# Can a non-binding minimum wage reduce wages and employment? *

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## 1 Introduction

In a perfect competition model, the introduction of a binding minimum wage implies a decrease in employment (Stigler, 1946). It has been argued, however, that the opposite may occur in models characterized by search frictions (see Manning (2003) and references therein). In this note we show that, in the large-firm search model (e.g. Cahuc et al. (2008)), employment may decrease even when the level of the introduced minimum wage lies below the equilibrium wage of the *laisser-faire* economy. Not only employment decreases, wages decrease too with the presence of the minimum wage.

The argument is based on multiple equilibria and the idea from the literature that, in a large-firm context, the representative firm may choose to overemploy workers in order to renegotiate lower wages. While the equilibrium is unique in the *laisser-faire* economy, another equilibrium may appear with a minimum wage. In this other equilibrium, the minimum wage acts as a focal point. Low-skilled workers anticipate to earn the minimum wage. Because wages are downward rigid by regulation, overemployment by the representative firm

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is weaker. This depresses labor demand for the low skilled and, because aggregate labor demand is too low, low-skilled workers cannot negotiate a wage equal to the equilibrium wage of the *laisser-faire* economy, which results into employment and wages of low-skilled workers being lower than in the latter equilibrium.

# 2 Model

We consider a framework similar to Bauducco and Janiak (2014). The economy is in steady state and time is continuous. Two types of workers operate on separate labor markets: high-skilled workers hired in quantity $H$ and low-skilled workers hired in quantity $L$. Each labor market is characterized by search and matching frictions *à la* Pissarides. Workers of type $i = \{H, L\}$ either choose to negotiate their wage at a rate $\breve{w}^i$ or earn the minimum wage $\bar{w}$. We allow workers to opt for mixed strategies and call $\chi_i$, $i = \{H, L\}$ the probability to negotiate the wage. Hence, the expected wage of $i$-type workers $w^i$ reads as

$$w^i = \chi_i \breve{w}^i + (1 - \chi_i)\bar{w}.$$

Negotiated wages are continuously renegotiated, with $\beta \in (0, 1)$ being the bargaining power of workers.

The value of the representative firm is

$$\Pi(H, L) = \max_{\{v_H, v_L\}} \frac{1}{1 + rdt} \left( \left[ F(\pi H, L) - w^H H - v_H c - w^L L - v_L c \right] dt + \Pi(H', L') \right),$$

subject to the constraints

$$\begin{aligned} H' &= (1 - s_H dt)H + q(\theta_H)v_H dt, \\ L' &= (1 - s_L dt)L + q(\theta_L)v_L dt, \end{aligned}$$

where $dt$ is an arbitrarily small interval of time, $r$ is the discount rate, $F$ is the production function, $\pi > 1$ are labor services provided by skilled workers, $v_i$ are the vacancies posted on labor market $i = \{H, L\}$, $c$ is the flow vacancy cost, $s_i$ is the exogenous separation rate for workers of type $i = \{H, L\}$, $\theta_i$ is the labor market tightness of labor market $i = \{H, L\}$ and the $q$ function gives the rate at which vacancies are filled. The $q$ function is derived from a standard matching function.

Notice that the representative firm is *large*, in the sense that it hires more than one worker of each type $i$. The firm internalizes the fact that the negotiated wage of an $i$-type worker will be influenced by the quantities $H$ and $L$ of workers hired through the effects of these quantities on the marginal product of labor. Consequently, the firm will take into account the effects of $H$ and $L$ over wages when deciding how many vacancies $v_i$, $i = H, L$, to post.

Finally, the value of an employed worker is

$$rW_i = w^i + s(U_i - W_i),$$

while the value of an unemployed worker is

$$rU_i = b + s(W_i - U_i),$$

for all $i = \{H, L\}$.

# 3 Equilibrium conditions

The equilibrium allocations in the economy can be identified by eight conditions. The two vacancy conditions for the two labor markets read:

$$\frac{c}{q(\theta_H)} = \frac{\Omega_H \frac{\partial F(\pi H, L)}{\partial H} - w^H}{r+s} \quad \text{and} \quad \frac{c}{q(\theta_L)} = \frac{\Omega_L \frac{\partial F(\pi H, L)}{\partial L} - w^L}{r+s}.$$

The two wage equations for the negotiated wages are:

$$\breve{w}^H = \beta \breve{\Omega}_H \frac{\partial F(\pi H, L)}{\partial H} + (1-\beta)b + \beta \theta_H c \quad \text{and} \quad \breve{w}^L = \beta \breve{\Omega}_L \frac{\partial F(\pi H, L)}{\partial L} + (1-\beta)b + \beta \theta_L c.$$

The two Beveridge curves are standard and read:

$$H = \frac{\theta_H q(\theta_H)}{s + \theta_H q(\theta_H)} \quad \text{and} \quad L = \frac{\theta_L q(\theta_L)}{s + \theta_L q(\theta_L)}.$$

The variables $\Omega_i$ and $\breve{\Omega}_i$, defined in Bauducco and Janiak (2014), are front-load factors resulting from the strategic behavior of agents. When $\Omega_i$ takes a value above 1, for $i = \{H, L\}$, the firm overemploys factor $i$, while it underemploys it when $\Omega_i < 1$. Over-employment and under-employment arise when the firm influences the value of the negotiated wage of $i$-type workers through the value of its marginal product by altering the quantities of each labor type hired. For example, consider the case in which $H$ and $L$ are complements in production, in the sense that the cross-derivatives of the production function are positive. In this case,

Table 1: Parametrization

| $b$ | $s$ | $r$ | $\beta$ | $c$ | $\eta$ | $m_0$ | $\alpha$ | $\rho$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.71 | 0.036 | 0.004 | 0.5 | 0.356 | 0.5 | 0.7 | 0.5 | 0.401 | 4.7 |

the firm has incentives to overemploy $H$-type workers to decrease the marginal product of labor of type $H$ workers, but at the same time it has incentives to underemploy $H$-type workers in order to decrease the marginal product of labor of type $L$ workers. Depending on which effect dominates, $\Omega_H \lesseqgtr 1$.

The optimal wage strategies for workers to determine the $\chi_i$'s are given by the following equilibrium condition:

**Equilibrium condition 1.** *Workers' wage strategy is optimal:*

- *The fraction* $\chi_i = 1$, $i = \{H, L\}$, *is an equilibrium if* $\breve{w}_i > \bar{w}$.
- *The fraction* $\chi_i = 0$, $i = \{H, L\}$, *is an equilibrium if* $\breve{w}_i < \bar{w}$.
- *The fraction* $\chi_i \in (0, 1)$, $i = \{H, L\}$, *is an equilibrium if* $\breve{w}_i = \bar{w}$.

Notice that Equilibrium condition 1 suggests the possibility of multiple equilibria.

# 4 Numerical exercise

We illustrate the result with a numerical example. The parametrization is as follows. We assume a CES form for the production function, i.e. $F(\pi H, L) = [\alpha(\pi H)^\rho + (1-\alpha)L^\rho]^{\frac{1}{\rho}}$ and a Cobb-Douglas specification for the matching function, i.e. $m(u, v) = m_0 u^\eta v^{1-\eta}$. Table 1 shows the values of the parameters used in the exercise. Most of these values are simply taken from Pissarides (2009). The value for $\rho$ corresponds to the estimation by Krusell et al. (2000), while the value for $\pi$ produces a skill premium of roughly 80% as shown in Krueger et al. (2010) for the US.

Under this parametrization, the equilibrium is unique absent a minimum wage, while the introduction of a minimum wage may give rise to multiple equilibria even though the minimum wage lies below the equilibrium wage of the *laisser-faire* economy. This latter claim is illustrated on Figure 1 for a value of the minimum wage set 2.5% below the equilibrium wage of $L$-type workers in the *laisser-faire* economy. The Figure compares the equilibrium negotiated wage of $L$-type workers (the solid line) with the minimum wage (the dashed line)

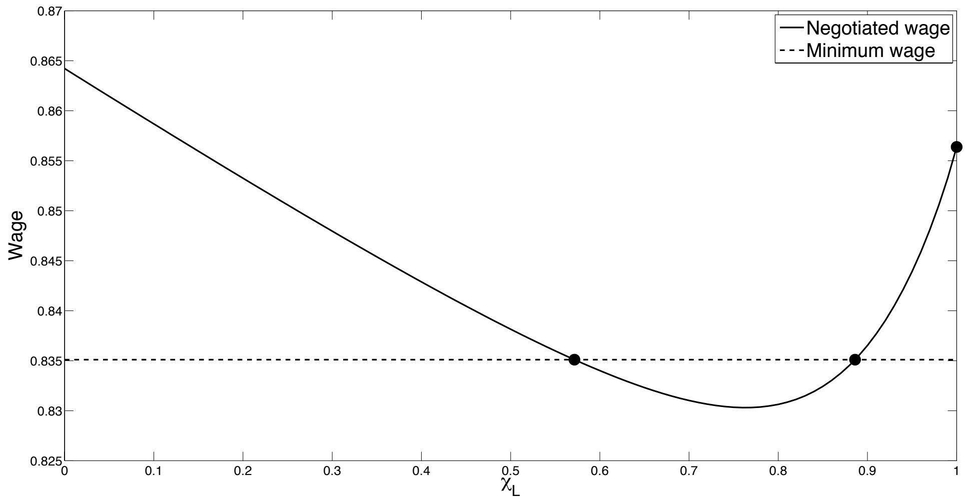


Figure 1: Determination of $\chi_L$ in the presence of a minimum wage

for several values of $\chi_L$ in order to identify the possible equilibrium value for $\chi_L$ as stated in Equilibrium condition 1. There are thus three equilibria: one in pure strategy in which all $L$-type workers negotiate the wage with the firm ($\chi_L = 1$) and two equilibria in mixed strategy, in which a fraction $0 < \chi_L < 1$ of $L$-type workers receives the negotiated wage, and a fraction $1 - \chi_L$ receives the minimum wage. Obviously, in these two equilibria, both wages coincide. In all three equilibria, $\chi_H = 1$.

Table 2 compares two equilibria: the high-wage equilibrium corresponds to the pure-strategy equilibrium of Figure 1, while the low-wage equilibrium is the left mixed-strategy equilibrium on the same Figure. The high-wage equilibrium is also the equilibrium of the *laisser-faire* economy.

The unskilled wage is 2.5% lower in the low-wage equilibrium since it is equal to the minimum wage. As explained above, the minimum wage acts as a focal point: in the low-wage equilibrium, unskilled workers anticipate to earn the minimum wage, hence the lower probability to negotiate the wage (57%). Because it is more difficult to influence the wage of the unskilled in the low-wage equilibrium, the representative firm underemploys unskilled workers: the front-load factor $\Omega_L$ is below 1, while it is 1 in the *laisser-faire* economy because of constant returns to scale as shown in Cahuc and Wasmer (2001). At the same time, the representative firm overemploys high-skilled workers in order to exploit the concavity of the production function in this factor ($\Omega_H$ lies above 1). As a result, unskilled unemployment increases when one moves from the high-wage equilibrium to the low-wage equilibrium, while

Table 2: Statistics describing two equilibria

| | $w^L$ | $w^H$ | $u_L$ | $u_H$ | $\frac{u_L+u_H}{2}$ | $\chi_L$ | $\chi_H$ | $\Omega_L$ | $\Omega_H$ |
|---|---|---|---|---|---|---|---|---|---|
| High wage equilibrium | 1 | 1.80 | 7.7% | 3.32% | 5.5% | 1 | 1 | 1 | 1 |
| Low wage equilibrium | 0.975 | 1.81 | 29.2% | 3.29% | 16.3% | 0.57 | 1 | 0.87 | 1.06 |

Note: values for wages are normalized by the wage level of low-skilled workers in the high wage equilibrium.

skilled unemployment decreases. Because the impact on the unskilled labor market is much more pronounced than on the skilled labor market (a 21.5% increase vs. a 0.03% decrease), the overall unemployment rate in the economy increases.

# 5 Conclusions

In this note we show that, within a framework in which there are search frictions in labor markets and firms may overemploy or underemploy workers in order to affect negotiated wages, the introduction of a minimum wage may have a negative effect on equilibrium wages and employment. While the effect on employment is present in other models of the labor market (i.e., the perfect competition framework), the effect on wages is novel.

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