

# The optimal rotation of a flammable forest stand when both carbon sequestration and timber are valued: a multi-criteria approach

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**Abstract** This paper proposes a multi-criteria approach that accounts for the risk of fire when determining the optimal rotation of a forest stand that is being managed for both timber production and carbon sequestration purposes. The multi-criteria framework uses in a combined way, multi-objective optimization and compromise programming methods. The proposed approach is computationally simple and allows for the quantification of conflicts between the criteria considered through the elicitation of the corresponding Pareto frontiers. Once the best portion or compromise sets of the Pareto frontiers are determined, then some indications of the increase in social welfare due to a potential reduction in the risk of fire are obtained. We illustrate the use of our methodology by applying it to an example that has previously been investigated in the forestry literature. Finally, some potential policy implications derived from the results obtained are highlighted.

**Keywords** Carbon capture · Compromise programming · Forestry · Multiple criteria decision making · Fire risk · Pareto frontiers

## 1 Introduction

The traditional problem of determining the optimal forest rotation age of a timber stand has evolved considerably since the classic formulation by Faustmann (1849). This evolution is

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due, among other things, to the fact that modern societies demand from their forests, not only industrial fiber in the form of timber that is sold in markets, but also public goods and services for which there are not always well-defined markets. This is precisely the case with the sequestration of carbon that is captured by stands that are used for timber production.

The seminal work by Hartman (1976) represents a stepping stone for determining the optimum forest rotation age for stands that produce different types of outputs. Romero et al. (1998) expanded upon Hartman's ideas by using multi-criteria methods to determine the optimal forest rotation age when economic returns and carbon capture are jointly considered. Unfortunately, neither of those approaches account for uncertain disturbance processes such as fire, insects and disease, or fluctuations in timber prices.

In this paper, we focus on uncertain fire loss. Martell (1980) developed a stochastic model for the case where the probability of burning is constant over time, to determine the reduction in the planned rotation age and of the underlying net present value (NPV) due to the risk of fire. Routledge (1980) produced similar results and also linked the reduction in the rotation age and NPV with the amount of timber salvaged in case of fire. Reed (1984) assumed fire occurrence was governed by a Poisson process and like Martell (1980) and Routledge (1980) found that the optimal planned rotation decreases as the fire arrival rate increases. Reed (1987) included fire protection in an extension of his 1984 model and assumed fire protection reduces the fire arrival rate, but not the damage that results from a fire. Amacher et al. (2005) focused on fuel management, including fire damage but assuming that fire arrival rate does not vary with fuel management effort. They found that there was no simple relationship between the level of fuel management activity and the optimal planned rotation.

There are many other extensions of the basic Faustmann approach that incorporate, besides the risk of fire, the production of non-timber goods and services. Englin et al. (2000), for example, determined the optimal planned rotation age taking into account the risk of fire and the production of amenities such as recreational canoe routes. The consideration of the recreational services increases the length of the planned rotation ages. Stollery (2005) generalized Reed's (1984) analysis to account for an increase in the fire arrival rate due to climate change and the capture of carbon from the perspective of a social optimum. Finally, Daigneault et al. (2010) address a similar problem without using a stochastic dynamic model. However, they also accounted for possible carbon sequestration credits as well as several silviculture strategies to mitigate wildfire risk. The consideration of credits for the carbon capture implies less intensive thinning as well as longer rotation ages, independently of the values associated with risk of fire and timber price.

A crucial problem in many countries is how best to deal with the risk of burning timber resources in forest management. The burning of timber implies not only a loss of merchantable timber, but also a loss of the carbon stored in the biomass that fire releases into the atmosphere. It is therefore important to consider the joint influence on forest rotation age of carbon capture that will increase the length of the rotation and of the risk of fire that will reduce it, and this paper addresses that issue. Our approach is to use multi-objective programming and compromise programming to develop a relatively simple procedure for determining the optimal planned rotation age for a forest stand when, besides timber production, the risk of fire and carbon capture are jointly considered. We illustrate the use of our method by applying it to a case study described in the forestry literature.

The use of multi-criteria techniques for incorporating the risk of fire in forest management is not common (see Diaz-Balteiro and Romero 2008). One exception is Teeter and

Dyer (1986) who developed a two-attribute additive utility function that can be used to help decide how much should be invested in fire management programs in national forests in the USA. They assumed that the number of large fires that occur in a national forest each year follow a Poisson distribution and calculated a fire risk index. They then derived a two-attribute utility function based on their risk index and the cost plus net value change of a fire program. In this way, possible outcomes of their risk index associated with seven management options were characterized using hypothetical lotteries obtained from a survey of fire management planners.

This paper is organized as follows. Section 2 presents the analytical framework based upon multi-criteria tools, for determining the optimal planned forest rotation age in the context of both carbon sequestration and fire risk. In Sect. 3, we illustrate how our approach works by applying it to a forest that is representative of those in the coastal forest region of the Province of British Columbia, Canada, which was studied by van Kooten et al. (1995). The results derived from that application are discussed in Sect. 4 where some possible policy implications are highlighted.

## 2 Methodology

We begin by stating the basic assumptions underlying the model, the general notation and the analytical procedure.

### 2.1 Modeling assumptions

*Assumption 1* The future price of timber is known and constant.

*Assumption 2* From the point of view of timber production, we consider an infinite planning horizon.

*Assumption 3* Given that, in the long-term, forest ecosystems are carbon neutral, only a single plantation cycle is considered in terms of carbon uptake.

*Assumption 4* The amount of carbon content in the timber biomass is a function of the age of the stand, and we assume its value is known and constant.

*Assumption 5* The amount of carbon that is released into the atmosphere when a stand burns or is harvested is known and constant and is computed according to the Kyoto Protocol assumptions (i.e., all sequestered carbon are re-emitted at harvest time).

*Assumption 6* There is no salvage harvesting of timber after a fire.

*Assumption 7* The probability that a stand will burn is constant (i.e., it does not vary over time or with stand age).

The above set of assumptions might be considered somewhat restrictive for some applications. For instance, the acceptance of assumption 6 can lead to shorter optimal rotations. However, we note that the primary purpose of our simplifying assumptions is to facilitate the development of a general model. When this model is applied to a particular instance, some of our simplifying assumptions could be relaxed in order to accommodate the features of the particular case being analyzed. In short, we aim to formulate a general framework that can be adapted to the particularities of different real situations.

## 2.2 Notation

The following notation will be used throughout the paper:

$t$  = age of the timber stand

$V$  = volume of timber

$p_B$  = annual probability that a stand will burn

$P$  = timber price

$i$  = discount rate

$\gamma$  = proportion of the timber biomass that is carbon

$\mu$  = fraction of timber harvested whose carbon content is released into the atmosphere

$\mu'$  = fraction of timber burned whose carbon content is released into the atmosphere

## 2.3 Analytical procedure

Let us begin by considering a growth curve that relates timber volume  $V$  to stand age  $t$ :

$$V = f(t) \quad f'(t) \geq 0 \quad f''(t) \leq 0 \quad t \geq 0 \quad (1)$$

Since we have assumed that the annual probability that the stand will burn is constant and equal to  $p_B$ , then we have a binomial distribution, and consequently, the probability that the stand will burn before year  $t$ ,  $\mathbf{P}_B$ , will be:

$$\mathbf{P}_B = [1 - (1 - p_B)^t] \quad (2)$$

The following two criteria will be considered. First, the economic return from timber without considering the risk of fire. This return will be measured by the net present value (NPV) of infinite plantation cycles according to Faustmann logic (e.g., Neher 1990, pp. 68–72), that is:

$$\text{NPV} = \frac{\text{Pf}(t)}{(e^{it} - 1)} \quad (3)$$

If the risk of the stand burning before its planned harvest date is considered, we can derive a simple estimate of  $\text{NPV}_B$ , i.e., the NPV, given the risk of fire, by assuming that if the stand burns before its planned rotation time, it is neither salvage harvested nor regenerated (Martell 1980):

$$\text{NPV}_B = \frac{\text{Pf}(t) - \text{Pf}(t) \times [1 - (1 - p_B)^t]}{(e^{it} - 1)} \quad (4)$$

which is equivalent to the following expression:

$$\text{NPV}_B = \frac{\text{Pf}(t) \times (1 - p_B)^t}{(e^{it} - 1)} \quad (5)$$

The second criterion is the carbon captured ( $C$ ) in a rotation cycle which, without considering the risk of fire will be equal to:

$$C = \gamma(1 - \mu)f(t) \quad (6)$$

When the risk of burning the stand is considered, the above carbon uptake  $C$  becomes:

$$C_B = \gamma(1 - \mu)f(t) - \gamma f(t)\mu' \times [1 - (1 - p_B)^t] \quad (7)$$

From the above equations, we can determine the pay-off matrices corresponding to the bi-objective programming problem when we are concerned with both NPV and carbon capture, with and without considering the risk of fire. Thus, by maximizing (3, 6) separately and then computing the value of each objective at each of the optimal solutions, the pay-off matrix corresponding to a situation without taking into account the risk of fire is obtained (see Table 1a).

The elements of the main diagonal of the above matrix  $(NPV^*, C^*)$  represent the ideal values. This point in the objective space is infeasible, but will play a crucial role as a point of reference. The elements of the other diagonal  $(NPV_*, C_*)$  represent the anti-ideal or worst case values for the two objectives considered. The differences between ideal and worst case values (i.e.,  $NPV^* - NPV_*$  and  $C^* - C_*$ ) provide the range of feasible variation for each objective. These ranges will play an important role in normalizing purposes. Finally, the comparison of the two rows of the above pay-off matrix will provide useful information concerning the degree of conflict between the two objectives considered. Implementing the same type of mathematical operations with equations (5, 7), the pay-off matrix for a risk of fire context is obtained (see Table 1b).

Now,  $(NPV_B^*, C_B^*)$  is the ideal vector and  $(NPV_{B^*}, C_{B^*})$  the anti-ideal vector, being the interpretations equivalent with respect to the pay-off matrix in Table 1a which is valid for a situation without considering the risk of fire. By comparing the elements of both matrices, an initial assessment of the losses caused by the risk of fire can be obtained.

Once the respective pay-off matrices have been calculated, our next step will be to determine the Pareto frontiers or trade-off curves between NPV and carbon capture in the two contexts considered (with and without risk of fire). This will be accomplished by applying a generating technique such as the constraint method (see Romero and Rehman 2003, pp. 52–53). Thus, by maximizing (3) while considering (6) as a parametric constraint, or vice versa, the efficient points that lead to the Pareto frontier between the two objectives considered without considering the risk of fire is established. In a similar way, by maximizing (5) while considering (7) as a parametric constraint, or vice versa, the other Pareto frontier when the risk of fire is considered will be obtained. In both cases, the range of variation of the right-hand side of the parametric constraint will be given by the closed interval defined by the respective anti-ideal and the ideal values provided by the respective pay-off matrices.

Once the two Pareto frontiers have been calculated, we can try to approximate portions of these curves more interesting for a rational decision-maker. In fact, according to a well-known axiom of behavior, the decision-makers will seek a point in the Pareto frontier as close as possible to the ideal point (see Zeleny 1974). In order to achieve this type of proximity, when the risk of fire is not considered, a family of  $p$ -metric distance functions is introduced, leading to the following compromise programming model (CP) (Yu 1973; Zeleny 1982).

$$\text{Min } L_p = \left[ W_1^p \times \left( \frac{NPV^* - NPV}{NPV^* - NPV_*} \right)^p + W_2^p \times \left( \frac{C^* - C}{C^* - C_*} \right)^p \right]^{\frac{1}{p}} \tag{8}$$

Subject to the Pareto frontier  $T(NPV, C) = K$

When the risk of fire is included, we have the following CP model:

$$\text{Min } L_p = \left[ W_1^p \times \left( \frac{NPV_B^* - NPV_B}{NPV_B^* - NPV_{B^*}} \right)^p + W_2^p \times \left( \frac{C_B^* - C_B}{C_B^* - C_{B^*}} \right)^p \right]^{\frac{1}{p}} \tag{9}$$

Subject to the Pareto frontier  $T(NPV_B, C_B) = K$

**Table 1** Pay-off matrix for the net present value and carbon capture without and with considering the risk of fire

	NPV (\$)	Carbon capture (tons)
a. No risk of fire		
NPV (\$)	NPV*	C*
Carbon capture (tons)	NPV*	C*
b. With risk of fire		
NPV (\$)	NPV <sub>B</sub> *	C <sub>B</sub> *
Carbon capture (tons)	NPV <sub>B</sub> *	C <sub>B</sub> *

where  $p$  represents the metric defining the family of distance functions and  $W_1$  and  $W_2$  are the preferential weights attached by a hypothetical decision-maker to the two criteria involved.

By solving model (8, 9) for  $p = 1$ , the  $L_1$  bound of the compromise set is obtained. This bound implies the best strategy from the point of view of the maximization of the average achievements, but this type of solution can be very biased against the achievement of one of the two objectives considered. The opposite solution, that is, the “most balanced” solution, can be obtained by solving (8, 9) with  $p = \infty$ , which leads to the following mathematical programming problem (Zeleny 1982):

$$\begin{aligned}
 & \text{Min } L_\infty = D \\
 & \text{s.t.} \\
 & W_1 \left( \frac{\text{NPV}^* - \text{NPV}}{\text{NPV}^* - \text{NPV}_*} \right) \leq D \\
 & W_2 \left( \frac{C^* - C}{C^* - C_*} \right) \leq D \\
 & T(\text{NPV}, C) = K
 \end{aligned} \tag{10}$$

when the risk of fire is not considered and when the risk of fire is considered, we have the following model:

$$\begin{aligned}
 & \text{Min } L_\infty = D \\
 & \text{s.t.} \\
 & W_1 \left( \frac{\text{NPV}_{B^*} - \text{NPV}_B}{\text{NPV}_{B^*} - \text{NPV}_{B^*}} \right) \leq D \\
 & W_2 \left( \frac{C_B^* - C_B}{C_B^* - C_{B^*}} \right) \leq D \\
 & T(\text{NPV}_B, C_B) = K
 \end{aligned} \tag{11}$$

Yu (1973) demonstrated that, for bi-criteria problems, the  $p = 1$  and  $p = \infty$  metrics define a portion of the Pareto frontier called the compromise set (i.e., the arc  $[L_1, L_\infty]$ ); i.e., something like a parametric line between the two points exists. The best-compromise solutions fall between solutions corresponding to these two metrics. Therefore, it is not necessary to solve models (8, 9) for other values of metric  $p$ .

On the other hand, under very general conditions, it has been demonstrated that the compromise set is a good surrogate for the social optimum (Ballesteros and Romero 1991). That is, the point of the Pareto frontier for which the unknown social welfare function  $U(\text{NPV}, C)$  or  $U(\text{NPV}_B, C_B)$  achieves its maximum value is a point in the respective compromise sets  $[L_1, L_\infty]$ . In the application presented in the next section, we illustrate how this idea can be applied. For instance, it will be shown how the difference between the surrogate social optimum in the two contexts considered, with and without risk of fire, can provide an indication of the increase in social welfare due to a technological change that reduces the risk of fire.

### 3 An application

To illustrate the theory presented in the preceding section, the following data from a coastal forest in British Columbia will be used (see van Kooten et al. 1995):

$$V = 0.000573t^{3.7819}e^{-0.030965t} \text{ [m}^3\text{/ha]}$$

$$P = 25 \text{ dollars/m}^3$$

$$i = 0.02$$

$$\gamma = 0.2 \text{ tons of carbon/m}^3$$

$$\mu = 0$$

$$\mu' = 1$$

$$W_1 = W_2 = 1$$

We will assume the probability that the stand will burn,  $p_B = 0.015$

For the above values, criteria functions (3, 6) valid for a situation without risk of fire are obtained:

$$NPV = \frac{25 \times 0.000573t^{3.7819}e^{-0.030965t}}{(e^{0.02t} - 1)} \tag{12}$$

$$C = 0.2 \times 0.000573t^{3.7819}e^{-0.030965t} \tag{13}$$

In the same way, criteria functions (5, 7), which are valid for situations where the risk of fire is considered, yield the following expressions:

$$NPV_B = \frac{25 \times 0.000573t^{3.7819}e^{-0.030965t}(0.985)^t}{(e^{0.02t} - 1)} \tag{14}$$

$$C_B = 0.2 \times 0.000573t^{3.7819}e^{-0.030965t}(0.985)^t \tag{15}$$

By maximizing alternatively (12–15) as indicated in the preceding section, the two pay-off matrices shown in Table 2 were obtained. All these computations and hereafter were solved using LINGO (2007) with a negligible computer time (less than a second). It should be noted, that these two matrices have been augmented with two columns conveying additional information related to forest rotation age and the timber volume harvested.

We can now approximate the Pareto frontier between net present value and carbon capture in the two contexts considered, by resorting to the constraint method as explained in the preceding section. We have the following two mathematical programming models:

#### 3.1 No risk of fire considered

$$\text{Max NPV} = \frac{25 \times 0.000573t^{3.7819}e^{-0.030965t}}{(e^{0.02t} - 1)} \tag{16}$$

s.t.

$$C \geq Z \in [109.40, 203.70]$$

where  $Z$  plays the role of a parameter that through its variation allows to determine the Pareto efficient combinations of NPV and carbon uptake when no risk of fire is considered.

**Table 2** Pay-off matrix for net present value and carbon capture without considering the risk of fire (bold figures denote ideal values and underlying figures anti-ideal values)

	NPV (\$)	C (tons)	t (years)	V (m <sup>3</sup> )
a. No risk of fire				
NPV (\$)	<b>5,243</b>	<u>109.4</u>	65	558
C (tons)	<u>2,489</u>	<b>203.7</b>	122	1,048
b. With risk of fire				
NPV (\$)	<b>2,256</b>	<u>29.1</u>	48	306
C (tons)	<u>1,360</u>	<b>45.3</b>	82	804

### 3.2 Risk of fire considered

$$\text{Max NPV}_B = \frac{25 \times 0.000573t^{3.7819} e^{-0.030965t} (0.985^t)}{(e^{0.02t} - 1)} \quad (17)$$

s.t.

$$C_B \geq Z_B \in [29.10, 45.30]$$

where  $Z_B$  plays again the role of a parameter that allows to determine the Pareto efficient combinations of NPV and carbon uptake, but now when the risk of fire is considered.

By solving the two parametric programming models defined by (16, 17), the Pareto frontiers shown in Table 3, as well as in Figs. 1 and 2, were obtained.

Now, if we solve CP models (8, 9) for  $p = 1$  to the above data, the  $L_I$  bounds of the compromise sets for the two situations considered will be obtained. Similarly, if the above data are substituted in CP models (10, 11), the  $L_\infty$  bounds of the respective compromise sets are obtained. These values are shown in Table 4.

It should be pointed out that for our case study, the two bounds of the compromise set almost coincide in the two situations analyzed (i.e., with and without risk of fire). In short, in our particular case, the compromise set is almost reduced to a single point. These compromise points, which as we noted earlier can be considered as surrogates of the social optimum, are especially marked in the two Pareto frontiers or trade-off curves shown in Figs. 1 and 2.

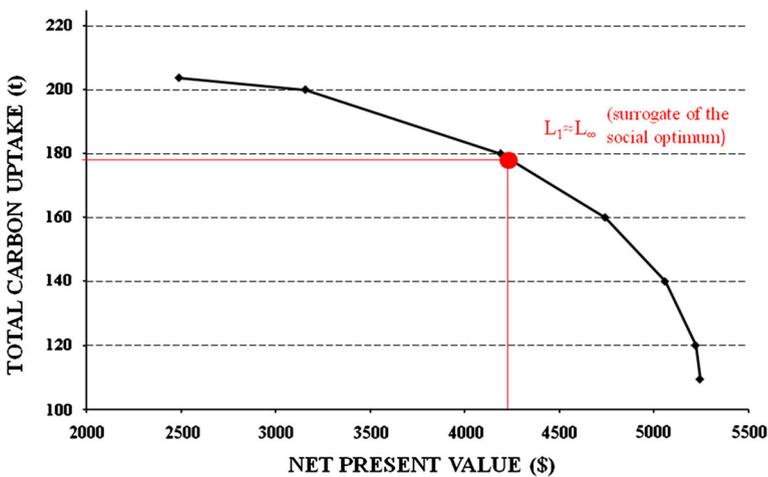
## 4 Discussion and conclusions

Analysis of the information contained in the pay-off matrices shown in Table 2 indicates the following:

1. There is a clear difference between the forest rotation ages that maximize the NPV and carbon sequestration. This conflict arises in the two situations considered. In short, the economic returns and the carbon sequestered are forest uses in clear conflict independently of the consideration of the risk of burning the stand.
2. The consideration of the risk of fire reduces the NPV associated with the optimum forest rotation age by more than 50 % with respect to the maximum NPV. The equivalent reduction in terms of carbon captures is even more dramatic, with a reduction of around 80 % with respect the maximum carbon uptake.

**Table 3** Approximation of the Pareto frontier between net present value and carbon capture without considering the risk of fire

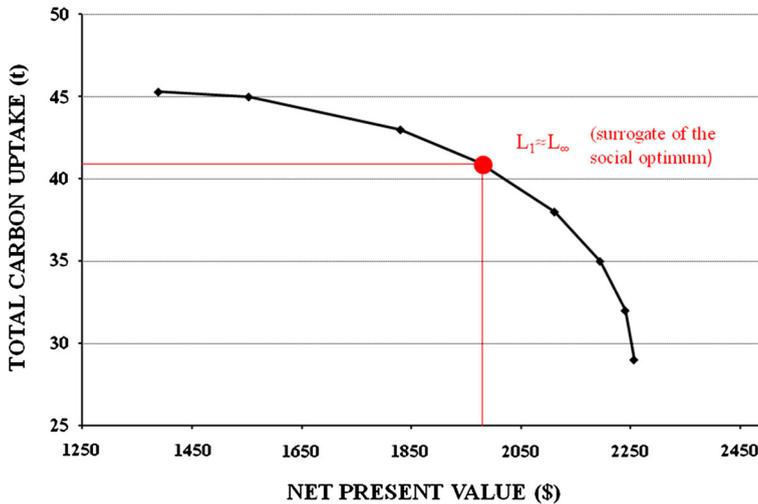
NPV (\$)	Carbon capture (tons)
a. No risk of fire	
5,243	109.4
5,220	120.0
5,058	140.0
4,742	160.0
4,190	180.0
3,157	200.0
2,489	203.7
b. With risk of fire	
2,256	29.1
2,240	32.0
2,194	35.0
2,111	38.0
1,973	41.0
1,830	43.0
1,360	45.3



**Fig. 1** Pareto frontier when the risk of fire is ignored

3. No solution generated by the single optimization of either of the two criteria considered implies a sensible solution. Therefore, in order to obtain an acceptable solution, it is necessary to look for the best-compromise solutions between the two criteria considered.

The Pareto frontiers shown in Table 3 and in Figs. 1 and 2 quantify the trade-offs or opportunity costs of NPV in terms of carbon capture and vice versa. To be more precise, the slopes of the segments of the straight lines linking the extreme efficient points that define the convex polygonal shown in Figs. 1 and 2 measure these opportunity costs.



**Fig. 2** Pareto frontier when the risk of fire is included

**Table 4** Compromise solutions as surrogates of the social optima

	NPV (\$)	C (tons)	$t$ (years)	$V$ (m <sup>3</sup> )
$L_1 \approx L_\infty$ compromise without risk of fire	4,389	173.7	90	869
$L_1 \approx L_\infty$ compromise with risk of fire	2,001	40.5	64	530

As indicated in the preceding section, the best-compromise solutions for  $p = 1$  and  $p = \infty$  almost coincide. Hence, there is one solution for a situation without considering the risk of fire and another solution for the case when the risk of fire is taken into account. In short, for this particular case study, the optimum solutions from the point of view of the average achievement between NPV and carbon capture and the optimum solution from the point of view of the maximum balance between the two objectives considered practically coincide. This situation has occurred because in this particular case, there is little conflict between the structure of preferences underlying metrics  $p = 1$  and  $p = \infty$ .

It is interesting to compare both the best-compromise solutions. In fact, these two equilibrium points, as mentioned in the preceding section, are good surrogates of the social optima. Hence, the corresponding numerical differences can provide a sensible estimation of the potential increase in social welfare due to a reduction in the risk of fire. Moreover, this increase in social welfare can be measured in monetary units of NPV ( $4389 - 2001 = 2,388$ ), in tons of carbon capture ( $173.70 - 40.50 = 133.20$  tons) or in cubic meters of timber ( $869 - 530 = 339$  m<sup>3</sup>). Whatever is the unit of measurement used, the loss of welfare due to the risk of fire is very significant.

We carried out sensitivity analysis concerning changes in the discount rate and in the probability of burning. Tables 5 and 6 show the main results that derive from that analysis. Since, we have not applied any discounting to the carbon uptake, changes in the value of the discount rate affects only the figures of NPV, and in our case in a very significant way. On the other hand, changes in the probability of burning have more significant effects in the carbon capture and on the timber volume.

**Table 5** Sensitivity analysis of the discount rate

	<i>i</i> = 0.01	<i>i</i> = 0.02	<i>i</i> = 0.03	<i>i</i> = 0.04
Objective: max NPV				
NPV	5,920	2,256	1,129	657
<i>t</i>	54	48	43	39
<i>C</i>	34	29	24	20
Vol	393	301	237	184
Objective: max <i>C</i>				
NPV	4,451	1,388	528	221
<i>t</i>	82	82	82	82
<i>C</i>	45	45	45	45
Vol	783	783	783	783

The base case is in italic

**Table 6** Sensitivity analysis of the probability of burning

	<i>p<sub>B</sub></i> = 0.005	<i>p<sub>B</sub></i> = 0.01	<i>p<sub>B</sub></i> = 0.015	<i>p<sub>B</sub></i> = 0.02	<i>p<sub>B</sub></i> = 0.025
Objective: max NPV					
NPV	3,789	2,897	2,256	1,755	1,410
<i>t</i>	58	53	48	44	41
<i>C</i>	67	43	29	20	14
Vol	457	374	301	252	209
Objective: max <i>C</i>					
NPV	2,009	1,652	1,388	1,126	938
<i>t</i>	105	92	85	74	67
<i>C</i>	156	70	45	31	21
Vol	1,006	913	783	696	597

The base case is in italic

As is usually reported in literature, optimal rotation is longer when carbon captured is maximized (Van Kooten et al. 1995; Bussoni Guitart and Estraviz Rodriguez 2010). This result has been showed for the two scenarios considered (with and without risk of fire). When risk of fire has been included, the optimal rotation decreases as some authors state (Martell 1980; Pasalodos et al. 2009). However, the influence on optimal rotation of damage caused by forest fires might not be the same if we consider other climate risks (Kuboyama and Oka 2000). It is also interesting to note that the relaxation of some of our simplifying assumptions, such as no post-fire salvage harvesting, might lead to longer optimal planned rotations.

We conclude by stating that the proposed approach for determining the optimal rotation age of a timber stand by integrating NPV, carbon capture and risk of fire is attractive for the following reasons. First, it is computationally very simple. Second, the proposed method not only allows one to determine the best-compromise rotation ages, but it also furthers our understanding and helps quantify the underlying conflict and interaction

between economic return, carbon capture and risk of fire. Finally, the best-compromise solutions obtained can be interpreted as surrogates of the social optima. This economic interpretation implies that the optimal figures obtained can provide tentative measures of the social welfare increase due to a potential technological change that reduces the risk of fire, which could be useful from the point of view of policy making.

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