Mecanism design for speed scaling scheduling

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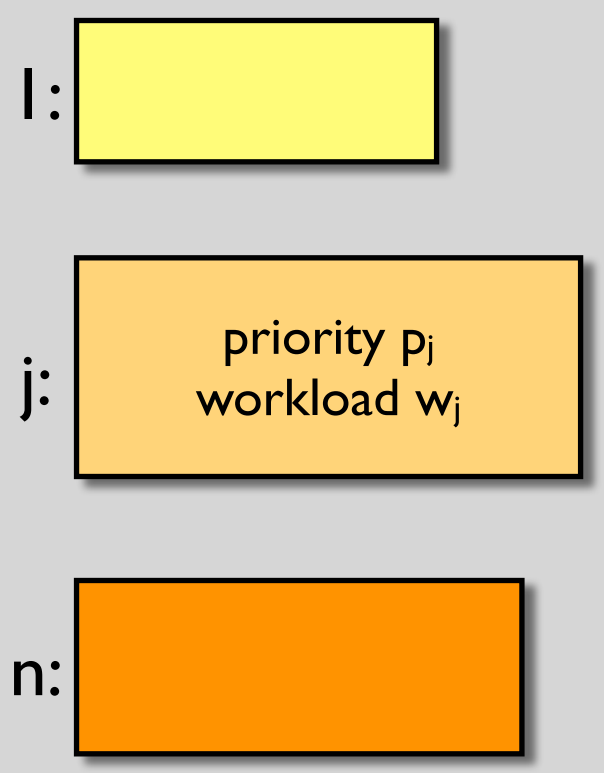
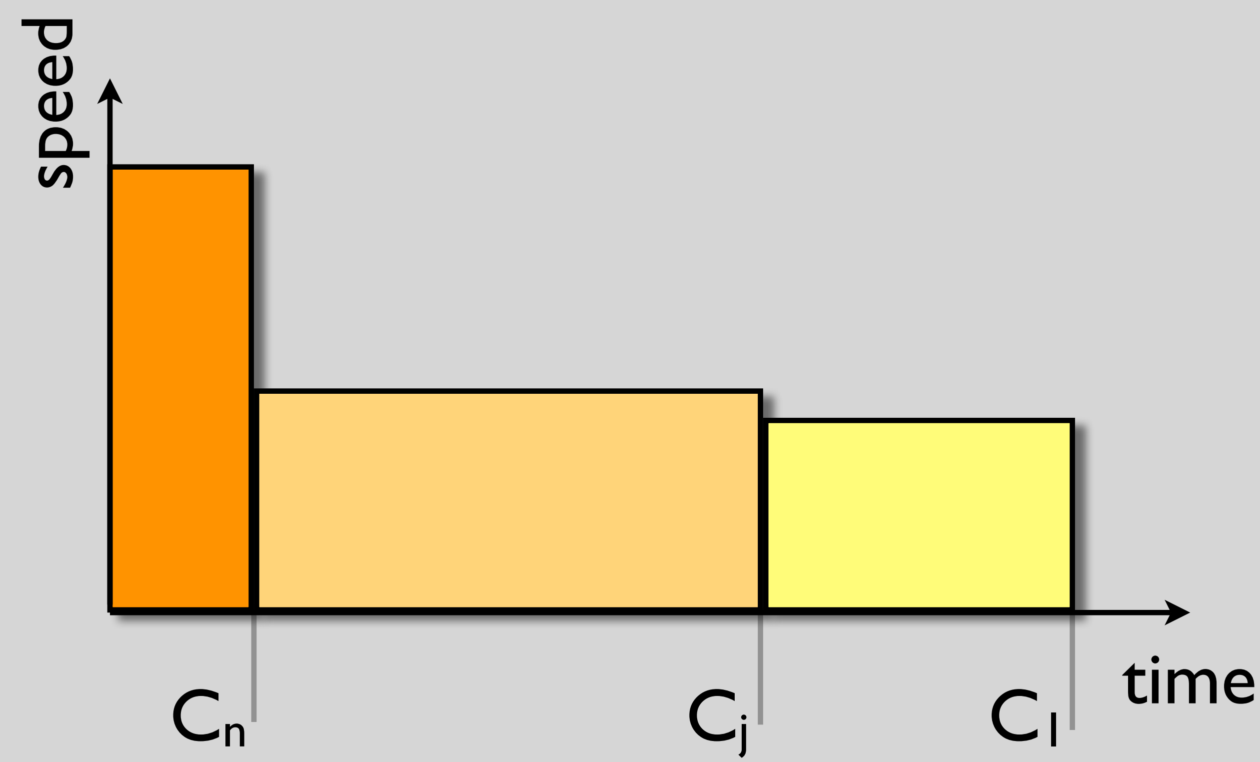
Speed scaling scheduling

n given jobs



single machine



decide on order on jobs and speed
in order to **minimize**

energy consumption + weighted completion time

$= \int s(t)^\alpha \, dt \quad + \Sigma\, p_j C_j$

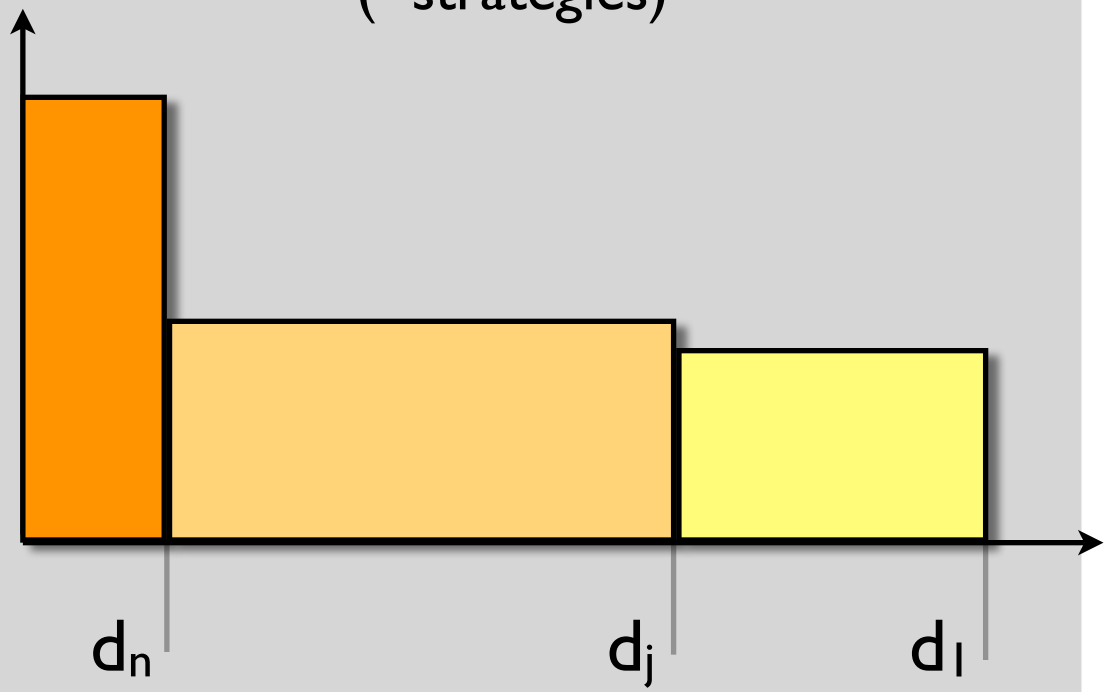
for a physical constant $2 \leq \alpha \leq 3$

computational complexity is open
but an approximation scheme exists [megow,verschae'13]

Define a strategic game

deadline game

players decide on the deadline of their job (=strategies)



- compute minimum energy schedule=easy
- need to charge consumed energy to players

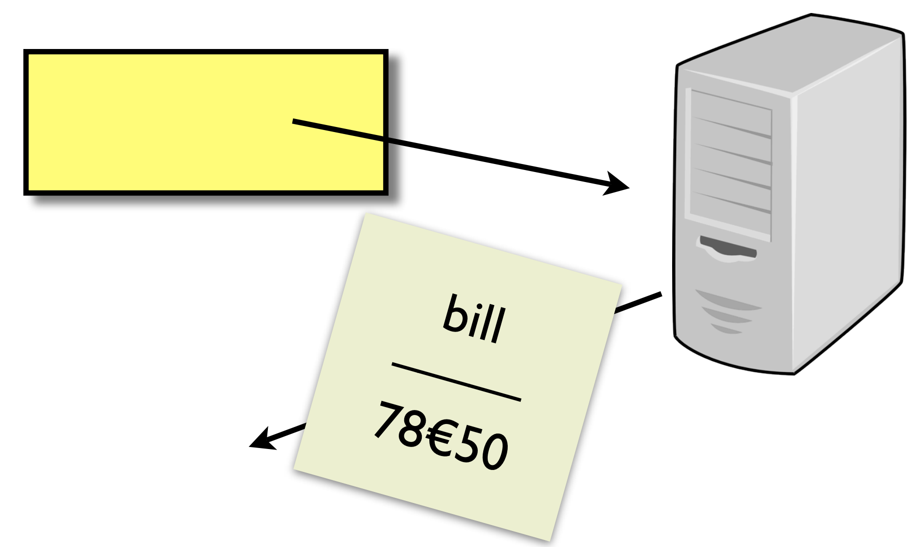
penalty game

players announce a deadline penalty $\tilde{p}_j$ (=strategies)

- strategy proof is needed (dominant strategy should be $\tilde{p}_j = p_j$)
- compute minimum energy schedule = hard because we have to decide on the job order
- need to charge consumed energy to players

What do we want from a charging scheme ?

1. compute optimal schedule (or approximate)
2. charge every user i a value b_i
3. player i wants to minimize $p_i C_i + b_i$

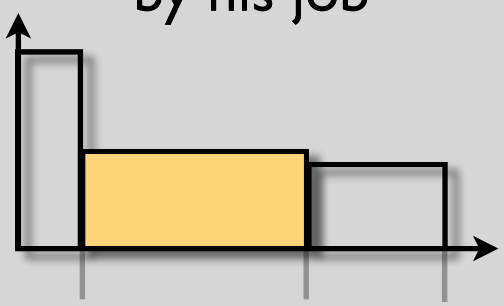


- pure Nash equilibria should exist
- … and be computable in polynomial time
- total amount charged should cover energy consumption and not exceed it by more than a constant factor
(**O(1)-budget balanced**)
- social cost of equilibria should be close to social optimum
(**price of anarchy**)

deadline game

proportional charge

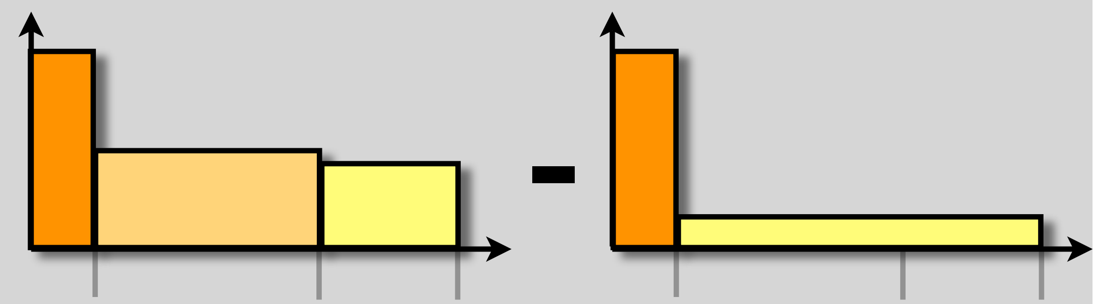
player i pays exactly the energy consumed by his job



- is clearly budget balanced
- does not garanty pure Nash equilibria

marginal charge

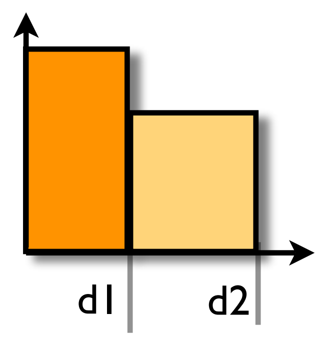
player i pays the difference of the optimal schedule with and without him



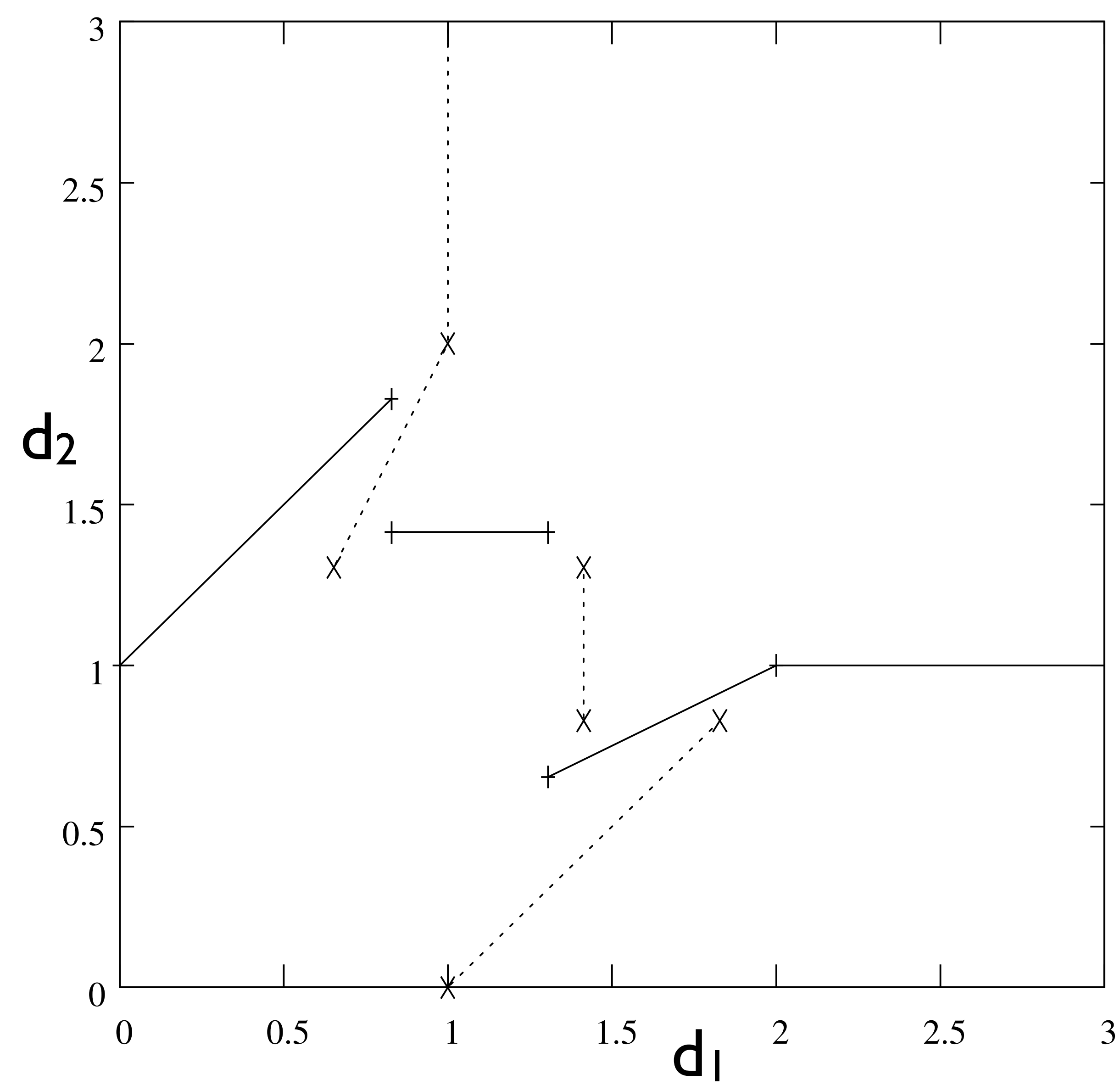
- every player pays at least the energy consumed by his job and at most α times that value
- is a potential game
  → pure Nash equilibria exist,
  and can be found by best response dynamics,
  time of convergence has not been analyzed yet
- price of anarchy has not been analyzed yet

deadline game
proportional cost sharing

- example with 2 identical jobs
- but any schedule creates an asymetry between jobs

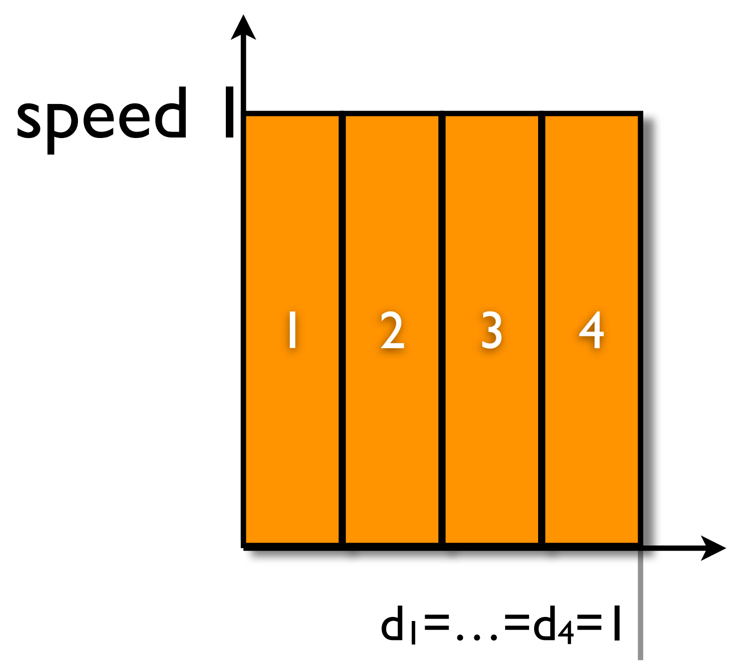


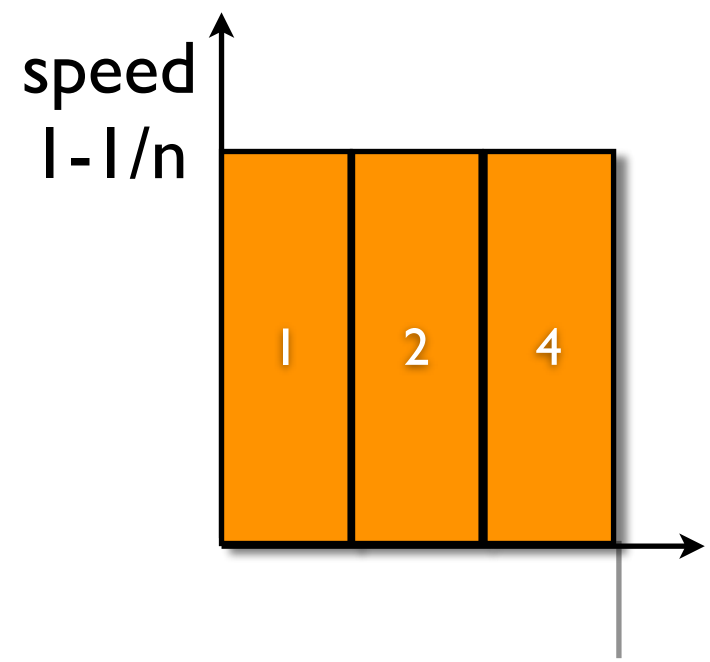
- every strategy profile (d_1,d_2) is a point in $R^+ \times R^+$
- best response functions have no fix point
- there is no pure Nash equilibrium already for this simple game



deadline game, marginal cost share

- every player pays at least the energy consumed by his job and at most α times that value
- **tight example**: n jobs with deadline 1, workload 1/n.





- every player is charged $1-(1-1/n)^\alpha$
- which is $\lim_{n\to\infty} 1-(1-1/n)^\alpha = \alpha/n$

deadline game, marginal cost share

- OPT(d) = optimal energy consumption of a schedule for all players
- OPT(d_{-i}) = … all players but i
- cost share for player i = OPT(d)-OPT(d_{-i})
- her total penalty is $p_i d_i$+OPT(d)-OPT(d_{-i})
- but social cost is $\Sigma p_i d_i$+OPT(d)
- so if a player changes strategy and improves by Δ so does the social cost
- this is a **potential game** → pure Nash equilibria exist

penalty game
work in progress

- we need to fix an order on the jobs (arbitrary or random)
- then computing energy optimal schedule is easy
- cost share for player i = $\alpha(OPT(\tilde{p})-OPT(\tilde{p}_{-i})) - \tilde{p}_i C_i$
- her total penalty is $(p_i-\tilde{p}_i)C_i + \alpha OPT(\tilde{p}) - \alpha OPT(\tilde{p}_{-i})$
- dominant strategy is $\tilde{p}_i = p_i$ (strategy proof)
- cost share is at least energy consumption of her jobs and at most $\alpha+1$ times that value

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