Subexponential parameterized algorithms

Fedor V. Fomin*

Abstract

We present a series of techniques for the design of subexponential parameterized algorithms for graph problems.

Examples of parameterized problems are:

- The k-Vertex Cover problem, which is to decide, given a graph G and a positive integer k, whether G has a vertex cover of size at most k.
- The k-Dominating Set problem, which is to decide, given a graph G and a positive integer k, whether G has a dominating set of size at most k.
- The k-Longest Path problem, which is to decide, given a graph G and a positive integer k, whether G contains a path of length at lest k.

By subexponential parameterized algorithm we mean an algorithm that for a given input of length $O(n)$ and parameter k, outputs solution the parameterized problem in $2^{O(\sqrt{k})} \cdot n^{O(1)}$ steps.

The design of such algorithms usually consists of two main steps: first find a branch- (or tree-) decomposition of the input graph whose width is bounded by a sublinear function of the parameter and, second, use this decomposition to solve the problem in time that is single exponential to this bound. The main tool for the first step is Bidimensionality Theory. Here we present the potential, but also the boundaries, of this theory. For the second step, we describe recent techniques, associating the analysis of sub-exponential algorithms to combinatorial bounds related to Catalan numbers. As a result, we have $2^{O(\sqrt{k})} \cdot n^{O(1)}$ time algorithms for a wide variety of parameterized problems on graphs, where n is the size of the graph and k is the parameter.

*Department of Informatics, University of Bergen, N-5020 Bergen, Norway. Email: fedor.fomin@ii.uib.no. Supported by the Norwegian Research Council.