Equilibrium and learning in traffic network games

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Traffic in congested networks is often described as an equilibrium or steady state that emerges from the adaptive behavior of drivers. While this intuitive view tends to be confirmed in experiments, traditional models such as *Wardrop Equilibrium* or *Stochastic User Equilibrium* are stated directly in the form of equilibrium equations and are not tied to an underlying adaptive dynamics. The notion of *Markovian equilibrium* incorporates some dynamical features by assuming that passengers move towards their destination through a sequential process of arc selection based on a random discrete choice model at every intermediate node in their trip: route selection is the outcome of this sequential decision process while network flows correspond to the invariant measures of the corresponding Markov chains. Despite a certain dynamical flavor, this equilibrium notion remains essentially static since it is not tied to an adaptive behavior of the players.

In this talk we discuss a discrete time stochastic process that represents a plausible model for the adaptive behavior of finitely many users in a simple traffic network. The dynamics are based on a minimal piece of information: each player observes only the travel time for the specific route chosen on any given day, and future decisions depend on the history of past individual observations. Since travel times depend on the congestion conditions imposed collectively by all the player's decisions, the iteration of this process progressively reveals to each player the congestion conditions throughout the network. In the long run, the temporal evolution of these dynamics lead the system to coordinate on a steady state that may be characterized as a Nash equilibrium for a particular steady state game. In this way the approach provides a unified framework where a stochastic model for user behavior gives rise to a continuous dynamical system and leads ultimately to a consistent notion of equilibrium.

The approach bears some connections with the literature on adaptive learning in repeated games and specific tools such as fictitious play and no-regret dynamics. However it presents substantial differences in the form of the dynamics as well as in the interpretation of the state variables. In particular we observe a clear distinction between the original repeated game and the steady state game.

Keywords: congested networks, learning dynamics, stochastic equilibrium, variational formulation