Coloring mixed graphs

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With a motivation arising from scheduling problems, the classical graph coloring model has been extended to mixed graphs, i.e., graphs having (unoriented) edges and (oriented) arcs.

In scheduling applications the colors are in fact numbers representing periods of time. For each edge $[i, j]$ one simply requires that the colors assigned to its vertices satisfy $c(i) > c(j)$ or $c(i) < c(j)$. For each arc (i, j), one requires $c(j) > c(i)$. So this extended model can handle precedence constraints as well as disjunctive constraints.

Colorings in mixed graphs have been studied by several authors for this purpose. We shall review some properties of mixed graph colorings and derive some bounds on the generalized concept of chromatic number called mixed chromatic number.

Complexity will also be studied. It is known in particular that the mixed chromatic number of trees can be computed in polynomial time, while its determination for bipartite graph is difficult.

Besides the above requirements for the colors of the vertices of arcs and edges, there is also a concept of weak mixed coloring where for each arc (i, j), one simply requires that $c(j) \geq c(i)$.

This concept will also be studied here and bounds of the weak mixed chromatic number will be derived.

Some coloring techniques will be designed to get bounds on the mixed chromatic number and they will provide some heuristic procedures for such colorings.

Finally some open questions will be presented.

Keywords: graph coloring, mixed graph, scheduling, precedence constraint, disjunctive constraint.

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