Economic Performance, Wealth Distribution and Credit Restrictions under variable investment: The open economy

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Abstract

Potential entrepreneurs require capital for investment in projects. They are differentiated by wealth and can abscond with the funds from a loan. In this setting, agents with little wealth are unable to fund their projects, those with intermediate levels of wealth can fund inefficiently sized projects and only wealthy entrepreneurs can attain the efficient firm size. We show that improvements in the legal framework for loans improves economic efficiency, by improving access to credit and by increasing the size of loans for projects. We also examine the effects of wealth redistribution. The effects depend on the aggregate wealth of the economy; in countries with low wealth, redistribution may reduce economic efficiency and GDP, while it may increase economic efficiency and GDP in a wealthy economy. Next, we consider an economy with labor and risky projects. We recover the main results of the simpler model, and we examine the effects of having priority of workers in bankruptcy. We show that it leads to conflicting interests between workers in large and small firms as well as conflicts between small and large entrepreneurs with respect to improvements in the financial system.

Keywords: Financial development, wealth and firm size distribution, efficiency

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1 Introduction

This paper studies the effects of credit market imperfections on the performance of an economy where inequality is relevant. Since the late 90's, it has been recognized that credit market restrictions impact on the economy and on growth.[1] In this paper we study the effects on the performance of an economy of credit restrictions caused by differences in wealth of potential entrepreneurs. These differences imply that some potential entrepreneurs have no access to credit, others receive credit for their projects, but the credit is insufficient to attain the efficient firm size. A third group of wealthier potential entrepreneurs face no restrictions on credit and are able to operate efficient and more profitable firms. Restricted firms can be interpreted as the medium and small sized firms that usually have problems with the access to credit, or face expensive costs of credit and operate less efficiently as a result.[2] Since potential entrepreneurs have profitable projects, the fact that they do not receive credit or it is too small to reach the efficient firm size reduces the efficiency of the economy.

Unlike most previous theoretical models which analyze the effect of financial market imperfections on economic performance, this model incorporates non-linear variable investment decisions.[3] This allows us to model SME's as firms which have access to credit, but cannot achieve the efficient firm size due to credit constraints. In this setting the effects of financial market improvements will have both an effect on the extensive margin –how many potential entrepreneurs can get loans to start their firms– as well as on the intensive margin –entrepreneurs whose credits were inefficiently sized face a relaxation of this constraint and become closer to the efficient size.[4]

In the model, there is a continuum of potential entrepreneurs with heterogenous wealth. Capital is combined in variable proportions with one unit of nontangible and unalienable specific capital owned by the agent (an idea for a profitable project, human capital). Banks are competitive and can obtain funds abroad at a fixed rate,. There are market imperfections in lending, which leads to credit rationing. An entrepreneur that invests is always successful, so there is no risk for lenders, except for the risk of the borrower absconding with the funds.[5]

In this setup, we examine the effects of improvements in financial market institutions on economic performance. Moreover, we look at the effects of changes in the distribution of wealth in this economy. We study the impacts of a redistribution of wealth among agents without changing in aggregate wealth, that is, the pure effects of redistribution. The effects of redistribution on various macro variables depend on the aggregate wealth of the economy. In wealthy societies, redistribution tends to improve growth, while in the case of poor societies, the effect may be reversed.[6] We show that the effects of redistribution

[1] Levine (2005) collects the literature on finance and growth and concludes that more developed financial markets improve growth by easing financial constraints.

[2] See Beck and Demirgüç-Kunt (2008), for example, for evidence of reduced productivity due to credit restrictions in small and medium enterprises (SMEs).

[3] Among papers using a fixed investment size (see Hoshi et al. (1993) and Holmstrom and Tirole (1997). Many papers that examine financial market imperfections in the context of a single firm use non-linear investment, for example Burkart and Ellingsen (2004). Tirole (2006) analyzes the case of variable investment with a constant productivity of investment, except in some exercises.

[4] Fracassi et al. (Forthcoming) show that small business loans for small firms increase their probability of success.

[5] We defer the introduction of the possibility of projects failing to section 5, because it adds to the complexity of the model without any relevant results, except when we include labor in the model.

[6] See Barro (1999) for empirical results going in the opposite direction. On the other hand, Forbes (2000) shows that increased

can also depend on the quality of the financial market, as measured by the loan recovery rate. When the financial system works well, redistribution in relatively less wealthy countries tends to improve growth, while growth is lower after redistribution in countries with malfunctioning financial systems, even if they are relatively wealthy. We also show that an improvement in the quality of the financial market –measured by the loan recovery rate– leads to an improved *ex post* distribution of income, in terms of Generalized Lorenz Dominance (Shorrocks, 1983).

In the extensions, we add labor to the model. There is a threshold level of wealth such that agents with less wealth have no access to credit and become workers, while the rest become entrepreneurs, with a discrete jump in individual welfare. Thus, changes in the financial market parameters or in the wealth distribution alter the amount of labor in this economy. We also allow for failed projects, leading to new sources of credit imperfections due to inefficient bankruptcy procedures. In particular we examine the effects of preferential conditions for workers in bankruptcy, a very common situation in countries with civil law.[7]

An important result is that we show that there are conflicts between small sized firms and large firms.[8] Large firms are reluctant to improve the performance of financial markets, because it increases the size of loans available to restricted firms and allows the entry of some previously excluded entrepreneurs. Both effects increase the demand for workers, thus raising wages and reducing profits. For small firms, the effects of increased efficiency – due to a larger plant size– compensate for the higher wage, whereas large firms only observe the negative effect on wages because they are already at the efficient plant size.[9] Our explanation for the opposition to financial development by large firms is based on the effect on factor prices, and the effect depends on the distribution of wealth. This argument is considered by Rajan and Ramcharan (2011) in their study of financial development in early XX century US agricultural counties.[10] Several papers have hypothesized that the reason for the opposition to financial market reforms in many countries is due to the reduction in expropriation possibilities by managers or controlling shareholders (Rajan and Zingales, 2003; La Porta et al., 2000). These results provide a complementary explanation.

Moreover, we show that increased protection for workers in bankruptcy also drives a wedge between workers in large and small firms. It has a negative effect on small firms, whose access to credit and size and efficiency falls. Thus demand for labor and wages fall. In large firms, demand for labor fall less, because the higher cost of protection for workers is balanced by lower wages. Hence both large firms and their workers may not be averse to these types of worker protection, even though it harms workers in SME's (and the unemployed).

Various implications of the model are verified by empirical research, as we have mentioned and show in the literature review below. Other predictions of the model have not been studied empirically (as far as we have been able to ascertain). An example is the result that the effects of improvements of credit protection

inequality raises growth, independently of the income level.

[7]This is the case of French bankruptcy law, an epitome of civil law countries (Davydenko and Franks, 2005).

[8]The case for medium sized firms is ambiguous.

[9]Even though the plant size is affected by the raised wages, they can adapt their plants to the new optimal size.

[10]In our open economy model, we do not have effects through a higher interest rate due to increased demand for capital. In his dissertation Huerta (2014) analyzes the case of the closed economy, and observes that large firms oppose financial liberalization due to the effect on interest rates, a result obtained in a different setting by Shleifer and Wolfenzon (2002).

parameter on the efficiency of the economy are larger in more unequal countries, if the countries are either very wealthy or very poor.[11] The impacts are reversed for economies with intermediate levels of average wealth.

1.1 Literature Review

Unlike many previous theoretical models, which have analyzed the effect of financial market imperfections on the performance of an economy using fixed investment choices see Hoshi et al. (1993), Hoshi et al. (1993), Repullo and Suarez (2000), this model incorporates non-linear variable investment decisions. This allows us to include firms which have access to credit, but are inefficient because they cannot achieve the efficient size due to credit constraints. Moreover, the effects of financial market improvements changes will have both an extensive margin –more potential entrepreneurs can get loans to start their firms– as well as an intensive margin –entrepreneurs whose credits were inefficiently sized face a relaxation of this constraint and become closer to the efficient size.

This is important because many studies have documented the high returns to capital in SME's. Many of these studies are collected in the Global Financial Development Report 2014.[12] This paper incorporates these effects, and examines the impact of financial market improvements on the performance of the economy considering the effect of the improvements on relaxing the constraints facing SME's. Various studies have noted that movable collateral with centralized registries are measures that improve financial markets. Credit bureaus are also important by helping to reduce adverse selection and moral hazard problems of borrowers and thus reducing credit market imperfections, see Japelli and Pagano 2002, Miller 2003. Djankov et al. (2007) show that these instruments are effective in increasing the ratio of private credit to GDP. The quality of insolvency regimes is another measure of the quality of financial markets and also helps improve access to credit as shown in the Report.

The paper also examines the effects of changes in the wealth distribution on the performance of the economy through the action of the credit constraints on the efficiency of firms.[13] This is related to the approach of Banerjee (2009), who studies the effect of wealth inequality on economic performance, acting through financial market imperfections on the inefficiency of firms –both not allowing efficient entrepreneurs to start firms and by implicitly subsidizing the prices of factors (as this paper does in the section with labor). Galor and Zeira (1993) study the effect of inequality and credit constraints on the acquisition of human capital, leading to reduced growth. Benabou (1996) examines the effect of inequality on growth acting through capital taxation in response to political pressures. Empirically the evidence is varied. Forbes (2000) finds that inequality is positively related to growth, while Barro (1999) finds a U-shaped relationship.

Finally, there are several papers that point out that the interests of small and large businesses are at odds. We have already mentioned several papers on this topic, and we must add Shleifer and Wolfenzon (2002) and the results that are reviewed in Morck et al. (2005).

[11] The intensifier "*very*" has a precise definition in section 4.

[12] Demigürç-Kunt, ed (2014, p.116 ff). See also Beck and Demirgürç-Kun (2006) and Beck et al. (2008).

[13] Balmaceda and Fischer (2010) obtained similar results in a model with a fixed investment size. Note that Tirole (2006, p. 474) describes similar effects in the simpler case of two levels of wealth.

2 The model

We examine a static model of an open economy with heterogeneous agents and variable-investment decisions. The single period is divided into four stages (see Figure 1). In the first stage, a continuum of agents indexed by $z \in [0, 1]$ are born, each endowed with one unit of inalienable specific capital (an idea, an ability or a project) that cannot be transferred or sold. Each entrepreneur is also born with different amounts of observable wealth or mobile capital K_z. The cumulative wealth distribution among the population of agents is given by $\Gamma(\cdot)$, which has a continuous density and full support.

During the second stage, agents go to the credit market to either deposit their mobile capital or to borrow funds for their projects. In the third stage, agents who receive a loan either invest in a firm or abscond, committing *ex-ante* fraud. As in Burkart and Ellingsen (2004), if an agent absconds with a loan, a fraction $1 - \phi$ of the loan is recovered by the legal system. Therefore, $1 - \phi$ represents the degree of *ex-ante* creditor protection or the *loan recovery rate.* Agents who do not need a loan always invest in their project. Agents who are unable to obtain loans may choose to loan their wealth, losing the contribution of their specific capital. In the last stage, deposits are repaid and payoffs are realized.

Figure 1: Time line.

There is only one good in this economy, with $f(\cdot)$ its production function such that $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = +\infty$ and $f(0) = 0$. Thus the model incorporates the assumption of decreasing returns to scale to capital investment. Agents are assumed to be price takers in the credit and output market. We normalize the price of the single good.

Agents who operate a firm try to maximize their total utility from consumption given by:

$$U(C_z) = U(K_z, D_z) = \begin{cases} f(K_z + D_z) - (1+r)D_z - \theta & \text{if the agent forms a firm} \\ (1+r)K_z & \text{if the agent deposits her wealth with a bank.} \end{cases} \quad (1)$$

Here θ is a sunk startup cost of a firm, D_z is the amount loaned or borrowed by entrepreneur z, $(1+r)K_z$ is the return on wealth in the competitive banking system and r is the competitive interest rate charged by banks. The domestic interest rate $r = \rho$, the international rate, because of our assumption of an open economy. In section 5 firms can fail, and the equality no longer holds.

The profit of a firm is:

$$\pi(K_z + D_z) = f(K_z + D_z) - (1+r)(K_z + D_z) - \theta \quad (2)$$

Using this definition, the utility function can be rewritten as:

$$U(K_z, D_z) = \pi(K_z + D_z) + (1+r)K_z \quad (3)$$

Without credit market imperfections, all agents, no matter how small their initial capital stock, would have access to the credit market. Thus, all entrepreneurs would be able to borrow as much as they wanted at the interest rate r, and therefore, would be able to operate their firms at the profit maximizing capital level K^*:

$$f'(K^*) = 1 + r \tag{4}$$

However, not all entrepreneurs will be able to reach the optimal capital level, because there are market imperfections and loans are limited by moral hazard. The borrower may decide to abscond in order to finance non-verifiable personal consumption. Thus, we assume that investment decisions are non-contractible, and that loans used to finance personal benefits are only repaid to the extent that creditor rights are enforced. Since the legal system is able to recover only a fraction $1 - \phi$ of the amounts loaned, we interpret an increase in $1 - \phi$ as an improvement in *ex-ante* creditor protection or in the loan recovery rate.

In contrast, those entrepreneurs who decide to invest all their borrowed capital plus their initial wealth in a firm, enjoy returns only after repaying their obligations, i.e. output and sales revenue are verifiable and can be pledged to investors. Furthermore, all these agents would like to operate their firms at the optimal capital level K^*, but due to moral hazard and credit market imperfections, some agents will have partial access to credit market and may decide to operate their business using a lower amount than optimal capital stock. Moreover, poorer agents may not have access to the credit market. In other words, there is credit rationing: a rationed borrower may be willing to pay a higher interest rate to lenders in order to get a loan or a higher loan, but investors do not want to grant such a loan, because they cannot trust the borrower.[14]

Therefore, the model characterizes two types of constrained entrepreneurs: those that do not have enough capital stock to access to the credit market and that may decide to loan their wealth instead of forming a firm (see proposition 1) , and those agents who have partial access to credit market who get a loan that allows them to operate their firms, but at a sub-optimal level. On the other hand, there are two types of unconstrained agents: those who have enough capital stock to get a loan that allows them to operate efficiently, and those richer entrepreneurs who own more than the optimal capital level, who form an efficient firm and decide after to loan their surplus capital. In summary, the model distinguishes between four types of agents.

The demand for credit originates in agents who own less than the optimal capital stock K^*. Note that two types of agents deposit money: agents who do not have access to credit and decide to not form a firm, and by those richer entrepreneurs who own more than the optimal capital level K^*.

Because of competition in the banking market, banks have losses if they lend to agents who commit fraud. In order to assure that fraudulent behavior never occurs in the equilibrium, we define the following incentive compatibility constraint, that must be satisfied for all agents who want to get a loan from the credit market:

[14] In our basic setting, if a bank decides to loan to an entrepreneur, it simply receives its return. In section 5 we allow for project riskiness, but all projects are equally risky. Consequently, our approach is does not include many informational considerations relevant for bank credit.

$$f(K_z + D_z) - (1+r)D_z - \theta \geq \phi D_z \tag{5}$$

Condition (5) assures that the utility received by an agent who receives a loan D_z if she decided to not abscond, is at least the same that she would obtain if she did. In addition, this inequality implies that the marginal return for getting a loan is at least $1 + r + \phi$, i.e. returns for borrowers are between this value and $1 + r$. Note that under constant returns to scale all firms are equally profitable and in that case loans are unneeded. In the case of perfect loan recovery, i.e. if $\phi = 0$, all agents have perfect access to the credit market.

Additionally, the following breakeven constraint or participation constraint must be satisfied:

$$\pi(K_z, D_z) \geq 0 \tag{6}$$

Condition (6) ensures that the profit of the firm is not negative. Note that this condition is the same as asking that the utility of the entrepreneur for operating a firm is at least what she will obtain from loaning all her capital:

$$U(K_z, D_z) = f(K_z + D_z) - (1+r)D_z - \theta \geq (1+r)K_z. \tag{7}$$

2.1 Critical capital levels

In order to study the behavior of entrepreneurs we need to define several regimes that are clearly differentiated by the capital levels of entrepreneurs. There will be critical capital levels such that agents that belong to the intervals between these capital levels behave and are treated similarly by banks. The first critical capital level is the lowest level of capital for a firm to receive a loan. This critical capital, denoted by K_d, separates agents with and without access to loans. Agents with $K_z < K_d$ are excluded from the capital markets. We show below that for these agents it is better to lend their scarce holdings, rather than to start a firm with so little capital.

A second set of agents are those with limited access to capital. These entrepreneurs receive loans, but these loans are not large enough to lead to efficient investments in plant size. Hence these firms operate suboptimally. Let K^* denote the optimal level of operations (or plant size), i.e such that $f'(K^*) = 1 + r$. Then the agents with inefficient investment are those with capital endowments above K_d, but without sufficient capital to obtain loans sufficient to reach the optimal plant size. If the capital level of the poorest entrepreneur that can obtain loans that let it achieve the optimal plant size is K_r, then the constrained entrepreneurs are those in the interval $K_z \in [K_d, K_r)$. Entrepreneurs with capital stocks between K_r and K^* are able to attain the efficient firm size, and those entrepreneurs with more than K^* in capital deposit the surplus. This is shown in figure 2.

In order to determine the critical capital levels we define the following auxiliary function:

$$\psi(K_z, D_z) \equiv f(K_z + D_z) - (1 + r + \phi)D_z - \theta \tag{8}$$

which will allow us to define the minimum capital level needed to obtain a loan K_d, the associated debt

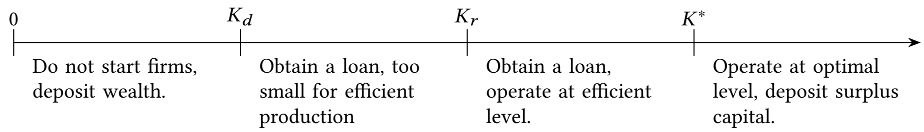

Figure 2: Entrepreneurs' decisions.

D_d and the critical capital level K_r that allows a firm to achieve the optimal size (and also the maximum loan). Note that this function is concave. We begin by noting that $\psi(K_z, D_z) = 0$ defines the debt D_z that a potential entrepreneur with wealth K_z can have that leaves him indifferent between operating a firm and absconding with the loan and committing *ex ante* fraud; and the agent will default on large loans.

In order to define the minimum debt D_d note that by definition, any larger debt will lead the agent with K_d to abscond. Assuming K_d known, the minimum debt is the amount of debt that maximizes the auxiliary function at K_d, subject to the auxiliary function being nonnegative, so that the incentive compatibility constraints are satisfied. In addition, the minimum capital stock K_d defines the first agent (i.e. with the smallest capital) who is able to get the minimum loan without having incentives to abscond. Therefore, the pair (K_d, D_d) is determined as the solution to the following *minimax* problem:

$$\min_{K\geq 0}\max_{D\geq 0} \psi(K, D) \geq 0$$

To simplify the problem, note that minimization of $\psi(K, D)$ leads to $\psi(K, D) = 0$, because otherwise the incentive compatibility constraint is violated. Thus minimization over K leads to a binding incentive constrain and we can rewrite the *minimax* problem as:

$$\max_{D\geq 0} \psi(K_d, D)$$
$$\text{s.t. } \psi(K_d, D) = 0.$$

This is a simple problem, since the objective function is continuous and concave. Since the incentive compatibility constraint is binding, and the function $\psi(K_d, \cdot)$ is maximized at D_d, the entrepreneur cannot obtain a larger loan, and a smaller loan also violates the incentive compatibility constraints.[15] Taking the lagrangian leads to the following definition:

Definition 1 *The minimum debt $D_d \geq 0$ and the minimum capital stock $K_d \geq 0$ are defined by the following two conditions*[16]*:*

$$\psi(K_d, D_d) = f(K_d + D_d) - (1 + r + \phi)D_d - \theta = 0 \tag{9}$$

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0 \tag{10}$$

From (10), the marginal return to investment of the first agent with access to capital is $1 + r + \phi$. We show below that agents with no access to loans (i.e., with $K_z < K_d$) do not for firms and prefer to loan

[15]When $\theta = 0$, then $K_d = 0$ and finding D_d corresponds to finding the solution of $\psi(0, D_d) = 0$.
[16]We assume that the minimum capital stock to get a loan is positive ($K_d > 0$). If $\theta > 0$, $K_d > 0$.

their money. Thus, this is the highest return to investment and as K_z increases, the return falls, eventually to $1 + r$.

In order to determine the critical capital level K_r, we impose the condition that the maximum debt corresponding to K_r allows the firm to attain exactly the optimal capital level K^*. Therefore, the incentive compatibility constraint binds, and the maximum debt of an entrepreneur who owns K_r is $K^* - K_r$.

Definition 2 *The critical capital level K_r of the first agent who is able to invest in the optimal plant size is defined by:*

$$\psi(K_r, K^* - K_r) = f(K^*) - (1 + r + \phi)(K^* - K_r) - \theta = 0 \tag{11}$$
$$\Big(\Leftrightarrow \pi(K^*) + (1 + r)K_r = \phi(K^* - K_r)\Big)$$

Note that $D_r \equiv K^* - K_r$ is the maximum level of debt of a firm in this economy.

2.2 The optimal choices of entrepreneurs

The three threshold values of wealth that we have found allow us to define four categories of agents:

1. Agents that do not form firms; with $K_z \in [0, K_d)$
2. Agents with inefficient firms, with $K_z \in [K_d, K_r)$
3. Agents that borrow up to the efficient size: $K_z \in [K_r, K^*]$
4. Agents that forms efficient firms and deposit their surplus assets: $K_z > K^*$.

Agents in the first group have the option of setting up firms with their own resources, but it is easy to show that they prefer to deposit their resources:

Proposition 1 *Agents with $K_z \in [0, K_d]$ do not form firms. They prefer to deposit their capital.*

Proof: We have that $\psi(K_d, D_d) = 0$ from (9) and also that $d\psi(K_d, D)/dD > 0$ for $D < D_d$ by concavity of f and (10). Thus, $\psi(K_d, D) < 0$ for $D < D_d$. Therefore $\psi(K_d, 0) < 0$, which implies that $\psi(K, 0) < 0$ for $K < K_d$ because f is increasing. ■

Agents who have access to the credit market, that is, those with $K_z > K_d$ will invest the amount of capital that maximizes their utility while satisfying the participation and incentive constraints. They solve the problem:

$$\max_{I_z} U(K_z, D_z) \tag{12}$$
$$\text{s.t. } \psi(K_z, D_z) \geq 0$$
$$\pi(K_z, D_z) \geq 0$$

It is easy to solve this problem using Lagrangians, but we obtain more insight by a more intuitive approach. First, note that agents with wealth above K^* do not want to invest more than K^* in their projects, since the return on the additional investment is lower than $1 + r$, which they would obtain by depositing the excess above K^*. Second, for those agents in the range $[K_r, K^*)$, which can get a loan big enough to invest the efficient amount, any bigger loan means they pay more for the loan than the profits from the additional investment. Similarly, investing less than K^* means that their returns fall by more than the cost of the additional investment. In the case of agents with wealths in the range $[K_d, K_r)$, any additional debt they can achieve generates more profits than its cost, so they get the largest loan they can and are constrained by the incentive constraint. Thus, in the range $K_z \in [K_d, K_r]$ we have that the optimal debt $D_z(K_z)$ satisfies:

$$\psi(K_z, D_z(K_z)) = f(K_z + D_z(K_z)) - (1 + r + \phi)D_z(K_z) - \theta = 0 \tag{13}$$

Given this behavioral pattern, it is convenient to think ~~of~~ a firm associated to agents with $K_z > K^*$ as Large firms, with sufficient resources to achieve the optimal plant size and invest their surplus in the credit market. Those entrepreneurs with $K_z \in [K_r, K^*)$ can be identified with Larger Medium sized enterprises, which can produce efficiently. Entrepreneurs in the range $K_z \in [K_d, K_r)$ are not efficient producers and can be associated to small and medium sized enterprises (SMEs). The remaining agents do not form enterprises. The characteristics of the debt function associated to the different types of firms is described by the following result:

Proposition 2 *The effective debt curve $D_z(K_z)$ satisfies the following properties:*[17]

1. $\frac{\partial D_z(K_z)}{\partial K_z} > 1$ *if* $K_z \in (K_d, K_r]$.
2. $D_z(K_z) > K_z$ *if* $K_z \in (K_d, K_r]$.
3. $D_z(K_z)$ *is concave in* K_z.

Proof:
Differentiation of equation (8) leads to:

$$\frac{\partial \psi(K_z, D_z)}{\partial K_z} + \frac{\partial \psi(K_z, D_z)}{\partial K_z} \frac{\partial D_z}{\partial K_z} = 0$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z} = -\frac{\psi_K}{\psi_D} \tag{14}$$

Using equations (9) and (10), which define K_d and D_d we obtain that:

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0. \tag{15}$$

Moreover, from the definition of ψ in (8) we have

$$\psi_K(K_z, D_z) = f'(K_z + D_z) > 0 \tag{16}$$

[17] We do not write the dependence of D_z on K_z when clear.

Note that if $K_z > K_d$, $f'(K_z + D_d) < 1 + r + \phi$ (because $f''(\cdot) < 0$). Thus $\psi_D < 0$ to the right of K_d.[18] Using these facts in (14) we conclude that:

$$\frac{\partial D_z}{\partial K_z} = -\frac{f'(K_z + D_z)}{f'(K_z + D_z) - (1 + r + \phi)} > 1$$

For the second item, to show that $D_z(K_z) > 0$, note that differentiating (13) at $K_z = K_d$ and assuming $D_z = 0$ leads to $f'(K_d, D_z) = 0$. On the other hand, if we use (10), one of the two equations that define (K_d, D_d), we have that $f'(K_d) = 1 + r + \phi$, a contradiction. Thus $D_d > 0$.

To show that $D_z(K_z) > K_z \in [K_d, K_r]$, note that we can rewrite (13) and the incentive compatibility constraint (5) respectively as:

$$U(K_z, D_z(K_z)) = \phi D_z$$
$$U(K_z, D_z(K_z)) \geq (1 + r)D_z$$

Comparing, we see that $D_z(K_z) \geq [(1 + r)/\phi]\, K_z$. The result $D_z(K_z) > K_z$ follows, since $0 \leq \phi < 1$.
For the last item, note that differentiating (14) with respect to K_z leads to: $\frac{\partial^2 D_z}{\partial K_z^2} = \frac{f''(K_z+D_z)(1+r+\phi)}{(f'(K_z+D_z)-(1+r+\phi))^2} < 0.$■

What is most interesting about this result is that the loan size is discontinuous at K_d, jumping from zero to the left of K_d to a value $D_z(K_z) > K_z$. Moreover, the loan size continues to be larger than the own capital until K_r, i.e., the agent that can attain the optimal size through a loan. In financial terms, the leverage ratio of undercapitalized firms is higher than 1. As entrepreneurs have more capital, past K_r, the loan sizes decrease and leverage falls until it becomes smaller than one and eventually disappears in large firms. Note that these results are consistent with the literature showing that due to credit limitations, SMEs have lower productivity (Banerjee (2009), or Demigürç-Kunt, ed (2014), for a recent review of the evidence, apart from the papers on the topic mentioned in the Introduction)

With the results of proposition 2 we can depict the optimal loans as a function of the capital of the entrepreneur.[19] The figure shows (by proposition1) that entrepreneurs with $K_z < K_d$ do not form firms (with no loans) and prefer to deposit their small capital.[20]

Associated to this optimal loan function, there is a utility function associated to each level of entrepreneurial capital. Figure 4 shows this. In particular, there is a jump in entrepreneurial utility at K_d, when entrepreneurs can obtain loans and form firms. The slopes for low wealth ($K_z < K_d$) poor and wealthy entrepreneurs ($K_z > K^*$) are the same and grow at the rate $(1 + r)$.

[18] Recall that this derivative can only be defined to the right of K_d, because there is a discontinuity to the left of K_d.

[19] In figure 3, negative values correspond to saving deposits by agents.

[20] This is easy to see, since otherwise we would have that $f(K_z) - \theta > (1 + r)K_z$. But then there exists a small loan $D_z \approx 0$ such that $f(K_z + D_z) - (1 + r)D_z - \theta > (1 + r)K_z > \phi K_z$, satisfying the incentive constraint. Hence K_d was not the least level of capital which allows a loan.

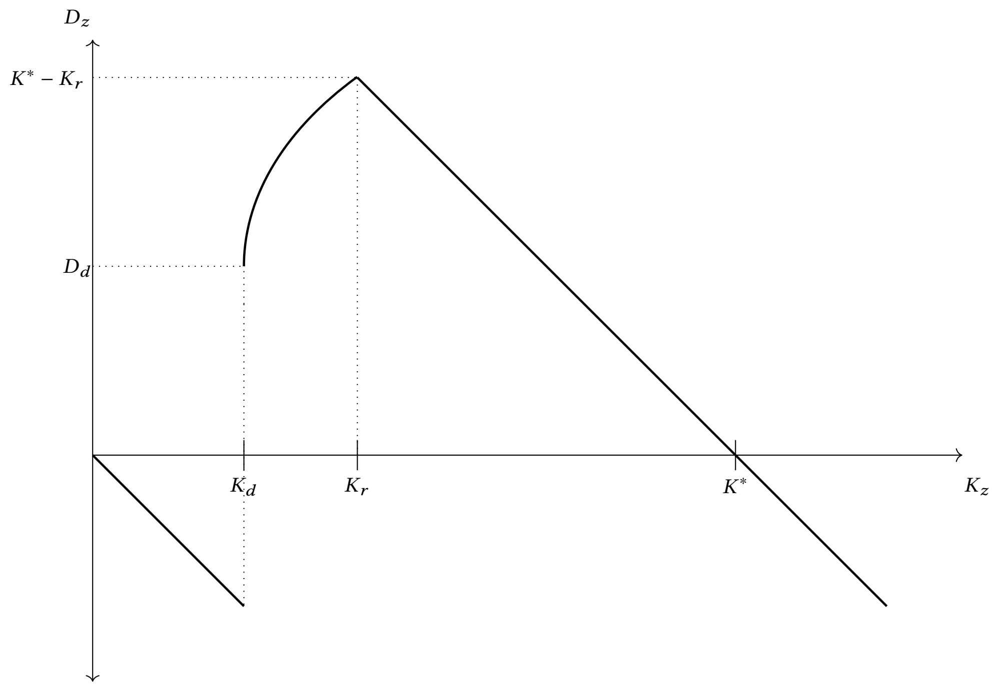

Figure 3: Effective loan curve.

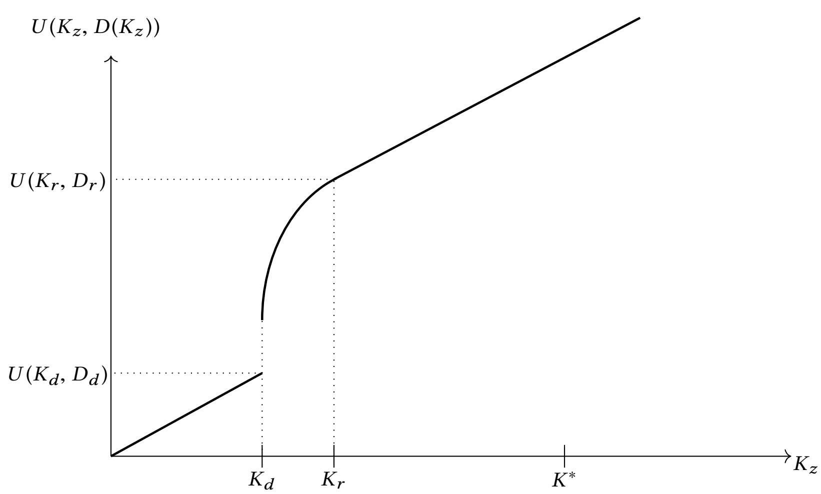

Figure 4: Utility function.

3 Comparative statics

In this section we examine the effects of changes in the fundamentals of the model: improvements in creditor protection ($\phi \downarrow$), reductions in the fixed costs of forming a firm (θ) and changes in the international interest rate. Note that in the small open economy, the adjustments require inflows or outflows of capital.

3.1 Effects of changes in ϕ, θ and r.

We can easily show that:

Lemma 1 *I a small open economy, an improvement in* ex ante *protection ($\phi \downarrow$), a reduction in fixed costs θ or a fall in the interest rate r lead to:*

1. *A reduction in K_d.*
2. *An increase in the maximum loan D_z, for $K \in [K_d, K_r]$.*
3. *The improvement in protection or reduction of fixed costs lead to a fall in K_r.*

Proof: For the results related to K_d, we use the two equations defining D_d, K_d, (9) and (10). Totally differentiating (9) with respecto to ϕ, θ and using (10),

$$\frac{\partial K_d}{\partial \phi} = \frac{D_d}{1+r+\phi} > 0$$
$$\frac{\partial K_d}{\partial \theta} = \frac{1}{1+r+\phi} > 0$$
$$\frac{\partial K_d}{\partial r} = \frac{D_d}{1+r+\phi} > 0$$

For the case of K_r, we use the defining expression $f(K^*) - (1+r+\phi)(K^* - K_r) - \theta = 0$ (see (11)) and $f'(K^*) = 1 + r$. Totally differentiating and recalling that K^* depends only on r, we have that

$$\frac{\partial K_r}{\partial \phi} = \frac{\overbrace{K^* - K_r}^{D_r}}{1+r+\phi} > 0$$
$$\frac{\partial K_r}{\partial \theta} = \frac{1}{1+r+\phi} > 0.$$

We consider the effects of improvement in *ex ante* protection ($\phi \downarrow$) for capital stocks in the range $K_z \in [K_d, K_r]$. We use equation (5) with equality, as we showed that in this range entrepreneurs choose the largest loan they can get. Totally differentiating, with fixed K_z, we have:

$$\frac{\partial D_z}{\partial \phi} = \frac{\partial D_z}{\partial r} = \frac{D_z}{f'(K_z + D_z) - (1+r+\phi)} < 0.$$

Note that the denominator is negative because $f'(K_z + D_z) < (1 + r + \phi)$ for $K_z \in [K_d, K_r]$.[21] Similarly,

$$\frac{\partial D_z}{\partial \theta} = \frac{1}{f'(K_z + D_z) - (1 + r + \phi)} < 0.$$

■

Using these results we obtain the following conclusion:

Proposition 3 *In a small open economy, the distance between K_r and K_d becomes smaller as the credit recovery rate $1 - \phi$ improves.*

Proof: Using the results of the previous lemma:

$$\frac{\partial(K_r - K_d)}{\partial \phi} = \frac{D_r - D_d}{1 + r + \phi} > 0$$

■

That is, the first entrepreneur that gets a loan, and thus can start a firm, needs less capital as the credit recovery rate improves.[22] (including income and wealth) Moreover, the first entrepreneur whose firm attains the efficient size, also shifts to the left. More interestingly, the proposition implies that the range of values of capital consistent with SME's is smaller. On the other hand, a change in the fixed costs of establishing a firm has no effect on this difference, even though both K_r, K_d also shift to the left.

Another simple fact is that as the credit recovery rate improves, the smallest firm increases in size, and thus becomes more efficient. To see this, we use (10), $f'(K_d + D_d) - (1 + r + \phi) = 0$: as ϕ falls, plant investment $K_d + D_d$ increases. Not only does an improvement in the *ex ante* loan recovery rates means that entrepreneurs with less wealth have access to loans, but the size of the loans they can obtain is so much larger that they can invest in a more efficient plant size. Moreover, all SME's benefit, since all of them are credit constrained and they can obtain bigger loans. Since they operate in a range in which the marginal cost of loans is smaller than the marginal addition to profits, that all do better. Non-credit constrained entrepreneurs, on the other hand, are not affected by the improvement in this parameter.

An important result with political economic implications is the following:

Proposition 4 *In a small open economy, an improvement in ex-ante creditor protection raises profits of firms with $K_z \in [K_d, K_r]$, while the profits of other firms remain constant.*

The proof follows by inspection of (13), which is applicable to firms in this range. Since the RHS of the equation falls, firms invest more, and since they were constrained before, profits must increase.[23] The

[21] In the range $[K_r, K^*]$, loans also increase with falls in ϕ:

$$\frac{\partial D_z}{\partial \phi} = \frac{\partial(K^* - K_r)}{\partial \phi} = -\frac{\partial K_r}{\partial \phi} < 0.$$

[22] Fabbri and Padula (2004) show empirically, that in Italy, as the quality of legal enforcement of debt contracts improves, the probability of obtaining a loan increases, other things equal.

[23] Note that a reduction in the fixed costs θ of setting up firms increases the profits of all firms with the smaller firms benefiting from the additional effect of looser credit constraints.

intuition is that the reduction in agency costs allow inefficiently sized firms to grow larger and therefore more efficient, boosting profits. On the other hand, unconstrained firms are not affected by the change.

This result suggests that there might be differences in the position of large and small firms with respect to measures that promote legal improvements protecting creditors, as described by La Porta et al. (2000) and Rajan and Zingales (2003). These authors suggest that incumbent –large– firms oppose financial development because it creates competition and raises the cost of finance. In our open economy case, these effects do not occur, but see Huerta (2014), who also studies the closed economy case and where this opposition to financial improvement does occur. This paper's result is consistent with Rajan and Zingales (2003), who suggest that opposition of incumbents to increased protection of creditors will be weaker if the economy allows both cross-border trade and capital flows. Therefore, an open economy is more likely to undertake reforms benefiting financial development. Moreover, openness stands to be an important determinant of creditor proteection. See also similar results in Balmaceda and Fischer (2010). At this stage of our modelling procedure, we observe a divergence, but no opposition between large and small firms. Once we include labor (see section 5), however, this changes, because the increased size of restricted firms leads to increased demand for labor, raising wages, and thus lowering profits for large firms.

As an immediate corollary of these results, a reduction in the costs of setting up a firm or an improvement in *ex ante* protection translates into an influx of funds into the economy, as expected.

Definition 3 *We define GDP as follows:*

$$GDP = \int_{K_d}^{K_r} [f(K_z + D_z) - (1+r)D_z - \theta]\partial\Gamma(K_z) \\ + \int_{K_r}^{K^*} [f(K^*) - (1+r)(K^* - K_z) - \theta]\,\partial\Gamma(K_z) + (f(K^*) - \theta)(1 - \Gamma(K^*)) \quad (17)$$

Total investment is:

$$I = \int_{K_d}^{K_r} (K_z + D_z)\partial\Gamma(K_z) + K_z(1 - \Gamma(K_r)) \quad (18)$$

and Gross Output is:

$$GO = \int_{K_d}^{K_r} f(K_z + D_z)d\Gamma(K_z) + f(K^*)(1 - \Gamma(K_r)) \quad (19)$$

We are led to the following result:

Proposition 5 *In a small open economy, an improvement in ex-ante protection ($\phi \downarrow$) leads to an increase in the following macroeconomic variables:*

1. *Gross Output and GDP,*
2. *Total investment,*
3. *Total debt and credit penetration.*

Proof: Differentiating GDP defined by (17) in terms of x:

$$\frac{\partial GDP}{\partial \phi} = \int_{K_d}^{K_r} \left(\overbrace{[f'(K_z + D_z) - (1+r)]}^{>0} \frac{\partial D_z}{\partial \phi} \right) \partial \Gamma(K_z) - \overbrace{[f(K_d + D_d) - (1+r)D_d - \phi]}^{>0} \frac{\partial K_d}{\partial \phi} \gamma(K_d) < 0 \quad (20)$$

where we have used the fact that $\frac{\partial D_z}{\partial \phi} < 0$, $\frac{\partial K_d}{\partial \phi} > 0$ and $\partial K^*/\partial \phi = 0$. The proof for Gross output is similar. For the second item, we differentiate total investment with respect to x:

$$\frac{\partial I}{\partial \phi} = \int_{K_d}^{K_r} \frac{\partial D_z}{\partial \phi} \partial \Gamma(K_z) - \frac{\partial K_d}{\partial \phi}(K_d + D_d)\gamma(K_d) < 0 \quad (21)$$

For the last result we use that total debt is given by:

$$D_T = \int_{K_d}^{K_r} D_z \, \partial \Gamma(K_z) + \int_{K_r}^{K^*} (K^* - K_z) \, \partial \Gamma(K_z) \quad (22)$$

Differentiating condition (22) with respect x:

$$\frac{\partial D_T}{\partial \phi} = \int_{K_d}^{K_r} \frac{\partial D_z}{\partial \phi} \partial \Gamma(K_z) - D_d \frac{\partial K_d}{\partial \phi} \gamma(K_d) < 0 \quad (23)$$

Note that credit penetration is defined as follows:

$$CP = \Gamma(K^*) - \Gamma(K_d), \quad (24)$$

that is, as the measure of entrepreneurs that receive loans. Differentiating this condition:

$$\frac{\partial CP}{\partial \phi} = -\frac{\partial K_d}{\partial \phi} \gamma(K_d) < 0 \quad (25)$$

■

Observation: The same theorem and results apply to increases in the fixed costs θ.

The interpretation of this result is simple. An improvement in *ex ante* protection for loans improves GDP, investment, credit penetration and total debt. The effects on GDP, investment and total debt have two sources: first, there is an inframarginal effect as those agents that received loans that were not large enough to attain the efficient investment size now receive larger loans and become more efficient producers, and there is a marginal effect, because additional agents receive loans.

Note that this result is consistent with the empirical results of the seminal paper by La Porta et al. (1997) (and more recent papers, such as Djankov et al. (2007) and La Porta et al. (2008)), who found that better protection for creditor rights increased lending in the economy.[24]

[24]Because of our assumption of an open economy, and the fact that there are no other factors of production, the entry of

4 Changes in the wealth distribution

One of the advantages of the present modelling structure is that it is possible to evaluate the effects of the distribution of wealth on the performance of an economy. In order to isolate the effects due to pure wealth distribution, independent of any real wealth effects, we consider Mean Preserving Spreads (MPS) of an original wealth distribution. As two distributions, the second being a MPS of the first, have the same mean, any effects we derive are solely due to the increase in wealth inequality in the second distribution.

Recall that a MPS of any distribution implies a single-crossing property at the mean of the distribution.

Definition 4 *Consider two distributions $\Gamma_1(K_z)$ and $\Gamma_2(K_z)$ with the same expected value. The distribution $\Gamma_1(K_z)$ is said to be a MPS of the initial wealth distribution $\Gamma_0(K_z)$, if the following both conditions are satisfied:*

1. $\Gamma_1(K_z) > \Gamma_0(K_z)$ *if* $K_z < E(K_z)$
2. $\Gamma_1(K_z) < \Gamma_0(K_z)$ *if* $K_z \geq E(K_z)$

In order to get strong results, we impose an additional condition on the two distributions:

Assumption 1 (Double crossing condition) *The density functions associated to the two distributions cross at only two points.*

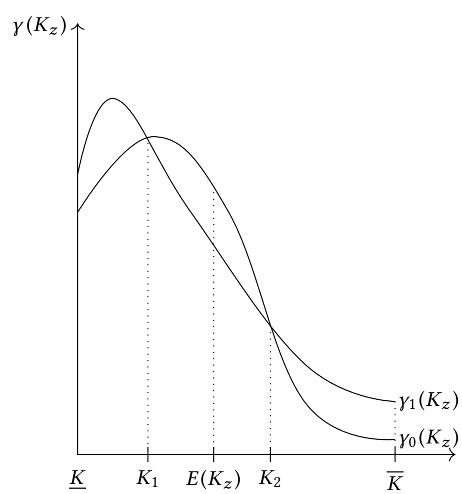

Figure 5: Densities associated to two MPS Distributions that satisfy the double crossing condition.

Consider figure 5. The densities γ_0, γ_1 are associated to the distributions Γ_0 and Γ_1 respectively, have the same expectation and cross at only two points. At K_1, the positive difference $\Gamma_1(K_z) - \Gamma_0(K_z)$ is maximized, while at K_2 it is minimized. The points $K_1, E(K_z), K_2$ define 4 intervals which will help prove the following results, and which we denote as the first, second, third and fourth intervals. In the first and fourth interval

additional entrepreneurs does not affect wealthy entrepreneurs, who will not oppose the improvements. However, in a closed economy, financial market improvements increase the interest rate and should generate the opposition of large firms (Huerta, 2014). See below for the addition of labor and the reappearance of this type of conflict.

we have that $\gamma_1(K_z) - \gamma_0(K_z) > 0$ and in the second and third intervals, $\gamma_1(K_z) - \gamma_0(K_z) < 0$. Most common distributions, including the lognormal, satisfy these conditions for appropriate MPS.[25]

To proceed, we define $\Gamma_\lambda = \lambda\Gamma_1 + (1-\lambda)\Gamma_0$, where $\lambda \geq 0$ and Γ_1 is a MPS of Γ_0. Notice that as λ increases we obtain a sequence of riskier (i.e., more unequal) distributions that transform Γ_0 continuously into Γ_1. The following result describes the effects of an increase in wealth inequality on various measures of the performance of an economy.

Proposition 6 *Consider a small open economy such that $K_d > E(K_z)$ and with an initial wealth distribution $\Gamma(K_z)$. Suppose that $\Gamma_1(K_z)$ is a Mean Preserving Spread (MPS) of $\Gamma_0(K_z)$. Then the following macroeconomic variables will be higher in the economy with more inequality:*

1. *Gross Output.*
2. *Total investment.*
3. *GDP*

Otherwise, if $K_r \leq E(K_z)$, Gross Output and Total Investment are lower. If in addition $K^* < E(K_z)$, GDP is also smaller.

Proof: Differentiating the various quantities with respect to λ and evaluating at $\lambda = 0$:

$$\frac{\partial GO}{\partial \lambda} = \int_{K_d}^{K_r} f(K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - f(K^*)(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{26}$$

$$\frac{\partial I}{\partial \lambda} = \int_{K_d}^{K_r} (K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - K^*(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{27}$$

$$\frac{\partial GDP}{\partial \lambda} = \int_{K_d}^{K_r} U(K_z, D_z)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_r}^{K^*} U(K_z, K^* - K_z)(\partial\Gamma_1 - \partial\Gamma_0) - U(K^*, 0)(\Gamma_1(K^*) - \Gamma_0(K^*)) \tag{28}$$

The proofs consist on finding upper or lower bounds for the different terms of these expressions and simplifying, using the properties of the differences of the distributions and densities in the different intervals. We show this in the case of GDP, by considering the various possible arrangements and considering appropriate bounds.

Case 1: $K_d, K_r, K^* \in [E(K_z), K_2]$. We have that $\gamma_1(K_z) - \gamma_0(K_z) < 0, \forall K_z \in [K_d, K^*]$. Hence $(\partial\Gamma_1 - \partial\Gamma_0) < 0$ in (28) and replacing the integrand by $U(K^*, 0)$ we have a lower bound. Simplifying we obtain

$$\frac{\partial GDP}{\partial \lambda} > -U(K^*, 0)\underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{<0} > 0.$$

[25] Two distributions defined on the same range and having the same expected value necessarily cross at least twice. Two Pareto distributions with the same expectation have different ranges so this condition does not apply. For other uses of the double crossing condition, Benassi et al. (2002). In fact, the proof can be easily generalized to any number of crossings between the densities.

Case 2 $K_d, K_r \in (E(K_z), K_2)$; $K^* > K_2$ Expression (28) can be written as:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} &= \int_{K_d}^{K_r} U(K_z, D_z)\underbrace{(\partial\Gamma_1 - \partial\Gamma_0)}_{<0} + \int_{K_r}^{K_2} U(K_z, K^* - K_z)\underbrace{(\partial\Gamma_1 - \partial\Gamma_0)}_{<0} \\ &+ \int_{K_2}^{K^*} U(K_z, K^* - K_z)\underbrace{(\partial\Gamma_1 - \partial\Gamma_0)}_{>0} - U(K^*, 0)(\Gamma_1(K^*) - \Gamma_0(K^*))\end{aligned}$$

A lower bound for this expression:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} &> \int_{K_d}^{K_r} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_r}^{K_2} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_2}^{K^*} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) \\ &\qquad - U(K^*, 0)(\Gamma_1(K^*) - \Gamma_0(K^*)) \\ &= \underbrace{(U(K_2, D_2) - U(K^*, 0))}_{<0}\underbrace{[\Gamma(K^*) - \Gamma_0(K^*)]}_{<0} - U(K_2, D_2)\underbrace{[\Gamma(K_d) - \Gamma_0(K_d)]}_{<0} > 0\end{aligned}$$

Case 3: $K_d \in (E(K_z), K_2)$; $K_r, K^* > K_2$

Using the same trick again we obtain a positive lower bound for this expression:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} &> \int_{K_d}^{K_2} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_2}^{K_r} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_r}^{K^*} U(K_2, D_2)(\partial\Gamma_1 - \partial\Gamma_0) \\ &\qquad - U(K^*, 0)(\Gamma_1(K^*) - \Gamma_0(K^*)) \\ &= \underbrace{(U(K_2, D_2) - U(K^*, 0))}_{<0}\underbrace{[\Gamma(K^*) - \Gamma_0(K^*)]}_{<0} - U(K_2, D_2)\underbrace{[\Gamma(K_d) - \Gamma_0(K_d)]}_{<0} > 0\end{aligned}$$

Case 4: Since $K_r > K_d > K_2 > E(K_z)$ we have that $(\partial\Gamma_1 - \partial\Gamma_0) > 0$ in (28). Hence, replacing the integrands by $U(K_d, D_d)$ we have a lower bound. Collecting terms we have

$$\frac{\partial GDP}{\partial \lambda} > \underbrace{(U(K_d, D_d) - U(K^*, 0))}_{<0}\underbrace{(\Gamma_1(K^*) - \Gamma_0(K^*))}_{<0} - U(K_d, D_d)\underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{<0} > 0$$

We conclude that $\frac{\partial GDP}{\partial \lambda} > 0$ if $K_d > E(K_z)$. The other proofs are similar. The proof can be generalized to more than two crossings of the density function. ■

First note that if $K_d > E(K_z)$ the last agent to receive a loan has more than the average capital in the economy, i.e., only fairly rich agents have access to the credit market. Hence by concentrating wealth, SMEs will be more efficient and perhaps some excluded entrepreneurs now have the capital to obtain a loan and start a firm. Hence, GDP increases.

Contrariwise, if $K_r \leq E(K_z)$, even relatively poor agents can invest and achieve the efficient plant size. These economies then are wealthy. A redistribution will reduce the capital stock of relatively poor agents, either excluding them or reducing the size and efficiency of their firms. Hence gross output and total investment decrease.

Note also that it could be that, for a given societal wealth $E(K_z)$, we could have $K_d \lesseqgtr E(K_z)$ due to changes in ϕ, which we interpret as different levels of financial development. Hence the direction of changes macroeconomic variables due to changes in the distribution of wealth could also depend on the degree of legal protection for creditors: for example, financial markets that work better (smaller ϕ) implies that gross output increases with less inequality.

Corollary 1 *If $E(K_z) \in (K_d, K^*)$ then credit penetration $CR \equiv \int_{K_d}^{K^*} 1 \; \partial\Gamma$ will decrease with an increase in inequality.*

Proof:

$$\frac{\partial CP}{\partial \lambda} = \underbrace{(\Gamma_1(K^*) - \Gamma_0(K^*))}_{<0} - \underbrace{(\Gamma_1(K_d) - \Gamma_0(K_d))}_{>0} < 0$$

■

This result shows that when loans are easily given, because the loan recovery rates are high, so entrepreneurs with less wealth than the average receive loans, and when interest rates are low, so the optimal size of a firm is large, an increase in inequality reduces credit penetration.

As we have seen above, there is a relationship between the distribution of wealth and the loan recovery rate. This is easy to show in the case of very capital abundant ($K_r < K_1$) or very capital poor ($K_d > K_2$) economies, in which case we show the impact of decreases in the loan recovery rate on various macroeconomic variables, for countries with different degrees of inequality.

Proposition 7 *Consider two small open economies A and B such that the wealth distribution is an MPS of that in B, and and which have the same credit protection parameter. If in both countries $K_d > K_2$ (or $K_r < K_1$), then the following macroeconomic variables improve relatively more in the more unequal country A when creditor protection improves ($\phi \downarrow$):*

1. *Investment.*
2. *Gross output.*
3. *GDP.*
4. *Total Debt.*
5. *Credit Penetration ($K_d < K_1$ or $K_d > K_2$).*

Otherwise, if $K_d, K_r \in (K_1, E(K_z))$ or $K_d, K_r \in (E(K_z), K_2)$ then Investment, Gross output, GDP, Total Debt and Credit Penetration [26] *rise more in country B after an improvement of ex-ante creditor protection.*

Proof: We prove one case, given that the others are fundamentally the same:

$$\frac{\partial^2 GDP}{\partial\phi\partial\lambda} = \int_{K_d}^{K_r} [f'(K_z + D_z) - (1+r)] \frac{\partial D}{\partial \phi}(\partial\Gamma_1 - \partial\Gamma_0) - U(K_d, D_d)\frac{\partial K_d}{\partial \phi}(\gamma_1(K_d) - \gamma_0(K_d)) \tag{29}$$

[26] For credit penetration we just need that: $K_d \in (K_1, E(K_z))$ or $K_d \in (E(K_z), K_2)$.

because in the range $K_z \in [K_r, K^*]$, we have $\partial D_z/\partial\phi = \partial(K^* - K_z)/\partial\phi = 0$, so the derivative of the second integral is zero. Since $\frac{\partial D_z}{\partial\phi} < 0$ by footnote 21 in the second integral. The conditions imply that both K_d and K_r) lie within either the first or fourth interval determined by the crossings of the density functions and the expected value of the distribution. In the two cases we have that $\gamma_1(K_z) - \gamma_0(K_z) > 0, \forall K_z \in [K_d, K_r]$. Moreover, we showed in lemma 1 that $\partial D_z/\partial\phi < 0$ and $\partial K_z/\partial\phi > 0$. Thus the integral is negative and the second term is positive. Then $\frac{\partial^2 GDP}{\partial\phi\partial\lambda} < 0$. ■

One interpretation of the condition that K_d lies in the fourth interval is that the average wealth in this economy is very low, so that most potential entrepreneurs do not have access to the credit market. In that case, the result shows that if we consider two equally poor economies, with one of them having more concentrated wealth, the positive effect of an improvement in the loan recovery rate is larger in the economy with a better distribution of wealth.

Noting the two terms in (29) helps to interpret the result. The first term is the intensive effect of the change in the loan recovery rate $1 - \phi$. It measures the change in the contribution to GNP due to the changed size of loans of agents that already had loans. The second term adds the contribution of the new agents that have access to loans due to the change in the loan recovery rate.

A better distribution of wealth implies that more agents have wealth that is close to the level required for a loan. The improvement in the loan recovery rate allows them to obtain credit. In the economy with more concentrated wealth, more of the benefit accrues to entrepreneurs which already had credit and can now obtain larger loans. Since the marginal increase in productivity is higher for agents with less capital (or who just got a loan), the effect in the first case is larger.

This result is reversed when both K_d and K_r belong to either the second or third intervals in the range of K_z. In that case both the intensive and the extensive components to the change in GDP are negative in the more unequal economy with the increase in ϕ. Hence it is the economy that benefits most from an improvement in the loan recovery rate.

We now show that an improvement in the loan recovery rates ($\phi \downarrow$)leads to a better distribution of wealth in the Generalized Lorenz sense, which means that the new distribution is "better" in a well defined sense than with the original value of ϕ.

Definition 5 *(Shorrocks, 1983) The Generalized Lorentz (GL) Curve is defined as:*

$$GL(K_z) = \int_0^{K_z} U(K_z, D_z)\partial\Gamma(K_z) \tag{30}$$

The Generalized Lorentz curve induces an ordering among distributions of income that satisfies reasonable welfare properties. Consider two distributions of income F, G. If the GL curve associated F lies above and does not cross the GL curve of G, then F Second Order Stochastic Dominates G. Moreover, F is preferred to G by all symmetric utilitarian welfare functionals with increasing and concave utility, even when their means differ (Kleiber and Kraemer, 2000).

Figure 6 (derived from figure 4) shows the effect of the improvement in loan recovery rates on the utility of the different agents. The primed variables show the new values on the axis, while the dark curves show the displacements. The next result shows that the Generalized Lorentz curve with improved loan recovery rates lies above the original Lorenz curve, and thus leads to an unequivocal improvement in social welfare.

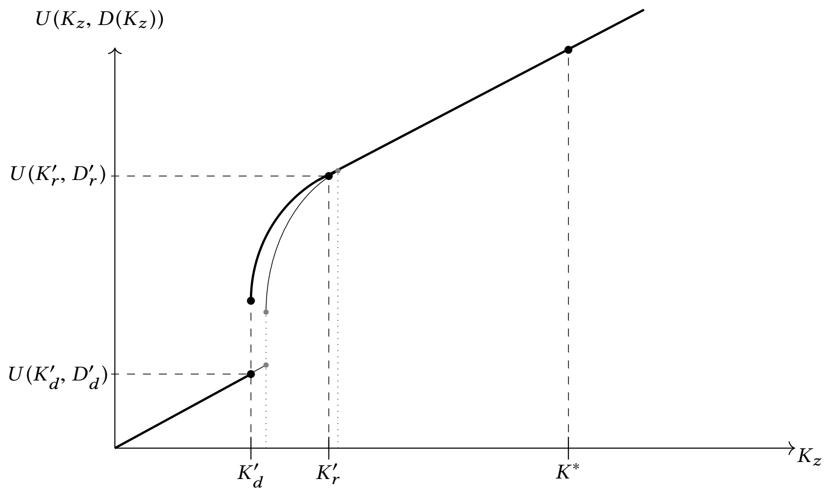

Figure 6: Shift in the utility function due to the change in ϕ.

Proposition 8 *Consider two open economies A and B with the same initial wealth distribution, but which differ in their ex-ante protection parameter ϕ_A and ϕ_B respectively. If $\phi_A < \phi_B$ then $GL(K_z, \phi_A) \geq GL(K_z, \phi_B), \forall K_z$.*

Proof:

For $K_z < K_d$ it is straightforward to see that $\frac{\partial GL(K_z)}{\partial \phi} = 0$. Similarly utility does not change with ϕ if $K_z \geq K^*$ (because these agents do not require loans) so $\frac{\partial GL(K_z)}{\partial \phi} = 0$ in that range.

If $K_z \in [K_d, K_r)$ then:

$$\frac{\partial GL(K_z)}{\partial \phi} = \underbrace{[(1+r)K_d - U(K_d, D_d)]}_{<0} \gamma(K_d) \underbrace{\frac{\partial K_d}{\partial \phi}}_{>0} + \int_{K_d}^{K_z} \underbrace{(f'(K_z + D_z) - (1+r))\frac{\partial D_z}{\partial \phi}}_{<0} \partial \Gamma(K_z) < 0$$

Similarly, if $K_z \in [K_r, K^*)$ we obtain that: $\frac{\partial GL(K_z)}{\partial \phi} < 0$. ■

Galor and Zeira (1993) in a dynamic model with credit constraints in education and altruistic bequests, were apparently the first to show that the evolution of income distribution and aggregate variables in an economy depend on the initial wealth of society. Our model, while static, allows us to analyze the effects of a general MPS of the distribution of income on the aggregate variables and show that these depend on the aggregate wealth of society..

5 A model with bankruptcy and labor

We consider here an expansion of the model model where firms requiere labor and capital for production. In this economy, entrepreneurs unable to obtain a loan, lend their services for a wage. Thus now all agents are identical, with the potential to create enterprises and reap profits. However, only those agents that can obtain loans receive these benefits and thus wealth and its distribution will affect welfare. Note that in an economy where there are many firms, there is a large demand for labor, so salaries will tend to be relatively high. Thus, any factor that increases the number of firms will tend to benefit even agents who do not form enterprises. This also means hat there is a potential for conflicts between different agents. We assume the production function is $f(K, L)$, satisfying $f_K, f_L > 0,\ f_{KK}, f_{LL} < 0,\ f_{KL} > 0$.

The profits of a firm are now: $f(K_z + D_z, L_z) - (1 + r_z)D_z - \theta - wL_z$, where wL_z are the total wages. We make the simplification of assuming that from the point of view of the firm, labor is continuous (i.e., it is behaves like capital). On the other hand, when considering the welfare of workers employed by the different types of firms we assume that workers are attached to specific firms. When we consider the welfare of labor as a factor of production, again we assume labor is continuous.

In order to incorporate additional features in the model, we assume there is a probability p of success of the project, and that it goes bankrupt with probability $(1 - p)$. In general some assets will survive bankruptcy, and can be used to pay some of the debt owed to creditors. We assume, that the value of assets that survive bankruptcy depend on the quality of bankruptcy legislation (or alternatively, the hardness of the assets in a sector) and the appropriability of assets following bankruptcy. Moreover, we assume banks select the return and riskiness of loans, as in Allen and Gale (2004).

When a firm fails, a fraction η of total investments $(K_z + D_z)$ is recovered. The fraction η depends on the quality of bankruptcy laws −the time it takes to resolution, for example−, and on the hardness of the sector. The notion of hardness follows Braun (2005), i.e., sectors in which recovery is higher because of sector characteristics. Examples would be properties versus machinery, for example. Moreover, in cases of bankruptcy, the quality of creditor protection in the case of bankruptcy or liquidation, denoted by τ, is important. It represents the fraction of the value recovered in bankruptcy that the entrepreneur can pledge to lenders. In some countries, the entrepreneur obtains a fraction of the value recovered in bankruptcy, and τ represents the part that he cannot appropriate [27].

A final aspect that we consider is the fraction of value in bankruptcy that is due to workers. In some countries, when faced with bankruptcy, the wages that are owed to workers are considered normal debt. In other countries it has priority in bankruptcy, which means that workers are paid first from whatever assets survive bankruptcy, and if anything is left over, it goes to pay other creditors. We simplify by assuming that the obligation to workers can be written as $\Theta wL_z, \Theta \in [0, 1]$, and that any remaining debt to workers is cancelled in bankruptcy. Since in general, the priority of workers has limits, we will parameterize the fraction of wages that are prioritized. Taking all of this into account, we obtain the following expected utility for an entrepreneur with wealth K_z:

$$U_e = p[f(K_z + D_z, L_z) - (1 + r_z)D_z - \theta - wL_z] + \max\{(1 - p)(1 - \tau)\left(\eta(K_z + D_z) - \Theta wL_z\right), 0\} \qquad (31)$$

In this expression, the first term corresponds to the profits in case the project is successful. Note that now the interest rate is differentiated, and depends on the agent. The reason is that in case of failure, the return

[27] Thus the parameter τ can also be interpreted as *ex post* credit protection.

depends on the amount that can be rescued from failure. Apart from parameters that correspond to the quality of the legal system (τ and η) the return in case of failure depends on the original capital invested in the project and on the secure fraction of wages, which have priority over general debt.

The profits of a representative bank are:

$$U_b = p(1 + r_z)D_z + \max\{(1 - p)\tau(\eta(K_z + D_z) - \Theta wL_z), 0\} - (1 + \rho)D_z$$

The zero profit condition on banks determines the interest rate charged to each entrepreneur z:

$$(1 + r_z) = \frac{1 + \rho}{p} - \frac{1}{pD_z}\max\{(1 - p)\tau(\eta(K_z + D_z) - \Theta wL_z), 0\}$$

which we use in (31) to obtain the utility of entrepreneur z:

$$U_e = p[f(K_z + D_z, L_z) - \theta - wL_z] + (1 - p)[\eta(K_z + D_z) - \Theta wL_z] - (1 + \rho)D_z$$

In order to determine K_d, D_d, we define the following auxiliary function:

$$\begin{aligned}\Psi(K_z, D_z) &\equiv p[f(K_z + D_z, L_z) - (1 + r_z)D_z - \theta - wL_z] + (1 - p)[(1 - \tau)(\eta(K_z + D_z) - \Theta wL_z)] - \phi D_z \\ &\Leftrightarrow p[f(K_z + D_z, L_z) - \theta - wL_z] + (1 - p)[\eta(K_z + D_z) - \Theta wL_z] - (1 + \rho + \phi)D_z\big) \qquad (32)\end{aligned}$$

The conditions which determine K_d, D_d are:

$$\begin{aligned}\Psi(K_d, D_d, L_d) &= 0 \qquad (33)\\ \Psi_D(K_d, D_d, L_d) &= 0\\ \partial U_e(K_d, D_d, L_d)/\partial L_d &= 0\end{aligned}$$

where the derivative Ψ_D exists. An entrepreneur with access to the credit market solves:

$$\begin{aligned}&\max_{D_z, L_z} U_e(K_z, D_z, L_z) \qquad (34)\\ &\text{s.t.} \quad \Psi(K_z, D_z, L_z) \geq 0\end{aligned}$$

To simplify the analysis, we assume that $\eta(K_z + D_z) - \Theta wL_z > 0$ in what follows, i.e., that after bankruptcy there always remains enough left over to pay workers what they are owed, if not the banks. In that case, it is easy but cumbersome to show that there is a range $[K_z, K_r]$ where entrepreneurs are credit constrained. Though these credit constrained firms hire labor efficiently, given their investments, the fact that investment is nonoptimal implies that there is less hiring than otherwise. It can be shown that for credit constrained firms, labor demanded by a firm increases with the capital stock of the entrepreneur. The intuition is that in the credit constrained range total investment $K_z + D_z$ by a firm increases with K_z, thus increasing the marginal productivity of labor.

5.1 Labor supply

In order to find the equilibrium in the labor market, we assume for an agent z that chooses to be a worker, the cost of providing an amount l_z of labor is $\varsigma(l_z)$, where $\varsigma' > 0, \varsigma'' < 0$ with $\varsigma(0) = 0, \varsigma(+\infty) = \infty$. To simplify, we assume that agents are expected utility maximizers and can work or become entrepreneurs. The utility of a worker that provides l units of labor to a firm (and deposits his capital K_z) is:

$$U_w = (1 + \rho)K_z + pwl_z + (1 - p)\Theta wl_z - \varsigma(l)$$

This implies that for an agent to become an entrepreneur, $U_e(K_z, D_z, L_z) \geq U_w(K_z, l_z)$. Without additional conditions on ς, we cannot show that this constraint is nonbinding for agents that form a firm. There are then two possible cases. In the first case (nonbinding), society can be divided among those that have sufficient wealth to obtain a loan and start their firm, and those that have to work, because they cannot develop their project. Moreover, having enough wealth for a loan implies a discreet increase in wellbeing relative to an agent who has to sell his labor. In the second, binding case, there is no jump in utility of the first worker that becomes an entrepreneur.

However, we can show that labor supply is increasing in wages and that labor demand is decreasing in wages, so there is a labor market equilibrium.

5.2 Results with labor

We can reproduce most of the previous results in this economy with labor (see the appendix to this paper). In addition. we show that as the cost of giving workers priority in bankruptcy increases (given by the parameter Θ), the minimum capital level increases, thus lowering the access to credit of smaller firms. In addition, increasing the preference of workers in bankruptcy lowers wages. This effect results from the combination of two channels . First, because raising the payment Θ to workers in case of bankruptcy shifts K_d to the right. Some potential employers cannot obtain credit to start a firm and must become workers, thus increasing the supply of labor. A second effect occurs because restricted entrepreneurs obtain smaller loans and therefore hire fewer workers, again lowering wages. Hence total hours supplied also fall.

Similar effects on wages and labor demand occur if there is less *ex ante* ($\phi \uparrow$) credit protection or worse ($\eta \downarrow$) bankruptcy procedures. On the other hand, improvements in *ex post* creditor protection (here represented by the parameter τ) have no effect on these variables. The only effect of a reduction in τ (the fraction of the value of the failed firm that can be pledged to the lender) is to increase the interest rate r_z that is charged to firms, according to the level of wealth of entrepreneur z. The reason is that changes in this variable have effects that are anticipated by competitive lenders.

Improvements in financial markets, create a wedge between types of entrepreneurs. Credit constrained entrepreneurs are better off, some because they have access to credit, and others because they have access to more credit, leading to more efficient firms. On the other hand, non-constrained entrepreneurs are worse off, because they were unconstrained and operating efficiently before, but now have to pay higher wages to workers due to increased demand for labor by constrained firms. The opposition to improvements in finance due to, among others, the effects on factor prices (specially labor) is mentioned by Rajan and

Ramcharan (2011, p. 1897) in their study of farming in early twentieth farming in the US.[28]

An increase in protection for workers in case of the failure of a firm (Θ) reduces the welfare of those entrepreneurs now unable to form firms, and who go back to being workers. Secondly, firms that are financially constrained will obtain smaller loans, reducing their productive efficiency and their labor demand. Firms that are well capitalized and that continue to use the efficient level of capital after the increase in Θ are less affected, because the reduction in wages compensates for the increase in Θ. Thus they will tend to be less opposed to proposals to raise Θ, creating another wedge with the interests of SMEs.

Note that an increase in Θ, which supposedly protects workers in case of the failure of a firm, has ambiguous effects on their welfare, which depend on the type of firm in which they work. While "on average" workers are better off (since total compensation rises), not all workers are better off. Workers in smaller firms are worse off, in some cases because the firms close down because they do not obtain financing under the new conditions. Second, SME's that survive have to shrink, because their obtain smaller loans and hire less labor, so workers in those firms can also be worse off. There is a cutoff level $\hat{K}_z$ such that workers in SMEs with less capital than the cutoff level are worse off, but the effect of the increase in Θ for the welfare of workers in larger SMEs is ambiguous.[29] It is easy to show that workers in large firms are always better off. Thus the model predicts that there will be a conflict of interests regarding Θ between workers in small and larger firms.

While similar qualitative results were obtained in Balmaceda and Fischer (2010), here we do not only have an adjustment through the exit of firms, but also by the effect of the increase in Θ on the size of credit constrained firms, which can invest less, become less efficient and employ less labor.

6 Conclusions

This paper has examined an open economy model with credit constrained entrepreneurs, differentiated by their initial level of wealth. Agents with little initial wealth cannot obtain loans to develop their projects, while those with more wealth can get loans to create SMEs or large enterprises. SMEs are credit constrained and inefficient.

We examine the effect of increased efficiency of financial markets –understood as improvements in the loan recovery rate– on the various types of firms, and we examine the effects of increases in wealth inequality on the performance of the economy. These effects depend on the aggregate wealth of the economy or, alternatively, on the initial quality of the financial system. We find that for countries that have little capital or that have deficient financial systems, a regressive redistribution of resources could improve investment and gross and net output. On the other hand, in wealthy economies or with well functioning financial systems, redistribution will increase investment and gross output.

The model is adapted to incorporate continuous labor as an independent factor of production. We also

[28] As we have mentioned before, a primary motive for the opposition to financial market improvement is the effect on economy-wide interest rates, as in Shleifer and Wolfenzon (2002), which we cannot examine here by our assumption of an economy that ~~i~~ is open to international financial flows. See Huerta (2014) for a model of a closed economy along these lines, where the effect on interest rates appears.

[29] Adding the assumption $f_{LLK}, f_{KLK} < 0$ we can show that above the cutoff workers in SMEs are better off. This assumption is verified by a Cobb-Douglas technology.

add the possibility of the failure of the firm, leading to bankruptcy. The main results continue to hold, and additional effects appear. When labor is present, the interests of SMEs and large firms diverge with respect to improvements in the financial markets, an effect similar to the one studied in Rajan and Ramcharan (2011). Moreover, the interests of workers in small and large firms also diverge with respect to measures to increase labor protection when the firm fails.

An interesting extension of this paper would be to examine how these results are altered in the closed economy. Huerta (2014) has done preliminary work in this direction. Another relevant extension is to examine the effects of imperfect competition in the financial market under the current settings, in particular the effects of market opening on welfare under different degrees of competition.

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A Appendix: Proof for the results of the model with bankruptcy and labour

Lemma 2 *The maximum debt D_z satisfies the following conditions:*

1. $\frac{\partial D_z}{\partial K_z} > 1$ *if* $K_z > K_d$.
2. $D_z > 0$ *and* $D_z > K_z$ *if* $K_z \geq K_d$.

Proof:

Recall the condition which defines $D(K_z)$:

$$\Psi(K_z, D_z, L_z) = 0 \tag{35}$$

Differentiation of this condition leads to:

$$\frac{\partial \Psi(K_z, D_z, L_z)}{\partial K_z} + \frac{\partial \Psi(K_z, D_z, L_z)}{\partial D_z}\frac{\partial D_z}{\partial K_z} + \frac{\partial \Psi(K_z, D_z, L_z)}{\partial L_z}\frac{\partial L_z}{\partial K_z} = 0 \tag{36}$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z} = -\frac{\Psi_L \frac{\partial L_z}{\partial K_z} + \Psi_K}{\Psi_D} \tag{37}$$

where:

$$\Psi_K = f_K + \frac{1-p}{p}\eta > 0 \tag{38}$$

$$\Psi_L = f_L - w + \frac{1-p}{p}\Theta w = 0 \tag{39}$$

$$\Psi_D = f_K - \left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right) \leq 0 \tag{40}$$

Therefore, if $K_z > K_d$, we conclude that:

$$\frac{\partial D_z}{\partial K_z} = -\frac{f_K + \frac{1-p}{p}\eta}{f_K - \left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)} > 0 \tag{41}$$

Moreover, because $f_K(K_z + D_z, L_z) \in \left[\frac{1+\rho+\phi-(1-p)\eta}{p}, \frac{1+\rho-(1-p)\eta}{p}\right)$, we have that $\frac{\partial D_z}{\partial K_z} > 1$ if $K_z > K_d$. For the second item note that the conditions that define K_d are not satisfied at $D_d = 0$. Using the compatibility constraint and the participation constraint jointly:

$$
\begin{aligned}
\Psi(K_z, D_z, L_z) = U_e(K_z, D_z, L_z) - \phi D_z = 0 \\
U_e(K_z, D_z, L_z) \geq U_w(K_z, l_z) \\
\Rightarrow \phi D_z \geq U_w(K_z, l_z) = (1+\rho)K_z + wl_z[p + (1-p)\Theta] - \varsigma(l) \\
\Rightarrow D_z \geq \frac{(1+\rho)}{\phi} K_z > K_z
\end{aligned}
$$

Lemma 3 *The level of labour that a firm contracts L_z increases with K_z.*

Proof: Differentiating the FOC for L_z with respect K_z we obtain that:

$$f_{KL}\frac{\partial D_z}{\partial K_z} + f_{LL}\frac{\partial L_z}{\partial K_z} = 0 \tag{42}$$

$$\Rightarrow \frac{\partial L_z}{\partial K_z} = -\frac{f_{KL}\frac{\partial D_z}{\partial K_z}}{f_{LL}} > 0 \tag{43}$$

Lemma 4 *The supply of labour is downward sloping in w, while the demand of labour is increasing in w.*

Proof: Differentiating the optimal level of labour of workers (l_z):

$$\frac{\partial l_z}{\partial w} = \frac{p + (1-p)\Theta}{\varsigma''(l)} > 0 \tag{44}$$

From the FOC of labour:

$$\frac{\partial L_z}{\partial w} = \frac{1 + \frac{(1-p)}{p}\Theta - f_{LK}\frac{\partial D_z}{\partial w}}{f_{LL}} < 0 \tag{45}$$

where we have used the fact that $\frac{\partial D_z}{\partial w} = \frac{L_z(1+\Theta)}{f(K_z+D_z,L_z)-\left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)} < 0$. Total differentiation of condition (35) leads to:

$$\frac{\partial \Psi}{\partial D_d}\frac{\partial D_d}{\partial w} + \frac{\partial \Psi}{\partial L_d}\frac{\partial L_d}{\partial w} + \frac{\partial \Psi}{\partial K_d}\frac{\partial K_d}{\partial w} + \frac{\partial \Psi}{\partial w} = 0 \tag{46}$$

Replacing terms of previous conditions we obtain that:

$$\underbrace{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)}_{=0}\frac{\partial D_d}{\partial w} + \underbrace{\left(f_L - w - \frac{1-p}{p}\Theta w\right)}_{=0}\frac{\partial L_d}{\partial w} + \frac{\partial K_d}{\partial w}\left(f_K + \frac{(1-p)}{p}\eta\right) = \frac{L_d}{p} + \frac{1-p}{p}L_d\Theta \tag{47}$$

$$\Rightarrow \frac{\partial K_d}{\partial w} = \frac{L_d(1+(1-p)\Theta)}{pf_K + (1-p)\eta} > 0 \tag{48}$$

Now, differentiating the left-hand side of the labour market equilibrium condition we obtain:

$$\frac{\partial \mathcal{S}_L}{\partial w} = \int_0^{K_d(w)} \frac{\partial l_z}{\partial w}\partial\Gamma(K_z) + \frac{K_d}{\partial w}l_d\gamma(K_d) > 0 \tag{49}$$

For the demand of labour we have:

$$\frac{\partial \mathcal{D}_L}{\partial w} = \int_{K_d(w)}^{+\infty} \frac{\partial L_z}{\partial w} \partial\Gamma(K_z) - \frac{K_d}{\partial w} L_d \gamma(K_d) < 0 \tag{50}$$

Proposition 9 *The equilibrium wage w rises after:*

1. *An improvement in* ex-ante *creditor protection* $1-\phi$.
2. *An increase in* ex-post *protection* η.
3. *An decrease in firing costs* Θ.
4. *A decrease in fixed costs* θ.

   while it remains constant after an improvement in the appropiability parameter τ.

Proof: In order to simplify calculations we define $x = \phi, \eta, \Theta, \theta, \tau$. From equilibrium labour market condition we have[30][31]

$$\int_0^{K_d(w)} \frac{\partial l_z}{\partial w}\frac{\partial w}{\partial x} \partial\Gamma(K_z) + l_d \gamma(K_d) \left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x} \right) - \left[\int_{K_d(w)}^{+\infty} \frac{\partial L_z}{\partial w}\frac{\partial w}{\partial x} \partial\Gamma(K_z) - L_d \gamma(K_d) \left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x} \right) \right] = 0 \tag{51}$$

For $x = \phi$, $x = \Theta$ or $x = \theta$ the direct effect on K_d is: $\frac{\partial K_d}{\partial x} > 0$. If $\frac{\partial w}{\partial x} > 0$ then all terms would be positive and the labour market equilibrium condition would be violated. On the other hand, if $\frac{\partial w}{\partial x} < 0$, we will have terms with opposite signs and the market condition will be satisfied. Therefore, the equilibrium wage decreases after an increase in ϕ, Θ or θ. If $x = \eta$ then we have that: $\frac{\partial K_d}{\partial x} < 0$. Using the same argument we conclude that $\frac{\partial w}{\partial x} > 0$.

Now, if $x = \tau$ then $\frac{\partial K_d}{\partial x} = 0$. Thus, the equilibrium market condition is not affected by τ, i.e. $\frac{\partial w}{\partial \tau} = 0$.

Lemma 5 *The minimum capital level K_d decreases if:*

1. Ex-ante *creditor protection* $1-\phi$ *improves.*
2. *Firing cost* Θ *decreases.*
3. *The appropiability of investment after bankruptcy* η *decreases.*
4. *Fixed costs* θ *decreases.*

while it remains constant after an improvement of bankruptcy procedures τ.

[30]Note that l_d is how much labour will supply an entrepreneur who owns K_d and decides to work instead of forming a firm, while L_d is the amount of labour demanded by a firm which owns K_d.

[31]Notice that total differentiation of $K_d(w)$ with respect any measure x incorporates a direct effect and a indirect effect (given by the change in w): $\frac{\partial K_d(w)}{\partial x} = \left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x} \right)$

Proof: In order to simplify calculations we define $x = \phi, \eta, \Theta, \theta, \tau$. Differentiating condition (35) at (K_d, D_d, L_d) we obtain:

$$\frac{\partial \Psi(K_d, D_d, L_d)}{\partial K_d}\frac{\partial K_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial D_d}\frac{\partial D_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial L_d}\frac{\partial L_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial x} = 0$$

$$\Rightarrow \left(f_K + \frac{(1-p)}{p}\eta\right)\frac{\partial K_d}{\partial x} + \underbrace{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)}_{=0}\frac{\partial D_d}{\partial x} + \underbrace{\left(f_L - w - \frac{1-p}{p}\Theta\right)}_{=0}\frac{\partial L_d}{\partial x} = -\frac{\partial \Psi(K_d, D_d, L_d)}{\partial x}$$

$$\Rightarrow \frac{\partial K_d}{\partial x} = -\frac{\frac{\partial \Psi(K_d, D_d, L_d)}{\partial x}}{pf_K + (1-p)\eta}$$

Differentiating and replacing terms we obtain that:

$$\frac{\partial K_d}{\partial \phi} = \frac{D_d + \frac{\partial w}{\partial \phi}L_d(p + (1-p)\Theta)}{pf_k + (1-p)\eta} \tag{52}$$

$$\frac{\partial K_d}{\partial \eta} = \frac{-(1-p)(K_d + D_d) + \frac{\partial w}{\partial \eta}L_d(p + (1-p)\Theta)}{pf_k + (1-p)\eta} \tag{53}$$

$$\frac{\partial K_d}{\partial \Theta} = \frac{L_d\left(\frac{\partial w}{\partial \Theta}((1-p)\Theta + p) + w(1-p)\right)}{pf_k + (1-p)\eta} \tag{54}$$

$$\frac{\partial K_d}{\partial \theta} = \frac{p + \frac{\partial w}{\partial \theta}L_d(p + \Theta(1-p))}{pf_k + (1-p)\eta} \tag{55}$$

Note that condition (51) implies that $|\frac{\partial w}{\partial \phi}| < \frac{D_d}{L_d(p+(1-p)\Theta)}$, $|\frac{\partial w}{\partial \Theta}| < \frac{(1-p)w}{p+(1-p)\Theta}$,$|\frac{\partial w}{\partial \eta}| < \frac{(1-p)(K_d+D_d)}{L_d(p+(1-p)\Theta)}$ and $|\frac{\partial w}{\partial \theta}| < \frac{1}{L_d(1+\Theta(1-p))}$, otherwise the equilibrium condition will be violated. Therefore we conclude that: $\frac{\partial K_d}{\partial \phi} > 0$, $\frac{\partial K_d}{\partial \Theta} > 0$, $\frac{\partial K_d}{\partial \eta} > 0$ and $\frac{\partial K_d}{\partial \theta} > 0$. The case for $x = \tau$ is straightforward.

Lemma 6 *If Θ increases then D_z and L_z decrease.*

Proof: Differentiating condition (35) at (K_z, D_z, L_z) we obtain:

$$\frac{\partial D_z}{\partial \Theta} = \frac{\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right)}{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)} < 0 \tag{56}$$

From the FOC of labour we obtain:

$$f_{KL}\frac{\partial D_z}{\partial \Theta} + f_{LL}\frac{\partial L_z}{\partial \Theta} = \frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w) \tag{57}$$

$$\Rightarrow \frac{\partial L_z}{\partial \Theta} = \frac{\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w - f_{KL}\frac{\partial D_z}{\partial \Theta}}{f_{LL}} < 0 \tag{58}$$

where we have used the fact that $\frac{\partial D_z}{\partial \Theta} < 0$ and $\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w > 0$.

A.1 Proof of the results related with changes in welfare of workers and firms

In order to compare the effects of Θ and ϕ among the different entrepreneurial groups we define the profits of a firm of size $K_z + D_z$ as follows:

$$\Pi(K_z + D_z, L_z) = p[f(K_z + D_z, L_z) - \theta - wL_z] + (1-p)[\eta(K_z + D_z) - \Theta w L_z] - (1+\rho)(D_z + K_z) \quad (59)$$

Proposition 10 *If firing costs Θ increase then:*

1. *All firms experience a decrease in their profits, and there exists a threshold $K_\Theta \in (K_d, K_r)$ such that firms with $K_z \in (K_d, K_\Theta]$ are worse off than firms with $K_z \geq K_r$.*

2. *Individual workers experience an increase in their utilities.*

Proof: For firms with $K_z \in [K_d, K_r)$ differentiation of condition (59) with respect Θ leads to:

$$\frac{\partial \Pi(K_z + D_z, L_z)}{\partial \Theta} = \frac{\partial \Pi}{\partial D_z}\frac{\partial D_z}{\partial \Theta} + \underbrace{\frac{\partial \Pi}{\partial L_z}}_{=0}\frac{\partial L_z}{\partial \Theta} + \frac{\partial \Pi}{\partial \Theta} \quad (60)$$

$$\Rightarrow \frac{\partial \Pi(K_z + D_z, L_z)}{\partial \Theta} = \underbrace{\left(f_K - \left(\frac{1+\rho-(1-p)\eta}{p}\right)\right)}_{>0}\underbrace{\frac{\partial D_z}{\partial \Theta}}_{<0} - L_z\underbrace{\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right)}_{>0} < 0 \quad (61)$$

For firms which produce optimally ($K_z \geq K_r$) we have that:

$$\frac{\partial \Pi(K^*, L^*)}{\partial \Theta} = \frac{\partial \Pi}{\partial K^*}\frac{\partial K^*}{\partial \Theta} + \underbrace{\frac{\partial \Pi}{\partial L^*}}_{=0} + \frac{\partial \Pi}{\partial \Theta}$$

$$\Rightarrow \frac{\partial \Pi(K^*, L^*)}{\partial \Theta} = p\underbrace{\left(f(K^*, L^*) - \frac{(1+\rho)-(1-p)\eta}{p}\right)}_{=0}\frac{\partial K^*}{\partial \Theta} - L^*\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right) < 0$$

Note that if $K_z = K_d^+$ then $\frac{\partial \Pi}{\partial \Theta} \to -\infty$. Else if $K_z \geq K_r$ then $\frac{\partial \Pi}{\partial \Theta} = -[\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w]L^* > -\infty$. Since $\frac{\partial \Pi}{\partial \Theta}$ is continuous in $(K_d, +\infty]$, there exists an interval $K_z \in (K_d, K_\Theta]$ such that $\frac{\partial \Pi}{\partial \Theta}$ is lower than when $K_z \geq K_r$.

For an individual worker we have that:

$$\frac{\partial U_w}{\partial \Theta} = \frac{\partial w}{\partial \Theta}l(p + (1-p)\Theta)) + w\frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + wl(1-p)\Theta - \varsigma'(l)\frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta} \quad (62)$$

$$\Rightarrow \frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta}\underbrace{(p + (1-p)\Theta)w - \varsigma'(l)}_{=0} + l\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + w(1-p)\right) > 0 \quad (63)$$

Proposition 11 *If ex-ante creditor protection $1-\phi$ improves then:*

1. *There exists a threshold $K_\phi \in (K_d, K_r)$ such that all firms with $K_z \in (K_d, K_\phi)$ are better off, while firms with $K_z \geq K_r$ are worse off.*

2. *Individual workers are better off.*

Proof: Differentiating (59) with respect ϕ we have:

$$\frac{\partial \Pi(K_z + D_z)}{\partial \phi} = \left(f_K - \left(\frac{1 + \rho - (1-p)\eta}{p} \right) \right) \frac{\partial D_z}{\partial \phi} - L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta)) \tag{64}$$

where:

$$\frac{\partial D_z}{\partial \phi} = \frac{D_z + L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta))}{f_K - \left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)} \tag{65}$$

Replacing this last expression in (64) we obtain that:

$$\frac{\partial \Pi(K_z + D_z)}{\partial \phi} = \underbrace{\frac{D_z \left(f_k - \frac{(1+\rho)-(1-p)\eta}{p}\right)}{f_k - \frac{(1+\rho+\phi)-(1-p)\eta}{p}}}_{<0} + \underbrace{\frac{\frac{\phi}{p} L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta))}{f_k - \frac{(1+\rho+\phi)-(1-p)\eta}{p}}}_{>0} \tag{66}$$

For firms which produce optimally we obtain that:

$$\frac{\partial \Pi(K^*)}{\partial \phi} = p \underbrace{\left(f(K^*) - \frac{(1+\rho) - (1-p)\eta}{p} \right)}_{=0} \frac{\partial K^*}{\partial \phi} - \frac{\partial w}{\partial \phi} L^*(p + (1-p)\Theta) > 0 \tag{67}$$

Notice that the sing of expression (66) is ambiguous. However, note that if $K_z \to K_d^+$ then $\frac{\partial \Pi}{\partial \phi} \to -\infty$. Since $\frac{\partial \Pi}{\partial \phi}$ is continuous in $(K_d, +\infty)$, there exists some cutoff K_ϕ such that $\frac{\partial \Pi}{\partial \phi} < 0$ if $K_z \in (K_d, K_\phi)$.

For individual workers we obtain:

$$\frac{\partial U_w}{\partial \phi} = l_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta) < 0 \tag{68}$$

Now we are interested in studying the effects of Θ on workers of different entrepreneurial groups. We define the total welfare of workers of firm z as:

$$\tilde{U}_w(L_z) = pwL_z + (1-p)\Theta w L_z - \varsigma(L_z) \tag{69}$$

Proposition 12 *If firing costs measured by Θ increase then there exists a cutoff $\tilde{K}_\Theta \in (K_d, K_r)$ such that:*

1. *The representative worker of a firm with $K_z \in (K_d, \tilde{K}_\Theta)$ is worse off.*

2. *The representative worker of a firm with $K_z \geq K_r$ is better off.*

Proof:

Differentiating condition (69) with respect Θ:

$$\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} = L_z \underbrace{\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right)}_{>0} + \underbrace{\frac{\partial L}{\partial \Theta}}_{<0} \left[pw + (1-p)\Theta w - \varsigma'(L_z)\right] \tag{70}$$

Notice that the sign of this last expression will depend on the sign of the marginal benefit of working one additional hour: $B(L_z) \equiv pw + (1-p)\Theta w - \varsigma'(L_z)$.

The labour market equilibrium condition can be rewritten as:[32]

$$l(1 - \Gamma(K_d)) = \int_{K_d}^{+\infty} L_z \partial \Gamma(K_z)$$

This condition implies that:

$$l(1 - \Gamma(K_d)) = \int_{K_d}^{+\infty} L_z \partial \Gamma(K_z) > \int_{K_d}^{+\infty} L_d \partial \Gamma(K_z)$$
$$\Rightarrow l > L_d$$

Similarly we conclude that $L^* > l$. Now note that by definition we have that $B(l) = 0$. Since $B(\cdot)$ is decreasing in L_z, we have that $B(L^*) < 0$ and $B(L_d) > 0$. Note that when $K_z \to K_d^+$ then $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} \to -\infty$, while if $K_z \geq K_r$ then $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} > 0$. Since $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}$ is a continuous function in $(K_d, +\infty)$ we conclude that there exists a cutoff $\tilde{K}_\Theta$ such that the representative worker of firms with $K_z \in (K_d, \tilde{K}_\Theta)$ are negatively affected by an increase in Θ. On the other hand, workers of firms with $K_z \geq K_r$ are better off after an increase in Θ.

Corollary 2 *If $f_{LL,K} < 0$ and $f_{KL,K} < 0$, then* $\frac{\partial\left(\frac{\partial L_z}{\partial \Theta}\right)}{\partial K_z} > 0$.

Proof:

In order just to simplify notation we define: $h(K_z, L_z) \equiv f_K - (1 + \rho^* + \phi^*)$ with $1 + \rho^* + \phi^* \equiv \frac{1+\rho+\phi-(1-p)\eta}{p}$ and $g(\Theta) \equiv \frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)$.

From proposition 6 and equation (56) we have that:

$$\frac{\partial L_z}{\partial \Theta} = g(\Theta)\left(\frac{1}{f_{LL}} - \frac{f_{KL}}{f_{LL} h(K_z, L_z)}\right) \tag{71}$$

Differentiating this condition with respect K_z and assuming that $f_{LL,K} < 0$, $f_{KL,K} < 0$ we have:

$$\frac{\partial\left(\frac{\partial L_z}{\partial \Theta}\right)}{\partial K_z} = g(\Theta)\left(\underbrace{-\frac{f_{LL,K}}{(f_{LL})^2}}_{>0} - \underbrace{\left[\frac{\overbrace{f_{KL,K} f_{LL} h(K_z, L_z)}^{<0} - f_{KL}\overbrace{(f_{LL,K} h(K_z, L_z) + f_{LL} f_{KK})}^{>0}}{(f_{LL} h(K_z, L_z))^2}\right]}_{<0}\right) > 0$$

Proposition 13 *If $f_{LL,K} < 0$ and $f_{KL,K} < 0$, there exists a threshold $\hat{K}_\Theta$ such that:*

[32] We have used the fact that in equilibrium all workers supply the same amount of labour($l_z = l$).

1. *The representative workers of firms with* $K_z \in (K_d, \hat{K}_\Theta)$ *are worse off.*

2. *The representative workers of firms such that* $K_z \geq \hat{K}_\Theta$ *are better off.*

Proof: Under corollary 2 and using the Intermediate Value Theorem the demonstration is straightforward.