



# A Priori and A Posteriori Aggregation Procedures to Reduce Model Size in MIP Mine Planning Models

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## Abstract

Mine planning models have proved to be very effective in supporting decisions on sequencing the extraction of material in copper mines. These models have been developed for CODELCO, the Chilean state copper mine and used successfully. Here, we wish to develop a corporate model, including all mines of CODELCO. The original models are extremely large MIPs. In order to run a global model, the original models need to be reduced significantly. We develop an approach to aggregate the models. The aggregation is done both on the original data of the mine as well as on the MIP original models. The aggregation is based on clustering analysis. Promising results were obtained with data of a large underground mine.

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# 1 Introduction

Models have been developed for planning in copper mines for CODELCO, the state owned mine corporation, one of the main world producers of copper. CODELCO owns seven mines, both open pit and underground. In the planning process, detailed models are used. Consider the largest underground mine, El Teniente. The mine is divided into sectors and sub-sectors. Each sector is divided into columns of extraction points, we call PEXTs. These columns are composed of blocks, usually of 20 by 20 by 20 meters, called UBCs, which are the basic units of extraction. The planning process considers a 25 year horizon, and the sequence to follow points of extraction is given. The decisions to be made are the period (yearly) in which each PEXT (or column) is started, and how many UBCs will be extracted. Typically the % of copper decreases as the extraction goes up the column, so it often is more convenient to leave some UBCs not extracted and move to the next PEXT. The extraction is carried out through gravity, using explosives to loosen the material, which is then carried through a transportation system to downstream plants. In each PEXT then, the lower UBC is extracted first and the extraction follows upward. A MIP model was developed to support decisions for planning extraction of UBCs, and leads to decisions of exact timing for each UBC. Using the model to support decisions increased the value of the mine over 100 million dollars. Similar models have been developed for open pit mines, mostly in northern Chile. Again, these are large scale MIP models (Epstein et al. 2003 [3], Caro et al. 2007 [2]).

We are interested in developing a model which integrates all mines for corporate decisions, to determine extraction from each sector, in each mine, each period. In particular, given price uncertainties the corporation might want to develop a stochastic model to incorporate these uncertainties. In this case, we will consider a 5 year horizon model.

The original models are too large, too slow to run to include them directly in such a corporate model. Also, at corporate level there is no need to define in such detail the decisions. While at one level, planning for each mine is carried out in detail, and given the very large amounts involved, this planning is a vital issue by itself, there is also need to consider the whole enterprise. CODELCO owns seven mines, between underground and open pit. Some, very large like the one we are presenting in this study. At the corporation level there are global constraints that are not present at mine level. CODELCO produces a significant percentage of the world copper, thus its actions influence the market. So, the corporation will need a strategy on how much copper it

wants to produce as a sum of all its mines. It is also important that the model corresponding to each mine runs fast. We are considering developing an approach to incorporate prize uncertainty into the decision process. We want to develop approaches, like robust programming (Berktsikas et al. 2003 [1]) or coordinated branching (Alonso et al. 2003 [4]) to incorporate prize uncertainty. Any of these approaches requires running each of the mine problems multiple times, hence the need for fast solutions.

For this reason it is convenient to consider aggregate models. We will use cluster analysis to do the aggregation. The advantage of rigorous aggregation processes is in the preservation of feasibility once the aggregate solution is disaggregated to detailed level. Two types of aggregations will be considered. One, *a priori*, which will aggregate the UBCs, and *a posteriori*, which will aggregate columns of the original model.

## 2 The *A Priori* Aggregation

We consider first the *a priori* aggregation. We need to aggregate UBCs that are connected to each other and in such a way that it is possible to extract each cluster independently from the others. In this way we can assign one extraction variable to each cluster. We need to group the UBCs according to similarities. We chose basic characteristics that describe UBCs to establish a way to measure the dissimilarity between two UBCs. These are Tonnage, Percentage of Copper, Percentage of Molybdenum and Speed of Extraction, which best describe a UBC. Each characteristic has a different importance, so a set of weights associated to the characteristics was defined. This was done with support of a knowledgeable source.

The dissimilarity measurement between two UBCs is defined as the sum of the difference between the numerical values of the characteristics of both UBCs divided by the average value taken by the characteristic through all UBCs, multiplied by the weight of each characteristic. Then, the dissimilarity measurement of a cluster is defined as the sum of the dissimilarity measure between every pair of UBCs in the same cluster.

In order to group UBCs into clusters we developed a  $K$ -means type algorithm. To use this type of algorithm an initial partition was created, based on a greedy selection of clusters according to similarities and spatial location. Note the importance of the spatial location, as clusters defined must be feasible to be extracted independently.

As an output a parameter, delta, is given, corresponding to the total dissimilarity of the partition. This is the sum of the total dissimilarity of each

cluster.

After the initial step a the  $K$ -means algorithm is applied. It is based on exchanging UBCs between clusters in order to reduce the value of delta. Combinations of exchanges are tested to find those that reduce the value of delta. If no improving exchange is found, the algorithm stops.

Note that only exchanges that create feasible clusters are considered.

The value of each characteristic in every cluster . These are calculated as a weighted average proportional to the tonnage of each UBC.

Once the clusterization process was finished, the aggregate model was created.

The main variables relate to extraction of clusters from subsectors in given time periods (0-1) and the tonnages of copper and molybdenum extracted from each sector and period.

The constraints that need to be satisfied are: Each cluster can be extracted only once, satisfy the defined sequence of extractions, the allowable speed and capacity of extraction, as well as conservation of flows and logical relationships between variables. The objective function is to maximize the profit. The costs are those of extraction, activation of a PEXT, the cost due to increase or decrease of production between periods, and the transportation costs. The benefits are related to sales of copper and molybdenum.

We then solved the aggregate model. The solution was fed into the original model by indicating productions by sector, so we could compare the solution of the original model with the solution of the detailed model where productions by sector and period follow the aggregated model solution. This was implemented in the underground mine of El Teniente. A five year model was created, that considered the eleven sectors that could be extracted in the next 5 years.

When comparing the original model with the disaggregation of the aggregate model, the percentage error in the value of the objective function was 3,62%. The reduction of execution time was of 73,68%. The model dimension was reduced by 90%.

So the solutions found are coherent with the original model and the reduction of time is significant. The reduction of time will be more significant when the model is larger (more periods). Note that when the number of periods goes to 25 years, as in the case of the models presently used, solving the original model takes two hours of CPU.

### 3 The *A Posteriori* Approach

This approach consists in aggregating the original mine planning model through clustering techniques.

In order to develop this aggregated model we use *a posteriori* aggregation. This type of aggregation procedures is based on the original standard linear problem expressed in matrix form  $Max\{Z = cx : Ax \leq b, x \geq 0\}$ , where  $A$  is a  $m \times n$  matrix;  $c, x \in \mathbb{R}^n$ , and  $b \in \mathbb{R}^m$ .

Again cluster analysis is used for the aggregation process based on similarities. Each column is associated to a variable of the problem. We used a column aggregation procedure proposed by Zipkin, 1980 [6]:

- We consider a partition in subsets of  $K$  columns. Therefore  $\sigma = \{S_k : k = 1, 2, \dots, K\}$ , is a *partition* of  $\{1, 2, 3, \dots, n\}$  where  $S_k$  has  $n_k$  elements.
- To obtain the model parameters we use a method called fixed-weight combination. It involves a convex weighting of a cluster of elements of cardinality  $n_k$  by an  $n_k$ -vector  $g_k$ , whose components are nonnegative and sum to unity.
- Let  $A'_k = A_k g_k$ ,  $c'_k = c_k g_k$ ,  $x'_k = x_k g_k$ ,  $k = 1, \dots, K$

$$c' = (c'_1, c'_2, \dots, c'_k), A' = (A'_1, A'_2, \dots, A'_k), \text{ and } x' = (x'_1, x'_2, \dots, x'_k).$$

Then, the aggregate problem AP is  $Max\{Z' = c'x' : A'x' \leq b, x' \geq 0\}$ .

In order to give the same importance to the values of the columns these were normalized and then components were normalized again within columns, to give an adequate importance to its values, for example, the component of the speed of extraction constraint is less important for the aggregation criteria than the  $X$  position of the UBC.

The similarity criteria defined was:  $1 - \cosin(B, C)$ , where  $B$  y  $C$  are two modified vectors of the  $A$  matrix corresponding to the extraction variables and  $\cosin(X, Y) = \frac{\langle X, Y \rangle}{\|X\| \cdot \|Y\|}$ .

The clustering method used was a modification of the Hartigan's Leader Type Method [5]:. This procedure was chosen due to the low computational effort required and reasonable quality of solution.

After the aggregation process is carried out, we can solve the smaller aggregate problem, using a commercial code.

This type of aggregation is highly sensitive to the weights  $g_k$  used. The aggregation and weights used considered the spatial characteristics of the UBCs to insure feasible solutions to the aggregate problem.

The disaggregation process uses the vector  $g_k$  to obtain results of the disag-

gregate problem, using the fixed weights. Feasibility is assured by this process

Finally, Zipkin provides for this method an a posteriori error bound for the objective function using data from the original problem and the solution of the aggregate problem [6], but does not need information from the solution of the original problem so, once the aggregate problem is solved, it is easy to obtain an a posteriori bound. Results show that the amount of variables was reduced to around 15% of the number of variables of the original problem. The model was solved in 27 seconds, a 88% solve time reduction.

The error bound obtained was 3% which is very similar to the real error of 2.93%.

This aggregation procedure had difficulties in defining aggregations and weights that insured feasible solutions, but finally led to reasonable good solutions.

In conclusion, it appears that the proposed cluster analysis approach, in the a priori and a posteriori cases, can lead to reduced corporate models, which approximate reasonably well the original large scale, detailed models. In this form, we can determine smaller models, which run in reduced CPU times, as needed for corporate models, where each mine model needs to be run several times. Further research is needed to integrate both types of aggregation.

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