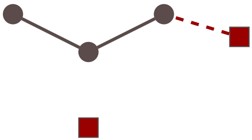

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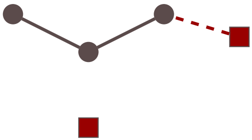

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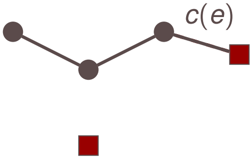

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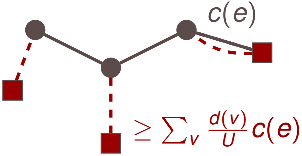

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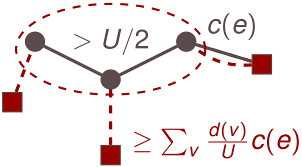
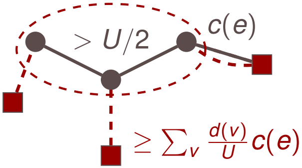


capacity:	$d(T) \leq U$	for all $T \in \mathcal{T}$
length:	$\ell(T[v,w]) \leq (2\alpha_{\mathrm{ST}} + 1)L$	for all $T \in \mathcal{T}$
cost:	$c(\mathcal{T}) + \phi(F) \leq c(T^*) + \phi(F^*) + 2\sum_{v \in \mathcal{C}} \tilde{c}(v, F^*)$	

Analysis

- returned solution $(\mathcal{T}, F)$
- initial tree T^*, UFL solution F^*



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Theorem

There is an $(O(\log n), O(\log n))$-approximation for UFL-CLT.

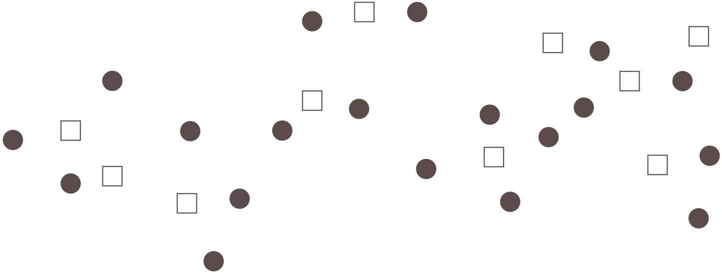
- use SLST matching augmentation and UFL-greedy

Approximation Guarantees

	length	cost	
general	$O(\log n)$	$O(\log n)$	
	$3+\varepsilon$	$O(\log^2 n)$	$(*)$
$\ell = c$, metric	3α	$(1+\frac{2}{\alpha-1})(\beta_{\mathrm{ST}}+\beta_{\mathrm{UFL}}^{\alpha})+1$	
$\ell \equiv 1$, c metric	$1+\varepsilon$	$O(\log n)$	

$(*)$ quasi-polynomial run-time

Greedy Cover



Algorithm

1. $F = \emptyset$
2. while $\exists v \in \mathcal{C}$ with $\ell(v, F) > 3L$
   - $F_v = \{w \in \mathcal{F} : \ell(v, w) \leq L\}$
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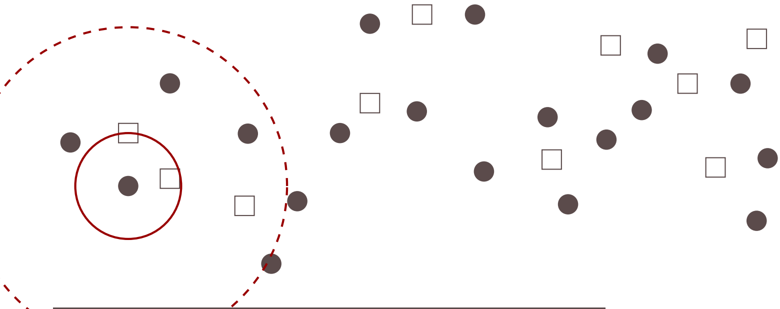

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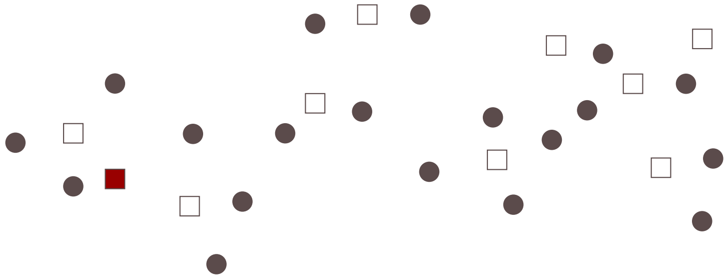

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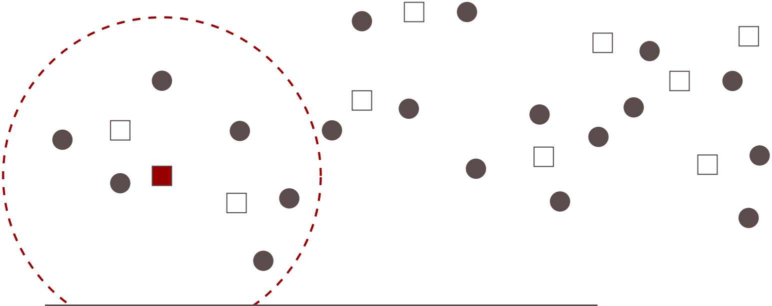

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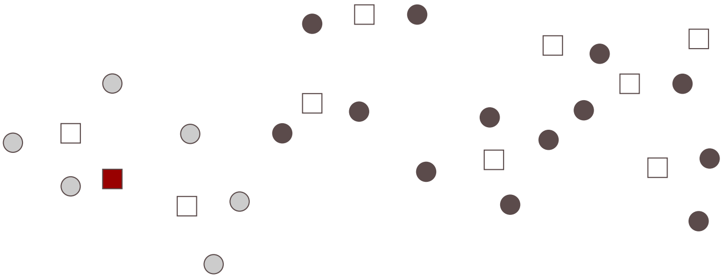

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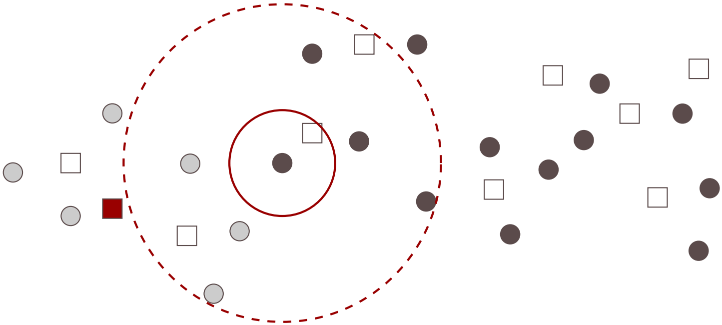

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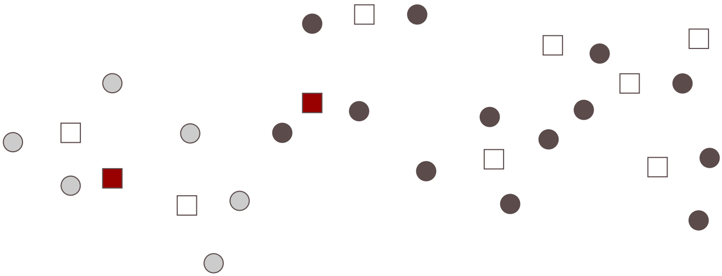

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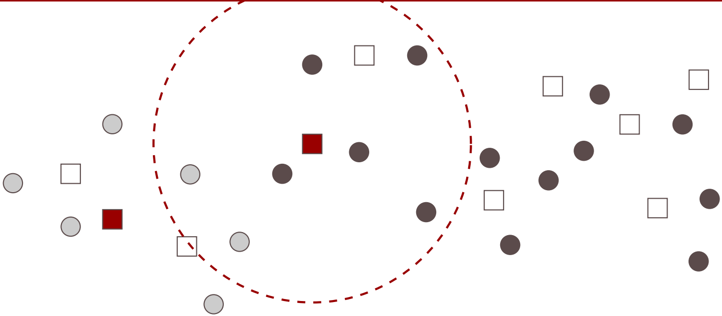

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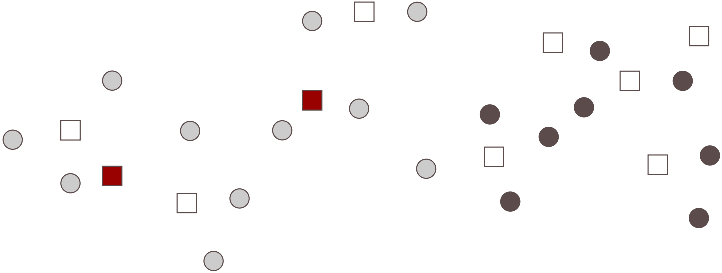

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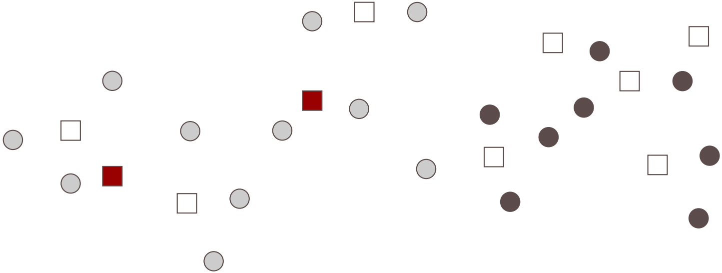

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If ℓ is a metric, then $\phi(F) \leq OPT$.

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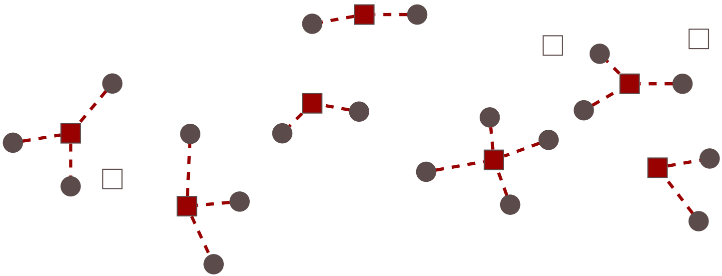

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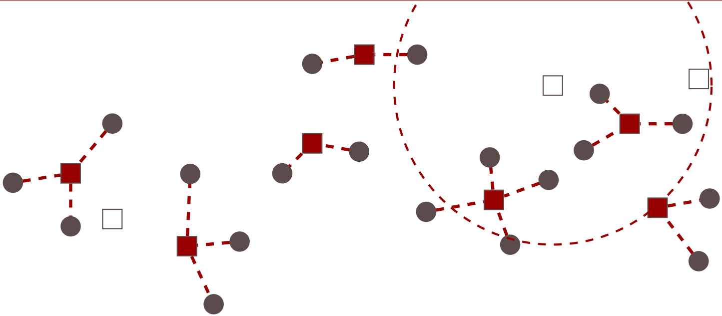

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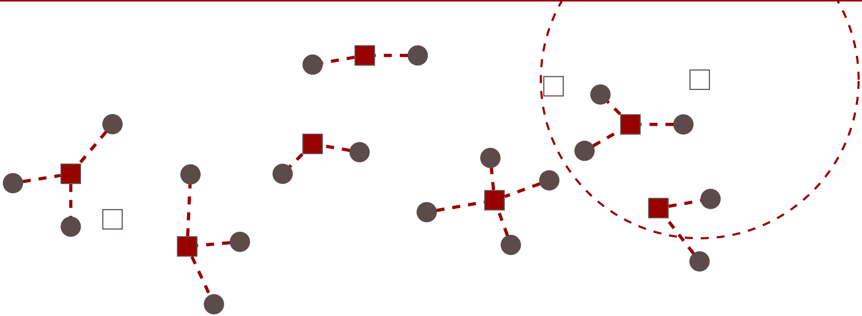

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Length-dependent Costs

- let $\ell = c$ be a metric

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Light Approximate Shortest-path Tree (LAST) [Khuller et al. 1995]

Given a tree T with root r, compute tree T' s.t.

- $c(T'[v, r]) \leq \alpha SP(v)$ ($SP(v) =$ length of shortest r-v-path)
- $c(T') \leq (1 + \frac{2}{\alpha - 1})c(T)$.

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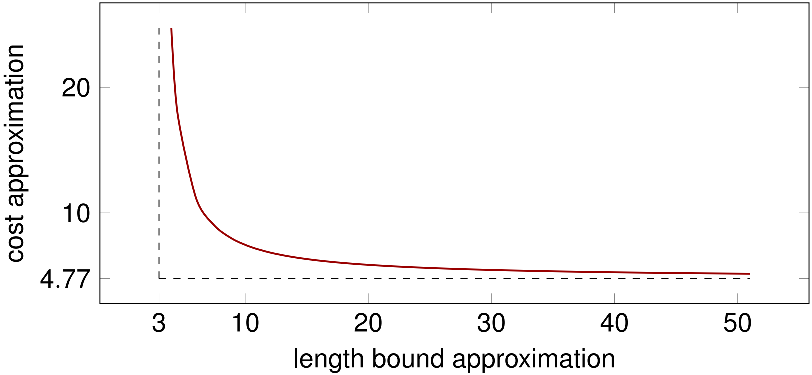
Idea

- ignore length bound for SLST and UFL
- compute greedy cover
- apply LAST algorithm

Length-dependent Costs



Approximation Factor

- **length:** $3\alpha \cdot L$
- **cost:** $((1 + \frac{2}{\alpha-1})(\beta_{\mathsf{ST}} + \beta^{\alpha}_{\mathsf{UFL}}) + 1)OPT$

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- let $\ell \equiv 1$, c be a metric

Approximation for Hop-constrained Trees

- spanning tree: $(1, O(\log n))$ [Althaus et al. 2005]
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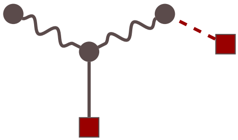
Modified Relieve Procedure

- when relieving subtree rooted at v, connect facility directly to v

! no increase in depth of tree



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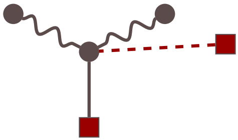

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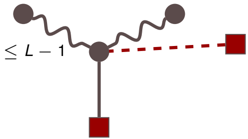

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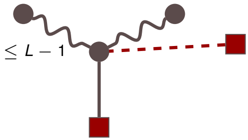

- $(1 + \varepsilon,\ O(\log n))$-approximation

Conclusion

- facility location + capacitated trees + length bound
- flexible approximation framework combining several lower bounds

Open questions

? capacitated facilities
? uncertain demands
? $(O(1), O(1))$-approximation for shallow-light Steiner tree

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Thank you!